ADINC: An Implicit Lagrangian Hydrodynamics Code

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THIS DOCUMENT IS BEST QUALITY PRACTICABLE. THE COPY FURNISHED TO DDC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.
This paper presents and describes a new software package called ADINC (ADiabatic and INCompressible flows) which is designed to solve the mass, momentum, and adiabatic energy equations for a rather general one-dimensional fluid system. A fully Lagrangian, conservative hydrodynamics algorithm is incorporated so that interfaces between different materials can be followed accurately. The Courant timestep limit for sound waves is overcome by implicit time differencing with a quadratically convergent iteration for nonlinear time centering. Listings of the pertinent ADINC routines, and a general driver test program are included in the appendices. (Continued)
20. ABSTRACT (Continued):

A rather general analytic equation of state routine is included which handles adiabatic gases, slightly compressible solids, and even the incompressible limit in adjacent cells. The test program has provision for up to five layers of material. Three test problems are presented in the appendices with representative outputs for code verification.
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<thead>
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<tr>
<td>$r_i$</td>
<td>position (cm) of the $i$-th cell interface</td>
</tr>
<tr>
<td>$V_i$</td>
<td>velocity (cm/sec) of the $i$-th cell interface</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>density (gm/cc) of the fluid in the $i$-th cell between interfaces $i-1$ and $i$</td>
</tr>
<tr>
<td>$P_i$</td>
<td>pressure (erg/cc) of the fluid in the $i$-th cell</td>
</tr>
<tr>
<td>$t$</td>
<td>time variable (sec)</td>
</tr>
<tr>
<td>$r$</td>
<td>generalized 1D coordinate (cm)</td>
</tr>
<tr>
<td>$\rho(r,t)$</td>
<td>fluid density as a continuum function</td>
</tr>
<tr>
<td>$V(r,t)$</td>
<td>fluid velocity as a continuum function</td>
</tr>
<tr>
<td>$P(r,t)$</td>
<td>fluid pressure as a continuum function</td>
</tr>
<tr>
<td>$\rho_0(r)$</td>
<td>density at zero pressure, a Lagrangian fluid property</td>
</tr>
<tr>
<td>$S(r)$</td>
<td>entropy constant, a Lagrangian fluid property</td>
</tr>
<tr>
<td>$\gamma(r)$</td>
<td>adiabatic gas constant, a Lagrangian fluid property</td>
</tr>
<tr>
<td>$A$</td>
<td>cell volume</td>
</tr>
<tr>
<td>$A(r)$</td>
<td>cross sectional area of the 1D computational &quot;volume&quot; as a function of position</td>
</tr>
<tr>
<td>$V(r)$</td>
<td>used briefly (Section II) as an integrated volume variable, do not confuse with velocity</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$1, 2, 3,$ or $4$ to determine ADIWC geometry</td>
</tr>
<tr>
<td>$G_1, \ldots, G_5$</td>
<td>GEOMCO = geometry coefficients for power series coordinates</td>
</tr>
<tr>
<td>$N$</td>
<td>$#$ of fluid cells in the computational domain</td>
</tr>
<tr>
<td>$A_i$, LAMC(I)</td>
<td>volume (cm$^3$ = cc) of the $i$-th cell</td>
</tr>
<tr>
<td>$A_i$, AREA(I)</td>
<td>area (cm$^2$) of interface $i$</td>
</tr>
<tr>
<td>$R_i$, RADC(I)</td>
<td>cell $i$ center location (cm)</td>
</tr>
<tr>
<td>$r_{LNEW}$</td>
<td>left bounding interface location (cm)</td>
</tr>
<tr>
<td>$r_{RNEW}$</td>
<td>right bounding interface location (cm)</td>
</tr>
</tbody>
</table>
\( V_L = \text{VLNEW} \)  
- left boundary interface velocity (cm/sec)

\( V_R = \text{VRNEW} \)  
- right boundary interface velocity (cm/sec)

\( \delta t = \text{DT,DTIN} \)  
- computational timestep increment (sec)

\( \epsilon_r = \text{EPSR} \)  
- explicitness parameter, position equation

\( \epsilon_v = \text{EPSV} \)  
- explicitness parameter, momentum equation

\( o \)  
- old value, as a superscript

\( n \)  
- new value, as a superscript

\( p \)  
- most recent approximation to the new value, as a superscript

\( <p \delta r>_{i+1/2} \)  
- average mass per unit area (gm/cm\(^2\)) in the vicinity of interface \( i \)

\( P^* \)  
- special acceleration averaged pressure at interface \( i \)

\( h \)  
- value averaged at the new and old values, as a superscript

\( a_i, b_i \)  
- coefficients in the momentum Eq. (13) and following

\( \Lambda_{p^{(cos)}} = \text{LAMEOS}(l) \)  
- estimated new volume of cell \( i \) using the equation of state, should equal \( r \)

\( \Lambda_{p^{(fd)}} \)  
- estimated new volume of cell \( i \) using the fluid dynamic motion

\( S_i = \text{ENTC}(l) \)  
- entropy constant (erg/cc) of cell \( i \)

\( \left[ \frac{\partial A}{\partial P} \right]_{p^{(cos)}} = \text{DLAMDP}(l) \)  
- rate of change of cell \( i \) volume with pressure from the equation of state.

\( d^+, d^-, c_i \)  
- coefficients in Eq. (18) and following

\( A^+, B^+, C^+, D^+ \)  
- coefficients in the tridiagonal equation derived for \( \{P^*_n\} \), Eq. (22).

\( \Delta M_i = \text{MASSC}(l) \)  
- fluid mass (gm) in cell \( i \)

\( C_s \)  
- sound speed in the fluid

\( \delta t_{\text{val}} = \text{DTVAL} \)  
- timestep limit (sec) to prevent interface crossing

\( \delta t_{\text{max}} = \text{DTMAX} \)  
- the maximum timestep (sec) permitted in the
calculation, chosen by the user

\[ \delta t_{\text{min}} = \text{DTMAX} \]
- the minimum timestep (sec) permitted in the calculation, chosen by the user

\[ \gamma_i = \text{GAMMAC}(i) \]
- adiabatic gas constant, a Lagrangian fluid property of cell \( i \)

\[ \delta V = \text{DVEL} \]
- velocity perturbation (cm/sec) in standard Test \#1, Eq. (28), and following

\[ E_{\text{therm},i} = \text{ETHERM}(i) \]
- internal energy density (erg/cc) in cell \( i \)

\[ \lambda^+, \lambda^- \]
- partial volumes of cell \( i \) (cm\(^3\))

\[ E_{\text{KE},i} = \text{EKINE}(i) \]
- kinetic energy density in cell \( i \) (erg/cm\(^3\))

\[ k \]
- wavenumber cm\(^{-1}\) in Eq. (35)

\[ M_s, \rho_s, V_s \]
- slug mass, density and velocity in standard Test \#2, a nearly incompressible fluid

\[ P_R(t), P_L(t) \]
- pressure in the gas regions to the right and left of the slug respectively

\[ \gamma_R, \gamma_L \]
- adiabatic gas constants in the regions to the right and left of the heavy slug

\[ W_R(t), W_L(t) \]
- width (cm) of the gas regions to the right and left of the slug

\[ S_R, S_L \]
- entropy constant in the gas regions to the right and left of the slug

\[ \omega \]
- angular frequency of slug oscillation

\[ M_R, M_L \]
- mass (gm) of the gas regions to the right and left of the slug.

\[ L_c \]
- the size (cm) of the system when \( \rho = \rho_c \)

\[ T \]
- nonlinear limiting period (sec) of the oscillating slug
ADINC: An Implicit Lagrangian Hydrodynamics Code

I. INTRODUCTION

This paper presents and describes a new software package called ADINC (ADIabatic and INCompressible flows) which is designed to solve the mass, momentum, and adiabatic energy equations for a rather general one-dimensional fluid system. To permit accurate representation of material interfaces, a fully conservative, Lagrangian hydrodynamics algorithm has been incorporated. Implicit time differencing with a quadratically convergent iteration for non-linear time centering permits ADINC to take long timesteps exceeding the Courant sound speed limit.

ADINC is designed as a user-oriented package with a flexible equation of state which may vary from cell to cell and provision for four different geometries. This write-up not only describes the techniques employed and three different benchmark test problems, it serves as documentation for the package. To these ends, listings and representative outputs are included in the appendices.

ADINC was written to circumvent in a single software package a number of common numerical difficulties which arise in the simulation of many different fluid dynamic systems. In some fluids the physical phenomena under study vary slowly compared to the time it takes sound waves to cross the system. Nevertheless, substantial compressions and expansions occur so the incompressibility assumption is invalid. In other systems rather sharp interfaces between chemical species or between different temperature regions must be maintained, yet the fluids interact dynamically across the interface.

The first class of problems demands an implicit treatment of sound waves in general although the asymptotic "slow-flow" approach\(^1-3\) works well in many cases. In many combustion systems, for example, heat is being added slowly so the gas expands locally where the heat is being released. The expansion is so structured that it guarantees continued spatial constancy

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of the pressure. Such pressure fluctuations as do occur\textsuperscript{4} are consistent with driving the fluid flows which bring about the required expansion. In these slow flow systems the heating and cooling rates from chemical reactions, thermal diffusion, and radiation emission and absorption control the hydrodynamics and hence determine the local density and temperature. The pressure evolution equation can be solved algebraically for the divergence of the velocity at any instant of time while the vorticity evolution equation is integrated to advance the curl of the velocity.\textsuperscript{5} Given the curl, the divergence, and reasonable boundary conditions, almost everything we need to know about the flow is known. When the pressure fluctuations, i.e. the sound waves, themselves are required, a stiff, implicit hyperbolic wave equation results. Such equations can be solved numerically but in multidimensions the cost is often high and the best formalism is by no means clear.

In the study of ablation and deflagration phenomena, pressure variations are important even though the fluid profiles themselves vary slowly. Here again the sound speed in the high temperature region would require very small timesteps if an explicit integration algorithm is used. Even though the slow-flow approximation of constant pressure is not valid here, the generalization to implicit hydrodynamics is valid— as long as shocks are absent. In atmospheric flows, in turbulence modelling for reactive systems and in studies of Rayleigh-Taylor instabilities it is the pressure fluctuation gradients specifically which interact with the local density gradients to generate vorticity. Thus, again, some form of implicit hydrodynamics or implicit pressure solution is required in these and similar cases.

In imploding liner and pellet fluid dynamics, solid and higher density "fluids" such as compressed DT, heavy pusher shells, and liquid metal rotating liners move in a way which is often shock free and which generally requires non-trivial equations of state. In LINUS\textsuperscript{6-8}, for example, an electrically conducting cylindrical liquid metal shell is imploded to compress a fusion plasma. The material along the inner surface of the liner starts out almost perfectly incompressible but compresses by as much as 50\% when the dynamic pressure from geometric
convergence reaches its maximum. The liquid compression is important and hence so are the
detailed dynamics leading up to that compression. The sound waves in the liquid metal lead to
a "water hammer" effect which may have severe engineering consequences.

The second class of numerical problems, maintaining sharp interfaces between disparate
materials or between materials in widely disparate states, often demands a Lagrangian descrip-
tion so that the exact interface location is available at any time. Eulerian computations end up
with the real interface spread over several zones from the numerical diffusion needed for stabil-
ity. While this numerical smearing of gradients is acceptable in many cases, in many others it
is not acceptable. The ADINC package uses an implicit, fully Lagrangian algorithm to over-
come these two classes of problems. The system as currently modelled is assumed to have no
non-ideal effects such as viscosity. Thus energy conserving shocks are not recovered even
though large amplitude acoustic waves are handled accurately. The entire package is structured
so that rather general equations of state, boundary conditions, and flow geometry are permitted.
Other non-ideal effects can be introduced to the calculation via time-step splitting.

The Courant timestep limit is overcome by using an implicit finite difference algorithm
with adjustable explicitness coefficients in both the position and velocity equations. Since the
basic equations (Section II) are nonlinear, iteration is required to obtain convergence. ADINC,
unlike some implicit formulations, continually re-evaluates the nonlinear terms after each itera-
tion to obtain an accurate as well as a stable algorithm when changes per timestep are large.
Since this increased accuracy has the computational penalty that all coefficients for the iteration
must be re-evaluated each iteration cycle, care has been taken to develop a quadratically con-
vergent iteration encompassing both fluid dynamics and equation of state variations simultane-
ously.

The interface resolution problem is handled by making the calculation fully Lagrangian.
No rezone or remap capability is included in the basic package so numerical diffusion from spa-
tial differences is absent from the model. Future additions to the ADINC package will include
a fully Lagrangian adaptive gridding package so that adequate resolution can be maintained in
regions where sharp gradients develop or where zone sizes become unacceptably large due to
fluid expansion.

Section II of this paper discusses the basic dynamical equations solved by the ADINC
package and the various choices of geometry which are available. Section III presents the
numerical algorithms used to integrate these equations along with the various tricks imple-
mented to maintain accuracy in bizarre situations. The timestep conditions built into the pack-
age are also discussed.

Section IV is devoted to the structure of the ADINC package and reviews the three major
routines. It gives calling sequences and argument lists for the nine entries. ADINC has been
constructed as a utility package to advance the four hydrodynamic variables:

\[ r_i = \text{position (cm) of the } i\text{-th cell interface} \]
\[ V_i = \text{velocity (cm/sec) of the } i\text{-th cell interface} \]
\[ \rho_i = \text{density (gm/cm}^3\text{) in the } i\text{-th cell between interfaces } i-1 \text{ and } i, \text{and} \]
\[ P_i = \text{pressure (erg/cm}^3\text{) in the } i\text{-th cell between interface } i-1 \text{ and } i. \]

ADINC has been cast into a form resembling that of an ordinary differential equations package.
The user requests integration of the system of equations to a certain point in time and ADINC
then selects the number and length of timesteps necessary to span this interval. The user also
has control of various error and integration parameters without having to modify the ADINC
code.

Section V tells how to use ADINC but a prospective user will probably find Section VI
which describes three test problems and the Appendices equally instructive. The three test
problems were designed to give a prospective user a useful background of experience with the
program in different regimes and with different problems. The Appendices contain listings of
the ADINC package (Appendix A) and a rather general test program with initialization and I/O routines (Appendix B). The present version of ADINC and its test program are written entirely in 64 bit floating point arithmetic. Thus convergence to better than 1 part in $10^7$ is possible for problems with near incompressibility and/or extreme density discontinuities which require this accuracy. A quadratically convergent algorithm is used to speed joint convergence of the nonlinear fluid dynamics and equation of state physics. The only portion of the code not appearing in the appendices are the vectorized tridiagonal solvers and these are documented by Boris in NRL Memorandum Report 3408, November, 1976.

The test program is arranged to handle up to five distinct layers of fluid composed of up to 200 individual finite difference cells. Problems in one of four geometries can be set up: Cartesian coordinates, cylindrical coordinates, spherical coordinates, and a variable power series coordinates for treating one-dimensional nozzle-like geometries. The boundary conditions are controlled by specifying the position and velocity of the first and the last cell interface. Piston-like conditions coupling the fluid system to external sources and sinks of energy are easy to set up.

Appendices C, D, and E reprint actual output from the three test problems for code verification and illustration purposes. ADINC has been optimized and vectorized for the Texas Instruments' ASC system at NRL.
II. THE BASIC EQUATIONS

ADINC solves the following equations for mass and momentum transport in one dimension:

\[
\frac{dp}{dt} = -\rho \nabla \cdot \mathbf{V} \tag{1}
\]

and

\[
\rho \frac{d\mathbf{V}}{dt} = -\nabla P. \tag{2}
\]

The energy evolution equation is eliminated by using an adiabatic equation of state in which the entropy is assumed constant throughout the numerical integration. Non-adiabatic processes such as external heating, thermal conduction, and chemical energy release can be added to Eqs. (1-2) using time step splitting provided sufficiently short timesteps are used to make the splitting procedure accurate. In the reference version of ADINC, reproduced as Appendix A of this report, the equation of state of the fluid in each cell of the calculation is

\[
\rho(P, S, \ldots) = \rho_c + (P/S)^{1/\gamma}. \tag{3}
\]

This equation of state with \( \rho_c = 0 \) is correct for adiabatic compression and expansion of an ideal gas. In that case \( 1.2 \leq \gamma \leq 1.7 \). When \( \rho_c \neq 0 \), Eq. (3) gives an adequate representation of a mildly compressible liquid. Water, for example, has \( \rho_c = 1 \text{ gm/cc} \) and \( S \approx 2.5 \times 10^{11} \) in CGS units. Thus in this crude model a pressure of 250 Kbar (2.5 \times 10^{11} \text{ dynes/cm}^2) causes a substantial increase in the compression.

During an ADINC timestep \( \rho_c, \gamma \), and \( S \) are treated as constants; only the variation of \( \rho \) with \( P \) is considered. ADINC does not use the temperature \( T \) anywhere and uses Eq. (3) in the form specified. Rather than knowing \( \rho \) and asking what the pressure is, ADINC calculates the fluid density given an approximation to the pressure. This equation of state density is compared to the density derived from the fluid dynamics via Eq. (1). This difference is iterated to zero using a quadratically convergent implicit solution of Eq. (2) which delivers an improved pressure approximation. During this iteration the analytic derivative \( \frac{dA}{dP} \) is used where \( A \) is
the volume of a computational cell.

\[
\frac{1}{\Lambda} \frac{d\Lambda}{dP} = -\frac{1}{\gamma \rho P} (P/S)^{1/\gamma}
\]  

(3)

for the particular equation of state (3).

The ADINC package is written in a sufficiently modular form that replacement of Eq. (3) with another equation of state should be quite straightforward. To do this the common block /ADICOM/ would have to be modified to include other constants of the fluid motion for the various materials being represented. Thermochemistry and thermophysical properties of realistic gases could be included, for example, so the effective gas constant \(\gamma\) can be made to display the correct variation with \(T\). As another example, more involved equations of state for solid and liquid materials can be included. We plan to use Gardner's model\(^9\) in studies where the transition from solid to plasma must be treated accurately.

ADINC uses the equation of state in the form \(\rho(P, S, \ldots)\) because the density is a much less sensitive function of the pressure than the pressure is of the density for liquids and solids. During the iteration process, finite errors in pressure and density are expected. In the other form \(P(\rho, S, \ldots)\), the errors in density \(\rho\) would appear as wild fluctuations in the pressure. For gases and plasma the two forms are basically of the same accuracy. There is a second related reason why ADINC uses the equation of state in the case \(\rho(P, S, \ldots)\), a specific form of which appears in Eq. (3). The ADINC package is specially designed to deal with discontinuities in zone size and density. When a gas-solid interface is encountered, the pressure is continuous but the density need not be. Therefore finite differences in the pressure are bound to be more accurate than transformed differences in the density.

Equations (1-3) are solved in the form shown without non-dimensionalization or scaling. The package is designed with CGS units in mind but these appear nowhere explicitly in the calculation. Therefore non-dimensional calculations are possible without modifying the program but \textit{caveat emptor}. 

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To complete the basic equations of the system, the geometry of the calculation must be given. ADINC, as presented, allows four one-dimensional systems to be treated by selection of the integer \( \alpha \). Figure 1 shows a general one-dimensional region of variable cross section area \( A(r) \). The volume of the region is \( V(r) = \int_{r_1}^{r} A(r') \, dr' \). The numerical algorithm, discussed in the next section, uses interface areas and cell volumes exclusively to convey geometry information. Therefore substitution of other 1D geometries than those provided for should be straightforward.

\( \alpha = 1 \), Cartesian coordinates

\[
\begin{align*}
A(r) &= 1.0 \\
V(r) &= r
\end{align*}
\]  

(4.1)

\( \alpha = 2 \), Cylindrical coordinates

\[
\begin{align*}
A(r) &= 2\pi r \\
V(r) &= \pi r^2
\end{align*}
\]  

(4.2)

\( \alpha = 3 \), Spherical coordinates

\[
\begin{align*}
A(r) &= 4\pi r^2 \\
V(r) &= \frac{4}{3} \pi r^3
\end{align*}
\]  

(4.3)

\( \alpha = 4 \), Power series (nozzle) coordinates

\[
\begin{align*}
A(r) &= G_1 + G_2 r + G_3 r^2 + G_4 r^3 + G_5 r^4 + G_6 r^5 \\
V(r) &= G_1 r + G_2 \frac{r^2}{2} + G_3 \frac{r^3}{3} + G_4 \frac{r^4}{4} + G_5 \frac{r^5}{5}
\end{align*}
\]  

(4.4)

In principle the volume is ambiguous up to an additive constant, the volume \( V(r_1) \). In practice ADINC deals only with volume differences to determine the incremental volume \( \Lambda \) of a computational cell so this constant cancels out conveniently. Using the symbol \( \Lambda \) also means that confusion with the velocity \( V \) cannot occur.
III. THE NUMERICAL ALGORITHM

Figure 1 shows a schematic diagram of the computational region treated by the ADINC package. There are \( N \) cells of volume \( \Lambda_i \), \((i = 2, 3, \ldots, N + 1)\) bounded by \( N + 1 \) interfaces of area \( A_i \), \((i = 1, \ldots, N+1)\). The interfaces are located at \( r_i \), \((i = 1, \ldots, N+1)\) so \( A_i \equiv A(r_i) \) where \( A(r) \) is given by one of the choices \( \alpha = 1, 2, 3, 4 \) in Eq. (4). The cell volumes \( \Lambda_i \equiv V(r_i) - V(r_{i-1}) \) are the difference of the volume integral from Eq. (4) at the two cell interfaces. In the development to follow we will also need the cell center locations

\[
R_i \equiv \frac{A_i r_{i-1} + A_{i-1} r_i}{A_i + A_{i-1}}
\]

which always lies between \( r_{i-1} \) and \( r_i \), provided the interface areas are positive.

In ADINC the first physical cell is \( i = 2 \) and it lies between interfaces \( r_1 \) and \( r_2 \). The last physical cell is \( i = N + 1 \) and lies between interfaces \( r_N \) and \( r_{N+1} \). Cells 1 and \( N + 2 \) are not used currently by ADINC but have been left available for future use in complicated boundary conditions or extrapolations. Interface 1 is the left hand boundary of the system \( (r_1 \equiv r_L) \) and interface \( N + 1 \) is the right hand boundary of the system \( (r_{N+1} \equiv r_R) \). The interfaces are treated in a fully Lagrangian manner and therefore the interface velocities \( \{V_i\} \) are defined as well as the interface positions \( \{r_i\} \). The left hand boundary velocity \( (V_L \equiv V_1) \) is established externally as is the right hand boundary velocity \( (V_R \equiv V_{N+1}) \). The interior interface locations and velocities are the quantities which ADINC integrates from one discrete time \( t \) to the next \( t + \delta t \) given the masses, entropies, and other cell quantities which are conserved during the motion.

The cell interface positions \( \{r_i\} \) satisfy

\[
\frac{dr_i}{dt} \equiv V_i
\]

which has a straightforward discretization

\[
r_i^n = r_i^0 + \delta t [\epsilon_i V_i^0 + (1 - \epsilon_i) V_i^0].
\]
ADINC GRID STRUCTURE AND VARIABLE PLACEMENT

Figure 1 — Grid structure and variable definition for the ADINC package. Position \( r_i \) and Velocity \( V_i \) are defined at cell interfaces while density \( \rho_i \) and pressure \( P_i \) are defined at all centers. Interface area \( A_i \) and cell volume \( \Lambda_i \) are derived from the instantaneous interface positions \([r_i]\).
In Eq. (5') and throughout the remainder of the paper the superscript "n" indicates variables at
the "new" time \( t + \delta t \) while superscript "o" indicates variables at the "old" time \( t \). The quantity
\( \epsilon_r \) is the explicitness parameter for the interface position, \( 0 \leq \epsilon_r \leq 1 \). When \( \epsilon_r \leq 1 \), the
method is at least partially implicit. When \( \epsilon = 1/2 \) the method is centered and nominally most
accurate. If long timesteps are contemplated, \( \epsilon \leq 1/2 \) is required for Courant stability with
strict inequality usually required to deal with nonlinear effects. When \( \epsilon_r = 0 \), the calculation is
fully implicit, i.e. fully forward differenced. This case is most stable but is only first order accu-
rate. ADINC uses the same value of \( \epsilon_r \) at every interface but this one value is varied from
cycle to cycle. Generalization to a spatially varying \( \{ \epsilon_r \} \) is quite possible but left for the future.

