APPROXIMATION FOR SATELLITE DOPPLER FREQUENCY --\text{ETC}(U)

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1-80
The Doppler principle has been used to track satellites since the early 1960s. When the data sampling interval was less than one second, this data has been treated as frequency data in orbit determination programs. This method results in an approximation when the number of cycles counted in an interval divided by the time interval is treated as a frequency in the equations of condition. The result of this truncated series is that it produces station heights that are about 1.4 meters too low. Therefore the data must be treated as range differences, even for sampling intervals as low as
FOREWORD

Since the early 1960s artificial earth satellites have been tracked using the Doppler principle. Until the early 1970s a large part of this data was processed in orbit determination programs as frequency data, a procedure which resulted in a small error due to a truncated time series. Today, the data from only a few stations is processed by this method. This report describes a better way to process this data and gives the differences in results for the two methods.

This work was accomplished in the Satellite Geodesy Branch, Astronautics and Geodesy Division. The report was reviewed by R. J. Anderle, Head, Astronautics and Geodesy Division.

Released by:

ROBERT T. RYLAND, JR., Head
Strategic Systems Department
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INTRODUCTION

Doppler observations of artificial earth satellites have been taken since the early 1960s to determine the satellite's orbit if the positions of the observing sites are known or to determine the position of the observing site if the satellite orbit is known. The Doppler observations are made at the observing stations by the station receiving a continuous wave signal generated by a satellite at a known frequency. The received frequency is beat against a frequency generated by a precise ground-station oscillator. The method of counting these beat cycles differs. Until 1971 most of the data was taken by counting a preset number of cycles and recording the time required for the count. The count was typically done once every 4 seconds, and the number of cycles to count was chosen so that the count interval would last just under 1 second. This type of data is usually labeled as Sampled Doppler. Today only the OPNET stations located in Maine, Minnesota, California, and Hawaii take data using this method, for in 1970-71 the other stations were converted to a more automatic mode of operation. As part of this change, the counting device was modified so that the time for the completion of one count is now identical to the time for the start of the next count. The number of cycles to be counted is set so that the counting interval lasts from 10 to 30 seconds. This type of data is labeled Continuous-Count Integral Doppler (CCID). The Sampled Doppler data has been processed as frequency data in orbit determination programs, although the technique for doing this leads to an error in position due to an approximation when the number of cycles counted in an interval divided by the time interval is treated as a frequency in the equations of condition. This truncated series results in an error that has previously been neglected. The behavior of this error was evaluated by treating this Sampled Doppler data as range difference rather than frequency. This report gives the magnitude of the error involved and a way of avoiding it.

PROCEDURE

Filtered frequency observations and a readily modifiable station position determination program were initially used for this study. Their use provided the most expedient way to obtain the magnitude of the error to determine if it was of enough consequence to pursue and to justify modifying our large-scale orbit determination program, CELEST.* Here we had available the time of the midpoint of the interval over which the Sampled Doppler data was taken, and a corresponding frequency observation that had been computed from the ratio of the Doppler count to the time interval. Needed were the times at the beginning and at the end of the counting interval and a way of converting the frequency value of observation to a range difference observation.

To obtain the length of the count interval

\[ \Delta t = \frac{N_C}{(f_R - f_S) - \Delta f} \]

where \( \Delta t \) = time to count \( N_C \) cycles

\( N_C \) = number of cycles to count

\( f_R \) = station reference frequency

\( f_S \) = transmitted frequency

\( \Delta f \) = frequency data observation value = \( f_R - \frac{N_C}{\Delta t} - f_S \)

from which the begin and end times of the sample interval are obtained. The range difference value may then be computed by

\[ \Delta R = \frac{c}{f_S} \left[ N_C - (f_R - f_S) \Delta t \right] \]

where \( \Delta R \) = range difference observation value

\( c \) = speed of light

This method was sufficient to indicate that there was an error of significant magnitude to justify the modification of our main orbit computation program, CELEST. The best way to do the conversion is by using the raw observational data because one has available the actual time required to count the preset number of cycles and the time the count started. This is the way the conversion will be made in practice and the method used for the later parts of the study.

