PREDICTED CRACK REPAIR COSTS FOR AIRCRAFT STRUCTURES

University of Dayton Research Institute
Dayton, Ohio 45469

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Copies of this report should not be returned unless return is required by security considerations, contractual obligations, or notice on a specific document.
This report presents the results of a study designed to (1) prepare a computer program for use in predicting expected repair costs of the cracks which develop during the operational usage of a structure; (2) to provide a document which describes the use of the computer program and guides a potential user in the specification of the required input; and (3) to use data which is reasonably representative of Air Force experience as input for use in.
determining the sensitivity of expected maintenance costs to variations in input. Results are provided which compare expected costs for changes in inspection intervals, quality of inspection, quality of repair, operational usage, and equivalent initial flaw size distributions.
FOREWORD

This report was prepared by the Aerospace Mechanics Division of the University of Dayton Research Institute as a requirement of Air Force Contract F33615-77-C-0800. The work was administered by the Structures Division, Air Frame Directorate, Deputy for Engineering, Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio. Mr. Stephen Guilfoos (ASD/ENFSL) was the Air Force Project Monitor. The effort covered by this report was conducted between June 1977 and September 1978.

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SECTION I
INTRODUCTION

A significant percentage of the life cycle cost of a fleet of aircraft is due to the cost required to maintain the aircraft for the safe performance of its required mission. Let structural cost of ownership denote that portion of the total life cycle cost which is attributable to the inspections and repair of the primary aircraft structure. This structural cost of ownership depends on the loads environment that will be encountered by the individual aircraft of the fleet as well as damage tolerance properties that resulted from the design and manufacturing processes.

The routine use of an operational aircraft results in the structure being subjected to a large number of significant load cycles which occur randomly in time and which have randomly varying magnitudes. These repeating load cycles can eventually weaken the structure to the extent that an applied load which is within the original design capability will cause failure of the structure. The fatigue damage resulting from the applied load cycles is currently being characterized in terms of the growth of cracks at the critical locations in the structure. The crack growth depends on the material and geometry of the structural element, the crack length at the time of a stress cycle, and the magnitude and order of the stress cycles imposed by the random loads on the aircraft. Thus, given a structural detail with a crack of specified initial length and the stress history which will be imposed on the detail, crack length as a function of flight time can be calculated. This function plays a key role in the decision making processes employed to insure the structural integrity of aircraft.
Structural reliability analyses have been derived which present methods of calculating the probability the structure will not fail for T flight hours in an operational environment. (References 1 through 5). These analyses take into account some or all of the following factors: time to crack initiation; initial flaw size distribution; random load sequences representative of the real environment; random effect of inspections; repair quality; and material variability. However, the practical application of these methods require extensive data bases which, in general, are not available. Thus, although analytical models are available for calculating structural reliability under different inspections, they cannot be applied.

To assure flight safety of structure, the Air Force has adopted an inspection interval which is based on predicted crack growth of the largest flaw in a critical location which could pass the quality control inspection. To allow for variability in crack growth and inspection reliability, the structure must be inspected within half the time interval required for the maximum flaw to grow to an unstable length. To date, this flight safety criteria has been the determining factor in scheduling structural inspections.

Now during the periodic inspections, all cracks which are detected are repaired and, in general, the cost of repair depends on the size of the crack. Thus, the structural cost of ownership of a fleet of aircraft is the cost of all inspections and repairs plus the cost of any aircraft which may be lost due to catastrophic failure. Since the number of cracks that are detected and repaired at an inspection is a random variable (depending on both the distribution of crack sizes at the time of the inspection and the probability of detecting the crack for the inspection method being employed), the structural cost of ownership is also a random variable and should be described in statistical terms.

This report presents the results of a study to estimate the structural cost of ownership due to the detection and repair of cracks. A statistical model formulated by Yang (Reference 6)
constituted the essence of the approach. This model accounted for an initial flaw size distribution, flaw growth by a deterministic equation, probability of flaw detection as a function of flaw size, renewal of repaired flaws, and costs of inspection and repair. The basic model was generalized to permit variability in flaw growth due to flight to flight variation of usage (Reference 7). Since the model simulates the growth and repair of a distribution of cracks in a structural detail, the output of the model is expressed in statistical terms. The results cannot be used to infer crack size or cost of a repair for a specific airplane tail number.

Section II presents a summary of the method of calculating the crack size distribution at each inspection interval and the expected cost of inspection and repair. Also included is a discussion on the particular method selected for modeling crack growth and for accounting for the flight to flight variation of usage. The program for calculating predicted costs is presented in Section III with a description of methods for estimating the statistical parameters of the distributions of input parameters. Examples of the use of the program are presented in Section IV. The examples are actually a series of sensitivity studies to investigate the effect of variations in input parameters on the expected cost of ownership.
SECTION II
CRACK REPAIR COST MODEL

The objective of the structural cost of ownership model is to estimate the fleet costs associated with the inspection and repair of potential cracks at a critical location on the airframe. Since total maintenance costs can be obtained by summing the estimated costs for the individual locations, this study has been restricted to a single location. The following paragraphs present a summary of the estimation method. The basic approach is due to Yang (Reference 6). However, the method of calculating crack growth and the method of accounting for the effect of flight to flight variability are different from Yang's approach and a discussion of these features is also presented.

2.1 MATHEMATICAL FORMULATION

Assume that a fleet of aircraft contains $N$ structural details which will be subjected to statistically equivalent stress environments. There may be one structural detail (critical location) on each aircraft or there may be multiple details on each aircraft. The important elements are that the properties which determine crack growth are equivalent for all of the details. Since no two aircraft will be flown the same, the actual stress histories endured by the aircraft will be different. However, it is assumed that stress environments are statistically equivalent in their effect on crack growth.

As the individual aircraft perform their operational missions, cracks will initiate and grow at the critical point. Inspections will be performed at increments of $T$ flights (or an equivalent number of flight hours) and, at each inspection, all detected cracks will be repaired. The cost of repair will be a function of the size of the crack and it is assumed that there are 4 ranges of crack size with a constant repair cost in each range. It is assumed that the initial quality of the details can be characterized by an equivalent initial flaw size distribution which depends on the initial manufacturing and quality control processes.
(References 8 and 9). Similarly, the repaired details are characterized by an equivalent repaired flaw size distribution.

At the ith inspection cycle (after \(i T\) flights), the cost associated with the inspection and repair of the detail under consideration is given by

\[
C_i = I_i + \sum_{j=1}^{4} n(i,j) C(i,j)
\]

where

- \(I_i = \) cost of ith inspection
- \(n(i,j) = \) number of level \(j\) cracks discovered and repaired at the ith inspection
- \(c(i,j) = \) cost of repairing one level \(j\) crack at ith inspection

Since \(n(i,j)\) depends on the distribution of crack sizes immediately prior to the ith inspection and the probability of crack detection for the inspection method, the cost of the maintenance action is a random variable and must be described in statistical terms.

To model the random variable \(n(i,j)\), assume that crack growth and the probability of crack detection are statistically independent across structural details. Let \(p(i,j)\) denote the probability of detecting (and repairing) a level \(j\) crack at the ith inspection. Then, \(n(i,j)\) will have a binomial distribution with parameters \((N, p(i,j))\). The expected value and variance of the number of detected \(j\) level cracks at inspection \(i\) are given by

\[
\bar{n}(i,j) = N p(i,j)
\]

\[
\sigma^2_n(i,j) = N p(i,j) [1-p(i,j)]
\]

Thus, the expected value and variance of the maintenance costs at the ith inspection are given by
\[ E(C_i) = I_i + \sum_{j=1}^{4} C(i,j) \cdot N \cdot p(i,j) \] (3)

\[ S^2(C_i) = \sum_{j=1}^{4} C^2(i,j) N p(i,j) [1-p(i,j)] \]

and the expected value and variance of the total maintenance costs through the ith inspection (for all i T flights) is given by

\[ E(C_{TOT}(i)) = \sum_{k=1}^{i} E(C_k) \]

\[ \text{var}(C_{TOT}(i)) = \sum_{k=1}^{i} S^2(C_k) \] (4)

If \( N p(i,j) \) and \( N (1-p(i,j)) \) are both greater than 5, then the normal distribution may be used to approximate the binomial and the distribution of costs at the ith inspection and for the total of all inspections can be characterized by the normal distribution with parameters as specified in (3) or (4).

The calculation of \( p(i,j) \) depends on the distribution of crack sizes at the ith inspection and the probability of detecting a crack as a function of crack size for the particular inspection technique being used. Let \( f_i^-(a) \) represent the density function of crack sizes immediately before the ith inspection. Let \( H(a) \) denote the probability of detecting a crack of length \( a \). Then

\[ p(i,j) = \int_{a_{j-1}}^{a_j} H(y) f_i^-(y) \, dy \] (5)

where the \( a_j, j = 0,1,2,3,4 \) define the crack size levels and \( a_0 = 0, a_4 = \infty \).

