Technical Note

Restoration of Speckle Images

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FOR THE COMMANDER

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RESTORATION OF SPECKLE IMAGES

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In this report, several techniques to reduce speckle noise (more generally signal independent multiplicative noise) in images are studied. The techniques include gray scale modification, frame averaging, low-pass filtering in the intensity and density domain, and application of the short space spectral subtraction image restoration technique in the density domain. Some discussions on the theoretical basis of the techniques studied are given and their performances are illustrated by way of examples.
I. Introduction

The 'speckle effect' (1) is commonly observed in images generated with highly coherent laser light. As will be illustrated later (Figure 2), it appears as a multiple of tiny spots (or 'speckles') of varying intensity, superimposed on the true image. Although this can be useful in certain applications (2), speckle is generally regarded as a degrading effect. For example, speckle in an optical radar system can reduce the probability of target detection. Thus, the elimination of speckle is of vital concern in such systems.

In this report, we consider the restoration of images degraded by a multiplicative noise model (3) for speckle. The restoration techniques that we studied include gray scale modification, multi-frame averaging, low-pass filtering both in the intensity and density (log intensity) domain and application of the short-space spectral subtraction image restoration technique (4) in the density domain. As will be illustrated by way of examples, the techniques that we studied with the exception of low-pass filtering are generally useful in enhancing images degraded by speckle noise.

In section II, we present some statistical properties of speckle noise on which the study reported here is based. In section III, we discuss techniques applied to enhance images degraded by speckle in the case when only one frame of a degraded image is available. The case when more than one frame of a degraded image are available for processing is discussed in section IV.
II. Statistical Model for Speckle Noise

In this report, we consider a model for speckle noise which is adequate when the degraded image has been sampled coarsely enough such that the degradation at any point can be assumed to be independent from all other points. Specifically, this model, derived both theoretically and experimentally (5), gives for a point with intensity $x(n_1,n_2)$ the corresponding point $y(n_1,n_2)$ in the degraded image as an independent sample from the following density:

$$p(y(n_1,n_2)) = \frac{1}{x(n_1,n_2)} \cdot \frac{y}{x(n_1,n_2)} \cdot e^{-\frac{y}{x(n_1,n_2)}} \cdot u(y)$$  \hspace{1cm} (1)

where

$$u(y) = 1 \text{ for } y \geq 0$$

$$0 \text{ otherwise}$$ \hspace{1cm} (2)

From the density of equation (1), speckle can be modelled (3,6) as a multiplicative noise $w(n_1,n_2)$ such that

$$y(n_1,n_2) = x(n_1,n_2) \cdot w(n_1,n_2)$$  \hspace{1cm} (3)

where

$$P_{w(n_1,n_2)}(w) = e^{-w} \cdot u(w)$$ \hspace{1cm} (4)
With this model for speckle, it can be shown that the average of $N$ frames of the same image degraded by independent speckle for each frame is the maximum likelihood estimate (MLE) of the undegraded image. Furthermore, the $N$ frame average can also be modelled as the original image degraded by a multiplicative noise. Let $y_1(n_1,n_2), \ldots, y_n(n_1,n_2)$ be the respective observed values of an undegraded pixel $x(n_1,n_2)$ in $N$ independently degraded images.

Then the joint density, $P_{y_1(n_1,n_2), \ldots, y_N(n_1,n_2)}$ is given by

$$P_{y_1(n_1,n_2), \ldots, y_N(n_1,n_2)} = \frac{1}{(x(n_1,n_2))^N} \cdot e^{-\frac{1}{x(n_1,n_2)} \cdot (y_1+\ldots+y_N)} \cdot u(y_1,\ldots,y_N)$$

where $u(y_1,\ldots,y_N) = u(y_1) \cdot u(y_2) \ldots u(y_N)$

This expression is maximized when

$$\hat{x}(n_1,n_2) = y'(n_1,n_2) = \frac{1}{N} \sum_{i=1}^{N} y_i(n_1,n_2)$$

Thus, the average of $N$ speckle frames is the MLE of the undegraded image. Since $y_1(n_1,n_2) = x(n_1,n_2) \cdot w_1(n_1,n_2)$

