PARALLELED POWER SUPPLIES
A comparison of methods of paralleling power supplies with the method used in the AEGIS low-voltage power system

C. W. Rosengrant
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Final Report
Prepared for
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Washington DC 20362

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A Comparison of Methods of Paralleling Power Supplies with the Method Used in the AEGIS Low-Voltage Power System

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SUPPLEMENTARY NOTES

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AEGIS
Reliability

ABSTRACT (Continue on reverse side if necessary and identify by block number)
This report was prepared as part of the reliability studies of the AEGIS low-voltage power supply and compares methods of paralleling power supplies to determine the method most suitable for the AEGIS system. The two fundamental methods of paralleling power supplies are the automatic crossover method and master/slave method. The automatic crossover method has two subdivisions: (1) automatic crossover method (used by AEGIS system); and (2) the spoiler resistor method. The master/slave method has three subdivisions: (1) master pass element drive; (2) master voltage reference; and (3) current sense. The degree of current sharing that may be expected from these methods varies and depends on the method and how it was used.

(Continued)
20. Abstract (Continued)

The automatic crossover method is characterized by unequal current sharing and closely regulated power supplies; however, if regulation can be sacrificed, a degree of current sharing may be achieved. The master/slave method theoretically offers equal current sharing but, because of inescapable component differences, only the current sense method offers a reliable method for achieving this.

A comparison of the methods of paralleling power supplies show advantages and disadvantages for each; the master/slave method may have a slightly higher individual power supply reliability because of its equal load-sharing characteristics; however, this reliability advantage does not consider the additional control circuitry that would reduce reliability.

Using the reliability of the AEGIS power supply, the bus reliabilities of three methods of paralleling power supplies are calculated and are compared: automatic crossover, master/slave I, and master/slave II. Master/slave I represents a design whereby failure of the master power supply causes catastrophic bus failure. Master/slave II represents a design that causes a slave unit to take over as the master supply. The comparison method shows that the only possible improvement to the automatic crossover method is through the master/slave II method. The degree of improvement would have to be determined by actually designing and testing such a power supply system, which is beyond the scope of this report.

If a master/slave II system were designed and built for the AEGIS low-voltage power system, there is no assurance that it would achieve the advantage that the master/slave II method potentially has over the present automatic crossover method. Time, plus engineering design and development costs, would be required to acquire a master/slave II power supply system. Meanwhile, the already available automatic crossover method power supplies could be improved in reliability through field experience and improved parts.

It is concluded that, for the conditions considered, the equal load-sharing advantage of the master/slave II method of paralleling is marginal when compared to the present automatic crossover method, and that the two methods are roughly the same in both bus and power supply reliability. It is recommended that the present automatic crossover method of paralleling low-voltage power supplies in the AEGIS system be retained.
OBJECTIVE

Determine the power supply paralleling method that is most suitable for the AEGIS low-voltage power system.

RESULTS

Five methods of paralleling power supplies are described and compared:

- Automatic crossover
- Snubber resistor
- Master pass element drive
- Master voltage reference
- Current sense.

The first two methods are automatic crossover methods and the last three are master/slave methods. Of these methods, only the current sense method offers a reliable method of equal load sharing.

Three methods of paralleling power supplies were defined for bus reliability calculations: automatic crossover, master/slave I, and master/slave II. For the conditions considered in the report the bus reliabilities of the automatic crossover and master/slave II methods were improved by approximately 10.25%, for a 5% increase in power supply reliability. Both of these methods have bus reliabilities significantly better than the master/slave I method.

RECOMMENDATION

Retain the AEGIS system’s automatic crossover method of paralleling low-voltage power supplies.
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INTRODUCTION

Paralleled power supplies may be a desirable feature in some power applications. Some of the advantages that may be gained through paralleling are high bus reliability, simplified maintenance, simplified logistics, and system flexibility. These and other advantages depend on the method of paralleling used.

There are several methods that may be used in paralleling power supplies. The paralleling methods described in this report are automatic crossover, spoiler resistor, master pass element drive, master voltage reference, and current sense. The first two methods are in the automatic crossover category and the last three in the master/slave category. The variations within the categories offer equal and unequal current sharing plus other advantages and disadvantages. The choice of the method to use would be based on a comparison of the power system requirements and the characteristics of the possible methods.

Only the basic power supply circuits are discussed in this report, but appendix A is included to briefly describe how the high-voltage and reverse-voltage protection is provided to the AEGIS low-voltage power supplies.

