EFFECTS OF CAVITATION ON UNDERWATER SHOCK LOADING. PART 2. PLAN--ETC(U).

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Effects of Cavitation on Underwater Shock Loading - Plane Problem, Part 2

by

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**ABSTRACT:**

Addition of a structural code to the previously developed finite element fluid code is described. Preliminary conclusions concerning conditions needed to induce cavitation and its effects on structural response are reported.
EFFECTS OF CAVITATION ON UNDERWATER SHOCK LOADING -
PLANE PROBLEM, Part 2

1. Introduction

Development of a finite element fluid code for solving plane problems involving shock-induced cavitation was described in Ref. 1. The program uses displacement potential as the dependent variable as described in Ref. 2. A structural program (Ref. 3) providing a plane model for a ring-stiffened cylindrical shell has recently been completed. In the following sections, coupling of the structural and fluid programs is described and results thus far are summarized.

2. Structural Program

2.1 Program Features. The structural program (STRUK5) was developed by Jack T. Waller, a thesis student of mine, and is described in Ref. 3. A shell structure having specified radius, area per unit axial length, radius of gyration, material density, and modulus of elasticity is modeled. Trigonometric series are used to represent the radial and tangential components of shell displacement. Taking account of symmetry, the half-circumference of the shell is divided into N equal segments. The fluid mesh is matched with the structural model, providing an interface node at each point of division. Using fluid pressures calculated at these nodes by the fluid program, the coefficients for trigonometric series representation of the interface pressure are found by STRUK5.
For each trigonometric order $m$ there is a term $c_m \cos m\theta$ for interface pressure, a term $a_m \cos m\theta$ for radial displacement and a term $b_m \sin m\theta$ for tangential displacement. Because the structure is rotationally symmetric, there is no coupling between coefficients with different orders $m$. For a given $m$, differential equations governing $a_m$ and $b_m$ are coupled.

Time integration is performed using an explicit central difference algorithm. A chosen time step, which may be suitable with a particular fluid mesh, may result in instability for structural modes with large $m$. Accordingly the program limits the number of structural modes retained. For this purpose the extensional modes are considered separately from the flexural modes.

2.2 Nonstructural Mass. The shell structure of a submarine typically has a mass no greater than 20 percent of the displaced water. The remaining mass is connected to the shell through relatively flexible structure and/or isolation mountings. Although the early-time response to shock loading is little affected by this additional internal mass, its omission from the structural model will be significant at later times. Since cavitation effects on shock response may occur at late times, it was deemed important to include the additional mass.

A rigid body displacement of the shell can be described by trigonometric terms of order $m=1$. For modeling purposes the spring forces between shell and contents are distributed in corresponding fashion. Thus there is a set of radial
forces varying as \( \cos \theta \) and a set of tangential forces varying
as \( \sin \theta \). This produces equations of motion in which the
displacement of the nonstructural mass couples only with
the shell displacements described by \( a_1 \) and \( b_1 \).

Modification of the equations of motion to include this
nonstructural mass ("contents" mass) was accomplished in
the following way. The notation of Waller (Ref. 3) is used
with the following additions.

\[ u_c = \text{contents displacement (in } \theta = 180^\circ \text{ direction)} \]
\[ m_c = \text{contents mass} \]
\[ k_c = \text{common stiffness of radial and tangential spring sets} \]

The increments to elastic potential energy and kinetic energy,
\( \Delta U \) and \( \Delta T \), are given by

\[ \Delta U = \frac{1}{2} k_c \left[ (a_1 - u_c)^2 + (b_1 - u_c)^2 \right] \tag{1} \]
\[ \Delta T = \frac{1}{2} m_c \dot{u}_c^2 \tag{2} \]

When these additions are included the motion equations for
\( a_1 \) and \( b_1 \) each contain both \( \dot{a}_1 \) and \( \dot{b}_1 \). Algebraic manipulation
allows the equations to be written in a form suitable for
explicit time integration as

\[ (1+2z)\dot{a}_1 + \frac{c^2}{R^2}(1+z)(a_1 + b_1) + \frac{k_c}{\pi \rho A R}[(1+z)a_1 - zb_1 - (1+2z)u_c] = (1+z)\frac{c_1}{\rho A} \tag{3} \]

\[ (1+2z)\dot{b}_1 + \frac{c^2}{R^2}(1+z)(a_1 + b_1) + \frac{k_c}{\pi \rho A R}[-za_1 + (1+z)b_1 + (1+2z)u_c] = -\frac{z c_1}{\rho A} \tag{4} \]