$\hat{x}(n_1,n_2)$ or $y'(n_1,n_2)$ in equation (7) can be written as

$$\hat{x}(n_1,n_2) = y'(n_1,n_2) = x(n_1,n_2) \cdot w'(n_1,n_2)$$

where

$$w'(n_1,n_2) = \frac{1}{N} \sum_{i=1}^{N} w_i(n_1,n_2)$$
From equation (9), the MLE $x(n_1,n_2)$ or $y'(n_1,n_2)$ can also be modelled by the noise-free image $x(n_1',n_2')$ degraded by multiplicative noise $w'(n_1,n_2)$. From equation (10), if only a single frame is available such that $N=1$, $w'(n_1,n_2)$ equals $w(n_1,n_2)$ in equation (4) and $y'(n_1,n_2)$ equals $y(n_1,n_2)$.

From equations (3) and (9), it follows that if $y(n_1,n_2)$ or $y'(n_1,n_2)$ is logarithmically transformed such that

$$\log y(n_1,n_2) = \log x(n_1,n_2) + \log w(n_1,n_2)$$

and

$$\log y'(n_1,n_2) = \log x(n_1,n_2) + \log w'(n_1,n_2)$$

the noise component of $\log w(n_1,n_2)$ or $\log w'(n_1,n_2)$ is an additive one. Image restoration techniques such as a spectral subtraction technique (4) require the knowledge of the second order statistics of additive noise. Results on the statistics for logarithmically transformed speckle (3) are summarized below.

(R1) For a single speckle frame, the mean and variance of $\log w(n_1,n_2)$ are given by

$$m = \text{Euler's constant} = 0.577...$$

$$\sigma^2 = \pi^2/6 = 1.64$$

(R2) For $N$ speckle frames that are averaged, the mean of $\log w'(n_1,n_2)$ is the same as in (R1) and the variance of $\log w'(n_1,n_2)$ for $N >> 1$ can be approximated by

$$\sigma^2 \approx 1/N$$
(R3) If $\bar{D}$ is the mean intensity of the logarithm of the speckle image and signal-to-noise ratio is defined as

$$S/N = 20 \log \frac{\bar{D}}{\sigma^2}$$

then $S/N$ is on the order of 0 to 1 dB for most speckle images.

(R4) For $N$ averaged speckle frames

$$S/N = 20 \log (\bar{D} \sqrt{N})$$

These statistics of speckle were used for generating degraded images as well as choosing appropriate parameters for the restoration techniques. The rest of this report describes these techniques and the results obtained from them.
III. Techniques for Reduction of Speckle Noise: Single Frame Case

In this section, we discuss various techniques that we have studied to enhance images degraded by speckle noise when only one frame of speckle image is available for processing. The techniques that we have considered are gray scale modification, low-pass filtering in the intensity and density domains and short space spectral subtraction in the density domain and they are discussed in sections III.1, III.2 and III.3, respectively.

1. Gray Scale Modification

To illustrate a basis for gray scale modification, the histograms of an original image and the same image degraded by artificially generated speckle noise are shown in Figures 1(a) and 1(b). From the figures, it can be seen that speckle noise tends to shift the image to a darker side in the luminance domain, and the overall brightness of an image degraded by speckle noise is generally much darker than the original image. A common technique used to correct such a problem is some form of gray scale modification (7,8). In Figure 1(c) is shown a threshold clipping technique for gray scale modification and in Figure 1(d) is shown the histogram of an image obtained by modifying the gray scale of the image shown in Figure 1(c). In general, we have found that a simple gray scale modification improves the quality of both unprocessed and processed images and consequently will be used in illustrating all the examples in this report.

