THE ALLOCATION OF RUNWAY SLOTS BY AUCTION. VOLUME III: THEORY.

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The Allocation of Runway Slots by Auction

Theoretical and Technical Issues for Implementation

LEVEL III

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The allocation of runway slots at the high-density airports by means of an auction is studied. Previous approaches to slot auctions have not allowed for the interdependency of slot values to the air carriers—a single slot for a landing of an aircraft is likely to be of little value without a corresponding slot for a subsequent take-off of that aircraft. A Slot Exchange Auction is designed, its theoretical properties and practical implementation discussed. It is shown to allow the slot market to reach an efficient equilibrium under competitive conditions. The Airline Management Game is used to create a simulation test of the Slot Exchange Auction and its associated continuous market, the slot exchange.
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The Airline Deregulation Act of 1978 became law on October 24, 1978. Its spirit is to open the industry—albeit gradually—to the usual market forces by encouraging price competition and allowing free entry and exit from the market, that is, permitting carriers and commuters to open or to close service on any route. In November of 1978 the press reported the CAB's and FAA's growing concern with the fact that existing capacity limitations at four major airports together with the potential for new congestion due to new or increased service could effectively impede competition and could well subvert the intent of the Act.

Some ten years ago the FAA determined hourly limits ("quotas") on IFR operations at the four congested airports, and the CAB granted an antitrust exemption to allow carriers to agree among themselves on how to allocate slots. The allocation of slots at quota airports are set twice yearly by scheduling committees made up of carrier representatives, at least one day being devoted to each of the four airports. However, the antitrust exemption is not considered as consistent with the spirit of the law. Moreover, FAA projections show as many as 40 airports experiencing congestion by 1985. So the problem will proliferate.

The Act specifically directs the CAB to take into consideration

"the desirability of a variety of price and service options such as peak and off-peak pricing or other pricing mechanisms to improve economic efficiency and provide low-cost air-service."

There is, it seems, little argument over the desirability of using a "pricing mechanism" to effect the allocation of scheduled slots: but no specific, realistic, operational method for so doing was known.

To devise a pricing or market mechanism, the "good" which is to be sold and/or traded must be defined. Current usage has it that twice a year the Airline
Scheduling Committees meet to distribute slots for the four congested airports and slots are committed to air carriers for a stated period of six months. Each slot becomes, in effect, an option giving the holder the right to schedule an operation at a given hour and airport for the six month period. The complex environment of airline scheduling requires that these rights should be vested for sufficiently long periods of time. Slots are used to schedule flights, flights represent the markets in which air carriers sell their services, these services require investments in support facilities, advertisement and the like which cannot be altered in the very short run. We therefore consider a slot to be an option vesting its owner with the right to schedule an operation at a given time and place for a period of six months. An efficient market mechanism is necessary to allocate slots well in advance of each six month period. In addition, since the demand for air travel, the financial positions of individual carriers, the general state of the industry and the condition of the economy as a whole may change, the holders of slot options should be allowed to trade—to buy and/or sell—their options. A carrier having used some options to schedule a particular flight might decide, after two months of service, to drop that service and sell the four months-options which remain to other parties.

The problem at hand is the design of these two linked competitive mechanisms to first allot then facilitate the trade of slots.

In this volume we analyze a specific mechanism for allocating slots between competing air carriers based on a sequential auction procedure. Recent theoretical work has strongly reinforced an old idea on the attainment of a competitive equilibrium in markets where prices are free to respond to demand and supply forces. The mechanism for conducting a slot auction is a tatonnement process: it is shown to achieve an efficient solution where one exists. In case there is no
efficient solution, we propose that the slot market remain open continuously throughout the six months of operations so that air carriers can exchange slots on the open slot market in order to improve the balance between slot allocations and flight schedules. The continuous slot exchange has the additional advantage that it allows changes in the allocation of scarce runway capacity in response to changing economic and air transportation conditions. In the proposed method of organizing the slot market there is no essential difference in method between the initial slot auction and the continuous slot exchange. Thus there is no incentive to air carriers to adopt predatory or deceptive strategies in the initial auction of slots in an attempt to achieve competitive advantages in the subsequent trading on the slot market.
2. THE AIRLINE INDUSTRY AND THE SLOT PROBLEM

2.1 History of the Quotas

Air travel became an increasingly popular and important transportation mode during the 1960s. The massive upsurge in passenger demand for air travel and the accompanying increase in the number of flight operations soon outstripped the existing runway capacity at several metropolitan airports. The required growth of runway capacity at key airports was retarded by several factors including lack of space, long lead times for runway construction, environmental questions and other concerns of the affected communities. By the late 1960s the situation became intolerable, as both the explicit and hidden costs of delay and congestion became prohibitive.

In order to provide immediate relief, the Federal Aviation Administration (FAA) established the "High Density Quota Rule" which restricted the number of instrument flight rule operations (takeoffs and landings) per hour for certain metropolitan airports. It should be emphasized that the purpose of the rule was to provide relief from excessive delays and not to correct a safety problem. The quota rules became effective on an interim basis on June 1, 1969 and after several limited extensions, became permanent on October 25, 1973.

The rule sets a quota for each of three classes of user: certified air carrier, scheduled air taxi (commuter) and general aviation. Table 2.1 lists the current quotas for the four congested airports. The quota rules, in effect, limit the number of scheduled operations, but unscheduled operations (e.g., general aviation) flying by visual flight rules can be accommodated as conditions permit. The allocation of the limited number of landing and takeoff slots for scheduled air carriers is done by
TABLE 2.1  CURRENT HIGH DENSITY QUOTA RULES (OPERATIONS PER HOUR)

<table>
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<tr>
<th>CLASS OF USER</th>
<th>JOHN F. KENNEDY AIRPORT (NOTE a)</th>
<th>LAGUARDIA AIRPORT</th>
<th>O'HARE AIRPORT (NOTE a)</th>
<th>WASHINGTON NATIONAL AIRPORT</th>
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<tr>
<td>AIR CARRIERS EXCEPT AIR TAXIS</td>
<td>b/ 70</td>
<td>48</td>
<td>115</td>
<td>c/ 40</td>
</tr>
<tr>
<td>SCHEDULED AIR TAXIS</td>
<td>5</td>
<td>6</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>OTHER</td>
<td>5</td>
<td>6</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>TOTAL</td>
<td>80</td>
<td>60</td>
<td>135</td>
<td>60</td>
</tr>
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a/ QUOTAS APPLY AT KENNEDY AND O'HARE ONLY BETWEEN 3 P.M. AND 8 P.M. LOCAL TIME.

b/ BETWEEN 5 P.M. AND 8 P.M., 80 SLOTS ARE RESERVED FOR AIR CARRIERS, 5 FOR AIR TAXIS, AND 5 FOR "OTHER."

c/ REDUCTION TO 36 IS PROPOSED BY FAA IN NPRM 80-2.

the airline scheduling committee meetings which will be discussed in the next section. The air taxis have a seniority system, under which incumbent users can hold their slots indefinitely. The rule grants a greater priority to certified air carriers who provide common carriage service. The concept of "first come-first serve" in landings and takeoffs remains valid until capacity limitations compel a choice, in which case the public service offered by the common carrier is preferred.4

In arriving at the quota for each airport, a number of factors were considered including airport ground facilities, weather conditions, noise abatement procedures, aircraft mix, uniformity of flow, runway combinations and proximity of alternate airports. The current quotas do not completely eliminate congestions and delay, as the quotas in some cases exceed the capacity of airports to handle traffic in IFR conditions. The FAA found it preferable to fix the number higher and accept delays that may occur under the most severe weather conditions, rather than
employing the lower quotas which might result in unused capacity during the good weather conditions that prevail most of the time.\(^5\)

As might be expected, the "High Density Quota Rules" have been subject to some controversy over the past ten years, as discussed below. Criticism of the quotas comes from both: a) general aviation interests who believe the quotas violate a basic freedom of access, and b) scheduled air carriers, including air taxis, who are unable to provide as many flights as they would like.

1. Legal Issues Surrounding Quotas

When the FAA first instituted quotas in 1969, the Aircraft Owners and Pilots Associations (AOPA) sued to have the rules withdrawn, arguing that quotas favored the airlines and violated the "freedom-to-airspace" principle stated in the Federal Aviation Act of 1958.\(^6\) The courts sided with the FAA and decided that quotas properly balanced general aviation, air carrier and public interests.

2. Pressures to Change Quotas Among User Classes

The recent dramatic growth in commuter airline traffic at the congested airports has put pressure on airport operators and the FAA to increase the air taxis quotas. For example, in 1978 the Office of Aviation Policy (AVP) prepared an issue paper\(^7\) which recommended that the air taxi quota be increased from eight to ten, and the general aviation quota be reduced from 12 to 10 at Washington National Airport. It should be noted that the FAA operates Washington National Airport while the other congested airports are operated by local airport authorities. In April 1979 FAA Administrator Langhorne Bond proposed that the air carrier quota be reduced from 40 to 36 and the commuter quota be increased further from eight to 12 at National Airport. Notice of proposed rulemaking, NPRM 80-2, suggests such changes currently are in process. Many have suggested that small "relierver
airports" may be a way to reduce peak period general aviation traffic of the congested airports.

3. Validity of Runway Capacity Estimates

As pointed out before, the high density quota rules are based on an analysis of runway capacity which was made in 1968. Since the validity of the runway capacity estimates is subject to question, the quotas can also be similarly questioned. At National Airport, for example, more than 70 operations per hour under VFR are possible, as opposed to the current quota of 60 per hour under IFR. The claim that the capacity estimates are too conservative is usually supported by pointing out that the air traffic control (ATC) systems now in operation are more effective than those in operation during the late 1960s. Critics of the current quotas also claim that the efficiency of groundside operations has improved over the past ten years.

4. Quota System Does Not Solve Long-Term Congestion Problems

The quota system does keep demand within capacity indefinitely, thereby keeping aircraft delays at a minimum. But by restricting traffic, the quotas change the air transportation service provided and do not give planners accurate information on the need for additional runway capacity. In short, it could be claimed that quotas are and can only be a short-term remedy. On the other hand, as discussed throughout this report, slot allocation under a quota system can be administered by market methods which do provide accurate information on the need for additional runway capacity.

2.2 Airline Scheduling Committees

In 1968, the Civil Aeronautics Board (CAB) authorized the establishment of certified airline scheduling committees to allocate runway operation quotas to
specific carriers. The intent of the committees, comprised of airline representatives, is to obtain unanimous agreement on an allocation of slots among air carriers which meet the high density quota rules.

For a given airport the scheduling committee meets twice a year to settle the winter and summer schedules. Prior to each meeting, the carriers submit their requests for runway operation reservations (slots) to the scheduling committee staff. These requests are tabulated and distributed to members at the start of the meeting. Submissions generally exceed slots available. Through negotiation, the committee generally first seeks to reduce total requests by all carriers to the daily quota (for example 620 at National by scheduled certified carriers). The second step of the negotiation process is to equate hourly reservation requests to hourly quotas by having the carriers "slide" their operations across hours of the day. Under the guidelines set down in the CAB antitrust exemption for the scheduling committees, committee representatives are not permitted to discuss operations with respect to particular markets or routes. Should a committee fail to reach an agreement, the responsibility for a decision rests with the FAA. Although the scheduling committees have always reached a unanimous agreement, the flexibility airlines now have in entering new markets, afforded by the Deregulation Act of 1978, has considerably lengthened the process. New entrants are less willing than established carriers to agree to proportional reductions in slot requests in order to meet the quotas. Voluntary reductions in slot requests by an airline in the face of proportional reductions by competitors has in the past been the "sine qua non" of obtaining unanimous agreements which meet the quota rules. It should be noted that the CAB can lift the antitrust exemption at any time. In fact, as has been publically stated, the CAB is interested in ending the scheduling committees. The CAB believes that the current setup is inconsistent with the goals of the Airline
Deregulation Act since the committees constrain airlines from entering and exiting key metropolitan markets served by the high density airports.

The commuter scheduling committee operates by a rule of seniority. Slots in unsaturated hours (excess supply of slots) are allocated on first come-first served basis until all slots are allocated. Slots are made available in a saturated hour only when a slot is voluntarily vacated by an incumbent carrier. The vacancy is issued on a seniority basis according to a seniority list. Seniority is based on the original date of request for an allocation of slots provided such requests are renewed and updated on an annual basis.

2.3 Costs of Congestion

The material in this section is based on the 1979 GAO report to the Congress on aircraft delays. Aircraft congestion is generally the result of excessive air traffic and bad weather. In 1977 aircraft delays caused U.S. airlines to use an additional 700 million gallons of fuel, which was over 8 percent of their total consumption. In addition to increased fuel consumption, delays force airlines to pay for extra crew time and, in some cases, accommodations for stranded passengers. In 1977 aircraft delays cost U.S. airlines over $800 million, and cost passengers over $750 million.

