AFAPL-TR-78-6
Part III

ROTOR-BEARING DYNAMICS TECHNOLOGY DESIGN GUIDE
Part III
Tapered Roller Bearings

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AIR FORCE/56780/23 August 1978 — 140
This report is an update of the original Part IV of the Rotor-Bearing Dynamics Design Technology Series, AFAPL-TR-65-45 (Parts I through X). A computer program is given for preparation of tapered roller bearing stiffness data input for rotodynamic response programs. The complete stiffness matrix is calculated including centrifugal effects. Considerations such as elastohydrodynamic and cage effects are not included since they have little influence on the calculation of tapered roller bearing stiffness. The resulting program is reasonably small and easy to use.
FOREWORD

This report was prepared by Shaker Research Corporation under USAF Contract No. AF33615-76-C-2038. The contract was initiated under Project 3048, "Fuels, Lubrication, and Fire Protection," Task 304806, "Aerospace Lubrication," Work Unit 30480685, "Rotor-Bearing Dynamics Design."

The work reported herein was performed during the period 15 April 1977 to 15 November 1978, under the direction of John B. Schrand (AFAPL/SFL) and Dr. James F. Dill (AFAPL/SFL), Project Engineers. The report was released by the authors in December 1978.
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<tr>
<td>b_x</td>
<td>Semi-width of contact ellipse at x</td>
<td>in.</td>
</tr>
<tr>
<td>B_1</td>
<td>Corner break at roller small end</td>
<td>in.</td>
</tr>
<tr>
<td>B_2</td>
<td>Corner break at roller big end</td>
<td>in.</td>
</tr>
<tr>
<td>B_{ij}</td>
<td>Damping component, change of force in i direction due to velocity in j direction; i = x, y, z; j = x, y, z.</td>
<td>lb-sec in</td>
</tr>
<tr>
<td>B_N</td>
<td>Damping matrix</td>
<td>lb-sec in</td>
</tr>
<tr>
<td></td>
<td>\begin{bmatrix} B_{xx} &amp; B_{xy} \ B_{yx} &amp; B_{yy} \end{bmatrix} &amp; \begin{bmatrix} (B_N)<em>{\text{lineal}} &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; (B_N)</em>{\text{angular}} \end{bmatrix}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Damping matrix due to lateral velocities</td>
<td>lb-sec in</td>
</tr>
<tr>
<td></td>
<td>Damping matrix due to angular velocities</td>
<td>in-lb-sec radian</td>
</tr>
<tr>
<td>C_i</td>
<td>A constant, C_i = \begin{cases} 1 &amp; \text{for } i = 1 \ -1 &amp; \text{for } i = 2 \end{cases}</td>
<td>-</td>
</tr>
<tr>
<td>d</td>
<td>Roller diameter at midpoint of effective length of roller</td>
<td>in.</td>
</tr>
<tr>
<td>d_x</td>
<td>Roller diameter at x</td>
<td>in.</td>
</tr>
<tr>
<td>E</td>
<td>Pitch diameter at midpoint of effective length</td>
<td>in.</td>
</tr>
<tr>
<td>E_E</td>
<td>Modulus of elasticity for roller</td>
<td>lbs/in^2</td>
</tr>
</tbody>
</table>
\[ E_R \] Modulus of elasticity for race body \[ \text{lbs/in}^2 \]

\[ E_x \] Pitch diameter at \( x \) \[ \text{in.} \]

\[ F_c \] Roller centrifugal force \[ \text{lbs.} \]

\[ F_i \] External applied force, \( i = x, y, z \) \[ \text{lbs.} \]

\[ F'_i \] Reaction force, positive in direction opposite to displacements, \( i = x, y, z \) \[ \text{lbs.} \]

\[ F \] Force Matrix \[ \text{lbs.} \]

\[ G \] Distance along roller cone element from extreme end of effective length to point where crown drop is measured \[ \text{in.} \]

\[ H \] Roller crown radius minus the rise of the arc at midpoint of effective length \[ \text{in.} \]

\[ I_{cg} \] Moment of inertia about roller center of gravity \[ \text{lbs-in}^2 \]

\[ K \] Roller-race stiffness \[ \text{lbs/in.} \]

\[ K_{ij} \] Stiffness component, change of force in \( i \) direction due to displacement in \( j \) direction. \( i = x, y, z; \ j = x, y, z \) \[ \text{lbs/in.} \]

\[ K_N \] Stiffness matrix

\[
\begin{bmatrix}
(K_N)_{\text{lineal}} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & (K_N)_{\text{angular}}
\end{bmatrix}
\]

\[ (K)_{\text{lineal}} \] Stiffness matrix due to lateral displacements \[ \text{lbs/in.} \]

\[ (K)_{\text{angular}} \] Stiffness matrix due to angular rotations \[ \text{in-lb/rad} \]

---

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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l )</td>
<td>Perpendicular distance from the line of action of flange reaction, ( P_3 ), to roller centerline at midpoint of effective length</td>
<td>in.</td>
</tr>
<tr>
<td>( l_e )</td>
<td>Effective length of roller load carrying surface</td>
<td>in.</td>
</tr>
<tr>
<td>( l_F )</td>
<td>Length of flat portion of roller measured along roller cone element</td>
<td>in.</td>
</tr>
<tr>
<td>( l_T )</td>
<td>Total length of roller measured parallel to roller axis between sharp intersections of end faces with roller cone elements</td>
<td>in.</td>
</tr>
<tr>
<td>( m )</td>
<td>Mass of roller</td>
<td>lbs.</td>
</tr>
<tr>
<td>( M_G )</td>
<td>Roller gyroscopic moment</td>
<td>lbs-in.</td>
</tr>
<tr>
<td>( M_i )</td>
<td>External applied moment, ( i = x, y, z )</td>
<td>lbs-in.</td>
</tr>
<tr>
<td>( M_i' )</td>
<td>Reaction moment, ( i = x, y, z )</td>
<td>lbs-in.</td>
</tr>
<tr>
<td>( M_1 )</td>
<td>Outer race/roller contact moment</td>
<td>lbs-in.</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>Inner race/roller contact moment</td>
<td>lbs-in.</td>
</tr>
<tr>
<td>( n )</td>
<td>Number of rollers</td>
<td></td>
</tr>
<tr>
<td>( N_1 )</td>
<td>Outer ring rotational speed</td>
<td>rad/sec.</td>
</tr>
<tr>
<td>( N_2 )</td>
<td>Inner ring rotational speed</td>
<td>rad/sec.</td>
</tr>
<tr>
<td>( P_x )</td>
<td>Contact unit loading</td>
<td>lbs/in.</td>
</tr>
<tr>
<td>( P_x' )</td>
<td>Current estimate of contact unit loading</td>
<td>lbs/in.</td>
</tr>
<tr>
<td>( P_D )</td>
<td>Diametral clearance</td>
<td>in.</td>
</tr>
<tr>
<td>( P_{1q} )</td>
<td>Outer contact load on ( q )th roller</td>
<td>lbs.</td>
</tr>
<tr>
<td>( P_{2q} )</td>
<td>Inner contact load on ( q )th roller</td>
<td>lbs.</td>
</tr>
<tr>
<td>( P_{3q} )</td>
<td>Flange reaction on ( q )th roller</td>
<td>lbs.</td>
</tr>
<tr>
<td>( q )</td>
<td>Roller position index</td>
<td></td>
</tr>
<tr>
<td>( R_C )</td>
<td>Roller crown radius</td>
<td>in.</td>
</tr>
<tr>
<td>( R_E )</td>
<td>Roller big-end spherical radius</td>
<td>in.</td>
</tr>
<tr>
<td>( V )</td>
<td>Radial distance from roller center line to flange reaction</td>
<td>in.</td>
</tr>
</tbody>
</table>
\[
\begin{bmatrix}
\delta_x \\
\delta_y \\
0_x \\
0_y
\end{bmatrix}
\]

\( x, y, z \)  
Bearing coordinate system  
in.

\( x_0 \)  
Static component of displacement  
in.

\( x' \)  
Dynamic component of displacement  
in.

\( X_A \)  
Big-end extremity of contact pattern measured parallel to roll axis from the midpoint of the effective length

\( X_B \)  
Small end extremity of contact pattern measured parallel to roller axis from the midpoint of the effective length

\( X_A' \)  
Maximum permissible distance of big-end pattern extremity from midpoint of effective length measured along race

\( X_B' \)  
Maximum permissible distance of small-end pattern extremity from midpoint of effective length measured along race

\( Z_N \)  
Impedance matrix  
\( K_N + i \omega N \)

Other notations as defined in text.
**GREEK SYMBOLS**

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<th>Description</th>
<th>Units</th>
</tr>
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<tr>
<td>( \alpha )</td>
<td>Angle between roller axis and line of action of flange reaction</td>
<td>radians</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Outer ring contact angle</td>
<td>radians</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>( d_x \cos \beta/E_x )</td>
<td>-</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>( d_x \cos(\beta-\tau)/E_x )</td>
<td>-</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Displacement</td>
<td>in.</td>
</tr>
<tr>
<td>( \delta_x )</td>
<td>Linear displacement in ( x ) direction</td>
<td>in.</td>
</tr>
<tr>
<td>( \delta_y )</td>
<td>Linear displacement in ( y ) direction</td>
<td>in.</td>
</tr>
<tr>
<td>( \delta_z )</td>
<td>Linear displacement in ( z ) direction</td>
<td>in.</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>Approach of inner race to outer race at midpoint of effective length</td>
<td>in.</td>
</tr>
<tr>
<td>( \Delta_x )</td>
<td>Approach of roller to race at ( x )</td>
<td>in.</td>
</tr>
<tr>
<td>( \Delta_{1q} )</td>
<td>Approach of roller to outer race (cup) at ( q )th roller</td>
<td>in.</td>
</tr>
<tr>
<td>( \Delta_{2q} )</td>
<td>Approach of roller to inner race (cone) at ( q )th roller</td>
<td>in.</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Residues of simultaneous equations</td>
<td>-</td>
</tr>
<tr>
<td>( \eta_E )</td>
<td>Roller elastic constant ( = \frac{4(1 - v_E^2)}{E_E} )</td>
<td>( \text{in}^2 \text{/lb.} )</td>
</tr>
<tr>
<td>( \eta_R )</td>
<td>Race elastic constant ( = \frac{4(1 - v_R^2)}{E_R} )</td>
<td></td>
</tr>
<tr>
<td>( \theta_x )</td>
<td>Angular rotation about ( x ) axis</td>
<td>radians, (^0)</td>
</tr>
<tr>
<td>( \theta_y )</td>
<td>Angular rotation about ( y ) axis</td>
<td>radians, (^0)</td>
</tr>
<tr>
<td>( \theta_z )</td>
<td>Angular rotation about ( z ) axis</td>
<td>radians, (^0)</td>
</tr>
<tr>
<td>( v )</td>
<td>Frequency of rotation</td>
<td>rad/sec.</td>
</tr>
<tr>
<td>( v_E )</td>
<td>Poisson's ratio for roller</td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------------------------------</td>
<td>------------</td>
</tr>
<tr>
<td>$v_R$</td>
<td>Poisson's ratio for race</td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>Material density</td>
<td>lbs/in$^3$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Included roller cone angle</td>
<td>radians,$^\circ$</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Circumferential roller position</td>
<td>radians,$^\circ$</td>
</tr>
<tr>
<td>$\omega_R$</td>
<td>Angular velocity of roller about its own center</td>
<td>rad/sec.</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Orbital velocity of roller</td>
<td>rad/sec.</td>
</tr>
<tr>
<td>$V$</td>
<td>Crown drop</td>
<td>in.</td>
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<td>b</td>
<td>Refers to bearing</td>
</tr>
<tr>
<td>cg</td>
<td>Refers to center of gravity</td>
</tr>
<tr>
<td>E</td>
<td>Refers to roller</td>
</tr>
<tr>
<td>F</td>
<td>Refers to flat</td>
</tr>
<tr>
<td>g</td>
<td>Refers to gyroscopic</td>
</tr>
<tr>
<td>i</td>
<td>Index, i = 1, 2, 3 or i = x, y, z</td>
</tr>
<tr>
<td>i,j</td>
<td>Refers to index of stiffness matrix; i.e., force in i direction due to displacement in j direction</td>
</tr>
<tr>
<td>p</td>
<td>Refers to pedestal</td>
</tr>
<tr>
<td>q</td>
<td>Refers to roller circumferential position</td>
</tr>
<tr>
<td>R</td>
<td>Refers to roller</td>
</tr>
<tr>
<td>T</td>
<td>Refers to total</td>
</tr>
<tr>
<td>x</td>
<td>Refers to x direction</td>
</tr>
<tr>
<td>y</td>
<td>Refers to y direction</td>
</tr>
<tr>
<td>z</td>
<td>Refers to z direction</td>
</tr>
<tr>
<td>1</td>
<td>Refers to outer race</td>
</tr>
<tr>
<td>2</td>
<td>Refers to inner race</td>
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SECTION I
INTRODUCTION

The original Rotor-Bearing Dynamics Design Technology Series AFAPL-TR-65-45 (Parts I through X) included a volume, Part IV(1), which presented design data for typical deep-groove and angular contact ball bearings. The data was presented in graphical form and consisted of direct radial stiffness, load carrying capacity, and load levels. In addition design guidelines and limitations were discussed. The major deficiencies of this original volume were that centrifugal effects due to high speed were ignored, and axial and angular stiffness information were omitted.

Subsequent to the publication of Part IV, several extensive treatments of rolling element bearings including elastohydrodynamic, thermal, and cage effects have been published. The computer program of Mauriello, LaGasse, and Jones (2) considers both elastohydrodynamic and cage effects for ball bearings. The more recent computer based design guide prepared by Crecelius and Pirvics (3) treats elastohydrodynamic, thermal, and cage effects for a system of ball and roller bearings.

Thus, very sophisticated analytical tools are available for the design and application of rolling element bearings. Neither of these tools, however, provide the user with the stiffness matrix required for solution of rotor dynamics problems. In addition both computer programs are very large and require an extensive computer facility for use.

Part II(4) of the revised series provided an update of the original Part IV(1). Those aspects of the original Part IV(1) which treated general design aspects of ball bearings, load capacity, speed limitations, etc. were deleted since their coverage is superficial compared to the more sophisticated computer tools now available (2,3). Only those parts directly connected with preparation of input for the rotodynamic response
programs (Part I(5) of the revised series) were retained. The stiffness data included in the original Part IV were also updated.