The momentum equation (2) for an interface velocity is as follows:

\[
\frac{dV_i}{dt} = \frac{-1}{\rho_{\text{interface}}} \frac{\partial P}{\partial r}_{\text{interface}}
\]

where the density and pressure gradient are needed at cell interfaces. The discretization used in
ADINC is

\[
V_i^n = V_i^o - \frac{\delta t \epsilon_r}{<\rho \delta r>_{i+1/2}} (P_{i+1}^o - P_i^o)
- \frac{\delta t (1 - \epsilon_r)}{<\rho \delta r>_{i+1/2}} (P_{i+1}^o - P_i^o)
\]

(6')

where \( \epsilon_r \) is the explicitness parameter for the interface velocity and has the same properties
described above for \( \epsilon_r \). The quantities \( \epsilon_r \) and \( \epsilon_r \) are distinct in ADINC but no reason has been
uncovered to date for using different values in an actual calculation. The interface average
indicated as \( <\rho \delta r>_{i+1/2} \) is both a spatial and temporal average as described below. Physical
considerations are used to define \( <\rho \delta r>_{i+1/2} \) so the discretization in Eq. (6') is insensitive to
numerical errors arising from large density discontinuities at the interfaces.

Figure 2 shows two cells \( i \) and \( i + 1 \) which straddle interface \( i \). The pressures \( P_i \) and \( P_{i+1} \)
are defined at \( R_i \) and \( R_{i+1} \) as shown and the densities \( \rho_i \) and \( \rho_{i+1} \) are assumed constant
throughout their respective cells. Because \( \rho_i \) and \( \rho_{i+1} \) differ spatially (ignore their time
ACCELERATION MATCHING ALGORITHM

REQUIRE:

\[-1 \frac{P_{i+1} - P_i^*}{\dot{q}_{i+1} + \frac{R_i}{r_i}} = a_i^+ = a_i^-\]

\[-1 \frac{P_i^* - P_i}{\dot{q}_i + \frac{r_i - R_i}{R_i}}\]

Figure 2 — Acceleration matching trick for density discontinuities at interfaces. An intermediate interface pressure $P_i^*$ is defined such that the acceleration of material to the right and to the left of the interface is matched. Most of the pressure gradient thus appears across the denser fluid.
variation for a moment), the straight line pressure gradient shown would impart a different acceleration to the fluid just to the right and to the left of interface $i$. If the fluid were permitted to move according to these distinct accelerations, the fluids would either overlap or a gap would open up at interface $i$ after a short while. To prevent this a fictional pressure $P_i'$ is defined at interface $i$ such that the acceleration calculated from the left equals the acceleration calculated from the right.

$$P_i' = \frac{P_{i+1} f_{i+1} + P f_i^+}{f_{i+1} + f_i^+}$$  \hspace{1cm} (7)

where

$$f_i^+ \equiv \frac{1}{\rho_i (r_i - R_i)} , \quad f_{i+1} \equiv \frac{1}{\rho_{i+1} (R_{i+1} - r_i)} .$$ \hspace{1cm} (8)

In terms of the indicated average $<\rho \delta r>_{i+1/2}$ in Eq. (6) we can eliminate $P_i'$ completely from further consideration and use

$$<\rho \delta r>_{i+1/2} \equiv \rho_{i+1} (R_{i+1} - r_i) + \rho_i (r_i - R_i)$$ \hspace{1cm} (9)

to define the spatial part of the free average.

The question of how to evaluate the average (9) in time arises and has not been fully settled. The major points to consider are momentum conservation, nonlinear instability of the overall algorithm, and time-centering accuracy. Equation (6'), when multiplied by $<\rho \delta r>_{i+1/2}$ and summed, yields

$$\sum_i V_i' <\rho \delta r>_{i+1/2} = \sum_i V_i' <\rho \delta r>_{i+1/2} + \text{boundary terms}$$ \hspace{1cm} (10)

where the telescoping pressure terms cancel except at the boundary of the computational region. One would like to use the "old" time values on the right and the "new" time values on the left to give a true momentum "integral" — at least in Cartesian coordinates. Since the quantity $<\rho \delta r>_{i+1/2}$ is conserved in Cartesian Lagrangian coordinates, however, it doesn’t really matter at what time we evaluate $<\rho \delta r>_{i+1/2}$, it is just the mass associated with interface $i$. In non-Cartesian coordinate systems the momentum integral has little meaning and the
quantity \( <p\delta r>_{i+1/2} \) is not really a constant of the motion... \( \rho, \Lambda \), is. ADINC uses an exactly time centered average for the geometric parts of \( <p\delta r>_{i+1/2} \). The density which appears in the expression is our best approximation (latest iteration) to the new density for reasons of numerical stability.

Return to Eq. (9). ADINC actually uses

\[
<p\delta r>_{i+1/2} \equiv \left[ \rho_i^n(R_{i+1}^n - r_i^n) + \rho_i^n(r_i^n - R_i^n) \right]
\]

where superscript "P" stands for "previous" and indicates the latest iterated approximation to the "new" value of the variable, in this case \( \{\rho_i^n\} \). The superscript "H" is used to indicate the exact "half" time average. In Eq. (11)

\[
r_i^h \equiv \frac{1}{2}(r_i^n + r_i^p), \quad R_i^h \equiv \frac{1}{2}(R_i^n + R_i^p).
\]

We make no representation that these are the best averages or that extensive testing of this aspect of ADINC has been performed. Perhaps the freedom remaining in this region of the calculation can be used to further improve the accuracy and veracity of the algorithm. We do note that no problems arising from this particular choice have been observed to date in numerous test calculations using ADINC.

Returning to the complete algorithm, we wish to find a tridiagonal equation for \( \{P_i^n\} \). The momentum equation (6') can be simplified as follows:

\[
V_i^n = a_i - b_i(P_{i+1}^n - P_i^n) \text{ for } i = 2, \ldots, N
\]

where

\[
a_i \equiv V_i^n - \frac{\delta t \epsilon_i}{<p\delta r>_{i+1/2}} (P_{i+1}^n - P_i^n),
\]

\[
b_i \equiv \frac{\delta t(1 - \epsilon_i)}{<p\delta r>_{i+1/2}}.
\]

The equation of state is introduced by requiring that the cell volume, \( \Lambda_i^n(eo) \), computed from the equation of state using the new time values of pressure, equal the new cell volume computed from the fluid dynamics, \( \Lambda_i^n(\delta t) \). At any iteration \( P \) the difference is
\[ \delta \Lambda_i^P \equiv \Lambda_i^{P%(con)}(P_i^P, S_i, \ldots) - \Lambda_i^{P%(id)}([r_i^D]) \]  
(15)

and should be iterated to zero. Changing \( P_i^P \) to \( P_i^F \) varies both terms in Eq. (15). In the fluid
dynamics contribution \( r_i^D \) converges to \( r_i^F \) as a function of the pressure through Eq. (13) and
Eq. (5'). We use effectively a Newton-Raphson approach to obtain a quadratically convergent
iteration to the desired solution at time \( t + \delta t \)

\[ \Lambda_i^{%(con)}(P_i^F, S_i, \ldots) = \Lambda_i^{%(id)}([r_i^F]). \]  
(16)

The difference between \( \Lambda_i^{%(con)} \), which is known at each iteration, and the desired
\( \Lambda_i^{%(id)} = \Lambda_i^{%(con)} - \Lambda_i^{n} \) can be written in terms of the cell interface areas and the desired new
fluid velocities at the cell interfaces.

\[ \Lambda_i^{n} - \Lambda_i^{%(id)} \approx (1 - \epsilon_i) \delta t [A_i^{h}(V_i^n - V_i^P) - A_{i-1}^{h}(V_{i-1}^n - V_{i-1}^P)]. \]  
(17a)

The same treatment of the equation of state gives

\[ \Lambda_i^{n} - \Lambda_i^{%(con)} \approx (P_i^n - P_i^P) \left( \frac{\partial \Lambda}{\partial P} \right)^{%(con)}. \]  
(17b)

Let

\[ d_i^+ \equiv -(1 - \epsilon_i) \delta t A_i^{h} \] and \( d_i^- \equiv -(1 - \epsilon_i) \delta t A_{i-1}^{h}. \)  
(18)

Then Eq. (17a) becomes

\[ \Lambda_i^{n} - \Lambda_i^{%(id)} \approx -d_i^+(V_i^n - V_i^P) + d_i^-(V_{i-1}^n - V_{i-1}^P). \]  
(17a')

Equating \( \Lambda_i^{n} \) in Eqs. (17a) and (17b) gives

\[ \delta \Lambda_i^P + (P_i^n - P_i^P) \left( \frac{\partial \Lambda}{\partial P} \right)^{%(con)} \approx -d_i^+(V_i^n - V_i^P) + d_i^-(V_{i-1}^n - V_{i-1}^P). \]  
(19)

Define

\[ c_i \equiv P_i^n \left( \frac{\partial \Lambda}{\partial P} \right)^{%(con)} + d_i^+ V_i^n - d_i^- V_{i-1}^P - \delta \Lambda_i^P. \]  
(20)

Then Eq. (19) becomes

\[ P_i^n \left( \frac{\partial \Lambda}{\partial P} \right)^{%(con)} + d_i^+ V_i^n - d_i^- V_{i-1}^P \approx c_i. \]  
(19')
This is our tridiagonal, implicit, linear equation for the estimated new pressures \( P^n \) when Eq. (13) is used to eliminate \( V^n \) in terms of \( P^n \). Expanding this out for completeness gives

\[
P^n \left[ \frac{\partial \Lambda}{\partial P} \right]^{P_{\text{eos}}} + d^+ \left[ a_i - b_i (P^n_{i+1} - P^n_i) \right] - d^- \left[ a_{i-1} - b_{i-1} (P^n_i - P^n_{i-1}) \right] \approx c_i. \quad (21)
\]

Equation (21) is the basic equation solved by ADINC. Iteration is necessary because Eqs. (17a') and (17b) are only equalities to first order. The iteration is quadratically convergent, however, because only second order terms are neglected at each stage of the iteration. For the interior cells \( i = 3, 4, \ldots, N \) Eq. (21) may be written as

\[
A_i^+ P^n_{i-1} + B_i^+ P^n_i + C_i^+ P^n_{i+1} = D_i^+. \quad (22)
\]

where

\[
\begin{align*}
A_i^+ & \equiv -d^- b_{i-1}, \\
B_i^+ & \equiv \left[ \frac{\partial \Lambda}{\partial P} \right]^{P_{\text{eos}}} - C_i^+ - A_i^+, \\
C_i^+ & \equiv -d^+ b_i, \quad \text{and} \\
D_i^+ & \equiv c_i - d^+ a_i - d^- a_{i-1}.
\end{align*}
\]

At the right and left boundaries of the system the new velocities are given externally for Eq. (19), (19'). In terms of the tridiagonal coefficients of Eq. (23),

\[
\begin{align*}
D^n_r & \equiv D^n_r \text{(above)} + d^n_r V^n_i \quad \text{and} \\
D^n_{i+1} & \equiv D^n_{i+1} \text{(above)} - d^n_{i+1} V^n_i, \quad \text{and} \\
A^n_i & \equiv 0, \quad C^n_{i+1} \equiv 0.
\end{align*}
\]

When integrating the fluid dynamic equations, ADINC assumes that each interface moves in a fully Lagrangian manner according to Eq. (5). The change in density from one timestep to the next in a cell is therefore given simply by the change in cell volume according to the mass conservation equation

\[
\rho_i^n \Lambda_i^n \equiv \Delta M_i \equiv \rho_i^n \Lambda_i^n. \quad (25)
\]

When individual species number densities must be followed, they are also advanced by Eq.
The accuracy and stability of ADINC presupposes the monotonicity of the interface positions \( \{ r_i \} \) with increasing \( i \) and, of course, the positivity of the interface areas \( \{ A_i \} \). Since the algorithm is based on a discretization of the continuum fluid dynamic equations, the possibility exists for numerical error and even instability caused by non-physical crossing of cell interfaces even though the implicit algorithm given above is nominally stable for sound waves at arbitrary timestep. To prevent interface crossings, a Courant condition must still be satisfied for the flow velocities \( \{ V_i \} \) even though \( |V| \ll C_i \equiv \sqrt{P/\rho} \) throughout the fluid (gas). In the reference version of ADINC, reproduced in Appendix A, the maximum timestep which still prevents interfaces crossing adjacent interface positions is calculated from the formula

\[
\delta t_{\text{max}} = \frac{1}{2} \min_{i-1,N} \left( \frac{(r_i - r_{i-1})}{|V_i|}, \frac{(r_{i+1} - r_i)}{|V_i|} \right)
\]

where a very small number is added to \( |V| \) to prevent dividing by zero.

Equation (26) is conservative and even includes the factor of \( \frac{1}{2} \) in case two interfaces are moving toward each other. Generally longer timesteps are quite acceptable. By rights only one of the two terms should be included since an interface can be moving either to the right or the left but not both at the same time. Furthermore, interfaces only cross due to a differential velocity, not an absolute one. The denominators in Eq. (26) should be \( |V_{i+1} - V_i| \) and \( |V_i - V_{i-1}| \) when a net motion is superimposed on relative expansions and contractions.

Situations exist where the timestep calculated from Eq. (26) can lead to trouble. From Eq. (5') one can see that the cell interface positions are advanced using an average of the new and old velocities. Since the timestep has to be estimated at the beginning of a cycle, it is quite possible for \( \{ V^n \} \) to be small or zero while \( \{ V^m \} \) can be large. If \( V^n \) becomes large enough, \( r^n \) can cross adjacent interfaces even though the original timestep estimate should prevent this. The test program reproduced in Appendix B also contains a user-supplied maximum timestep.
\( \delta t_{\text{max}} \) to provide a limit externally because no fully satisfactory algorithm has yet been developed for ADINC to estimate the limit internally. An estimate of this effect would require dealing with accelerations as well as velocities. This venue may be impractical because the acceleration calculation must be implicit and would substantially increase the computational cost of the subroutine. In the current algorithm non-convergence of the pressure iteration by a certain amount (discussed below) is signalled through a diagnostic message printed by ADINC but the calculation continues. In principle non-convergence after a certain number of iterations could cause re-initiation of the computational cycle with a smaller timestep. This is not included in the current version for three reasons:

1. Extra data would have to be stored to re-initiate the calculation at additional space and time expense.

2. The ADINC algorithm already subcycles the fluid dynamic calculation when the user specifies a longer integration interval than Eq. (26) permits. Thus the logic is already quite complicated.

3. Crossing of interfaces is usually accompanied by rather drastic numerical errors, not nonconvergence, therefore recovery is problematical at best. The problem usually blows up in the equation of state calculation.

For these reasons I've chosen to put the onus on the user to avoid "sneaky" interface crossings. In practice the timestep given by Eq. (26) is, in fact, conservative not overly ambitious. Few problems have arisen to date.

ADINC is designed to be used as a specific partial differential equation integrator in much the same spirit as many ordinary differential equation packages are used. It attempts to adjust the timestep and number of iterations at each step internally to maximize speed with good accuracy. Using the timestep estimation algorithm given above, ADINC subcycles the
hydrodynamics as often as necessary to integrate over the full time interval which the user has specified. Since the opportunity exists for an inordinate number of subcycles if something unexpected happens during the calculation, the number of subcycles has arbitrarily been limited to 100. If ADINC performs any subcycles at all, it prints a message to that effect. If the package detects that more than 100 subcycles seem to be required, it terminates the calculation printing a suitable diagnostic message. A user may wish to change these limits.

Within each timestep (or subcycle) the package performs a convergence calculation on the iterated new pressure solution. The convergence condition used for each cell of the calculation is

\[
\frac{|P_i^n - P_i^m|}{P_{\text{max}}} < 10^{-9} \sqrt{\frac{M_{\text{max}}}{M_{\text{min}}}} \tag{27}
\]

where \(P_{\text{max}}\) is the largest pressure in the system, \(M_{\text{max}}\) is the largest cell mass in the system, and \(M_{\text{min}}\) is the smallest cell mass in the system. Equation (27) is largely heuristic but has been found to work extremely well. Using the timestep condition given above, convergence is often obtained in two iterations in \([P_i^n]\) and almost always in three or four. The maximum number of iterations is limited to six because this should be sufficient to obtain full double precision accuracy using the quadratically convergent algorithm described.
IV. STRUCTURE OF THE ADINC PACKAGE

The ADINC package consists of three subroutines containing three entries each. These FORTRAN subroutines are vectorized for the Texas Instruments ASC system at the Naval Research Laboratory and are reproduced in their entirety in Appendix A. A rather general test program and several simple utility subroutines are reproduced as Appendix B. Here we discuss the three major routines of the package and the interactions between them. The next section considers more practical aspects of how to use ADINC.

The three routines deal with the geometry of the problem, the equations of state of the fluid, and the fluid dynamics of the problem. The geometry of a problem is established and controlled by the subroutine SETGEO and its two entries USEGEO and DTFLOW. The calling sequences and arguments for these entries are as follows:

CALL SETGEO (ALPHA, GEOMCO)
arguments:
   ALPHA integer 1 = Cartesian coordinates
                2 = Cylindrical coordinates
                3 = Spherical coordinates
                4 = Power Series coordinates
   GEOMCO real*8 array contains the five coefficients $G_1 - G_5$ of Eq. (4.4) on entry
and is only used when ALPHA = 4

CALL USEGEO (RAD, AREA, RADC, LAMC, N)
arguments:
   RAD real*8 array dimension at least N+1 — contains N+1 interface positions on entry
   AREA real*8 array dimension at least N+1 — contains N+1 interface areas on exit (Eq. (41))
   RADC real*8 array dimension at least N+2 — contains N cell center positions on exit
   in locations 2, ..., N+1 (Eq. (51))
   LAMC real*8 array dimension at least N+2 — contains N cell volumes on exit
   in locations 2, ..., N+1 (Eq. 41)
   N integer contains the number of interior cells on entry

CALL DTFLOW (RAD, VEL, DTVAL, N)
arguments:
   RAD real*8 array dimension at least N+2 — contains N+1 interface positions on entry
   VEL real*8 array dimension at least N+2 — contains N+1 interface velocities on entry
   DTVAL real*8 contains estimated max timestep to prevent numerical error (on exit)
   N integer contains the number of interior cells in entry

SETGEO is used to establish the geometry of the problem at the beginning of a calculation. Thereafter USEGEO is called to fill the geometric arrays AREA, RADC and LAMC every time a new set of interface positions is obtained during the Lagrangian calculation. The routine remembers the value of ALPHA (and the coefficients GEOMCO where appropriate) until SET-
GEO is called to set up a new problem.

DTFLOW is used to estimate a maximum timestep based on Eq. (26) for use in the fluid dynamic calculation performed by ADINC. The reference version presented in Appendix A calculates a timestep which prevents a new interface position \( r_i' + V_i'\Delta t \) from crossing one of the original adjacent interface locations, \( r_{i+1} \) or \( r_{i-1} \).

The equation of state of the fluids in each cell of the computation is calculated by the subroutine SETMAT and its two entries SETEOS and USEEOS. The calling sequences and arguments for these entries are as follows:

**CALL SETMAT (MATER,MASS,GAMMA,RHOCON,N)**

- **MATER** real *8 array, dimension at least \( N+2 \) — contains \( N \) cell material identifiers (\( i=2, \ldots, N+1 \))
- **MASS** real *8 array, dimension at least \( N+2 \) — contains cell masses on entry
- **GAMMA** real *8 array, dimension at least \( N+2 \) — contains \( N \) cell \( \gamma \) values from Eq. (3)
- **RHOCON** real *8 array, dimension at least \( N+2 \) — contains \( N \) cell \( \rho \) values from Eq. (3)
- **N** integer, contains the number of interior cells on entry

**CALL SETEOS (RHO,PRE,N)**

- **RHO** real *8 array, dimension at least \( N+2 \) — contains \( N \) cell densities on entry
- **PRE** real *8 array, dimension at least \( N+2 \) — contains \( N \) pressures on entry
- **N** integer, contains the number of cells on entry

**CALL USEEOS (RHO,PRE,LAMEOS,DLAMDP,N)**

- **RHO** real *8 array, dimension at least \( N+2 \) — contains \( N \) cell densities \( \rho(P, S, \ldots) \) on exit
- **PRE** real *8 array, dimension at least \( N+2 \) — contains \( N \) pressures on entry
- **LAMEOS** real *8 array, dimension least \( N+2 \) — contains \( N \) cell volumes on exit
- **DLAMDP** real *8 array, dimension at least \( N+2 \) — contains \( dN/dP \) for each cell on exit
- **N** integer, contains the number of interior cells on exit

SETMAT is used to place the equation of state cell constants into the common block /ADI-COM/ which communicates equation of state information among the three routines (nine entries) of the ADINC package. These constants are those conserved quantities and local properties of the fluid cells used to connect the grid configuration at one time to that at another adiabatically. Entry SETEOS is called to determine the entropy for given values of cell density, cell pressure and interface positions. Since thermal conduction, external heating, chemical energy release, etc. all change the entropy but not the other constants in the equation of state, a separate entry from SETMAT is provided to reduce the cost of resetting the equation of state to account for these non-ideal phenomena.
USEEOS is the entry provided to determine the cell density expected, given the cell pressures and all the equation of state constants in ADICOM. Also returned to the user is the cell volume expected and the rate of change of change of cell volume with pressure. These quantities depend on the equation of state only and are used in the ADINC iteration which reconciles the cell volumes computed from the fluid dynamic equations with those needed to satisfy the equation of state. While SETMAT and SETEOS need only be called only once by the user in many types of calculations, USEEOS is called at least two times per iteration by ADINC and therefore entails the lion’s share of the computational expense.

The common block /ADICOM/ is declared as follows:

```
PARAMETER NPT = 202
INTEGER NCELLS
REAL *8 MATERC(NPT), MASSC(NPT), GAMMAC(NPT)
REAL *8 ENTC(NPT), RHOC(NPT)
COMMON /ADICOM/ MATERC, MASSC, GAMMAC, ENTC, RHOC, NCELLS
```

and appears in the three ADINC routines in exactly the same form. If a user wishes, more complicated formulae may be used for an equation of state. The additional arrays needed would then be added to each realization of /ADICOM/ and the appropriate computations to SETMAT, SETEOS, and USEEOS. The version provided allows a relatively wide selection of problems to be tackled with a relatively simple formulation that is inexpensive to compute.

The quantity MATERC(i) is the material and shell identification of cell i. It is a floating point number of the form L.MM where 0 < L < 10 is the layer identifier and 0 < MM < 100 is the local material identifier. The layer number is used by the diagnostics to perform various partial sums. The layer can contain different materials from cell to cell but the layer number must increase monotonically from cell 2 to cell N+1. The material identifier MM is not used in the current version but is available for flagging which of several computationally distinct equations of state are to be used in the future. MASSC(i) is the mass (nominally in grams) of cell i. GAMMAC(i), RHOC(i), and ENTC(i) are the equation of state conserved constants for cell i which appear in Eq. (3).

Double precision is used throughout the floating point computations so that problems with
large disparity in cell mass or with nearly incompressible fluids can be treated accurately. The basic convergence criterion of $10^{-9}$ for the quadratically convergent iteration in ADINC is far more accurate than single precision on the ASC permits. We are using a 64 word length here with an 8 bit (hexidecimal) exponent. The versions of the equation of state and geometry routines are also fully vectorized for efficient computation on the ASC.

The basic fluid dynamic calculation is performed by ADINC and its two entries ADINCO and SETEPS.