It is not clear what fraction of the congestion costs could be avoided even if the number of operations were reduced at the four high density airports. In short, are delays generally caused by bad weather or are they due to excessive traffic? A 1974 FAA study on airport capacity at eight major airports concluded that nearly all delays were attributable to weather problems and most severe delays were weather-related and largely unavoidable. However, a 1976 report by Chicago O'Hare's Delay Task Force, comprised of FAA, airport and airline officials, questioned whether delays were largely unavoidable. According to the study,
delays may result from a series of controllable factors, such as air traffic control procedures and excessive demand, which can cause severe system delays when compounded by weather problems. Data gathered by Atlanta Hartsfield's Delay Task Force in 1978 supports this conclusion. The data indicates that weather is significant, but does not cause the majority of aircraft delays. Sixty percent of total annual aircraft delays at Hartsfield occurred during good weather conditions. Forty percent of total annual delays occurred during poor weather conditions, which were present 12 percent of the time.

According to Chicago O'Hare's Delay Task Force, delays at O'Hare alone annually cost the airlines $44.3 million, burn an additional 67 million gallons of fuel and delay passengers 4.6 million hours. FAA and three air carriers—Eastern, United and American—are currently developing a standard method for air carriers to report delays. Figures from these three represent about one-third of all airline delays in the United States.

2.4 The Deregulation Act and Its Implication for Slot Allocation

The Airline Deregulation Act, which became law in October 1978, will gradually end forty years of federal protection for the airlines. The act will end the Civil Aeronautics Board's (CAB) power over routes and fares by 1983 and abolish the agency entirely by 1985. The intent of the new law is to open the industry to the forces of competition by encouraging price competition and importantly allowing unrestricted entry and exit from markets.

From the standpoint of slot allocation at the high density airports, deregulation has two important implications: a) the scheduling committees which are apparently inconsistent with the spirit of the act, may have to be replaced with a new method of slot allocation; b) the current and anticipated rise in both the demand for air travel and the number of competing airlines operating at a given
airport (including importantly commuter airlines) will likely increase the number of scheduled aircraft operations. For the high density airports, the need to accommodate additional operations will put pressure on authorities to change the quotas. For the remaining airports, an increase in operations will cause congestion requiring the eventual use of quotas, other things being equal. Each of these points will be discussed in order below.

**Scheduling Committees**

The deregulation action of 1978 specifically calls for "the encouragement of entry into air transportation markets by new air carriers... (and) additional... markets by existing carriers..."\(^1\) The scheduling committees, which are permitted by a temporary CAB sanctioned antitrust exemption, tend to limit entry into the high density airports. As pointed out in Section 2.2 on the scheduling committees, the airline representatives must reach a unanimous agreement on feasible allocation. Convergence to the final allocation is not guided by considerations of economic efficiency nor public service. The deliberations are rather influenced mostly by: a) historical market share and thence a particular airline's "rightful" share of the slots and b) fear of committee default. This is not to say that smaller airlines or new entrants are always at a disadvantage. Though slot allocation by historical share obviously favors the established larger airlines, fear of committee default can benefit an aggressive new entrant who refuses to reduce his request for slots. Airlines generally fear the spectre of committee default since the allocation is then decided by the FAA using an unknown rule.

On the other hand, though new entrants may be able to gain some access to high density airports, the number of operations or slots afforded to the various airlines is not determined by competitive means. A lack of competition in determining the extent of airport (market) access will decrease the overall
competitiveness of the industry, which is counter to the spirit of the Deregulation Act.

Increase in Number of Operations

Deregulation will likely result in an increase in the number of operations at hub airports since the Act: a) will eventually remove all impediments to gaining new route authority, and b) will tend to keep fares down and thereby increase demand. In addition, trunk airlines may abandon certain feeder routes only to be replaced by a larger number of commuter airlines.

Recent reports suggest that there will be severe airside congestion problems over the next ten years unless improvements are made. By 1985 as few as four or as many as 40 airports could experience severe airside congestion. Severe congestion is generally defined as a situation where annual aircraft delays exceed six minutes per aircraft. By 1990 the number of airports experiencing severe congestion could be as high as 60. In the past airports experiencing severe congestion have imposed operations quotas.

2.5 Various Proposals for Slot Allocation

Over the past ten years various proposals have been advanced for allocating slots at the high density airports. The purpose of this section is to briefly enumerate several of these with an eye to examining the relative merits of each proposal. What are the characteristics of a desirable allocation scheme? There are many, but they may be classified broadly as: 1) pragmatic--how difficult is it to implement, operate over several years; 2) economic efficiency for airlines--could a feasible allocation be reorganized such that well-being of one or more airlines is increased without decreasing the well-being of another; 3) public service--how well does the allocation serve the public good and/or meet national
transportation goals and 4) acceptance by airlines and flying public. It should be noted that these goals are not mutually exclusive.

The various allocation schemes may be broadly organized into two categories: administrative or noneconomic methods and price-rationed or economic schemes. The administrative methods include: first come-first served; administrative allocation according to priorities; lotteries and scheduling committees (which have been covered previously). The discussion that follows is based on Ref. 14.

**First Come-First Served**

First come-first served is basically a seniority system where seniority is established by the order of requesting service. The ranking of requests to conduct an operation can be done while aircraft are waiting in a queue or at the reservation office well in advance of the actual time of use. In general, the former method is currently used to regulate operations by general aviation aircraft. The latter method is used by commuter airlines where the request for service is dated to when the airline first began operations at a particular airport.

**Administrative Allocation According to Priorities**

Allocation priorities can be determined by either evaluating a weighted combination of characteristics of a given airline or an optimization procedure to maximize the value of resource use. Air carrier characteristics frequently mentioned as useful in establishing runway capacity allocation criteria include the historical level and diurnal pattern of individual air carrier operations, individual air carrier passenger enplanement or deplanements (historical or projected), individual air carrier load factors (historical or projected), the routes served by individual air carriers (historical or proposed), and the profitability of individual airline operations (historical or projected).
The FAA's Office of Aviation Policy (AVP) has been exploring an administrative procedure whereby runway operation capacity is allocated according to priorities determined by a weighted combination of factors—potential public service, historic share of operations at the airport and airline scheduling preferences. The procedure is divided into two subproblems: 1) determining each user's "fair share" of the total daily operation ceiling at the airport and 2) determining an hour-to-hour allocation of operations among users.

Under the potential procedure each user's "fair share" is calculated as a weighted combination of components based on their historic share of operations and public service rendered. New carriers' operation requests are evaluated and used to determine their "fair share." Carrier enplanements and deplanements at the subject airport are used as a surrogate measure of carrier public service.

To assign hour-by-hour allocations, carriers would be polled on their scheduling preferences. Each carrier would provide several schedule choices. A computer program is then used to analyze the set of choices provided by the carriers to identify feasible scheduling solutions and to select the feasible solution which maximizes attainment of carrier scheduling preferences.

Another potential form of allocation could be derived by means of solving the problem of maximizing the value of airline operations. Through mathematical modeling it is theoretically possible to determine the total value of operations proposed by specific airlines (represented by net profits), and to select the set of operations which comes closest to the set representing the theoretical maximization of the value of airport operations subject to airport operational constraints including runway capacity, terminal capacity, noise constraints and stage lengths.
Lottery

The allocation of a slot or sets of slots could be done by a random selection process.

- Lotteries have several characteristics which can be varied to create alternative methods for allocating runway capacity. Parameters which can be varied include:
  - Conditions imposed on eligibility for lottery participation
  - Probability of individual participants "winning" a capacity allocation
  - Ability of individual participants to influence the probability of winning
  - Nature of the capacity award.

For example, to qualify for participation, candidates may be required to have CAB authority to serve a given airport and some form of FAA certification as an air carrier, and/or might be required to make some form of money payment or deposit (buying a chance). Probabilities of a specific participant winning a capacity allocation could be varied according to the number of lottery participants, the total number of historical or desired operations by individual participants, the number of estimated enplaned and deplaned passengers or in other ways. It is possible to permit participants to directly influence the probability of "winning" an allocation through the sale of multiple lots or chances. Finally, the nature of the award or prize can be varied--a single operation in a given time period, a pair of operations in a given time period, the right to a position in the sequence of choices among single operations identified by a time period, or a position in the sequence of choices or pairs of operations identified by time period.

Broadly speaking, the first come-first served lottery and priority methods have the advantage of being fairly easy to administer. The first come-first served method is currently in use for general aviation aircraft and the commuter airlines.
On the other hand, only by chance would these methods yield an economically efficient allocation. Furthermore, the priority method of administrative allocation may appear subjective and thus may not gain acceptance by vested interests.

The price-rationed or economic methods to allocate slots include time-differentiated landing fees, auctions and the computerized slot exchange. Since both auctions and the slot exchange concepts are well covered elsewhere in the report, only time-differentiated landing fees will be covered here.

Time-differentiated landing fees can be used with or without a quota system. When there is no quota system, landing fees can theoretically be set such that each additional flight at an airport pays for all the additional congestion costs it imposes on passengers and airlines. The marginal cost pricing approach has been addressed by many authors including Carlin and Park, Fitzgerald and Aneuryn- Evans and, finally, de Neufville and Mira. The approach is based on the theory that when an airport is continuously busy each user imposes some delay on all users until the end of the busy period. Each "additional user shoves those following him one space back in the queue, and the effect persists until the queue dissipates." Though computation of marginal delay costs for a given level of demand is possible, it would be quite difficult to analytically determine the set of equilibrium landing fees over a wide range of traffic patterns. As Carlin and Park point out, to calculate equilibrium fees "we would need to know with some confidence and precision what the pattern of traffic would be under different sets of prices...we do not." (Emphasis added.) In short, the demand elasticity of traffic patterns with respect to landing fees is not known.

When landing fees are used in conjunction with quotas, the object is to set landing fees such that the quotas are precisely met. The same difficulty arises
since the traffic demand elasticities are unknown and unmeasurable over time without actually instituting landing fees over an extended period of time.
3. AUCTIONS, EXCHANGE MARKETS AND THEIR APPLICATION

3.1 Auctions as Competitive Market Mechanisms

Auctions, in one form or another, have a long history dating back to ancient times. They gained prominence in the commodity exchanges of the sixteenth century, and are still found in active use today for art sales, some commodity markets, oil lease sales, the sale of timber and mineral rights by the federal government and the sale of Treasury notes. Auctions are very familiar and easy to describe, but difficult to define precisely. Perhaps the most notable feature of auctions is the passiveness of the sellers. While buyers are actively bidding for the item(s) that are for sale, the seller waits passively for the highest bid, or accepts the first bid in a Dutch auction where descending prices are announced. In most auctions, the sale is final as soon as a winner has been determined. This is a great timesaver, and perhaps the major reason for the popularity of auctions. In other types of markets, buyers and sellers engage in protracted negotiations; they may break off negotiations without reaching an agreement or contract; they may seek a better deal by trying to find other traders who will offer more (accept less); they may seek to resell items just purchased; and so on. An auction usually provides a speedy contract between a seller with an item to sell and a buyer interested in that item by allowing a large number of interested buyers to bid simultaneously for the same contract. The auction may be of the sealed-bid variety, in which case the buyers are unaware of the buyer's bids; or it may be "open," in which case all the buyers know each other's bids. In either case, once the auction is closed, the

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*Auctions are used more in commodity futures markets than in spot markets. See Table 3.1.*
TABLE 3.1 LOCATIONS OF MAJOR COMMODITY FUTURES AUCTIONS

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SOURCE: WALTER C. LABYS, "BIDDING AND AUCTIONING ON INTERNATIONAL COMMODITY MARKETS" IN Y. AMIHUD'S 1976 BOOK BIDDING AND AUCTIONING FOR PROCUREMENT AND ALLOCATION, NEW YORK UNIVERSITY PRESS.

buyers must live with their mistakes. They may have paid "too much"—that is, they could have obtained the item for a lesser price. They may have failed to obtain an item by not bidding enough. In either case the auction does not provide for recontracting. Buyers can attempt to negotiate privately with other buyers in an aftermarket if one exists, but as far as the original seller is concerned, the auction contract is final.

3.2 Federal Use of Auctions

The purpose of this section is to report on existing markets and auctions administered by the federal government. The section will also point out similarities and differences between these markets and the potential market for airport runway slots.
3.2.1 Timber Cutting Rights

The timber sales/management division of the U. S. Forest Service sells companies the right to cut and haul timber on National Forest Lands for a specified period. A sale is administered as follows:

1. The Forest Service places an advertisement six months to one year prior to the sale which describes the tract, reservation price and sale date. Interested parties can: a) inspect government survey records which estimate extent and nature of the timber on the tract, b) request permits in order to conduct their own survey, and c) study the proposed contract of sale which specifies length of the contract and may include special provisions on required road construction and watershed provisions.