2. Low-pass Filtering

From equations (3) and (4), speckle noise can be modelled by an independent multiplicative noise \( w(n_1, n_2) \). Since speckle noise at a point in space is assumed to be statistically independent of any other points, the multiplicative noise \( w(n_1, n_2) \) is wide band random noise. Suppose we
Fig. 1(a-d). (a) Histogram of a noise-free image (a picture of a clock). (b) Histogram of the same image of Fig. 1(a) degraded by speckle noise. (c) A gray scale modification technique by threshold clipping. (d) Histogram of the degraded image of Fig. 1(b) enhanced by the gray scale modification technique of Fig. 1(c).
decompose the noise-free image \( x(n_1, n_2) \) into two components, one obtained by low-pass filtering and the other obtained by high-pass filtering so that

\[
x(n_1, n_2) = x_L(n_1, n_2) + x_H(n_1, n_2)
\]

(13)

where \( x_L \) and \( x_H \) represent the components obtained by low-pass filtering and high-pass filtering respectively. Similarly, \( w(n_1, n_2) \) can be decomposed into two components such that

\[
w(n_1, n_2) = w_L(n_1, n_2) + w_H(n_1, n_2)
\]

(14)

Combining equations (6), (13) and (14),

\[
y(n_1, n_2) = x_L(n_1, n_2) \cdot w_L(n_1, n_2) + x_L(n_1, n_2) \cdot w_H(n_1, n_2) + x_H(n_1, n_2) \cdot w_L(n_1, n_2) + x_H(n_1, n_2) \cdot w_H(n_1, n_2)
\]

(15)

Qualitatively speaking, from equation (15) low-pass filtering \( y(n_1, n_2) \) approximately leads to \( x_L(n_1, n_2) \cdot w_L(n_1, n_2) \). Since the image \( x(n_1, n_2) \) generally has large amplitude low-frequency components relative to high-frequency components while speckle noise \( w(n_1, n_2) \) is wide-band random noise, low-pass filtering \( y(n_1, n_2) \) may be viewed as an operation that attempts to improve the S/N.

We have applied various different types of low-pass filters to \( y(n_1, n_2) \). In general, the lower the cut-off frequency, the more speckle noise appears to be reduced, but at the same time, the resulting image is noticeably blurred. As an example of low-pass filtering to reduce speckle noise, in Figure 2 are shown two noise-free images. In Figure 3 are shown the two images in Figure 2 degraded by artificially generated speckle noise and in Figure 4 are shown the two images in Figure 3 processed by low-pass filtering.
Fig. 2. Original images
Fig. 3. Images in Fig. 2 degraded by speckle noise

Fig. 4. Images in Fig. 3 processed by low-pass filtering in the intensity domain
In the above discussions, we have considered low-pass filtering in the intensity domain. Low-pass filtering in the density domain also has a theoretical basis. Specifically, from equation (11), speckle noise is an additive component in the density domain. For typical images, \( \log x(n_1,n_2) \) has large amplitude low-frequency components relative to high-frequency components while the additive component \( \log w(n_1,n_2) \) is wide-band random noise. Therefore, low-pass filtering \( \log y(n_1,n_2) \) generally reduces \( \log w(n_1,n_2) \) more than \( \log x(n_1,n_2) \) thus leading to a S/N improvement. Filtering in the density domain so that the multiplicative component becomes an additive component is known as homomorphic filtering (9). Homomorphic filtering to reduce speckle noise has been considered in (6,10).

We have applied various different types of low-pass filters to \( \log y(n_1,n_2) \) and as a typical example the two images in Figure 3 processed by low-pass filtering in the density domain are shown in Figure 5. Like low-pass filtering in the intensity domain, we have found that speckle noise is reduced but the resulting images are noticeably blurred. Due to the high degree of image blurring relative to the amount of speckle noise reduction, low-pass filtering in the density or intensity domain does not appear to be a useful technique in practical applications.

3. Short Space Spectral Subtraction Technique

Since speckle noise can be modelled by an additive random noise in the density domain, in addition to simple low-pass filtering, there exists a variety of other restoration techniques such as Wiener filtering (11,12) and power spectrum filtering (13) that may be applied to reduce the additive random noise. One technique which has been particularly successful in reducing additive random noise at relatively high S/N is short space spectral subtraction image restoration (SSIR) technique*. This technique requires only the knowledge of the power spectrum of the additive random noise.