The additional equation of motion for the contents mass is
\[ m_c \ddot{u}_c + k_c (2u_c - a_1 + b_1) = 0 \]  

(5)

The contents mass \( m_c \) is determined within the program from the stipulation that shell plus contents be neutrally buoyant. Instead of specifying \( k_c \) directly, the fixed-base frequency \( f_c \) of the contents is specified and \( k_c \) is found from

\[ k_c = f_c^2/(8\pi^2 m_c) \]  

(6)

2.3 Curvature Correction. When the just-described process for including nonstructural mass in the model was first implemented it was discovered that effects of shell rotatory inertia appeared in the equation for rigid body motion of the shell (obtained by putting \( b_1 = -a_1 \) in the \( m=1 \) equations). Since this behavior is clearly incorrect, the equations were carefully re-examined to discover the cause. Two related errors were discovered. Both appear first in my own notes and are clearly my responsibility, not Waller's.

The first mistake is that the expression used for rotational velocity \( \omega \) of the shell is incomplete. The correct form is

\[ \omega = \frac{1}{R} \left( \frac{\partial \dot{w}}{\partial \theta} - \dot{\nu} \right) \]  

(7)

The second term is omitted in Ref. 3. The other mistake is in the expression for shell bending curvature \( \kappa \). The correct form is

\[ \kappa = \frac{1}{R^2} \left( \frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta} \right) \]  

(8)
In Ref. 3 the second term in parentheses is +w. The latter form is correct only if the ring deformation is inextensional.

The reason that these errors were not discovered during initial testing of Waller's program is that they have negligible effects on results. Nevertheless, the corresponding changes have been made in the structural program. The new version of the program, which also includes the nonstructural mass as detailed in Section 2.2, is called STRUK5.

Revisions to the equations of Ref. 3 resulting from the corrections to rotational velocity (Eq. 7) and curvature (Eq. 8) are given in Appendix A.

3. Fluid Program

Principal features of the fluid program were described in Ref. 1. Additional details are given in Ref. 4, including discussion of the staggered solution procedure which is used to couple fluid and structure for a time marching solution.

4. Results

Two kinds of results have been obtained from program tests conducted to date. The first kind give no new information concerning cavitation effects, but simply demonstrate that the coupled response of the structure is in agreement with known analytic results. These data, which afford supporting evidence concerning the validity of the modeling, are summarized in Appendix B.
Results of the second kind concern effects of cavitation upon the severity of shock induced structural loads. For the cases studied to date results are summarized in Ref. 4 and quoted below.

Examination of results from a large number of cases studied up to this time reveals that the ratio of shock pressure increment to hydrostatic pressure \( \frac{p_s}{p_h} \) must exceed three times the ratio of decay length to structure radius \( \frac{L}{R} \) in order to induce cavitation. Cavitation begins about one decay length in front of the structure at about the time when the initial scattered wavefront reaches this location.

Extreme values of bending moments and circumferential compressive force in the ring have been found to occur before the shock front has completely traversed the structure. Since cavitation has not yet begun during this interval, the extreme structural loads are not affected by cavitation. There is, to be sure, a shock pulse generated by cavity closure. The initial magnitude of this pulse is of the same order as hydrostatic pressure. After travelling to the structure the pulse is significantly attenuated and the resulting loads are not significant.

5. Conclusions

Cases investigated thus far have shown no instance in which the occurrence of cavitation increases the maximum stresses in the structure. A wider range of encounter parameters needs to be investigated before it is possible to assert with confidence that cavitation induced by structural
response to underwater shock will not increase the likelihood of structural damage or collapse.

Considering the related phenomenon of bulk cavitation, evidence to date does not rule out the possibility that it may intensify shock-induced structural loading. The finite element method should be used to investigate effects of bulk cavitation on structural loads.
APPENDIX A

Corrections for Equations of Reference 3

Corrected forms for equations of Ref. 3 based on expressions for \( \omega \) and \( \kappa \) given in Eqs. 7 and 8 of this report appear below. Equation numbers are those of Ref. 3.