2. Parties are then required to submit sealed bids with a check greater than or equal to the reservation price, along with the appropriate documentation on the qualifications of the bidder. The number of bidders ranges from one to 15.

3. On the appropriate day, the Forest Service publicly opens the bids and first determines which of the bidders are qualified. For example, a bidder may not be qualified if he is unable to build the access roads properly.

The determination of the winning bidder differs from sale to sale. In approximately 50% of the sales, the highest qualified bid wins, losing bids are returned and if the bidder agrees to the terms of the contract, the sale is completed. But, if the tract lies in certain designated areas (especially in the West) the sale is conducted differently. All qualified parties submitting a bid greater than or equal to the reservation price are invited to a final oral auction. The auctioneer goes around the table sequentially asking each party for a bid. The bidding stops and a winner is declared when only one bidder remains active. The last bid received is the final price. Note that there is no motive to bid higher than reservation price in the sealed bid round. The magnitude of winning bids ranges from one hundred dollars to several million dollars.
The final oral auction was introduced in early 1978 after several small sawmills sued the U.S. Forest Service. It seems that with the sealed bid scheme, a "small" sawmill could be "blind sided" by competitors and consequently depress the economy of towns dependent on the losing sawmill company. With an oral auction, however, such behavior by a competitor could be spotted before it was too late. The importance of the smaller companies is further underscored by the government's commitment to tailor the tract sizes to the local industry, so that companies with limited resources can compete effectively.

The Forest Service has been conducting sealed bid auctions for about twenty years (about a million acres a year are auctioned), and have had few problems outside of the suit discussed above. Mr. George Leonard of the Timber Management staff felt that sealed bid auctions were preferable since: a) the possibility of collusion was less due to uncertainty over the bids of noncolluders, and b) the sealed bid auction is easier and cheaper to administer than an oral auction.

3.2.2 Off-Shore Oil Drilling

The Bureau of Land Management (BLM) distributes the right to drill for oil and gas on specified tracts by competitive bidding. The sales are conducted by a sealed bid auction where the highest qualified bidder wins. The details of the auction are quite complex as the government has attempted to address a wide range of issues.

Timing

Three years before a proposed sale, the government nominates a given set of off-shore tracts for development. The three years are required to allow time for filing of environmental impact studies, public hearings and exploration of the tracts by prospective bidders. If all goes well, a subset of the nominated tracts is

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*This refers to the possible surprise to the small local firm when a much larger firm without local roots enters the bidding unexpectedly and wins by virtue of its high amount bid for the tract.*
announced thirty days before the submission deadline. The announcement 1) describes the bidding system to be used (discussed below), and 2) contains a copy of the sample contract which specifies environmental restrictions, length of contract, etc.

**Bidding Systems**

"Bidding system" refers to the type and nature of bids that the government will accept in a given auction. The bid types may be a fixed or variable percent royalty on revenue (called fixed or sliding scale royalties), a one-time cash payment (called a bonus), a promise to spend a fixed amount per year in exploration and development (called a work commitment bid) or a fixed percent of the net profit. The BLM can choose any one of the bidding systems to encourage or discourage certain behavior by the drilling companies. In some cases hybrids are used which are designed to avoid ambiguities over the winning bid. For example, the BLM may require that all parties bid the same fixed bonus, but compete via sliding scale royalties. The Department of Interior designed the current bidding systems and has the responsibility to design alternative methods when necessary.

**Joint Bidding**

If a company has annual revenues of less than $1.6 billion, joint bids for a given tract with qualified parties are allowed. Otherwise joint bidding is forbidden.

**Survey Information and Government Reservation Price**

The U.S. Geological Survey estimates the size of the oil and gas contained in each tract, but does not release the information publicly. Only an estimate of the total quantity of oil in all tracts is released. The survey is used to establish a government reservation price which is also not released to bidders. All bids
received may be lower than the reservation price and the tract is not awarded. In fact, Mr. Larson of BLM estimates that 10 to 15 percent of off-shore tracts open for competitive bidding are not awarded.

**Financial Commitment by Bidders**

All bidders are required to file a $350,000 bond with the government. In addition, each sealed bid in a bonus bidding system auction must be accompanied by 20 percent of the bid. The remaining 80 percent is required if the bidder is awarded the tract and, of course, the 20 percent is refunded if the bid fails.

There is one comment to be made here when comparing the current off-shore drilling market with the potential market for runway slots. The two markets have an interesting similarity in that the value of each object being auctioned has a high degree of interdependence. The temporal and spatial interdependence of the slot auction is obvious. The complementarity among off-shore tracts is due to uncertainty over the size and location of the oil fields. Unfortunately, for purposes of comparison, the off-shore drilling market is a sealed bid auction and not an oral auction and, therefore, one cannot observe the potential bid variations that might result from the complementarity.

### 3.2.3 Coal Mining Leases

The Bureau of Land Management distributes the right to mine coal on federal lands by competitive sealed bidding by qualified parties. The mechanism is quite similar to the one reported under off-shore drilling except that it is simpler and the government releases more information.

**Bidding Systems**

The bidding systems allowed are fixed or variable bonus and fixed or variable royalty though there is a floor on royalties. For example, royalty bids on fields to be mined by surface techniques must be greater than 16.5 percent. Underground
mines must pay more than 8 percent. It should be noted that though work commitment bidding systems are not used, i.e., winners must have the mine under development within ten years of the sale date.

Survey Information and Reservation Price

The U.S. Geological Survey estimates the size of the coal field and releases the information to prospective bidders. In addition, the reservation price is published with the sale announcement.

Financial Commitment by Bidders

As before, each sealed bid in a bonus bidding system auction must be accompanied by 20 percent of the bid.

Present Status

Due to a court injunction, the BLM is required to file an environmental impact statement before a given field is nominated for development. As a result, the whole bidding mechanism in the West (surface mining) is at a standstill until the Secretary of the Interior issues a set of guidelines. No fields have been leased by competitive bidding in the West since 1971. The Office of Coal Management has been operating under a set of emergency provisions that 1) allows awards only to companies that need the coal to satisfy contracts made before September 27, 1977, and 2) allows awards if the lack of coal development would disrupt small communities in the West.

In the East (underground mining) fields are leased by competitive bidding but there has been little activity over the past ten years. It seems that there were plenty of leases transacted prior to 1971, and coal mine operators would prefer to develop the West since the unit costs are so much lower with surface mining.
3.2.4 Auctioning of Government Debt

The Treasury Department holds two types of auctions: one for the short-term debt, treasury bills, and the other for long-term treasury notes. The bills are auctioned weekly in lots by treasury personnel who accept oral bids from qualified bonded parties. The bids are on the discounted value of the bill, given its announced face value and time to maturity. There is no recontracting at the end of the session and as such the implied interest rates change with each new round of auction. The use of auction to sell short-term government debt is well-established and has been in use almost as long as the Treasury Department.

The use of oral auctions to sell longer-term notes, however, is fairly recent (1973) and has been conducted at random intervals. The nature of the note auctions are necessarily different from the bill auctions since notes pay a coupon value at well-defined intervals.

To oversimplify, let $C_t$ be the coupon payment per period, $i$ the interest rate, $A$ the face value, $N$ the time to maturity and $P$ the market price of the bond (present value). Then, if only average values are considered (i.e., the flows are not discounted), we have

$$\sum_{t=1}^{N} C_t \frac{A - P}{N} = \frac{A + P}{2}$$

as the fundamental relationship.

Until recently the Treasury Department fixed $A$ and $N$ and, as market conditions warranted, announced a particular value for $C_t$. Bidders submitted bids on $P$ and received an annual interest rate of $i$. Currently the government fixes $A$
and N and accepts bids only on the variable i. By this method the Treasury is free
to choose a variety of values for \( P \) and \( C_1 \) at later time.

3.3 An Example of an Exchange Market: The National Association of Securities
Dealers' Automated Quotation System Service (NASDAQ)

NASDAQ was established in February 1971 to serve the over-the-counter
(OTC) market. NASDAQ is a system of computers and terminals that provide
instantaneous bid-and-asked quotations to various groups of dealers, traders and
market makers. To understand how NASDAQ works and why it is important
requires first an understanding of the OTC market.

The OTC market provides a mechanism for trading various types of
securities including bank stocks, mutual funds, U.S. government securities,
municipal bonds, industrial and utility stocks, corporate bonds and some foreign
securities. The OTC market is not located in any one central location like the
NYSE, but rather consists of thousands of security houses located all over the
United States.

The securities houses are called broker-dealers and engage in buying and
selling securities usually for their own account and risk. They also buy and sell for
the accounts of others and, of course, charge a commission. In order to transact
business in a widely disaggregated market it is necessary to have a fairly
sophisticated communications network such as NASDAQ.

It is important to note that the OTC market is a negotiated market rather
than an auction market. Prices are arrived at by dealers negotiating with other
dealers in order to arrive at the best price. An exchange or auction market like
the NYSE is an auction market in which brokers bid or offer successively higher or
lower prices until a common price is reached and the transaction is completed.
Therefore the OTC market, in order to operate, requires a much more sophisticated quotation system than an auction market since the bid-ask quotations from literally thousands of different trades must be compiled and reported. An additional complexity in the OTC market is the use of firm or subject quotations. A subject market is a quotation in which the prices are subject to confirmation. Firm market prices are those at which a security can actually be bought and sold. The firm market is referred to as the actual market. Obviously, firm bids or firm offers are prices at which a dealer is committed to buy or sell a specified amount of securities, whether for a brief moment only or for a given period of time. A work-out market represents an indication of prices at which it is believed a security can be bought or sold within a reasonable length of time. All of this information is presented via NASDAQ to broker-dealers and traders.

The NASDAQ system proper consists of a central processing complex in Trumbell, Connecticut; four regional concentrator sites in New York, Atlanta, Chicago and San Francisco; and 1,061 Level 3 and Level 2 terminals. Level 3 terminals are used by market makers or dealers in NASDAQ securities. These terminals display the quotations of NASDAQ market makers for those securities for which the terminal is registered. This prevents a dealer from dealing in a security for which he is not registered. The terminals also permit the dealers to enter or update their quotations on the securities in which they are registered to make markets. Level 2 terminals are used by institutions and brokerage firms. They too display the quotations of all market makers, but do not permit the entry of information into the NASDAQ system. The Central Processing complex also delivers the representative bid and asked prices (the middle point of all quotes of all dealers in a security) on each NASDAQ security and summary data to several market data vendor companies. These companies distribute the information to
Level 1 terminals throughout the United States and the world. Level 1 terminals are located principally in the retail branches of brokerage firms to inform clients of representative bid and asked prices.

The market works as follows. A client observed the representative bid and asked prices on a Level 1 terminal. If he decides to buy (for example) he places the order including price and quantity information with his broker-dealer. The broker-dealer goes to a Level 3 terminal and observes the sell quotations offered by the various market makers registered in the given securities. The screen identifies the market makers and shows only price quotations for a standard lot size, usually 100 units of the security. Upon seeing a favorable price, the dealer phones the relevant market maker and begins negotiating. If he buys the order, the market works as follows. A client observes the representative bid and asked prices on a Level 1 terminal. He decides to buy 500 shares of XYZ Corporation only if he can get them for not more than $3.00 a share. The client phones the order into his dealer, who, while watching the Level 3 terminal, appraises his client of the market conditions. Below is a dramatic representation of the terminal screen as viewed by the broker.

**Quotations for XYZ Corporation**

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<th>Market Maker</th>
<th>Sell</th>
<th>Buy</th>
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<tr>
<td>Doe Brothers</td>
<td>3.20</td>
<td>3.10</td>
</tr>
<tr>
<td>Smith and Smith</td>
<td>3.50</td>
<td>3.40</td>
</tr>
<tr>
<td>Apple and Cherry</td>
<td>3.75</td>
<td>3.65</td>
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The broker is convinced that he can buy the stock for $3.00 a share and accepts the order. He phones Doe Brothers and points out that the quote is $3.20 for 100 shares, but his client wants 500 shares. Doe Brothers agrees and gives the broker a
transaction number. Additionally, Doe Brothers feels that the market trend is going against XYZ Corporation and decides to update its buying and selling price. One of the Doe Brothers goes to his Level 3 terminal and enters $3.10 as a new selling price and $3.00 as the new buying price. The accuracy of the displayed price quotation is guaranteed by periodic reviews of all market maker transactions by NASDAQ. If there is a discrepancy between what is displayed and the actual transaction prices accomplished over the phone, the dealer is subject to disciplinary action.