* A brief description of this technique is given in the Appendix. A more detailed discussion can be found in (4).
noise. The additive noise $\log w(n_1,n_2)$ is broad-band and its spectral amplitude can be determined from (R1). In Figure 6 are shown the two images in Figure 3 processed by short space SSIR technique. It is clear from Figure 6 that speckle noise is reduced without noticeably blurring the image. However, the remaining degradation looks like a moiré pattern superimposed on the original image. Informal tests show that improvement in image quality by such processing is debatable. This result is partly due to the fact that there is no known algorithm which effectively enhances images degraded by additive random noise at a very low S/N such as 0 or 1dB. When the S/N is increased by frame averaging, a more promising result is obtained as will be discussed in the next section.
Fig. 5. Images in Fig. 3 processed by low-pass filtering in the density (log intensity) domain

Fig. 6. Images in Fig. 3 processed by a short space spectral subtraction image restoration technique
IV. Techniques for Reduction of Speckle Noise: Multiple Frame Case

When N frames of the same image but with independent speckle noise are available for processing, the MLE of the noise-free image is the average of N frames given by equation (7). The frame averaging technique to reduce speckle noise has been considered in (2,14). In Figure 7 are shown $y'(n_1,n_2)$, the results of averaging four frames of independently degraded speckle images of Figure 3. Consistent with the theoretical results of (R1) - (R4), comparison of Figures 3 and 7 clearly shows that frame averaging increases the S/N and improves the image quality and intelligibility.

From equations (9) and (10), $y'(n_1,n_2)$, the result obtained by frame averaging, can again be viewed as an image degraded by a broad-band multiplicative noise. Consequently, all the techniques discussed in section III may be applied to further reduce speckle noise.

We have applied various different types of low-pass filters to $y'(n_1,n_2)$ both in the intensity and density domains. As a typical example of low-pass filtering applied to $y'(n_1,n_2)$, in Figure 8 are shown two images in Figure 7 processed by low-pass filtering. The results obtained by low-pass filtering in the density domain are shown in Figure 9. As in the single frame case discussed in section III, we have found that low-pass filtering reduces the multiplicative noise but at the same time noticeably blurs the resulting images.

We have also applied short space SSIR technique to log $y'(n_1,n_2)$ with the spectral amplitude of log $w'(n_1,n_2)$ obtained from (R2) and (R4). The results are shown in Figure 10. Even though the signal correlated degradation that looks like a moiré pattern is still visible in the figure, the amplitude of the degradation is smaller and is not too apparent in those regions without uniform intensity. Furthermore, reduction of the multiplicative noise without noticeably blurring the image is evident in the figure. This is consistent with the result (4) that short space SSIR technique is more effective in restoring images degraded by additive random noise at relatively high S/N.
As an additional example, we have considered the case when eight frames of independently degraded speckle images are available for processing. Figures 11, 12, 13 and 14 are equivalent to Figures 7, 8, 9 and 10 with the difference that eight frames rather than four frames have been used. Our discussions in the four frame case are also applicable to the eight frame case.
Fig. 7. Results of averaging four frames of independently degraded speckle images of Fig. 3

Fig. 8. Images in Fig. 7 processed by low-pass filtering in the intensity domain
Fig. 9. Images in Fig. 7 processed by low-pass filtering in the density domain.

Fig. 10. Images in Fig. 7 processed by a short space spectral subtraction image restoration technique.
Fig. 11. Results of averaging eight frames of independently degraded speckle images of Fig. 3

Fig. 12. Images in Fig. 11 processed by low-pass filtering in the intensity domain
Fig. 13. Images in Fig. 11 processed by low-pass filtering in the density domain

Fig. 14. Images in Fig. 11 processed by a short space spectral subtraction image restoration technique
V. Conclusion

In this report, we have considered the problem of restoring images degraded by speckle noise. The specific techniques that we have studied are gray scale modification, frame averaging, low-pass filtering in the intensity and density domain, and short space SSIR technique in the density domain.

