THEORETICAL STUDIES OF THREE DIMENSIONAL TRANSONIC FLOW THROUGH A COMPRESSOR BLADE ROW

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Section 1

INTRODUCTION

Three-dimensional flow effects play an important role in the performance of axial-flow fans and compressors that operate at transonic speeds. The coupling between transonic and three-dimensional effects limits the applicability of the two-dimensional analysis methods that have been in use for some years.

Under a previous contract with Calspan Corporation, AFOSR sponsored a study of the applicability of finite-difference computational methods to this problem. That study led to the development of a computer program which used a relaxation method to solve the problem in the nonlinear small-disturbance approximation. Use of this approximation facilitated the adaptation of external-flow computational methods to the internal-flow case.

The present research program was undertaken with the aim of extending these numerical techniques, so as to handle more fully the nonlinearity of the problem. Thus, heavily loaded blades with large turning angles are to be considered, and the simplifications of small-disturbance theory (such as satisfaction of boundary conditions on mean-chord surfaces, neglect of trailing vortex-sheet deformation) will not be used.

This Annual Technical Report contains, in Sections 2 through 7, the items listed in the report specifications (Paragraph 2a of the Contract Document).

Section 2
OBJECTIVES

The general goal of this program is to devise a numerical method for solving the equations of inviscid flow through a compressor blade row, in an approximation that retains the nonlinearity of the problem. This method is to be applied in the form of a computer code and associated users' guide. The level of approximation envisioned for the study is that of the full nonlinear potential equation, but the use of the original Euler equations is not excluded.

The specific statement of work contained in the contract is:

Develop a method for calculating the inviscid three-dimensional flow field through a turbo-machine blade row for the case of heavily loaded blades. The full nonlinear potential equation will be solved using the relaxation method with a full accounting for the three dimensional distortions of the trailing vortex sheets.

Examine methods that can be used to speed up the rate of convergence for two dimensional calculations for possible adaptation to three dimensional turbo-machinery flows.

Examine the suitability of using implicit time marching methods for future studies of the three dimensional flutter problem.

It should be noted that the contributions to the solution coming from highly deformed trailing vortex sheets are to be included, and that some attention is to be given to the possibility of speeding up calculations.
Section 3

PRESENT STATUS

The first and second quarters were devoted to a review of the problem formulation. The full set of Euler equations describing the inviscid nonlinear flow through a compressor blade row were re-derived, in the coordinate system fixed to the blades. If it is further assumed that the flow is isentropic, and uniform at the inlet, and if the Mach number upstream of any shock waves that are present is less than around 1.3, then the problem can be further simplified to the potential-flow approximation. This equation is presented in the first quarterly progress report (1).

During the second quarter, a detailed review was made of numerical methods that might be used to solve the various sets of equations (embodying various levels of approximation) derived during the first quarter. It had been hoped that methods suitable for solving the full potential equation would be available; indeed the first paragraph of the work statement envisions such an approach (see Section 2).

The review done during the second quarter led to the conclusion, however, that the required methods were not ready. Many promising techniques had been reported, especially for two-dimensional, external-flow problems. But in all cases, the extension of these methods to the present problem clearly would entail further development of numerical techniques rather than adaptation of existing techniques. Accordingly, further attention was given to the implicit time-marching methods, which had been referred to in the original work statement as applicable only to future studies. It was found, on close examination, that these methods were much farther advanced, especially for three-dimensional problems, to the point where a minimum amount of technique development was required. Furthermore, these methods make it possible to remove the assumptions required for the potential-flow approximations (isentropic flow, low supersonic Mach numbers). For these reasons, a
decision was made at the end of the second quarter to bypass the potential-flow approximation entirely, and to move on to the full Euler-equation formulation.

The strong-conservation-law forms of the Euler equations were derived, and are presented in the second quarterly\(^\text{(2)}\). Also outlined in this report was the derivation of the implicit finite-difference algorithm. This algorithm envisions the use of a coordinate system in which the blade surfaces lie along constant planes of one of the coordinates. The entire third quarter was given over to a study of a conformal-mapping method for effecting such a transformation. The particular method chosen was that of Ives and Liutermoz\(^\text{(3)}\); each of the steps in their transformation was worked through in detail. Figure 1 shows the mapping found for a specific blade row at one radial station\(^\text{(4)}\). The two families of lines shown map into a rectangle in the transformed plane, with the blade surface lying along one side.

During the fourth quarter, effort on this contract was reduced substantially, while awaiting the outcome of a parallel investigation being conducted at Calspan under sponsorship of Naval Air Systems Command (Contract No. N00019-78-C-0603). In that work, it was found recently that certain solutions calculated by the computer program developed under a previous AFOSR contract at Calspan\(^\text{(5)}\) may fail to yield periodic solutions. It was feared that this discrepancy might be due to a basic flaw in the problem formulation, possibly shared by the formulation used in the present work. Accordingly, it was decided to curtail effort on the present work while awaiting resolution of the problem.

The problem has been resolved\(^\text{(6)}\), and it was found that there is no flaw in the present formulation. It was found that certain finite-difference formulas must be avoided, when dealing with the potential equation in a skewed coordinate system. These problems do not affect the implicit time-marching method under development here.
Section 4

PUBLICATIONS

There have been no journal publications during this reporting period.
Section 5

PERSONNEL

The principal investigator for this effort is Dr. William J. Rae. He has been assisted by Drs. J.C. Erickson, Jr., John A. Lordi, Gregory F. Homicz, and Joseph P. Nenni. On matters related to computer programming methods, he has had the assistance of Mr. John R. Moselle.
Section 6

INTERACTIONS

During this contract year, the principal investigator attended the following meetings:


2. NASA Lewis Research Center, Workshop on Computational Fluid Dynamics Applied to Turbomachinery; Cleveland, OH, November 14-15, 1978.


In addition, copies of the third quarterly progress report were sent to Dr. David C. Ives (Pratt & Whitney), Dr. Peter M. Sockol (NASA Lewis), and Dr. W. Habashi (Concordia University, Montreal, Quebec). A copy of the computer-program deck was also sent to Dr. Habashi.

As a related matter, a copy of the computer deck from the previous AFOSR contract (5) was sent to Dr. David L. Whitfield (AEDC, Tullahoma, TN).
Section 7

INVENTIONS

There have been no inventions or patent disclosures stemming from this research effort.
Figure 1  COORDINATE GRID IN Z-PLANE
REFERENCES