THE AUTOMATIC CROSSOVER METHOD OF PARALLELING POWER SUPPLIES

The automatic crossover method of paralleling power supplies takes advantage of the V/I characteristic curve of the paralleled power supplies. For this reason, there is no need for interconnecting control wires among the paralleled supplies. Unequal current sharing is a characteristic of this method, but under certain conditions some current sharing may be achieved.

AUTOMATIC CROSSOVER METHOD

Figure 1 shows the V/I characteristic curve of power supplies used in parallel operation in the automatic crossover mode. The voltage is well regulated until a specific current is reached and current regulation takes effect. A detailed description of the AEGIS low-voltage power supply V/I characteristic curve is given in appendix B.

Figure 1. Power supply V/I characteristic curve.
Parallel-connected power supplies that have V/I characteristics such as that of figure 1 will normally have close but different values of voltage regulation. The result is that the power supply with the highest value of voltage regulation will supply current to the load until it crosses over into current regulation. For additional current, the supply with the next lower value of voltage regulation will begin to contribute current to the load. This process repeats until the load's current requirement is met.

As an example, the V/I characteristic curves of three identical power supplies with different values of voltage regulation are shown in figure 2. The basic circuit of a power supply is illustrated in figure 3 and the paralleled configuration of the power supplies is illustrated by figure 4. When the load current from power supply 1 reaches the current regulation value for power supply 1, I_1, power supply 2 will begin to contribute current to the load at a reduced voltage. The total load current becomes I_1 plus whatever power supply 2 is contributing. When the current increases beyond the capacity of power supplies 1 and 2, power supply 3 will begin to contribute current in the same way. Note that in each case the output voltage drops slightly so that instead of being V_1 when only power supply 1 is contributing current, it finally becomes V_3 when power supply 3 is contributing current.

In the automatic crossover mode, one power supply will be constantly carrying a partial or full load while the others could be in standby and not supplying any current. Thus, unequal sharing of the load current is inherent with the circuits and V/I characteristics illustrated in figures 2, 3, and 4.

Figure 2. V/I characteristics of circuit with three paralleled power supplies.

Figure 3. Basic power supply circuit.
SPOILER RESISTOR METHOD

Another method of paralleling power supplies related to the automatic crossover method employs the use of "spoiler" resistors to achieve equal load sharing (figure 5).

The operation requires that a spoiler resistor $R_s$ be inserted in series with each of the paralleled supplies. The spoiler resistors are adjusted to achieve equal current sharing and may be large or small, depending on the ability of the power supply to hold its voltage over the life of the supply. This method is useful for fixed-load situations but sacrifices efficiency, regulation, and low source resistance because of the resistance of the series resistor. The spoiler resistors may be less than 1 ohm when adjusted.

CURRENT SHARING IN THE AUTOMATIC CROSSOVER METHOD

Equal current sharing is theoretically possible in the configuration described by figures 2, 3, and 4 if the V/I characteristics of each paralleled power supply are identical. Practically, this is very difficult if not impossible. Very fine adjustments would have to be made to achieve the necessarily identical output voltages, and even then equal current sharing could not be assured because of the tolerance of the voltage measuring equipment. Furthermore, factors that cause unequal current sharing would be introduced sooner or later by temperature differences and by component aging. Finally, the greater the voltage regulation for the power supplies, the greater the difficulty there will be in achieving equal current sharing with the automatic crossover method.

A degree of current sharing among power supplies in the automatic crossover method may be achieved by introducing an impedance into the output of each of the power supplies. This impedance will cause a decrease in regulation — the change in the supply

Figure 4. Automatic crossover configuration.

Figure 5. Spoiler resistor configuration.
output voltage as the output current increases. The decrease in voltage is called droop and will cause current sharing among power supplies that are set to approximately the same voltage. This situation, the case where there are three paralleled power supplies with two intended to supply current and one intended to be in standby, becomes a case in which all three share the load current to some degree. The V/I characteristic curves in figure 6 illustrate this principle. The total output current is the sum of $I_1, I_2, I_3$, and the degree of current sharing is determined by the droop and by precisely how the output voltages $V_1, V_2, V_3$ are calibrated.