Immediately after the repair of the cracks the density function of the crack sizes is a mixture of the density function of the cracks which were not found and the equivalent repaired crack size density function. If \( f_i^+(a) \) represents the density function of crack sizes immediately after repair, then

\[ f_i^+(a) = P(i) f_R(a) + [1-H(a)] f_i^-(a) \] (6)
where \( f_R(a) \) = equivalent repaired crack size density function

and

\[ P(i) = \text{proportion of all cracks which were repaired} \]

\[ = \sum_{j=1}^{4} P(i,j) \]

\[ = \int_{0}^{\infty} H(y) f_i^-(y) \, dy \]

Using equations (4) through (7), the expected value and variance of the inspection and repair costs can be obtained at each inspection period and the distribution of the crack sizes immediately after the inspection can be calculated. The only remaining element which must be modeled is the density function of the crack sizes immediately before the inspection. This distribution is obtained iteratively by assuming cracks grow in accordance with the equation

\[ a(T) = a_i e^{b x(T)} \]

where \( a_i = \text{crack length after ith inspection and repair} \)

\( b = \text{constant depending on material and detail geometry} \)

\( x(T) = \text{normally distributed random variable with parameters} \)

\( \mu_T \text{ and } \sigma_T \) which accounts for the random stress environment encountered by a particular detail during the \( T \) flights.

This crack growth model is discussed in detail in paragraph 2.2.

Given the distribution of flaw sizes immediately following an inspection and repair, \( f_i^+(a) \), and the crack growth equation, (8), the density function of flaw sizes immediately before the next inspection (after \( T \) flights) can be obtained as follows. First the cumulative distribution of flaw sizes after \( T \) flights
is calculated

\[ F_{i+1}(a) = P\{a_{i+1} \leq a\} \]

\[ = P\{a_i \exp(-b \cdot x) \leq a\} \]

\[ = \int_{-\infty}^{\infty} \left[ \int_{0}^{a \exp(-b \cdot x)} f_i^+(y) dy \right] g(x) \, dx \tag{9} \]

where \( g(x) \) is the normal density function

\[ g(x) = \frac{1}{\sqrt{2\pi} \sigma_T} \exp\left[-\frac{(x-\mu_T)^2}{2\sigma_T^2}\right] \tag{10} \]

The density function is then obtained as

\[ f_{i+1}(a) = \frac{d}{da} F_{i+1}(a) \]

\[ = \int_{-\infty}^{\infty} \exp(-b \cdot x) f_i^+(a \exp(-b \cdot x)) g(x) \, dx \tag{11} \]

Further, letting

\[ z = x - \mu_T / \sigma_T \]

then, for all practical purposes

\[ f_{i+1}(a) = \int_{-3}^{3} \exp(-b \cdot (\mu_T + z \sigma_T)) f_i^+(a \exp(-b \cdot (\mu_T + z \sigma_T))) g(z) \, dz \tag{12} \]

with

\[ g(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \]

The above mathematical model for simulating crack growth, inspection, and cost of repair has been programmed to provide output at each of the pre-specified inspection cycles through a pre-defined lifetime. The simulation process starts with an equivalent initial crack size distribution from which the distribution
of flaw sizes immediately before the first inspection is calculated, equation (12). The costs of inspection and repair for this first inspection cycle are calculated using equations (5) and (3). (At subsequent inspections, total costs are also calculated using equation (4).) Equations (7) and (6) then provide the density function of flaw sizes immediately after the first inspection and this density is used to initiate the second cycle of simulated operation. The process is repeated for a specified number of cycles (aircraft life). This process is described in greater detail in Section III, which presents a description of the computer program and its use.

2.2 DISCUSSION

The mathematical model presented in the previous paragraph depends on basic assumptions regarding the process of crack growth and repair during operational usage. The assumption regarding independence of crack detection between details is a probabilistic assumption which is non-controversial and permits characterization of the cost of repairs in terms of the binomial distribution. A second class of assumptions are those which relate to the equivalent distribution of crack sizes. This method of characterizing initial and repair quality has met with general acceptance in the Air Force and the only difficulty in the application is the specification of the proper distributional family and the values of the parameters of the distribution. Similarly, the concept of the probability of detection of a crack as a function of crack size is well accepted but a specific distribution form and specific parameter values for an NDI method may not be known. The mathematics of the crack repair cost model, however, do not depend on the specific family or its parameters for either of the latter types of distributions.

The method of simulating crack growth in equation (8) is, however, a different concept which requires discussion. In the following two paragraphs, a heuristic argument is presented for the use of this model.
2.2.1 Crack Growth

The crack growth model of Yang is based on the assumption that incremental crack growth during the ith flight is given by

$$\Delta a_i = b a_{i-1}^c, \ c \neq 1$$

(13)

After t flights, a crack of initial length $a_0$ would be given by

$$a = a_0 [1 + (1-c) b t a_0^{c-1}]^{1/(1-c)}$$

(14)

In particular, Yang states that for aluminum, $c = 2$, so that

$$a = \frac{a_0}{1-b t a_0^2}$$

(15)

This model is completely deterministic in that a given initial crack length completely determines the history of the growth of the crack.

Now crack growth tests from simulated operational usage have indicated that a model of the form

$$a = a_0 \exp (b t)$$

(16)

also fits the data reasonably well for values of t less than the design life. As an example, Figure 1 presents crack length data (with a least squares fit of the exponential model superimposed) from crack growth tests during the F-5 damage tolerance assessment, Reference (10). This example is typical of the early stages of crack growth where the growth is very slow. Thus, there is evidence to indicate that the exponential model may provide a reasonable (if not good) approximation for crack length as a function of time. This is particularly true for the earlier stages of crack growth (up to half crack life) which are the periods of major interest in the calculation of the expected cost of crack repair.

The exponential model results from the assumption that the crack growth during flight i is directly proportional to
Figure 1. Crack Length Data From Tests of F-5 Structure.
the crack length at the start of the flight:

$$\Delta a_i = b^{a_{i-1}}$$  \hspace{1cm} (17)

Thus, the exponential model is the special case of equation (13) with $c = 1$. Given a crack-length versus number-flights data set for a detail of interest under the anticipated loading history, the parameter of the model can easily be calculated, e.g., by a least squares estimate.

To consider the crack growth model from another approach, assume that

$$\frac{da}{dn} = b (\Delta K)^m$$  \hspace{1cm} (18)

where

$$\Delta K = \beta (a) \sqrt{\pi a} \Delta \sigma$$  \hspace{1cm} (19)

$$\beta (a) = \text{factor dependent on crack length and geometry.}$$

This crack geometry model has often fit empirical data in the mid ranges of crack growth and provides an acceptable fracture mechanics model for fleet modeling where actual stress time histories in each aircraft will not be available (Reference 11). For aluminum, values of $m$ in the range of 3 to 4 are commonly observed.

During flight $i$ an aircraft will experience $n_i$ load cycles and the crack will grow an increment $\Delta a_i$ where

$$\Delta a_i = \sum_{j=1}^{n_i} \Delta a_{i,j}$$  \hspace{1cm} (20)

Assuming negligible error results if crack length at start of flight is used to calculate all increments during the flight, substituting from (18) and (19) fields

$$\Delta a_i = \sum_{j=1}^{n_i} b \left[ \sqrt{\pi} \beta(a_i) \sqrt{a_i} \Delta \sigma_j \right]^m$$

$$= b^{m/2} \left[ \beta^m (a_i) a_i^{m/2} \right] \sum_{j=1}^{n_i} (\Delta \sigma_j)^m$$  \hspace{1cm} (21)
Since the exponential model has been observed to fit crack length data in some applications, it may be reasonable to assume that for the $\Delta a$ ranges of interest that

$$b^m (a_i^{1/m}) a_i^{m/2} = a.$$  

Thus, there exists a constant $b$ such that the crack growth during the $i$th flight is given by

$$\Delta a_i = b a X_i$$

where

$$X_i = \frac{n_i}{\sum_{j=1}^{n_i} (\Delta a_j)^m}$$

Since the number and magnitude of the stress cycles to be encountered in a randomly selected flight are not known, $X_i$ can be considered as a random variable which accounts for the stress cycles of the flight. If the crack growth tests for the structural detail were based on a random spectrum which meets some baseline usage, the exponential crack growth curves could be considered as resulting from the statistical model

$$E(\Delta a_i) = b E(X_i) a.$$  

The statistical approach introduced by equation (23) will be further developed in the next paragraph.