CALL ADINC (RAD,VEL,RHO,PRE,N,DTIN,CYCLE,RRNEW,RLNEW,VRNEW,VLNEW)
Arguments:
- RAD real *8 array dimension at least N+1 - contains N+1 interface positions
- VEL real *8 array dimension at least N+1 - contains N+1 interface velocities
- RHO real *8 array dimension at least N+2 - contains N cell densities
- PRE real *8 array dimension at least N+2 - contains N cell pressures

These four arrays of physical variables contain the old timestep values on entry and will be filled with the new timestep values DTIN later on exit. Users should store old values in the external program if desired. The test program shows how to do this.

- N integer contains the number of interior cells on entry
- DTIN real *8 the integration timestep on entry
- CYCLE integer the timestep number for identification
- RRNEW real *8 on entry the right boundary position
- RLNEW real *8 on entry the new left boundary position
- VRNEW real *8 on entry the new right boundary velocity
- VLNEW real *8 on entry the new left boundary velocity

The boundary interface values are specified at the end of the desired timestep since they are assumed to be driven externally. Usually the positions are fixed and the velocities zero. RAD(N+1), RAD(1), VEL(N+1), (VEL) should contain the corresponding old time values.

CALL ADINCO (MODE, CYCLE)
Arguments:
- MODE integer = 0 if reinitializing NCALL
- CYCLE integer the timestep number for identification

CALL SETEPS (ERO, EVO, MDAMP, EROUT, EVOUT)
Arguments:
- ERO real *8 basic position explicitness parameter $\epsilon$, on entry
- EVO real *8 basic velocity explicitness parameter $\epsilon$, on entry
- MDAMP integer number of cycles + 1 over which initial filtering by ADINC is desired
- EROUT real *8 on exit the most recent value of $\epsilon$, used by ADINC
- EVOUT real *8 on exit the most recent value of $\epsilon$, used by ADINC

The next section discusses the use of SETEPS to vary the explicitness parameters. The arguments EROUT and EVOUT allow the user to query ADINC about the current values of $\epsilon$, $\epsilon$, being used as well as to change them. The need to monitor the initial filtering feature (discussed in the next section) and the need to print out the time averaged velocities and
pressures actually used to advance the positions (Eq. (5')) and the velocities (Eq. (6')) makes the inclusion of EROUT and EVOUT as arguments to SETEPS a necessity.

Entry ADINCO is provided so the user can monitor the performance of ADINC. Each time ADINCO is called, the number of calls to ADINC, the number of timesteps performed by ADINC, and the number of iterations used in these timesteps is printed out. Each call to ADINCO with MODE = 0 reinitializes these counters so we only get the accumulated calls, timesteps, and iterations since the previous call to ADINCO. MODE = 1 controls a special initial filtering facility described in the next section and MODE > 1 is currently not permitted.

The major routine ADINC advances the physical variables \( \{r, V, \}, \{p, J, \}, \) and \( \{P, \} \) from one timestep to the next. ADINC uses the geometry and equation of state routines and controls all the logic of the iteration and timestep subcycling. The whole package has been written, as nearly as possible, to conform with styles and usage of coupled ordinary differential equation packages. Thus the user can ask the package to integrate over an interval requiring many timesteps. The user can also change the convergence criteria and the order of the algorithm via the \( \epsilon, \) and \( \epsilon, \) coefficients. Unlike ordinary differential equations, however, partial differential equations require boundary as well as initial conditions. Furthermore, the generally large number of cells and concommitant large number of coupled equations force corresponding restraints as the algorithm. Because the number of simultaneous equations is large, the algorithm must be simple and optimized so the computation time is acceptable. ADINC is second-order accurate at best but is fully vectorized for parallel computation on machines such as the NRL Texas Instruments ASC.

The algorithm described in Section III is a single-step algorithm so numerous copies of the physical variables at different time levels do not need to be maintained. Multiple level predictor-corrector schemes are the norm for ODE packages but require an inordinate amount of computer storage when many equations are coupled as in fluid dynamic problems. The next section considers the use of ADINC for the rather general case provided by the test program in
Appendix B and supported by the utilities there. These utilities perform auxiliary functions useful in the ADINC computation but not crucial to it such as evaluating internal and kinetic energies and initializing a rather general multilayer test problem. These routines and their use are described via comments in the listings in Appendix B. They would not necessarily be used when ADINC is being applied as a package within the context of a more complex computation.
V. USING ADINC

Appendix B contains the listing of a rather general and useful test program for the ADINC package as well as the various utility programs not called by the user directly. Use of most of the ADINC facilities is illustrated there and a thorough understanding of this program and the examples provided in Appendices C, D, and E amounts to a thorough understanding of how to use the ADINC package. Here I present a general discussion designed to give the potential user an overall view of the calculation and such specific information as does not appear elsewhere in this write-up. The three test problems and variations discussed in the next section will illustrate better than any general discussion how physical problems are solved accurately and reliably using ADINC.

ADINC is applied to a user defined fluid problem via a series of structured calls to the 9 entries in the three major routines dealing with geometry, equation of state, and fluid dynamics. Figures 3 and 4 outline the correct sequence for this structured series of calls. Figure 3 contains the computations and routine references required to initialize the package and Figure 4 contains those calculations and calls to be performed during the timestep loop.

The first step in the initialization (Fig. (3)) is to establish the geometry within which the calculations is to proceed. The integer \( a \) is set and the values \([G_i] \) for \( i = 1, 2, 3, 4, 5 \) are defined if required. Then entry SETGEO is called to transmit these values to ADINC. Then the initial cell interface positions are set by the user so that a call to USEGEO can calculate the interface areas, cell center positions, and cell volumes for the initial grid geometry.

The third step is to set the desired explicitness parameters \( \epsilon_r \) and \( \epsilon_e \) into the ADINC package. This is done via the entry SETEPS where the number of timesteps in the initial damping transient is also communicated to ADINC.

Next the physical problem on the grid must be initialized and communicated to ADINC via the variables in common block /ADICOM/. The physical system has already been divided...
Figure 3. To Initialize the ADINC Package

- Determine geometry (i = 1, 2, 3, or 4 and G_i if needed)
  CALL SETGEO (ALPHA, GEOMCO)

- Determine the initial interface positions r_i and velocities V_i;
  CALL USEGEO (RAD, AREA, RADC, LAMC, N)
  to initialize the interface areas, cell centers, and volumes

- Set the desired explicitness parameters and damping period
  CALL SETEPS (ERO, EVO, MDAMP, EROUT, EVOUT)
  to pass these values on to ADINC

- Determine the desired fluid properties and constants which are conserved
  with the cell motion
  CALL SETMAT (MATER, MASS, GAMMA, RHOCON, N)
  to set these cell constants into common block /ADICOM/

- Determine the initial cell densities and pressures
  CALL SETEOS (RHO, PRE, N)
  to set the current value of the cell entropies ENTC into /ADICOM/. This has
  to be repeated in the timestep loop whenever irreversible and dissipative
  phenomena change the entropy.

Figure 3 — Schematic for initializing the ADINC package. The rather general case presented mirrors the test program
of Appendix B. The test program presents, however, a number of additional facilities for use in a scientific research
program which are not part of ADINC proper. These do not appear in the schematic.
Figure 4. Application of ADINC During a Timestep

- Establish and limit a monotonically increasing cycle number—
  DO 9999 ISTEP = 1, MAXSTP (in the test program)

- Set a timestep DELTAT which depends on geometry, flow and other
  considerations. This involves:
  CALL DTFLOW (RAD, DABSV, DTVAL, N)
  where DABSV is an array containing the maximum absolute value of the
  velocity in the previous two timesteps (DABSV = VEL) works fine in most
  circumstances. Other timestep limiting calculations should also be
  performed at this time.

- Perform diagnostics on the geometry and physical variables. This usually
  involves another
  CALL USEGEO (RAD, AREA, RADC, LAMC, N)
  to update AREA, RADC, and LAMC consistent with current cell interface
  locations in RAD
  any other diagnostics; including ERGPRT, are performed

- Advance the hydrodynamic variables by first:
  setting the new boundary positions RLNEW, RRNEW and velocities
  VLNEW, VRNEW before
  CALL ADINC (RAD, VEL, RHO, PRE, N, DELTAT, ISTEP, RRNEW, RLNEW, VRNEW, VLNEW)

- Reset the $\epsilon_r$, $\epsilon_v$ values by changing EPSRO, EPSVO in
  CALL SETEPS (EPSRO, EPSVO, MDAMP, EPSR, EPSV)
  9999 CONTINUE (go back for another timestep)

Figure 4 — Schematic for using the ADINC package to integrate the Lagrangian equations of motion. The
upper portion of the figure shows the use of ADINC in diagnostics and the lower portion, its use in the
integration proper. Additional physics can be added via timestep splitting. This is indicated in the test pro-
gram.
into a number of computational cells. For each of these Lagrangian cells a number of quantities must be established which are constant throughout the ADINC integration. These can be initialized directly into the /ADICOM/ variables MATERC, MASSC, GAMMAC, and RHOC by the user or auxiliary variables MATER, MASS, GAMMA, RHOCON can be setup and the entry SETMAT called to place these quantities into the common block in the desired locations. The test program fills /ADICOM/ directly but demonstrates the call to SETMAT anyway.

Remember that the $N+1 \equiv NCELLS+1$ interfaces and associated interface quantities reside in the first through the $N+1$-st locations of interface arrays but the $N$ cells and associated cell quantities must reside in locations 2 through $N+1$. This offset leaves "cell" 1 and "cell" $N+1$ available for fancy boundary conditions or other ghost cell applications. In the test problem, printouts of Appendices C, D and E these undefined quantities appear as the number $2. \times 10^{68}$ or $\ldots$ depending on the print out format. Leaving these undefined quantities in the arrays has no adverse effects in the calculations, as will be seen, and does have a potential advantage. Any off-by-one errors introduced in new code by the user will show up as arithmetic exceptions involving the use of these undefined cell quantities.

Once the initial density and pressure in each cell have been set, the cell entropy ENTC can be determined and placed in /ADICOM/. The entry SETEOS performs this calculation using the equation of state function. A separate entry is provided here for efficiency because I anticipate resetting the entropy whenever non-ideal, dissipative, or energy source terms are present. In more complicated systems such as detailed chemically reactive flows even the values of $\{\gamma_i\}$ (GAMMAC) change. The utility DUBLOG is provided and used in SETEOS and USEEOS because the current ASC system has an error in the double precision logarithm routine which prevents vectorization. Efficiency in the SETEOS and USEEOS calculations, particularly the latter, is extremely important because these transcendental calculations seem to dominate the ADINC execution time.

The repetitive computations within an ADINC simulation cycle are listed in Fig. 4. The
first part of a cycle is to find the correct timestep, $\delta t$ (DELTAT in the test program). Entry DTFLOW is provided in the ADINC package to calculate an estimated timestep which will prevent interface crossing but several other timestep limitations, which are normally problem dependent, have to be included. Even though the sound speed is nominally not part of the timestep calculation, it certainly affects the accuracy and the tendency toward non-linear instability. The test program illustrates a number of the considerations involved in timestep selection.

Once the timestep has been chosen I usually recommend that all I/O, dumps, and diagnostics be performed. Since actual integration of the first timestep has not occurred, performing the diagnostics here within the timestep loop allows the initial conditions to be printed when $\text{ISTEP} = 1$. Thus diagnostic tests and subsidiary calculations have to appear only one place in the code. As part to these diagnostics the geometry variables should be updated as well via USEGEO.

Before calling ADINC to integrate the fluid dynamic equations in the loop, the boundary conditions for the integration step must be established. The values of $\text{RRNEW}$, $\text{RLNEW}$, $\text{VRNEW}$, and $\text{VLNEW}$ convey this information to ADINC. On entry $[r]$, $[V]$, $[\rho]$, and $[P]$ contain the physical quantities at time $t$, the beginning of the integration step. ADINC is being asked to integrate an internal $\delta t$ up to time $t + \delta t$. At this time the interior values of $[r]$ and $[V]$ will be determined but the boundary velocities $V_1$ and $V_{X+1}$ and hence the boundary positions depend on external factors. Thus

$$\text{RRNEW} \equiv r_{n+1} (t + \delta t),$$
$$\text{RLNEW} \equiv r_{i} (t + \delta t),$$
$$\text{VRNEW} \equiv V_{n+1} (t + \delta t),$$
$$\text{VLNEW} \equiv V_{i} (t + \delta t).$$

Nominally $\text{VRNEW}$ and $\text{VLNEW}$ can be anything but for internal consistency $\text{RRNEW}$ should satisfy
\[ r_{N+1}(t + \delta t) = r_{N+1}(t) + \epsilon_1 \delta t V_{N+1}(t) + (1 - \epsilon_1) \delta t V_{N+1}(t + \delta t) \]

with a similar equation for \( r_1(t + \delta t) \). The test program uses a simplified version of this to avoid the complications of calling SETEPS again just to set the boundary condition. Instead the previous values \( \epsilon_1 \) and \( \epsilon_2 \) are used as these will apply unchanged throughout the calculation except possibly in the first few cycles.

If the values \( \{P_1\} \) and \( \{V_1\} \) are stored in auxiliary arrays \( \{P_a\} \) and \( \{V_a\} \) before ADINC is called, the average velocities and pressures used to advance the interface positions and velocities can be calculated as shown in the test program. Even though this information can be reconstructed without special calculation if two successive timesteps are printed out, these intermediate values are the basis of the Lagrangian dynamics so the prospective user of ADINC should be familiar with the contents and purpose of DO LOOP 110.

The user of ADINC also has control over the values of the explicitness parameters \( \epsilon_1 \) and \( \epsilon_2 \) for the interface position and velocity integrations. When \( \epsilon_1 \) or \( \epsilon_2 = 0 \), the corresponding equations is fully forward differenced and therefore implicit. When \( \epsilon_1 \) and \( \epsilon_2 \) are both equal to \( \frac{1}{2} \), the calculation is fully centered and hence will be second-order accurate in time. The use of \( \epsilon_1 > \frac{1}{2} \) or \( \epsilon_2 > \frac{1}{2} \) can lead to numerical instability and is therefore discouraged. The effect of using different values for \( \epsilon_1, \epsilon_2 \) is discussed in the next section as part of the sound wave test problem. At any point during the calculation the values of \( \epsilon_1, \epsilon_2 \) can be changed by calling the entry SETEPS with the desired values as the first two arguments. The default values are \( \epsilon_1 = 0.45, \epsilon_2 = 0.45 \) so the calculation will be slightly damped. This setting is good for moderately long timesteps relative to sonic transit times but will eventually damp finite speed sound waves appreciably so the flexibility to change \( \epsilon_1, \epsilon_2 \) closer to \( \frac{1}{2} \) is important.

A special facility has been built into ADINC to damp out initial transients when long timesteps are being used. The facility is controlled by the third argument of SETEPS, the integer MDAMP. When MDAMP = 1, the facility is disabled making the values of \( \epsilon_1, \epsilon_2 \)
constant until the next time they are changed under user control by calling SETEPS with new values.

Before describing exactly what this facility does, it is important to understand why such a facility can be necessary in some problems. Consider a relatively slow flow in which we wish to keep $\delta t$ large for computational efficiency but where $\varepsilon_r, \varepsilon_t \sim \frac{1}{2}$ is desired for numerical accuracy, i.e., we want to keep the numerical damping small. In such a problem any initial condition of pressures, positions, and velocities can be resolved in high frequency sound waves and slow flow components such as might be driven by thermal conduction, chemical energy release, etc. With $\varepsilon \sim 1/2$, the high frequency components will oscillate rapidly and can mask the desired slowly varying solution even though these oscillations are stable. The problem arises because the various initial conditions are slightly incompatible with each other and this incompatibility does not decay away quickly when $\varepsilon_r, \varepsilon_i$ are near $1/2$. Rather than expending a great deal of effort to filter the initial conditions, it is often perfectly adequate to damp the first few cycles of the ADINC calculation strongly by using $\varepsilon_r, \varepsilon_t < \frac{1}{2}$.

The algorithm implemented linearly increases $\varepsilon_r, \varepsilon_t$ from ERO/MDAMP, EVO/MDAMP to ERO, EVO in MDAMP-1 successive calls to ADINC. The default values are MDAMP = 10, ERO = 0.45, EVO = 0.45 if the SETEPS entry is not used. The implementation is quite obvious in Appendix A. Clearly MDAMP = 1 circumvents the changing values of $\varepsilon_r, \varepsilon_t$ entirely. Since it may be desirable to perform this filtering operation more than once during a calculation, the facility is provided to reset the counter for the MDAMP-1 filtering steps. Every time ADINCO is called with MODE = 0, the integer NCALL gets reset to zero after the three integers are printed out. When MODE $\geq 1$, NCALL does not get reset and hence the filtering operation is not performed anew.

The ADINCO entry was provided so the user can diagnose the performance of ADINC at any point in a calculation. When called, the total number of calls to ADINC, the total number
of subcycle steps performed, and the total number of iterations performed since the last called to ADINCO are printed out. Each time ADINCO is called with MODE = 0, the counters are all reset. Using ADINCO to control re-initialization of the filtering facility MDAMP ensures that diagnostic prints are obtained every time the solution is re-filtered.
VI. DISCUSSION OF TEST PROBLEMS

Appendices C, D, and E provide selected printouts from three test calculations using ADINC. These test are chosen to illustrate the use of ADINC in several different and interesting cases, to demonstrate the flexibility and accuracy of the package, to demonstrate some of the control features the user has at his command, and to allow positive code verification via known answers for users implementing the code for themselves. In addition to providing evidence that ADINC and its associated subroutines do something reasonable and accurate in several cases, these test problems provide a jumping off point for many user applications and may be directly adaptable with little or no reprogramming.

The three problems are:

#1. An Adiabatic Sound Wave Test,

#2. An Incompressible Slug Between Adiabatic Gases, and

#3. A LINUS Simulation.

Each test has a main calculation, for which results are printed in the appendices, along with the corresponding data for code verification. In this section a number of auxiliary calculations are also reported to establish properties of the ADINC package with respect to convergence, accuracy, and stability.

#1. An Adiabatic Sound Wave Test (see Appendix C)

The first test problem concerns the ability of ADINC to capture the properties of a sound wave in an ideal gas. The gas is uniform with density 1.4 and pressure 1.0 in the absence of the sound wave. The gas constant \( \gamma \equiv 1.4 \) ensures a sound speed \( C_s = 1.0 \equiv \sqrt{\gamma P/\rho} \). The gas is contained between two rigid, impermeable walls at \( x=0 \) and \( x=1.0 \) so a sound wave propagates from \( x=0 \) to \( x=1 \) and back in a period of time \( t=2.0 \). The standard test #1 has half a
wavelength between the walls, the velocity being initialized sinusoidally according to

\[ V(x,0) = \delta V \sin \pi x \]  

where \( \delta V(DVEL) = 0.01 \) for the standard test. The initial grid is uniform with \( \delta x = 0.1 \) and the timestep is \( \delta t = 0.1 \). The standard calculation is thus performed with 20 cells per wavelength and 20 timesteps per period.

The data for this standard test #1 are given in the following table:

<table>
<thead>
<tr>
<th>Namelist /CONTRL/ (program control)</th>
<th>MAXSTP = 26</th>
<th>DTMIN = 0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPRINT = 1</td>
<td>DTMAX = 0.1</td>
<td></td>
</tr>
<tr>
<td>ALPHA = 1</td>
<td>EPSRO = 0.5</td>
<td></td>
</tr>
<tr>
<td>N = 10</td>
<td>EPSV0 = 0.5</td>
<td></td>
</tr>
<tr>
<td>GEOMCO = (1.0,0.0,0.0,0.0,0.0,0.0)</td>
<td>(not used if ( \alpha = 1 ))</td>
<td></td>
</tr>
<tr>
<td>LZONE = .FALSE.</td>
<td>LCND = .FALSE.</td>
<td></td>
</tr>
<tr>
<td>LCHEM = .FALSE.</td>
<td>LTPRT = TRUE.</td>
<td></td>
</tr>
<tr>
<td>LDIFF = .FALSE.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Namelist /SHLINI/ (layer (shell) initializer)</th>
<th>NSHELL = 1</th>
<th>MODE = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>RN = 1.0x10^{-20}</td>
<td>DRHO = 0.0</td>
<td></td>
</tr>
<tr>
<td>VN = 0.0</td>
<td>DVEL = 0.01</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Namelist /SHLDAT/ (first and only layer)</th>
<th>LCELLS = 10</th>
<th>MATERS = 1.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>RN = 1.0</td>
<td>GAMMAS = 1.4</td>
<td></td>
</tr>
<tr>
<td>VN = 0.0</td>
<td>RHOCS = 0.0</td>
<td></td>
</tr>
<tr>
<td>RHOS = 1.4</td>
<td>PRES = 1.0</td>
<td></td>
</tr>
<tr>
<td>POWS = 1.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Because the peak velocity is small compared to the sound speed, \( \delta V/C_s = 10^{-2} \), the evolution of nonlinear effects will be slow and the profiles will remain sinusoidal for a long time. The period of the oscillation is determined by noting when \( r_n \), which starts at 0.5, passes through 0.5 the second time. Since the numerical period computed by ADINC will differ from the exact theoretical period due to finite difference truncation error, interpolation is necessary between timesteps to find the numerical period. This interpolation process has an error associated with it so the numerical period is never found exactly.

The standard test #1 with 20 cells per wavelength and \(-20\) timesteps per cycle has the
crossing of \( r_b \) through 0.5 occur between cycles 20 and 21. At \( r = 2.0, r_b = 0.49976 \) and at \( r = 2.1 \) the code gives \( r_b = 0.50074 \). The period computed from these values is \( \tau_{\text{code}} = 2.0245 \pm 0.001 \), in error by \( \sim 1\% \). This error arises from several sources, the most important of which are the finite timestep and grid size of the calculation. The error from interpolating for \( r_b = 0.5 \) is twenty times smaller.

Table 2 below summarizes the period computed using ADINC for several values of grid size (initial) and timestep. The number of cells per wavelength and timesteps per period, the important non-dimensional quantities, are also shown.

<table>
<thead>
<tr>
<th>( \delta x )</th>
<th>( \delta t )</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>( \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>cells per wavelength</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.2</td>
<td></td>
<td></td>
<td>2.0526</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.1</td>
<td>2.0714</td>
<td>2.0245</td>
<td>2.0122</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.05</td>
<td></td>
<td>2.0184</td>
<td>2.0061</td>
<td></td>
</tr>
<tr>
<td>( \infty )</td>
<td>0</td>
<td></td>
<td></td>
<td>2.0000</td>
<td></td>
</tr>
</tbody>
</table>

Since the values of \( \epsilon_r \), and \( \epsilon_e \), are both 0.5, the time integration is centered as well as the spatial differences. Therefore we expect full second order accuracy. The coefficients in time and space are different of course since the spatial and temporal algorithms are different. The functional form of the computed period is

\[
\tau_{\text{code}}(\delta x, \delta t) = \tau_{\text{theory}}(1 + \alpha(\delta x)^2)(1 + \beta(\delta t)^2) + \text{higher order terms}
\]

(29)

where \( \alpha \sim 1.6 \) and \( \beta \sim 3.2 \) when \( \delta x \) and \( \delta t \) are measured in units of wavelengths and wave periods respectively. That the above data in Table 2 satisfy Eq. (29) can be seen by the fact that the error in the 20-20 calculation is four times the error in the 40-40 calculation.

In these calculations the layer summaries printed by the test program include energy sums. Subroutine ERGPRT and the diagnostics section of the main program show how these sums are calculated. The interval energy density \( E_{\text{int}} \) is given by
for each cell of volume $\lambda_i$. The kinetic energy density is a little more difficult to compute because the interface velocities are known, not the cell center velocities. Averaging the velocities to the cell center introduces a damping term which appears as an overall small amplitude oscillation in the total energy as the wave cycles between potential and kinetic energy.