3.4 The Application of Auction Procedures to the Runway Slot Allocation Problem

The justification for rationing slots by price lies in the spirit of deregulation currently being espoused by the CAB. Through differential ticket pricing (in which the airlines have only recently been able to freely engage), the cost of the various slots can be passed through, at least in part, to the air travelers. Consequently, demand for travel at various times of day will affect, and be affected by, ticket prices; at equilibrium, an economically efficient allocation of travel facilities is a likely result.

Two types of auction procedures have received most of our attention. Under one, the airlines bid directly for slots or bundles of slots; under the other, they bid for the right to select a slot or bundle. Each of these procedures can be implemented within a variety of auction mechanisms.

There are also several fundamentally different ways of dealing with the results of an auction. These results can be taken as final, obligating the airlines to certain activities over a period of time. Alternatively, if the results of the auction involve minor inefficiencies which the airlines wish to rectify, they can be allowed to revise the auction outcome through a private after-market (which deals in
"property rights" for slots), or to "back out" of certain tentative commitments (after which a secondary auction may be held), or to revise their original bids (which consequently revises the resulting allocation of slots).

A principal feature of the slot allocation problem is the fact that operations slots are used in pairs (for every landing, there is an eventual take-off). Therefore, for example, any auction procedure under which an airline might find itself allocated an odd number of slots should be viewed as the initial part of an allocation mechanism which permits after-auction adjustment of the slot assignments.

There are a number of attractive aspects to procedures which involve the airlines in bidding for the right to select slots rather than bidding directly for the slots. Indeed, if all operations slots were distinguishable (that is, were attached to specific times), such procedures would probably be the best available. However there are two critical disadvantages to these procedures. They are informationally complex, calling for each airline to make a large number of individual decisions in an uncertain environment. They also are highly sensitive to minor misperceptions of demand. As a result, a slot (more precisely, the right to choose this slot) in a given one-hour period may sell early in the auction for an amount substantially higher or lower than the price of an identical slot later in the auction after the total demand for such slots becomes clearer. Hence, the variance in price of identical slots may be unacceptably high.

Alternative procedures involve bidding directly for slots. Most such procedures can be viewed as dynamic-pricing mechanisms. All the slots within any given one-hour interval (of course, intervals other than one hour can also be used) are allocated simultaneously. Coordination of allocations across intervals, in order
to satisfy slot-pairing requirements, is handled through subsequent adjustments to the initial within-interval allocations.

The allocations of slots within an interval can be determined by direct adjustment of prices. Assume that the expressed demand for slots exceeds supply at a per-slot price of zero. Then the price can be gradually raised; demand will decrease as the price increases. The "equilibrium" price, at which demand first equals supply, can be tentatively established, and the slots allocated accordingly. An equivalent version of the price-adjustment process has each competitor submit a price-quantity demand curve, or (under an appropriate assumption of convexity), a list of bids for an initial slot and for additional slots. The equilibrium price will be equal to the highest rejected bid.

A drawback to this approach can be seen through an example. Assume there are bids of 8, 8, 8, 7 and 6 for a supply of four slots; further assume that the first three bids are all entered by the same airline, and the fourth and fifth bids by two other airlines. The first airline will then receive a bundle of slots of (subjective) value 24, at a cost of 18 units. However, an alternative strategy for this airline would be to misrepresent its demand, entering only two positive bids of 8 units each. Slots would now be priced as free goods (there would be no excess expressed demand), and the airline under consideration would net a gain of 16 units for the two slots purchased rather than only 6 for the three slots purchased.

An alternative pricing mechanism, which does not suffer from this drawback, is presented in the next section. In its simplest form, it involves eliciting bid lists, awarding slots according to the highest bids and charging each airline the sum of the bids (other than its own) it has caused to be rejected. An airline with k winning bids would be charged the sum of the k highest rejected bids; obviously this cannot exceed the total amount bid by the successful bidder. In the preceding example
this mechanism would charge the same amount, 6 units, to both winning airlines. The incentive for misrepresentation of demand would be eliminated.

An open adjustment period, in which the airlines are allowed to revise their bid lists in view of the across-interval slot allocations, could be permitted to facilitate resolution of the slot-pairing problem. Details of such secondary mechanisms are discussed in a subsequent section.

3.5 General Demand-Revealing Mechanisms

Principal concerns of the FAA are that operations slots be allocated equitably and efficiently. A constraint related to equity considerations is that the mechanism to be adopted must treat all competitors neutrally, with the possible exception of an initial reliance on historical operations patterns. The current spirit of deregulation further requires that the entry or exit of any competitor into or from a market must be immune to being blocked by other competitors, except at substantial cost to them.

Several types of efficiency must be considered. Computational efficiency requires that the allocation procedure can be administered in a reasonable amount of time. Informational efficiency requires that each competitor, when called upon to act, must face rationally manageable decisions. Economic efficiency requires that, after the allocation procedure is completed, the various competitors (individually and in groups) must be left with little desire for retrospective changes in their actions.

The mechanism adopted must be privacy-respecting. That is, although the mechanism may request information from the competitors, it cannot involve any exogenous procedure for determining the "accuracy" of the information. Therefore it is highly desirable to employ a mechanism which provides direct economic incentive for truthful revelation of demand. Such "incentive-compatible" mechanisms
have drawn substantial attention to the past few years (see the special supplement to *Public Choice*, Spring 1977).

The basic procedure for designing such a mechanism is to establish a charge structure referred to as a "Clarke tax." Each competitor is asked to assign a value to every state of the world. The state of greatest total declared value is the one adopted. Each competitor pays the amount he declared as his value for the adopted state. However, each competitor also receives a rebate. Assume that a particular competitor had not been present to participate in the choice of a state of the world. Then the declared values of the remaining competitors would have possibly determined some alternative state. The competitor's rebate is the difference between the total declared value (to all competitors) of the originally chosen state with that competitor's choice included, and the total value (to the others) of the alternative state without his choice. (Equivalently, each competitor simply pays the total amount that his presence cost the others.)

A simple example concerns the sale of a single object, which is of different (subjective) value to each of a group of bidders. Each is called upon to make a sealed bid, which represents the value to him of the "state of the world" in which he possesses the object; it is assumed that a value of zero is associated with the state in which someone else owns the object. The above-described mechanism then gives the object to the highest bidder and charges him the amount of the second-highest bid; all other bidders pay nothing. It is not difficult to verify that no bidder has incentive to misrepresent the value of the object to him.

How does this approach relate to the problem at hand? We need merely call upon each airline to assign a value to every possible bundle of operations slots. Subsequently, the partition of slots among airlines is chosen to maximize total
declared value; charges are assessed subsequent to the solution of the partitioning problems which arise from the exclusion of individual competitors.

This procedure is clearly equitable and is economically efficient. However, the computational and information inefficiencies are manifest--this scheme cannot be practically implemented.

Consider the problem of allocating slots within a particular one-hour period. In this case the homogeneity of the slots makes it possible to devise a reasonable demand-revealing mechanism. Have each airline assign a total value to every number of slots, from zero up to the hourly quota. Allocate the slots and apportion charges as described above. For example, assume that the marginal value of additional slots is decreasing. Then instead of soliciting total values, one can ask for the value of a single slot and for the marginal value of each additional slot. Treating these values as individual bids, one assigns all slots to the highest bidders. A competitor who receives k slots will be charged the sum of the k highest (other than his own) rejected bids.

This last mechanism, due to the incentive-compatibility of truthful revelations, is a most reasonable candidate for a slot auction procedure. An additional distinguishing feature of the mechanism is known as the "no regret" property or "ex post optimality:" even if the prospective bids of all the other competitors were known, an individual would have no incentive to deviate from the strategy of truthful revelation. This is in stark contrast to the situation indicated in the previous section for the highest-rejected bid pricing scheme and indeed for any uniform pricing mechanism. Therefore a slot-allocation scheme such as the one we are discussing, which permits the repeated revision of bids so as to coordinate allocations across time intervals, will tend to be more stable when based on the auction mechanism just described than when based on a uniform pricing scheme.
3.6 The Sequence of Sales

Suppose that tradition is followed and that all slots within any given one-hour interval are regarded as identical. Then an initial decision must be made concerning the sequence in which the day's time intervals are dealt with. One possibility, which circumvents this sequencing decision, is to sell the slots in all time intervals simultaneously by requiring the airlines to submit all of their bids at once for slots throughout the day. A similar procedure is currently used for the leasing of off-shore oil rights, where several hundred leases may be simultaneously sold through sealed bids. This method has recently drawn criticism due to the problems which may arise when a competitor can't make his actions in some auctions contingent upon the outcome of other auctions; for instance an airline's demand for off-peak slots depends upon the number of peak-hour slots which can be obtained. The solution to these problems is found in the design of the simultaneous auction as a series of nonbinding rounds of bidding as described more fully in Section 3.8. For the remainder of this section we consider sequences of auctions on the supposition that the simultaneous auction is unavailable in practice.

Sequencing possibilities include treating the time intervals in temporal order, or in accordance with a randomly-generated, pre-announced order, or through sequential randomization in which the next-to-be-auctioned time interval of slots is determined just prior to its treatment. Traditionally, the common sense approach has been to allocate slots in periods of high demand first and to subsequently allocate for spill-over demand in the intervals outside of the high-demand period. In particular, such sequencing alleviates the potential need of an airline to purchase unneeded slots at low-demand times in an effort to protect itself against an unfavorable result in the subsequent allocation of high-demand
slots. Therefore the appropriate basis for the sequencing of time-intervals in the slot auction is in accordance with the excess demand for slots in these intervals.

How is a preliminary determination of demand to be made if the auction has not yet been held? Two alternatives suggest themselves. Either the periods of highest demand can be determined from historical data (records from previous scheduling committee meetings or, eventually, from the previous slot auctions), or the airlines can be asked to submit tentative demands for slots as free goods (much as is currently done), and these submissions can be used to determine the sale sequence.

It is possible that demand will peak at two or more noncontiguous periods in the day (a bimodal distribution). In this case spillover demand could cause irregularities in the sale of slots in intervening periods. For a simple example, suppose that the time intervals 11 A.M.-12 noon and 1 P.M.-2 P.M. experience substantially higher demand than the 12 noon-1 P.M. interval. If slots in the two former intervals were to be sold first, the spillover from both periods could so increase demand in the middle interval as to yield higher slot prices than in the adjacent intervals. Problems of this sort seem to be unavoidable when slot auctions are organized in a temporal sequence.

3.7 The Auction Mechanism

As previously noted, we wish to construct a sale mechanism which minimizes the airlines' incentives to misrepresent their demands (i.e., to "game" the system), and which consequently places a relatively light strategic burden on the competing airlines. To this end we employ an incentive-compatible mechanism.

There are two approaches which can be taken. In the simpler of the two, each airline is asked to submit a list of bids (more formally, a descending sequence of bids) for slots in the time interval. If \( k \) slots are available, then they are
assigned in accordance with the k highest bids. Each winning bidder who obtains no slots is charged the sum of the m highest bids rejected.

For example, assume that three airlines are competing for a total of ten slots.

<table>
<thead>
<tr>
<th>Airlines</th>
<th>Bids</th>
<th>Awards</th>
<th>Payments</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10, 10, 10, 7, 6, 2, 1, 0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>12, 11, 9, 2, 0</td>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>C</td>
<td>20, 8, 8, 8, 0</td>
<td>4</td>
<td>18</td>
</tr>
</tbody>
</table>

The sizable difference in charges to A and B (which, after all, receive the same number of slots) is an artifact of this example, and is due to the rapid drop-off in the demands of B and C just beyond the numbers of slots allocated to them. However, the purpose of this example is to argue that the difference in charges is justified, even in this extreme case. Underlying much of economic theory is the fact that the price we pay for a commodity is the amount we must pay in order to deny that commodity to a competitor. At the margin, the final slot "consumed" by A is sought only by B and is valued at only 2 units. However, the final slot consumed by B is valued by A at 7 units. Hence, B is required to pay more at the margin than is A. Pricing out the other allotted slots in this manner leads to the proposed charges.

It is vitally important to note that, both in this example and in general, there is no airline which, having bid truthfully, could have improved its lot by submitting some other list of bids. This "no regret" property of the truthful-revelation strategy being ex post optimal is what motivates our choice of an auction mechanism.

We consider next an alternative, more general slot auction format, in which each airline submits a price-quantity demand schedule (that is, a list of bids, one for every number of slots in the time interval under consideration). Available slots
are allocated according to the sum maximizing partition, and each winning bidder is again charged the cost to the others of his presence. For example, assume that eight slots are available.