2.2.2 Flight by Flight Variability

In the previous paragraph, a heuristic argument was presented for modeling the incremental crack growth during flight $i$ by the equation

$$\Delta a_i = b a_{i-1} X_i$$

where $b$ is a constant determined from specimen test, $a_{i-1}$ is the crack length after $i-1$ flights and $X_i$, defined by equation (23), is a random variable which characterizes the non-deterministic stress history to be encountered during the $i$th flight. Using this relation, it is easily shown that the crack length after $t$
flights is given by

\[ a_t = a_0 \frac{t}{\prod_{i=1}^{t}(1+b X_i)} \]  

(25)

from whence

\[ \ln a_t - \ln a_0 = \sum_{i=1}^{t} \ln (1+b X_i) \]

Since \( b X_i = \frac{\Delta a_i}{a_{i-1}} \ll 1 \) over the crack range of interest, it can be shown that

\[ \ln a_t - \ln a_0 = b \sum_{i=1}^{t} X_i \]

or

\[ a_t = a_0 \exp \left( b \sum_{i=1}^{t} X_i \right) \]

\[ = a_0 e^{bX(t)} \]

(26)

where

\[ X(t) = \sum_{i=1}^{t} X_i. \]

By the Central Limit Theorem, when \( t \) is large (>50), \( X(t) \) can be assumed to have a normal distribution with mean and variance given by

\[ \mu_t = t E(X_i) \]

\[ \sigma_t = t \text{var}(X_i) \]  

(27)

Thus, it can be assumed that the random usage factor of the crack growth model has a normal distribution as specified in equation (10).

Now the crack growth rate parameter, \( b \), will be estimated from a baseline spectrum which represents the anticipated stress environment. Rather than calculate \( E(X_i) \) for this spectrum to obtain a growth rate parameter that is independent of the baseline spectrum, a single growth rate parameter which includes the baseline environment can be calculated and the flight by flight variation can be expressed in terms of the coefficient of variation. Thus, if \( b \) is
calculated from the baseline spectrum and the aircraft of the fleet are flying this spectrum, then

\[ u_t = t \]

\[ \sigma_t = \sqrt{E} \cdot \frac{\sigma_{X_1}}{E(X_1)} \]  

In the absence of specific stress history data, neither \( E(X_1) \) or \( \sigma_{X_1} \) can be known. However, coefficients of variation of the Miner's damage/flight have been calculated in a few cases and these values provide an indication of the relative magnitude of the variability of crack growth per flight. These values range from about unity (\( \sigma = \mu \)) for specific well defined missions to approximately 4 (\( \sigma = 4\mu \)) for composites of several missions of a fighter/attack aircraft. Such large coefficients of variations would also be expected for the random variable \( X_1 \), equation (23), since both metrics are a function of the number and magnitude of the same random number of random stress cycles.

Since the crack growth model is directly dependent upon \( E(X_1) \), equation (27) suggests a simple test of equality of sample means as a method for determining if operational usage is different from that of the baseline spectrum. Sample size considerations could also be expressed in terms of number of flights to be monitored to obtain a pre-determined precision in the value of \( E(X_1) \). These statistical considerations have been considered in Reference (12) which used average Miners damage/flight as the monitoring parameter. However, the statistical considerations would transfer directly to the new parameter \( E(X_1) \).

Note that the model for accounting for flight to flight variability in crack growth during operational usage depends on the assumption that the stress environment encountered by a particular aircraft during a flight is a random selection of all such flights of all aircraft of the fleet. If a particular aircraft flies a spectrum that is biased toward the severe side, it will, of course, experience crack growth greater than that predicted by the
model. Individual aircraft tracking is required to account for specific aircraft usage.
SECTION III
APPLICATION OF MODEL

A computer program was written to calculate the predicted crack repair costs (PCRC) in accordance with the model expressed in the previous paragraphs. Figure 2 presents a schematic of the calculations and indicates the iterative nature of the computation. The issue specifies a set of input data related to flaw sizes, cost of inspections and repairs, and number of inspection cycles. At each inspection, the program outputs the percentage of cracks in each repair interval which was identified by the inspection process, the total percentage of cracks found, the expected value and standard deviation of the inspection and repair cycles, and the cumulative cost totals. At the users option, the program will also print out the density functions of the crack sizes immediately before and after the inspections.

In writing the program, specific assumptions regarding the equational form of distributions had to be made. In the following paragraphs, all of the input parameters required by the program are discussed. The input parameters are considered in three groupings: (1) flaw size parameters, (2) inspection and repair parameters, and (3) program parameters. The last paragraph of this section presents the program and its user's manual.

3.1 FLAW SIZE PARAMETERS

The input parameters related to flaw size are the equivalent initial and repaired flaw size distributions and the crack growth parameters. Each of these is discussed in the following.

3.1.1 Initial Flaw Size Distributions

The equivalent initial flaw size distribution is a concept used to quantify the concept of manufacturing quality. Direct measurements of initial flaw sizes cannot be made so that the distributional family that is used to represent initial quality must be inferred from other data. One method is to measure the cracks in a structural item after exposure to a stress environment...
Figure 2. Flow Schematic for Predicting Crack Repair Costs.
and using that stress environment to back calculate the "initial" crack size at time zero. Reference 8 presents the distributions obtained from A-7D and F-4 C/D aircraft. The A-7D initial hole quality data is represented reasonably well by the Weibull family of distributions. The F-4 C/D data deviate from the Weibull family particularly for the very small crack sizes. (The F-4 C/D have been fit by a Johnson $S_u$ model but the use of this particular 4 parameter family cannot be justified for any other data set. Further, there is some question regarding the homogeneity of the data points of the F-4 C/D data set (Reference 7).)

Since the two parameter Weibull family of distributions provides four different slopes of the density function, since it is a commonly accepted and used distribution, since it is analytically easy to manipulate, and since it is a reasonable model for at least one data set (the A-7D), this family was programmed in the model for the specification of equivalent initial hole quality. If the user prefers another two-parameter distribution a new FORTRAN equation must be entered for the FUNCTION FO(X) subroutine with the parameters defined as FOALPH and FOBETA. If a distribution with three or more parameters is desired, additional changes to the input logic will be required.

The density function and cumulative distribution function of the two-parameter Weibull distribution are given by

\[ f(y) = \frac{\alpha}{\beta} \left(\frac{y}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{y}{\beta}\right)^\alpha\right] \]

\[ F(y) = 1 - \exp\left[-\left(\frac{y}{\beta}\right)^\alpha\right] \]

The shape of the distribution is determined from $\alpha$ while $\beta$ is a scale parameter and 63 percent of the distribution is always less than $\beta$ (i.e., $F(\beta) = 0.63$). $\beta$ is also known as the characteristic value. Figure 3 presents plots of the Weibull density functions for 5 values of the shape parameter. For each value of $\alpha$, the scale parameter, $\beta$, was determined such that

\[ F(0.050) = 0.999 \]
Figure 3. Weibull Density Functions With 99.9 percent Less Than 0.050 in.
This value was selected since it is representative of the type of equivalent initial quality that is required by Mil Std 1530A. Note that for $\alpha<1$, the density function is asymptotic to the vertical axis and a high percentage of the cracks is very small. For $\alpha=1$, the exponential distribution results. For $\alpha>1$, the density function will have 1 or 2 points of inflection and as $\alpha$ increases, the scatter about the median decreases and the median increases. The data from the A-7D and F-4 C/D initial hole quality studies result in values of $\alpha$ between 2 and 3.

Given a set of equivalent initial hole quality data which is applicable to the structural detail of interest, the Weibull parameters can be estimated through graphical (Weibull probability paper) or analytical (maximum likelihood) methods. Given the data points the appropriateness of the Weibull family could also be tested. In the absence of data, however, the values of the parameters must be inferred from other information. For example, if a quality control process has been shown to be effective at detecting flaws of a given magnitude with a high probability, this information can be interpreted as a point on the cumulative function.

In general, either $\alpha$ or $\beta$ can be calculated from a specified value of the other and from one specified point on the cumulative distribution functions. Given $\alpha$ and the point $F(Y_p) = P$, then

$$\beta = \frac{Y_p}{[-\ln(1-P)]^{1/\alpha}}$$ \hspace{1cm} (30)

For example, if it is assumed that the shape parameter, $\alpha$, is two in accordance with the A-7D data and it is shown that 99.9 percent of all initial flaws are less than 50 mills, i.e., flaws greater than 50 mills are eliminated by quality control, then

$$F(0.050) = 0.999$$
and

\[
\beta = \frac{0.050}{\sqrt{-\ln(1-0.999)}}
\]

\[
= 0.019 \text{ inches}
\]

Given \( \beta \) and the point \( F(y_p) = P \), then

\[
\alpha = \frac{\ln [\ln (1-p)]}{\ln y_p - \ln \beta}
\]  \hspace{1cm} (31)

For example, if assumed as before that 99.9 percent of all initial flaws are less than 50 mills and that the characteristic value of an initial flaw size is 20 mills, then

\[
\alpha = \frac{\ln [\ln (1-0.999)]}{\ln (0.050)-\ln (0.020)}
\]

\[
= 2.11
\]

If two points on the cumulative distribution function are specified \((y_1, P_1)\) and \((y_2, P_2)\) then

\[
\beta = \exp\left[\frac{\ln y_1 - \ln y_2}{A - 1}\right]
\] \hspace{1cm} (32)

where

\[
A = \frac{\ln [\ln (1-P_2)]}{\ln [\ln (1-P_1)]}
\]

and

\[
\alpha = \frac{\ln [\ln (1-P_1)]}{\ln y_1 - \ln \beta}
\] \hspace{1cm} (33)

For example, if 99.9 percent of initial flaws are less than 50 mills
and 50 percent of the initial flaws are less than 20 mills, then

\[ A = \frac{\ln(-\ln(0.001))}{\ln(-\ln(0.50))} = 5.27306 \]

\[ \beta = \frac{\frac{A \ln(0.020) - \ln(0.050)}{A^2}}{A-1} = 0.0231 \]

\[ \alpha = \frac{\ln(-\ln(1-0.50))}{\ln(0.020) - \ln(0.0231)} = 2.51 \]

In the program, the shape and scale parameters of the initial crack quality distribution are denoted by FOALPH and FOBETA, respectively.