There is a definition of the kinetic energy which is more consistent with ADINC’s finite difference algorithms and which does not display the energy changes that the velocity average definition does. This simple diagnostic uses the fact that ADINC defines cell center positions $\{R_i\}$ and matches accelerations across cell interfaces. Let $\lambda_i^+$ and $\lambda_i^-$ be the cell partial volumes to the right and the left of the cell center position respectively.

$$\lambda_i = \lambda_i^+ + \lambda_i^-$$

(31)

In the test program $\{\lambda_i^+\}$ and $\{\lambda_i^-\}$ are calculated as a simple average but more accurate values can be determined by USEGEO for non-Cartesian geometries. The kinetic energy density in each cell is given by the formula

$$E_{\text{kin},i} = \frac{1}{2} \rho \left[ V_i^2 \lambda_i^+ + V_{i-1}^2 \lambda_i^- \right].$$

(32)

The code diagnostics print out the total energy to nine significant digits and to this accuracy energy is conserved identically. Since the algorithm is nominally reversible, good energy conservation is expected for this sound wave test problem. Essentially perfect energy conservation here means consistent thermal and kinetic energy definitions have been found.

A separate calculation with 20 cells was performed in which the individual zones varied in thickness by a factor of 100, alternating between $\delta x = \frac{10}{101}$ and $\delta x = \frac{1}{101}$ all the way from $x=0$ to $x=1$. The timestep was taken to be $\delta t = 0.05$ and the period was determined to be 2.0121, in error by ~ 0.5%. The conservation of energy for this calculation was again good to 1 part in $10^9$ even though ADINC had to subcycle twice at each timestep because some of the
cells were so small. The wide disparity in cell sizes did not adversely affect the accuracy of the calculation. Of course the large cells, not the small ones, determine the overall accuracy of the calculation but more realistic problems with widely varying characteristic scale lengths and/or imbedded discontinuities will be able to take good advantage of this additional flexibility in the ADINC algorithm.

A series of calculations was next performed (with uniform zoning again) where the values of $\epsilon_1$ and $\epsilon_2$ were varied to determine the amount of sound wave damping implied by forward differencing. It is important to understand that the stability properties usually invoked as benefits of implicit differencing come at stiff price.$^{10-13}$ A user of ADINC should perform calculations with $\epsilon_1$, $\epsilon_2$ as close to 0.5 as possible so accuracy can be maintained as well as stability. To measure wave damping the energy diagnostic was used to filter out the oscillations. Twenty cells per wavelength and 20 timesteps per cycle were again chosen as the standard conditions. Figure 5 and Table 3 summarize a number of computations performed with different values of $\epsilon_1$ and $\epsilon_2$, the ADINC explicitness parameters. The wave amplitude relative to the initial wave amplitude, $\mathcal{A}/\mathcal{A}_0$, is plotted versus the time measured in wave periods (theoretical). $\mathcal{A}(t)$ is determined by subtracting from the total energy, the thermal energy that the system would have with zero velocity. The linearity of $\mathcal{A}/\mathcal{A}_0$ versus $t/\tau$ on the semi-logarithmic scale demonstrates that the numerical damping is exponential as expected. Even with the very modest damping introduced by $\epsilon_1 = \epsilon_2 = 0.45$, the sound wave has decayed by more than a factor of 2.5 in only 10 oscillation periods. This means that the use of fully forward differenced schemes for numerical stability makes the meaningful simulation of sound waves essentially impossible.

Table 3 below summarizes these damping calculations in terms of the per period damping coefficient. The physical sound wave being studied should undergo no damping at all so the loss of energy marks a numerical error which is used here to measure the accuracy of the ADINC algorithm. Variation of damping with $\epsilon_1$, $\epsilon_2$, timestep $\delta t$, and grid size $\delta z$ is shown.
WAVE DAMPING IN ADINC

\[ \varepsilon_r = \varepsilon_v = 0.3 \]

The values of the explicitness parameters are shown. With \( \varepsilon_r = \varepsilon_v = 0.45 \), for example, the wave has damped by a factor of >2.5 in ten periods of the wave.

Figure 5 — Wave amplitude as a function of time illustrates damping introduced by forward differencing. Amplitudes are measured in units of the initial amplitude.
Table 3. Variation of wave damping coefficient (per period) with numerical control parameters. The physical wave should be undamped.

<table>
<thead>
<tr>
<th>$\epsilon_r$</th>
<th>$\epsilon_v$</th>
<th>cells per wavelength</th>
<th>timesteps per period</th>
<th>damping coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>0.50</td>
<td>20</td>
<td>20</td>
<td>&gt;.998</td>
</tr>
<tr>
<td>0.475</td>
<td>0.475</td>
<td>20</td>
<td>20</td>
<td>.953</td>
</tr>
<tr>
<td>0.45</td>
<td>0.45</td>
<td>20</td>
<td>20</td>
<td>.909</td>
</tr>
<tr>
<td>0.40</td>
<td>0.40</td>
<td>20</td>
<td>20</td>
<td>.826</td>
</tr>
<tr>
<td>0.30</td>
<td>0.30</td>
<td>20</td>
<td>20</td>
<td>.683</td>
</tr>
<tr>
<td>0.40</td>
<td>0.50</td>
<td>20</td>
<td>20</td>
<td>.908</td>
</tr>
<tr>
<td>0.50</td>
<td>0.40</td>
<td>20</td>
<td>20</td>
<td>.909</td>
</tr>
<tr>
<td>0.40</td>
<td>0.40</td>
<td>10</td>
<td>20</td>
<td>.830</td>
</tr>
<tr>
<td>0.40</td>
<td>0.40</td>
<td>20</td>
<td>40</td>
<td>.907</td>
</tr>
</tbody>
</table>

There are no surprises in these data. They are a quantitative as well as qualitative verification of the linearized ADINC dispersion relation

\[
\frac{\mathbf{A}(t+\delta t)}{\mathbf{A}(t)} = \frac{1-\beta^2/2 \left[ \epsilon_r (1-\epsilon_v) + \epsilon_v (1-\epsilon_r) \right] \pm i \beta \sqrt{1-\frac{\beta^2}{4} \left[ \epsilon_r (1-\epsilon_v) - \epsilon_v (1-\epsilon_r) \right]^2}}{[1+\beta^2 (1-\epsilon_v) (1-\epsilon_v)]}
\]

and

\[
\left| \frac{\mathbf{A}(t+\delta t)}{\mathbf{A}(t)} \right|^2 = \frac{1+\beta^2 \epsilon_v \epsilon_r}{[1+\beta^2 (1-\epsilon_v) (1-\epsilon_v)]}
\]

These formulae predict the symmetric behavior of wave damping and wave period with $\epsilon_r$ and $\epsilon_v$, they also predict the relative insensitivity of the wave damping to the cell size when $\delta x$ is small in

\[
\beta = \frac{2C_s \delta t}{\delta x} \sin \frac{k \delta x}{2}
\]

The next set of tests performed concerned the nonlinear behavior of sound waves computed by ADINC. Three variations of the standard test #1 were performed in which $\delta V$, the initial sinusoidal velocity maximum, was increased to 0.1, 0.3, and 0.5 times the speed of sound in the system. Figure 6 shows four normalized velocity profiles at cycle 40 in the three calculations, two linear wave periods after the calculation is initialized. The linear (sinusoidal) profile
Figure 6 — Nonlinearity in Standard Test #1. Without damping, increasingly nonlinear initial sound waves develop short wavelength oscillations proportionately faster. The profiles are scaled; the sinusoidal linear solution after two periods is also shown for comparison.
is shown as the solid line for comparison.

In the $\delta V = 0.1$ case the calculation required more iterations than in the previous cases and ADINC had difficulty with convergence criteria during some of the timesteps. The choice of $\epsilon_r = \epsilon_l = 0.5$ for these calculations seems to be permitting the nonlinear growth of short wavelength perturbations which are not preferentially damped in the calculation. At proportionately larger wave amplitudes these perturbations get larger. In the $\delta V = 0.5$ undamped case, strong evidence of an even-odd type of grid instability can be seen. Clearly some damping is necessary for nonlinear cases like these even before shocks can be expected to form.

The $\delta V = 0.5$ calculation was repeated with $\epsilon_r = \epsilon_l = 0.45$, with 30 cells in the system (60 cells per wavelength), and with $\delta t = 0.02$. Figure 7 shows the velocity profile at a series of times during the calculation half a theoretical linear wave period apart. In the physical problem simulated here two large amplitude sound waves of equal amplitude are passing through each other travelling in opposite directions. As the pressure increases in the wave so does the sound speed. Thus the two waves each steepen until a shock (actually a number of shocks) forms. Even though there is some linear damping in the calculation of Fig. 7, it is not enough to provide the required conversion of kinetic to thermal energy through the shock. Future emphasis of research on the ADINC package itself will be on these nonlinear aspects of implicit hydrodynamics. This is a very difficult problem. Implicit differencing means that the numerical "characteristic" travels at infinite speed. Thus the possibility always exists for information about the shocked fluid to propagate nonphysically ahead of the advancing shock. My current recommendation for treating shocks accurately is to use the FCT algorithms described in refs. 14-16.

#2. An Incompressible Slug Between Adiabatic Gases (see Appendix D)

The second test problem is illustrated in Figure 8. There are three layers bounded on the left and on the right by rigid impermeable walls. The center region is an incompressible slug. The left and right regions are each adiabatic gas layers of exactly the same properties as the sin-
Figure 7 — Velocity profiles of nonlinear sound wave steepening. The four curves are taken half a (theoretical linear) period apart. The $\tau/2$ and $3\tau/2$ data are inverted + for − and flipped left for right on the plot so that the nonlinear evolution is evident. With $\varepsilon_1 = \varepsilon_2 = 0.45$ as in this calculation, there is not enough dissipation to maintain monotonicity at the shock.
# STANDARD TEST #2 — HEAVY SLUG BETWEEN ADIABATIC GAS LAYERS

| \( \rho_L = 1.4 \text{ gm/cm}^3 \) | \( M_s = 140 \text{ gm/cm}^2 \) | \( \rho_R = 1.4 \text{ gm/cm}^3 \) |
| \( \gamma_L = 1.4 \) | \( V_s = 0.001 \text{ cm/sec} \) | \( \gamma_R = 1.4 \) |
| \( P_L = 1.0 \text{ erg/cm}^3 \) | | \( P_R = 1.0 \text{ erg/cm}^3 \) |

Figure 8 — Standard Test #2 — A heavy Slug Between adiabatic gas layers. A slug of density 140 and thickness \( W_s = 1 \) is situated between two adiabatic gas layers. Each adiabatic layer is exactly as treated in the first test problem.
gle gas layer considered as standard test #1. The large density discontinuities across the two layer interfaces makes this a test of the acceleration-matching part of the ADINC algorithm. Without this correction the effective mass of the slug would vary by half a cell or so hence finite difference errors would be of order 5%.

Detailed comparisons require an analytic solution. Let the slug have a mass $M_s$ per cm$^2$ and a density $\rho_s$. The right and left hand adiabatic gas layers have densities which are determined by the location of the moveable slug. At the initial time $t=0$ the slug is at the equilibrium position but moving with a nonzero velocity $V_s$. The initial pressures are equal, $P_R = P_L$, but the densities, $\rho_R$ and $\rho_L$, the width of the regions, $W_R$ and $W_L$, and the gas constants, $\gamma_R$ and $\gamma_L$, can all be different. This system is soluble both in the linear and in the asymptotic nonlinear limit and thus permits a detailed evaluation of the accuracy of the numerical techniques.

Let the displacement of the slug from equilibrium be denoted by $X(t)$. Then

$$W_L(t) = W_L + X(t) \quad P_L(t) = S_L W_L^{-\gamma_L}(t)$$
$$W_R(t) = W_R - X(t) \quad P_R(t) = S_R W_R^{-\gamma_R}(t)$$

where the fixed entropies $S_L$ and $S_R$ can be determined from the initial conditions. There are two equations to be solved,

$$\frac{dX(t)}{dt} = V(t) \quad \text{and} \quad M_s \frac{dV(t)}{dt} = P_L(t) - P_R(t).$$

Linearizing gives $P_L(t) \approx P_L - \gamma_L S_L W_L^{-\gamma_L-1} X$. With a similar expression for $P_R(t)$. The equilibrium terms cancel and

$$M_s \frac{dV(t)}{dt} = -X(t) \left[ \frac{\gamma_L P_L}{W_L} + \frac{\gamma_R P_R}{W_R} \right]$$

Let $X(t) = X_s \sin \omega t$ where $V_s = \omega X_s$. The resulting dispersion relation is

$$\omega^2 = \left[ \frac{\gamma_L P_L}{W_L} + \frac{\gamma_R P_R}{W_R} \right] / M_s.$$
The test calculations performed on this problem with ADINC have \( W_R = W_L = 1.0 \text{cm} \) and the slug density \( \rho_s = 140 \text{ gm/cm}^3 \), the slug length is 1.0 cm, and \( M_s = 140 \text{ gm/cm}^3 \). The initial gas layer pressures are 1.0 dyne/cm\(^2\) and \( \gamma_L = \gamma_R = 1.4 \). With these parameters the frequency \( \omega = \sqrt{2.8/140} = 0.141421 \) and the period \( \tau = \frac{2\pi}{\omega} = 44.4288 \) is expected.

Actually we should not neglect the mass of the gas because ADINC won't. When the density ratio is large as it is here, the velocity will vary roughly linearly across the gas regions in order that the density and pressure stay spatially constant. The effective mass of the slug therefore increases by half of the mass of each of the gas regions. Equation (39) becomes

\[
\omega^2 = \frac{[\gamma_L P_L/W_L + \gamma_R P_R/W_R]}{[M_s + \frac{1}{2} M_L + \frac{1}{2} M_R]}
\]

which predicts a period \( \tau = 44.6504 \text{ sec} \). Here I will not attempt still higher order corrections which arise from the curvature of the gas layer profiles, i.e. the fact that the density cannot be constant if there is a pressure variation across the gas layer. These are of order \( 1:10^{-4} \) and are about as large as the interpolation error in evaluating the data, \( \pm 0.001 \text{ sec} \).

The data for the standard test #2 are given in the following table:

<table>
<thead>
<tr>
<th>Namelist/CONTRL/ (program control)</th>
<th>DIMIN = 1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAXSTP = 26</td>
<td>DTMAX = 1.0</td>
</tr>
<tr>
<td>IPRINT = 1</td>
<td>EPSR0 = 0.5</td>
</tr>
<tr>
<td>ALPHA = 1</td>
<td>EPSV0 = 0.5</td>
</tr>
<tr>
<td>N = 10</td>
<td></td>
</tr>
<tr>
<td>GEOMCO = (1.0, 0.0, 0.0, 0.0, 0.0) (not used if ( \alpha=1 ))</td>
<td></td>
</tr>
<tr>
<td>ZONE = .FALSE.</td>
<td>LTCND = .FALSE.</td>
</tr>
<tr>
<td>LCHEM = .FALSE.</td>
<td>LTPRT = .TRUE.</td>
</tr>
<tr>
<td>LDIF = .FALSE.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Namelist /SHLINI/ (layer (shell) initializer)</th>
<th>MODE = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSHELL = 3</td>
<td>DRHO = 0.0</td>
</tr>
<tr>
<td>RN = 1.0 \times 10^{-20}</td>
<td>DVEL = 0.0</td>
</tr>
<tr>
<td>VN = 0.0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Namelist /SHLDAT/ (first layer data)</th>
<th>RN = 1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCELLS = 5</td>
<td>VN = 0.001</td>
</tr>
<tr>
<td>MATERS = 1.01</td>
<td>POWS = 1.0</td>
</tr>
<tr>
<td></td>
<td>RHOCS = 0.0</td>
</tr>
</tbody>
</table>
In the second layer, absolute incompressibility is not forced so it can be used as a diagnostic of ADINC’s accuracy. The density in this slug satisfies

$$\rho = 140 + (P/S)^2$$  \hspace{1cm} (41)

where the initial value of $\rho_\circ = 140[1 + 10^{-12}]$ is used in conjunction with $\rho_\circ = 140$ in the initializer to determine $S = 8.4516 \times 10^4$. In this second test, therefore, density fluctuations of order $10^{-16}$ are expected physically. This is smaller than roundoff error. The calculations described below all have the slug density constant to at least 1 part in $10^{10}$, about the convergence criterion for the nonlinear iteration in ADINC.

The ADINC package, of course, accepts a wide continuous range of values for $\rho_\circ$, $\gamma$, and $S$ in the formulae for the equation of state but for only a few values (i.e., gamma is the energy integral readily computable (I believe). When $\rho_\circ = 0$ in Eq. (3), for example, the energy density $E$ is given by the familiar formula

$$E = P/ (\gamma - 1).$$  \hspace{1cm} (42)

when $\rho_\circ \neq 0$, $E$ is given by the integral

$$E = \frac{S}{L} \int_L (\rho(L') - \rho_\circ)^\gamma dL'$$  \hspace{1cm} (43)

where

$$\rho(L') = \rho_\circ L'/L$$  \hspace{1cm} (44)

and $L_\circ$ is the size of the system when $\rho = \rho_\circ$. The integral (43) is basically just an evaluation of
the \( \int PdV \) work done on the fluid.

When \( \gamma = \frac{1}{2} \), as in the test case, the integral can be performed in closed form.

\[
E = \frac{P \tan^{-1} \left( \frac{\rho}{\rho_c - 1} \right)^{1/2}}{\left( \rho/\rho_c - 1 \right)^{1/2}} - 1
\]

This formula for the internal energy density goes to zero in the zero pressure \( \rho = \rho_c \) limit. As will be seen in the test calculation of Appendix D, the internal energy calculated for the slab is thirteen orders of magnitude smaller than the thermal energy of the gas layers. This indicates near perfect effective incompressibility of some finite difference cells immediately adjacent to cells whose equation of state requires compression. Equation (45) is implemented as an energy diagnostic for all cells having \( \gamma = \frac{1}{2} \) in the test program so that future users may see examples of how the solution quality varies as a result of varying both physical and numerical parameters in the calculation.

Again temporal and spatial resolution are expected to play major roles in determining the accuracy of the computed solution. Table 5 below summarizes the results of a number of test calculations with varying timestep and cell size.

<table>
<thead>
<tr>
<th>NCELLS</th>
<th>( \delta x )</th>
<th>( \delta t )</th>
<th>(~10)</th>
<th>(~20)</th>
<th>(~40)</th>
<th>(\infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1.000</td>
<td>4.0</td>
<td>45.7972</td>
<td>44.9430</td>
<td>44.7234</td>
<td>--</td>
</tr>
<tr>
<td>15</td>
<td>0.200</td>
<td>2.0</td>
<td>45.7269</td>
<td>44.8727</td>
<td>44.6534</td>
<td>--</td>
</tr>
<tr>
<td>(\infty)</td>
<td>0</td>
<td>1.0</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>(\sim 44.6504)</td>
</tr>
</tbody>
</table>

As can be seen, the timestep plays a much larger role here in the accuracy than the spatial resolution. The errors in the temporal integration are about the same as for the pure sound wave when \( \delta t \) is measured in units of the appropriate period. Here the slug is very heavy so the period is much longer than in test #1 reported above. At \(\sim 20\) timesteps per period the error is
again between 0.5% and 1% even though the period is theoretically more than 22 times longer for the slug than for the sound wave. This suggests a useful rule of thumb. Expect θ 1% errors in computing phenomena with a characteristic period of twenty finite difference timesteps. Faster phenomena will be less accurate, slower phenomena more accurate. Computed periods seem to be slower than theoretically expected.

The acceleration matching algorithm in ADINC is the reason that the computed slug period is insensitive to the spatial resolution. The interfaces are perfectly resolved and any density discontinuities are automatically treated with high accuracy as long as pressure variations are linear between cell centers and adjacent interfaces.

As with standard test #1, the nonlinear limit of this oscillatory slug problem is interesting. When the pressure in the gas layers is low relative to the kinetic energy of the slug, the slug travels from one wall to the other at essentially constant velocity where it then rebounds specularly. Therefore, to lowest order anyway, the period of oscillation will be

\[ \tau = \frac{2(W_L + W_R)}{V_s} \]  

Equation (46) gives only the limiting value, of course, valid when the gas layer compresses to vanishing thickness during rebound. Figure 9 plots the slug frequency 1/τ determined numerically as a function of the maximum slug velocity. Clearly the correct asymptote is being approached. As the slug velocity approaches the sound speed of the gas in the adiabatic layers, however, the flow becomes complicated and shock dissipation (which ADINC cannot handle properly) will become important.

As in standard test #1, further investigations of steepening sound waves will be deferred until a future paper. New users of ADINC might find it very instructive to boost the sound speed (or lower the pressure) in the gas layers to allow a faster slug velocity without generating a shock in the buffer layers. In principle the density of these layers can always be dropped far enough with \( P = 1 \) so that any given slug velocity \( V_s \) will be subsonic throughout the
OSCILLATION FREQUENCY
VS
PEAK SLUG VELOCITY

Figure 9 — Oscillation frequency versus peak slug velocity. For \( V_0 \) near zero the frequency is essentially constant at the theoretical value given by Eq. (40). For large velocities the slug always travels close to but below the peak velocity so the actual frequency lies somewhat below the asymptotic limiting curve.
compression and rarefaction phases of the slug oscillation. When shocks do develop, I recom-
mend the FCT algorithms discussed in refs. 14-16.

#3. A LINUS Simulation (see Appendix E)

The problem which necessitated writing ADINC in its current form, more than any other, is the simulation of the slightly compressible liquid liners used in Linus^6-^8. Original treatment of the plasma/magnetic field/liner system assumed the cylindrical plasma/magnetic field core was being compressed by a constant density, incompressible, ideal liquid flowing radially inward. The compressibility of the liner actually can have a substantial effect on the dynamics, energet-
ics and dwell time of the implosion. Hence the desire to simulate acoustic phenomena accu-
trately for time periods long compared to a sonic transit time became a necessity. Fully forward differenced schemes and even the quasi-static iteration used originally for Linus^{10-13} would not perform this calculation well. Therefore ADINC was developed with the capability for central as well as forward differencing under programmer control.

The geometry of the calculation is shown in Figure 10 with the reference values of the coordinates displayed. Five cylindrical shells are used with a 100 atmosphere pressure in the driver gas plenum. The input data for the standard test #3 are given in the following table:

<table>
<thead>
<tr>
<th>Namelist/CONTRL/ (program control)</th>
<th>Namelist/SHLINI/ (layer (shell) initializer)</th>
<th>Namelist/SHLDAT/ (plasma layer data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAXSTP = 1001</td>
<td>NSHELL = 5</td>
<td>LCELLS = 3</td>
</tr>
<tr>
<td>IPRINT = 50</td>
<td>RN = 10^{-20}</td>
<td>RN = 10.0</td>
</tr>
<tr>
<td>ALPHA = 2</td>
<td>VN = 0.0</td>
<td>VN = 0.0</td>
</tr>
<tr>
<td>N = 30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOMCO = (1.0, 0.0, 0.0, 0.0, 0.0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LZONE = .FALSE.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LCHEM = .FALSE.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LDIF = .FALSE.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DMIN = 10^{-7}</td>
<td>MODE = 1</td>
<td></td>
</tr>
<tr>
<td>DMAX = 10^{-4}</td>
<td>DRHO = 0.0</td>
<td></td>
</tr>
<tr>
<td>EPSR0 = 0.45</td>
<td>DVEL = 0.0</td>
<td></td>
</tr>
<tr>
<td>EPSV0 = 0.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEOMCO = (1.0, 0.0, 0.0, 0.0, 0.0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LTEND = .TRUE.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LTPRT = .FALSE.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Several outputs from the complete calculation are included in Appendix E. A number of ad hoc modifications of the test program were also included for this particular calculation. In the timestep control portion of the program, the initial timestep is taken as the geometric mean of the minimum and maximum values specified. In a problem where geometric convergence is strong, as in standard test #3, the spread between $\delta t_{\text{min}}$ and $\delta t_{\text{max}}$ has to be large. Thus the initial timestep is unspecified when all the velocities are zero.