![Figure 6. Exaggerated V/I characteristic curves of three automatic-crossover, paralleled power supplies.](image)

**THE MASTER/SLAVE METHOD OF PARALLELING POWER SUPPLIES**

Three possible master/slave configurations of paralleling power supplies are (1) master pass element drive; (2) master voltage reference; and (3) current sense. Control is by voltage feedback in the first two configurations, while the third uses current feedback. The fundamental principle behind the master/slave method is that only one element of the system controls all of the other elements. Each of the master/slave methods will theoretically cause equal current sharing, but because of the effects of component differences, only the current sense method can be expected to deliver equal currents reliably.
MASTER PASS ELEMENT DRIVE

A single voltage-regulated power supply may consist of a comparison amplifier, A, that is driving a pass element, Q. This is illustrated by the circuit of figure 7.

![Circuit diagram of figure 7](image)

Figure 7. Single voltage-regulated power supply.

In the circuit of figure 7, $E_O$ is proportional to $E_T$, $R_R$, and $R_C$ in accordance with equation 1 that is derived in appendix C:

$$E_O = E_T \frac{R_C}{R_R}$$

(1)

Thus, if $I_L$ increases while $R_L$ is a constant, a corresponding increase in $E_O$ will take place. This will be divided in turn across $R_C$ and $R_R$. This increase across $R_R$ will be sensed by comparison amplifier A, inverted, and will cause a decrease in current through the pass element, Q.

If two or more identical power supplies are placed in parallel, control may be achieved by having a comparison amplifier drive the pass elements of all the power supplies. This configuration is illustrated in figure 8. Under ideal conditions, the same comparison amplifier is driving identical pass elements and the pass elements will conduct equally and generate the same terminal voltages. In reality, however, identical pass elements do not exist and differences in output voltage and current will occur at each pass element. The number of pass elements or power supplies that can be controlled by this method depends on the amount of drive that the comparison amplifier is capable of producing. Other factors that would cause differences in the currents would be aging and changes in the load.

![Circuit diagram of figure 8](image)

Figure 8. Master pass element drive configuration.
MASTER VOLTAGE REFERENCE

Paralleled power supplies may use a master control supply that acts as a common reference to control all the paralleled power supplies. The master supply is not required to deliver any power to the load since it is used only to maintain the desired reference voltage.

The configuration for this type of control is illustrated in figure 9. $E_M$ is the master voltage reference and is equal to the desired output voltage. Any increase in load current with $R_L$ constant will show up as an increase in $E_O$. The voltage increase of $E_O$ will appear across the terminals of the comparison amplifiers, be inverted, and cause a decrease in current through the transistor pass elements. Under ideal conditions for identical power supplies adjusted to deliver equal currents, any change in $E_O$ will cause an equal change in all the power supplies. In reality, however, there are no identical power supplies, and because of differences among the pass elements and comparison amplifiers, the delivery of equal currents would not occur. Factors in this configuration that would cause differences in the currents delivered are component aging, pass element differences, and comparison amplifier differences. Furthermore, because of these factors, as the load changes the current ratios will also change.

CURRENT SENSE

The previous two examples described master/slave control methods that used voltage feedback. In the current sense method, a master unit is controlled by the voltage-control method described earlier but the slave units are controlled by current sense resistors in the emitter circuits. This configuration is illustrated by the circuit in figure 10.

The output voltage is determined by the ratio of equation (1). Once this is set, the ratio of the currents shared by the three parallel power supplies of figure 10 is determined by the $R_S$ of the slave units. If $R_S$ is the same for each power supply, the load current will be shared equally among them. A larger $R_S$ for a slave unit will cause that unit to contribute a proportionately smaller amount of current.

The current sense method offers the greatest possibility for equal current sharing in the master/slave control method since it is not affected by differences among pass elements or comparison amplifiers and changes in the load.
CURRENT SHARING IN THE MASTER/SLAVE METHOD

In the three master/slave methods described, the current sense method would provide the most equal sharing of the load current at the required voltage. Currents delivered in the current sense method will typically be within 10% of each other for light loads and 2% for heavy loads. The other two methods that employ voltage control are subject to wider difference in the parallel currents, but under normal operating conditions the current differences would not cause significant differences in stress among the paralleled power supplies. Table 1 summarizes the effects that certain influences will have on current sharing in master/slave methods of paralleling. For equal current sharing, the order of preference would be current sense, master pass element drive, and master voltage reference.

Table 1. Comparison of methods of master/slaving with factors that affect current sharing.