The present volume (Part III of the revised series) extends the treatment of rolling element bearings to the tapered roller bearing. The complete stiffness matrix is calculated including centrifugal effects. Considerations such as elastohydrodynamic and cage effects are not included since they have little influence on the calculation of tapered roller bearing stiffness. The resulting program (Appendix) is reasonably small and easy to use.
SECTION II
ANALYSIS

2.1 General Bearing Model and Coordinate System

Accurate calculation of the lateral dynamic response of a high-speed rotor depends on realistic characterization of the support bearings. In the most general case, both linear and angular motions are restrained by the support bearings at the attachment location. In the analytical model, the reaction force and the reaction moment of each bearing are felt by the rotor through a single station of the rotor axis. As schematically illustrated in Figure 1a, a coil spring restraining the lateral displacement and a torsion spring which tends to oppose an inclination are attached to the same point of the rotor axis. A complete description of the characteristics of the support bearings, however, involves much more than the specification of the two spring constants. This is because:

- The lateral motion of the rotor axis is concerned with two displacement components and two inclination components.
- The restraining characteristics may include cross coupling among various displacement/inclination coordinates.
- The restraining force/moment may not be temporally in phase with the displacement/inclination.
- The restraining characteristics of the bearing may be dependent on either the rotor speed or the frequency of vibration, or both.
- Bearing pedestal compliance may not be negligible.

To accommodate the above considerations, the support bearing characteristics are described in Reference 5 by a four-degrees-of-freedom impedance matrix as defined in Equation (1):
FIG 1a) Bearing Stiffness Model

FIG 1b) Bearing Location Coordinate System
$$\mathbf{R}_N = -\mathbf{Z}_N \cdot \mathbf{W}_N$$  \hspace{1cm} (1)$$

where \(\mathbf{W}_N\) is a column vector containing elements which are the two lateral displacements \((\delta_x, \delta_y)\) and the two lateral inclinations \((\theta_x, \theta_y)\) of the rotor axis at the bearing station \(N\).

Employing a right-handed Cartesian representation in a lateral plane as depicted in Figure 1b, the z-axis is coincident with the spin vector of the rotor. The x-axis is oriented in the direction of the external static load, and the y-axis is perpendicular to both z and x axes forming the right-handed triad \((x, y, z)\). \((\delta_x, \delta_y)\) are respectively lateral lineal displacement components of the rotor axis along the \((x, y)\) directions. \((\theta_x, \theta_y)\) are lateral inclination components respectively in the \((z-x, z-y)\) planes. Note that \(\theta_y\) is a rotation about the y-axis, while \(\theta_x\) is a rotation about the negative x-axis.

\(\mathbf{Z}_N\) is a complex \((4 \times 4\) matrix), and in accordance with the common notation for stiffness and damping coefficients, may be expressed as

$$\mathbf{Z}_N = \mathbf{K}_N + i\nu\mathbf{B}_N$$  \hspace{1cm} (2)$$

where \(\mathbf{K}_N\) is the stiffness matrix and \(\mathbf{B}_N\) is the damping matrix. \(\nu\) is the frequency of vibration. Most commonly, lateral lineal and angular displacements do not interact with each other so that the non-vanishing portions of \(\mathbf{K}_N\) and \(\mathbf{B}_N\) are separate \(2 \times 2\) matrices. That is

$$\mathbf{K}_N = \begin{bmatrix}
(K_{N_{\text{lineal}}}) & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & (K_{N_{\text{angular}}}) 
\end{bmatrix}$$  \hspace{1cm} (3)$$
Accordingly, a total characterization of a support bearing would include sixteen coefficients which make up the 4 (2 x 2) matrices:

\[
\begin{bmatrix}
(B_N \text{ lineal}) & 0 & 0 \\
0 & 0 & (B_N \text{ angular})
\end{bmatrix}
\]

\[
B_N =
\]

Accordingly, a total characterization of a support bearing would include sixteen coefficients which make up the 4 (2 x 2) matrices:

\[
(K)_{\text{lineal}} =
\begin{bmatrix}
K_{xx} & K_{xy} \\
K_{yx} & K_{yy}
\end{bmatrix}
\]

\[
(B)_{\text{lineal}} =
\begin{bmatrix}
B_{xx} & B_{xy} \\
B_{yx} & B_{yy}
\end{bmatrix}
\]

\[
(K)_{\text{angular}} =
\begin{bmatrix}
K_{xx} & K_{xy} \\
K_{yx} & K_{yy}
\end{bmatrix}
\]

\[
(B)_{\text{angular}} =
\begin{bmatrix}
B_{xx} & B_{xy} \\
B_{yx} & B_{yy}
\end{bmatrix}
\]

In the event that the pedestal compliance is significant, then the effective support impedance can be calculated from

\[
Z_N = (Z_b^{-1} + Z_p^{-1})
\]

where subscripts "p" and "b" refer to the pedestal and bearing respectively. Note that both pedestal inertia and damping may be included in \(Z_p\).
2.2 General Bearing Support Characteristics

The function of a bearing is to restrict the rotor axis to a nominal axis under realistic static and dynamic load environments. Deviation of any particular point of the rotor axis from the nominal line can be characterized by three linear and two angular displacements. These may be designated as \((\delta_x, \delta_y, \delta_z, \Theta_x, \Theta_y)\) in accordance with a right-handed Cartesian reference system. The \(z\)-coordinate is coincident with the reference axis and is directed toward the spin vector. \((\Theta_x, \Theta_y)\) are rotor axis inclinations respectively in the \(z\)-\(x\) and \(z\)-\(y\) planes. The \(x\)-coordinate is directed toward the predominant static load; e.g., earth gravity. Ideally, the bearing would resist the occurrence of any displacement so that the reaction force system imparted by the bearing to the rotor is generally expressed in matrix notation as

\[
F = Z \cdot x
\]  

(10)

\(F\) is a column vector comprising the five reaction components \((F_x, F_y, F_z, M_x, M_y)\), while \(x\) is the displacement vector \((\delta_x, \delta_y, \delta_z, \Theta_x, \Theta_y)\). \(Z\) is a \((5 \times 5)\) matrix containing the elements \(Z_{ij}\) with both indices \((i, j)\) ranging from 1 to 5. The values of \(Z_{ij}\) characterize how rotor displacements are being resisted by the bearing.

From the standpoint of dynamic perturbation, distinction is made between a static equilibrium component and a dynamic perturbation component for both the displacements and the reactions. Thus,

\[
x = x_o + x'; \quad F = F_o + F'
\]

(11)

\((x', F')\) are respectively presumed to be infinitesimal in comparison with \((x_o, F_o)\). Accordingly, \(Z_{ij}\) are regarded as dependent on \(x_o\) but not on \(x'\).

To illustrate the idea of perturbation linearization, one may examine the one-dimensional load-displacement curve shown in Figure 2.
Figure 2. Linearization of Tapered Roller Bearing Stiffness
As illustrated, the load-displacement relationship is a 10/9 power law in accordance with the Hertzian point contact formula. It is not possible to describe the entire range by a linear approximation. However, if a small dynamic perturbation is taken around a static equilibrium point, \( \delta_x' < \delta_x \), the small segment of the load-displacement curve can be approximated by a local tangent line. The corresponding force increment is

\[
F' = \left. \frac{\partial F}{\partial \delta_x} \right|_{\delta_x} \delta_x'
\]  

(12)

where \( \delta_x' \) is the incremental displacement. \( \frac{\partial F}{\partial \delta_x} \) will depend on the amplitude of \( \delta_x \).

The question of history dependence is resolved by regarding \( x' \) as periodic motions at any frequency \( \nu \) of interest, and \( Z_{ij} \) accordingly would have both real and imaginary parts and may also be dependent on both the rotor speed \( \omega \) and the vibration frequency \( \nu \).

To avoid notational clumsiness, the primes will be dropped from \((F', x')\) which are understood to be dynamic perturbation quantities unless the subscript "o" is used to designate the static equilibrium condition.

2.3 Tapered Roller Bearing Characterization

In many ways the tapered roller bearing is much simpler to model from a rotor dynamic point of view than a fluid film bearing. In general, the following two simplifications can be made:

- The restraining characteristics do not include cross coupling among the various displacement/inclination coordinates.
- The restraining force/moment is normally temporally in phase with the displacement/inclination.
Figure 3 shows a tapered roller bearing referred to in an orthogonal xyz coordinate system. The outer ring is fixed but the inner ring may move with respect to the coordinate system. Both rings are free to rotate about their axes.

Three lineal displacements, \( \delta_x, \delta_y, \delta_z \), and two angular displacements, \( \Theta_x, \Theta_y \), are required to define the spatial position and attitude of the inner ring when it is displaced from its initial position. For purposes of derivation the initial situation is that existing when the bearing's end play is just taken up in the thrust direction. Figure 4 shows these displacements in the positive sense. Figure 5 establishes the convention of the roller-position index \( q \).

2.3.1 Stiffness

The total characterization of a tapered roller bearing's stiffness can be expressed by the matrix.

\[
[K] = \begin{bmatrix}
\frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} & \frac{\partial F_x}{\partial z} & \frac{\partial F_x}{\partial \Theta_x} & \frac{\partial F_x}{\partial \Theta_y} \\
\frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} & \frac{\partial F_y}{\partial z} & \frac{\partial F_y}{\partial \Theta_x} & \frac{\partial F_y}{\partial \Theta_y} \\
\frac{\partial F_z}{\partial x} & \frac{\partial F_z}{\partial y} & \frac{\partial F_z}{\partial z} & \frac{\partial F_z}{\partial \Theta_x} & \frac{\partial F_z}{\partial \Theta_y} \\
\frac{\partial M_x}{\partial x} & \frac{\partial M_x}{\partial y} & \frac{\partial M_x}{\partial z} & \frac{\partial M_x}{\partial \Theta_x} & \frac{\partial M_x}{\partial \Theta_y} \\
\frac{\partial M_y}{\partial x} & \frac{\partial M_y}{\partial y} & \frac{\partial M_y}{\partial z} & \frac{\partial M_y}{\partial \Theta_x} & \frac{\partial M_y}{\partial \Theta_y}
\end{bmatrix}
\]  

(13)
Figure 3. Tapered Roller Bearing
Figure 4. Bearing Coordinate System

\[
\begin{align*}
\delta_x & \quad \delta_y \\
F_x' & \quad F_y' \\
W_x' & \quad W_y'
\end{align*}
\]
Figure 5. Tapered Roller Bearing Index, $q$
The lineal and angular stiffness matrices (Equations 5 and 7) can be derived from Equation (13). For example:

\[
(K)_{\text{lineal}} = \begin{bmatrix}
\frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} \\
\frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y}
\end{bmatrix}
\] (14)

\[
(K)_{\text{angular}} = \begin{bmatrix}
\frac{\partial M_x}{\partial \theta_x} & \frac{\partial M_x}{\partial \theta_y} \\
\frac{\partial M_y}{\partial \theta_x} & \frac{\partial M_y}{\partial \theta_y}
\end{bmatrix}
\] (15)

Note that although the axial components of stiffness are not utilized by the lateral rotor dynamics program (5), they have been retained in the general tapered roller bearing stiffness matrix, Equation (13). The axial stiffness would be required, for example, if the reader was calculating the axial natural frequency of a tapered roller bearing mounted shaft.

2.3.2 Damping

An extensive search of the literature revealed no experimental damping data for tapered roller bearings. As the current state-of-the-art does not permit accurate calculation of tapered roller bearing damping, no damping data is included in this report.
2.4 Tapered Roller Bearing Under Combined Loading

Solution for the stiffness matrix of a tapered roller bearing under combined loading is a tedious problem and requires the use of a digital computer. In this section, the derivation of the solution is described. A computer program for obtaining the solution is included in the Appendix.

2.4.1 Bearing Applied Forces and Moments

As the result of the five displacements described previously in Figures 3 and 4, there are the reactions \( F'_x, F'_y, F'_z \) and \( M'_x, M'_y, M'_z \) are forces. \( M'_x \) and \( M'_y \) are moments. All are shown in their positive sense in Figure 4. External forces \( F_x \) and \( F_z \) may be applied at the inner ring center. The senses of the signs are the same as for the reactions \( F'_x \) and \( F'_z \).

2.4.2 Roller Geometry

Figure 6 shows the boundary dimensions of a typical tapered roller. Roller mass, moment of inertia, and location of the center of gravity are calculated assuming the roller is a flat-ended, truncated cone bounded by \( R_1, R_2, \) and \( \ell_1 \).

In general, the big-end face of the roller is not flat but is a sphere having the radius \( R_e \) which is generally a proportion of the slant height, \( \ell_s \), of the untruncated roller cone. Roller crown and corner breaks are also omitted from mass and moment of inertia calculations as their contributions are second order.

Figure 7 is a more complete sketch of the roller showing the details of the roller crown. The big-end spherical surface is neglected here also.

\( \tau \) is the included angle of the roller cone and is obtained by iteration of

\[
\frac{\tau}{2} = \tan^{-1} \left( \frac{d}{E} \sin \left( \beta - \frac{\tau}{2} \right) \right)
\]  

(16)
Figure 6. Boundary Dimensions of Typical Tapered Roller
Figure 7. Dimensions of Roller Profile and Crown
From Figure 7

\[ H = \sqrt{R_c^2 - (\frac{e}{2})^2} \]  

(17)

where \( R_c \) is the crown radius and \( e \) the length of the flat portion of the roller profile. In a fully crowned roller, the flat length is zero.

\( e \) is the effective length of the roller load-carrying surface. The actual working length for any loading must lie within \( e \). \( B_1 \) and \( B_2 \) are the corner breaks. Their shapes are unimportant as long as they blend smoothly into the crowned surface.

\( V \) is the drop of the crown and is measured at the extremes of the effective length of the roller.

\[ V = H - \sqrt{R_c^2 - (\frac{e}{2})^2} \]  

(18)

\[ H_0 = \frac{d}{2\tan(\frac{e}{2})} \]  

(19)

\[ H_1 = H_0 - \frac{e}{2} \cos(\frac{e}{2}) + V\sin(\frac{e}{2}) - B_1 \]  

(20)

\[ H_2 = H_0 + \frac{e}{2} \cos(\frac{e}{2}) + V\sin(\frac{e}{2}) + B_2 \]  

(21)

\[ R_1 = H_1 \tan(\frac{e}{2}) \]  

(22)

\[ R_2 = H_2 \tan(\frac{e}{2}) \]  

(23)

Let the cone corresponding to \( H_1 \) have the mass \( m_1 \) and a moment of inertia about its center of gravity \( I_{1cg} \).

Let the cone corresponding to \( H_2 \) have the mass \( m_2 \) and a moment of inertia about its center of gravity \( I_{2cg} \).
\[ m_1 = \frac{\pi R_1^2 H_1 \rho}{3 \times 326.4} \]  

\[ m_2 = \frac{\pi R_2^2 H_2 \rho}{3 \times 326.4} \]  

\[ I_{1_{cg}} = \frac{3m_1}{5} \left( \frac{R_1^2}{4} + \frac{H_1^2}{16} \right) \]  

\[ I_{2_{cg}} = \frac{3m_2}{5} \left( \frac{R_2^2}{4} + \frac{H_2^2}{16} \right) \]  

where \( \rho \) is the material density in lb/in\(^3\).

