PURPOSES AND PROBLEMS OF STRUCTURAL FATIGUE

D.G. FORD

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SUMMARY

This is a qualitative description of the abstraction which has led to the problems of applied probability theory treated in structural fatigue. It also describes the relation of this to the lifting philosophies of safe-life, fail-safe and damage tolerant design. The experimental data needed to apply structural fatigue is also discussed.

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PURPOSE AND PROBLEMS OF STRUCTURAL FATIGUE.

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1. INTRODUCTION

Unified structural fatigue theory is now being programmed at the Aeronautical Research Laboratories for routine prediction of fatigue lives. This theory, abbreviated below to "structural fatigue", originated as an abstract framework for the collation of various processes occurring during fatigue. It is now some 15 - 20 years since development of the general theory began; the ARL work has wider scope and greater generality than standard methods. The earliest roots (unpublished) are related to the combined advent of Griffith-Irwin Fracture Mechanics, Bastenaire Damage Theory, and the application of reliability theory.

Since then, it has been the subject of a moment theory (Ref. 1) which was revised later for probability densities, after which reliability effects (Pefs. 2 & 3) were added, including hijacking. The complete theory (still unpublished) now includes these effects, without moments, formulated for several cracks which interact but are kept apart.

Because of this long gestation, much of the earlier reasoning which produced the mathematical models has receded to the background in more recent work with the result that the relevance of solutions in the complete theory is not always self-evident.

This Memo is meant to restate this earlier, sometimes unconscious, chain of reasoning so that abstract structural fatigue problems may be related to every-day life estimation. For example, because the theory is most useful in the abstract form, it is not generally appreciated that it ultimately relies on experimental fatigue data. Moreover, much of this data, because the theory represents an advance, is by nature different from the results of present day tests. On the other hand, much present data can still be used, especially that relating to crack growth, and it is possible to also include current research results that are not yet being applied.

Supporting details are presumed to be supplied in a way consistent with reasonable engineering judgement. We reject the notion that fatigue life prediction must always be entirely experimental; this disagreement with some other workers is not defended here, nor do we ignore experimental results per se.

2. BASIS OF THEORY

The development of structural fatigue may be traced through Refs. 1, 4, 5 & 3, which do not consider detailed data requirements. In these references, and structural fatigue generally, "theory" does not refer to complete or partial solutions of isolated problems, but to a connected body of knowledge in the way, for example, that engineering bending theory applies to a variety of particular problems.
As in other branches of structural engineering, we focus our attention upon doom and catastrophe— the buckled panel, the broken tie, the onset of flutter. In fatigue, this cataclysm is undoubtedly the final failure; although not desired, it is the focus of the mathematical modelling.

Unlike the other examples, fatigue life is essentially random. Therefore, theory must allow for this and predict life distributions. The natural setting for this is reliability theory, as used for complex assemblies and in less integrated fatigue predictions (Refs. 2 & 5).

Reliability theory in turn implies risk estimation over a practical range of lives. In a particular case, this risk depends on load Kollektiv and crack length. Both of these depend on data and possible ancillary prediction. In addition however, the crack length for these fixed times is a random variable, so that in turn its distribution must be estimated. This follows from the continuity equation for crack length probability, which is a degenerate form of the Fokker-Planck equation for probability densities governed by diffusion and time. When applied to crack length over life this partial differential equation has boundary conditions determined by the onset of cracks: the initial lives.

Like other fatigue phenomena, initial lives are random variables. Thus, detailed consideration of random lives to catastrophic failure has led us to require distributions of initial lives. This reasoning implies a distinction between the times before and after crack initiation, and of course implies that the latter can be defined. A case for this has been argued elsewhere (Refs. 1 & 6); it is based on grounds of physical processes, statistical considerations and ease of computation.

The chain of reasoning above has led via crack growth to the concept of initial lives. Cracking is now subject to intensive investigation; the distribution of initial life follows from the neglected subject of damage accumulation. In structural fatigue, this is supplied by Bastenaire theory (Ref. 7). Experimental data for this is particularly sparse.

3. RELATION TO PHILOSOPHIES

The above refers to a typical random structure which begins its life intact. It stays that way (for engineering purposes) until the onset of a crack which then proceeds for some non-zero time until final failure intervenes. In the current form of structural fatigue, crack growth is supposed to be deterministic so that randomness of final failure is supplied only by initiation and the reliability part of the model.
Obviously, each separate phase is important though various ones may dominate for different situations. The distinctive feature of the various philosophies popular in different eras since World War II, is that a single aspect of the normal sequence of events has been arbitrarily selected for exclusive attention. We will now consider some of these:

3.1 Safe-Life or Classical Comparison

This was the earliest philosophy of lifeing. Its practice is based on Miner-Palmgren damage (Refs. 8 & 9), mis-applied by substituting final failure for initiation. In the context of structural fatigue theory, this amounts to assuming infinite crack growth at initiation, or equivalently an initial crack length comparable with the size of structure. This bypasses the need for predicting crack growth and the assessment of reliability, with corresponding losses in accuracy. Not unnaturally, it has been found that a damage theory serving also for the crack growth stage has performed indifferently.