<table>
<thead>
<tr>
<th>Method</th>
<th>Aging</th>
<th>Comparison Amplifier</th>
<th>Pass Element</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Sense</td>
<td>Small</td>
<td>Small</td>
<td>Small</td>
<td>Small</td>
</tr>
<tr>
<td>Master Pass Element Drive</td>
<td>Significant</td>
<td>Small</td>
<td>Significant</td>
<td>Significant</td>
</tr>
<tr>
<td>Master Voltage Reference</td>
<td>Significant</td>
<td>Significant</td>
<td>Significant</td>
<td>Significant</td>
</tr>
</tbody>
</table>

COMPARISON OF THE AUTOMATIC CROSSOVER AND MASTER/SLAVE METHODS OF PARALLELING POWER SUPPLIES

The automatic crossover method of paralleling well regulated power supplies does not require the special internal circuitry or external control interconnections that are required in the master/slave method. Since the individual supplies are all the same and interchangeable, the system is easily expanded to accommodate increased power requirements. In systems designed for fault tolerance, one supply will normally operate in a standby mode ready instantly to assume the load of a power supply that has been removed.
from the power supply circuit because of failure. Unequal load sharing would be expected in the automatic crossover method of paralleling, but this may not be significant as long as the power supplies are not overstressed. If necessary, a degree of load sharing may be created by increasing the output impedance of the power supplies and sacrificing close voltage regulation.

The master/slave method of paralleling power supplies has the potential of being able to cause each power supply in parallel to provide roughly equal amounts of current to the load, and to provide for less system stress than that experienced in the automatic crossover method. In reality, however, only the current sense method may provide equal currents because the other master/slave methods are subject to differences in components that cause unequal currents. The master/slave method uses two types of power supplies, but the use of only one is possible if it has provisions to be set either as a master or a slave prior to insertion in the system. These supplies would require additional control circuitry and a fail-safe method for insuring that only one master supply is inserted.

BUS RELIABILITY OF PARALLELED POWER SUPPLIES

When a power supply fails in the fault-tolerant, automatic crossover method of paralleling and is removed electronically from the bus, the redundant power supply will take over the failed supply’s load. For the master/slave method, the failure of the slave supplies and master supply must be considered separately. When a slave supply fails, the other paralleled supplies assume its load. When a master supply fails, there are two possible results that depend on two designs. These designs will be called master/slave I and master/slave II. Master/slave I represents a design in which failure of the master supply causes catastrophic bus failure. Master/slave II represents a design that causes a slave supply to take over as the master supply.

Bus reliabilities, which are expressed as mean times between failure (MTBF), are calculated in appendix D for the automatic crossover, master/slave I, and master/slave II methods of paralleling power supplies. The results of the calculations for MTBF with repair are summarized in table 2. The conditions that were used to obtain the results are listed below:

- A failed power supply is removed electronically from the bus.
- n power supplies are paralleled, but n – 1 power supplies are necessary to sustain the bus.
- 25 000- and 26 250-hour MTBFs are required for power supplies. (These figures represent a 5% difference in reliability and reflect RCA’s reported difference in AEGIS power supply reliabilities under equal and unequal local sharing conditions; RCA reply to AWS PRDR action item 2.) The calculations are based on the reliability of the AEGIS automatic crossover power supply, but do not consider the additional circuitry that would be used in a master/slave method.
Table 2. MTBF with repair (hours).

<table>
<thead>
<tr>
<th></th>
<th>$n = 3$</th>
<th>$n = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MTBF$_1$</td>
<td>MTBF$_2$</td>
</tr>
<tr>
<td>Automatic Crossover</td>
<td>208.33M</td>
<td>229.68M</td>
</tr>
<tr>
<td>Master/Slave I</td>
<td>24.999k</td>
<td>26.249k</td>
</tr>
<tr>
<td>Master/Slave II</td>
<td>208.33M</td>
<td>229.68M</td>
</tr>
</tbody>
</table>

The following are some of the conclusions that may be drawn from table 2:

- As $n$ increases, all bus reliabilities decrease.
- For power supplies of the same reliability, the master/slave I method has the least reliable bus among the three methods. (The master/slave I bus reliability can be no greater than the reliability of the master power supply.)
- For power supplies of the same reliability, the automatic crossover and master/slave II methods have the same bus reliability.
- For the automatic crossover or master/slave II methods, and $n = 3$, an increase of 5% in power supply reliability will cause an increase of 10.25% in bus reliability.
- For the automatic crossover or master/slave II methods, and $n = 4$, an increase of 5% in power supply reliability will cause an increase of 10.24% in bus reliability.
- As $n$ increases, the redundant power supply has a decreasing effect on the bus reliability.
- As $n$ increases, the percent difference in bus reliability between the automatic crossover and master/slave II methods will decrease.