<table>
<thead>
<tr>
<th>Airlines</th>
<th>Numbers of Slots</th>
<th>Awards</th>
<th>Payments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 1 2 3 4 5 6 7 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bids</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0 10 20 28 33 35 35 35</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>B</td>
<td>0 0 20 20 30 30 34 34</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>0 10 25 25 28 28 28 28</td>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>

In this example airline B presents the case of an airline which wishes to operate only in this time interval and therefore requires an even number of operations slots. (This situation is represented by the zero marginal utility to B of incremental odd slots.)

The allocation is of value $33 + 20 + 25 = 78$, which is greater than the value of any other allocation. The charges to A are computed as follows. Had A not been present, the resulting allocation would have given six slots to B and two to C for a total value of $34 + 25 = 59$. However, in actuality B and C each receive two slots for a total value of 45. Hence A pays $59 - 45 = 14$. The other charges are computed similarly.

The advantage of this procedure, over the one previously discussed, is that it permits the airlines to more accurately represent their demands through their bids, and hence will lead to a more economically efficient allocation of slots. The principal disadvantage lies in the computational complexity of the set-partitioning problems which must be solved in order to determine the allocation and charges from the bid lists. This disadvantage may be relatively small because the special structure of the partitioning problems may facilitate their solution.
If the airlines assign decreasing marginal value to incremental slots (as A did in the most recent example), then the two procedures are, in fact, identical. In the second procedure, the set-partitioning problem can be solved by a device known as the "greedy algorithm" which leads directly to the first procedure. This decreasing marginal value assumption, which is equivalent to the concavity of the value functions, seems not too unreasonable (that is, not too far from the actual case). Since we anticipate the need for a secondary market in slots, or aftermarket in order to resolve such issues as post-auction schedule coordination, it appears likely that the first and simpler of the two auction procedures could serve acceptably for the initial slot allocation process.

3.8 The Stabilization Process

An airline cannot be certain of the precise value to it of a particular operations slot unless it also knows its allocation of slots from other time intervals. Therefore, after slots in several time intervals have been allocated, an airline may wish to revise bids which it made in previous rounds. Through the use of an incentive-compatible auction mechanism, we avoid in principle the situation in which an airline wishes, immediately after a round of bidding, to revise its bids in that round. However, to cover the case where an airline misperceives its strategic situation when the auction procedure is first used, we do not rule out the possibility of some competitor wishing to revise its bids immediately after the round is over. Because of these potential events it may be of value to offer the possibility of reopening the bidding for a set of slots at some time subsequent to the initial bidding for those slots.

What incentive will an airline have to truthfully reveal its demands through its bids if the resulting allocation is not final? In order to preserve the truthful-revelation incentive, there must always be the possibility that the bidding for a
particular set of slots will not be reopened. What criteria should determine whether a round of bidding is reopened? A general dissatisfaction among the competitors with the outcome of that round, or the strenuous objections of a specific competitor, should be required.

In some experimental settings procedures of indefinite duration are terminated upon a consensus of the participants. By analogy, one might propose the use of some type of voting procedure to finally ratify the results of a round of bidding. However, this seems an unnecessarily rigid approach, especially when the need to avoid giving any competitor veto power (so that ratification is always considered a possibility), while still recognizing the strong objections of individual competitors, is taken into account. Rather, we propose that there be an auction administrator with the authority to reopen (or refuse to reopen) any round of bidding at any time. Hence, in particular, the auction administrator will always hold the power to conclude the entire slot-allocation process at any moment after at least one round of bidding has been held for every time interval of slots.

Through this device the competitors will be able to present their cases for the reopening of any round of bidding, but must face the possibility of rejection if they fail to make a sufficiently strong case. Clearly this places substantial power in the hands of the auction administrator. How should this administrator be chosen? One requirement is obvious: the administrator must not have revenue-maximization as his primary goal. Indeed, this observation is in keeping with the results of recent research, which indicates that in a quite general setting allocation mechanisms which maximize revenues must involve the possibility of economically-inefficient outcomes. (In a sense, the threat of an inefficient allocation is needed to extract high revenues from the participants. But the threat will not be effective unless it is sometimes carried out).
3.9 **Summary of the Auction Process**

We envision a slot-allocation procedure developing somewhat as follows. The FAA, or some industry-based association such as the ATA, appoints an auction administrator. The administrator asks all airlines to submit requests for slots, much as is currently done. If all requests in all time intervals can be fulfilled, then slots are allocated accordingly as free goods, and no auction takes place. However, if it is impossible to satisfy all of the requests (as is likely, in the case of the current high-density airports), the auction commences.

The administrator announces to all participants the preliminary aggregate demand for slots in each time interval. Using this data, and historical data as well, he selects the set of slots to be initially auctioned. Bid lists (lists of marginal values, as in the first of the discussed auction procedures) are submitted by the airlines, and the available slots are allocated and charges are assessed accordingly. At this point the allocations and assessments are only tentative. The administrator next receives requests for reopening of the bidding and, as a result of these requests, either reopens the bidding or proceeds to the allocation of a new set of slots. If the bidding is immediately reopened, most airlines will probably make no changes or only minor changes in their bid lists. Hence, a round of bid revision should not consume too substantial an amount of time.

The definition of the sets of slots offered in a sequence of auctions is of great importance and embodies a number of different considerations. Theoretically, as discussed in detail in Chapter 4, the best choice is to auction off all slots at the high-density airports at once. This allows a maximum of coordination of slot purchases with schedule requirements. In practice it may be desirable to hold separate auctions for separate airports, in line with the traditional manner of conducting the airline scheduling committee meetings. An idea for easing the
transition from committee methods to the slot auction has been suggested: auction off only a percentage of the slots at quota airports, the rest having been previously allocated administratively or by some other methods. This idea leads naturally to the related idea of holding a sequence of slot auctions during each of which a share of the remaining unallocated slots is offered. In either case the set of slots offered for auction is obtained by taking a fraction of the unallocated slots, either across the board—all airports and hours—or selectively for particular airports and hours.

The slots are auctioned off in a sequence of bidding rounds; under one method these are final but apply only to a subset of the slots available; under another method the bids are treated as tentative and nonbinding, and all slots are subject to bidding. In either case we are interested in the convergence of the sequence to a market-clearing equilibrium, and in the equitability and efficiency of the final slot allocation. Not enough is known yet to make a choice of methods based on rigorous theoretical analysis of the properties of auctions. Nevertheless, based on recent unpublished work by P. Dubey at Yale University, we feel that there are strong reasons to consider a particular method of organizing the sequence of bidding rounds. This will be fully described in Chapter 4.

3.10 The Secondary Market

At the conclusion of the auction some airlines might still seek minor changes in their allotments of slots (that is, they may seek to acquire additional slots or to relinquish slots which are currently held). Also, in the span of time between slot auctions, the economic environment might change in such a manner as to make some airlines seek revision of their allotments.

These adjustments can be handled through a formal market mechanism or through a more flexible administrative system. An amalgam of these two approaches appears most attractive. If at any time two airlines wish to exchange
sets of slots, perhaps involving slots at different high-density airports and perhaps with one airline providing financial compensation to the other, they may propose this transaction to the slot allocation administrator. This might well be the same individual who administers the slot auctions. The administrator has the authority to either approve or reject this proposal.

Concurrently, the administrator monitors an open-book bidding process in which airlines register the amounts they are willing to pay for currently unavailable slots. The high outstanding bids are publicly known and establish "market values" for slots. An airline wishing to relinquish possession of any slot is allowed to turn that slot back to the administrator, who then, after a public posting of the availability of the slot, to allow for last-minute bids, awards the slot to the airline currently having the highest registered bid for a slot in this time interval. The revenues from this sale are used to compensate the airline which relinquished the slot, but on a prorated bias and only up to the marginal cost of that slot to the airline when it was originally obtained. The use of such a limited rebate scheme ensures that no profits are to be had from speculation in slots.

The details of the secondary market just presented are not critical; after all, the goal of the original slot auction procedure is to diminish as much as possible the need for such a market. However, as indicated above, the key features of any secondary market mechanism should be administrative flexibility in the approving of private slot transactions and a resale mechanism for unwanted slots, which does not encourage speculation during the slot auctions.

3.11 Sequential One-Time Auctions and the Prisoners' Dilemma

Consider the following example. There are two airlines, A and B, and two slot-option markets, 1 (JFK at 0900) and 2 (O'Hare at 1300). Each airline wishes to schedule two flights through the two airports, thus each needs two slots at each of
the two airports. It turns out that both of the airlines wish to land and take off within the same hourly periods. The quotas are four slots at each airport.

Let \((x_1, x_2)\) represent the allocation made to one of the airlines with \(x_i\) the number of slots in market \(i\). Assume identical utilities

\[ u(x_1, x_2) = K \min \{x_1, x_2, 2\} - p_1 x_1 - p_2 x_2, \]

where \(p_i\) is the price of a slot in market \(i\) and \(K\) the profit of one flight. There is an obvious unique efficient allocation: \((2, 2)\) for each airline (which need not be reached by a simultaneous "one-time" auction).

Consider a sequential auction with the market 2 auction following that of the market 1 auction. Each airline can make its strategy in market 2 contingent upon the outcome of market 1. For example, A's strategy could be as follows: to bid for three slots at price \(\bar{p}\) in market 1; if obtained, then bid for one slot in market 2 at price \(\bar{p}\); if not obtained, then bid for four slots at price \(p^\ast(\bar{p})\). And B's strategy could be: to bid for one slot at price \(\bar{p}\) in market 1; if obtained, then bid for three slots in market 2 at price \(\bar{p}\); if not obtained, then bid for four slots at price \(p^\ast\).

These strange strategies comprise a Nash equilibrium which yield the allocation \((1, 3)\) to A and \((3, 1)\) to B which is not efficient. Admittedly, both A and B are using irrational strategies; however, the point of the example is to show that one participant may be trapped as a victim of another's irrationality. This is precisely the phenomenon so dramatically capsuled in the well-known and well-baptized "prisoner's dilemma" example.

3.12 The Inefficiency of One-Time Auctions Followed by an Aftermarket

Consider the following example. There are five airlines (A, B, C, D and E) and one congested airport. The quota of the airport is four. Each airline wishes to route flights through the airport at particular times of day, first landing then
taking off. Each airline knows the "value" of each flight, which represents the total dollar amount it is willing to pay to acquire the necessary slots. The data of the problem are as follows. The notation x indicates the slots needed to realize the flights in question.

<table>
<thead>
<tr>
<th>AIRLINE</th>
<th>FLIGHT</th>
<th>VALUE</th>
<th>0900</th>
<th>1000</th>
<th>1100</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A1</td>
<td>305</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>A</td>
<td>A2</td>
<td>305</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>B1</td>
<td>320</td>
<td>xx</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>B2</td>
<td>305</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>B3</td>
<td>295</td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>C</td>
<td>C1</td>
<td>302</td>
<td>xx</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>C2</td>
<td>320</td>
<td></td>
<td>xx</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>C3</td>
<td>300</td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>D</td>
<td>D1</td>
<td>300</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>D2</td>
<td>305</td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>E</td>
<td>E1</td>
<td>324</td>
<td></td>
<td>x</td>
<td>xx</td>
</tr>
</tbody>
</table>

C2, for example, is to land and take off in the 1000 period; D2 to land in the 1000 period and take off in the 1100 period.

We analyze a one-shot auction approach. Suppose that the (simultaneous) bidding is as follows.

<table>
<thead>
<tr>
<th>AIRLINE</th>
<th>FLIGHT</th>
<th>VALUE</th>
<th>0900</th>
<th>1000</th>
<th>1100</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A1</td>
<td>305</td>
<td>$170*</td>
<td>$135</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>A2</td>
<td>305</td>
<td>$140</td>
<td></td>
<td>$165*</td>
</tr>
<tr>
<td>B</td>
<td>B1</td>
<td>320</td>
<td>$160*, $167*</td>
<td>$155*</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>B2</td>
<td>305</td>
<td>$150</td>
<td></td>
<td>$155*</td>
</tr>
<tr>
<td>B</td>
<td>B3</td>
<td>295</td>
<td></td>
<td>$145*</td>
<td>$150</td>
</tr>
<tr>
<td>C</td>
<td>C1</td>
<td>302</td>
<td>$151, $151</td>
<td>$160*, $160*</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>C2</td>
<td>320</td>
<td>$160*, $160*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>C3</td>
<td>300</td>
<td>$136</td>
<td></td>
<td>$164*</td>
</tr>
<tr>
<td>D</td>
<td>D1</td>
<td>300</td>
<td>$157*</td>
<td></td>
<td>$143</td>
</tr>
<tr>
<td>D</td>
<td>D2</td>
<td>305</td>
<td></td>
<td>$135</td>
<td>$170*</td>
</tr>
<tr>
<td>E</td>
<td>E1</td>
<td>324</td>
<td></td>
<td></td>
<td>$162*, $162</td>
</tr>
</tbody>
</table>

Then, the price in the 0900 market is $157, in the 1000 market is $145 and in the 1100 market is $162. The winners of slots are starred (*). Thus, A wins one slot at
0900, one at 1100; B wins two at 0900 and two at 1000; C wins two at 1000 and one at 1100; D wins one at 0900 and one at 1100; and E wins one at 1100. The initial endowments are indeed unbalanced. Most flights cannot be realized.