3.1.2 Crack Growth Parameters

There are four parameters in the program which are identified as crack growth parameters. These are (1) the number of flights between inspections, UT; (2) the average crack growth rate constant for the baseline usage, UB; (3) the bias factor which reflects change in usage from baseline, UM; and (4) the flight by flight variability factor which is the coefficient of variation of the flight by flight measure of the stress environment, US.

The number of flights between inspections is entirely at the discretion of the user. If inspection intervals are planned in terms of flight hours rather than number of flights, the average flight length provides an acceptable conversion to number of flights. This parameter is considered as a crack growth parameter since it is required in the statistical distribution which describes effect of flight to flight variability on crack growth, equations (28) and (12).
The crack growth rate parameter $b$, or $UB$ in the program, can be estimated from the results of random, flight by flight crack growth tests. A least squares fit of equation (16) would provide an estimate of $b$ and if multiple specimens were tested to the same spectrum, an average of $b$ values would be used in the model. Similarly, a crack length vs. number of flights curve could be generated by a cycle by cycle crack growth equation for the baseline spectrum and $b$ could be determined to provide a fit of equation (16) to this curve.

In the absence of data or for analytical studies not related to specific details, reasonable values of $b$ can be inferred from assumptions regarding the expected safe life of a detail and the initial quality distribution of a detail. For example, if it has been demonstrated or assumed that flaws of a magnitude greater than $a_0$ will be detected by the manufacturer, that $a_c$ is the critical crack length for the detail, and that the life of a detail is $T_c$ flights, then

$$a_c = a_0 e^{bT_c}$$

and

$$b = \frac{\ln a_c - \ln a_0}{T_c}$$

(34)

If the safe life estimate $T_c$ is conservative (short) then the crack growth parameter will be conservative (large).

To provide an indication of the magnitude of the crack growth rate parameter, Table I presents the least squares estimates obtained from specimen crack growth test for the F-5 E/F aircraft, Reference 10. Two specimens were tested under identical test conditions so that the observed differences in $b$ values for the duplicates are due to uncontrolled sources of variation. Note that each individual crack growth curve was fit very well by the exponential model but that for some specimen types, significant differences in estimated $b$ values were obtained from duplicate tests.
The usage bias and flight by flight variability parameters can be considered together. If the anticipated stress environment is equivalent to that of the baseline spectrum, there will be no usage bias and the usage bias factor is unity. If the anticipated environment is different from the baseline, the ratio of anticipated usage severity to baseline is the usage bias parameter. Equation (23) expressing a flight by flight usage parameter in terms of the summation of a power of stress changes is a parameter which may provide a measure of this usage. Further development is required on the application of this parameter.

**TABLE I**

<table>
<thead>
<tr>
<th>Specimen Identification</th>
<th>Max Spectrum Stress</th>
<th>(a_0)</th>
<th>(b (\times 10^4))</th>
<th>(r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>01 W 1</td>
<td>32910</td>
<td>.020</td>
<td>2.93</td>
<td>.999</td>
</tr>
<tr>
<td>01 W 2</td>
<td>32910</td>
<td>.018</td>
<td>2.40</td>
<td>.999</td>
</tr>
<tr>
<td>02 W 1</td>
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<td>.999</td>
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<tr>
<td>02 W 2</td>
<td>29120</td>
<td>.024</td>
<td>1.68</td>
<td>.999</td>
</tr>
<tr>
<td>03 W 1</td>
<td>30150</td>
<td>.029</td>
<td>2.62</td>
<td>.978</td>
</tr>
<tr>
<td>03 W 2</td>
<td>30150</td>
<td>.029</td>
<td>2.20</td>
<td>.999</td>
</tr>
<tr>
<td>04 W 1</td>
<td>33730</td>
<td>.027</td>
<td>2.62</td>
<td>.999</td>
</tr>
<tr>
<td>04 W 2</td>
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<td>1.56</td>
<td>.998</td>
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<td>.037</td>
<td>2.77</td>
<td>.996</td>
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<tr>
<td>05 W 2</td>
<td>30810</td>
<td>.036</td>
<td>4.38</td>
<td>.992</td>
</tr>
<tr>
<td>06 W 1</td>
<td>± 29030</td>
<td>.054</td>
<td>1.12</td>
<td>.993</td>
</tr>
<tr>
<td>06 W 2</td>
<td>± 29030</td>
<td>.049</td>
<td>1.19</td>
<td>.994</td>
</tr>
<tr>
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<td>36880</td>
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<td>.995</td>
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<td>08 W 2</td>
<td>36880</td>
<td>.017</td>
<td>5.35</td>
<td>.996</td>
</tr>
</tbody>
</table>

The flight by flight variability parameter required by the program is the coefficient of variations of the flight by flight usage parameter. No experience is available for the usage parameter defined by equation (23) but Miners damage calculations indicate that any realistic metric of flight by flight structural damage will, in general, have an erratic distribution with a standard deviation at least equal to the mean. For Miners damage
and some mission types on fighter aircraft, coefficients of variation greater than 3 have been observed, c.f. Reference 12. Actually, in the examples of Section IV it will be shown that the expected cost of crack repair is extremely insensitive to this parameter. This point will be discussed later.

3.1.3 Repaired Flaw Size Distribution

After a crack has been detected and repaired, it is assumed that the detail has an equivalent crack that is described by the repaired flaw size distribution. Thus, the exact crack size for a particular detail is not known; but knowing the distribution would permit calculation of the probability that the equivalent crack is in a specific interval. Further, if a large group of details is repaired, the repaired flaw size distribution would provide the percentage of the details that have equivalent cracks in a specified interval.

In the program, it is assumed that the equivalent repaired flaw size distribution is from the two-parameter Weibull family. The reasons for choosing the Weibull family are the same as those for selecting the Weibull family for the equivalent initial flaw size distribution. If a different two-parameter family is desired, the change can be accomplished by making the appropriate changes in the FUNCTION PR(X) subroutine. The methods for estimating the parameters of the repaired flaw size distribution are the same as those presented for the initial flaw size distribution in Paragraph 3.1.1. Note that the model assumes a constant repair quality for each inspection and for the repair of all crack sizes.

3.2 Inspection and Repair Parameters

The input data which are identified as inspection and repair parameters are the cost of each inspection, the function which specifies the probability of crack detection, and the costs of repair for the details identified as having cracks.
3.2.1 Cost of Inspection

The model uses a predetermined cost of inspection at each inspection cycle. This cost is independent of crack size and represents a particular detail's share of the total cost of preparing the airplane for inspection as well as the cost of the time required to inspect the actual detail. Since the location of the critical detail could greatly influence the time required for its inspection, the estimate of the inspection cost can only be based on an analysis of the total inspection plan. For example, an accessible critical location may be easy to inspect in the field while an inaccessible critical location may be inspected only during a PDM at a time when many other maintenance actions are performed.

The program is designed to permit a different cost of inspection at each cycle. This feature could be used to account for different inspections occurring in the field or in the depot. In order to set up an array for multiple inspection costs, the user is asked to specify whether or not multiple inspection costs will be specified. If not, the user enters the single value. If so, the user enters one value for each inspection cycle.

3.2.2 Probability of Crack Detection

The non-destructive inspection techniques currently in use in the Air Force are not capable of detecting all cracks in the zone of inspection. Since the likelihood of a crack being detected depends on a large number of factors (length, shape, location of crack; NDI method; ability of inspector; location of inspection; time of day; etc.) crack detection capability has been characterized in terms of the probability of detection (POD) as a function of crack length. The other factors which influence crack detection are accounted for either by degrading the POD for a particular crack length due to the scatter introduced by the factors or by determining POD curves for major controllable stratifications of the population of all NDI inspections. Thus, there may be a POD curve for each combination of inspection technique and structure type but the personal factors which influence
the inspector (time of day, day of week, mental attitude, etc) would have to be measured in terms of scatter in POD for a crack of fixed length.

For the computer program of the model it was assumed that the probability of crack detection as a function of crack length can be described by a cumulative Weibull distribution. This family of distributions was selected since it includes the exponential distribution which has been used in the past as a model (Reference 6), and also since it was used to characterize POD in a recently completed study for the Air Force (Reference 13). Further, due to the extreme scatter in the percentage of inspectors who detect cracks of almost equal length, there are many families of distributions that could not be rejected as models for the data. Since the Weibull family has an analytically simple form, it provides a convenient and acceptable model. If the user desires, any other two-parameter family can be substituted by making changes to the FUNCTION H(X) subroutine. It should be noted, however, that H(X) must approach one as X increases to insure the consistency of the mathematics of the model. Probability of detection curves are often shown as being asymptotic to a value less than one. If such a curve is desired, it must be set equal to unity, at some crack length. The incorporation of such a POD would require significant changes in the programmed model.