This type of calculation starts with an implosion. The liner accelerates radially inward until the plasma-magnetic field core becomes highly compressed. When the pressure interior to the liner is high enough, the implosion comes to rest and is then converted to an accelerating explosion. At "turn around" the liner comes to rest. At this time, the timestep computed by DTFLOW is large because the fluid is moving slowly even though the accelerations are maximum. To avoid the problem of taking too large a timestep, users might be well advised to include in the timestep control portion of their codes, an estimate of the change in velocity expected during the step as well as the actual velocity at the beginning of the step. In the test program here this problem has been tackled in two ways. Both the old velocity and the current...
LINUS SIMULATION INITIAL CONDITIONS

\[ \gamma_{\text{plasma}} = \frac{5}{3} \]
\[ \gamma_{\text{B field}} = 2 \]
\[ \gamma_{\text{driver}} = 1.4 \]
\[ P_{\text{driver}} = 10^8 \]
\[ P_{\text{piston}} = 10^6 \]
\[ P_{\text{liner}} = 10^6 \]
\[ P_{\text{plasma}} = 10^6 \]
\[ P_{\text{B field}} = 10^6 \]

\[ \rho_{\text{liner}} = 1 \text{ gm/cc} + (P/S \text{ liner})^2 \]
\[ \rho_{\text{piston}} = 7.8 \text{ gm/cc} + (P/S \text{ piston})^2 \]

INITIAL VELOCITIES ZERO

Figure 10 — A schematic of the LINUS simulation initial conditions. A five layer calculation involving liquids, gases, and plasma. Parameters of the calculation are close to those used in recent LINUS 1 experiments at NRL with the exception of replacing the plasma and magnetic field with a \( \delta = 1.4 \) gas.
velocity are used to get the smallest estimate of $\delta t$ available. Then the estimated $\delta t$ is limited to be no more than 1.1 times the previous timestep. This last operation ensures that the step doesn't automatically become large when the liner comes to rest.

The test program also has a section of code at the beginning of the diagnostics portion which reduces the print out interval near liner turnaround. Since this occurs after cycle 650, the special tests have been left in the main program. The other tests performed for problem #1 and problem #2 generally require less than a couple of hundred steps so this problem dependant code is never invoked. During the deceleration and turnaround period the explicitness parameters $\epsilon$, and $\epsilon$, are also set to zero, an ad hoc fix for the nonlinear instability and non-convergence problems which arise when a centered, non-dissipative algorithm is subject to fast transients. The limitation here seems to be the fact that the solid equation of state cannot permit $\rho < \rho_c$, a situation which occurs behind the water hammer unless strongly damped.

Figure 11 displays the radial position of the inner edge of the liner as a function of time near turn around. Some asymmetry seems evident - possibly correlated with the weak compressions of the liner material noted near turnaround in the appendix.
Figure 11 — Trajectory $r$ vs $t$ of the inner edge of the liner at turnaround. A very slight compression of the liner material is expected for these parameters with a corresponding modification of the trajectory near turnaround.
VII. DISCUSSION

ADINC was originally motivated by four related but distinct computational problems. The first of these was the need to extend our detailed reacting shock model\textsuperscript{17} to treat flame propagation and similar reacting flow problems where chemistry and fluid dynamics interact on timescales appreciably slower than the sonic transit time.\textsuperscript{18} The self consistent energy release of many reacting flows occurs in extremely narrow reaction zones. Therefore Eulerian algorithms are dangerous because their unavoidable numerical diffusion\textsuperscript{14-16} usually swamps real molecular diffusion unless the grid resolution is unacceptable fine. Thus the Lagrangian aspect of the calculation is also important. We could have used the 1D analog of our 2D slow flow algorithm\textsuperscript{5} for many of the problems but felt that the ability to follow sound waves as well would be important in studies of the deflagration to detonation transition.

The second problem which motivated ADINC was the potential importance of acoustic waves (weak shocks) and modest compressibility in the physics of imploding liner systems (LINUS\textsuperscript{6-9}) as discussed above. Sonic transit times in the liquid metal liner are up to two orders of magnitude shorter than the implosion time thus implicit integration is again important. Furthermore, the large density discontinuities, coupled with the diverse materials and discontinuous properties in even the simplest LINUS system clearly demand a Lagrangian treatment of at least the material interfaces.

The third problem which led to ADINC was the desire to develop a fully compressible version of our two-dimensional hydrodynamics code SPLISH\textsuperscript{19,20} which would allow discontinuous material interfaces and reasonable approximations to real equations of state. SPLISH is Lagrangian and uses a reconnectable grid of triangles to permit long time integration of strongly sheared flows. Because of the complexity of the SPLISH variable geometry, it was clearly going to be profitable to perform tests of the proposed compressible algorithms in one dimension first. ADINC fills this role as well.
The fourth problem which motivated ADINC is the detailed simulation of a laser-driven ablation. The detailed analysis of the structure of this ablation predicts a thickness for the ablation layer of less than 0.1 μ but the sound speed in this region can exceed 10 cm/sec. Thus picosecond timesteps would be required for explicit courant stability — a strong inducement to look for accurate implicit algorithms since 10 nsec and longer experiments have to be simulated. Here, as in the LINUS problem, a single calculation must span the range of densities from solid or greater to a tenuous gas. Furthermore, the vast span of zone sizes required for different regions of the flow clearly requires special treatment to retain accuracy.

The three test problems presented above were designed to give the prospective user of ADINC a useful background of experience with the program in different regimes and with different problems. Various types of errors were tested and discussed but clearly a lot of work is required on shocks and other strong nonlinear sonic phenomena. In particular, until an adaptive rezoning algorithm is included, even the flexible initial zoning allowed by varying the parameter POWS in Namelist /SHLDAT/ is not adequate to deal accurately with a number of problems. Auxiliary reports in this series will present and test additions to the ADINC package for this rezoning, for more realistic equations of state, and for propagating discontinuities and shocks.

ADINC is an evolving package and its users are part of the evolution process. Your comments and suggestions for improving or simplifying the calculation will certainly be taken into account in future editions of the package. Comments on omissions and inaccuracies in this documentation will also be gratefully received and incorporated.

ACKNOWLEDGEMENT

This work was supported by the Office of Naval Research under projects RR014-03 and RR011-09-41 and by the Naval Material Command, project ZF43-451-001. The author would like to acknowledge useful conversations with K. Hain, E. Oran, M. Fritts, W. Jones and J.
Gardner during the course of this work and the contributions of Elliod Dent to the programming of some of the routines in Appendix B.
REFERENCES AND FOOTNOTES


4. Pressure fluctuations less than or of order $\frac{1}{10} \frac{V^2}{C_s^2}$ relative to the constant background pressure are generally quite large enough to maintain constancy of the pressure.


Appendix A
THE ADINC PACKAGE SUBROUTINES

SUBROUTINE ADINC (RAD, VEL, RHO, PRE, K, DTIN, CYCLE,
                   RNEW, RLEN, VNEW, VLEN)

ADINC HAS BEEN CONSTRUCTED AS A UTILITY PACKAGE TO ADVANCE THE
FOUR HYDRODYNAMIC VARIABLES.

RAD(I) = POSITION (RADIUS) OF THE I-TH CELL INTERFACE (CM)
VEL(I) = VELOCITY OF THE I-TH CELL INTERFACE (CM/SEC)
RHO(I) = DENSITY IN CELL I BETWEEN INTERFACES I,I+1 (GM/CC)
PRE(I) = PRESSURE IN THE I-TH COMPUTATIONAL CELL (ERG/CC)

LAGRANGIAN FLUID DYNAMICS EQUATIONS ARE SOLVED INCLUDING A FLEXIBLE
EQUATION OF STATE WHICH CAN VARY FROM CELL TO CELL IN THE
DISCRETIZED REPRESENTATION OF THE FLUID. THE EQUATIONS SOLVED ARE

\[ \frac{D}{D \tau} \cdot \nabla V = \frac{1}{\rho} \frac{\partial P}{\partial \tau} \]

AND THE EQUATION OF STATE ..

\[ \rho = \rho_0 \left( \frac{P}{\rho_0} \right)^{\gamma} \]

ALL NON-Ideal EFFECTS WHICH MIGHT BE INCLUDED IN AN ADINC CALCULATION
HAVE TO BE INCLUDED SEPARATELY EITHER BY PHENOMENOLOGICALLY
INCLUDING A SIMPLE MODEL IN THE CALCULATION OR BY Timestep
SPLITTING.

EACH FULLY LAGRANGIAN CELL HAS SEVERAL QUANTITIES THAT ARE
CONSERVED MOVING WITH THE FLUID AS LONG AS DIFFUSIVE AND OTHER
NON-Ideal EFFECTS AND SOURCE TERMS ARE NOT INCLUDED IN THE CALCULATION.
THE EQUATION OF STATE IN EACH FLUID CELL MAY DIFFER.

THE QUANTITIES INVOLVED IN THE EQUATION OF STATE ARE INITIALIZED
BY THE TWO ENTRAILS SETMAT AND SETEPS AND THE EQUATION OF STATE IS
EVALUATED BY CALLING USEECs. THE EQUATION OF STATE QUANTITIES
ARE COMMUNICATED THROUGHOUT THE ADINC PACKAGE IN COMMON BLOCK
ADINC/. THESE "CONSTANTS" VARY FROM CELL TO CELL ACCORDING TO
THE INITIAL CONDITIONS. FOLLOWING ARE THE DEFINITIONS OF THESE
QUANTITIES ..

MATC(I) = CELL IDENTIFIER = L,MM WHERE 0 < L < 10 IS THE
          LAYER NUMBER AND 0 < MM < 100 IS THE
          MATERIAL IDENTIFIER.

MCD(I) = CELL MASS = RHO(I)*LAM(I) - HELD CONSTANT IN ADINC

GAMMA(I) = CELL ADIABATIC GAS CONSTANT - HELD FIXED IN ADINC

ENTC(I) = CELL ENTHALPY - CONSTANT DURING ADINC HYDRODYNAMICS

RHO(I) = DENSITY CONSTANT IN THE EQUATION OF STATE (GM/CC)
The variable gamma and entropy are used in each Lagrangian cell. An implicit pressure iteration requires linearity, stability, but high
frequency phenomena are inaccurately integrated when the time steps
are chosen to be appreciably longer than the Courant timestep. The
nonlinear terms are iterated with a quadratically convergent algo-

Problems in one of four geometries can be set up for ADINC by
changing the integer alpha in the call to SETGE... 

\begin{align*}
\text{Alpha} = 1 & \quad \text{Cartesian coordinates}, \\
\text{Alpha} = 2 & \quad \text{Cylindrical coordinates}, \\
\text{Alpha} = 3 & \quad \text{Spherical coordinates}, \\
\text{Alpha} = 4 & \quad \text{Power series coordinates}.
\end{align*}

ADINC uses the utility used to determine the instantaneous grid
quantities. Call SETGE calculates the interface areas, the cell
volumes, and the cell center positions. ADINC does not update all
the geometry quantities automatically. The user must also be called externally. In general, ADINC does not have access
to the user-defined arrays for area, RADC, and LAMC.

The boundary conditions treated by ADINC are quite general.
The positions and velocities of the region bounding interfaces
RADC(1) and RADC(N+1) can be externally determined functions of time
and other physical variables during the calculation. The boundary
conditions are communicated to the ADINC package via the four
arguments...

\begin{align*}
\text{RRNEW} & = \text{right boundary position} \ RADC(N+1) \ \text{at end of timestep} \\
\text{FLN} & = \text{left boundary position} \ RADC(1) \ \text{at end of timestep} \\
\text{VRNEW} & = \text{right boundary velocity} \ VEL(N+1) \ \text{at end of timestep} \\
\text{VLN} & = \text{left boundary velocity} \ VEL(1) \ \text{at end of timestep}
\end{align*}

Several auxiliary variables are used by ADINC itself or the
ADINC routines which should also be explained to the user.

\begin{enumerate}
\item \text{n = ncells} \quad \text{number of fluid cells in the ADINC integration}
\item \text{dtin} \quad \text{time interval for the ADINC integration which may}
\item \text{cycle} \quad \text{is an integer up to 100 times internally if needed for}
\item \text{epr} \quad \text{is an explicitness parameter for the position integration}
\item \text{epsy} \quad \text{is an explicitness parameter for the velocity integration}
\item \text{namp} \quad \text{is the number of cycles at the beginning of a calculation}
\item \text{eps} \quad \text{is an explicitness parameter for rad last used by ADINC}
\item \text{epsy} \quad \text{is an explicitness parameter for vel last used by ADINC}
\end{enumerate}

Parameter \text{npt}, here 202, must be at least two larger than
the number of finite difference cells being integrated by ADINC.

\begin{align*}
\text{parameter} & = \text{npt} = 202, \ \text{npt} = 2 \times \text{npt} \\
\text{integer} & = \text{cycle} \\
\text{real} & = \text{rad}(\text{npt}), \ \text{vel}(\text{npt}), \ \text{rho}(\text{npt}), \ \text{pre}(\text{npt}) \\
\text{real} & = \text{rad}(\text{npt}), \ \text{rho}(\text{npt}), \ \text{pre}(\text{npt}), \ \text{pre}(\text{npt}) \\
\text{real} & = \text{lamb}(\text{npt}), \ \text{lamb}(\text{npt}), \ \text{radc}(\text{npt}), \ \text{radc}(\text{npt}) \\
\text{real} & = \text{rvel}(\text{npt}), \ \text{lvel}(\text{npt}), \ \text{lvel}(\text{npt}), \ \text{lam}(\text{npt}) \\
\text{real} & = \text{rvel}(\text{npt}), \ \text{lvel}(\text{npt}), \ \text{lam}(\text{npt}) \\
\end{align*}
REAL *8 AA(NPT), BB(NPT), CC(NPT), DD(NPT)
REAL *8 A1(NPT), B1(NPT), C1(NPT), D1(NPT)
REAL *8 RN(NPT), SC(NPT), SCB(NPT), DLAMDP(NPT)
REAL *8 AREAX(NPT), AREAH(NPT), AREAN(NPT)
REAL *8 ERLIM, ERRMAX, ERROR, AMNAS
REAL *8 PREMAK, PREMIN, APPREM, DT
REAL *8 VP0LD, VLM0LD, RPM0LD, PLNEH
REAL *8 EPL0LD, EPS0LD, EPE0LD,
REAL *8 EPSR, EPSPSR, EPSV, OMEPSV
REAL *8 EPSX, EPSVR, EPSV, OMEPSV
REAL *8 EPSX, EPSVR, EPSV, OMEPSV
REAL *8 EPSX, EPSVR, EPSV, OMEPSV

DECLARATIONS FOR COMMON BLOCK /ADICOM/ APPEAR THROUGHOUT THE ADINC
PACKAGE ROUTINES.
REAL *8 MASSC(NPT), ENTC(NPT), RHOC(NPT), MATERC(NPT)
REAL *8 GAMMAC(NPT)
COMMON /ADICOM/ MATERC, MASSC, GAMMAC, ENTC, RHOC, NCELLS

DATA NDAMP /10/, EPSR, EPSV /0, 0.00, 0.00/
DATA NCALL, NITER, NTIME /0, 0, 0/
DATA ERLIM, EPSRO, EPSV0, ITMAX/1.00, 0.450, 0.450, 6/

EQUIVALENCES (OTORHO (1), RDRI(1), (AREA hostile), AREAN hostile)

ADINC FORMATS FOR DIAGNOSTIC AND ERROR PRINTS.
1001 FORMAT ('ADINC TIMESTEP PROBLEM AT CYCLE', IS, 'DT = ',
1002 FORMAT ('ADINC TIMESTEP PROBLEM AT CYCLE', IS, 'DTVAL = ',
1003 FORMAT ('ADINC TIMESTEP PROBLEM AT CYCLE', IS, 'DTVAL = ',
1004 FORMAT ('ADINC TIMESTEP PROBLEM AT CYCLE', IS, 'DTVAL = ',
1005 FORMAT ('ADINC TIMESTEP PROBLEM AT CYCLE', IS, 'DTVAL = ',
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1007 FORMAT ('ADINC TIMESTEP PROBLEM AT CYCLE', IS, 'DTVAL = ',
1008 FORMAT ('ADINC TIMESTEP PROBLEM AT CYCLE', IS, 'DTVAL = ',
1009 FORMAT ('ADINC TIMESTEP PROBLEM AT CYCLE', IS, 'DTVAL = ',
1010 FORMAT ('ADINC TIMESTEP PROBLEM AT CYCLE', IS, 'DTVAL = ',

CHECK THE INPUT FOR ADINC FOR REASONABILITY.
166 IF (N .LE. 2 .OR. M .GT. NPT-1) WRITE (6, 1004) CYCLE, N, NPT
167 IF (E .LE. 2 .OR. M .GT. NPT-1) STOP
168 IF (PRN0W LE. 0 ) WRITE (6, 1005) CYCLE, PLNEH, PRN0W
169 IF (PRN0W .LE. NP0W) STOP
170 IF (N .LE. 2 .OR. M .GT. NPT-1) STOP
171 IF (PRN0W LE. 0 ) WRITE (6, 1005) CYCLE, PLNEW, PRN0W
172 IF (PRN0W .LE. NP0W) STOP
173 IF (N .LE. 2 .OR. M .GT. NPT-1) STOP
174 CALL DAXMI (M(2), N, PREMAK, IMAX, PREMIN, IMIN)
175 IF (PREMIN .LE. 0.000) WRITE (6, 1006) CYCLE, PREMIN, IMIN
176 IF (PREMIN .LE. 0.000) STOP
177 CALL MAXMIN (PHO(2), N, PREMAX, IMAX, PREMIN, IMIN)
178 IF (PREMIN .LE. 0.000) WRITE (6, 1007) CYCLE, PREMIN, IMIN
179 IF (PREMIN .LE. 0.000) STOP
180 CALL MAXMIN (PRE(2), N, PREMAX, IMAX, PREMIN, IMIN)
181 IF (PREMIN .LE. 0.000) WRITE (6, 1008) CYCLE, PREMIN, IMIN
182 IF (PREMIN .LE. 0.000) STOP
183 C
184 C ESTABLISH INTEGRATION AND CONTROL CONSTANTS FOR THIS CYCLE.
185 N1 = N + 1
186 CALL MAXMIN (HASSC(2), N, AMXMAS, IMAX, AMXMAS, IMIN)
187 EPRR = DSRST(AEXMAS/AMXMAS)*ERRM
188 RROL = RAD(N1)
189 RROLD = RAD(I)
190 VLRL = VEL(N1)
191 VLRLD = VEL(I)
192 NCALL = NCALL + 1
193 MCALIM = MINU(NCALL, NDAMP)
194 EPSR = DFLAT(MCALIM)EPSR/DFLAT(NDAMP)
195 EPSV = DFLAT(MCALIM)EPSV/DFLAT(NDAMP)
196 OMEPSR = 1.000 - EPSR
197 OMEPSV = 1.000 - EPSV
198 C
199 C CHECK THE TIMESTEP FOR SUBCYCLING AND NOTE ANY PROBLEMS.
200 CALL DTFLOW (RAD, VEL, DTVAL, N).
201 NSTEP = 1
202 DT = DTIN
203 IF (DTVAL .GT. DTIN) GO TO 40
204 IF (DTVAL .LT. 0.0100*DTIN) GO TO 30
205 WRITE (6, 1002) CYCLE, DTVAL, DTIN
206 DT = DTIN/DFLAT(NSTEP)
207 GO TO 40
208 30 WRITE (6, 1003) CYCLE, DTVAL, DTIN
209 STOP
210 C
211 C INITIALIZE VARIABLES FOR THE SUBCYCLING AND ITERATIONS.
212 DO 55 I = 2, N
213 OVRRLD = (VRNEW - VRRLD)/DFLAT(NSTEP)
214 CVLRLD = (VLNEW - VLRLLD)/DFLAT(NSTEP)
215 ORRRLD = (RNEW - RRRLD)/DFLAT(NSTEP)
216 ORLRLD = (RLNEW - RRLRLD)/DFLAT(NSTEP)
217 DT 50 I = 2, N
218 RIN(I) = RAD(I) + DT*VEL(I)
219 VEL(I)
220 PRE(I) = PRE(I)
221 VEL(I)
222 PRE(I) = PRE(I)
223 VEL(I)
224 PRE(I) = PRE(I)
225 VEL(I)
226 CALL USEGES (RAD, AREAS, RADC, LAMO, N)
227 CALL USEGES (RMIN, AREAN, RADCN, LAMN, N)
228 CALL USEGES (RHOP, PRPS, LAMPS, DLAMP, N)
229 CALL USEGES (RHOP, PRPN, LAMPS, DLAMP, N)
230 CALL USEGES (RHOP, PRPN, LAMPS, DLAMP, N)
231 PPREM = 1.000/PPREM
232 DO 60 I = 2, N
233 60 PRE(I) = 0.000
234 C
235 C PERFORM TIMESTEP SUBCYCLING
236 DO 550 ISTEP = 1, NSTEP

65
PERFORM THE ITERATION FOR NEW VALUES RHMN, LAMN, PREN, PIN AND VIN

C 500 ITER = 1, ITMAX
NITER = NITER + 1

CALCULATE QUANTITIES USED IN TRIDIAGONAL EXPRESSIONS.

DA 200 I = 1, N1
AREA(I) = 0.500*(AREA(I) + AREA(I))
RADP(I) = 4.500*(RAD(I) + RIN(I))
DA 205 I = 2, N1
DLAM(I) = LAMPS(I) - LAMN(I)
RBAR(I) = 0.500*(RADCM(I) + RADCM(I))
DA 210 I = 2, N1
PRE(I) = THON(I)*RADP(I) - RBAR(I) + PHON(I+1)*RBAR(I+1)

1 = PRE(I)

CALCULATE QUANTITIES USED IN THE TRIDIAGONAL COEFFICIENTS.

SOLVE THE TRIDIAGONAL SYSTEM AND CONSTRUCT THE NEW VALUES AT TIME

TRIDIAG, THE SCALE VERSION, IS USED IN THIS REGIME INSTEAD.

IF (N .LT. 15) CALL TRIDOS (N, AA(Z), BR(Z), CC(Z), DD(Z),
PRE(N), SCA(Z), SCA(Z))

IF (N .GE. 15) CALL TRIDDV (N, AA(Z), BR(Z), CC(Z), DD(Z),
PRE(N), SCA(Z), SCA(Z))

IN SOME CIRCUMSTANCES IT MAY BE APPROPRIATE TO LET THE PRESSURE GO
NEGATIVE, IT IS SUCH CASES THE FOLLOWING LAMP MUST BE REMOVED OR
PREMIN MUST BE ALLOWED TO GO NEGATIVE.