Consider A. It paid $157 for one slot at 0900 for A1, and so is prepared to pay at most $305 - $157 = $148 for a slot at 1000. B paid $145 for a slot at 1000, needs a slot at 1100 to accommodate its flight B1, but notes the price, $162, is too high since $162 + $145 = $307 which is more than its value. So B is willing to sell one of its 1000 slots for at least $145. B, therefore, sells an 1000 slot to A at some price between $145 and $148, say $146.50. Similarly B is willing to offer up to $160 for a 0900 slot, and D to sell a 0900 slot for at least $157. So B purchases from D a 0900 slot for, say, $158.50. Similarly, E purchases an 1100 slot from A at $162.

At this point, A has acquired the slots for A1; B has acquired the slots for B1 and B2; C has acquired the slots for C1 and C2; and E has acquired the slots for E1. C and D each hold one 1100 slot. No one airline is willing to make any departures from its present holdings; the solution is a Nash equilibrium.

However, the solution is inefficient and not at a competitive equilibrium. D and E cannot use their slots since they do not possess the corresponding slots necessary to complete the flights. The 0900 and 1000 markets are saturated, but the 1100 market accommodates only two flights, not the quota of four. The total "value" of the five realized flights is $1,574. The total paid at the auctions is $1,832.

There does exist an efficient, competitive equilibrium determined by the following prices: $151 in the 0900 market; $155 in the 1000 market; and $150 in the 1100 market. At these prices, A obtains the slots for A1, B obtains the slots for B1; C obtains the slots for C1 and C2; D obtains the slots for D2; and E obtains the slots for E1. No other flight is economically viable. The quota of each period
is saturated. The total value of the six realized flights is $1,874. The total sum paid at the prices named is $1,824.
4. THE SLOT EXCHANGE AUCTION AND MARKET

4.1 Approach

A semi-annual auction for the peak-hour slots at each quota airport has been proposed by various authors. The slot auction allows for competitive pricing and allocation of the slots. When slots are in high demand, the price of a slot will be high; conversely, for hours when there is less demand for slots at a quota airport, the price will drop towards zero and will become zero if there is excess supply. Airlines would be invited to bid for the slots which they need to complete their seasonal schedules. Each airline would decide (privately) how much it was willing to pay for these slots. During the course of the auction, the airlines would learn if the amounts they bid were sufficient to "win" their desired slots or not. Actual slot prices set by the auction might be higher or lower than the bids.

The value of each slot, as noted in the previous section, depends on the associated slots required to schedule a flight. This presents an airline with a bidding problem at the slot auction. It is clear that this is a very difficult problem, one which probably cannot be solved without some coordination between the auctions for different peak hours and different quota airports. Our approach solves this problem as follows. In a Slot Exchange Auction, the airlines bid on all peak hours at all quota airports together. They prepare their bids in packages of slots which correspond to flights, so that the value to the airline of the whole package can be related to the profitability of the flight. Bids are submitted confidentially by all airlines simultaneously. The auctioneer then determines a slot price for each hour and an allocation of all the slots. The

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* Or nominal, to avoid certain administrative problems with free slots.
** The details of this determination are provided in Section 4.2.
prices and allocations are, however, only tentative; they are subject to revision by the rebid process. After examining the slot prices and the allocation of slots, the airlines may change any or all of their bids, and make a new sealed-bid submission. Each round of bidding has a conditional outcome: the slot prices and allocations are nonbinding on condition that the round is not the final one. At the final round, they become binding.

Over a sequence of bidding rounds several things happen. One, the airlines "discover" how much they have to pay for slots at various times of day at the quota airports. Two, the bids for slots can be coordinated across hours and airports so that the package of slots "won" makes good sense to the winner. Three, the market for slots is cleared and an equilibrium is established. Four, no airline pays more for a slot than it is worth (to that airline).

There are some caveats. Because of the integer nature of the demand and supply of slots, there will occasionally be a need for another mechanism to allocate slots at the margin. Random allocation is the simplest mechanism. Some adjustment after the auction may be required for the airlines which receive a random allocation of slots. For instance, an airline may have bid for two slots at $50.00 but only received one at $50.00 because there were 37 bids of $50 or more and the quota was 36. The problem is inescapable. The solution appears to be: let the airlines trade slots on an aftermarket.

Another potentially more serious problem with the Slot Exchange Auction is the possible elimination of air service for small communities from peak hours at quota airports. Adequate service to the small community can only be guaranteed in a competitive slot allocation if some slots are set aside for this purpose. To some extent this already happens in the way that quota rules
are implemented: separate allocations are made for commuter airlines. But for those airlines which are bidding for the air carrier (401 certificate) slots in order to serve small communities in competition with airlines serving major hubs, the problem is not resolved.

4.2 Concept

Each airline comes to the initial round of the Slot Exchange Auction with its desired schedules in hand and some appreciation for the total expenditure (the "value") it is prepared to make to realize a flight or cycle. Each airline is requested to prepare sealed bids for the slots that it requires. This means that for each hour at each congested airport--at each of what we will call a "trading post"--it prepares bids for those slots it desires. Assuming, as we did earlier, four congested airports, two permitting scheduled operations in 16 hourly periods, the other two in five hours, there are 42 trading posts which make up the slot exchange. These first bids are made "in the dark," just as is the case in the "one-time" auction. The airlines should bid any prices for slots realizing a flight whose totals do not surpass the value it attaches to the flight. One can imagine that an airline makes its bids at the 42 trading posts by filing prices on forms, one page containing all the trading posts of each congested airport, as in Figure 4.1. An airline wishing to make six bids at trading post Washington National 0700-0759, three at $150, one at $100, one at $50, and one at $0 would complete the corresponding line as in Figure 4.2. So, each airline expresses its individual demand for slots--the quantity it desires and the prices it is willing to pay--at each of the 42 trading posts. The individual demand curve of the airline of Figure 4.2 is given on the left of Figure 4.3. Accumulating the individual demands at each of the 42 trading posts then yields the aggregate demand of the carriers at each post. A typical aggregate demand curve is given on the right of Figure 4.3.
### Washington National Trading Posts

<table>
<thead>
<tr>
<th>Trading Post</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>W. NTL. 1</td>
<td>0600-0659</td>
</tr>
<tr>
<td>W. NTL. 2</td>
<td>0700-0759</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>W. NTL. 16</td>
<td>2100-2159</td>
</tr>
</tbody>
</table>

**Figure 4.1 Possible Form for Airlines' Bids**

### Individual Demand Curve vs. Aggregate Demand Curve

**Individual Demand Curve:**

- Price vs. No. of Slots
- Price: 150, 100, 50
- No. of Slots: 3, 1, 1

**Aggregate Demand Curve:**

- Price vs. No. of Slots
- Price: 150, 100, 50
- No. of Slots: 150, 100, 50

**Figure 4.3 Individual and Aggregate Demand at One Trading Post**
The slot auction administrator determines the aggregate demands at each trading post. Either—as on the left of Figure 4.4—the total demand is less than the quota of that trading post, or—as on the right of Figure 4.4—the total demand is at least as great as the quota. The dotted line in effect expresses the supply curve, so the trading post price should be determined by where demand and supply intersect. In the case of excess supply (left Figure 4.4) the price should be $0; in the case of excess demand (right Figure 4.4) the price should lie in the interval comprised between the price of the $q^{th}$ highest bid and the $(q+1)^{st}$ highest bid. It may be that an aggregate demand curve is as in Figure 4.5, showing that there are several bids at the trading post price.

At this point the slot auction administrator reveals to all airlines the aggregate demand at each of the trading posts. Moreover, if these aggregate demands truly expressed the demands of the airlines, then each of those airlines having made bids at or above the trading post price $p$ (determined to make supply equal to demand) would receive slots for those bids; each of those having made bids lower than $p$ would not. In the case of Figure 4.5, where more than one bid is made at the trading post price $p$ but the quota is such that not all bids at that price or higher can be awarded, then some random allocation among those bidding $p$ would be made. The administrator would announce these conditional allocations and the trading post prices. Were this to be the final slot exchange auction, then each airline winning a slot at a trading post would pay $p$, the price at that post.

If the conditional allocations and trading post prices are agreeable to all airlines, then an efficient, competitive equilibrium allocation has been found. Typically—and certainly after the first round of bidding—many airlines will be dissatisfied with the conditional solution. The results of the first round are
precisely equivalent to what is produced by the "one-time" auction and so its defects are known.

This is why the administrator announces conditional allocations and trading post prices, and the current expression of total demand at each trading post. The first round gives each bidder information concerning the demand pressures which exist at each trading post. On the basis of these and of each airline's individual needs, each airline prepares new bids in a second round of the slot auction. The administrator accumulates the sealed, secret individual bids, and by the same
procedure, announces new conditional allocations, trading prices and total demands. The process is iterated so that instead of a one-time auction there are repeated auctions. Each step increases the information available to the airline and each adds to the airlines' insight into the demand pressure over all trading posts. If, at any step, no airline announces the wish to change its bid, then the process terminates at an equilibrium solution. The temporal and network interdependencies are accounted for directly in the process of the simultaneously repeated auctions.

One of the paramount outcomes of the repeated auctions would be the trading post prices. These would reflect the marginal values of slots at each hour and each congested airport. Comparison between airports would reveal where the demand pressure is greatest and where measures to alleviate congestion are the most pressing. Demand for slots is, in fact, driven by the demand of passengers to fly to and from certain airports at certain times of day. Therefore airlines should theoretically pass on the extra costs of purchasing slots to passengers, increasing the price of flights which land or take off at congested hours. The trading post prices could both guide and provide the rationale for determining passenger ticket prices differentiated over time. Ultimately the expression of passenger demand for flights using congested hours—as estimated by the airlines—will determine the bids of the airlines and so the final trading post prices. It may be quite true that today the airlines will be reluctant to engage in this estimation exercise, for detailed information concerning the elasticity of demand for peak-hour travel is not known; but, in the new era of deregulation, the airlines may be pushed to acquire the necessary knowledge. The costs of slots will, ideally, be borne by the airlines and taken from their profits; they should be absorbed by the traveling public which puts priority on peak-hour arrivals and departures.
A by-product of the trading post prices, were the same procedure used to allocate slots between commuters, would be an economic evaluation of the worth of an extra slot to commuters as versus carriers. The extent of the subsidy made of commuters as versus carriers in the determination of their respective quotas would be quantified and so available to guide the choice of those quotas.

Before proceeding we summarize the Slot Exchange Auction. 1) Airlines prepare their bids privately and submit them sealed. 2) Airlines bid for as many slots as they wish at all trading posts simultaneously. 3) The slot auction administrator announces, after every round of bidding, the conditional allocations, trading post prices and total demands. 4) When the administrator closes the slot auction—when the conditional solution is announced to be the final solution—the airlines must accept the slots awarded them and the obligations of payment at the final trading post prices. 5) If more than one bid is made at the trading post price \( p \) and not all can be awarded without exceeding the quota, then the administrator uses a lottery to determine the winners among those bidding \( p \). 6) The administrator may, at any stage, declare a conditional solution to be final on the basis of a pre-established convention or stopping rule.

4.3 Theory

The outcome of the slot exchange auction is that each airline is endowed with the ownership of a collection of slot options. Typically the auction procedure will not result in a perfect equilibrium: some airlines will wish to acquire several slots, some to dispose of several. And, as the six-month period of vested rights elapses, the desire to acquire more slots or to dispose of more may develop. Therefore a NASDAQ "open book" slot exchange (of precisely the same type as recommended in the Polinomics report) is maintained continuously until the expiration of the six-month period. An airline is free at any time to express its willingness to offer
slots for sale at stated prices or to bid to purchase slots at stated prices at each of
the 42 trading posts. The global slot exchange is operated by the administrator
who announces every two weeks—to maintain the periodicity of schedule changes—
the accumulated supply and demand curves of each trading post, the trading post
prices determined by the intersection of supply and demand, and the trades which
take place. The bids are prepared in precisely the same form as the auctions,
except that offers to sell are made in addition to offers to buy. The identities of
the bidders are not revealed until the exchanges and prices are determined. Each
trading post will have a total demand curve D and total supply curve S which may
be in any one of four essentially different qualitative forms pictured in Figure 4.6.
In case (d) the offers to sell are all at prices above the offers to buy, so no
exchanges take place. Otherwise, in cases (a), (b) and (c), q* slots are bought and
sold at some trading post price p* which lies between p_D* and p_S* (e.g.,
half way between). In these cases all sellers who announce a price higher than p*
sell nothing. All buyers who quote a lower price buy nothing. If there is excess
demand at p* (case (a) with say p_D* = p_S*) or excess supply at p* (case (b) with
say p_D* = p_S* = p_D) then those who bid p* are rationed by lottery.