Then the distance \( \bar{X} \) from the big end of the roller at \( H_2 \) to the center of gravity of the roller is \( \bar{X}' \)  

\[ \bar{X}' = \frac{\frac{m_2 H_2}{4} - m_1 \left( H_2 - \frac{3H_1}{4} \right)}{m_2 - m_1} \]  

The moment of inertia \( I_{cg} \) of the tapered roller about its center of gravity at \( \bar{X} \) is  

\[ I_{cg} = I_{2_{cg}} + m_2 \left( \frac{H_2}{4} - \bar{X}' \right)^2 - I_{1_{cg}} - m_1 \left( H_2 - \frac{3H_1}{4} - \bar{X}' \right)^2 \]  

Later the distance \( \bar{X} \), being the distance left from \( H_0 \) to the center of gravity of the roller, will be required.  

\[ \bar{X} = H_2 - H_0 - \bar{X}' \]  

The slant height, \( \ell_s \), of the truncated roller cone is shown in Figure 6 and is  

\[ \ell_s = \frac{R_2}{\sin \left( \frac{\pi}{2} \right)} \]  

and the big-end spherical radius, \( R_E \), is a proportion of \( \ell_s \).
V is the radius from the roller centerline to the point of contact of the roller and inner-race guide flange and the flange reaction. It is directed at an angle, \( \alpha \), where

\[
\alpha = \sin^{-1} \left( \frac{V}{R_e} \right) \tag{32}
\]

The lever arm of the flange reaction about the midpoint of the working surface of the roller at \( H_0 \) is

\[
h = \left[ \sqrt{R_e^2 - R_2^2} - H_2 + H_0 \right] \sin \alpha \tag{33}
\]

Figure 8 is an enlarged view of the race profile showing the crown drop \( V \) which is measured at a distance \( G \) from the end of the effective length. The contour is the same at both ends of the roll. If the radius \( R_c \) is known, the drop at \( G \) is

\[
V' = H - \sqrt{R_c^2 - \left( \frac{e}{2} - G \right)^2} \tag{34}
\]

If the drop is known and the radius \( R_c \) is not, the radius is

\[
R_c = \sqrt{\left[ \frac{\left( \frac{e}{2} - G \right)^2 - \left( \frac{e}{2} \right)^2 - V'^2}{2V'} \right]^2 + \left( \frac{e}{2} - G \right)^2} \tag{35}
\]

2.4.3 Roller Equilibrium

Figure 9 shows the forces and moments acting on a roller which is in contact with both outer and inner races and with the inner ring guide flange.

In the following discussion, the subscripts 1 and 2 refer to the outer and inner contacts, respectively.

\( P_1 \) and \( P_2 \) are the contact loads. \( M_1 \) and \( M_2 \) are contact moments resulting from nonuniform loading along the roller's length. \( F_c \) is the centrifugal force and \( M_G \) is the gyroscopic moment. The
latter acts at the center of gravity of the roller which is located the distance \( \bar{X} \) from the central plane of the roller which contains the midpoint of the effective length.

The centrifugal force and the gyroscopic moment are

\[
F_c = (m_1 + m_2) \left( \frac{E}{2} + \frac{X \sin(\beta - \frac{I}{2})}{E} \right)^2
\]

(36)

\[
M_G = I_{cg} \Omega_E \omega_R \sin(\beta - \frac{I}{2})
\]

(37)

where \( \Omega_E \) is the orbital velocity of the roller and \( \omega_R \) the angular velocity of the roller about its own center, both in radians/second.

\[
\Omega_E = \frac{1}{2} \left[ \Omega_1 \left(1 + \frac{d \cos(\beta - \frac{I}{2})}{E}\right) + \Omega_2 \left(1 - \frac{d \cos(\beta - \frac{I}{2})}{E}\right) \right]
\]

(38)

\[
\omega_R = \frac{E}{2d} \left[ (\Omega_1 - \Omega_2) \left(1 - \frac{d \cos(\beta - \frac{I}{2})}{E} \right)^2 \right]
\]

(39)

\( \Omega_1 \) and \( \Omega_2 \) are the input angular velocities of outer and inner rings in radians/second. \( P_3 \) is the reaction of the inner-ring flange on the roller.

In the present problem, we are concerned with external forces applied to the bearing inner ring along \( x \) and/or \( z \) (Figure 4) only. There may also be initial linear displacements along any or all of the coordinate axes, \( x \), \( y \), and \( z \); and initial rotations about \( x \) and \( y \). These initial displacements do not change when external forces are applied along \( x \) and/or \( z \). However, when initial rotations are present about \( x \) or \( z \), operating displacements may occur along \( x \) and/or \( y \) as the case may be. The system, therefore, has the possibility of three degrees of freedom; i.e., working linear displacements along any or all of the axes \( x \), \( y \), and \( z \). If initial displacements exist about \( x \) or \( y \), working displacements in these modes are prevented.
The approach of the inner race to the outer race along the line defined by $\phi$ for a roller at azimuth $\phi$ is

$$\Delta = (\delta_z + \delta''_z)\sin\phi + \{(\delta_x + \delta''_x)\cos\phi + (\delta_y + \delta''_y)\sin\phi - \frac{D_1}{2}\}$$

$$\cos\phi + \frac{1}{2}\{E\sin\phi + d\sin(\frac{\phi}{2})\}\{(\delta_x + \delta''_x)\sin\phi + (\delta_y + \delta''_y)\cos\phi\}$$

(40)

$P_D$ is the diametral clearance or the total diametral play of the inner ring relative to the outer ring before loading.

The azimuth angle, $\phi$, is related to the roller position index, $q$, through

$$\phi = \frac{2\pi(q-1)}{n}$$

(41)

where $n$ is the number of rollers.

The double-primed items in Equation (40) are the initial displacements in the several modes.

Also, as a result of the initial misalignments which may exist about $x$ and/or $y$, the inner race at the $q$th roller may be misaligned the amount $\theta$.

$$\theta = (\theta_x' + \theta''_x)\sin\phi + (\theta_y + \theta''_y)\cos\phi$$

(42)

If $\Delta_1$ is the approach of the roller to the midpoint of the outer race, the approach $\Delta_2$ of the inner ring to the roller at its midpoint is

$$\Delta_2 = \frac{(\Delta - \Delta_1)\cos(a - \frac{\phi}{2})}{\cos(a + \frac{\phi}{2})}$$

(43)

If $\theta_1$ is the misalignment of the roller relative to the outer race, the misalignment $\theta_2$ of the inner race relative to the roller is
Misalignment is positive if it tends to squeeze the big end of the roller more than the little end when the big end is at the left.

Figure 10 illustrates the geometric intersection of a roller and raceway.

The profiles of race and roller bodies are referred to an XY coordinate system. Note that the X axis is positive to the left of the origin.

The equation of the race surface is

\[ Y = 0 \]  

(45)

The equation of the flat portion of the roller or the element of the basic roller cone is

\[ Y = A_i + X \tan \theta_i \]  

(46)

The equation of the crowned portion of the roller profile is

\[ (X - H \sin \theta_i)^2 + (Y + H \cos \theta_i - A_i)^2 = R_c^2 \]  

(47)

The subscript \( i \) is 1 for an outer contact and 2 for an inner contact.

The intersections of the race and the crowned roller surface occur at \( X_{A_1} \) and \( X_{B_1} \)

\[ X_{A_1} = \sqrt{R_c^2 - (H \cos \theta_i - A_i)^2} + H \sin \theta_i \]  

(48)

\[ X_{B_1} = -\sqrt{R_c^2 - (H \cos \theta_i - A_i)^2} + H \sin \theta_i \]  

(49)
$X_{A_1}$ and $X_{B_1}$ must be within the projected extremities of the roller crown. That is

$$X_{A_1} < X^*_{A_1} \quad (50)$$

$$X_{B_1} > X^*_{B_1} \quad (51)$$

where

$$X^*_{A_1} = \frac{F}{2} \cos \theta_1 + V \sin \theta_1 \quad (52)$$

$$X^*_{B_1} = -\frac{F}{2} \cos \theta_1 + V \sin \theta_1 \quad (53)$$

If the quantity under the radical in Equations (48) and (49) is zero or negative, there is no contact between roller and race.

If $\frac{F}{2} \cos \theta_1 \leq X_{A_1}$, there is also no contact.

If $X_{A_1} > X^*_{A_1}$, $X_{A_1}$ is set equal to $X^*_{A_1}$.

If $X_{B_1} < X^*_{B_1}$, $X_{B_1}$ is set equal to $X^*_{B_1}$.

If $\frac{F}{2} \cos \theta_1 > X_{B_1} > -\frac{F}{2} \cos \theta_1$ and $X_{A_1} > \frac{F}{2} \cos \theta_1$,

the value of $X_{B_1}$ is $X^*_{B_1} = \frac{\Delta_1}{\tan \theta_1} \quad (54)$

Equations (55) and (56) are a set of simultaneous nonlinear equations in which the variables are $\Delta_1$ and $\theta_1$ at the outer contact of the...
particular roller.

The flange reaction $P_3$ is obtained by taking moments about the roller midpoint.

$$P_3 = \frac{-M_1 + M_2 - M_0 + F_c \bar{x} \cos(\beta - \frac{\pi}{2}) - \frac{d}{2} (P_1 - P_2) \sin(\frac{\pi}{2})}{I}$$  \hspace{1cm} (57)

From Figure 10 the intrusion of the roller into the race is

$$\Delta_x = \Delta_1 + X \tan \theta_1 \hspace{1cm} |X| \leq \frac{\alpha}{2} \cos \theta_1$$  \hspace{1cm} (58)

$$\Delta_x = \sqrt{R_c^2 - (X - H \sin \theta_1)^2} - H \cos \theta_1 + \Delta_1 \hspace{1cm} |X| > \frac{\alpha}{2} \cos \theta_1$$  \hspace{1cm} (59)

The derivatives of $\Delta_x$ with respect to $\theta_1$ will be required later and are

$$\frac{d\Delta_x}{d\theta_1} = \frac{X}{\cos^2 \theta_1} \hspace{1cm} |X| \leq \frac{\alpha}{2} \cos \theta_1$$  \hspace{1cm} (60)

$$\frac{d\Delta_x}{d\theta_1} = \frac{(X - H \sin \theta_1) H \cos \theta_1}{\sqrt{R_c^2 - (X - H \sin \theta_1)^2}} + H \sin \theta_1 \hspace{1cm} |X| > \frac{\alpha}{2} \cos \theta_1$$  \hspace{1cm} (61)

Lundberg (6) gives the approach $\Delta_x$ of two cylindrical bodies pressed together with the uniform loading $p_x$ as

$$\Delta_x = \frac{(\eta_R + \eta_E)}{2\pi} p_x \{1.8864 + \ln \left(\frac{X_A - X_B}{2b_x}\right)\}$$  \hspace{1cm} (62)

$\eta_R$ and $\eta_E$ are elastic constants for race and roller, respectively, having the form

$$\eta_{R,E} = \frac{4(1 - \nu^2)}{E_{R,E}}$$  \hspace{1cm} (63)

where $\nu$ is Poisson's Ratio and $E$ is the modulus of elasticity.
b is the semi-width of the pressure area in the rolling direction.

\[
b_x = \left[ \frac{(n_R + n_E)}{2\pi} p_x d_x (1 + C_i Y_i) \right]^{1/2} \tag{64}
\]

\( C_i \) is 1 for \( i = 1 \), corresponding to an outer contact; and -1 for \( i = 2 \), corresponding to an inner contact.

\[
Y_1 = \frac{d_x \cos \beta}{E_x} \tag{65}
\]

\[
Y_2 = \frac{d_x \cos(\beta - \gamma)}{E_x} \tag{66}
\]

where

\[
d_x = \frac{d + 2X \sin(\frac{\gamma}{2})}{\cos(\frac{\gamma}{2})} \tag{67}
\]

\[
E_x = E + 2X \sin \beta + d \cos(\beta - \frac{\gamma}{2}) - d_x \cos \beta \tag{68}
\]

The value of \( p_x \) corresponding to \( \Delta_x \) is required. This cannot be obtained from Equation (62) in closed form. It can be obtained numerically in the following manner.

Let \( p_x' \) be an estimate of \( p_x \). A good starting value is

\[
p_x' = 5 \times 10^7 \Delta_x^{10/9} \frac{X_A - X_B^{1/9}}{(X_A - X_B^{1/9})} \tag{69}
\]

An improved value of \( p_x \) is

\[
p_x = p_x' - \frac{(\Delta_x' - \Delta_x)}{d\Delta_x'/dp_x} \tag{70}
\]

\( \Delta_x' \) is the approach of race and roller bodies calculated for the current estimate of \( p_x' \) using Equation (62).
\[ \frac{d\Delta_x'}{dp_x'} = \frac{(n_R + n_E)}{2\pi} \left[ 1.3864 + \ln \left( \frac{X_A - X_B}{2b_x} \right) \right] \] (71)

Iteration of Equation (70) yields \( p_x \) to any desired accuracy.

The contact force, \( P \), and the moment, \( M \), are

\[ P_1 = \int_{X_B}^{X_A} p_x dX \] (72)

\[ M_1 = \int_{X_B}^{X_A} X_p dX \] (73)

Equations (55) and (56) may now be solved for \( \Delta_1 \) and \( \Theta_1 \), the displacements at the outer contact. Again, a closed-form solution cannot be obtained and numerical techniques are employed.

If estimates are made of the variables \( \Delta_1 \) and \( \Theta_1 \), Equations (55) and (56) may not be satisfied and there will be the residues \( \epsilon_1 \) and \( \epsilon_2 \) for Equations (55) and (56), respectively. Differentiating Equations (55) and (56) gives:

\[ \frac{dc_1}{d\Delta_1} = -\cos \beta \frac{dP_1}{d\Delta_1} + \cos(\beta - \tau) \frac{dP_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta_1} \frac{dP_3}{d\Delta_1} - \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_1} \] (74)

\[ \frac{dc_1}{d\Theta_1} = -\cos \beta \frac{dP_1}{d\Theta_1} + \cos(\beta - \tau) \frac{dP_2}{d\Theta_2} \frac{d\Theta_2}{d\Theta_1} \frac{dP_3}{d\Theta_1} - \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Theta_1} \] (75)

\[ \frac{dc_2}{d\Delta_1} = \sin \beta \frac{dP_1}{d\Delta_1} + \sin(\beta - \tau) \frac{dP_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta_1} + \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_1} \] (76)
\[
\frac{dc_2}{\delta_1} = \sin \beta \frac{dp_1}{\delta_1} + \sin(\beta - \tau) \frac{dp_2}{\delta_1} \frac{do_2}{\delta_1} + \cos(\beta - \tau - \alpha) \frac{dp_3}{\delta_1}
\]

(77)

From Equations (43) and (44)

\[
\frac{d\Delta_2}{d\delta_1} = -\frac{-\cos(\alpha - \frac{1}{2})}{\cos(\alpha + \frac{1}{2})}
\]

(78)

\[
\frac{do_2}{d\delta_1} = -1
\]

(79)

And, from Equation (57)

\[
\frac{dp_3}{d\delta_1} = \frac{-\frac{dM_1}{d\delta_1} + \frac{dM_2}{d\delta_1} \frac{d\Delta_2}{d\delta_1} - \frac{d}{2} \left( \frac{dp_1}{d\delta_1} - \frac{dp_2}{d\delta_1} \frac{dp_3}{d\delta_1} \right) \sin(\frac{\tau}{2})}{2}
\]

(80)

\[
\frac{dp_3}{d\delta_1} = \frac{-\frac{dM_1}{d\delta_1} + \frac{dM_2}{d\delta_1} \frac{do_2}{d\delta_1} - \frac{d}{2} \left( \frac{dM_1}{d\delta_1} - \frac{dM_2}{d\delta_1} \frac{do_2}{d\delta_1} \right) \sin(\frac{\tau}{2})}{2}
\]

(81)

If \( \Delta_1 \) and \( \alpha \) are current estimates, improved estimates are:

\[
\Delta_1 = \Delta_1' - \frac{1}{\frac{d\epsilon_1}{d\delta_1} \frac{d\epsilon_1}{d\delta_1} - \frac{d\epsilon_2}{d\delta_1} \frac{d\epsilon_2}{d\delta_1}}
\]

(82)
The determinants in Equations (82) and (83) are calculated at current estimates.