The interval over which the sample was taken was varied to see if the residuals of fit would vary. It was expected that the residual differences would vary quadratically with the interval change since powers of the reciprocal square of the time interval and higher are ignored in the frequency computation. The interval was varied as the frequency data was converted to range difference. The range difference data was then refit to obtain new residuals. The sample intervals used were the full interval of the original data, one-half the interval, and one-fifth the full interval.

Two days of data from the Navy navigation satellite, 1970-67A, were used for the test. This is a polar satellite with a perigee height of about 970 km and an eccentricity of 0.017. Other spans of data were also used as checks to be sure the discrepancies were not span-dependent.
RESULTS

When changing from frequency to range-difference data the resultant station positions varied by about 1.4 meters in height with small changes in latitude and longitude for a sample interval close to 1 second. When the sample interval was decreased, the station height residual differences decreased approximately as the square of the interval change. This change is seen in the pass-by-pass differences in Table 1, or more readily in Table 2, the combined pass solution with the differences given in three components. These changes are also seen in Figures 1-6 where the first three show the longitude differences as the interval decreases and the second three show the corresponding latitude changes. The longitude difference plots also show on which side of the station the satellite was, and the latitude difference plots show the direction of the pass. These are given to show that there is not a direct correlation between the pass geometry and the station position differences. Table 3 and Figures 7 and 8 give the station navigation differences in the alongtrack and radial frames. Figure 7 radial differences show the bias caused by the difference between processing as frequency and range difference. Table 3 and the corresponding figures (7 and 8) were produced by CELEST. CELEST was modified after the station position differences were shown to be of a significant magnitude.

CONCLUSION

The major resultant differences between the two processing techniques is that processing the data as frequency produces station heights that are about 1.4 meters too low for Doppler count intervals of about 1 second. In producing today's satellite orbits or point positions that are accurate to a few meters, this error is too large to ignore even though there are currently only a few observing stations that record this type of data. In processing data for periods prior to 1971, when most stations were taking frequency data (such as when doing a general geodetic solution), this discrepancy should not be overlooked as an error source.

LIMITATIONS OF THE METHOD

Least-squares solutions for geodetic parameters involve the partial derivatives of the data with respect to the parameters. For range-difference data, partial derivatives for range are usually formed at the beginning and end of the count interval and are differenced to obtain the partial derivative for range difference. For count intervals of 1 second or less, computer word length begins to introduce significant errors in the partial derivatives. An approach taken to avoid this problem will be the subject of a later report.
Table 1. Station Position Residual Differences

<table>
<thead>
<tr>
<th>Station</th>
<th>Day</th>
<th>Hour</th>
<th>Elevation (Deg)</th>
<th>Frequency - Range Difference (m)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Full Sample Interval Interval/2 Interval/5</td>
</tr>
<tr>
<td>321</td>
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<td>1</td>
<td>63</td>
<td>Long 3.55 1.16 0.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lat -0.42 -0.16 -0.10</td>
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<td>Long -0.70</td>
<td>Lat -0.17 -0.06 -0.01</td>
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<tr>
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<td>Long 3.58</td>
<td>Lat -0.06 -0.01 0.00</td>
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<tr>
<td>321</td>
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<td>Long -1.52</td>
<td>Lat -0.37 -0.12 -0.11</td>
</tr>
<tr>
<td>330</td>
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<td>23</td>
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<td>Long 2.73</td>
<td>Lat 0.10 -0.03 -0.04</td>
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<tr>
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<td>15</td>
<td>52</td>
<td>Long 2.43</td>
<td>Lat -0.04 0.00 0.00</td>
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<tr>
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<td>Long -3.22</td>
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<td></td>
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<td>Lat 1.48 1.58 1.61</td>
</tr>
</tbody>
</table>