The probability of crack detection in the program input requires the specification of the shape (HALPH) and scale (HBETA) parameters. These values should be estimated for the type of structure and NDI method that will be used during the inspections. Figure 4 presents the probability of detection for four values of the shape parameter and a common characteristic value of 0.1 in. The data of Reference 13 indicate that a characteristic value of 0.1 in. would be reasonable for eddy current surface scans around countersunk fasteners in skin and stringer wing segments. This data also indicates that the smaller shape parameters (long tail) are more descriptive of current capabilities than the less spread distributions which result from larger shape parameters.
Figure 4. Weibull Probabilities of Crack Detection.
3.2.3 **Cost of Repair**

Three types of data are required in the input related to cost of repair: the number of details in the population, the crack sizes which determine the four levels of cost of repair, and a cost value for each of the repair levels.

The number of details in the population is required in the calculation of the expected value and standard deviation of the costs of repair at an inspection. This is the total number of details in the fleet which are being subjected to an equivalent stress environment with equivalent crack detection probability and inspection and repair costs. In the simple case of one critical location per aircraft, this parameter is the number of aircraft in the fleet. In the program, the parameter is entered as the number of items, N.

The calculation of the expected cost of repair allows for four ranges of crack size with a (possibly) different repair cost for a crack in each range. This provision is based on the assumption that it may be more expensive to repair large cracks than small cracks. For example, oversizing a fastener hole may be an inexpensive and viable method for repairing a small crack but if a crack exceeds a certain length, an entire panel may require replacement. Further, if the replacement cost of the aircraft is entered as the cost of repair for the largest critical flaw size, the expected costs of catastrophic failure are also included in the model.

There are four critical flaw sizes which define the four ranges of cost repair. In the input to the program, these are denoted as $AA(i), i=1,4$. The four ranges are: $AA(1)$ to $AA(2)$; to $AA(3)$; $AA(3)$ to $AA(4)$; and greater than $AA(4)$. If any crack that is discovered is repaired, then $AA(1)=0$. In the case of cracks from fastener holes typical values for $AA(2)$, $AA(3)$, and $AA(4)$ would be determined such that repair is accomplished by oversize drilling and installing a new fastener. For example $AA(2)=0.016$, $AA(3)=0.032$, and $AA(4)=0.064$. For this set it would be assumed that
replacement (maximum repair) is required for cracks longer than 0.064.

The cost to repair a crack in each of the ranges would depend on the location and would be estimated from current experience. It is possible that different types of repair could be performed at different costs and the result would be different equivalent repaired crack size distributions. Cold working after oversize drilling would be an example of increasing a repair cost and obtaining a better repaired crack size distribution.

The repair costs are defined in the input as COST(J), J=1,4. Four cost values are required even if some are repeated.

3.3 PROGRAM PARAMETERS

Two other parameters must be specified as part of the data input. These are the crack size increment, DELTA, and the number of iterations of the inspection and repair cycle to be performed, I CYCLE. These parameters influence only the operation of the program.

DELTA controls the number of points at which the $f^-(a)$ and $f^+(a)$ functions are calculated. Since an integration is required at each a value, DELTA should not be chosen arbitrarily small. However, computational accuracy can be lost if DELTA is chosen too large. A value of DELTA approximately equal to 1/100 of the crack range was judged to provide sufficient accuracy and acceptable run times.

I CYCLE merely limits the number of iterations of the inspection/repair calculations to the desired number. The product of I CYCLE and UT (inspection interval) determines the fleet life being covered in the simulation.
3.4 PROGRAM LISTING

The program users manual and listing are presented in Appendices A and B. The users manual is on a separate card deck and does not necessarily have to be considered as a part of the program. Appendix A presents instructions for calling the users manual if it is entered in the computer, a listing of the manual, and sample output from one cycle of a representative computer run.

Appendix B contains the complete program listing for PCRC (Predicted Crack Repair Costs). This program was written for the CDC 6600 computer located in the Aeronautical Systems Division Computer Center, Area B, Wright-Patterson Air Force Base. It is possible that differences in computer systems could prevent the direct running of this program on other computers. The program also utilizes four programs from the International Mathematical and Statistical Library (IMSL). These routines are used in the calculation of integrals (DCADRE and DCSQDU) and for interpolation by spline function fitting (ICSICU) and evaluation (ICSEVU).

The first 277 lines of code control the input, execution, and output of the program. These are followed by the definition of 8 subroutines, of which, only 2 have not been discussed in detail in the preceding paragraphs. Function U2D is a subroutine which evaluates the argument in the calculation of FMINUS, equation (12). Function FPLIN controls the spline fitting used in interpolating to obtain intermediate functional values in the numerical integration routines.

Results from the programs have been verified under a variety of input conditions. However, due to the nature of the numerical computations, it is possible that inconsistent, ill-defined, or illogical input conditions could be specified which would cause failure of the program. Since such failures would not necessarily preclude completion of the run, the output should always be evaluated to insure that the results are reasonable.
SECTION IV
EXAMPLE RESULTS/SENSITIVITY STUDY

To illustrate the use of the Predicted Crack Repair Cost computer program, a series of example problems were run which also demonstrate the effect of variations in input parameters on the predicted. This section presents an example for defining an input data set and the results of the limited sensitivity study which was conducted.

The approach taken in the sensitivity study was to define a set of "standard" input conditions which are reasonably realistic with respect to Air Force experience and to vary only a small portion of the parameters during any one particular set of runs. The large number of possible combinations of the input parameters prohibited a more general approach in which all possible combinations are studied. Thus, the conclusions of this sensitivity analysis must be interpreted in this restricted framework. In particular, the costs of inspection and repair can greatly influence the sensitivity to some parameters and the allocation of a portion of the total cases of inspection and repair for an airplane to a single structural detail is not uniquely defined. If a user prefers any major deviation from the input "standard" conditions, a new set of computer runs should be generated to test the variables of interest.

The crack growth parameter values for the representative "standard" set were derived from the assumptions that the critical crack length, $a_0$, for the structural details was 0.24 in., that the probability of an equivalent initial crack being greater than 0.050 in. and passing the manufacturer's quality control system was 0.001, and that the design life of the aircraft was 6000 flights. Based on these assumptions, the rationale for the assignment of specific parameter values (in the order requested by the program) is as follows:
- Inspection interval: UT=2000; convenient fraction of design life
- Growth rate: UB=0.0002614; from equation (34) with above assumptions
- Usage Bias: UM=1.0; design usage
- Flt. by flt. variability: US=3.0; representative value observed in Miners damage
- \( F_0 \) shape parameter: FOALPH=2; representative values observed in F-4 data
- \( F_0 \) scale parameter: FOBETA=0.019; from assumptions and equation (30)
- \( F_R \) shape parameter: FRALPH=2; repaired flow distribution is equivalent to initial
- \( F_R \) scale parameter: FRBETA=01019; repaired flaw distribution is equivalent to initial
- \( H \) shape parameter: HALPH=1; representative from inspection reliability data
- \( H \) scale parameter: HBETA=0.1; assumed value for ultrasonic roto-scanner with fastener in
- Crack Length Increment: DELTA=0.004; sufficient resolution for density functions
- Number of Inspections: ICYCLE=6; perform calculations to 12000 flight lifetime
- Number of Items: N=1000; assume a fleet of 1000 aircraft
- Inspection flaw sizes: AA(I), I=1,4 =0.000, .016, .032, .064; repair by oversize drilling unless crack is greater than 0.064 in. which then requires replacement of entire panel
- Repair Costs: COST(j), J=1,4=3, 3, 3, 100; cost of oversize drilling and new bolt for cracks less than 0.064 in; proportionate share of panel replacement (among many fasteners) for cracks over 0.064 in.
- Inspection cost: AIC=5000; assume proportionate share of ultrasonic roto-scanner inspection with fastener in is $5.00 per detail for each of the 1000 details.
Variations from the above set of standard conditions were investigated for inspection intervals, inspection capabilities, repair quality, baseline usage, and repair costs. In addition, variations in equivalent initial and repaired flaw quality were analyzed by changing shape parameters for the fixed 99.9 percentile and also by reprogramming two other families of distributions. The results of these studies are presented in the following paragraphs. To display the changes in the crack size density distributions, Figure 5 displays the densities for the standard conditions immediately after the repair cycles.

4.1 COMPARING INSPECTION INTERVALS

To compare the effect of time between inspection intervals on the expected costs of inspection and repair, all parameters were fixed at standard values except T. In separate runs, the cost output was generated for inspection intervals of 1000, 2000, 3000, 4000, and 6000 flights. The results are summarized in Table II which presents the total percentage of cracks which were identified and repaired, the expected value and standard deviation of the cost of each inspection and repair cycle, and the expected value and standard deviation of the cumulative inspection and repair costs. The calculations were carried through a lifetime of 12000 flights to include the effects of utilization past original design life. Note that these calculations do not include any expected costs which could arise due to catastrophic failure from cracks exceeding the 0.24 in critical crack size.