NOW I = 2, N1
PREMIN = MAX1 (0,0.001*PREMIN, PREMIN)

NOW I = 2, N1
VIN(I) = AI(I) - BI(I)*(PREN(I+1) - PREN(I))

NOW I = IAI(I) + LT*(EPS*VEL(I) + MEPS*VIN(I))
CALL USEREG (RIN, AREAN, RADCN, LAMN, N)
CALL USEREG (RNCH, PREH, LAMCN, DLAMP, N)
C
C CHECK ON WHETHER THE ITERATION HAS CONVERGED.
C
DM 400 I = 2, N1
SCA(I) = PREH(I) - PRE(I)
SCA(I) = UABS(SCA(I)) - PPREX
SCD(I) = ERR0R
PRE(I) = PREH(I)
MTCON = 0
ICELL = 0
ERMAX = 0,000
DM 405 I = 2, N1
IF (SCA(I) .LT. SCF(I)) GO TO 405
MTCON = MTCON + 1
IF (SCA(I) .GT. ERMAX) ICELL = I
ERMAX = EMAX (ERMAX, SCA(I))
CONTINUE
IF (MTCON .EQ. 0) GO TO 505
C
500 CONTINUE
C
501 CONTINUE
C PRINT OUT IF WE HAVE NOT CONVERGED.
C IF (MOD(CYCLE, 10) .EQ. 0)
1 WRITE (6, 101) CYCLE, DT, ITMAX, EPREH, ICELL, ERMAX,
2 CONTINUE
505 CONTINUE
C
C SET UP FOR ANOTHER SUBCYCLE TIMESLAP.
C
C IF (ISTEP .EQ. NSTEP) GO TO 550
C
DM 520 I = 1, N1
ARLAN(I) = AREAN(I)
VEL(I) = VIN(I)
RAD(1) = PIN(I)
DM 525 I = 1, N1
VIN(I) = FAD(I) + DT*VEL(I)
VIN(I) = VEL(I)
RAD(I) = RAD(1) + DRPLRD
VIN(I) = VEL(I) + DVRLD
VIN(I) = VEL(N1) + DVRLD
DM 530 I = 2, N1
RHO(I) = RHOH(I)
PCH(I) = PCHH(I)
RHO(I) = RHOH(I)
CALL USEREG (RIN, AREAN, RADCN, LAMN, N)
C
550 CALL USEREG (RNCH, PREH, LAMCN, DLAMP, N)
C
550 CONTINUE
C
C CLEAN UP FOR EXIT.
C
DM 600 I = 1, N1
VEL(I) = VIN(I)
RAD(I) = PIN(I)
DM 601 I = 2, N1
RHO(I) = RHOH(I)
RETURN
C
C
C ENTRY ADINC (HOLE, CYCLE)
---
C
ADINC PRINTS OUT THE NUMBER OF CALLS TO ADINC, THE NUMBER OF
TIMESTEPS INCLUDING SUBCYCLING PERFORMED BY ADINC DURING THOSE
CALLS, AND THE TOTAL NUMBER OF ITERATIONS PERFORMED SINCE THE LAST
CALL TO ADINC.

MODE = 0  PRINT OUT AND RESET THE COUNTERS IN ADINC.
MODE = NE. 0  RESET ALL BUT NCALL COUNTER FOR FILTERING OPERATION.
CYCLE  = TIMESTEP NUMBER FOR IDENTIFICATION PURPOSES

WRITE (6, 1009) CYCLE, NCALL, NTIME, NITER
IF (MODE .EQ. 0) NCALL = 0
NTIME = 0
NITER = 0
RETURN

ENTRY SETEPS (EWO, EVO, MDAMP, EOUT, EVOUT)

SETEPS PERMITS THE USER TO RESET THE EXPLICITNESS PARAMETERS FOR
DIFFERENT TYPES OR STAGES OF FLUID PROBLEM. THE EXPLICITNESS
PARAMETERS ARE MAPPED INTO THE RANGE 0 <= EPS <= 1.

ERO  = THE NEW POSITION EXPLICITNESS PARAMETER
EVO  = THE NEW VELOCITY EXPLICITNESS PARAMETER
MDAMP = THE NEW VALUE OF NDAMP, # OF DAMPING CYCLES
EROUT = THE MOST RECENT POSITION EXPLICITNESS PARAMETER
EVOUT = THE MOST RECENT VELOCITY EXPLICITNESS PARAMETER

REAL*8  ERO, EVO, EROUT, EVOUT

NDAMP = MAXO (MDAMP, 1)
EPSRV = MINI (EFO, 1.000)
EPSWV = MINI (EVO, 1.000)
EPSHC = MINI (EPSRV, 0.000)
EPSRE = MAXI (EPSHC, 0.000)
EOUT = EPSR
EVOUT = EPSV
RETURN
END
SUBROUTINE SETGLE (ALPHA, GEOMCO)

SETGLE CONTROLS THE GEOMETRIC ASPECTS OF AN ADINC INTEGRATION.
SETGLE IS CALLED BY THE USER TO TELL THE ADINC PACKAGE EXACTLY
WHICH OF THE FOUR POSSIBLE GEOMETRIES HE WISHES TO USE.

ALPHA = 1 CARTESIAN COORDINATES.
ALPHA = 2 CYLINDRICAL COORDINATES.
ALPHA = 3 SPHERICAL COORDINATES.
ALPHA = 4 POWER SERIES COORDINATES.

GEOMCO(1..5) = ARRAY OF FIVE COEFFICIENTS IN THE EXPRESSION FOR
CELL AREA AND INTEGRATED VOLUME USED BELOW.

SETGLE INITIALIZES THE QUANTITIES GALPHA, HALPHA, G(I), AND H(I)
FOR LATER REPEATED USE IN ENTRY USEGE0-run.

PARAMETER NPT = 202
INTEGER ALPHA, ALPHH
REAL*8 GALPHA, HALPHA, G(S), H(S)
REAL*8 GLOCMC(S), RIAMC(NPT), DELR(NPT), TVOL(NPT)
REAL*8 TDMIN(NPT), ABSV(NPT), DRMIN, PI
EQUIVALENCE (DMIN(N1), TVOL(N1), (ABSV(N1), DELR(N1))
EQUIVALENCE (RIAM(N1), TVOL(N1))

CHECK THE INPUT TO SETGLE AND INITIALIZE.
IF (ALPHA.LT.1 OR. .ALPHA.GT.4) WRITE (6, 1001) ALPHA, GEOMCO
IF (ALPHA.LT.1 OR. .ALPHA.GT.4) STOP

1001 FORMAT ('SETGLE INPUT PROBLEM, ALPHA OUT OF RANGE.', ')
14, 2X, 1PE012, 4)
PI = 3.14159265358979D0
ALPHH = ALPHA
GO TO (10, 20, 30, 40), ALPHH

MODE = 1 RESET ADINC FOR CARTESIAN COORDINATES.
GALPHA = 1.0D0
HALPHA = 1.0D0
RETURN

MODE = 2 RESET ADINC FOR CYLINDRICAL COORDINATES.
GALPHA = PI
HALPHA = 2.0D0*PI
RETURN

MODE = 3 RESET ADINC FOR SPHERICAL COORDINATES.
GALPHA = 4.0D0*PI/3.0D0
HALPHA = 4.0D0*PI
RETURN

MODE = 4 RESET ADINC FOR POWER SERIES COORDINATES.
DO 41 I = 1, 5
G(I) = GEOMC0(I)
41 H(I) = G(I)/DFloat(I)
RETURN

ENTRY USEGE0 (RAD, AREA, RADC, LAMC, N)
C
ENTRY USEGE0 (RAD, AREA, RADC, LAMC, N)
C
GIVEN A MONOTONICALLY INCREASING SET OF CELL INTERFACE POSITIONS,
THE INTERFACE AREAS, CELL CENTER LOCATIONS, AND CELL VOLUMES ARE
CALCULATED IN A FULLY VECTORIZED MANNER, THIS GEOMETRIC UTILITY
IS USED BY ADIC, DIAGNOSTICS ROUTINES AND THE MAIN PROGRAM - 
WHenever THE CELL INTERFACE CONFIGURATION IS CHANGED - TO UPDATE
THE GEOMETRIC QUANTITIES.

RAD(I)  = POSITION OF THE I-TH INTERFACE (I = 1, N+1) (CM)
AREA(I) = AREA IN THE COMPUTATIONAL DOMAIN OF THE I-TH CELL
INTERFACE (CM**2)
RADC(I) = POSITION OF THE I-TH CELL CENTER (I = 2, N+1) (CM)
LAMC(I) = VOLUME OF CELL I BETWEEN INTERFACES I, I+1. (CM**3)
N = NUMBER OF INTERIOR CELLS IN THE SYSTEM

REAL*8 RAD(NPT), AREA(NPT), RADC(NPT), LAMC(NPT)

CHECK THE INPUT TO USECEO FOR REASONABLENESS.

IF (N.LE.1 .OR. N.GT.NPT-2) WRITE (6, 1002) N, NPT
IF (N.LE.1 .OR. N.GT.NPT-2) STOP

DO 50 I = 1, N
50 TVOL(I+1) = RAD(I+1) - RAD(I)

CALL MAXMIN (TVOL(2), N, TVOL(1), IMAX, DPl'IN, IMIN)
IF (DPl'IN .LE. 0.0000) WRITE (6, 1003) np
IF (DPl'IN .LE. 0.0000) STOP

FORMAT ('NO USECEO INPUT PROBLEM, N OUT OF RANGE. ', N)
FORMAT ('CELL SIZE NEGATIVE', N)

1002 'CALCULATION STOPPED.'

1003 'CALCULATION STOPPED.'

C

C CHECK THE INPUT TO USECEO FOR REASONABLENESS.

C

DO 50 I = 1, N
50 TVOL(I+1) = RAD(I+1) - RAD(I)

C

C FOR THE REGULAR GEOMETRIES CALCULATE THE AREA AND VOLUME.

C

FOR THE POWER SERIES (NOZZLE) COD RDI 1IAT ES

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FOR THE POWER SERIES (NOZZLE) COD RDI 1IAT ES

C FOR THE REGULAR GEOMETRIES CALCULATE THE AREA AND VOLUME.
AREA(I) = AREA(I) + G(I)
TVOL(I) = TVOL(I-1) + (TVOL(I) + H(I))

CALL MAXMIN (AREA(I), N, RADC(I), IMAX, DMIN, IMIN)
IF (DMIN .LE. 0.000) WRITE (6, 2001) DMIN, IMIN
IF (DMIN .LE. 0.000) STOP

2001 FORMAT ('DUSTFLOW PROBLEM, NEGATIVE AREA ', 1PD12.4, ' AT CELL ', I4, ' CALCULATION STOPPED.')

DR 601 I = 2, IP
LAM(I) = TVOL(I) - TVOL(I-1)

601 RAUC(I) = (RADC(I) - AREA(I-1) + RAUC(I-1) * AREA(I)) / (AREA(I) + AREA(I-1))

ENTRY DFLOW (ROU, VEL, DTVL, N)

-------------

DFLOW CALCULATES A PERMISSIBLE TIME STEP DTVL GIVEN THE SET OF
N + 1 CELL INTERFACES AND THEIR VELOCITIES.

ROC(I) = POSITION OF THE I-TH CELL INTERFACE (CM)
VEL(I) = VELOCITY OF THE I-TH CELL INTERFACE (CM/SEC)
DTVAL = THE ESTIMATED VALUE OF A PERMISSIBLE TIME STEP WHICH
PREVENTS INTERFACE CROSSING ASSUMING THE "ST CROPPED"
N = THE NUMBER OF INTERIOR CELLS IN THE CURRENT SYSTEM

REAL*8 RMAX(N), VEL(N), DTVL, EPS, DVSAFE, DTMAX

CHECK THE INPUT TO DFLOW FOR REASONABLENESS.

IF (N .LE. 2 .OR. N .GT. NPT-2) WRTE (6, 1004) N, NPT
DPT = N + 1
EPS = 0.0100
DEL(I) = ROC(I) - ROC(I-1)
CALL MAXMIN (DEL(I), N, DELR(I), IMAX, DMIN, IMIN)
IF (DMIN .LE. 0.000) WRITE (6, 1005) DMIN, IMIN
IF (DMIN .LE. 0.000) STOP

1004 FORMAT ('DUSTFLOW INPUT PROBLEM, N OUT OF RANGE. ', 14, I4, 
1005 FORMAT ('DUSTFLOW INPUT PROBLEM, CELL SIZE NEGATIVE ', 14, I4, 
1006 FORMAT ('DUSTFLOW INPUT PROBLEM, CMI SSIVE TIME STEP STOOED.'))

REQUIRE VEL*DT < MINIMUM CELL WIDTHS DELP.

720 DTMIN(I) = EPS*DMINI*(DEL(I), DELR(I)*I+1))
725 ABS(I) = LAIR (VEL(I))
730 DTVL = 1.0040
735 DO 730 I = 2, N

END
SUBROUTINE SETMAT (MATER, MASS, GAMMA, RHOCON, N)

THIS ROUTINE PROVIDES A VECTORIZED DOUBLE PRECISION EQUATION
OF STATE CALCULATION FOR ADINC. WHEN THE EQUATION OF STATE IS
CHANGED, A NUMBER OF OTHER ROUTINES MUST BE MODIFIED AS WELL.

EQUATION OF STATE ... PHM = RHOCON * (PRES/ENTC)**(1/GAMMA)

MATER(I) = CELL IDENTIFIER = L,MM WHERE 0 < L < 10 IS THE
LAYER NUMBER AND 0 < MM < 100 IS THE MATERIAL IDENTIFIER.

MASSC(I) = CELL MASS = PHM(I)*LAM(I)

GAMMA(I) = CELL ADIABATIC GAS CONSTANT — FIXED DURING ADINC

ENTC(I) = CELL ENTROPY — CONSTANT DURING ADINC HYDRODYNAMICS

RHO(I) = DENSITY CONSTANT IN THE EQUATION OF STATE

PARAMETER NPT = 20

REAL DELM(NPT), DELV(NPT), DMAX, DRMIN
REAL MATL(NPT), MASS(NPT), GAMMA(NPT), RHOCON(NPT)
REAL MASSC(NPT), ENTC(NPT), RHOCON(NPT), MATER(NPT)
COMMON /ADICOM/ MATER, MASSC, GAMMA, ENTC, RHO, NCELLS

CHECK THE INPUT TO SETMAT FOR REASONABLENESS.

IF (MATER(I) .LT. 0 .OR. MATER(I) .GT. 10) WRITE (6, 1001) MATER(I), NPT
IF (NCELLS(I) .LT. 2) STOP
CALL MAXMIN (MATER(I), N, DMAX, DRMIN, IMAX, IMIN)
IF (DRMIN .LE. 0.000) WRITE (6, 1002) DRMIN, IMIN
IF (DRMIN .LE. 0.000) STOP
CALL MAXMIN (MASSC(I), N, DRMAX, IMAX, DRMIN, IMIN)
IF (DRMIN .LE. 0.000) WRITE (6, 1003) DRMIN, IMIN
IF (DRMIN .LE. 0.000) STOP
CALL MAXMIN (RHOCON(I), N, DRMAX, IMAX, DRMIN, IMIN)
IF (DRMIN .LE. 0.000) WRITE (6, 1004) DRMIN, IMIN
IF (DRMIN .LE. 0.000) STOP

1001 FORMAT ('SETMAT INPUT PROBLEM: N OUT OF RANGE', I4, I4)
1002 FORMAT ( 'CALCULATION STOPPED.' )
1003 FORMAT ( 'SETMAT INPUT PROBLEM: CELL MATERIAL NEGATIVE', I4)
1004 FORMAT ( 'SETMAT INPUT PROBLEM: CELL RHOCON NEGATIVE', I4)
1005 FORMAT ( 'CALCULATION STOPPED.' )

NP = N + 1
DO 100 I = 2, NP
MATERC(I) = MATER(I)
MASSC(I) = MASSC(I)
GAMMA(I) = GAMMA(I)
100 RHOCON(I) = RHOCON(I)

NCELLS = N
RETURN

---

ENTRY SETEOS (RHO, PRE, N)

SETEOS COMPUTES THE ENTROPY CELL CONSTANTS (ENTC) GIVEN KNOWN
VALUES OF THE DENSITY AND PRESSURE IN THE CELLS. SETEOS IS USED
AT THE BEGINNING OF CALCULATIONS FOR INITIALIZATION AND DURING
CALCULATIONS WHENEVER NON-IDEAL, SOURCE, OR DISSIPATIVE EFFECTS

72
HAVE CHANGED THE CELL ENTROPIES, SINCE ADINC IS PREDICATED ON THE
CONSTANCY OF THE LAGRANGIAN CELL ENTROPY DURING A FLUID TIMESTEP.
THIS AMOUNTS TO A FORM OF TIMESTEP SPLITTING.

\[ \text{RHO}(I) = \text{DENSITY OF MATERIAL IN CELL I (I = 2, N+1) (GM/CC)} \]

\[ \text{PPE}(I) = \text{PRESSURE GIVEN IN CELL I BETWEEN INTERFACES I AND} \]

\[ \text{I-I1. THESE VALUES ARE INPUT TO USEEES (ERG/CC)} \]

\[ n = \text{NUMBER OF INTERIOR CELLS IN THE SYSTEM} \]

\[ \text{REAL*8 RHO(NPT), PPE(NPT)} \]

\[ \text{CALL THE INPUT TO USEEES FOR REASONABLENESS.} \]

\[ \text{IF (N.LT.2 .OR. N.GT.NPT-2) WRITE (6, 1005) N, NPT0} \]

\[ \text{CALL \text{MAXMIN} (RHO(2), N, DMAX, IMAX, DMIN, IMIN) \text{IF}} \]

\[ \text{IF (DMIN .LE. 0.000) WRITE (6, 1006) DMIN, IMIN} \]

\[ \text{1005 FORMAT ('USEEES INPUT PROBLEM, N OUT OF RANGE', 14, 14)} \]

\[ \text{1006 FORMAT ('USEEES INPUT PROBLEM, CELL DENSITY NEGATIVE', 14, 14,} \]

\[ \text{1007 FORMAT ('USEEES INPUT PROBLEM, CELL PRESSURE NEGATIVE', 14, 14,} \]

\[ \text{1008 IF (N.LT.2 .OR. N.GT.NPT-2) STOP \text{CALL MAXMIN (PPE(2), N, DMAX, IMAX, DMIN, IMIN) \text{IF}} \]

\[ \text{IF (DMIN .LE. 0.000) WRITE (6, 1006) DMIN, IMIN} \]

\[ \text{1008 IF (N.LT.2 .OR. N.GT.NPT-2) STOP \text{C}} \]

\[ \text{IF (PPE(1) .LE. 0.000) WRITE (6, 1007) PPE1, I1} \]

\[ \text{1007 IF (PPE(1) .LE. 0.000) WRITE (6, 1007) PPE1, I1} \]

\[ \text{SHEX = N + 1 \text{DELRHO}(I) \text{= DMAX (1.00-30) \text{RHO(I) = RHO(I)}) \text{CALL DUBLUG (DELRHO(2), ENTC2), N) \text{IF}} \]

\[ \text{ENTC(I) = ENTC(I)*GAMMAC(I) \text{CM 215 I = 2, IP}} \]

\[ \text{215 ENTC(I) = UEXP(ENTC(I)) \text{CM 220 I = 2, IP}} \]

\[ \text{220 ENTC(I) = FRL(I)/ENTC(I) RETURN \text{EN}} \]

\[ \text{ENTRY USEEES (RHO, PPE, LAMEOS, DLAMDP, N) \text{-----}} \]

\[ \text{USEEES TAKES THE SET OF CELL Pressures (PPE) AND CALCULATES THE} \]

\[ \text{CELL DENSITIES EXPECTED FOR THAT PRESSURE BASED ON THE EQUATION} \]

\[ \text{OF STATE CONSTANTS FOR THE CELL STORED IN COMMON BLOCK /ADICOM/} \]

\[ \text{THE EXPECTED CELL VOLUME (LAMEOS) IS COMPUTED FOR EACH CELL AS IS} \]

\[ \text{THE JACOBIAN DERIVATIVE DLAMDP,} \]

\[ \text{RHO(I) = DENSITY OF MATERIAL IN CELL I (I = 2, N+1) (GM/CC)} \]

\[ \text{PPE(I) = PRESSURE GIVEN IN CELL I BETWEEN INTERFACES I AND} \]

\[ \text{I-I1. THESE VALUES ARE INPUT TO USEEES (ERG/CC)} \]

\[ \text{LAMEOS(I) = THE CELL VOLUME REQUIRED BY THE EQUATION OF STATE} \]

\[ \text{GIVEN THE CELL PRESSURE PPE(I) AND E.A.S. CONSTANTS} \]

\[ \text{DLAMDP(I) = RATE OF CHANGE OF CELL VOLUME WITH PRESSURE IN THE} \]

\[ \text{EQUATION OF STATE} \]

\[ \text{N = NUMBER OF INTERIOR CELLS IN THE SYSTEM} \]
REAL*8 LAMENS(NPT), DLANDP(NPT)

C CHECK THE INPUT TO USEEMS FOR REASONABLENESS.
C
C \texttt{NPT} = NPT
C
IF (N.LT.2 .OR. N.GT.NPT-2) WRITE (6, 1009) N, NPT
IF (N.LE.2 .OR. N.GT.NPT-2) STOP
C
CALL MAXMIN (PRE(2), N, DMAX, IMAX, DMIN, IMIN)
C
IF (DMIN .LE. 0.0000) WRITE (6, 1009) DMIN, IMIN
IF (DMAX .LE. 0.0000) STOP
C
1008 FORMAT ('USEEMS INPUT PROBLEM. N OUT OF RANGE.', I, I, I, I, I, I)
C

C
130 C M = N + 1
140 DO 300 I = 2, M
141 300 DELV(I) = DMAXP(PRE(I),0.0000)/ENTC(I)
C
143 CALL DUPLOG (DELV(2), DELR(3), N)
144 DO 320 I = 2, M
145 320 DELP(I) = DELP(I)/GAMMAC(I)
146 DO 340 I = 2, M
147 340 DELR(I) = DEXP(DELPH(I))
148 C
149 C EVALUATE THE EQUATION OF STATE \( \texttt{RHO} = \texttt{RHO}^C + (\texttt{PRE}/\texttt{ENTC})**1/\texttt{GAMMAC} \)
150 DO 360 I = 2, M
151 360 RHO(I) = RHO(I) + DELR(I)
152 DO 380 I = 2, M
153 380 LAMENS(I) = LAMENS(I)/RHO(I)
154 DO 380 I = 2, M
155 380 DLANDP(I) = - LAMENS(I) * DELR(I) / (RHO(I) * PRE(I) * GAMMAC(I))
156 C
157 RETURN

END
Appendix B

TEST PROGRAM FOR ADINC AND UTILITIES

ADINC - SIMPLIFIED FIVE LAYER TEST PROGRAM  MARCH 1979

ADAPTATIC THROUGH INCOMPRESSIBLE FLOW IN ONE DIMENSION SANS SHOCK

THIS PROGRAM DEMONSTRATES USE OF THE ADINC PACKAGE TO SOLVE
A NUMBER OF ONE-DIMENSIONAL, LAGRANGIAN FLUID DYNAMICS PROBLEMS.
A WIDE CLASS OF PROBLEMS CAN BE SET UP USING THE DATA DRIVEN INITIALIZATION.
FLEXIBLE DIAGNOSTICS FOR EACH OF THE FLUID LAYERS ARE
INCLUDED. SPECIAL ATTENTION HAS BEEN PAID TO TREATING DENSITY DIS

CONTINUITIES AND ZONE SIZE DISCONTINUITIES ACCURATELY. THE PRESENT
VERSION OF ADINC AND ITS TEST PROGRAM ARE WRITTEN ENTIRELY IN 64
BIT (DOUBLE PRECISION) FLOATING POINT ARITHMETIC. THUS CONVERGENCE
TO BETTER THAN 1 PART IN 10^7 IS POSSIBLE FOR PROBLEMS WITH NEAR
INCOMPRESSIBILITY AND/OR EXTREME DENSITY DISCONTINUITIES WHICH
REQUIRE THIS ACCURACY, FOR SOMEWHAT LESS EXTREME FLUID SYSTEMS,
32 BIT FLOATING POINT COMPUTATIONS SHOULD BE ADEQUATE AND THE
CORRESPONDING SINGLE PRECISION TRIDIAGONAL SOLVERS ARE AVAILABLE.
ADINC HAS BEEN CONSTRUCTED AS A UTILITY PACKAGE TO ADVANCE THE
FOUR HYDRODYNAMIC VARIABLES.

