THE EFFECT OF CARRIER PHASE UNCERTAINTY
IN CORRELATION SYSTEMS USING HETERODYNING FOR DETECTION

by

William Lian
and
C. S. Clay

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ABSTRACT

The merit of employing signal bandshifting to transmit and detect a message for correlation purposes has been studied. It was found that the phase uncertainty of the returned signal may cause either a disappearance or a phase inversion of the supposedly detected message due to the process of heterodyning. This means the detectability of the correlation system is impaired. The analytic results have been verified by experiments.
One common technique to measure distance is to correlate the signal returned from the target with the transmitted signal, suitably delayed. If modulation is used, which is the case in most systems, the usual engineering requirement for the radiating source is to preserve the modulating signal bandwidth $\Delta \omega$ at a carrier frequency $\omega_0$ for a given power requirement. On reception, this band of information $\Delta \omega$ must be converted to some quantity, say, voltage, of the same frequency range as the original signal. This signal can be in any form: periodic, random, or coded. The modulation to this sinusoidal carrier can be of any type, i.e., basically AM or FM. Of whichever type, the modulation must be extracted from the carrier before correlation with the delayed signals.

One convenient way to extract the modulation is by heterodyning the returned signal with the carrier frequency. Often, the information to be carried is in a pulse code form to gate the carrier rather than in the strict form of modulation. So long as the signal is to be heterodyned, the problems anticipated will be similar to this.

Basically, when a signal is bandshifted from one center frequency to another, the quality of the bandwidth is different although superficially the amount of frequency variation may appear to be the same. In communication systems where only a reproduction of the general form of the original
signal is desired and where the center frequencies are much higher than the signal frequencies, the shifting of the band from one to another, plus the further aid of partial envelope detection, does not present any serious problems as far as the ultimate result is concerned. However, if the center frequency of a received signal is low relative to the bandwidth carried, any shifting of this band to an even lower region like the ultimate detected output information to be correlated would make the meaning of this band of information somewhat different from what it was. This is significant to most underwater sound experiments. Intuitively one can feel that, for instance, when a received signal in the range of 500-530 cps is beaten down to 0-30 cps by a local frequency, a slight distortion in the carriers may mean an entirely different signal in the low-frequency range. A detailed analysis is given in the latter part of this report. At this point let us simply analyze the waveforms by a common sense method, and let us take a simple case for example.

A carrier frequency of $X$ cps is gated by a $T$ sec pulse and ocean propagated. The returned signal is decoded by means of heterodyning it with the generated carrier. When the two are perfectly in phase, a zero beat results with a positive dc level lasting $T$ sec. Thus, if the heterodyned output is fed into a low pass filter and an amplifier
clipper, the original gate pulse may be reproduced.

However, if the returned signal is delayed by \( \pi \) rad with respect to the local signal, a zero frequency beat with a negative dc level will result, and this means that the detected pulse becomes a negative one. This could present a serious problem to signal processing systems dealing with coded signals and employing digital correlation detection techniques. The worst situation will occur at times when the returned signal is at quadrature phase angle with the beat signal, in which case the result of heterodyning would be at absolute zero level for a zero frequency beat. This means our information has disappeared.

To sum up, phase distortions in the returned signal would definitely create situations of having the code information either phase flipped or killed. This certainly endangers the detectability of the correlation function. A little simple mathematics may help, at least in this case, to clarify things.

Let the returned signal be

\[
e_R(t) = A \cos \left( \omega t - \frac{2(2\pi)}{\lambda} + \phi_0 \right)
\]

\[
= A \cos \left( \omega t - \frac{2\xi}{V} + \phi_0 \right),
\]
where \( x \) is single distance, \( V \) is the wave velocity, and \( \phi_0 \) is the initial phase angle. The phase difference between the returned and transmitted signals is

\[
\Delta \phi = \frac{2\omega}{V} ax
\]

At \( \omega = \omega_0 \) = constant, obviously \( \Delta \phi \) varies with \( \Delta x \). However, if \( \omega = \omega_0 + \Delta \omega \), then the phase difference varies with frequency. Taking differentials we have

\[
d\Delta \phi = \frac{2\alpha}{V} d\omega = \frac{\pi \Delta x}{4} df
\]

This means when the frequency varies by the amount of \( \frac{V}{\Delta x} \), the phase varies by \( \pi/2 \) and at \( df = \frac{V}{4\Delta x} \), the phase varies by \( \pi \).