Power supply reliability depends on design, and for equivalent power supplies it is reasonable to assume that a power supply designed for the master/slave method of paralleling would have more circuitry and less reliability than a power supply designed for the automatic crossover method of paralleling. This would be especially true of the master/slave II method of paralleling because of the additional control circuitry that is needed to switch master power supplies.

In the calculations that produced table 2, comparisons are made between bus(es) that have a 5% difference in power supply reliability. The basis for this difference in reliability was the equal versus unequal current sharing of the master/slave and automatic crossover methods, respectively. Not considered was the effect that any additional control circuitry would have. Since the 5% difference in power supply reliability would be reduced if the additional control circuitry were considered, the difference in bus reliability would also be reduced from 10.25% for $n = 3$. The amount that the power reliability would be reduced cannot be determined without a specific design, but it is possible that it could be more than 5%, which would make the automatic crossover power supply and bus more reliable.
CONCLUSIONS AND RECOMMENDATIONS

This report has shown that the automatic crossover method of paralleling power supplies is characterized by unequal current sharing and closely regulated power supplies; however, if regulation can be sacrificed, a degree of current sharing can be achieved. The master/slave method is shown to offer theoretically equal current sharing but, because of inescapable component differences, only the current sense method offers a reliable method of achieving this.

A comparison of the methods of paralleling power supplies shows advantages and disadvantages for each. The master/slave method may have a slightly higher individual power supply reliability because of its equal load-sharing characteristics; however, this reliability advantage does not consider the additional control circuitry that would reduce reliability.

The bus reliabilities of three methods of paralleling power supplies are compared: automatic crossover, master/slave I, and master/slave II. Master/slave I represents a design in which failure of the master power supply causes catastrophic bus failure. Master/slave II represents a design that causes a slave unit to take over as the master supply. The comparison shows that the only possible improvement to the automatic crossover method is through the master/slave II method. The degree of improvement would have to be determined by actually designing and testing such a power supply system, which is beyond the scope of this report.

If such a master/slave II system were designed and built, there is no assurance that it would achieve the advantage that the master/slave II method potentially has over the automatic crossover method. Time, plus engineering and design and development costs, would be required to acquire a master/slave II power supply system. Meanwhile, the already available automatic crossover method power supplies could be improved in reliability through field experience and improved parts.

It is concluded that, for the conditions considered here, the equal load-sharing advantage of the master/slave method of paralleling is marginal when compared to the present automatic crossover method, and that the two methods are roughly equivalent in both bus and power supply reliability. It is recommended that the present automatic crossover method of paralleling low-voltage power supplies in the AEGIS system be retained.
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APPENDIX A
HIGH-VOLTAGE AND REVERSE-VOLTAGE PROTECTION
IN THE AEGIS POWER SUPPLIES

When paralleling power supplies, high-voltage and reverse voltage protection must be considered. Without the protection, either the power supply electronics or the electronics being served, or both, will be damaged. High-voltage protection is needed when one of several paralleled power supplies fails and goes into over-voltage without shutting itself down. Reverse voltage protection is needed when positive and negative voltage power supplies are connected with a common ground and are then turned on at different times.

The RCA AEGIS low-voltage power supply is a closely regulated unit that operates in the automatic crossover mode and approaches the protection problems by using two techniques. First, for high bus voltages, the voltage is sensed and an SCR crowbar circuit on the output is triggered. Crowbars on other power supplies in parallel with this one are also triggered. Next, for reverse voltages, a hot carrier diode on the output handles the potentially damaging voltage and only the reverse voltage of one diode drop in voltage (less than 1 V) is seen by the power supply electronics. The diode and SCR arrangement is illustrated in simplified form in figure A-1.

![Figure A-1. Diode and SCR on output of power supply.](image)
APPENDIX B
AEGIS LOW-VOLTAGE POWER SUPPLY
V/I CHARACTERISTICS

The RCA AEGIS low-voltage power supply V/I characteristic curve is illustrated in figure B-1. Notice the hump at the beginning of the curve.

\[ V_{\text{HUMP}} \]

\[ \Delta V \]

\[ V \]

\[ V \]

\[ 0.1 I \]

\[ I \]

\[ V \]

\[ 2-4\% \text{ OF } V \]

\[ V_{\text{HUMP}} \]

\[ \Delta V \]

\[ \text{OUTPUT VOLTAGE TOLERANCE} \]

\[ \text{RATED VOLTAGE} \]

\[ \text{RATED CURRENT} \]

\[ \text{SINGLE POWER SUPPLY IDLE CURRENT} \]

Figure B-1. V/I characteristic curve of RCA AEGIS low-voltage power supply.