The results of Table II indicate that inspection intervals of 2000, 3000, 4000 and 6000 flights have essentially the same expected value and standard deviation of total costs over a lifetime of 12000 flights while an inspection interval of 1000 flights is slightly more expensive. Over a lifetime of 6000 flights, however, there appears to be a cost advantage to a 3000 flight inspection interval. Figure 6, which presents the expected total costs at each inspection interval, shows these results and also indicates that the expected cost differences
Figure 5. Density Functions of Crack Sizes Immediately After Inspection and Repair Cycles.
<table>
<thead>
<tr>
<th>Inspection Interval</th>
<th>Aircraft Life (Flights)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2000</td>
</tr>
<tr>
<td>1000 P(i)</td>
<td>0.219</td>
</tr>
<tr>
<td>C1</td>
<td>6200</td>
</tr>
<tr>
<td>S_i</td>
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<td>S_i</td>
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<td>3000 P(i)</td>
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<td>C_TOT</td>
<td>10800</td>
</tr>
<tr>
<td>S_TOT</td>
<td>700</td>
</tr>
<tr>
<td>4000 P(i)</td>
<td>0.361</td>
</tr>
<tr>
<td>C1</td>
<td>19400</td>
</tr>
<tr>
<td>S_i</td>
<td>1100</td>
</tr>
<tr>
<td>C_TOT</td>
<td>19400</td>
</tr>
<tr>
<td>S_TOT</td>
<td>1100</td>
</tr>
<tr>
<td>6000 P(i)</td>
<td>0.515</td>
</tr>
<tr>
<td>C1</td>
<td>44200</td>
</tr>
<tr>
<td>S_i</td>
<td>1500</td>
</tr>
<tr>
<td>C_TOT</td>
<td>44200</td>
</tr>
<tr>
<td>S_TOT</td>
<td>1500</td>
</tr>
</tbody>
</table>

TABLE II. Variation in Time Between Inspections—Standard Values for Other Parameters
Figure 6. Expected Total Maintenance Costs for Various Inspection Intervals.
between intervals of 2000, 3000, 4000, and 6000 may be negligible.

The higher costs for the 1000 flight inspection interval can be attributed to the relative costs of the inspection to the costs of the repairs being encountered at this interval. Changing the relative magnitude of these maintenance costs could change the conclusions regarding inspection intervals. For example, the high cost of repairing a crack over 0.064 in. in length is due to the necessity of replacing an entire panel. If the cost of the panel is $20,000 and the entire cost must be allocated to the critical location being simulated, then \( C(4) = 20,000 \) and the expected values and standard deviations for various inspection intervals up to a 6000 flight life would appear as in Table III. These results clearly indicate the significant advantage of the shorter inspection intervals over the longer as well as the greatly increased expected maintenance costs. Note, however, that the data of Table II pertain to only one fastener hole while the panel replacement which causes the high costs of Table III repairs several hundred fastener holes.

4.2 COMPARING INSPECTION CAPABILITIES

To compare inspection capabilities, three types of inspections were hypothesized with different relative costs and relative capabilities. These are summarized as follows:

<table>
<thead>
<tr>
<th>Inspection Type</th>
<th>Relative Cost/Detail</th>
<th>Relative Capability (Characteristic Value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visual-fastener in</td>
<td>$1.25</td>
<td>0.2 in.</td>
</tr>
<tr>
<td>Roto-scanner Ultrasonic-fastener in</td>
<td>$5.00</td>
<td>0.1 in.</td>
</tr>
<tr>
<td>Motorized eddy current-fastener out</td>
<td>$12.50</td>
<td>0.05 in.</td>
</tr>
</tbody>
</table>

These values were judged to be representative of Air Force experience. (The roto-scanner ultrasonic parameter values are "standard" as previously defined).
TABLE III. Variation in Time Between Inspections - High Cost for Repair of Cracks over 0.064 in.

<table>
<thead>
<tr>
<th>Inspection Interval</th>
<th>Aircraft Life (Flights)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>/000</td>
</tr>
<tr>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>$c_1$</td>
<td>18,000</td>
</tr>
<tr>
<td>$s_1$</td>
<td>15,700</td>
</tr>
<tr>
<td>$c_{TOT}$</td>
<td>18,000</td>
</tr>
<tr>
<td>$s_{TOT}$</td>
<td>15,700</td>
</tr>
<tr>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>$c_1$</td>
<td>199,300</td>
</tr>
<tr>
<td>$s_1$</td>
<td>61,900</td>
</tr>
<tr>
<td>$c_{TOT}$</td>
<td>199,300</td>
</tr>
<tr>
<td>$s_{TOT}$</td>
<td>61,900</td>
</tr>
<tr>
<td>3000</td>
<td></td>
</tr>
<tr>
<td>$c_1$</td>
<td>1,009,300</td>
</tr>
<tr>
<td>$s_1$</td>
<td>138,100</td>
</tr>
<tr>
<td>$c_{TOT}$</td>
<td>1,009,300</td>
</tr>
<tr>
<td>$s_{TOT}$</td>
<td>138,100</td>
</tr>
<tr>
<td>6000</td>
<td></td>
</tr>
<tr>
<td>$c_1$</td>
<td>7,777,600</td>
</tr>
<tr>
<td>$s_1$</td>
<td>308,300</td>
</tr>
<tr>
<td>$c_{TOT}$</td>
<td>7,777,600</td>
</tr>
<tr>
<td>$s_{TOT}$</td>
<td>308,300</td>
</tr>
</tbody>
</table>
The results using the costs of repair from the standard set of parameters are presented in Table IV. For these relative costs of inspections and repairs, the expected maintenance costs are less for the cheaper, less effective inspection procedures. This cost savings is due to the reduced cost of the inspections and may also reflect a false savings of not detecting (and not repairing) the cracks greater than 0.064 in. Note that these cost figures do not reflect expected losses that may occur when the cracks reach the critical size of 0.24 in. Due to the assumptions which determine the standard parameters, less than 0.1 percent of the aircraft will have a crack size greater than 0.24 in. at 6000 flights.

Table V presents the expected cost data when the repair cost for a crack longer than 0.064 in. is $20,000. At the 6000 flight aircraft life, the best and highest cost inspection procedure is the most cost effective. At the earlier inspection, the high cost of the expensive inspection procedure is attributable to the discovery of more of the large cracks. Over the 6000 flight life of the aircraft, the early repair of these cracks becomes evident. Again, the addition of expected loss due to catastrophic failures would reinforce the cost efficiency of the expensive inspection procedure.

4.3 COMPARING REPAIR QUALITY

To compare repair quality, it was assumed that after oversizing the fastener hole, it was either cold worked or a fatigue fastener was installed. It was assumed that this operation would cost twice as much ($6 per repair) for cracks less than 0.064 in. and that the equivalent repair quality distribution would have a characteristic value of 0.0019 (ten times better). Repair of cracks greater than 0.064 in. are accomplished by panel replacement and would cost the same.

Table VI presents the summary of the maintenance cost data for the standard cost of repair of cracks greater than 0.064 in. (i.e., $100). In a 6000 flight lifetime, the more expensive repair
TABLE IV. Variation in Inspection Capabilities—Standard Values for Other Parameters

<table>
<thead>
<tr>
<th>Inspection Capabilities</th>
<th>Aircraft Life (Flights)</th>
<th>2000</th>
<th>4000</th>
<th>6000</th>
<th>8000</th>
<th>10000</th>
<th>12000</th>
</tr>
</thead>
<tbody>
<tr>
<td>HBETA=0.2</td>
<td>P(i)</td>
<td>0.130</td>
<td>0.190</td>
<td>0.249</td>
<td>0.285</td>
<td>0.289</td>
<td>0.227</td>
</tr>
<tr>
<td>I(c)=1250</td>
<td>C_i</td>
<td>2200</td>
<td>8100</td>
<td>17000</td>
<td>21900</td>
<td>22000</td>
<td>20300</td>
</tr>
<tr>
<td></td>
<td>S_i</td>
<td>240</td>
<td>780</td>
<td>1100</td>
<td>1300</td>
<td>1300</td>
<td>1200</td>
</tr>
<tr>
<td></td>
<td>C_TOT</td>
<td>2200</td>
<td>10300</td>
<td>27300</td>
<td>49200</td>
<td>71200</td>
<td>91500</td>
</tr>
<tr>
<td></td>
<td>S_TOT</td>
<td>240</td>
<td>820</td>
<td>1400</td>
<td>1900</td>
<td>2300</td>
<td>2600</td>
</tr>
<tr>
<td>HBETA=0.1</td>
<td>P(i)</td>
<td>0.239</td>
<td>0.315</td>
<td>0.360</td>
<td>0.369</td>
<td>0.362</td>
<td>0.357</td>
</tr>
<tr>
<td>I(c)=5000</td>
<td>C_i</td>
<td>6700</td>
<td>14300</td>
<td>22000</td>
<td>23600</td>
<td>22600</td>
<td>22000</td>
</tr>
<tr>
<td></td>
<td>S_i</td>
<td>300</td>
<td>900</td>
<td>1200</td>
<td>1200</td>
<td>1200</td>
<td>1200</td>
</tr>
<tr>
<td></td>
<td>C_TOT</td>
<td>6700</td>
<td>21000</td>
<td>43000</td>
<td>66600</td>
<td>89200</td>
<td>111200</td>
</tr>
<tr>
<td></td>
<td>S_TOT</td>
<td>300</td>
<td>900</td>
<td>1500</td>
<td>1900</td>
<td>2300</td>
<td>2600</td>
</tr>
<tr>
<td>HBETA=0.05</td>
<td>P(i)</td>
<td>0.408</td>
<td>0.471</td>
<td>0.486</td>
<td>0.484</td>
<td>0.481</td>
<td>0.480</td>
</tr>
<tr>
<td>I(c)=12500</td>
<td>C_i</td>
<td>15100</td>
<td>21700</td>
<td>25100</td>
<td>25100</td>
<td>24900</td>
<td>24800</td>
</tr>
<tr>
<td></td>
<td>S_i</td>
<td>380</td>
<td>860</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>C_TOT</td>
<td>15100</td>
<td>36800</td>
<td>61900</td>
<td>87000</td>
<td>111900</td>
<td>136700</td>
</tr>
<tr>
<td></td>
<td>S_TOT</td>
<td>380</td>
<td>940</td>
<td>1400</td>
<td>1700</td>
<td>2000</td>
<td>2200</td>
</tr>
</tbody>
</table>
TABLE V. Variation in Inspection Capabilities—
High Cost for Repair of Cracks Over 0.064 in.