The derivatives of $P_i$ and $M_i$ with respect to $\Delta_i$ and $\Theta_i$ are

$$\frac{dP_i}{d\Delta_i} = \int_{X_{B_i}}^{X_{A_i}} \frac{dP_x}{d\Delta_i} dX$$

$$\frac{dP_i}{d\Theta_i} = \int_{X_{B_i}}^{X_{A_i}} \frac{dP_x}{d\Theta_i} dX$$

$$\frac{dM_i}{d\Delta_i} = \int_{X_{B_i}}^{X_{A_i}} \frac{dM_x}{d\Delta_i} dX$$

$$\frac{dM_i}{d\Theta_i} = \int_{X_{B_i}}^{X_{A_i}} \frac{dM_x}{d\Theta_i} dX$$

The value of $\frac{dP_x}{d\Delta_x}$ is obtained from Equation (71) and the value of $\frac{d\Delta_x}{d\Delta_i}$ is unity.

If Equations (43), (44), (55), and (56) are differentiated with respect to $\Delta$, there results four equations which are linear in
$dA_1/d\Delta, dA_2/d\Delta, d\theta_1/d\Delta,$ and $d\theta_2/d\Delta$ and from which all four derivatives can be obtained. Of the four derivatives, only $dA_1/d\Delta$ and $d\theta_1/d\Delta$ are of interest here.

\[
\left[-\cos(\beta - \frac{\pi}{2} - \alpha) \frac{dP_1}{dA_1} + \cos(\beta - \tau) \frac{dP_2}{dA_2}\right] + \left[\cos(\beta - \frac{\pi}{2} - \alpha) \frac{dP_3}{dA_2} - \sin(\beta - \frac{\pi}{2} - \alpha) \frac{dP_1}{d\theta_1}\right]
\]

\[
\frac{d\theta_1}{d\Delta} + \left[\cos(\beta - \tau) \frac{dP_2}{d\theta_2} - \sin(\beta - \frac{\pi}{2} - \alpha) \frac{dP_3}{d\theta_2}\right] = 0 \tag{88}
\]

\[
\left[-\sin(\beta - \frac{\pi}{2} - \alpha) \frac{dP_1}{dA_1} + \cos(\beta - \frac{\pi}{2} - \alpha) \frac{dP_3}{dA_2}\right] + \left[-\sin(\beta - \frac{\pi}{2} - \alpha) \frac{dP_1}{d\theta_1} + \cos(\beta - \frac{\pi}{2} - \alpha) \frac{dP_3}{d\theta_1}\right]
\]

\[
\frac{d\theta_1}{d\Delta} + \left[\sin(\beta - \tau) \frac{dP_2}{d\theta_2} + \cos(\beta - \frac{\pi}{2} - \alpha) \frac{dP_3}{d\theta_2}\right] = 0 \tag{89}
\]

\[
\frac{dA_1}{d\Delta} + \frac{\cos(\alpha + \frac{\pi}{2}) d\Delta_2}{\cos(\alpha - \frac{\pi}{2})} = 1 \tag{90}
\]

\[
\frac{dA_1}{d\Delta} + \frac{dA_2}{d\Delta} = 0 \tag{91}
\]

Equations (88) through (91) are easily solved for $dA_1/d\Delta$ and $d\theta_1/d\Delta$. $dA_1/d\theta$ and $d\theta_1/d\theta$ are obtained in a similar manner.

2.4.4 Bearing Equilibrium

The reactions of the bearing on the shaft at the central plane of the roller are
Considering the three-degree-of-freedom system, the inner ring is acted upon by the external forces $F_x$ and $F_z$ and may have working displacements along $x$, $y$, and $z$. Equilibrium requires that

\[ F'_x + F_x = 0 \]  \hspace{1cm} (97)
\[ F'_y = 0 \]  \hspace{1cm} (98)
\[ F'_z + F_z = 0 \]  \hspace{1cm} (99)

Here the variables are $\delta'_x$, $\delta'_y$, and $\delta'_z$. Again, a direct solution is not possible and numerical methods must be employed.

For initial estimates $\delta'_x$, $\delta'_y$, $\delta'_z$ of the variables Equations (97), (98), and (99) may not be satisfied and there remain the residues $\epsilon_1$, $\epsilon_2$, and $\epsilon_3$. Improved values of the variables are

\[
\delta_x = \delta'_x - \frac{d\delta_x}{d\epsilon} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{bmatrix}
\]

(100)
\[
\delta_y = \delta'_y - \frac{d\varepsilon_1}{d\delta_x} \frac{d\varepsilon_1}{d\delta_y} \frac{d\varepsilon_1}{d\delta_z} - D \\
\delta_z = \delta'_z - \frac{d\varepsilon_2}{d\delta_x} \frac{d\varepsilon_2}{d\delta_y} \frac{d\varepsilon_2}{d\delta_z} - D
\]

where \(D\) is the determinant of the system.

\[
D = \begin{vmatrix}
\frac{dc_1}{d\delta_x} & \frac{dc_1}{d\delta_y} & \frac{dc_1}{d\delta_z} \\
\frac{dc_2}{d\delta_x} & \frac{dc_2}{d\delta_y} & \frac{dc_2}{d\delta_z} \\
\frac{dc_3}{d\delta_x} & \frac{dc_3}{d\delta_y} & \frac{dc_3}{d\delta_z}
\end{vmatrix}
\]

The right members of Equations (100) through (103) are evaluated at current estimates

\[
\frac{dc_1}{d(s_x, s_y, s_z)} = \frac{df_x'}{d(s_x, s_y, s_z)} \\
\frac{dc_2}{d(s_x, s_y, s_z)} = \frac{df_y'}{d(s_x, s_y, s_z)} \\
\frac{dc_3}{d(s_x, s_y, s_z)} = \frac{df_z'}{d(s_x, s_y, s_z)}
\]
Although only the above derivatives are required in determining the equilibrium of the system, the complete matrix is required for stiffness calculations.

\[
\frac{dF'_y}{d(\delta_x, \delta_y, \delta_z, \Theta_x, \Theta_y)} = \cos \beta \sum_{q=1}^{n} \cos \phi_q \frac{dP'_1}{d(\delta_x, \delta_y, \delta_z, \Theta_x, \Theta_y)}
\]

(107)

\[
\frac{dF'_z}{d(\delta_x, \delta_y, \delta_z, \Theta_x, \Theta_y)} = \cos \beta \sum_{q=1}^{n} \cos \phi_q \frac{dP'_1}{d(\delta_x, \delta_y, \delta_z, \Theta_x, \Theta_y)}
\]

(108)

\[
\frac{dF'_y}{d(\delta_x, \delta_y, \delta_z, \Theta_x, \Theta_y)} = \sin \beta \sum_{q=1}^{n} \frac{dP'_1}{d(\delta_x, \delta_y, \delta_z, \Theta_x, \Theta_y)}
\]

(109)

\[
\frac{dM'_x}{d(\delta_x, \delta_y, \delta_z, \Theta_x, \Theta_y)} = \sum_{q=1}^{n} \left[ \frac{1}{2} \left( E \sin \beta + d \sin \left( \frac{x}{z} \right) \right) \frac{dP'_1}{d(\delta_x, \delta_y, \delta_z, \Theta_x, \Theta_y)} \right] \sin \phi_q
\]

(110)

\[
\frac{dM'_y}{d(\delta_x, \delta_y, \delta_z, \Theta_x, \Theta_y)} = \sum_{q=1}^{n} \left[ \frac{1}{2} \left( E \sin \beta + d \sin \left( \frac{x}{z} \right) \right) \frac{dP'_1}{d(\delta_x, \delta_y, \delta_z, \Theta_x, \Theta_y)} \right] \cos \phi_q
\]

(111)

where

\[
\frac{dP'_1}{d(\delta_x, \delta_y, \delta_z, \Theta_x, \Theta_y)} = \left[ \frac{dP'_1}{d\delta_x^0} \frac{d\delta_x^0}{d\delta_x^1} \frac{dP'_1}{d\delta_y^0} \frac{d\delta_y^0}{d\delta_y^1} \frac{dP'_1}{d\delta_z^0} \frac{d\delta_z^0}{d\delta_z^1} \right] \frac{d\delta_x^0}{d(\delta_x, \delta_y, \delta_z, \Theta_x, \Theta_y)}
\]

\[
+ \left[ \frac{dP'_1}{d\delta_x^0} \frac{d\delta_x^1}{d\delta_x^0} + \frac{dP'_1}{d\delta_y^0} \frac{d\delta_y^1}{d\delta_y^0} + \frac{dP'_1}{d\delta_z^0} \frac{d\delta_z^1}{d\delta_z^0} \right] \frac{d\delta_x^1}{d(\delta_x, \delta_y, \delta_z, \Theta_x, \Theta_y)}
\]

(112)
\[
\frac{dM}{d(\delta_x, \delta_y, \delta_z, \theta_{x}, \theta_{y}, \theta_{z})} = \left[ \frac{dM}{d\delta_{1}} \frac{d\Delta_1}{d\delta_{1}} + \frac{dM}{d\theta_{1}} \frac{d\theta_1}{d\delta_{1}} \right] \frac{d\delta_{1}}{d(\delta_x, \delta_y, \delta_z, \theta_{x}, \theta_{y}, \theta_{z})}
\]

(113)

The derivatives of \( \Delta_1 \) and \( \theta_1 \) with respect to the inner-ring displacements are, from Equations (40) and (42)

\[
\frac{d\Delta_1}{d\delta_{1}} = \cos \delta \cos \theta
\]

(114)

\[
\frac{d\Delta_1}{d\delta_{1}} = \cos \theta \sin \theta
\]

(115)

\[
\frac{d\Delta_1}{d\delta_{1}} = \theta
\]

(116)

\[
\frac{d\Delta_1}{d\theta_{1}} = \frac{1}{2}(E \sin \delta + d \sin(\frac{\delta}{2})) \sin \theta
\]

(117)

\[
\frac{d\theta_{1}}{d\theta_{1}} = \frac{1}{2}(E \sin \delta + d \sin(\frac{\delta}{2})) \cos \theta
\]

(118)

\[
\frac{d\theta_{1}}{d(\delta_x, \delta_y, \delta_z, \theta_{x}, \theta_{y}, \theta_{z})} = 0
\]

(119)

\[
\frac{d\theta_{1}}{d\delta_{1}} = \sin \theta
\]

(120)

\[
\frac{d\theta_{1}}{d\delta_{1}} = \cos \theta
\]

(121)

2.4.5 Effect of Unloaded Roller

In some instances one or more rollers may be out of contact with the inner race while in contact with the outer race and the inner-ring flange. The conditions for equilibrium of such rollers are
\[-P_1 \cos \beta - P_3 \sin (\beta - \frac{T}{2} - \alpha) + F_c = 0 \tag{121}\]
\[-P_1 \sin \beta + P_3 \cos (\beta - \frac{T}{2} - \alpha) = 0 \tag{122}\]

where
\[P_3 = \frac{-M_1 - M_0 - \frac{1}{2} P_1 dsin(\frac{t}{2}) + F_c \bar{X} \cos (\beta - \frac{T}{2})}{t} \tag{123}\]

Here the variables are \(\Delta_1\) and \(\theta_1\). Initial estimates \(\Delta_1'\) and \(\theta_1'\) will generally fail to satisfy Equations (121) and (122), and there will be the residues \(\epsilon_1\) and \(\epsilon_2\).

Improved values are

\[\Delta_1 = \Delta_1' - \frac{\epsilon_1}{\frac{d\epsilon_1}{d\theta_1}} \frac{d\Delta_1}{d\theta_1} \frac{d\epsilon_2}{d\theta_1} \tag{124}\]

\[\theta_1 = \theta_1' - \frac{\epsilon_1}{\frac{d\epsilon_1}{d\theta_1}} \frac{d\theta_1}{d\theta_1} \frac{d\epsilon_2}{d\theta_1} \tag{125}\]

The right members of Equations (124) and (125) are evaluated at current estimates. Iteration of Equations (124) and (125) yield \(\Delta_1\) and \(\theta_1\) to any desired accuracy.
The derivatives required in Equations (124) and (125) are

\[
\frac{dc_1}{d\Delta_1} = -\cos \beta \frac{dP_1}{d\Delta_1} - \sin (\beta - \frac{\pi}{2} - \alpha) \frac{dP_3}{d\Delta_1}
\]

(126)

\[
\frac{dc_1}{d\theta_1} = -\cos \beta \frac{dP_1}{d\theta_1} - \sin (\beta - \frac{\pi}{2} - \alpha) \frac{dP_3}{d\theta_1}
\]

(127)

\[
\frac{dc_2}{d\Delta_1} = -\sin \beta \frac{dP_1}{d\Delta_1} + \cos (\beta - \frac{\pi}{2} - \alpha) \frac{dP_3}{d\Delta_1}
\]

(127)

\[
\frac{dc_2}{d\phi_1} = -\sin \beta \frac{dP_1}{d\phi_1} + \cos (\beta - \frac{\pi}{2} - \alpha) \frac{dP_3}{d\phi_1}
\]

(128)

where

\[
\frac{dP_3}{d\Delta_1} = -\frac{\frac{dM_1}{d\Delta_1} - \frac{1}{2} dsin(\frac{\pi}{2})}{\frac{dP_1}{d\Delta_1}}
\]

(129)

\[
\frac{dP_3}{d\phi_1} = -\frac{\frac{dM_1}{d\phi_1} - \frac{1}{2} dsin(\frac{\pi}{2})}{\frac{dP_1}{d\phi_1}}
\]

(130)

Rollers which are out of contact with the inner race must be considered in evaluating the bearing's reactions. They, however, contribute nothing to the stiffness matrix since \( P_1 \) and \( M_1 \) for these rollers do not change with changes in the inner-ring displacements.
SECTION III
APPLICATION OF COMPUTER PROGRAM

The analysis of Section II has been programmed in Fortran IV for a digital computer and is suitable for use on the CDC 6600. A program listing is presented in the Appendix.