The hump serves several purposes:

- Reverse currents are prevented from flowing back into idling or standby power supplies.
- The power supplies are prepared for a fast response to rapid load changes since they are “primed” and operating at approximately 5% rated current.
- Idling permits monitoring of the standby power supplies for any failures.
APPENDIX C
DERIVATION OF OUTPUT VOLTAGE FORMULA
FOR A VOLTAGE FEEDBACK-CONTROLLED POWER SUPPLY

![Circuit Diagram]

\[ E_O = E_C + E_R - E_t \]

\[ E_D = 0 \text{ at steady state, so} \]
\[ E_t = E_R \text{ and } I_c = \frac{E_R}{R_R} = \frac{E_t}{R_R} \]

Thus

\[ E_O = E_C \]
\[ E_C = I_c R_C \]

\[ E_O = \left( \frac{E_t}{R_R} \right) \]

\[ E_O = \left( \frac{R_C}{R_R} \right) \]
APPENDIX D
BUS RELIABILITY CALCULATIONS

These calculations estimate bus reliability for three methods of paralleling power supplies: (1) automatic crossover (AC); (2) master/slave I (M/S I); and (3) master/slave II (M/S II). AC in this appendix operates the way that is typical for this method, and any failed supply’s load is assumed by a redundant supply. M/S I is configured as a conventional equal load-sharing master/slave design, whereby failure of the master supply causes catastrophic bus failure. A redundant slave supply is included in the M/S I method to increase bus reliability. M/S II is the same as M/S I except that there is a provision that allows any slave supply to become a master supply if the master supply fails. In all the configurations, the number of power supplies that are paralleled is n; the number of paralleled power supplies necessary to sustain the bus is n − 1; and a failed power supply is assumed to be removed electronically from the bus. Two different power supply reliabilities and two different n’s are used in the calculation for comparison purposes.

CALCULATIONS FOR EQUIVALENT BUS MTBF
(MTBF_{EQ}) WITHOUT REPAIR

The first set of calculations does not consider reliability with repair, which would result in a higher MTBF. Instead, these calculations are the MTBF_{EQ} of a bus that has no power supplies replaced until the bus fails. The steps that this set of calculations go through are separated into sections for clarity.

PROBABILITY OF SURVIVAL (Ps)

Probability of survival (Ps) is the probability that a piece of electronic equipment will operate successfully for a given time (t) when it has a given failure rate (λ). The formula for calculating Ps is given in equation (D-1). These calculations are based on a typical operating time of 5000 hours for 1 year, and failure rates (λ) that are listed in table D-1.

\[
Ps = e^{-\lambda t}
\]

where

\[
e = 2.71828 \text{ (a constant)}
\]

\[
t = 5000 \text{ hours}
\]

\[
\lambda = \text{failures per } 10^6 \text{ hours}
\]
Ps₁ and Ps₂ are calculated from equation (D-1):  
Ps₁ = 0.81873075;  
Ps₂ = 0.82654576.

**MISSION SUCCESS**

There are several possible methods to determine the probability of a system’s successful accomplishment of its mission. The method used in this set of calculations uses a truth table that represents all possible modes of successful system operation. The probabilities of success (Ps) of all the modes are added to give the probability of successful system operation.

The truth tables and calculations that follow assume that n power supplies are electrically paralleled, but that only n-1 are required to sustain the bus successfully. In tables D-2 through D-8, Ps₁ and Ps₂ from previous calculations and n = 3 and n = 4 are used. A, B, C, and D represent individual paralleled power supplies in the AC, M/S I, and M/S II methods. From a reliability standpoint, M/S I is series parallel, and AC and M/C II are parallel.