RADO(I) = POSITION (RADIUS) OF THE I-TH CELL INTERFACE (CM)
VEL(I) = VELOCITY OF THE I-TH CELL INTERFACE (CM/SEC)
RHO(I) = DENSITY IN CELL I BETWEEN INTERFACES I-1+1 (GM/CC)
PRE(I) = PRESSURE IN THE I-TH COMPUTATIONAL CELL (GM/CC)

LAGRANGIAN FLUID DYNAMICS EQUATIONS ARE SOLVED INCLUDING A FLEX-
IBLE EQUATION OF STATE WHICH CAN VARY FROM CELL TO CELL IN THE
DISTRIBUTED REPRESENTATION OF THE FLUID. ADINC HAS BEEN CAST INTO
A FORM RESEMBLING THAT OF AN ORDINARY DIFFERENTIAL EQUATIONS
PACKAGE, THE USER CAN REQUEST INTEGRATION TO A CERTAIN TIME DEPE-
DANT OF THE NUMBER OF CYCLES REQUIRED IN ADINC. THE USER ALSO
HAS CONTROL OF VARIOUS ERROR AND INTEGRATION PARAMETERS WITHOUT
HAVING TO DIG INTO THE BouflS OF THE SOLUTION METHOD ITSELF.
THE EQUATIONS SOLVED ARE:

(DV/DT) = VEL,       (DV/DT) = -1
                        = ---- GRAD (PRE),
                        DT       RHO

AND THE EQUATION OF STATE:

RHO = RHO(I) + (- - - - -)
     ( PHRT ) ** 1/GAMMAC

THE TEST PROGRAM IS ARRANGED TO HANDLE UP TO FIVE DISTIN
LAYERED FLOWS COMPOSED OF UP TO 200 INDIVIDUAL FINE DIFFERENTI
CELLS, EACH CELL IS LAGRANGIAN AND HAS SEVERAL QUANTITIES THAT ARE
CONSERVED MOVING WITH THE FLOW AS LONG AS DIFFUSIVE AND OTHER
NON-IDEAL AND EXTERNAL SOURCE EFFECTS ARE IGNORED. THESE CONSTANTS ARE COMMUNICATED THROUGHOUT THE ADINC PACKAGE IN COMMON BLOCK /ADICON/. THESE "CONSTANTS" VARY FROM CELL TO CELL ACCORDING TO THE INITIAL CONDITIONS. FOLLOWING ARE THE DEFINITIONS OF THESE QUANTITIES.

MAT(1) = CELL IDENTIFIER = LMM WHERE 0 < L < 10 IS THE LAYER NUMBER AND 0 < MM < 100 IS THE MATERIAL IDENTIFIER.

MASS(1) = CELL MASS = R/(LMM) = HELD CONSTANT IN ADINC.

GAMMAC(1) = CELL ACIABATIC GAS CONSTANT = HELD FIXED IN ADINC.

ENTC(1) = CELL ENTRPY = CONSTANT DURING ADINC HYDRODYNAMICS.

RHC(1) = DENSITY CONSTANT IN THE EQUATION OF STATE.

NCILLS = NUMBER OF CELLS OF FLUID IN THE CALCULATION.

VARIABLE GAMMA AND ENTROPY ARE USED IN EACH LAGRANGIAN CELL AND AN IMPLICIT PRESSURE ITERATION ENSURES LINEAR STABILITY, IF NEEDED. SPLITTING OF TIME STEPS LONGER THAN THE CRUPTANT CONDITION. THE NONLINEAR TERMS ARE IERATED WITH A QUADRATICALLY CONVERGENT ALGORITHM. ADINC IS DESCRIBED IN NEL MEMORANDUM REPORT # , 1976. THE TRIDIAGONAL SOLVERS USED ARE DOCUMENTED IN NEL MEMORANDUM REPORT #3408, NOVEMBER 1976.

PROBLEMS IN ONE OF FOUR GEOMETRIES CAN BE SET UP FOR ADINC BY CHANGING THE INTEGER ALPHA IN THE CALL TO SETGEO. ALL NON-IDEAL EFFECTS HAVE TO BE TREATED SEPARATELY EITHER BY IMBEDDING OR BY Timestep Splitting.

ALPHA = 1 CARTESIAN COORDINATES.

ALPHA = 2 CYLINDRICAL COORDINATES.

ALPHA = 3 SPHERICAL COORDINATES.

ALPHA = 4 POWER SERIES (NOZZLE) COORDINATES.

THE BOUNDARY CONDITIONS TREATED BY ADINC ARE QUITE GENERAL. THE POSITIONS AND VELOCITIES OF THE REGION BOUNDING INTERFACES RAD(1) AND RAD(N+1) CAN BE EXTERNALLY DETERMINED FUNCTIONS OF TIME AND OTHER PHYSICAL VARIABLES DURING THE CALCULATION. THE BOUNDARY CONDITIONS ARE COMMUNICATED TO THE ADINC PACKAGE VIA THE FOUR ARGUMENTS...

RHE = RIGHT BOUNDARY POSITION RAD(N+1) AT END OF Timestep.

LHE = LEFT BOUNDARY POSITION RAD(1) AT END OF Timestep.

VHE = RIGHT BOUNDARY VELOCITY VEL(N+1) AT END OF Timestep.

VLE = LEFT BOUNDARY VELOCITY VEL(1) AT END OF Timestep.

SEVERAL AUXILIARY VARIABLES ARE USED BY ADINC AND THE CONTROLLING TEST PROGRAM WHICH SHOULD ALSO BE EXPLAINED TO THE USER.

N = NCILLS = NUMBER OF FLUID CELLS IN THE ADINC INTEGRATION.

Deltat = TIME INTERVAL FOR THE ADINC INTEGRATION WHICH MAY SUBCYCLE UP TO 100 TIMES INTERNALLY IF NEEDED FOR ACCURACY OR STABILITY.

ISTFP = TIMESTEP NUMBER USED BY ADINC FOR IDENTIFICATION.

EPSR = EPSRO = EPSVO = EXPLICITNESS PARAMETER FOR THE POSITION INTEGRATION.

HIAPP = NUMBER OF CYCLES AT THE BEGINNING OF A CALCULATION IN WHICH ADDITIONAL DAMPING/SMOOTHING IS APPLIED.

EPSR = EPSRO = EPSVO = EXPLICITNESS PARAMETER FOR VEL LAST USED BY ADINC.

MEOC(I,,5) = AREA OF FIVE COORDINATE COEFFICIENTS FOR ALPHA = 4.

AREA = AREA OF THE I-TH CELL INTERFACE.

RACC = POSITION OF THE I-TH CELL CENTER.
<table>
<thead>
<tr>
<th>Line</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>113</td>
<td>( V ) = volume of the i-th finite difference cell</td>
</tr>
<tr>
<td>114</td>
<td>( E_{\text{KE}}(i) ) = kinetic energy density in the i-th cell</td>
</tr>
<tr>
<td>115</td>
<td>( E_{\text{TH}}(i) ) = thermal energy density in the i-th cell</td>
</tr>
<tr>
<td>118</td>
<td>( \text{PARAMETER NPT = 202} )</td>
</tr>
<tr>
<td>120</td>
<td>( \text{INTG_RICAL LPTPT, LCHEN, LTCNO, LDIFF} )</td>
</tr>
<tr>
<td>129</td>
<td>( \text{REAL*8 MQS5C(NPT), ENTC(NPT), RHEO(NPT), MASSCA(NPT)} )</td>
</tr>
<tr>
<td>131</td>
<td>( \text{THE FOLLOWING DECLARATIONS APPEAR IN ADDING AND THE E.S.S. Routines} )</td>
</tr>
<tr>
<td>133</td>
<td>( \text{FOR THE COMMON DATA WHICH CONVEYS AND STORES E.S.S. INFORMATION, IT IS NEEDED IN THE MAIN PROGRAM FOR DIAGNOSTIC REASONS.} )</td>
</tr>
<tr>
<td>137</td>
<td>( \text{COMMON VACISO MATER, MASS, GAMMAC, ENTC, RHEO, NCELLS} )</td>
</tr>
<tr>
<td>138</td>
<td>( \text{DATA MAXSTP, IPRINT, IPRINT, ALPHA /26, 1, 0, 1/} )</td>
</tr>
<tr>
<td>142</td>
<td>( \text{DATA EPSHO, EPSVO, 0.50000, 0.50000, TIMEC /1*0.00000/} )</td>
</tr>
<tr>
<td>151</td>
<td>( \text{THE TIME STEP, DT = , DLIB, } )</td>
</tr>
<tr>
<td>153</td>
<td>( \text{FORMAT (11 AFTER STEP NO., '15, DT = , DLIB, } )</td>
</tr>
<tr>
<td>158</td>
<td>( \text{CELL VOLUME ENTER, THFAC, GAMMA, } )</td>
</tr>
<tr>
<td>165</td>
<td>( \text{FORMAT (10 MANY ZONES AT CYCLE ', } )</td>
</tr>
<tr>
<td>166</td>
<td>( \text{CONTROL PARAMETERS ARE INITIALIZED. CHANGE FOR DIFFERENT CASES.} )</td>
</tr>
<tr>
<td>170</td>
<td>( \text{CALL COMMON} )</td>
</tr>
<tr>
<td>172</td>
<td>( \text{TIME = 0.000} )</td>
</tr>
<tr>
<td>173</td>
<td>( \text{WRITE (6, 1003)} )</td>
</tr>
<tr>
<td>174</td>
<td>( \text{WRITE (6, COMMON)} )</td>
</tr>
</tbody>
</table>
**Commentary on the Code:**

175 **READ (5, COMPL)**
176 **WRITE (6, 100)**
177 **WRITE (6, COMPL)**
178 **LTPRE = DSORT (DMINX, MAXX)**
179 ****
180 ****
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182 ****
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---

176 **READ (6, COMPL)**
177 **WRITE (6, COMPL)**
178 **LTPRE = DSORT (DMINX, MAXX)**
179 ****
180 ****
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**Legend:**

- **C** indicates a comment block.
- **** denotes continuation of a line.
- **LTPRE** is a dummy parameter used for sorting.
- **DMINX** and **MAXX** are the minimum and maximum values of a parameter.
- **DSORT** is a subroutine for sorting.
- **WRITE** and **READ** are standard Fortran I/O statements.
- **COMPL** is a comment placeholder.
- **MINUTE, MAXTIME** are placeholders for time units.
- **PRINT** statements are used for printing intermediate results.
- **CONTINUE** is used to skip to the next iteration.
- **IF** statements are used for conditional execution.
- **CALL** statements are used for procedure calls.
- **LZON** is a flag for the Lagrangian zone facility.
- **DTIME** is a dummy parameter used for time.

---

**Explanation:**

The code appears to be a fragment from a numerical simulation or computational fluid dynamics (CFD) code. It involves initializing physical data for the calculation, including grid, material, and fluid properties. There is a user-defined problem initializer, and the code seems to be designed for performative diagnostic purposes. The various subsections are timed for LINUS simulation purposes, and there are warnings about the following block of code controlling the printout interval and explicitness parameters. The Lagrangian zone facility is to be developed, and the code includes calls to various functions and subroutines for data handling and printing. The logical tests are used to control the flow of the program, ensuring that specific conditions are met before proceeding to the next step.
CALCULATE THE TIMESTEP BASED ON FLOW VELOCITY AND EXTERNAL LIMITS.

CALL SECOND (1, DTIME)

IF (20 I 1 = 1, NP)

DABS(V(I)) = DABS(VEL(I))

DABS(V(I)) = DMAX1 (DABS(V(I)), DABS(VOLD(I)))

CALL DTFLOW (RAD, UABS, DTVAL, N)

DELTAT = DMIN1 (DTMAX, 0.3D0*DTVAL)

DELTAT = DMIN1 (DELTAT, 1.1D0*DTPRE)

IF (DELTAT .LT. DTMIN) WRITE (6, 1005) ISTEP, DELTAT, DTMIN

DELTAT = DMAX1 (DELTAT, DTMIN)

DTRE = DELTAT

CALL SECOND (0, DTIME)

TINE(2) = TINE(2) + UTIME

THE RESULTS AND DIAGNOSTICS ARE PRINTED BY CELL AND BY LAYER. THE
UTILITY EREPORT USES THE CELL IDENTIFIER MATERC(I) TO DISTINGUISH
LAYER BOUNDARIES FOR CERTAIN IMPORTANT CONSERVATION SUMS.

IF (IPRINT .EQ. 0) GO TO 30

IF (MOD(ISTEP-1, IPRINT) .NE. 0) GO TO 30

IF (MATERC(I) .NE. 0) CALL SECOND (1, DTIME)

CALL USEGEOM (RAD, AREA, RADC, LAMC, N)

IF (25 I 2 = 2, NP)

ETHRI(I) = PRE(I)/(GAMMAC(I) - 1.0D0)

THE FOLLOWING FUNCTION EVALUATES THE ENERGY INTEGRAL FOR A GAMMA
EQUALS 1/2 "SOLID" EQUATION OF STATE.

IF (RHOC(I) .NE. 0.0D0)

DARG = DMAX1 (RHO(I)/RHOC(I), 1.0D0)

1 = DARG*DATA/(DARG - 1.0D0)

LAMC(I) = (RAC(I) - RADC(I))*LAMC(I) + 0.5D0*(PANP(I) -

RAD(I-1))*AREA(I-1)/(PANP(I) - RAD(I-1))

EMKE(I) = 0.5D0*RHOC(I)*VEL(I)**2*LAMC(I) + VEL(I-1)**2*

1 - LAMC(I) - LAMC(I))/LAMC(I)

ETHRI(I) = EMKE(I) - ETHRI(I)

25 THEFAC(I) = ETHRI(I)/ETHRI(I)

JSTEP = ISTEP - 1

WRITE (6, 1001) JSTEP, DELTAT, TIME

WRITE (6, 1002) (RHO(I), I, RAD(I), VEL(I), AREA(I), SBE(I), ETHTL(I),

1 - LAMC(I), VAVG(I), LAMC(I), EMKE(I), THEFAC(I), GAMMAC(I),

2 I = 1, NP)

WRITE (6, 2001) JSTEP, TIME

WRITE (6, 2002) (TIME(JJ), JJ = 1, 9)

CHECK ON THE INTEGRATOR PERFORMANCE VIA ADINCO, THE ARGUMENT I
PREVENTS THE RESET OF THE REAL PRINTERS IN ADINCO TO ZEROS. THUS
SMOOTHING IS ONLY ENABLED ON THE INITIAL DATA.

CALL ADINCO (1, ISTEP)

WRITE (6, 1601)

CALL SECOND (0, DTIME)
TIMEC(3) = TIMEC(3) + DTIME

300 C
301 C ENERGY AND OTHER DIAGNOSTICS SHELL BY SHELL, THE MATERIAL CAN VARY
302 C WITHIN A SHELL BUT THE SHELLS MUST BE ENCOUNTERED IN NUMERICAL
303 C SEQUENCE FROM 1 TO AT MOST 5.
304 C
305 C CALL SECOND (1, DTIME)
306 C CALL FIRSTP (H0D, VEL, AREA, LAMC, RHO, PRE, N, EKINE, ETHER)
307 C CALL SECOND (2, DTIME)
308 C TIMEC(4) = TIMEC(4) + DTIME
309 C
310 C 30 CONTINUE
311 C
312 C** PERFORM GRAPHICS ** TO BE DEVELOPED
313 C
314 C IF (.NOT.LGRT) GO TO 35
315 C IF (.NOT.ISTERF) GO TO 35
316 C CALL SECOND (3, DTIME)
317 C
318 C**
319 C
320 C CALL SECOND (4, DTIME)
321 C TIMEC(5) = TIMEC(5) + DTIME
322 C 35 CONTINUE
323 C
324 C** INTEGRATE THE CHEMICAL KINETIC RATE EQUATIONS ** TO BE DEVELOPED
325 C
326 C IF (.NOT.LCHE) GO TO 40
327 C CALL SECOND (4, DTIME)
328 C
329 C**
330 C
331 C CALL SECOND (5, DTIME)
332 C TIMEC(6) = TIMEC(6) + DTIME
333 C 40 CONTINUE
334 C
335 C** PERFORM DIFFUSION CALCULATIONS ** TO BE DEVELOPED
336 C
337 C IF (.NOT.LCHEM) GO TO 50
338 C CALL SECOND (5, DTIME)
339 C
340 C**
341 C
342 C CALL SECOND (6, DTIME)
343 C TIMEC(7) = TIMEC(7) + DTIME
344 C 50 CONTINUE
345 C
346 C** PERFORM CONDUCTION AND ENERGY ADDITIONS ** TO BE DEVELOPED
347 C
348 C IF (.NOT.LCHEM) GO TO 60
349 C CALL SECOND (6, DTIME)
350 C
351 C**
352 C
353 C CALL SECOND (7, DTIME)
354 C TIMEC(8) = TIMEC(8) + DTIME
355 C 60 CONTINUE
356 C
357 C** CALCULATE THE AUGMENTED PRESSURE AND UPDATE THE ENTROPY IF NON-
358 C IDEAL PHYSICS IS INCLUDED; SAVE THE OLD PRESSURE AND VELOCITY.
359 C
360 C**
DO 400 I = 2, NP
362 C          ** **
363 C          ** **
364 C 400C CONTINUE
365 IF (LCHEM .OR. LDIFF .OR. LTOFD) CALL SETEPS (RHO, PRE, N)
366 DN 100 I = 2, NP
367 HOLD(I) = PRE(I)
368 100 VOLD(I) = VEL(I)
369 VOLD(I) = VEL(I)
370 C          ** **
371 C          ** **
372 C INTEGRATE A HYDRODYNAMIC TIMESTEP OF LENGTH DELTAT AFTER RESETTING
373 C THE BOUNDARY CONDITIONS TO THE END OF THE TIMESTEP.
374 C          ** **
375 IF (.NOT.LTPRT) GO TO 70
376 CALL SECOND (I, DTIME)
377 VRNEW = VEL(I)
378 RRNEW = PRNEW + DELTAT*VRNEW
379 VLNEW = VEL(I)
380 RLNEW = PRNEW + DELTAT*VLNEW
381 CALL ADINC (RAV, VEL, RHO, PRE, N, DELTAT, ISTEP, 
382 I, RRNEW, RLNEW, VRNEW, VLNEW)
383 70 CALL SECOND (I, DTIME)
384 9999 CONTINUE
385 C          ** **
386 C CALCULATE THE AVERAGE VELOCITY AND PRESSURE ACTUALLY USED BY ADINC
387 C TO ADVANCE THE POSITIONS (RAD) AND VELOCITIES (VEL) FOR DIAGNOSIS.
388 C          ** **
389 C          ** **
390 C          CALL SETEPS (EPSRU, EPSVO, MDAMP, EPSR, EPSV)
391 DN 110 I = 2, NP
392 VAVG(I) = EPSR*VOLL(I) + (1.000 - EPSR)*VEL(I)
393 PAVG(I) = EPSV*POLL(I) + (1.000 - EPSV)*PRE(I)
394 VAVG(I) = EPSR*VOLL(I) + (1.000 - EPSR)*VEL(I)
395 C TIME = TIME + DELTAT
396 9999 CONTINUE
397 C STOP
SUBROUTINE EGPRT (PAIR, VNEW, AREA, LAM, PNEW, PNEV, 
   1, 5X, RNEW, PNEW)

EGPRT TAKES THE CELL AND INTERFACE DATA AND USES THE MATERIAL 
IDENTIFIER ARRAY MATRC TO RETURATE VARIOUS PHYSICAL QUANTITIES 
BY LAYER RATHER THAN BY CELL, THE DIAGNOSTIC IS PARTICULARLY USE-
FUL FOR HETEROGENEOUS CALCULATIONS. ONLY FIVE LAYERS ARE PERMITTED 
CURRENTLY BUT THIS COULD BE INCREASED EASILY BY CHANGING SOME OF 
THE LAYER LIMITS AND FORMATS. THANKS TO ELLIPTIC TENT FOR DEVELOPING 
THIS ROUTINE FOR MLNC.

PARAMETER (NPT = 5)

REAL R(0:NPT), V(0:NPT), X(0:NPT), P(0:NPT), XA(0:NPT), XV(0:NPT), 
   2, REAL T(0:NPT), M(0:NPT), I(0:NPT)

COMMON /ADGNC/ MATRC, MASC, GAMMAC, ENTCC, RHOC, NCELLS

FORMAT THERMAL EPG 1, 1PE017.A)

106 FORMAT (KINETIC EPG 1, 1PE017.A)
107 FORMAT (TOTAL EPG 1, 1PE017.A)
109 FORMAT (' SHELL 1 ', 1X, ' TOTALS SHELL 2 SHELL 3 SHELL 4 SHELL 5 '

SHELL 4 SHELL 5')
106 FORMAT (INTERFACE POSITION 1, 1PE017.A)
107 FORMAT (INTERFACE VELOCITY 1, 1PE017.A)
106 FORMAT (INTERFACE AREA 1, 1PE017.A)
106 FORMAT (LAYER BOUNDARY 1, 10X, 'INTER 1 ', 10X, 'INTER 2 ', 10X, 
   10X, 'INTER 3 ', 10X, 'INTER 4 ', 10X, 'INTER 5 ', 10X, 
   10X, 'INTER 6 ')
106 FORMAT (VOL AVG PH 1, 1PE017.A)
106 FORMAT (VOL AVG PRE 1, 1PE017.A)
106 FORMAT (LAYER VOLUME 1, 1PE017.A)
106 FORMAT (LAYER MASS 1, 1PE017.A)
106 FORMAT (VOL AVG GAM 1, 1PE017.A)
106 FORMAT (VOL AVG DRT 1, 1PE017.A)
106 FORMAT (VOL AVG RHOC 1, 1PE017.A)
106 FORMAT (5X, ')

C C THE VARIOUS SUBNUMS ARE INITIALIZED TO ZERO.
C 47 NP = NX * 1
C 48 RMASS = 0.000
C 49 RTIME = 0.000
C 50 RTEN = 0.000
C 51 RTHO = 0.000
C 52 RTPRE = 0.000
C 54 TVOL = 0.000
C 55 TGAM = 0.000
C 57 RHER = 0.000
C 58 DMAP = 1.000
C 59 RMASS(1) = 1.000
C 60 RTIME(1) = 0.000
C 61 RTEN(1) = 0.000
C 62 RTHO(1) = 0.000

82
63 \( \text{XRHO}(1) = 0.000 \)
64 \( \text{XPRE}(1) = 0.000 \)
65 \( \text{XGAM}(1) = 0.000 \)
66 \( \text{XENT}(1) = 0.000 \)
67 \( \text{XRHO}(1) = 0.000 \)
68 \( \text{XRAD}(1+1) = 0.000 \)
69 \( \text{XVEL}(1+1) = 0.000 \)
70 \( \text{XAREA}(1+1) = 0.000 \)
71 \[50 \text{XVOL}(1) = 0.000 \]
72 C
73 C THE VARIOUS SUMMATIONS ARE PERFORMED.
74 ISHL = MATERC(I)
75 XRAD(I) = RALM(I)
76 XVEL(I) = VNEP(I)
77 XAREA(I) = AREA(I)
78 II = 1
79 DO 56 I = 2, NP
80 IHLD = I
81 ISHL = MATERC(I)
82 IF (ISHL .LT. 1) GO TO 300
83 IF (ISHL .EQ. 5) GO TO 301
84 IF (ISHL .GT. JSHL) GO TO 54
85 JSHEL = ISHL
86 XRAD(I+1) = RADM(I+1)
87 XVEL(I+1) = VNEP(I+1)
88 XAREA(I+1) = AREA(I+1)
89 XENEW(I) = XINEW(I) + XNEW(I)
90 II = II + 1
91 XNEW(I) = XNEW(I) + XNEW(I) + XNEW(I) + LAM(I)
92 XINEW(I) = XINEW(I) + XNEW(I) + LAM(I)
93 XRHOC(I) = XPHOC(I) + XNEW(I) + LAM(I)
94 XGAM(I) = XGAM(I) + GAMMA(I) + LAM(I)
95 XENT(I) = XENT(I) + ENTE(I) + LAM(I)
96 XRHOC(I) = XRHOC(I) + RHOC(I) + LAM(I)
97 XVOL(I) = XVOL(I) + LAM(I)
98 56 XMASS(I) = XMASS(I) + XNEW(I) + LAM(I)
99 XENEW(I) = XINEW(I) + XNEW(I)
100 IF II = II
101 XRAD(I+1) = RADM(NP)
102 XVEL(I+1) = VNEP(NP)
103 XAREA(I+1) = AREA(NP)
104 DO 60 I = 1, II
105 XNEW = XNEW + XINEW(I)
106 TKNEW = TKNEW + XNEW(I)
107 TENEW = TENEW + XENEW(I)
108 TRPH = TRPH + XPHR(I)
109 TPRE = TPRE + XPRE(I)
110 TXGAM = TXGAM + XGAM(I)
111 TENT = TENT + XENT(I)
112 TRHOC = TRHOC + XRHOC(I)
113 TVOL = TVOL + XVOL(I)
114 60 XMASS = XMASS + XMASS(I)
115 TRPH = TRPH + TRPH(I)
116 TPRE = TPRE + TPRE(I)
117 TXGAM = TXGAM + TXGAM(I)
118 TENT = TENT + TENT(I)
119 TRHOC = TRHOC + TRHOC(I)
120 TVOL = TVOL + TVOL(I)
121 IF 70 J = 1, II
122 XRPH(J) = XRPH(J) + XVOL(J)
123 XPRE(J) = XPRE(J) + XVOL(J)
124 XGAM(J) = XGAM(J) + XVOL(J)
THE LAYER-BY-LAYER CFSS TAPULATIONS ARE PRINTED.