<table>
<thead>
<tr>
<th>Inspection Capabilities</th>
<th>Aircraft Life (Flights)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2000</td>
<td>4000</td>
<td>6000</td>
<td></td>
</tr>
<tr>
<td>HBETA=0.2, I(c)=1250</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_I</td>
<td>115,600</td>
<td>1,302,600</td>
<td>3,097,300</td>
<td></td>
</tr>
<tr>
<td>S_I</td>
<td>47,600</td>
<td>156,000</td>
<td>228,800</td>
<td></td>
</tr>
<tr>
<td>C_TOT</td>
<td>115,600</td>
<td>1,431,200</td>
<td>4,515,500</td>
<td></td>
</tr>
<tr>
<td>S_TOT</td>
<td>47,600</td>
<td>163,100</td>
<td>280,900</td>
<td></td>
</tr>
<tr>
<td>HBETA=0.1, I(c)=5000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_I</td>
<td>199,300</td>
<td>1,720,200</td>
<td>3,288,000</td>
<td></td>
</tr>
<tr>
<td>S_I</td>
<td>61,900</td>
<td>177,100</td>
<td>234,300</td>
<td></td>
</tr>
<tr>
<td>C_TOT</td>
<td>199,300</td>
<td>1,920,000</td>
<td>5,208,000</td>
<td></td>
</tr>
<tr>
<td>S_TOT</td>
<td>61,900</td>
<td>187,600</td>
<td>300,100</td>
<td></td>
</tr>
<tr>
<td>HBETA=0.05, I(c)=1250</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_I</td>
<td>302,000</td>
<td>1,618,800</td>
<td>2,317,700</td>
<td></td>
</tr>
<tr>
<td>S_I</td>
<td>75,400</td>
<td>171,800</td>
<td>201,900</td>
<td></td>
</tr>
<tr>
<td>C_TOT</td>
<td>302,000</td>
<td>1,920,800</td>
<td>4,238,500</td>
<td></td>
</tr>
<tr>
<td>S_TOT</td>
<td>75,400</td>
<td>187,600</td>
<td>275,600</td>
<td></td>
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</tbody>
</table>
TABLE VI. Variations in Repair Quality Standard Values for Other Parameters

<table>
<thead>
<tr>
<th>Repair Quality</th>
<th>Aircraft Life (Flights)</th>
<th>2000</th>
<th>4000</th>
<th>6000</th>
<th>8000</th>
<th>10000</th>
<th>12000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RBETA=0.019</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(J)=3</td>
<td>P(i)</td>
<td>0.239</td>
<td>0.315</td>
<td>0.360</td>
<td>0.369</td>
<td>0.362</td>
<td>0.358</td>
</tr>
<tr>
<td></td>
<td>C_i</td>
<td>6700</td>
<td>14300</td>
<td>22000</td>
<td>23600</td>
<td>22600</td>
<td>22000</td>
</tr>
<tr>
<td></td>
<td>S_i</td>
<td>300</td>
<td>900</td>
<td>1200</td>
<td>1200</td>
<td>1200</td>
<td>1200</td>
</tr>
<tr>
<td></td>
<td>C_TOT</td>
<td>6700</td>
<td>21000</td>
<td>43000</td>
<td>66600</td>
<td>89200</td>
<td>111200</td>
</tr>
<tr>
<td></td>
<td>S_TOT</td>
<td>300</td>
<td>900</td>
<td>1500</td>
<td>1900</td>
<td>2300</td>
<td>2600</td>
</tr>
<tr>
<td>RBETA=0.0019</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(J)=6</td>
<td>P(i)</td>
<td>0.239</td>
<td>0.260</td>
<td>0.228</td>
<td>0.156</td>
<td>0.088</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td>C_i</td>
<td>7300</td>
<td>14400</td>
<td>19600</td>
<td>17000</td>
<td>11600</td>
<td>7900</td>
</tr>
<tr>
<td></td>
<td>S_i</td>
<td>300</td>
<td>900</td>
<td>1100</td>
<td>1000</td>
<td>800</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>C_TOT</td>
<td>7300</td>
<td>21700</td>
<td>41300</td>
<td>58300</td>
<td>69900</td>
<td>77800</td>
</tr>
<tr>
<td></td>
<td>S_TOT</td>
<td>300</td>
<td>900</td>
<td>1400</td>
<td>1800</td>
<td>1900</td>
<td>2000</td>
</tr>
</tbody>
</table>
is only slightly less expensive but beyond 6000 flights the effect of the improved quality of repair becomes significant. Table VII summarizes the repair cost data when the repair cost (panel replacement) of the cracks longer than 0.064 in. is $20,000, i.e., the total cost is attributed to the critical location. The cost effectiveness of the higher cost, higher quality repair under these conditions, is significant at the 6000 flight lifetime.

4.4 COMPARING DIFFERENCES IN AIRCRAFT USAGE

Differences in aircraft usage are modeled by the usage bias and flight by flight variability parameters. To determine the sensitivity of predicted maintenance costs to these parameters, the program was exercised for usage bias values of 0.8, 0.9, 1.0, 1.1, and 1.2 and for flight by flight variability (coefficients of variation) of 1.5, 3.0 and 4.5. The maintenance costs were calculated for both the high and the low panel replacement costs (repair costs for cracks larger than 0.064 in).

The effect due to usage bias is significant as expected. Table VIII shows the expected total inspection and repair costs at each inspection for both sets of panel replacement costs. The effect of deviations from the baseline are proportionately larger for the higher repair costs. If a baseline usage is assumed when the aircraft are being used in a 10 percent more severe environment, the expected maintenance costs over a 6000 flight life will be underestimated by 26 percent for the high repair costs and by 12 percent for the low repair cost.

The effect of flight by flight variability on the expected total costs was negligible. The range of values considered (1.5 to 4.5) encompasses the variability that would be expected in operation and the expected costs were within 1 percent of baseline for both sets of cost of repair. This result agrees with a similar conclusion regarding flight by flight variability of Miners damage (Reference 12). However, even though flight by flight variability may be unimportant in the calculation of expected costs, it is a necessary parameter for the estimation
TABLE VII. Variation in Repair Quality—High Cost for Repair of Cracks Over 0.064 in.

<table>
<thead>
<tr>
<th>Repair Quality</th>
<th>Aircraft Life (Flights)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2000</td>
</tr>
<tr>
<td>RBETA = 0.019</td>
<td></td>
</tr>
<tr>
<td>C(J) = 3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>199,300</td>
</tr>
<tr>
<td></td>
<td>61,900</td>
</tr>
<tr>
<td></td>
<td>199,300</td>
</tr>
<tr>
<td></td>
<td>61,900</td>
</tr>
<tr>
<td>RBETA = 0.0019</td>
<td></td>
</tr>
<tr>
<td>C(J) = 6</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>200,000</td>
</tr>
<tr>
<td></td>
<td>61,900</td>
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<tr>
<td></td>
<td>200,000</td>
</tr>
<tr>
<td></td>
<td>61,900</td>
</tr>
<tr>
<td>Panel Replacement Cost</td>
<td>Usage Bias</td>
</tr>
<tr>
<td>------------------------</td>
<td>------------</td>
</tr>
<tr>
<td></td>
<td>2000</td>
</tr>
<tr>
<td>C(4)=$100</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>6000</td>
</tr>
<tr>
<td>0.9</td>
<td>6300</td>
</tr>
<tr>
<td>1.0</td>
<td>6700</td>
</tr>
<tr>
<td>1.1</td>
<td>7100</td>
</tr>
<tr>
<td>1.2</td>
<td>7800</td>
</tr>
<tr>
<td>C(4)=$20,000</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>82,600</td>
</tr>
<tr>
<td>0.9</td>
<td>130,600</td>
</tr>
<tr>
<td>1.0</td>
<td>199,300</td>
</tr>
<tr>
<td>1.1</td>
<td>293,500</td>
</tr>
<tr>
<td>1.2</td>
<td>417,800</td>
</tr>
</tbody>
</table>
of the precision of the usage bias estimate. Thus, the importance of this parameter is in the determination and/or the evaluation of sample size considerations with respect to the estimate of the significant parameter of usage bias.