It must be noted that the Slot Exchange Auction and continuous slot exchange
which operates afterwards, embody precisely the same economic mechanism: the
law of supply and demand determines the equilibrium solutions in both cases with
the price and number of slots accorded or exchanged determined simultaneously.
Thus the same arguments sustain the relevance of both mechanisms. In fact there
is little controversy over the open-book NASDAQ type slot exchange: this is a well
practiced form of market.

The Slot Exchange Auction is relevant for the same reason. If one assumes
that each airline has well determined values on the totality of schedules that it
FIGURE 4.6 ALL POSSIBLE FORMS OF SLOT SUPPLY-DEMAND RELATIONSHIPS

may fly; that is, that between any two it can state its preference given prices for each slot, then the 42 trading posts which make up the slot auction must have a competitive equilibrium solution with an optimum allocation of slots. The airlines are, of course, players on the slot exchange: they will adopt strategies which attempt to optimize their individual returns. The strategies consist of distributing their investments over the diversity of slots they seek. Are they induced by the system itself or the mechanism, to misrepresent, to attempt to "corner" markets? The theory--based upon recent as yet unpublished results--answers: no. The equilibrium solution is Pareto optimal or efficient; that is, there is no solution under which all airlines are better off according to their own evaluations; it is a Nash equilibrium or noncooperative equilibrium; that is, no airline acting alone
could change its bid and thereby improve its position according to its own evaluations; and, finally, it is a strong Nash or strong noncooperative equilibrium; that is, no coalition of airlines acting in concert could change their bids together and thereby arrange to improve each of their positions according to their own evaluations. Thus, at equilibrium, no airline and no set of airlines are able to do better than what is offered: there is no inducement to posture.

The theory is established under very general conditions which explicitly includes the interdependence of the 42 (or whatever number) trading post markets. It is assumed that the marginal values of extra slots at any one trading post is nonincreasing: the bids of Figure 4.2 must be nonincreasing, the corresponding bids of sellers in the post-auction exchange must be nondecreasing. This is natural. Strictly speaking, the theory applies to commodities which are divisible, and slots are not. However, the uniformity of the slots of a trading post and the fact that there are sufficient numbers (at least 36) available, permits the use of the results. Indeed, if the individual preferences or utilities were additive, then the indivisibility of the slots makes no difference: the theory holds absolutely.

The slot auction procedure which we recommend is a "tatonnement" process: the idea is to "try" solutions and have participants react to them by bidding higher for slots desired but not obtained or to drop from the bidding for slots too dear. There is no mathematical guarantee that this process will converge to the sought for equilibrium. Experimental work of recent years, however, shows that bidding and auctioning approaches converge to competitive equilibria remarkably fast. The nature of the experiments is roughly this. Subjects are given supply and demand curves in the form of "incentive structures." They are instructed to keep these secret. The knowledge of the utilities of all subjects permits the experimenter to compute the equilibrium prices $p^*$ and quantities $q^*$. The subjects are then asked
to trade according to some protocol, such as an open-book NASDAQ type of "double auction," with bids to buy and sell being made simultaneously. Trades then take place. The experiment is repeated with the same players and then with different sets of players. Invariably solutions very close to competitive equilibrium are found: repetitions only improve this finding. These experiments have led Vernon Smith to remark: "There are no experimental results more important or more significant than that the information specifications of traditional competitive price theory are grossly overstated. The experimental facts are that no double auction trader needs to know anything about the valuation conditions of other traders, or have any understanding or knowledge of market supply and demand conditions, or have any trading experience (although experience may speed convergence)..."²⁶ (p. 57). In our approach, experience through repetition, information concerning the valuations of others, and revelation of the total demand pressures at each trading post through repeated auctions, can only help the convergence. On the other hand the complex interdependence of the trading posts—which is at the heart of the problem of the efficient allocation of slots—appears to be a factor which has not previously been studied experimentally. The experimental evaluation of the Slot Exchange approach (Volume II) strongly suggests that convergence can be obtained for all practical purposes within the time frame of the one auction. The practical considerations for implementing this approach are outlined in the next section.

4.4 Practical Considerations

We propose that the slots acquired at auction be paid for in monthly installments over the six-month period in which the slot options are vested. Thus slot options would be paid for during use: this amounts to an interest-free loan on the part of the authorities who offer the slots at auction. This ensures that the
procedure is fair to small or new airlines: no initial cash outlay is necessary. An economically efficient small airline, having limited capital or cash, can therefore compete with the large, well-established and well-endowed airlines. A small competitor would "lose out" only if unable or unwilling to match bids which it felt it could not cover from returns from flights: this is the very nature of efficient competition.

The Slot Exchange Auction and the succeeding continuous slot exchange must be completely computerized. Each certified participant should have a terminal tied directly to a central processor controlled by the slot auction administrator. Each airline would enter its bids directly instead of using forms such as presented in Figure 4.1. The processor would automatically provide each airline with their conditional allocations, the trading post prices and the total demands. Speed of processing is of the essence since the entire procedure rests upon the building of an information base to promote convergence. Each airline would have the comfort and convenience of bringing its in-house expertise to bear on the problem of preparing subsequent bids. The proposed system would be conceptually easy to design and implement, and relatively inexpensive to maintain. Precisely the same system would be used for the post-auction slot exchange.

The airlines have frequently voiced their wish for quotas to be increased. The Slot Exchange Auction admits, at least theoretically, the possibility for the airport authority to make quotas flexible, by making more available at a price. In effect each Slot Exchange Auction has many buyers (the airlines) and one supplier (the airport authority). The buyers' demand curve $D$ (solid line) and the seller's supply curve $S$ (jagged line) are as in Figure 4.7. The airport could well change its reflected-L supply curve into a different supply curve $S'$ (dashed line). No change in the auction rules is required. $S'$ simply expresses the airport authorities' wish to
impose a minimum slot price; then to increase this minimum as the total number of acquired slots becomes larger. It could be that at some airport the minimum price should remain at $0 up to 30 slots; then increase by a constant to 38; then by a goodly amount up to 42, above which no more would be allowed. The airport authority has the flexibility of imposing any "supply curve" it wishes subject to the physical limitations of the airport and its surroundings without changing the basic allocation mechanism. This points to the opportunity to adopt a supply strategy, somewhat relaxing the absolutely fixed nature of the quota limitations by introducing extra slots at higher prices. It may be that a rationale exists for setting these which truly reflects steeply increasing delay costs as the total number of slots per hour go up.

![Figure 4.7 Other Forms of Supply Curve](image)
5. MATHEMATICAL PROGRAMMING APPROACHES TO SLOT ALLOCATION

In this section we develop mathematical programming approaches to slot allocation, retaining however the use of an auction for obtaining information on values. The development of the approaches have been motivated by two considerations: 1) to adequately consider the interdependence of slot values to an airline, and 2) to explicitly consider "fairness" in slot allocation.

5.1 Slot Interdependence

One purpose in having airline scheduling committees to date has been for effective coordination in the allocation of slots. Airlines have to consider changes to an entire flight schedule when the lose certain slots at a congested airport. Each flight schedule is itself part of a larger cycle of operations involving crew scheduling and aircraft maintenance at specific centers. Developing and maintaining these schedules is a complex and difficult art.

It is recognized that airlines may have no value for slots independent of other slots. Only a combination of slots required to maintain a flight or a sequence of flights has some value. For example, a landing slot without a corresponding take-off slot has obvious deficiencies. The two together, however, may have some value to an airline. Thus a major motivation in constructing a scheme for allocation is that an airline be allocated all of the requested slots in a sequence defined by the airline or none at all.

5.2 Fairness in Allocation of Slots

A "fair" allocation of slots could be interpreted in a number of different ways. An airline operating a small aircraft carrying 80 passengers may consider it unfair if it has to pay the same price for a slot as an airline operating a larger
aircraft. Similarly, a short-haul carrier might consider it unfair if it pays the same price as a long-haul carrier. On the other hand, having differential prices for slots in the same period by type of aircraft or by arrival/destination city for each landing/take-off may be deemed unfair. In developing the mathematical programming approach, a careful specification of the principles of fairness being employed is made. It should be made clear at this point that the whole approach is a natural outgrowth of the "fairness principles" stated below as axioms. Although there may be disagreement over the choice of these particular axioms, we feel that the axioms we have chosen can be defended as reasonable and interesting.

5.3 The Axioms of Fairness

1. Any player can define an object, \( i \), made up of integral quantities of resources, \( K \), and state a bid price, \( \phi_i \), representing a measure of his willingness to pay for the object. The player can refuse any allocation of a fractional object when such an allocation is made to the player by the agency (the integral Leontief commodity axiom).

2. No player has to pay a price greater than the bid price for the object.

3. Every player in obtaining resources for an object is required to pay a fixed price, \( p_k \), for every unit of resource \( k \) (the nondiscriminatory pricing axiom).

4. The price of a unit resource allocated by the agency is set such that there is a non-negative excess demand at the price and yet there exists a feasible allocation (the passive agency axiom).

5. If the bid price for a particular object, \( i \), defined by a vector, \( M_i \), is \( 0 \) and the resource price defined by passive agency axiom is \( P \), then the rules of allocation are:

   a. All players with object, \( i \), such the \( \phi_i > PM_i \) are awarded the object.**

---

*Precisely, an object is an airline identification and a vector of integral quantities of resources (slots) and the object space is the Cartesian product of all airlines, identified by integers, with the resources space, \( K \).

**The vector notation is used: \( P' \) is the transpose of the vector \( P \), of all prices.
b. All players with object, i, such that \( \phi_i < P'M_i \) are denied the object.

c. All players with object, i, such that \( \phi_i = P'M_i \) may or may not be awarded the object.

In the context of the airlines, each player is an airline; the objects would be particular flights of that airline; the resources are slots at a particular time period in an airport; \( M_i \) is the vector of required slots to operate the flight, i; and the vector, \( P \), defines the slot prices.

The axioms of fairness can be mathematically stated as follows:

Let:

\[
\begin{align*}
\phi_i & \quad \text{a measure of the willingness to pay of bidding airline for object i} \\
K & \quad \text{index of time period at a particular airport} \\
M_{ki} & \quad \text{number of slots required by object i in time period k} \\
S_k & \quad \text{number of slots available at k} \\
P_k & \quad \text{price of a slot at k} \\
d_i(P) & \quad \text{demand for object i at price P} \\
y_i & \quad \text{1 if an allocation of object i is made to the bidding airline} \\
q_i & \quad \text{Max} \{ 0, \phi_i - P'M_i \}.
\end{align*}
\]

By the allocation axiom

\[
\begin{align*}
\phi_i > P'M_i \Rightarrow y_i = 1 & \quad (5.1) \\
\phi_i < P'M_i \Rightarrow y_i = 0 & \quad (5.2) \\
\phi_i = P'M_i \Rightarrow y_i = 0 \text{ or } 1 & \quad (5.3)
\end{align*}
\]

Introducing \( q_i \) we can rewrite (5.1), (5.2) and (5.3)
\[ y_i(P_i - P^*M_i - q_i) = 0 \]  
\[ q_i(1 - y_i) = 0 \]

By the definitions of \( d_i \), the following conditions also hold:

\[ d_i(P)(d_i - P^*M_i - q_i) = 0 \]  
\[ q_i(1 - d_i(P)) = 0 \]

Since \( d_i(P) \) is always a function of the same argument, we will use notation "\( d_i \)" for short. The demands and the allocations are further related by the following:

\[ y_i = 1 \Rightarrow d_i = 1 \]
\[ d_i = 0 \Rightarrow y_i = 0 \]

### 5.4 The Relationship Between the Fairness Axioms and Certain Linear Programs

The passive agency axiom, together with the assumption that all \( p_i \geq 0 \), requires:

\[ \sum_i m_{ki}d_i < s_k \Rightarrow p_k \leq 0 \]  
\[ \sum_i m_{ki}d_i > s_k \Rightarrow p_k > 0 \]

If (5.9) is a strict inequality, then there exists some \( i \) where \( q_i = 0 \) and \( d_i > 0 \). Let \( I = \{ i | q_i = 0 \text{ and } d_i > 0 \} \). Since \( q_i = 0 \) implies indifference, any feasible reduction in the magnitude of \( d_i \) for \( i \in I \), satisfies the definition of demand and, thus, if vector \( D(P) \) can be found such that:

\[ M \cdot D = S \]
and

\[ M \cdot Y \leq S, \quad (5.10) \]

then the price vector \( P \) and the allocation \( Y \) will satisfy all the axioms of fairness.