In general, some form of gray scale modification has been found to be useful in enhancing images degraded by speckle noise. If only one frame of speckle image is available, then neither low-pass filtering nor short space SSIR technique has been found to be effective in restoring images degraded by speckle noise. When multiple frames of speckle images are available for processing, frame averaging which corresponds to the maximum likelihood estimation of the noise-free image has been quite effective in increasing the S/N and improving the image quality and intelligibility. Furthermore, the short space SSIR technique applied to the results of frame averaging appear to have some usefulness in further reducing the degradation. Low-pass filtering, however, does not appear to be useful in the multiple frame case.
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APPENDIX. Short Space Spectral Subtraction Image Restoration Technique

In the short space SSIR technique, the degraded image \( r(n_1,n_2) \) is divided into many subimages each of which is restored separately and then the restored images are combined to form an estimate of the noise-free image \( f(n_1,n_2) \). More specifically, let \( r(n_1,n_2) \) be represented by

\[
r(n_1,n_2) = f(n_1,n_2) + d(n_1,n_2) \tag{A1}
\]

where \( d(n_1,n_2) \) denotes an additive random noise uncorrelated with \( f(n_1,n_2) \). By applying a 2-D window function \( w_{ij}(n_1,n_2) \) to equation (A1),

\[
r(n_1,n_2) \cdot w_{ij}(n_1,n_2) = f(n_1,n_2) \cdot w_{ij}(n_1,n_2) + d(n_1,n_2) \cdot w_{ij}(n_1,n_2) \tag{A2}
\]

Rewriting equation (A2),

\[
r_{ij}(n_1,n_2) = f_{ij}(n_1,n_2) + d_{ij}(n_1,n_2) \tag{A3}
\]

where \( r_{ij}(n_1,n_2) \) represents \( r(n_1,n_2) \cdot w_{ij}(n_1,n_2) \), and \( f_{ij}(n_1,n_2) \) and \( d_{ij}(n_1,n_2) \) are similarly defined. To estimate the noise-free subimage \( f_{ij}(n_1,n_2) \) from \( r_{ij}(n_1,n_2) \) in equation (A3), \( F_{ij}(\omega_1,\omega_2) \), the discrete space Fourier transform* of \( f_{ij}(n_1,n_2) \), is first estimated and then inverse

*The definition of discrete space Fourier Transform, power spectrum and energy spectrum, and the determination of the normalization constant "k" can be found in references (4) and (15).
Fourier transformed. The discrete space Fourier transform $F_{ij}(\omega_1, \omega_2)$ is estimated by a particular form of spectral subtraction (6):

$$\hat{F}_{ij}(\omega_1, \omega_2) = (|R_{ij}(\omega_1, \omega_2)|^2 - \alpha \cdot k \cdot P_d(\omega_1, \omega_2))^{1/2} \cdot e^{jR_{ij}(\omega_1, \omega_2)}$$  \hspace{1cm} (A4)

for $|R_{ij}(\omega_1, \omega_2)|^2 \geq \alpha \cdot k \cdot P_d(\omega_1, \omega_2)$ and 0 otherwise

where $\hat{F}_{ij}(\omega_1, \omega_2)$ represents an estimate of $F_{ij}(\omega_1, \omega_2)$, $R_{ij}(\omega_1, \omega_2)$ represents the discrete space Fourier Transform of $r_{ij}(n_1, n_2)$, $<R_{ij}(\omega_1, \omega_2)>$ represents the phase of $R_{ij}(\omega_1, \omega_2)$, "\(\alpha\)" is a constant, "\(k\)" is a scaling factor that normalizes the power and energy spectral densities, and $P_d(\omega_1, \omega_2)$ represents the power spectrum of the additive random noise. From the estimated $f_{ij}(n_1, n_2)$, an estimate of $f(n_1, n_2)$ is obtained by combining the restored subimages:

$$\hat{f}(n_1, n_2) = \sum \sum \hat{f}_{ij}(n_1, n_2)$$  \hspace{1cm} (A5)

where $\hat{f}_{ij}(n_1, n_2)$ represents the estimated $f_{ij}(n_1, n_2)$ and $f(n_1, n_2)$ is similarly defined.

In implementing the short space SSIR technique, in this report, a separable 2-D triangular window of size 16 x 16 pixels overlapped with its neighboring window by half the window duration in each dimension was used for $w_{ij}(n_1, n_2)$ and the value of "\(\alpha\)" was assumed to be approximately 1/2.
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In this report, several techniques to reduce speckle noise (more generally signal independent multiplicative noise) in images are studied. The techniques include gray scale modification, frame averaging, low-pass filtering in the intensity and density domain, and application of the short space spectral subtraction image restoration technique in the density domain. Some discussions on the theoretical basis of the techniques studied are given and their performances are illustrated by way of examples.