* Station Locations: 311 Prospect Harbor, Maine 320 Rosemont, Minnesota 321 Rosemont, Minnesota 330 Laguna Peak, California
Table 2. Combined-Pass Station Position Differences

<table>
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<tr>
<th>Station*</th>
<th>No. of Passes</th>
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<th>Full Sample Interval</th>
<th>Interval /2</th>
<th>Interval /5</th>
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* See Table 1.
Figure 1. Longitudinal Differences in Full Sample Interval Station Position Residuals (Frequency - Range Differences)
Figure 2. Longitudinal Differences in Station Position Residuals
Sample Interval/2 (Frequency – Range Differences)
Figure 3. Longitudinal Differences in Station Position Residuals
Sample Interval/5 (Frequency - Range Differences)
Figure 4. Latitudinal Differences in Station Position Residuals
Full Sample Interval (Frequency - Range Differences)
Figure 5. Latitudinal Differences in Station Position Residuals
Sample Interval/2 (Frequency - Range Differences)
Figure 6. Latitudinal Differences in Station Position Residuals
Sample Interval/5 (Frequency - Range Differences)
Table 3. CELEST Station Navigation Differences
(Frequency - Range Differences) (m)

<table>
<thead>
<tr>
<th>Station*</th>
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<th>Elevation (Deg)</th>
<th>Frequency - Range Difference (m)</th>
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<td>23</td>
<td>36</td>
<td>-0.7</td>
<td>0.1</td>
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* See Table 1.
Figure 7. CELEST Station Navigation Differences in the Radial Direction (Frequency - Range Differences)
Figure 8. CELEST Station Navigation Differences in the Along-Track Direction (Frequency - Range Differences)
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UNCLASSIFIED TITLE
EFFECT OF LINEAR APPROXIMATION FOR SATELLITE DOPPLER FREQUENCY IN GEODESY

ABSTRACT
(U) THE DOPPLER PRINCIPLE HAS BEEN USED TO TRACK SATELLITES SINCE THE Early SIXTIES LESS THAN ONE SECOND. THIS DATA HAS BEEN TREATED AS FREQUENCY DATA IN AN APPROXIMATION WHEN THE NUMBER OF CYCLES COUNTED IN AN INTERVAL EQUATED TO THE EQUATIONS OF CONDITION. THE RESULT OF THIS TRUNCATED SERIES AMOUNTED TO ABOUT 1.4 METERS TOO LOW. THEREFORE THE DATA MUST BE TREATED AS RANGE DATA AS SECOND. (AUTHOR)

INDEX TERMS ASSIGNED
DOPPLER SYSTEMS
FREQUENCY
POSITION FINDING
LINEAR SYSTEMS
SAMPLING

DATA SAMPLING INTERVAL
EARLY 1960S
STATION HEIGHTS
TRUNCATED SERIES

TERMS NOT FOUND ON NLD
DOPPLER PRINCIPLE
ORBIT DETERMINATION
TRACK SATELLITES
1.4 METERS
FREQUENCY IN GEODETIC POSITIONING.

SATELLITES SINCE THE EARLY 1960'S. WHEN THE DATA SAMPLING INTERVAL WAS FLEXIBLE, FREQUENCY DATA IN ORBIT DETERMINATION PROGRAMS. THIS METHOD RESULTED IN AN INTERVAL DIVIDED BY THE TIME INTERVAL IS TREATED AS A FREQUENCY. THE TRUNCATED SERIES IS THAT IT PROCUCES STATION HEIGHTS THAT ARE TREATED AS RANGE DIFFERENCES, EVEN FOR SAMPLING INTERVALS AS LOW.

INDEX TERMS ASSIGNED
- FREQUENCY
- GEODESY
- APPROXIMATION (MATHEMATICS)
- INTERVALS
- TIME INTERVALS

RMS NOT FOUND ON NLDS
- DOPPLER PRINCIPLE
- ORBIT DETERMINATION PROGRAMS
- TRACK SATELLITES
- 1.4 METERS