3.1 Sample Test Case

To illustrate a typical case consider the bearing in Figure 11. This is a tapered roller bearing assembly modified for high speed operation. The geometry of this sample bearing is summarized below.

- Number of rollers: 37
- Roller diameter at midpoint: .2913 in.
- Pitch diameter: 5.0 in.
- Contact angle at outer race: 14°40'
- Effective length of roller: .6001 in.
- Roller big-end spherical radius: 0.8 in.
- Radius from roller centerline to point of big-end spherical surface with inner race flange: 0.75 in.
- Roller crown radius: 100 in.
- Roller small-end corner break: .02 in.
- Roller big-end corner break: .03 in.
- Crown drop gage point: .03 in.

The operating conditions for the sample case are:

- Rotational speed: 20,000 rpm
- Load Condition #1
  - Thrust Load: 3,000 lbs.
- Load Condition #2
  - Thrust Load: 3,000 lbs.
  - Radial Load: 700 lbs.
Figure 11. Sample Tapered Roller Bearing Assembly
3.2 Input Format

Figure 12 presents the input data format and Figure 13 shows the actual input data for Load Conditions #1 and #2 of the sample case.

3.3 Output Format

Figure 14 presents the output data for Load Condition #1. The input data are summarized in Figure 14, followed by the output data including the internal load distribution as well as various other stress and displacement parameters. The stiffness matrix is given on the last page of Figure 14.

The output data for Load Condition #2 are presented in Figure 15.
<table>
<thead>
<tr>
<th>Number of Rolls (50 maximum)</th>
<th>Roll Diameter at Midpoint - Inches</th>
<th>Pitch Diameter - Inches</th>
<th>Contact Angle at Outer Race - Degree (Must Be Positive)</th>
<th>Total Length of Roll - Inches</th>
<th>Effective Length of Roll - Inches</th>
<th>Length of Flat Portion of Roll Working Surface - Inches</th>
<th>Roll Big End Surface Spherical Radius - Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>TITLE</td>
<td>Radius from Roll Centerline to Point of Big End Sphere Surf with Inner Race Flange</td>
<td>Roll Crown Radius - Inches</td>
<td>Roll Crown Drop - Inches</td>
<td>Roll Small End Corner P Rear Inches</td>
<td>Roll Big End Corner Break Inches</td>
<td>Crown Drop Cage Point - Inches</td>
<td>Geometrical Clearance - Inches</td>
</tr>
<tr>
<td>TITLE</td>
<td>Modulus of Elasticity Outer Ring - lb/in²</td>
<td>Modulus of Elasticity Inner Ring - lb/in²</td>
<td>Modulus of Elasticity Rolls Outer Ring (if blank assume 0.25)</td>
<td>Poisson's Ratio Inner Ring</td>
<td>Poisson's Ratio Rolls</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RPM - Outer Ring</td>
<td>RPM - Inner Ring</td>
<td>Force Along x - lb</td>
<td>Force Along y - lb Must Be Negative</td>
<td>Initial Displacement Along x - Inches</td>
<td>Initial Displacement Along y - Inches</td>
<td>Initial Displacement Along z - Inches</td>
<td>Initial Displacement About x - Radians</td>
</tr>
<tr>
<td>Initial Displacement About y - Radians</td>
<td>A 1. In these fields permit a working displacement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- If total length is given omit effective length
- If effective length is given omit total length
- Enter 1 to start printout at top of next page
- If crown radius is given omit crown drop. If drop is given omit radius.
- To run additional load cases with same bearing, repeat cards 4 and 7 directly after last card 7.
- To run new system place 2 blanks after last card 7 and repeat cards 1, etc.

Figure 12. Input Data Format
### Sample Problem Data Input

<table>
<thead>
<tr>
<th>FORTRAN Statement</th>
<th>FORTRAN Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>37. 2913159 5. 14.66667 6001</td>
<td>8</td>
</tr>
<tr>
<td>SAMPLE PROBLEM - TAPERED ROLLER BEARING</td>
<td></td>
</tr>
<tr>
<td>FULLY-CROWNED ROLLS</td>
<td></td>
</tr>
<tr>
<td>75 100.</td>
<td>02 03 03</td>
</tr>
<tr>
<td>BLANK CARD</td>
<td></td>
</tr>
<tr>
<td>20000</td>
<td></td>
</tr>
<tr>
<td>700.</td>
<td>3000.</td>
</tr>
<tr>
<td>BLANK CARD</td>
<td></td>
</tr>
<tr>
<td>BLANK CARD</td>
<td></td>
</tr>
<tr>
<td>BLANK CARD</td>
<td></td>
</tr>
</tbody>
</table>
### Sample Problem: Tapered Roller Bearing Analysis

#### Design Data for Bearing No. 1

<table>
<thead>
<tr>
<th>NO.</th>
<th>ROLL ROLLS</th>
<th>DIAMETER</th>
<th>DIAMETER</th>
<th>TAPER</th>
<th>LENGTH</th>
<th>LENGTH</th>
<th>LENGTH</th>
<th>FLEET</th>
<th>VALUE</th>
<th>VALUE</th>
<th>CROWN</th>
<th>CROWN</th>
</tr>
</thead>
<tbody>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>3.7605 x 10^-1</td>
<td>2.9352 x 10^-1</td>
<td>1.387 x 10^-1</td>
<td>1.243 x 10^-1</td>
<td>1.4505 x 10^-1</td>
<td>1.3505 x 10^-1</td>
<td>1.3005 x 10^-1</td>
<td>1.3005 x 10^-1</td>
<td>1.3005 x 10^-1</td>
<td>1.3005 x 10^-1</td>
<td>1.3005 x 10^-1</td>
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<td></td>
</tr>
<tr>
<td>3.5505 x 10^-1</td>
<td>2.8552 x 10^-1</td>
<td>1.1743 x 10^-1</td>
<td>1.0577 x 10^-1</td>
<td>1.1743 x 10^-1</td>
<td>1.0577 x 10^-1</td>
<td>1.0577 x 10^-1</td>
<td>1.0577 x 10^-1</td>
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<td>1.0577 x 10^-1</td>
<td>1.0577 x 10^-1</td>
<td>1.0577 x 10^-1</td>
</tr>
</tbody>
</table>

#### Table: Spherical Inclusion

<table>
<thead>
<tr>
<th>NO.</th>
<th>ROLL ROLLS</th>
<th>DIAMETER</th>
<th>DIAMETER</th>
<th>LENGTH</th>
<th>LENGTH</th>
<th>LENGTH</th>
<th>FLEET</th>
<th>VALUE</th>
<th>VALUE</th>
<th>CROWN</th>
<th>CROWN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1.3219 x 10^-1</td>
<td>8.8002 x 10^-1</td>
<td>7.4503 x 10^-1</td>
<td>1.3156 x 10^-1</td>
<td>1.2707 x 10^-1</td>
<td>1.2707 x 10^-1</td>
<td>1.2707 x 10^-1</td>
<td>1.2707 x 10^-1</td>
<td>1.2707 x 10^-1</td>
<td>1.2707 x 10^-1</td>
<td>1.2707 x 10^-1</td>
<td>1.2707 x 10^-1</td>
</tr>
</tbody>
</table>

#### Table: Single Recess Roller in a Spherical Inclusion

<table>
<thead>
<tr>
<th>NO.</th>
<th>ROLL ROLLS</th>
<th>DIAMETER</th>
<th>DIAMETER</th>
<th>LENGTH</th>
<th>LENGTH</th>
<th>LENGTH</th>
<th>FLEET</th>
<th>VALUE</th>
<th>VALUE</th>
<th>CROWN</th>
<th>CROWN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5005 x 10^-1</td>
<td>2.5005 x 10^-1</td>
<td>2.5005 x 10^-1</td>
<td>2.5005 x 10^-1</td>
<td>2.5005 x 10^-1</td>
<td>2.5005 x 10^-1</td>
<td>2.5005 x 10^-1</td>
<td>2.5005 x 10^-1</td>
<td>2.5005 x 10^-1</td>
<td>2.5005 x 10^-1</td>
<td>2.5005 x 10^-1</td>
<td>2.5005 x 10^-1</td>
</tr>
</tbody>
</table>

---

**Figure 14:** Output Data for Load Condition #1
**Table: Input Data for Load No. 1 Bearing No. 1**

<table>
<thead>
<tr>
<th>Bearing No. 1</th>
<th>Load Applied to Inner</th>
<th>Initial Displacements of Inner with Respect to Outer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load No. 1</td>
<td>2.0000 x 30</td>
<td>0.0000 x 30</td>
</tr>
<tr>
<td>Initial</td>
<td>0.0000 x 30</td>
<td>0.0000 x 30</td>
</tr>
<tr>
<td>Displacements</td>
<td>0.0000 x 30</td>
<td>0.0000 x 30</td>
</tr>
</tbody>
</table>

**Table: Output Data for Load No. 1 Bearing No. 1**

<table>
<thead>
<tr>
<th>Bedding No. 1</th>
<th>SLP Deflection Permitted</th>
<th>Total Displacements of Inner with Respect to Outer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load No. 1</td>
<td>0.0000 x 30</td>
<td>0.0000 x 30</td>
</tr>
<tr>
<td>SLP</td>
<td>0.0000 x 30</td>
<td>0.0000 x 30</td>
</tr>
<tr>
<td>Deflection</td>
<td>0.0000 x 30</td>
<td>0.0000 x 30</td>
</tr>
</tbody>
</table>

**Figure 14. (Continued)**
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>z</th>
<th>x'</th>
<th>y'</th>
<th>z'</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1243-02</td>
<td>-0.1243-02</td>
<td>0.1243-02</td>
<td>-0.1243-02</td>
<td>0.1243-02</td>
<td>-0.1243-02</td>
</tr>
<tr>
<td>0.1243-02</td>
<td>-0.1243-02</td>
<td>0.1243-02</td>
<td>-0.1243-02</td>
<td>0.1243-02</td>
<td>-0.1243-02</td>
</tr>
<tr>
<td>0.1243-02</td>
<td>-0.1243-02</td>
<td>0.1243-02</td>
<td>-0.1243-02</td>
<td>0.1243-02</td>
<td>-0.1243-02</td>
</tr>
<tr>
<td>0.1243-02</td>
<td>-0.1243-02</td>
<td>0.1243-02</td>
<td>-0.1243-02</td>
<td>0.1243-02</td>
<td>-0.1243-02</td>
</tr>
</tbody>
</table>

**Partial Derivatives of Reactions with Respect to Displacements**

---

**Figure 14.** (Continued)
<table>
<thead>
<tr>
<th>INPUT DATA FOR LOAD No. 1, REACTION No. 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>BRIEF NO ALONG X ALONG Y ALONG Z IPOD</td>
</tr>
<tr>
<td>1 2 3 4</td>
</tr>
<tr>
<td>----------------------------------------</td>
</tr>
<tr>
<td>0.0000 0.0000 0.0000</td>
</tr>
<tr>
<td>INITIAL DISPLACEMENTS OF INNER WITH RESPECT TO OUTER CARTILAGE REACTION</td>
</tr>
<tr>
<td>ALONG X ALONG Y ALONG Z</td>
</tr>
<tr>
<td>1 2 3 4</td>
</tr>
<tr>
<td>----------------------------------------</td>
</tr>
<tr>
<td>0.0000 0.0000 0.0000 0.0000</td>
</tr>
<tr>
<td>TOTAL DISPLACEMENTS OF INNER WITH RESPECT TO OUTER CARTILAGE REACTION</td>
</tr>
<tr>
<td>ALONG X ALONG Y ALONG Z</td>
</tr>
<tr>
<td>1 2 3 4</td>
</tr>
<tr>
<td>----------------------------------------</td>
</tr>
<tr>
<td>0.0000 0.0000 0.0000 0.0000</td>
</tr>
</tbody>
</table>

**Figure 15. Output Data for Load Condition #2**
<table>
<thead>
<tr>
<th>Cylindrical Number</th>
<th>Contact Length (in.)</th>
<th>Maximum Contact Width (in.)</th>
<th>Maximum Contact Deflection (in.)</th>
<th>Maximum Hertz Stress (ksi)</th>
<th>Location of Max. Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.177</td>
<td>-1.357</td>
</tr>
<tr>
<td>2</td>
<td>3.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.177</td>
<td>-1.357</td>
</tr>
<tr>
<td>3</td>
<td>3.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.177</td>
<td>-1.357</td>
</tr>
<tr>
<td>4</td>
<td>3.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.177</td>
<td>-1.357</td>
</tr>
<tr>
<td>5</td>
<td>3.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.177</td>
<td>-1.357</td>
</tr>
<tr>
<td>6</td>
<td>3.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.177</td>
<td>-1.357</td>
</tr>
<tr>
<td>7</td>
<td>3.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.177</td>
<td>-1.357</td>
</tr>
<tr>
<td>8</td>
<td>3.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.177</td>
<td>-1.357</td>
</tr>
<tr>
<td>9</td>
<td>3.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.177</td>
<td>-1.357</td>
</tr>
<tr>
<td>10</td>
<td>3.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.177</td>
<td>-1.357</td>
</tr>
<tr>
<td>11</td>
<td>3.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.177</td>
<td>-1.357</td>
</tr>
<tr>
<td>12</td>
<td>3.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.177</td>
<td>-1.357</td>
</tr>
<tr>
<td>13</td>
<td>3.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.177</td>
<td>-1.357</td>
</tr>
<tr>
<td>14</td>
<td>3.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.177</td>
<td>-1.357</td>
</tr>
</tbody>
</table>

Figure 15. (Continued)
<table>
<thead>
<tr>
<th>Reactions</th>
<th>Partial Derivatives</th>
<th>with Respect to Displacements</th>
</tr>
</thead>
<tbody>
<tr>
<td>D&lt;sub&gt;0&lt;/sub&gt;/D&lt;sub&gt;T&lt;/sub&gt;</td>
<td>D&lt;sub&gt;0&lt;/sub&gt;/D&lt;sub&gt;L&lt;/sub&gt;</td>
<td>D&lt;sub&gt;0&lt;/sub&gt;/D&lt;sub&gt;Y&lt;/sub&gt;</td>
</tr>
<tr>
<td>1.5498±0.7</td>
<td>1.869±0.2</td>
<td>1.869±0.2</td>
</tr>
<tr>
<td>1.5498±0.7</td>
<td>1.869±0.2</td>
<td>1.869±0.2</td>
</tr>
<tr>
<td>1.5498±0.7</td>
<td>1.869±0.2</td>
<td>1.869±0.2</td>
</tr>
<tr>
<td>1.5498±0.7</td>
<td>1.869±0.2</td>
<td>1.869±0.2</td>
</tr>
</tbody>
</table>