<table>
<thead>
<tr>
<th>Table D-2. Mission Ps for AC method and n = 3.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Table D-3. Mission Ps for AC method and n = 4.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
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<tr>
<td>1</td>
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<tr>
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<table>
<thead>
<tr>
<th>Table D-4. Mission Ps for M/S I method and n = 3.</th>
</tr>
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<tbody>
<tr>
<td>A</td>
</tr>
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</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
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<tr>
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</tr>
</tbody>
</table>

21
Table D-5. Mission Ps for M/S I method and n = 4.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Ps Function</th>
<th>P&lt;sub&gt;Ps1&lt;/sub&gt;</th>
<th>P&lt;sub&gt;Ps2&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>P&lt;sup&gt;4&lt;/sup&gt;</td>
<td>0.44932896</td>
<td>0.46673204</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>P&lt;sup&gt;3&lt;/sup&gt;(1-P)</td>
<td>0.09948267</td>
<td>0.09794576</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>P&lt;sup&gt;3&lt;/sup&gt;(1-P)</td>
<td>0.09948267</td>
<td>0.09794576</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>P&lt;sup&gt;3&lt;/sup&gt;(1-P)</td>
<td>0.09948267</td>
<td>0.09794576</td>
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</tbody>
</table>

Total: 0.74777697 0.76056932

Table D-6. Mission Ps for M/S II and n = 3.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Ps Function</th>
<th>P&lt;sub&gt;Ps1&lt;/sub&gt;</th>
<th>P&lt;sub&gt;Ps2&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>P&lt;sup&gt;3&lt;/sup&gt;</td>
<td>0.54881163</td>
<td>0.56467779</td>
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<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>P&lt;sup&gt;2&lt;/sup&gt;(1-P)</td>
<td>0.12150841</td>
<td>0.1185001</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>P&lt;sup&gt;2&lt;/sup&gt;(1-P)</td>
<td>0.12150841</td>
<td>0.1185001</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>P&lt;sup&gt;2&lt;/sup&gt;(1-P)</td>
<td>0.12150841</td>
<td>0.1185001</td>
</tr>
</tbody>
</table>

Total: 0.91333686 0.92017809

Table D-7. Mission Ps for M/S II method and n = 4.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Ps Function</th>
<th>P&lt;sub&gt;Ps1&lt;/sub&gt;</th>
<th>P&lt;sub&gt;Ps2&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>P&lt;sup&gt;4&lt;/sup&gt;</td>
<td>0.44932896</td>
<td>0.46673204</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>P&lt;sup&gt;3&lt;/sup&gt;(1-P)</td>
<td>0.09948267</td>
<td>0.09794576</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>P&lt;sup&gt;3&lt;/sup&gt;(1-P)</td>
<td>0.09948267</td>
<td>0.09794576</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>P&lt;sup&gt;3&lt;/sup&gt;(1-P)</td>
<td>0.09948267</td>
<td>0.09794576</td>
</tr>
</tbody>
</table>

Total: 0.84725964 0.85851508

Table D-8. Mission Ps summary.

<table>
<thead>
<tr>
<th>n = 3</th>
<th>n = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P&lt;sub&gt;Ps1&lt;/sub&gt;</td>
</tr>
<tr>
<td>M/S I</td>
<td>0.79182845</td>
</tr>
<tr>
<td>M/S II</td>
<td>0.91333686</td>
</tr>
</tbody>
</table>
BUS MTBF<sub>EQ</sub> WITHOUT REPAIR

The mission Ps from table D-8 and operating time of 5000 hours from the Ps calculations are used to determine the bus equivalent failure rate and MTBF<sub>EQ</sub> in tables D-9 and D-10. These are defined, respectively, as failure rate and MTBF that a serial item would need to have the same mission reliability.

Sample calculations:

\[ P_s = e^{-\lambda t} \]

\[ e = 5000 \text{ hours (typical 1-year operating time)} \]

\[ P_s = \text{table D-8 figures} \]

since

\[ P_s = e^{-\lambda t}, \]

\[ 0.91333686 = e^{-\lambda(5000)} \]

\[ \lambda = 18.13 \times 10^{-6} \text{ hours} \]

\[ MTBF_{EQ} = \frac{1}{\lambda_{EQ}} = 55157 \text{ HOURS.} \]

Table D-9. Bus failure rates per 10^6 hours.

<table>
<thead>
<tr>
<th></th>
<th>( P_{s1} )</th>
<th>( P_{s2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>18.13</td>
<td>16.64</td>
</tr>
<tr>
<td>M/S I</td>
<td>46.68</td>
<td>44.18</td>
</tr>
<tr>
<td>M/S II</td>
<td>18.13</td>
<td>16.64</td>
</tr>
</tbody>
</table>

Table D-10. Bus mean time between failure (hours).