The data for this are final lives of structures whose fatigue behaviour is deemed to resemble that in question. For this reason one may regard linear damage as a means of comparing the condition of the service structure with the condition of another structure, with known S-N data, under the same strain sequence.

Hence assessments based on damage calculations could also be described as comparisons between two structures. Choosing S-N data amounts to deciding which structure is similar and this choice replaces the detailed crack-following of structural fatigue or fail-safe approaches. The choice of such data is critical, and at least two comparisons should be made for confidence. Ford (Ref. 10) has extended this to a possible method of "vertical comparisons" with two or more levels of detailed stressing.

3.2 Fail-Safe

In 1954, McBrearty (Ref. 11) of Lockheed, suggested that cracks could be allowed provided that they did not extend catastrophically during any one flight. This allowed longer use of structures, avoiding the penalties of safe-life at the expense of more intense inspection. Because initiation was ignored, such inspection however had to proceed from the beginning, thus incurring another type of expense.
3.3 Damage-Tolerance

This is a recent development of the fail-safe philosophy, open to the same objections. It is now assumed that all critical locations contain small cracks whose possible growth is predicted and compared with fail-safe criteria.

These specified cracks are assumed to be present from the start and it is implied that they are fatigue-sharp and in the worst combination.

This standpoint is a volte-face from the safe-life philosophy but equally unrealistic in ignoring a fraction of life that can be more than 90%. The standardised flaws usually exceed the range of crack lengths in which most propagation time is spent and where most current research effort is rightly placed.

Because the flaws are more or less extreme values any statistical considerations for considering the safety of inspection or monitoring are vitiated at considerable cost for the same nominal safety level.

4. EXPERIMENTAL DATA

Before structural fatigue is applied, preliminary estimates of damage or initial life distribution, crack growth, and risk of failure are required. These are subsidiary problems which use experimental data at various levels.

Reliability depends on crack growth and directly on the load Kollektiv. Crack growth also depends on the load Kollektiv and da/dn data. The latter may or may not include retardation effects which could require further data if they are included. Obviously, the input from this area to structural fatigue will be affected by current work on retardation. The most complete work on this to date is that of Pellas, Baudin and Robert (Ref. 12), based on variability of threshold intensity.

If cracking is predicted by fracture mechanics, then one requires a finite initial crack length. The size of this critically affects the cracking time and the level of detail in stress analysis. It also affects the density of initial lives, which is natural since changes here amount to re-defining it. With suitable care however, the effect on cracking time and initial failure compensate one another to produce realistic total lives. Thus the choice of initial crack length is an engineering compromise based on knowledge of threshold intensity, effort available for stress analysis, and the nature of initial failure data.
As with classical lifing, the initial crack sizes determine a hierarchy of engineering formulations for a particular structure. The most accurate ones correspond to small initial cracks attended by greater problems of stress analysis.

If inspection is included, the data above must be augmented by characteristics relating the probability of repair to the size of cracks; in general, this should include fatigue characteristics of the repaired structure. At present, it is imagined that repaired structures have the same cracking properties but different initial lives.

5. CONCLUSIONS

The theory of structural fatigue is based upon the actual sequence of events leading to catastrophic failure by fatigue. Each of these is given equal weight but their estimation depends upon experimental data, some of which is presently lacking.

This data relates to the three stages:

(a) initiation, described by damage theory;

(b) crack growth, based on da/dn data, allowances for retardation and knowledge of threshold sizes; and

(c) the load Kollektiv for failure of the weakened structure. This also affects (a) and (b).

Current investigations of crack retardation have important application here, but more study of crack initiation or cumulative damage is needed urgently.
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APPENDIX

GENERAL APPLICATION OF STRUCTURAL FATIGUE
IN THE FUTURE

As a background to the previous discussion, we shall now outline the envisaged application of structural fatigue to a typical lifting problem. It is now presumed that adequate information is available. Because more results are obtainable from structural fatigue it is to be expected that more input is needed but this will be offset by the fact that the required data is more basic.

As one does now, the first step is to choose a mode of failure either from test results or past experience. In the present context, "mode" refers to a set of interacting control or critical points in the structure, but not to the order in which cracks appear at them.

After this, there would be further engineering judgement involved in the choice of initial crack lengths, damage laws, crack growth data and the inclusion of - and allowance for - various effects deemed relevant. This will depend on the usual norms of cost, available time, and required accuracy. These of course are closely considered at present, but the effectiveness of any choice is vitiated by over-restricted models.

Knowing the possible failure modes and materials data, it would now be possible to prepare input data for fatigue predictions from the known Kollektiv. For this, average damage rates are calculated at each critical point for a range of cracks elsewhere, including those still to appear. The implication of this step is that such data will allow continuous interpolation to a set of damage rates for any combination of crack lengths. Similar computations are necessary for crack growth rates. Such computations would involve the materials data and, consistent with the practical considerations above, would allow for plasticity in damage calculations and for retardation, threshold or corrosion effects on crack growth. If necessary, the damage data would be appropriate to a fretting environment.

The knowledge so gained, together with necessary interpolation rules, would then allow one to set up a formal fatigue problem to be solved for distribution of life, inspection intervals etc.
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