Figure 15. (Continued)
APPENDIX

COMPUTER PROGRAM FOR CALCULATING STIFFNESS MATRIX OF TAPERED ROLLER BEARING
<table>
<thead>
<tr>
<th>CARD</th>
<th>COL.</th>
<th>VIEW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>1-10</td>
<td>NUMBER OF ROLLS — 60 MAXIMUM</td>
</tr>
<tr>
<td>11-20</td>
<td>1-20</td>
<td>ROLL DIAMETER — IN. MEASURED AT MIDPOINT OF EFFECTIVE LENGTH. SEE Cols. 51-60 OF THIS CARD</td>
</tr>
<tr>
<td>21-30</td>
<td>21-30</td>
<td>PITCH DIAMETER — IN</td>
</tr>
<tr>
<td>31-40</td>
<td>31-40</td>
<td>CONTACT ANGLE AT OUTER RACE — DEG. MUST BE POSITIVE</td>
</tr>
<tr>
<td>51-60</td>
<td>51-60</td>
<td>EFFECTIVE LENGTH OF ROLL — IN. THE MAXIMUM WORKING LENGTH OF THE ROLL MEASURED ALONG THE ROLL CONE ELEMENT BETWEEN CORNER BREAKS</td>
</tr>
</tbody>
</table>

**NOTE**

IF TOTAL LENGTH IS GIVEN OUTF EFFECTIVE LENGTH

IF EFFECTIVE LENGTH IS GIVEN OUTF TOTAL LENGTH

| 61-70 | 61-70 | LENGTH OF FLAT PORTION OF ROLL WORKING SURFACE MEASURED ALONG THE ROLL ELEMENT — IN. FOR A FULLY-CROWNED ROLL THE FLAT LENGTH IS ZERO |
| 71-80 | 71-80 | ROLL BIG-END SURFACE SPHERICAL RADIUS — IN. IF NEGATIVE ITS ABSOLUTE VALUE MULTIPLIES THE ROLL CONE SLANT HEIGHT MEASURED FROM THE APEX TO THE SHARP INTERSECTION WITH THE BIG-END SURFACE TO GIVE THE SPHERICAL RADIUS |

| 2 | 1 | PUNCH 1, TO START THE PRINTOUT AT THE TOP OF A NEW PAGE |
| 2-80 | 2-80 | TITLE CARD, PUNCH ANYTHING |
| 3 | 1 | LEAVE BLANK |
| 4-80 | 4-80 | TITLE CARD, PUNCH ANYTHING |

| 4 | 1-10 | RADIUS FROM ROLL CENTERLINE TO POINT OF CONTACT OF BIG-END SPHERICAL SURFACE WITH THE INNER-RACE FLANGE — IN. IF NEGATIVE ITS ABSOLUTE VALUE MULTIPLIES THE BIG-END RADIUS FROM THE ROLL CENTERLINE TO THE SHARP INTERSECTION OF THE BIG-END SURFACE AND THE PROJECTED ROLL CONE ELEMENT TO GIVE THE DESIRED RADIUS |

| 11-20 | 11-20 | ROLL CROWN RADIUS — IN |

| 21-30 | 21-30 | ROLL CROWN DROP — IN. MEASURED FROM GAGE POINT. SEE Cols. 51-60 OF THIS CARD |

**NOTE**

IF CROWN RADIUS IS GIVEN OUTF CROWN DROP. IF DROP IS
GIVEN OMIT CROWN RADIUS

31-4u Roll small-end corner break - tv. view roll with axis horizontal and with big end at left. small-end corner break is the axial distance between the sharp intersection of the small end surface with the roll-cone element and the rightmost round of the effective length.

41-5u Roll big-end corner break - tv. view roll with axis horizontal and big end at left. big-end corner break is the axial distance between the sharp intersection of the big-end spherical surface with the roll-cone element and the leftmost round of the effective length.

51-6u Crown drop gage point - tv. reference point for the measurement of crown drop - tv. It is the distance measured along the projected roll-cone element from the outer edge of the effective length to the point of crown drop measurement. It is the same for both ends of the roll.

61-7u Diametral clearance - tv. the total diametral looseness or shake in the mounted bearing before loading. A negative value indicates tightness.

71-8u Roll material density - lb/in**3

5 1-10 Modulus of elasticity for outer ring - lb/in**2

11-2u Same for inner ring

21-3u Same for rolls

31-4u Poisson's ratio for outer ring

41-5u Same for inner ring

51-6u Same for rolls

6 1-10 RPM of outer ring

11-2u RPM of inner ring

21-3u Force along x - lb. OBSERVE SIGN

31-4u Force along y - lb. MUST BE NEGATIVE

41-5u Initial displacement along x - tv

51-6u Initial displacement along y - tv

61-7u Initial displacement along y - tv
INITIAL DISPLACEMENT ABOUT X - 0.0;
INITIAL DISPLACEMENT ABOUT Y - 0.0;
1-10 HERE PERMITS OPERATING DISPLACEMENTS ALONG Y.
11-20 HERE PERMITS OPERATING DISPLACEMENTS ALONG X.
21-30 HERE PERMITS OPERATING DISPLACEMENTS ALONG Y.
31-40 HERE PERMITS OPERATING DISPLACEMENTS ALONG X.

TO RUN ADDITIONAL LOAD CASES WITH THE SAME BEARING REPEAT
CARDS 6 AND 7 AS A UNIT DIRECTLY AFTER LAST CARD 7.

TO RUN NEW SYSTEM PLACE TWO BLANKS AFTER LAST CARD 7.

TO STOP PLACE THREE BLANKS AFTER LAST CARD 7.

COMMON ALPHA,AA(5,5),AL(5,5),AGIG, RFETA,R1,R2,RTA,BMTAU,BXTOP,
1RT2AI,GBTMP,RA(2,60), C(2),CROWN,CUTA,CTAU02,COMIU,COMJ02,
2CA2AI,CPH,COPR(5),CTH,COR(5), DMRP,DELU1(5),DF11(5),DFL(5),
3C7TV(5,5),DSCAL,DEL(2),DPEL(2),DVPEL(2),DPML(2),DMTH(2),HELPX,
4DTH,DX,DLX(2,60),DELSAV(2,60),DIPL(5),DIPL(5),STORE,DELX,
5R,REL2,ERR(5),EX,EX,CR(5), FRSL,FLTR,FRIE,FC, CAGE,JAM,
6NSTMG,NMGX, H1,H2,HT2Z(2,60),HR7,EI, TEDSTOP,IL2O,D1QUT,
7ICT,JPASS, KKK, NNOLOAD, ONE,OMP, PD,PR(2),PRP,PX(2),
8P(2,60),P3,PX3(60),P1OJT,P3OJT, RS,"HO,"MHL,"R1,"R2,"R3", COMB.
9CMTA,TSHA02,SLANT,SBMTA,TBAT2AL,SBAV1,SAV2,SAV3,SPH,STH,
10THD1(5),TAU02,TAU,TITA02,TITA02,TOL(5),THETA,TH2(2),TAH,
1THA(2,60),THET,TPRM00, VV, VV, WEIGHT, XN, XLT,YLE,XTA,
2XL02,XXL02,XXA,XXA,SMS1,XXS2,XXS1,XXS2,XXS1,IXCG,IXCG,IXCG,
3XAR02,XXAR,XXLEVER,XXALPHA,XY(2),XY(2),XY(2),XTH,YNEL,YX1,YX2,
4XN(12,60),XX(12,60),XNSAV(2,60),XNOUT,YX0(2),XH8IG, YM(2),
6YR
DOUBLE PRECISION 3B6MP,CTH,DELTA,DEL,DPML,DMTH,G2X,HQ,
1PX,P3,ST,THETA,TH,TAH,TH,TAH,TH,TAH,TH,TAH,TH,TAH,TH,TAH,
IRR=0.
C(1)=1.
C(2)=1.
10  READ(5,20)XN,DF,RETA,XLT,XLE,FLT,RS
20  FORMAT(AE10,0)
    IF(XN.EQ.0.)STOP
    READ(5,30)
30  FORMAT(AN)
    /AUH
    WRITE(6,30)
    READ(5,20)V,CROWN,DROP,A1,A2,GASE,PD,RHO,YM(1),YY(2),YMR,PR(1),PR(2),PRR
    ICT=0
    ISTOP=0
    I4R=I9R+1
    ILOAD=0
    BTA=BETA/57.29578
    SRTA=SIN(BTA)
    COTA=COS(BTA)
    DELD1(1)=SRTA
    TH01(1)=0.
    TH01(2)=0.
    TH01(3)=0.
    N=NX
    YM=29.56E
    PPR=.25
    DO 40 K=1,2
    YM(K)=29.56
    PR(K)=.25
40  EL(K)=636619A*(1.-PRR**2)/YMR+(1.-PR(K)**2)/YM(K)
    IF(RHO.EQ.0.)RHO=2.83
    GAM=D/E
    TAUO2=U.
    DO 50 I=1,30
    TEMP=TAUO2
    TAUO2=ATAN(GAM*SIN(BTA-1AUO2))
    IF(ABS(TAUO2-TEMP).LT.5.E-7)70,50,50
50  CONTINUE
    WRITE(6,60)
60  FORMAT(1AHU,MAIN PROGRAM 60)
    ISTOP=1
LET U=2.*TAU0
YTAU=57.23/7A+TAU
STA02=ST(I,TAU0)
CTAU0=CT(UL,TAU0)
YTAU0=CTAU0/CTAU0
IF(XLT)<100.00
X2=X*CTAU02+91+32
GO TO 100
XLT=(XLT-9-9)/CTAU02
100 XL02=.5*XLT
XLE02=.5*XLE
FLT02=1.5*FLT
CALL P=ALC(CROWN,DROP,FLT01,ALG=HR,XLE02)
XJ=2.5*J/CTAU02
H2=.5*J/XLE02+CTAU02+Y2AR*STAI02-=d1
H2=.5*J/XLE02+CTAU02+Y2AR*STAI02+92
R1=H2*ITAU02
R2=H2*ITAU02
XAM51=2.71039F-3+R1**2+H1**43
XAM52=2.71039F-3+R2**2+H2**43
XAM52=XAM52*YAM51
XNC6=1.6*XAM52*(11**2/4.*H1**2/16.)
X3AR=1/r1/4.
XAR2=H2/4.
X3AR=(XAM52=2*XAR2-XAM51*(XLT+XAR1)*1/YAM5S
XNC6=XNC6+YAM52*(Y2AR=2*XAR2)*2-XNC61-XAM51*(XLT+XAR1-X3AR)
1**2
X3AR=H2-X3AR=H0
SLANT=R2*/STAU02
IF(VLTV,0)V=AR5(V)*R2
IF(RS,LT,0.)RS=ABS(RS)*SLANT
ALPHA=ASIN(V/PS)
XALPHA=ALPHA*57.29578
XLEVER=(H0-H2+SQRT(RS**2-R2**2))*V/RS
R*TAU=B7A-1AU
SRMTAU=SIN(RMTAU)
CMTAU=COS(RMTAU)
RMT02=BTATAU
SRMT02=SIN(RMT02)
CMT02=CMT02
GAMM=CMT02*GAM
RT2AL=MT02+ALPHA
SR2AL=SIN(RT2AL)
CM2AL=COS(RT2AL)
RT2AL=SR2AL/CM2AL