<table>
<thead>
<tr>
<th></th>
<th>( P_{s1} )</th>
<th>( P_{s2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>55 157</td>
<td>60 096</td>
</tr>
<tr>
<td>M/S I</td>
<td>21 422</td>
<td>22 635</td>
</tr>
<tr>
<td>M/S II</td>
<td>55 157</td>
<td>60 096</td>
</tr>
</tbody>
</table>

23
CALCULATIONS FOR BUS MTBF WITH REPAIR

Equation D-2 is used to determine the approximate MTBF of a repairable redundant system.

\[
\text{MTBF} = u \left( \frac{u}{D} \right)^{n-r} / n \binom{n}{r-1} \quad (D-2)
\]

- **u** = MTBF of unit
- **D** = mean time to repair (MTTR) unit
- **n** = number of units
- **r** = number of units required for success.

Equation D-3 is the formula used to determine the number of combinations of **P** things taken **Q** at a time which is part of equation D-2.

\[
C^P_Q = \binom{P}{Q} = \frac{P!}{Q! (P - Q)!} \quad (D-3)
\]

\[ [P = \left( \frac{1}{0} \right) = 1 \text{ by definition.}]\]

In calculating the bus MTBF with repair, the following values will be used:

- **MTBF\(_1\)** = 25 000 hours
- **MTBF\(_2\)** = 26 250 hours (5% greater than **MTBF\(_1\)**)
- **D** = 30 minutes = 0.5 hour
- **n** = 3 or 4
- **r** = n - 1.

Reliability block diagrams are given in figures D-1 through D-6. Figures D-1 and D-5 represent three power supplies in operational redundancy where two are required for success. Figures D-2 and D-6 represent four power supplies in operational redundancy where three are required for success. Figure D-3 represents two power supplies in operational redundancy with one required for success, plus one power supply in series. Figure D-4 represents three power supplies in operational redundancy with two required for success, plus one power supply in series. The results of the calculations based on these reliability block diagrams are summarized in table D-11.

![Reliability block diagram](image)

Figure D-1. Reliability block diagram for three power supplies paralleled in the AC method.
Figure D-2. Reliability block diagram for four power supplies paralleled in the AC method.

Figure D-3. Reliability block diagram for three power supplies paralleled in the M/S I method.

Figure D-4. Reliability block diagram for four power supplies paralleled in the M/S I method.

Figure D-5. Reliability block diagram for three power supplies paralleled in the M/S II method.

Figure D-6. Reliability block diagram for four power supplies paralleled in the M/S II method.
Table D-11. MTBF with repair (hours).

<table>
<thead>
<tr>
<th></th>
<th>$n=3$</th>
<th>$n=4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MTBF&lt;sub&gt;1&lt;/sub&gt;</td>
<td>MTBF&lt;sub&gt;2&lt;/sub&gt;</td>
</tr>
<tr>
<td>AC</td>
<td>208.33M</td>
<td>229.68M</td>
</tr>
<tr>
<td>M/S I</td>
<td>24.999k</td>
<td>26.249k</td>
</tr>
<tr>
<td>M/S II</td>
<td>208.33M</td>
<td>229.68M</td>
</tr>
</tbody>
</table>

SAMPLE CALCULATIONS FOR MTBF WITH REPAIR

Example 1. Reliability Block Diagram:

\[ u = 25,000 \text{ hours} \]
\[ D = 30 \text{ minutes} = 0.5 \text{ hour} \]
\[ n = 3 \]
\[ r = 2 \]

\[ \text{MTBF} = u \left( \frac{u}{D} \right)^{n-r} \left/ \binom{n-1}{r-1} \right. \]
\[ = 25,000 \left( \frac{25,000}{0.5} \right)^{3-2} \left/ \binom{3}{2-1} \right. \]
\[ = 25,000 \times \frac{50}{3} \left( \frac{2}{1! \times 1!} \right) \]
\[ = 1250M/3(2) \]

\[ \text{MTBF} = 208.33 \text{ Mhours.} \]

Example 2. Reliability Block Diagram:

\[ \text{MTBF} = \frac{(\text{MTBF}_A) \times (\text{MTBF}_B)}{\text{MTBF}_A + \text{MTBF}_B} \]
\[ \text{MTBF}_A = 25,000 \text{ hours} \]
\[ \text{MTBF}_B = 208.33 \text{ Mhours} \]

\[ \text{MTBF} = \frac{(25k) \times (208.33M)}{25k + 208.33M} = \frac{5.2083 \times 10^{12}}{2.0836 \times 10^8} \]
\[ = 24.99 \text{ khours}. \]
BIBLIOGRAPHY