Rewriting, we have

\[ p_k(s_k - \sum_i m_{ki}d_i) = 0. \quad (5.11) \]

Conditions (5.4A), (5.5A) and (5.10) are the complementary slackness conditions to the following problems:

**Primal (P1)**

\[
\begin{align*}
\text{Max } z &= \sum_i \phi_i d_i \\
\text{subject to } & \sum_i m_{ki}d_i \leq s_k \\
& d_i \leq 0 \\
& d_i \geq 0
\end{align*}
\]

**Dual (P2)**

\[
\begin{align*}
\text{Min } v &= \sum_k p_k s_k + \sum_i q_i \\
q_i &\leq \sum_k m_{ki}p_k \geq \phi_i \\
p_k \geq 0 \\
q_i \geq 0
\end{align*}
\]
If a feasible solution exists, then the set of price vectors obtained in the dual problem are the only price vectors which satisfy all of the axioms of fairness.* Let \( p^* \) be such a price vector, and \( d^* \) be the corresponding primal solution vector.

Let

\[
y^*_i = 1 \text{ if } d^*_i = 1 \\
y^*_i = 0 \text{ if } d^*_i < 1
\]

The following results hold:

1. \( Y^* \) is feasible.
2. \( 0 < d^*_i < 1 \implies q_i = 0 \).
3. \( Y^* \) and \( P^* \) satisfy all the fairness axioms. (Proof is constructive.)
4. The number of fractional elements in \( D^* \) is at most \(|K|\).

Having established a mathematical procedure to determine \( Y^* \) and \( P^* \), we now turn our attention towards defining auction procedures. The first such procedure we discuss is a single-bid auction procedure.

### 5.5 Auction Procedures Based on Linear Programs

Having noticed the relationship between fair allocation and linear programming, we propose a practical procedure whereby the airlines could bid for flights, and obtain slot price information and slot allocations from the solution of the LP problems, \( P1 \) and \( P2 \). This approach differs from others in that the bids are expressed as willingness to pay for "objects", i.e., bundles of slots, which relate to airline scheduling requirements rather than bids for individual slots only.

The airlines submit bids, \( b_i \), for a set of objects, \( I \). The object \( i \in I \) is translated into slot requirements by a vector, \( M_i \), which is specified by the airline defining \( i \).

*Proof by analysis of complementary slackness condition on primal or dual problems.
The object, \( i \), could be
- A set of cycles
- A single cycle
- A pair of slots
- A slot.

A slot price is then determined using the previously specified linear programs \( P_1 \) and \( P_2 \), given in Section 5.4 above.

The slot prices are the dual prices on the corresponding slot constraints:

\[
\sum_{i} m_{k} \leq s_{i} \quad (5.12)
\]

An allocation of slots is made to all "winning bids" where \( d^* = 1 \). If \( d^* < 1 \), then no slot allocation is made for such \( i \).

Naturally, the outcome of a single-bid auction with associated LP solution may not prove completely acceptable to all airlines involved. This is because of errors in bidding (either \( b_i \) or \( M_i \) may be erroneous) or because some aspects of the LP solution may surprise the airlines. A method of dealing with this difficulty is to explicitly include "sliding" — that is, each airline can bid for a number of alternative objects corresponding to different flight schedules for the same aircraft. In so doing, the airline further specifies that at most one alternative object from the "sliding" subset should be awarded. And, if there is a choice of objects in sliding, the one \( (i) \) with the highest surplus, \( v_i \), should be awarded.

5.6 Single-Bid Scheme with Slides and Price Adjustment

Every airline makes a bid for all possible slides for a given aircraft. We then define the following problems:

Let \( \mathcal{S}(i) \) define all possible slides for \( i \).
(P3) \[ \text{Max } Z_p = \rho D \]
\[ M \cdot D \leq S \]
\[ \sum_{i \in J(i)} d_i \leq 1 \]

(P4) \[ \text{Min } Z_p = P^*S + \sum_i v_i \]
\[ P^*M_1 + V_{J(i)} \geq \rho \]
\[ V_{j(i)} \geq 0 \]
\[ P \geq 0 \]

In relation to the previous problems, P1 and P2, certain price adjustments may be required to ensure the satisfaction of the fairness axioms. A restricted problem is solved first to find out if price adjustments are in fact required. For example, if two alternative flights of one aircraft have the same surplus, say, $50, then the solutions of P3 and P4 may award half of each flight, which is not satisfactory to the airline. In order to take care of this contingency we modify the procedure as follows:

1. Solve the primal problem (P3). Partition \( I \) into three subsets \( R, A \) and \( U \) where

   a. \( i \in R : V_{J(i)} = 0 \)

   b. \( i \in A : V_{J(i)} > 0 \) and
   \[ \exists i \in J(i) : d_i = 1 \]

   c. \( i \in U : V_{J(i)} > 0 \) and
   \[ \exists i \in J(i) : 0 < d_i < 1 \]
2. Solve a restricted problem:

(P5) \( \max Z_{RP} = \sum_{l \in A} W_l \)

a. \( M_l W_l \leq S' \)

b. \( \sum_{l \in J(l)} W_l \leq 1 \)

c. \( W_l = 0, 1 \)

d. \( S' = S - \sum_{i \in A} M_i d_i \)

If \( Z_{RP} = |U| \) then allocation is feasible and \( P \) satisfies the passive agency axiom.

If in the solution of the restricted problem (P5), \( W_1^* = 1 \), then \( y_1^* = 1 \). And if \( d_1^* = 1 \), then \( y_1^* = 1 \). All other \( y_i^* \) are zero. \( Y^* \) and \( P^* \) satisfy the fairness axioms.

If \( Z_{RP} < |U| \), the allocation axiom, 5a, is violated; the prices must accordingly be adjusted upward until the surplus of one flight is driven to zero. This reduces \( |U| \) or \( |A| \) by at least one. The following procedure achieves the desired price adjustment.

Let

\[
\mu_i = \frac{\beta_i}{\sum_{i \in K} m_{ik} \cdot p_k}
\]

\( u^* = \min \{ \mu_i : i \in U \cup A \} \)

\( I^* = \{ i : u^* = \mu_i \} \)

\( U^* = U - I^* \)

\( A^* = A - I^* \)
\[ P_k^* = P_k + \mu^* P_k \quad V_k : m_{ik} > 0 \]

Following price adjustment, we solve the restricted problem again, but using \( U^* \), \( A^* \) in place of \( U, A \). Since \( U, A \) are finite sets and the procedure reduces its cardinality by at least one each iteration, this procedure converges in a finite number of iterations.

5.7 A Numerical Example

We use the same example with five airlines which was given in Section 3.12. The prices obtained there from a one-time auction were $157, $145 and $162 respectively for the 0900, 1000 and 1100 markets. This solution is inefficient and not at the competitive equilibrium. \( D \) and \( E \) cannot use their slots since they do not possess the corresponding slots necessary to complete the flights. The 0900 and 1000 markets are saturated, but the 1100 market accommodates only two flights, not the quota of four. The total "value" of the five realized flights is $1,574. The total paid for the slots is $1,832.

Now suppose that the airlines had entered a different auction in which they were asked to specify a single bid and the slot requirements for each flight. We may reasonably assume, for purposes of this example, that the bids were equal to the values shown in column three of the example. By solving the primal and dual linear programs, \( P1 \) and \( P2 \) (Section 5.4), for this problem we obtain the following results. The prices are $151 at 0900, $155 at 1000 and $150 at 1100. The flights for which slots are awarded are: \( A2, B1, C1, C2, D2 \) and \( E1 \). There are now four slots used each hour so that the quota is saturated. The total value of the six realized flights is $1874, which exceeds the total slot payments by $50. An
efficient, competitive equilibrium has been obtained by the linear programming approach for this small example.
6. EXTENSIONS, SUGGESTIONS AND RECOMMENDATIONS

The method of Section 4--the Slot Exchange Auction--has been subjected to limited testing (see Volume II) and has been shown to have informational feasibility. The test did not, however, permit the demonstration of convergence to an equilibrium since the time available for rebidding was inadequate. It is reasonable to expect, based on related work by V. L. Smith, that convergence will be attained after a finite number of rounds of bidding, but this needs to be tested carefully. Furthermore, there may be need of a stopping rule to prevent prices from "cycling," as discussed in Section 4.4.

The method of Section 5--the mathematical programming approach--has not been tested at all to date. While the theory guarantees a solution, the acceptability of that solution, its slot prices and slot allocations, to the airlines is not guaranteed. The mechanics of the method--LP, dual prices, fractional awards--also need to be developed and tested before it can be considered a practical alternative to the Slot Exchange Auction.

Any slot auction method will tend to generate revenues for the airport authority, and these revenues may be quite substantial. While the Department of Justice recommends development of a market-based system for allocating slots, they also warned that:

"The market mechanism chosen should result in the least added costs to carriers so that fares do not rise unduly;"

"The objective of the system should not be to generate as much revenue as possible, but rather, only to require the minimum amount of expenditure necessary to assure that slots go to the carriers that value them most;..."
In theoretical economic terms the use of such revenues to alleviate airport congestion is clearly indicated. As capacity expands the supply of slots at peak hours increases and slot prices, therefore also revenues, fall. Unfortunately this theory is not necessarily implementable, nor even entirely desirable, given the nature of the airport itself and the relationship of the airport to the surrounding community. The externalities to the runway congestion problem—air pollution, noise, landside traffic congestion and so on—dictate a different approach at least for some congested airports (notably Washington National). If the slot revenues collected at an airport cannot be used to expand runway capacity or improve the efficiency of operations at that airport, there is a necessity of deciding how those revenues will, in fact, be employed. The ability of a governmental body such as the FAA to charge fees and collect revenues from the air carriers hinges on their intended usage. In the case of the expected slot revenues at today's congested airports, this is an important issue.

A suggestion has been made that the slot auction be conducted with non-monetary points. Each air carrier would receive an initial allocation of points or entitlements which it would then use in bidding for actual slots. The initial point budget of each carrier would limit the bids of that carrier. A method of determining the initial budget equitably has to be found. One possibility is to use something like the first stage of the FAA Administrative Allocation to provide each air carrier with its point budget for each airport, leaving the carrier to decide how these points are to be allocated across the hours of the day in a subsequent bidding process. While this is an attractive possibility insofar as it avoids the problems associated with money transactions, a careful exploration of the economic properties of the entire process of allocating slots is called for. Theoretical work by Thomas Palfrey indicates that multi-object auctions with bidding
constraints may fail to have Nash equilibria; and that when a Nash equilibrium does exist it can realistically only be achieved if the bidders collude.

It is hard to make specific recommendations about the best form of market organization for slot allocation at this point; so many problems remain to be solved. If a true market for slots is to be established involving slot pricing as a particular feature, the Slot Exchange is our recommended candidate. As mentioned, a necessary condition for the successful operation of a slot market over the years is that a good policy for the employment of the slot revenues be developed. Once this policy exists the convergence of the Slot Exchange to an equitable and efficient solution needs further testing. We would recommend proceeding in two stages: 1) further laboratory testing with adequate time allowed for the bidding to move towards an equilibrium; 2) a full-scale test over one or two years in actual operations at one airport. The latter could be used as a proving ground for the slot auction; assuming that it is successful according to the measures established for testing the method, it becomes the first phase of a transition from the era of scheduling committees to the era of slot auctions. Problems with the auction method would be worked out before other airports were brought into the market.

On the other hand, failing a satisfactory resolution of the question of how the slot revenues will be used, there is less certainty to our recommendations. The nonmonetary slot auction—bidding with points—is probably worth considering for further examination and testing, but we are in no position to recommend it for implementation at this time. The idea must be subjected to both theoretical and experimental scrutiny starting with a thorough theoretical examination of the nature of the expected outcomes of a nonmonetary auction. This requires a reassessment of the antitrust exemption for the Airline Scheduling Committees. A nonmonetary alternative is being researched.
The FAA Administrative Allocation is, in our opinion, not a viable alternative. In the recent test (see Volume !!), it was found that the simultaneous submission of numerous plans revealing the airlines' willingness to "slide" was an impossible requirement. Furthermore, serious objections were raised regarding the point system for creating the initial allocation of slots to airlines without hourly assignments. The implementation of the FAA Administrative Allocation in the form which has been presented to date would lead to constraints on the efficiency of air service and inequities due to the restriction of access to runways at quota airports.
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