WEIGHT=MAX5*#86.4
TOL(1)=5.5-7
TOL(2)=5.5-7
TOL(3)=1.5-6
TOL(4)=1.5-6
TOL(5)=1.5-6
WRITE(6,110)I=R

110 FORMAT(30HU) DESIGN DATA FOR BEARING NO.,C3)
WRITE(6,120)
120 FORMAT(129940 NO. OF ROLL PITCH CONTACT TOT )
1AL EFFECTIVE FLAT VALUE VALUE CROWN
2 CROWN/129H ROLLS DIAMETER DIAMETER ANGLE L
3 LENGTH LENGTH LENGTH OF B1 OF B2 RADIUS
4 DROP/18X2IN,10X2HIN,9X7HDEG <7(12H IN))
WRITE(6,130)XN,D,BETA,XLT*XLE,FLR,RL,CROWN,DROP
130 FORMAT(1P11E12.4)
WRITE(6,140)
140 FORMAT(1259H VALUE SPHERICAL INCLUDED VALUE OF LOCATI)
1ON OF ROLL MOM. OF IN. ROLL MODULUS OF ELASTI
2CITY/129H OF V. END RADIUS ROLL ANGLE ALPHA CENTRO
3D WEIGHT ABOUT C.G. DENSITY OUTER INNER
4 ROLLS/131H IN IN deg deg deg deg deg
5N LB/IN*SEC*2 LB/IN*2 LB/IN*2 LB/IN*2
6 LB/IN*2)
WRITE(6,130)VR,STAU,XALPHA,XBAR,*WEIGHT,XINC,PHON,YM(1),YM(2),YMR
WRITE(6,150)
150 FORMAT(47HU) POISSON'S RATIO DIAMETRAL/47H 0
1UTER INNER ROLLS CLEARANCE/42X2HIN)
WRITE(6,130)PP(1),PP(2),PPR,PD
160 READ(5,20)RPM(1),RPM(2),XF(2),XF(1),DFL11(2),DFL11(3),DFL11(4),DFL11(5),FREE(2),FREE(3),FREE(1)
IF(ABS(RPM(1))+ABS(RPM(2))*.9,.0)GO TO 10
IF(ISTJP*,ST,0)GO TO 160
LOAD=LOAD*1+1
QUIT=0
TEMP1=0.05*O/5RATA
TEMP2=0.05*O/5RATA
DFL(1)=0,
DFL(2)=1
DFL(3)=0.
IF(XF(1),N,E,0,0)DFL(1)=SIGN(TEMP1*XF(1))
IF(XF(2),N,E,0,0)DFL(2)=SIGN(TEMP2*XF(2))
OME=5.5*(RPM(1)*(1.+GAM)+RPM(2)*(1.-GAM))
OMR=5.5*(RPM(1)-RPM(2))*(1.-GAM)**2/GAM
FC=MASS*5.5*(E+2.*XBAR**SRMT02)*(1.0+HAR*OME)**2
GM=XINC*OME*OMR*SRMT02*1.096623E-2
CALL OUTCON
IF(IQUIT,.EQ,0,1)GO TO 160
WRITE(0,165)LOAD,IBR
165 FORMAT(26H1 INPUT DATA FOR LOAD NO.,13,12H READING NO.,13)
WRITE(6,170)
170 FORMAT(13H0 RPM OF RPM OF LOADS APPLIED TO INNER IN
INITIAL DISPLACEMENTS OF INNER WITH RESPECT TO OUTER ORBITAL
2 ROTATIONAL/13H OUTER INNER ALONG X ALONG Y ALONG Z
3 ALONG X ALONG Y ALONG Z ABOUT X ABOUT Y VELOC
ITY VELOCITY/3H X9RHL9 LA IN IN
5 IN RADIANS RADIANS RPM RPM)
WRITE(6,130)RPM(1),RPM(2),XF(2),XF(1),DFL11(2),DFL11(3),DFL11(4),DFL11(5),FREE(2),FREE(3),FREE(1)
WRITE(6,180)GM,FC,FREE(2),FREE(3),FREE(1)
180 FORMAT(5H15H0 GYRO CENTRIFUGAL 1. = DEFLECTION PERMITTED/15AH MOTION FORCE ALONG X ALONG Y ALONG Z/20H
2 1B*IN LAS/1PS*12*4)
KKK=0
DO 190 K=1,3
IF(FREE(K),GT,0.)KKK=KKK+K**2
190 CONTINUE
IF(KKK,.EQ,0,0)GO TO 330
IF(FREE(1),.EQ.,0.) GO TO 330
SAV1=DFL(1)
SAV2=DFL(2)
SAV3=DFL(3)
DFL(1)=0.
IF(KKK.NE.,1) GO TO 210
IF(FCS3TA/CTA.LT.-XF(1)/XN) GO TO 320
WRITE(6,220)
200 FORMAT(A240 EXTERNAL THRUST IS NOT SUFFICIENT TO BALANCE INDUCED
1THRUST - PROBLEM ABANDONED)
GO TO 160
210 DO 290 ITX=1,20
NOLOAD=0
DO 220 K=1,5
XF1(K)=0.
DO 220 L=1,5
220 DTV(L*.K)=0.
DO 250 J=1,N
Y(J)=0.
JPASS=J
P1=6.23185*(XJ-1.)/XN
SPH=SIN(PHI)
CPH=COS(PHI)
CALL ROL01U
IF(1001)250,250,230
230 WRITE(6,240)ITX,J,(DFL(K),K=1,3)
240 FORMAT(19HU MAIN PROGRAM 240,216,1P3F1?,4)
60 TO 160
250 CONTINUE
IF(KKK.GT.,51) GO TO 260
F2=XF1(2)+XF(2)
COR2=EX2/DTV(2,2)
DFL(2)=DFL(2)-COR2
IF(ABS(COR2)-TOL(4))310,290,290
260 IF(KKK.GT.,10) GO TO 270
COR3=XF1(3)/DTV(3,3)
DFL(3)=DFL(3)-COR3
IF(ABS(COR3)-TOL(5))310,290,290
270 ED2=XF1(2)+XF(2)
X 14F1(3)
X I = X1 TV(2, 3) + XTV(3, 3) - XTV(3, 2) + XTV(2, 3)
IQR 2 = (ER2 + XTV(3, 3) - ER2 + XTV(2, 3))/
IQR 3 = (DT 2 + XTV(2, 2) + ER2 - DTV(7, 2) - ER2)/
FL(2) = FL(2) + COR 2
FL(3) = FL(3) + COR 3
1F (ABS(CO2) - TOL(4)) ARU, 290, 290
280 IF (ABS(COR 3) - TOL(5)) 31U, 290, 290
290 CONTINUE
WRITE (5, 300) KKK, (FL(K), K = 1, 3)
300 FORMAT (1AHU VAIN PROGRAM 300, I6, 1P3E12, 4)
GO TO 160
310 IF (XF1(l), LT, XE(1)) GO To 320
WRITE (6, 290)
GO TO 160
320 FL(1) = SAV 1
FL(2) = SAV 2
FL(3) = SAV 3
330 DO 530 IEXK = 1, 2
VLOAD = N
SN 340 K = 1, 5
iffin(K) EN.
SN 340 N = 1, 5
340 DT(V(L, K)) = N.
DO 370 L = 1, N
UPASS = J
X = U
P = 6.283185*(XJ-1) / XN
SPH = SIN (PHI)
CPH = COS (PHI)
CALL ROLLUP
IF (IQUIT) 70, 770, 350
350 WRITE (6, 360) IFR, KKK, J, (FL(K), K = 1, 3)
360 FORMAT (1A1H VAIN PROGRAM 360, I6, 1P3E12, 4)
GO TO 160
370 CONTINUE
Ifr (KKK = F, 0, U) GO TO 550
IF (KKK + GT, 1) GO TO 380
COR 1 = (XF1(1) + XF(1))/DTV(1, 1)
DFL(1)=DFL(1)-COR1
370 IF(ABS(COR1)-TOL(3))550,530,530
380 IF(KKK.GT.4)GO TO 390
390 COR2=(XFI(2)+XF(2))/DTV(2,2)
390 DFL(2)=DFL(2)-COR2
390 IF(ABS(COR2)-TOL(4))550,530,530
390 IF(KKK.GT.5)GO TO 410
390 ER1=XFI(1)+XF(1)
390 ER2=XFI(1)+XF(1)
390 DFT=DTV(1,1)*DTV(2,2)-DTV(2,1)*DTV(1,2)
390 COR1=(ER1*DTV(2,2)-ER2*DTV(1,2))/DET
390 COR2=(DTV(1,1)*ER2-DTV(2,1)*ER1)/DET
390 DFL(1)=DFL(1)-COR1
390 DFL(2)=DFL(2)-COR2
400 IF(ABS(COR1)-TOL(3))400,530,530
410 IF(KKK.GT.9)GO TO 420
410 COR3=XTF(1)/DTV(3,3)
410 DFL(3)=DFL(3)-COR3
410 IF(ABS(COR3)-TOL(5))550,530,530
420 IF(KKK.GT.10)GO TO 380
420 ER1=XFI(1)+XF(1)
420 ER3=XFI(3)
420 DFT=DTV(1,1)*DTV(3,3)-DTV(3,1)*DTV(1,3)
420 COR1=(ER1*DTV(3,3)-ER2*DTV(1,3))/DET
420 COR3=(DTV(1,1)*ER3-DTV(3,1)*ER1)/DET
420 IF(ABS(COR1)-TOL(3))430,530,530
430 IF(ABS(COR3)-TOL(5))550,530,530
440 IF(KKK.GT.13)GO TO 460
440 ER2=XFI(2)+XF(2)
440 ER3=XFI(3)
440 DFT=DTV(2,2)*DTV(3,3)-DTV(3,2)*DTV(2,3)
440 COR2=(ER2*DTV(3,3)-ER3*DTV(2,3))/DET
440 COR3=(DTV(2,2)*ER3-DTV(3,2)*ER2)/DET
440 DFL(2)=DFL(2)-COR2
440 DFL(3)=DFL(3)-COR3
450 IF(ABS(COR2)-TOL(4))450,530,530
450 IF(ABS(COR3)-TOL(5))550,530,530
460 ERR(1)=XFI(1)+XF(1)
DOUBLE PRECISION AGTMCP, CT, DELTA, DEL, DPH, GMCG, GMTH, GMX, HD
1, PX, PX, ST, THETA, TH, TANF, VV, XM, XTH, XDEL
DOUBLE PRECISION A1D, A2D, A3D, A4D, ATAN, BOPT, AMTA2D, BT2ALD, CRN,
1CTAUO2D, CATAD, CMAT2D, CMATUD, CAT2AD, DDEL, DTH, DSNRT, DSNNOCOS,
20YD, DBUG, NED, ELD, ELD (2), ELD, ELD, FLT2D, FCO, GM, PS1, PS2, PHDEL, P3TH
3, PS1DEL, PS1TH, PS2DEL, PS2TH, P3DEL1, P3TH1, P3DEL2, P3TH2, PDEL, STA2D,
4SATTAD, SATAD, SATEOD, SAT2AD, SMIND, SDN, TAU2O2D, TOLO (2), TEMP, TEMPO2O2D,
5X2AR, XLEV, XLO2O2D, XINC, XHD, XLO, X2D, Y1D, Y2N
DIMENSION PXS (2), XMS (2), NPDELS (2), DMDELS (2), NPHTS (2), NTMHS (2)
IF (ICT, GT, U) GO TO 5
ICT=1
BTAD=ATA
RMT2O2D=MT2O2
BMTAUD=MTAUD
BT2ALD=AT2AL
CRN=CROW
CTAUO2D=OCOS (TAUO2D)
CMATAD=CMATAD
CMATAUD=CMATAUD
CMAT2AD=OCOS (BT2ADO)
D7=0
E7=E
ELD (1)=EL (1)
ELD (2)=EL (2)
FLT2O2D=FLT2O2
ST2AD=DS1N (TAUO2D)
STAD=DS1N (ATAD)
SMAT2O2D=DS1N (MT2O2)
SMATAD=DS1N (MTAUD)
ST2ADO=DS1N (AT2ADO)
TAU2O2D=TAU2O2
TOLO (1)=TOL (1)
TLOO (2)=TLOO (2)
XPAR=XPAR
XLEVRO=XLEVRO
XLO2O2D=XLO2O2D
J=JPASS
F*0=FC
G/G=GM
VV=COS(ALPHA-TAU0)/COS(ALPHA+T(1/2))
DELTA=(DFL(1)+DFL(11(1)))*SRTA+5*(E*SRTA+0+STA02)*(DFL(11(4))*SPH+
1DFL(11(5))*CPH)+(DFL(2)+DFL(11(2)))*CPH+(DFL(3)+DFL(11(3)))*SPH+5*P)
2*SRTA
DFL(1)=.5540*DELTA
DFL(2)=(DELTA-DFL(1))*VV
THETA=DFL(11(4)*SPH+DFL(11(5))*CPH
TH(1)=.5014*THETA
TH(2)=THETA-TH(1)
60 140 IT=1,2,6
60 120 K=1,2
60 PY(K)=U0,00
60 X"(K)=U0,00
60 ANDEL(K)=0,00
60 BDEL(K)=0,00
60 ANTH(K)=0,00
60 BTH(K)=0,00
60 YTH=TH(K)
60 ETA=COS(X1H)
60 STH=SECTION(X1H)
60 T=STH/STH/STH
60 Y"EL=DEL(K)
C00 X1=REAL(CR4+CTH,FLT02D,HD*XK5,STH*X1,Y2D,XDFL*XLE*2D)
10 U=(KM)2,2,10
20 P(L,J)=P1OUT
30 X"M(1,J)=XM10UT
30 PX(1)=PIOUT
30 X"(1)=XM10UT
30 P(J)=0
30 PYS(1)=PIOUT
30 X"S(1)=XM10UT
30 NEXT=10
30 GOTO 270
20 EFLD=X10-X2D
20 XINC=DFL/30,00
20 XXY(1,K,J)=X15
20 XXY(2,K,J)=X2D
XHD=XID+XINCD
S'MINCD=1.0
S'TEMP=0.0
DO 110 L=1,31
SMD=3.00-S'MINCD
S'MINCD=S'MINCD
IF((L.EQ.1).OR.(L.EQ.31))SMD=1.00
XHD=XHD-XINCD
DELXD=DELX+XHD*TANH
D7THD=XHD/CTH**2
IF(DABS(XHD).LE.FLT020*CTH)GO TO 60
TEMPD=XHD-HD*STH
TEMP1D=CRN**2-TEMPD**2
IF(TEMP1D)>0.0050
30 I=QUIT=1
WRITE(6,40)IBR,ILOAD,IT,II,L,J,L,(DFL(W),M=1..3)
40 FORMAT(12H0,ROLOAD 40,6I6,1P3E12.4)
RETURN
50 TEMP1D=DSQRT(TEMP1D)
DELXD=TEMP1D-HD*CTH-XDEL
D7THD=TEMPD-HD*CTH/TEMP1D+HD*STH
60 IF(DELXD.LT.1.00-80)GO TO 110
DXD=DD+2.00*XHD*STAU2D/CTAU2D
EXD=ED+2.00*XHD*STAU2D/DD*CBMT2D=DXD*CATD
G*DXD*CATD/EXD
IF(K.EQ.2)GMX=-G*CBMT2D/EXD
TEMPD=DD*DELXD**1.111111/EFLO**1.11111
DO 70 ITF=1,20
IF(TEMPD.LE.0.00)GO TO 110
A1D=ELD(K)*DXD*TEMPD*(1.00+GVX)
A1D=DSQRT(A1D)
A2D=1.8640+DLOG(ELFLO**.500/A1D)
A3D=ELD(K)*A2D*TEMPD
A5D=(A3D-DELXD)/((A2D+.500)*ELD(K))
TEMPD=TEMPD-A5D
IF(DABS(A5D-DELXD).LT.ULO(1))90,70,70
70 CONTINUE
WRITE(6,80)IBR,ILOAD,IT,II,L,J,K,A4D,(DFL(W),M=1,3)
80 FORMAT(12H0,ROLOAD 80,5I6,1P4E12.4)
IUIT=1
RETURN

90 PX(K)=PX(K)+TFMPD*SMF
XY(K)=XY(K)+TFMPD*SMF
PDPL=SMF/(A20+50U)*FNY(K)
DPDEL(K)=PDDEL(K)*PDE
DPHY(K)=DPHY(K)+PDHY*PDDEL
DPYDEL(K)=PDYDEL(K)*XDEL*PDDEL
YPHY(K)=YPHY(K)+XDEL*PDHY*PDDEL
IF(TEMP0=90,10,110,110)
100 AGTMP=TFMPD
DLX(K,J)=EPLX
HTMTZ(K,J)=636619800*TFMPD/A1D
PA(K,J)=2.00*A1D
XHSAV(K,J)=XHD
110 CONTINUE
IF(PX(K),E0,0,0)GO TO 10
TEMP0=XINC0/3.00
PX(K)=PX(K)+TFMPD
XY(K)=XY(K)+TFMPD
DPDEL(K)=PDDEL(K)*TEMP0
DPHY(K)=DPHY(K)*TEMP0
DPYDEL(K)=PDYDEL(K)*TEMP0
YPHY(K)=YPHY(K)*TEMP0
120 CONTINUE
P3=(-X(1)+X(2)-GMO+FCO*SBAD*CAYT2D-(P Y(1)-P X(2))*.500*DG*ST. U2D
1)/XLEVKO
PS1=P X(1)*CSTAD+PX(2)*CAYTUD+FCO=P 5*SBTAD
PS2=PX(1)*SBTAD+PX(2)*SBTAD+P 3*GSM*
PDDEL=-(P Del(1)+PDDEL(2)*VY)*.500*DG*ST. U2D-6*Del(1)-4*Del(2) +V
1)/XLEVKO
P3TH=(-(DPHY(1)+DPHY(2))*.500*DG*STAU00-YPHY(1)-YPHY(2))/YLEVKO
PS1DEL=DPDEL(1)*CBTAD-PSDEL(2)*VY+CBTAD-P 3*CBTAD
PS1TH=DPHY(1)*CBTAD-PSHY(2)*VY+CBTAD-P 3*CBTAD
PS2DEL=DPDEL(1)*SBTAD-PSDEL(2)*VY+SBTAD-P 3*SBTAD
PS2TH=DPHY(1)*SBTAD-PSHY(2)*SBTAD+P 3*SBTAD
P3TH=PS1DEL*PS2TH=PS2DELP 51TH
Y10=(P S1*PS2TH=PS2*PS1TH)/DE0
Y30=(P S1DEL=PS2-P 52DELP 51)/DE0
DL(1)=DEL(1)-Y1D
DL(2)=(DELTA-DEL(1))*VV
TH(1)=TH(1)-Y2D
TH(2)=THETA-TH(1)
130 IF(DAR3(Y1D)-TOLD(1))130,140,140
140 CONTINUE
WRITE(6,150)IR,ILUAD,J,Y1D,Y2D,(DEL(M),M=1,3)
150 FORMAT(13H0,ROLOAD 150,3T6,1PS=12,4)
I=UIT=1
RETURN
160 DO 170 K=1,2
P(K,J)=PX(K)
XM(K,J)=XM(K)
HLSAV(K,J)=DEL(K)
170 THSAV(K,J)=TH(K)
P3(K,J)=P3
P3DEL1=(-PDEL1-PDEL(1)*500*DD*STAU2D)/XLEVPD
P3TH1=(-P3TH1-P3TH(1)*500*DD*STAU2D)/XLEVPD
P3DEL2=(PDEL2+PDEL(2)*500*DD*STAU2D)/XLEVRE
P3TH2=(P3TH2+P3TH(2)*500*DD*STAU2D)/XLEVRE
A(1,1)=-PDEL1*CRTAO-P3DEL1*SRT2A
A(1,2)=PDEL2*CRTUD-P3DEL2*SRT2A
A(1,3)=-P3TH1*CRTAO-P3TH1*SRT2A
A(1,4)=-P3TH2*CRTUD-P3TH2*SRT2A
A(2,1)=-PDEL1*SRTAD+P3DEL1*CRT2A
A(2,2)=PDEL2*SRTUD+P3DEL2*CRT2A
A(2,3)=-P3TH1*SRTAD+P3TH1*CRT2A
A(2,4)=P3TH2*SRTUD+P3TH2*CRT2A
A(3,1)=1.
A(3,2)=1./VY/VV
A(3,3)=0.
A(3,4)=0.
A(4,1)=0.
A(4,2)=0.
A(4,3)=1.
A(4,4)=1.
ER(1)=0.
EP(2)=0.
DPD1(L) = (DPDEL5(1) * DEL*DEL * DPTH5(1) * T4FL) * DEL1(L) + (DPDEL5(1) * T4FLH * DPTH5(1) * THTH) * TH1(L)