\[
U_B = \frac{1}{2} \int_0^{2\pi} \frac{1}{R^3} \left( \frac{\partial^2 w}{\partial \theta^2} - \frac{\partial^2 v}{\partial \theta^2} \right)^2 d\theta \tag{7}
\]

\[
U_T = \frac{1}{2} \int_0^{2\pi} \frac{1}{R^3} \left( \frac{\partial^2 w}{\partial \theta^2} - \frac{\partial^2 v}{\partial \theta^2} \right)^2 d\theta + \frac{1}{2} \int_0^{2\pi} \frac{AE}{R^3} (\partial^2 v + w)^2 d\theta \tag{9}
\]

\[
\delta U = \frac{\pi D}{R^3} \sum (n a_n + b_n) (n \delta a_n + \delta b_n) + \frac{\pi E A}{R^3} \sum (a_n + n b_n) (\delta a_n + n \delta b_n) \tag{10}
\]

\[
dT = \frac{1}{2} \rho (A R d\theta) (\dot{\omega}^2 + \dot{\psi}^2) + \frac{1}{2} \rho I R d\theta \left( \frac{\partial^2 w}{\partial \theta^2} - \dot{\psi}^2 \right) \tag{12}
\]

\[
\delta T = \pi \rho A \sum (\dot{a}_n \delta a_n + \dot{b}_n \delta b_n) + \frac{\pi E A}{R} \sum (\delta a_n + b_n) (n \delta a_n + \delta b_n) \tag{12}
\]

\[
(1 + n^2 z) \dddot{a}_n + n z \dddot{b}_n + (\frac{\partial}{\partial r} (1 + n^2 z) a_n + (\frac{\partial}{\partial r} (n^2 + 1 + n^2 z) b_n = \frac{c_n}{n^2} \tag{14}
\]

\[
nz \ddot{a}_n + (1 + z) \ddot{b}_n + (\frac{\partial}{\partial r} n(1 + n^2 z) a_n + (\frac{\partial}{\partial r} n^2 + 1 + z) b_n = 0 \tag{14}
\]

\[
n^2 + 1 + (n^2 - 1) \frac{2z}{n^2} \dddot{a}_n(F) + (\frac{\partial}{\partial r} (n^2 - 1)^2 z a_n(F) = \frac{c_n}{R^3} \tag{17}
\]

\[
b_n(F) = - \frac{a_n(F)}{n} \tag{17}
\]
Equations 14 each contain both $\tilde{a}_n$ and $\tilde{b}_n$ and are thus unsuitable for use with an explicit time integration algorithm. Some algebraic manipulation produces the following forms which are used in STRUK6.

\[
(1 + z + n^2 z) \tilde{a}_n + \left(\frac{\rho}{R}\right)^2 \left[1 + n^2 z + (n^2 - 1)^2 z\right] a_n + \left(\frac{\rho}{R}\right)^2 (1 + z) n b_n = (1 + z) \frac{c_n}{\rho A}
\]

\[
(1 + z + n^2 z) \tilde{b}_n + \left(\frac{\rho}{R}\right)^2 (1 - z + 2n^2 z) n a_n + \left(\frac{\rho}{R}\right)^2 (1 + n^2 z) n^2 b_n = -n z \frac{c_n}{\rho A}
\]
APPENDIX B

Modal Frequencies of a Submerged Cylinder

It is well known that late time response motions of submerged structures can be accurately forecast by treating the surrounding fluid as incompressible (Ref. 5). This permits calculation of the frequencies for modes involving either rigid body translation or shell flexure. Comparison between such calculated results and observed frequencies of the shock response motion of the finite element model provides a partial check on the accuracy of the modeling and the implementing programs.

Figure 1 shows the rigid body displacement $a_1$ of the shell and the displacement $u_c$ of the contents mass as a function of time in response to a shock with step pressure rise of 4MPa and decay time of 6ms. In view of the rapid shock pulse decay, the motion following the first half cycle may be considered free vibration. The succeeding cycle of motion has a period of 77ms or a frequency of 13.0 Hz. For the shell translation mode the added fluid mass is equal to the displaced mass. Taking this into account, the fixed base frequency $f_c = 10.0$ Hz for the contents mass gives a corrected value of 13.1 Hz in good agreement with the observed result.

Figure 2 shows results for a separate computer solution with $f_c = 5.0$ Hz. In this instance the observed period is 152ms, corresponding to a frequency of 6.6 Hz. This compares
Fig. 1. Shell and contents motion, $f_c = 10$ Hz.

Fig. 2. Shell and contents motion, $f_c = 5$ Hz.
closely with a calculated value of 6.5 Hz.

For the flexural modes corresponding to the trigonometric orders \( n = 2, 3, \) and 4, the corresponding added mass, based on maximum radial displacement amplitude, is \((\text{displaced mass})/n\). Table 1 compares theoretic frequencies with finite element method (FEM) results. It is apparent that agreement is satisfactory.

Table 1. Theoretic and Observed Modal Frequencies

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