Conversely, when \( \Delta x = \frac{V}{4\Delta f} \), we have a complete reversal in the returned signal, i.e., for

\[
V = 5000 \text{ ft/sec}, \quad \Delta f = 10 \text{ cps}
\]

\[
\Delta x = \frac{5000}{4 \times 10} = 125 \text{ ft}
\]

This indicates that the returned signal will change phase for every 125 ft of distance variation. If a wider band is used,
the returned signal will change phase more rapidly with respect to distance.

The process of heterodyning may include frequency multiplication and bandpass filtering. Let

\[ e(t) = A g(t) \cos (\omega_0 t + \phi_0) \]

be the transmitting signal in which the message \( g(t) \) is carried onto \( \omega_0 \), with \( \phi_0 \) as an initial phase. The returned signal

\[ e_R(t) = B g(t) \cos (\omega_0 t + \phi_0 + \phi_R) \]

is shifted by an angle \( \phi_R \). To heterodyne, multiply this by the carrier synchronized with the transmitting signal to get

\[ e_H(t) = [e_R(t)] \cos (\omega_0 t + \phi_0) \]

\[ = [\frac{B}{2} g(t) \cos(\omega_0 t + \phi_0 + \phi_R)] \cos(\omega_0 t + \phi_0) , \]

and the result is

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\[ e_H(t) = \frac{B}{2} [g(t) \cos \phi_R + g(t) \cos (2\omega_o t + 2\phi_o + \phi_R)] . \]

Extract by a low pass filter to get

\[ \text{Low} [e_H(t)] = \frac{B}{2} g(t) \cos \phi_R . \]

Obviously, when \( \phi_R = \frac{\pi}{2} \),

\[ \text{Low} [e_H(t)] = 0 . \]

and when \( \phi_R = \pi \),

\[ \text{Low} [e_H(t)] = -\frac{B}{2} g(t) . \]

This indicates, as stated previously, that the amplitude and sign of the heterodyned output signal are uncertain due to the effect of the phase uncertainty of the returned signal. Any attempt at bandshifting signals in this manner for correlation purposes is likely to face these difficulties of establishing a steady returned signal, and this is not something to be corrected in the signal correlation process. Therefore, it is felt that there is no need to bring any mathematical analysis
into the picture in order to investigate the outcome of the correlation function at this time.

Experimentally one may assume that a code of a given form has a positive phase carrier for the positive portions of the code and also a negative phase carrier for the negative portions of the code, as shown in Fig. 1. This signal is transmitted and then returned from the target. To decode, the local carrier is used to beat with the returned signal. Assuming no interference and distortion in propagation, a perfect reproduction of the original pulse code is expected if the two multiplying signals are locked in phase.

In obtaining the correlation function, one convenient way is to steer the returned code through a digital delay line (a shift register, for instance). If the delay line is properly weighted with respect to the code form, the outputs of the delay line at the weighted terminals can be summed up by means of resistive adding. Thus, for a perfect returned code, the code form should match the weighting function at a specific time delay, and the output of the resistive adding will be a maximum. Side lobes will be present, and they are determined by the code form and the weighting. An output from a perfect system is shown in Fig. 2. However, if the returned signal of Fig. 1 is delayed, say, by $\pi$ rad, the code will completely flip phase and the output of the resistive adding will be
meaningless. Of course, in principle there are means to obtain the correct code form from heterodyning detection, but they all amount to the use of an automatic phase-corrected beat frequency locked with the returned signal. This may mean the requirement of a large addition of equipment. Other than this, a partial envelope detection may be used if one is willing to accept the ambiguities introduced.

An experiment has been carried out in Long Island Sound at known basement structure areas. A simple code of one pulse was used. Employing heterodyning techniques for detection, the resultant correlation function shows great uncertainty in the intensity modulated printed output, as reprinted in Fig. 3A. The reflections are almost unreadable. The same course has been measured without using heterodyning, and the result shows clear bottom structure reflections, as given in Fig. 3B. This may be used to identify the phase changes and signal disappearances at various times in Fig. 3A where heterodyning was used.
FIG. 1

Pulse code +

Gated signal carrier

Beat frequency

Heterodyned output

FIG. 2

Detected code

Digital delay line

Weighted output through resistive adding
Fig. 3A  Heterodyning Detection

Fig. 3B  Signal Detection

Baseline Reflections Taken in Long Island Sound