II = II + 1

WRITE (6, 2000) (XHAD(I), I=1, IE)

WRITE (6, 2001) (XVEL(I), I=1, IE)

WRITE (6, 2002) (XAREA(I), I=1, IE)

WRITE (6, 2003) (XHOC(I), J=1, IE)

WRITE (6, 2004) (XPRE(I), J=1, IE)

WRITE (6, 2005) (XGAM(I), J=1, IE)

WRITE (6, 2006) (XINT(I), J=1, IE)

WRITE (6, 2007) (XASS(I), J=1, IE)

RETURN

PRINT A WARNING IF ANY SHELL POINTER IS LESS THAN ONE.

FORMAT ('ISHL = ', I3, ' LESS THAN 1 AT CELL ', I4)

RETURN

PRINT A WARNING IF ANY SHELL POINTER IS GREATER THAN FIVE.

FORMAT ('CISHL = ', I3, ' GREATER THAN 5 AT CELL ', I4)

RETURN

END
SUBROUTINE INITIAL (MATER, GAMMA, PHACAN, RAD, VEL, RH0, PRE, I)

INITIAL SETS UP A FAIRLY GENERAL "MULTILAYER FLUID DYNAMICS PROBLEMS". UP TO FIVE LAYERS OF CONSTANT (BUT DIFFERENT) DENSITY, PRESURE, AND LINEARLY VARYING VELOCITY CAN BE INITIALIZED. EACH LAYER CAN HAVE A DIFFERENT EQUATION OF STATE BUT WITHIN EACH LAYER PHACAN, GAMMA, AND THE MATERIAL MATER ARE FIXED, A VARIABLE SPACING ACROSS THE SHELL IS ALLOWED UNDER CONTROL OF THE PARAMETER PHAC.

PROVISION IS ALSO MADE FOR A SINUSOIDAL VELOCITY PERTURBATION.

PARAMETER NPT = 202
REAL*8 RH, V0
REAL*8 MATTERS, GAMMAS, PHAC, PHACCS
REAL*8 RHOSS, Deltas, VELS, PESC
REAL*8 MATH(NPT), GAMMA(NPT), PHACAN(NPT)
REAL*8 RHO(NPT), VEL(NPT), PH(NPT), PRE(NPT)

DATA SHELL, RN, VN /1, 1.0D-20, 0.000/
DATA MODE, DDEL, DRHO /1, 1.0D-2, 0.000/
DATA LCELLS, MATERS, RHOCS /1, 1.0D0, 0.000/

NAMELIST /SHELL/ SHELL, RN, VN, MODE, DRHO, DDEL
NAMELIST /SHLDAT/ LCELLS, RN, VN, MATTERS, GAMMAS, PHAC, PHACCS

FORMAT ('NAMELIST /SHELL/ DATA FOR LAYER I, I2) 1081

INITIALIZE THE LOOP OVER LAYERS.
FORMAT ('NAMELIST /SHELL/ DEFAULTS ...') 1003
WRITE (5, 1003)
WRITE (6, 1003)
READ (5, 'SHELL')

FORMAT ('NAMELIST /SHELL/ UPDATES ...') 1004
WRITE (6, 1004)
WRITE (6, 'SHELL')
RAD(1) = RH0
VEL(1) = VN
12 = 1
100 ISHELL = 1, SHELL
PHCS = 1.0D0
60 = RAD(12)
V0 = VEL(12)

READ IN THE DATA FOR THE SHELLS ONE AT A TIME.
READ (5, 'SHLDAT')
WRITE (6, 1001) ISHELL
WRITE (6, 'SHLDAT')
11 = 12 + 1
12 = 11 + LCELLS - 1

SET THE SHELL DATA INTO THE OUTPUT ARRAYS.

DO 200 I = 1, 12
PHACAN(I) = PHACCS
GAMMA(I) = GAMMAS
MATER(I) = MATTERS
PHAC(I) = PHAC
PRE(I) = PRE
Deltas = DDEL(I - 11 + 1)/DDEL(12 - 11 + 1)
VELTAR = DELTAV*PESC
RAD(I) = DELTAR*PH + (1.0D0 - DELTAR)*PH0

200 CONTINUE
VEL(I) = DELTAV*VN + (1.0d0 - DELTAV)*VN

64      CONTINUE
65    C
66      CONTINUE
67      N = 12 - 1
68      NP = N + 1
69    C
70    C
71      ADD IN A SINUSOIDAL VELOCITY PERTURBATION.
72      DO 300 I = 2, NP
73          ARG1 = 3.141592653589790d0*DFLAT(MODE)*RAD(I)/RAD(NP)
74          VEL(I) = VEL(I) + (VEL*DSIN(ARG1))
75      END
SUBROUTINE MAXMIN(A, N, AMAX, IMAX, AMIN, IMIN)

C THE MAXIMUM (AMAX) AND MINIMUM (AMIN) OF THE N DOUBLE PRECISION
C VALUES OF VECTOR A ARE DETERMINED AND THE CORRESPONDING INDICES IN
C THE ARRAY (AS WELL AS THE EXTREME VALUES) ARE RETURNED. THE MAXVAL
C AND MINVAL FUNCTIONS COMPILE AS INLINE VECTOR OPERATIONS BUT ARE
C RATHER UNWIELDY TO USE AND DO NOT COMPILe PROPERLY UNDER THE FX
C FORTRAN COMPILER.

PARAMETER NPT=202

REAL*8 A(N), AMAX, AMIN
INTEGER IMAX, IMIN

IMAX = MAXVAL(A) + 1
IMIN = MINVAL(A) + 1
AMAX = A(IMAX)
AMIN = A(IMIN)
RETURN

ENTRY CUBLOG (DA, DLNA, N)

THE (VECTORIZED) DOUBLE PRECISION LOGARITHMS OF THE N INPUT VALUES
IN REAL*8 ARRAY DA ARE COMPUTED AND RETURNED IN THE ARRAY DLNA.
AS OF SUMMER 1972, THE ASC, FOR SOME OBSCURE REASON, DID NOT HAVE
A DOUBLE PRECISION VECTORIZED LOGARITHM. THIS ROUTINE FILLS THAT
PURPOSE.

REAL*8 DA(N), DLNA(N), SCR(NPT), DL(NPT)
REAL SA(NPT), SLN(NPT)

DO 10 I = 1, N
SA(I) = DA(I)
10 SLNA(I) = ALOG(SA(I))

DO 20 I = 1, N
DLNA(I) = SLNA(I)
20 SCR(I) = DEXP(DLNA(I))

C

C

RETURN

END
Appendix C
STANDARD TEST #1

AN ALIARATIC SOUND WAVE TEST

```
&CRITIC
&ENUL
&SHLINI
&END
&SHLDAT
LCELLS=10, RN=1.000, GAMMA=1.400, RHO5=1.400, PRES=1.000,
EEN

THE TIMINGS FOR ADINC, EVEN THOUGH IT IS VECTORIZED, ARE A LOT
SLOWER THAN CORRESPONDING EXPLICIT AND EULERIAN CALCULATIONS. THE
ADINC PACKAGE IS CONSTRUCTED FOR FLEXIBILITY AND ACCURACY, NOT
HIGH SPEED. SEVERAL CALCULATIONS USING STANDARD TEST #1 HAVE BEEN
PERFORMED TO CONSTRUCT THE FOLLOWING TIMING TABLE. NOTE: MOST OF
THE COMPUTING IN ADINC OCCURS IN THE TRANSCENDENTAL FUNCTION
EVALUATIONS FOR THE EQUATION OF STATE.

<table>
<thead>
<tr>
<th>NCELLS</th>
<th>CPU TIME PER CALL</th>
<th>CPU TIME PER ITERATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.0062 SEC</td>
<td>0.0023 SEC</td>
</tr>
<tr>
<td>15</td>
<td>0.0099 SEC</td>
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<td>30</td>
<td>0.0132 SEC</td>
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<td>50</td>
<td>0.0175 SEC</td>
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<td>100</td>
<td>0.0276 SEC</td>
<td>0.0073 SEC</td>
</tr>
<tr>
<td>200</td>
<td>0.0475 SEC</td>
<td>0.0119 SEC</td>
</tr>
</tbody>
</table>
```
NAMELIST /CONTRL/ DEFAULTS ...
CONTRL
MAXSTF = 26,IPINTs = 1,ALPHA = 1,NTIMP = 0.1,00000000000000000000,CTMAX = 0.1,00000000000000000000,N = 10,EPBRO = 0,50000000000000000000,EPBRO = 0.50000000000000000000,
LZONE = F,LCHN = F,LLIF = F,LTCHN = F,LTFT = T,MCAPP = 1,
END
NAMELIST /CONTRL/ UPDATES ...
CONTRL
MAXSTF = 26,IPINTs = 1,ALPHA = 1,NTIMP = 0.1,00000000000000000000,CTMAX = 0.1,00000000000000000000,N = 10,EPBRO = 0,50000000000000000000,EPBRO = 0.50000000000000000000,
LZONE = F,LCHN = F,LLIF = F,LTCHN = F,LTFT = T,MCAPP = 1,
END
NAMELIST /SMLINI/ DEFAULTS ...
SMLINI
Nshell = 1,SH = \frac{1}{n},Dshell = 0.00000000000000000000,OM = 1,00000000000000000000,
END
NAMELIST /SMLINI/ UPDATES ...
SMLINI
Nshell = 1,SH = \frac{1}{n},Dshell = 0.00000000000000000000,OM = 1,00000000000000000000,
END
NAMELIST /SMLDAT/ DATA FOR LAYF 1
SMLDAT
LCELLS = 16,LL = 1.00000000000000000000,VL = 1.00000000000000000000,MTENS = 1.00000000000000000000,SUMMAS =
1.00000000000000000000,ARR = 1.00000000000000000000,PRE = 1.00000000000000000000,CHOS = 1.00000000000000000000,
END
AFTER STEP NO.  G DT = 0.00000000000000  

<table>
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<tr>
<th>POSITION</th>
<th>VELOCITY</th>
<th>DENSITY</th>
<th>PRESSURE</th>
<th>ENERGY</th>
<th>AVG PFER</th>
<th>AVG VELDC</th>
<th>CELL VOLUM</th>
<th>ELECTRY</th>
<th>TIMING</th>
<th>LAMMA</th>
</tr>
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<td>3.000000-03</td>
<td>1.400000-06</td>
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</tr>
</tbody>
</table>

TIMINGS AT CYCLE 1  TIME = 0.000000

REZONE TIMESTEP CELL PRINT LAYER PRINT GRAPHICS CHEMISTRY DIFFUSION CONDUCTION ADIC

ADIC FREQUENCY COUNTERS (SINCE LAST CHECK) AT CYCLL 1

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THERMAL ENG

KINETIC ENG

TOTAL ENG

VOL AVG PFR

VOL AVG PHF

VOL AVG GM

VOL AVG ENT

VOL AVG NHF

LAYER VOLUME

LAYER MASS

SHL II

SHL 3

SHL 4

SHL 5
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TIMING AT CYCLE 10 TIME = 1.500000
REZONE TIMES FOR CELL PRINT LAYER PRT GRAPHICS CHEMISTRY DIFFUSION CONDUCTION ACINC

ADINC FREQUENCY COUNTERS (SINGLE LAST CHECK) AT CYCLE 10

LAYER RECYCLE 1 INFER 1 INTERF 2 INTERF 3 INTERF 4 INTERF 5 INTERF 6
INTERFAI RE POSITION 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000
INTERFAI VELOCITY 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
INTERFAI AREA 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000

TOTALS SHELL 1 SHELL 2 SHELL 3 SHELL 4 SHELL 5
THERMAL ENE 2.500034X00 2.500034X00 2.500034X00 2.500034X00 2.500034X00 2.500034X00 2.500034X00
KINETIC ENE 1.192516X00 1.192516X00 1.192516X00 1.192516X00 1.192516X00 1.192516X00 1.192516X00
TOTAL ENE 2.500034X00 2.500034X00 2.500034X00 2.500034X00 2.500034X00 2.500034X00 2.500034X00
VOL AVG PRES 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000
VOL AVG PHASE 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000
VOL AVG VOLUME 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000
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VOL AVG PHASE 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000
VOL AVG VOLUME 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000
LAYER VOLUME 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000
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TIMES AT CYCLE 21
TIME = 21.0000
DEGREES TIMESTEP (FILL PRINT) LAYERS PRINT GRAPHICS CHEMISTRY DIFFUSION CONDUCTION HETEROGENEOUS UNITS (m,kg,sec) LAYER 4 TOTAL LAYERS=29 TOTAL LAYERS=29 TOTAL LAYERS=29 TOTAL LAYERS=29

ADVECTIVE FREQUENCY COLUMNS (TITLE LAST CHECK) AT CYCLE #4
COLUMNS = 29 LAYERS=1 TOTAL LAYERS=29 ITERATIONS=29

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INTERFACEN | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000

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Appendix D
STANDARD TEST #2

AN INCOMPRESSIBLE SLUG BETWEEN ADIABATIC GASES

1 2 &CTRL
3 UTMIN=1.000, UTHMAX=1.000,
4 &END
5 &SHLINI
6 NSHELL=3, UVEL=0.000,
7 &END
8 &SHLOAT
9 &SHLOAT
10 LCELLS=5, RN=1.000, VN=1.00D-3, HATERS=1.010D, GAMMAS=1.400D,
11 RHOS=1.400D, RHOC5=0.000D, PRES=1.000D,
12 &END
13 &SHLOAT
14 LCELLS=5, RN=2.000, VN=1.00D-3, HATERS=2.020D, GAMMAS=0.500D,
15 RHOS=1.000D, RHOC5=1.402D, PRES=1.000D, RHES=1.40000000000000D2,
16 &END
17 &SHLOAT
18 LCELLS=5, RN=3.000, VN=0.000D, HATERS=3.010D, GAMMAS=1.400D,
19 RHOS=1.400D, RHOC5=0.000D, PRES=1.000D,
20 &END
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**TIMINGS AT CYCLE 21:**
- **TYPE:** 4
- **REMARKS:**
  - GRAPHICS: CHEMISTRY, DIFFUSION, CUMULATION, ADICING
  - CUMULATION: ADICING FREQ. COUNTERS (SINCE LAST CHECK) AT CYCLE 21
  - CALL 1: 20
  - ITERATIONS: 3

**LAYER BOUNDARY:**
- INTER 1: 1,000,000
- INTER 2: 2,000,000
- INTER 3: 3,000,000
- INTER 4: 4,000,000
- INTER 5: 5,000,000

**INTERFASE POSITION:**
- LAYER 1: 1,000,000
- LAYER 2: 2,000,000
- LAYER 3: 3,000,000
- LAYER 4: 4,000,000
- LAYER 5: 5,000,000

**INTERFASE VELOCITY:**
- LAYER 1: 1,000,000
- LAYER 2: 2,000,000
- LAYER 3: 3,000,000
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  - KINETIC ENG: 5.00000
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TIMINGS AT CYCLE 28: TIME = 2.30100 01
REZONE TIME STEP CELL PRINT LAYER PRINT GRAPHICS CHEMISTRY DIFFUSION CONDUCTION ADIC
0.00000 00 0.0000000 01 0.0000000 01 0.0000000 01 0.0000000 01
ADIC FREQUENCY COUNTERS (SINGLE LAST CHECK AT CYCLE 28)
NR. CALLS = 23 NR. TIME STEPS = 1 TOTAL NR. ITERATIONS = 3

LAYER BOUNDARY INTERFACE 1 INTERFACE 2 INTERFACE 3 INTERFACE 4 INTERFACE 5 INTERFACE 6
INTERFACE POSITION 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000
INTERFACE VELOCITY 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000
INTERFACE AREA 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000 1.00000000

TOTALS
THERMAL ENERGY 5.00000000 00 0.00000000 00 0.00000000 00 0.00000000 00 0.00000000 00 0.00000000 00
KINETIC ENERGY 5.00000000 00 5.00000000 00 5.00000000 00 5.00000000 00 5.00000000 00 5.00000000 00
TOTAL ENERGY 5.00000000 00 5.00000000 00 5.00000000 00 5.00000000 00 5.00000000 00 5.00000000 00
VOL AVG RPM 0.00000000 00 0.00000000 00 0.00000000 00 0.00000000 00 0.00000000 00 0.00000000 00
VOL AVG GAM 0.00000000 00 0.00000000 00 0.00000000 00 0.00000000 00 0.00000000 00 0.00000000 00
VOL AVG HML 0.00000000 00 0.00000000 00 0.00000000 00 0.00000000 00 0.00000000 00 0.00000000 00
LAYER VOLUME 3.00000000 00 3.00000000 00 3.00000000 00 3.00000000 00 3.00000000 00 3.00000000 00
LAYER MASS 1.00000000 00 1.00000000 00 1.00000000 00 1.00000000 00 1.00000000 00 1.00000000 00

SPELL 1 SHELL 2 SHELL 3 SHELL 4 SHELL 5
## After Step No. 25

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### Timings at Cycle 26

| Time    | 2.50905081 |

### Zone Timet Step Cell Print Latex Print Graphics Chemistry Diffusion Conductivity Acetic

### Acetic Frequency Count (Since Last Cycle) at Cycle 26

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| Thermal Ene | 3.00000000 | 0.00000000 | 3.00000000 | 0.00000000 | 3.00000000 | 0.00000000 |
| Kinet Ene    | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| Vol Avg Pre  | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 |
| Vol Avg Pre  | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 |
| Vol Avg Pre  | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 |
| Vol Avg Pre  | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 |
| Vol Avg Pre  | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 |

### Layer Mass

| Layer Mass | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 |

### Total

| Shell 1 | 2.00000000 | 2.00000000 | 2.00000000 | 2.00000000 | 2.00000000 | 2.00000000 |
| Shell 2 | 2.00000000 | 2.00000000 | 2.00000000 | 2.00000000 | 2.00000000 | 2.00000000 |
| Shell 3 | 2.00000000 | 2.00000000 | 2.00000000 | 2.00000000 | 2.00000000 | 2.00000000 |
| Shell 4 | 2.00000000 | 2.00000000 | 2.00000000 | 2.00000000 | 2.00000000 | 2.00000000 |
| Shell 5 | 2.00000000 | 2.00000000 | 2.00000000 | 2.00000000 | 2.00000000 | 2.00000000 |
Appendix E
STANDARD TEST #3

A LINUS SIMULATION

&CTRL
UTMIN=1.0D-7, DTMAX=1.0D-4,
EPSR=0.4500, EPSV=0.4500,
MAXSTP=1001, IPRINT=50,
ALPHA=2, MDAMP=10.
&END
&SHLINT
NLHILL=5, OVELL=0.000.
&END
&SHLDAT
LCELLS=3, RN=10.000, VN=0.000, MATEPS=1.016, GAMMAS=1.666700,
RHOS=1.200-3, PHCS=0.000, PRES=1.000.
&END
&SHLDAT
LCELLS=2, RN=15.000, VN=0.000, MATEPS=2.020, GAMMAS=2.0000000,
RHOS=1.200-3, PHCS=0.000, PRES=1.000.
&END
&SHLDAT
LCELLS=15, RN=30.000, VN=0.000, MATEPS=3.030, GAMMAS=0.500000,
PHWS=2.000, RHCS=1.000, PRES=1.000, PHOS=1.000000000000.
&END
&SHLDAT
LCELLS=5, RN=35.000, VN=0.000, MATEPS=4.040, GAMMAS=0.500000,
PHWS=1.500, RHCS=7.800, PRES=1.000, RHOS=7.800000000080.
&END
&SHLDAT
LCELLS=5, PHWS=1.000, VN=0.000, MATEPS=5.050, GAMMAS=1.400000,
RHOS=1.200-1, PHCS=0.000, PRES=1.000, PHWS=1.000.
&END

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```
NAMELIST /CTRL1 /FAILANTS ...  
  &CTRL1  
  MAXSTP   = 280, IP1 = 1 
  INPERS = 1, EPS1 = 0.5, EPS2 = 0.5, 
  GEOME=1, LZONE = F, ILCHE = F, L1TFF = F, L1TNC = F, 
  END 

NAMELIST /CTRL1 /UPAATS ...  
  &CTRL1  
  MAXSTP   = 280, IP1 = 1 
  INPERS = 1, EPS1 = 0.5, EPS2 = 0.5, 
  GEOME=1, LZONE = F, ILCHE = F, L1TFF = F, L1TNC = F, 
  END 

NAMELIST /SHLINS /FAILANTS ...  
  &SHLINS  
  NSHELLS = 1, KMN = 0.0000000000000000, 
  OVEL = 0.0000000000000000, 
  END 

NAMELIST /SHLINS /UPAATS ...  
  &SHLINS  
  NSHELLS = 1, KMN = 0.0000000000000000, 
  OVEL = 0.0000000000000000, 
  END 

NAMELIST /SHLDATS /DATA FOR LAYEL 1  
  &SHELDAT  
  LCELLs = 2, RH = 0.0000000000000000, 
  MIS = 0.0000000000000000, 
  END 

NAMELIST /SHLDATS /DATA FOR LAYEL 2  
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  MIS = 0.0000000000000000, 
  END 

NAMELIST /SHLDATS /DATA FOR LAYEL 3  
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  MIS = 0.0000000000000000, 
  END 

NAMELIST /SHLDATS /DATA FOR LAYEL 4  
  &SHELDAT  
  LCELLs = 5, RH = 0.0000000000000000, 
  MIS = 0.0000000000000000, 
  END 

NAMELIST /SHLDATS /DATA FOR LAYEL 5  
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  LCELLs = 5, RH = 0.0000000000000000, 
  MIS = 0.0000000000000000, 
  END 
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### Table: Model Parameters

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### Model Parameters Summary

- **Total Energy**: 1,379,908
- **Kinetic Energy**: 6,000,000
- **Thermal Energy**: 6,000,000
- **Vol Avg Rho**: 2,000,000
- **Vol Avg Pre**: 2,000,000
- **Vol Avg Gam**: 2,000,000
- **Vol Avg Ent**: 2,000,000
- **Vol Avg Phc**: 2,000,000
- **Layer Volume**: 5,026,582
- **Layer Mass**: 1,022,732

---

**Notes**: The table above represents the model parameters for a simulation, detailing various energy, density, and pressure values across different positions. The total energy and other parameters are summarized at the end of the table.
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### Timings at Cycle 201
- **Time** = 1.1571-03
- **Total Wait Time** = 1.1571-03

### ADRC Frequency Counters (Since Last Check)
- **Total Wait Time** = 1.1571-03

### Layer Boundary
- **Total Wait Time** = 1.1571-03

### Interface Position
- **Total Wait Time** = 1.1571-03
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<th>ENERGY</th>
<th>PRESSURE</th>
<th>CELL VOLUME</th>
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After step 473, the time is 0.2463450792. The final step is 473.

Total iterations = 473.

At time 0.2463450792, the cell volume is 1.0000000000. The total area is 1.0000000000.

The final step is 473. The final cell volume is 1.0000000000. The total area is 1.0000000000.

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The final step is 473. The final cell volume is 1.0000000000. The total area is 1.0000000000.
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