DM101(L) = (DMDEL5(1) * DEL*DEL * DMT5(1) * THUL) * DEL1(L) + (DMDEL5(1) * T4FLH) * TH1(L)

XF(1) = XF(1) + PXS(1) * SRTA
XF(2) = XF(2) + PXS(1) * CDTA * CPN
XF(3) = XF(3) + PXS(1) * CDTA * SPN
TFMP = 5 * (E * SRTA + D * STAU02) + PXS(1) + XY5S(1)
XF(4) = XF(4) + TFMP + SPN
XF(5) = XF(5) + TFMP * CPN
IF (PX5(1), EQ, 0, ) RETURN
DO 280 L = 1, 5
DTV(1, 4) = DTV(1, 1) + DPD1(L) * SRTA
DTV(2, 4) = DTV(2, 1) + DPD1(L) * CDTA * CPN
DTV(3, 4) = DTV(3, 1) + DPD1(L) * CDTA * SPN
TFMP = 5 * (E * SRTA + D * STAU02) + DPD1(L) + DM101(L)
DTV(4, 4) = DTV(4, 1) + TFMP + SPN
DTV(5, 4) = DTV(5, 1) + TFMP * CPN
   RTURN
END

SURROUTINE OUTCON
COMMON ALPHAA, AA(5, 5), A(5, 5), A31G, BETA, P0, R2A, BMTA, RMT02,
1RT2AL, USTMP, BB(2, 60), C(2) CROWN, CDTA, STAUF, CBMTA, CBMT02,
2CT2AL, CPH, CORR(5), CTH, COR(5), DNMTP, DEL1(1), DEL1(1), DEL(3)
3, NT5(5, 5), DELTA, DEL(2), DPDDEL(2), DMDEL(2), DPTH(2), DMT(2), DEX,
4DTH, DX, DLT(2, 60), DELSAV(2, 60), T1D01(1), DM101(1), NELL, DELX,
5E, EL(2), ERR(5), EFL, EXE(5), FLT, FLT^2, FREE(3), FC, GAGE, GAM,
6GAM, GMG, GMX, H1, H2, HERTZ(2, 90), HP7, HP8, TBR, TSTOP, ILOAD, IQUIT,
7ICT, JPASS, KKK, NLOAD, OME, OQR, P0, PR(2), PRP, PX(2),
PR(2, 60), P3, PX3(60), P1OUT, P3OUT, RSP, PH0, RM, P1, R2, RP3(2),
COMMON SRTA, STAUF2, SLANT, SMTAU, S3TAP, SHA, SAVS, SAVT, SPH, STH,
1, THI01(5), TAUP2, TAU, TTAU01, TTA2L, TOL(5), THETA, TH(2), TANTH,
2THS(5, 60), THET, TMBIO, V, VV, W = TGH, XN, Y, YL, YTAU,
3XLT02, ALED2, XNAR, XMAS1, XMAS2, XMAS3, XINC1, XINC2, XINC3,
4XRAR, XRAR, XLEVER, XALPHA, XF(2), YF(1), X(M)2, YTH, XDEL, Y1, X2,
5XYM(1, 60), AX(2), AX(2, 60), XHSAV(2, 60), XM1OUT, XO(2), XI31G,
6Y(2), YX"R

DOUBLE PRECISION 36, TMP, CTH, DELTA, DEL, DNL, DMT, DPTH, DMT, GMX, HD
1, DX, P3, STH, THETA, TH, TANTH, VV, XN, XTH, XEL.
DOUBLE PRECISION C0N, C0NTHD, DLD, FLT0, SNT0, X0L, X01, X02
DFLL= UN5\#O
THETA= O.
C' 100 ITR=1, 25
P1OUT=O.
X'1OUT=O.
P1DEL=O.
P1TH=O.
X'1DEL=O.
X'1TH=O.
S'NTH=SIN(THET)
C'NTH=COS(THET)
CYN=CRWN
C0NTHD=CNT0
DLD=DELL
FLT0=FLT02
SNT0=SNT02
X0L=X0L02
CALL X'REME(CWN+C0NTHD,FLT0,H0,K4,SNT0,YN1,X02,DLD,X0L)
X1=X01
X2=X02
X0(1)=X1
X0(2)=X2
EFL=X1-X2
SNINC=1.
XINC=EFL/30.
XH=X1+XINC
T4PBIG=O.
DO 80 L=1, 31
XH=XH+XINC
DY=D+2.*XH+TTAU02
EX=E+2.*XH+STA+D*CBMTP*DX*CBTA
GA=2*DX*CBTA/EX
S'H=3.-SNINC
SNINC=-SNINC
IF((L.EQ.1).OR.(L.EQ.31))SM=1.
D=LX=DELL+XH+SNT0/CNTH
D7TH=XH/CNTH*2
IF(ABS(XH).LE.FLT02*CANTH)10 TO 40
7.

```plaintext
TEMP=YH-RMH*SNTH
TEMP1=CROWN***-TEMP**
IF(TEMP1)U,10,30
10 INUIT=1
WRITE(6,20)IBR,ILOAD,ITR,L,DELL,THET
20 FORMAT(12HU OUTCON 20*416*1P3E12*4)
RETURN
30 TEMP1=SORT(TEMP1)
NDLX=TEMP1-RMH*CNYTH+DELL
DTH=TEMP1+RMH+CNYTH/TEMP1+RMH*SNTH
40 IF(DELX,LT,1.E-8)GO TO 80
TEMP=5.E7*DELY**1.111111/EFL**1.111111
DO 50 ITO=1,20
IF(TEMP.LT.0.1)GO TO 80
A1=EL(1)*X*TEMP*(1.+GAMP)
A1=SORT(A1)
A2=1.864+ALOG(EFL*5/A1)
A3=EL(1)*2*TEMP
A4=(A3-DELX)/((A2-.5)*EL(1))
TEMP=TEMP-A4
IF(ABS(A3-DELX)-TOL(1))70,50,50
50 CONTINUE
WRITE(6,60)IBR,ILOAD,ITR,A4,DELL,THET
60 FORMAT(12HU OUTCON 60*316*1P3E12*4)
I*NUIT=1
RETURN
70 P1OUT=P1OUT+TEMP*SM
X1OUT=X1OUT+X*TEMP*SM
PDEL=SM/((A2-.5)*EL(1))
P1DEL=P1DEL+PDEL
X1DEL=X1DEL+X*PDEL
P1TH=P1TH+OTH*PDEL
X1TH=X1TH+X*OTH*PDEL
IF(TEMP,LT,TEMP1IG)GO TO 80
TEMP=TEMP1IG
AIG=2.*A1
DELX=DELX
X1=ROUND
H=TEMP/ADIG
```

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DOUBLE PRECISION RGTMP, CTH, DELTA, DEL, DPI, DMPI, PMI, GMX, HD
WRITE(0,10) LOAD, IBR
10 FORMAT(*$ OUTPUT DATA FOR LOAD NO.*$%,* BEARING NO.*%$)
WRITE(6,20)
20 FORMAT(*$ OUTPUT DATA FOR LOAD NO.*$%,* BEARING NO.*%$)
WRITE(6,30)
30 FORMAT(*$ OUTPUT DATA FOR LOAD NO.*$%,* BEARING NO.*%$)
WRITE(6,40)
40 FORMAT(*$ OUTPUT DATA FOR LOAD NO.*$%,* BEARING NO.*%$)
WRITE(6,50)
50 FORMAT(*$ OUTPUT DATA FOR LOAD NO.*$%,* BEARING NO.*%$)
WRITE(6,60)
60 FORMAT(*$ OUTPUT DATA FOR LOAD NO.*$%,* BEARING NO.*%$)
WRITE(6,70)
70 FORMAT(*$ OUTPUT DATA FOR LOAD NO.*$%,* BEARING NO.*%$)
190 FORMAT(1PSL12.4)
   WRITE(6,190)
190 FORMAT(10 DFY/DX DFY/DY DFY/DZ DFY/DALX DFY/DAL
   LAL/IN LBN/IN LAIN/IN LB/RAD L/R/RAD*)
   WRITE(6,190)DTV(3,2),DTV(3,3),DTV(3,1),DTV(3,4),DTV(3,5)
   WRITE(6,200)
200 FORMAT(10 DFZ/DX DFZ/DY DFZ/DZ DFZ/DALX DFZ/DAL
   LAL/IN LBN/IN LAIN/IN LB/RAD L/R/RAD*)
   WRITE(6,200)DTV(1,2),DTV(1,3),DTV(1,1),DTV(1,4),DTV(1,5)
   WRITE(6,210)
210 FORMAT(10 DMX/DX DMX/DY DMX/DZ DMX/DALX DMX/DAL
   LBN/IN LAIN/IN LAIN/RAD LBN/RAD*)
   WRITE(6,210)DTV(4,2),DTV(4,3),DTV(4,1),DTV(4,4),DTV(4,5)
   WRITE(6,220)
220 FORMAT(10 DMY/DX DMY/DY DMY/DZ DMY/DALX DMY/DAL
   LAIN/IN LAIN/IN LAIN/RAD LAIN/RAD*)
   WRITE(6,220)DTV(5,2),DTV(5,3),DTV(5,1),DTV(5,4),DTV(5,5)
   RETURN
END

SUBROUTINE RCALC(CROWN, DROP, FLT02, GAGF, HM, XLE02)
   DOUBLE PRECISION CRO, DRP, DSORT, FLT2D, GAG, HM, XLE02
   FLT2D=FLT02
   GAG=GAGF
   XLE02=XLE02
   IF (CROWN) 10, 10, 10
10 INPUT =DROP
   CPO=DSQRT(((XLE02-GAG)**2-FLT2D**2+DRP**2)/(2.*DM*DRP)**2+1)
   HCO=DSORT(CPO**2-FLT2D**2)
   CROWN=CPO
   RETURN
20 INPUT =HM
   HDM=DSORT(CPO**2-FLT2D**2)
   DROP=HDM-DSORT(CPO**2-(XLE02-GAG)**2)
   RETURN
END

SUBROUTINE XTREME(CRN, CTH, FLT02, HM, KM, SM, X1D, X2D, XDEL, XLE02)
   DOUBLE PRECISION CRN, CTH, DSORT, MABLA, FLT02, HM, SM, STAB, TEMP,
   XSTAR1, XSTAR2, X1D, X2D, XDEL, XLE02
CTHAR=JARK(5TH)
C"=0
TEMP=CX**2-(30*CTH-XDEL)**2
IF(TEMP)10,10,20
10 X"=1
RETURN
20 TEMP=CTH*PT(TEMP)
X1N=TP*4+TH*STHAR
X2N=TP*4+TH*STHAR
JABL=0D-150PT(CP)**2-XLE02D**2
XSTAR1=XLE02D*CTH+JNARL*STHAR
XSTAR2=XLE02D*CTH+JNARL*STHAR
IF(X2N,GT,XSTAR1)GO TO 1N
IF(X1N,LE,FLTN2D*CTH)GO TO 1N
IF(X1N,GT,XSTAR1)XID=XSTAR1
IF(X2N,LT,XSTARR2)XID=XSTARR2
IF((XPN,GT,FLTN2D*CTH),.AND.(X2N,LT,FLTN2D*CTH))XPD=CTH
1STHAR
IF(STH,GE,0.0)RETURN
TEMP=X1N
X1D=X2N
X"=TEMP
RETURN
END
SUBROUTINE SIMPLT(X1,N,PA,XX,KX)
DIMENSION AA(5,5),B(5),XX(5),KOL(5)
DOUBLE PRECISION A(5,5),B(5),X(5),ROW(5),TEMP,AMPY
DO 10 I=1,N
A(I)=AA(I)
DO 10 J=1,N
10 A(J,I)=AA(J,I)
DO 505 J=1,N
TEMP=C0.0
DO 505 K=1,N
IF(BK3.(A(J,K),-TEMP))5051,5051,5052
5052 TEMP=CANS(A(J,K))
5051 CONTINUE
DO 5053 K=1,N
5053 A(J,K)=A(J,K)/TEMP
5029 T= (KOL(1ROW)-KOUNT) 5029-5032
5030 GO TO 5031 JCOL=1+N
7ROW(I)=A(JCOL,1ROW)
A(JCOL,IROW)=A(JCOL,KOUNT)
5031 A(JCOL,KOUNT)=ROW(I)
KOL(KOUNT)=KOL(1ROW)
KOL(1ROW)=1ERASE
GO TO 5034
5032 CONTINUE
GO TO 5035
5034 CONTINUE
997 T=(XX-3)998,9000,998
9000 KY=0
RETURN
998 00 5042 IRROW=1+N
5040 X(IRROW)=0.00
5041 00 5042 JKOL=1+N
5042 X(IRROW)=X(IRROW)+A(IRROW,JKOL) * B(JKOL)
KY=0
GO TO 6000 I=1,N
6000 XY(I):X(I)
RETURN
5035 KY=1
RETURN
END
REFERENCES