4.5 COMPARING FLAW SIZE DISTRIBUTIONS

The equivalent flaw size distributions for either new or repaired structure cannot be ascertained by direct observations of cracks. Rather, observations representative of this distribution are obtained from a back calculation of an observed crack to time zero. The crack may or may not have been present at time zero but the structure responded as though it were and, hence, the equivalence of the crack at time zero.

Due to the difficulty in obtaining valid equivalent time zero crack lengths, relatively few such data points have been generated. There is very little data available for use in selecting an appropriate family of distributions for describing initial flaw sizes. As one step in assessing the relative importance of the possible shapes of this distribution, expected inspection and repair costs were calculated for three different families of distributions with three different medians (and, hence, shapes) for each family.

The three families of distributions that were compared for initial and repaired equivalent flaw size distributions were the Weibull, the gamma, and the lognormal. The equation of the Weibull density function is given in equation (29). The equation of the density function of the gamma distribution is given by

\[ f(y) = \frac{\beta^\alpha y^{\alpha-1}}{\Gamma(\alpha)} \exp(-[\beta y]) \]  

while that of the lognormal density is given by

\[ f(y) = \frac{1}{y \frac{a}{\sqrt{2\pi}}} \exp\left(-\frac{1}{2a^2} (\ln y - \beta)^2\right) \]
For each of the three families the parameters were determined such that 99.9 percent of the flaws were less than 0.050 in. and they had equal medians (50th percentiles). The parameter values used are given in Table IX. The resulting density functions are pictured in Figure 7. The distributions with a median of 0.0156 have a shape parameter approximately equal to that of the A-7D initial quality data, Reference 8.

The expected total costs of inspection and repair are presented in Table X for each of the three families, with the three different median initial (and repaired) cracks, and for the high and low cost for repair of cracks longer than 0.064 in. The differences in the expected costs can be interpreted with the aid of the density functions of Figure 7. The Weibull density function for the three medians (or the three shapes) considered is more diffuse and, hence, has greater area in the tails than the other two distributional types. Similarly, the gamma distribution is more diffuse than the lognormal. In the early life of the aircraft, these larger cracks will grow more rapidly, have a greater chance of being detected, and hence, will incur higher repair costs. However, when cracks around the central tendencies of the distributions have grown to more detectable sizes, the other distributions will have a larger percentage requiring repair. Hence, the expected costs cross over at an inspection time which depends on the median. The larger the median, the sooner the crossover occurs.

Note that for the lower cost of repair of the long cracks, the differences in expected total costs for the three families is not practically significant. Further, the differences for the higher costs of repair are not excessively large at 6000 flights for the two higher medians or for any of the medians at 8000 and 10000 flight lifetimes. However, the differences in expected total costs for the three medians is significant for all three families at each inspection throughout aircraft life. Therefore, from the viewpoint of maintenance costs it would be more important to determine the shape (or median) of the density.
TABLE IX. Parameter Values for Comparing Families of Initial Flaw Size Distributions

<table>
<thead>
<tr>
<th>Median</th>
<th>Weibull</th>
<th>gamma</th>
<th>lognormal</th>
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<td>1.347</td>
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<td></td>
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<tr>
<td></td>
<td>$\beta$</td>
<td>0.02413</td>
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Figure 7. Equivalent Initial Crack Size Density Functions.
TABLE X. Comparing Families of Equivalent Flaw Size Distributions—Expected Total Maintenance Costs

<table>
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<th>Panel Replacement Cost</th>
<th>Flaw Size Distribution</th>
<th>Aircraft Life (Flights)</th>
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<td>G</td>
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<td></td>
<td>LN</td>
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<tr>
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<td>W</td>
<td>6600</td>
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<td>G</td>
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<td>LN</td>
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<tr>
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<tr>
<td></td>
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<td>LN</td>
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<td></td>
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<td>LN</td>
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</table>

52
function of equivalent flow sizes than to determine the family of distributions. Note, however, that safety of flight considerations which are based only on the largest equivalent initial flaw sizes may well indicate the reverse order of priority.
SECTION V
SUMMARY AND RECOMMENDATIONS

There were three primary objectives of this study: (1) to write a computer program which could be used to predict the repair costs of cracks which develop during operational aircraft usage, (2) to provide a document which describes the use of the computer program and guides a potential user in the specification of input requirements, and (3) to use data which is reasonably representative of Air Force experience as input for use in determining the sensitivity of predicted crack repair costs to variations in the input parameters.

The statistical model which has been programmed to achieve these objectives computes at each inspection the crack size densities before the inspection and after the repair of cracks which have been found; the percentage of cracks in four ranges of crack size; the expected value and standard deviation of the cost of each inspection and repair cycle; and the expected value and standard deviation of the total costs of all inspection and repair cycles. The crack size distribution output is optional. The model is programmed with a Weibull distribution for the equivalent initial and repaired crack sizes and for the probability of crack detection as a function of crack size. It is assumed that cracks grow in the operational environment in accordance with an exponential model as derived from a baseline spectrum, and that deviations from baseline crack growth and flight by flight usage variability can be accounted for by a normal distribution. The model allows for different repair costs for up to four crack size ranges with the largest range usually accounting for the cost of panel replacement or, possibly, the cost of catastrophic failure. Changes in the family of distributions for the crack sizes and the probability of crack detection can be easily accomplished if the replacement family is defined in terms of 1 or 2 parameters.
The program was demonstrated by computed expected maintenance costs for different inspection intervals, inspection capabilities, repair quality, aircraft usage, and equivalent flaw size distributions. The computations were made for a set of input data that is reasonably representative of Air Force maintenance capabilities and costs. However, the risk of catastrophic failure was not included. The conclusions of the sensitivity studies are dependent on the values of the other parameters and only relatively few parameter combinations could be tested. The sensitivity computations were performed for a high and low cost of repairing long cracks since most results were definitely sensitive to this parameter. The following general conclusions resulted:

1) The effect of inspection interval depended on the cost of repairing long cracks. For the high repair cost, frequent inspections saved money but for the low repair cost of long cracks, a tradeoff between cost of inspection and cost of repair resulted in cost savings at the less frequent inspection intervals.

2) The effect of inspection capabilities is highly dependent on the relative magnitudes of the cost of inspection and cost of repair of long cracks. For the cost values used in this study, cheaper, less effective inspections resulted in significant (35 percent) cost savings for the low repair cost. For the high repair cost, however, the most expensive (most effective) inspection led to the least total maintenance cost while the intermediate inspection led to the highest total maintenance cost.

3) The two repair quality capabilities tested indicated that the more expensive, more effective repair is cost effective at the design life.

4) Deviations of 10 percent or more from baseline usage are significant with differences in expected maintenance costs of about 15 percent being observed for a 10 percent change in usage severity. Flight by flight variability in usage severity, however, has no practically significant effect on expected maintenance costs (less than 1 percent).
5) Three families of distributions for equivalent initial flaw sizes were tested. The differences in expected total costs at the design life from the three families were negligible in comparison to the location of the median flaw size within any one of the families. Therefore, it is more important to properly locate the distribution than to be concerned about the statistical model.

With respect to recommendations, there are two directions for further work that are indicated as a result of this study. First, the program can be modified to permit input being defined in more general terms. For example, it may be desirable to specify different parameter values at every inspection interval for every input item. Further, it may be desirable to pursue a random growth concept by allowing the crack growth parameters of the model to be a random variable. This concept would require changing the probability calculations of the model and would also necessitate some investigation as to the type of distribution that would be used to model this parameter.

The second direction suggested by the study is to pursue the usage parameter as a method for monitoring for change in usage. This direction would require the processing of stress histories (perhaps, from different aircraft types) and the calculation of crack length as a function of usage time. This direction is a distinct change from the current study but could have significance in other force management applications.
APPENDIX A

USERS MANUAL
IN ORDER TO LIST THE USER MANUAL, AFTER A COMMAND REQUEST
THE USER SHOULD ENTER:

```
ATTACH, FILE1, PCRC DOCUMENTATION
EDITOR
```

THE USER IS NOW IN THE EDITOR MODE. AFTER A DOUBLE DOT IS
RETURNED, ENTER:

```
EDIT, FILE1, S
LIST, ALL
```

THE LISTING WILL LOOK LIKE THIS:

```
USER MANUAL
```

(NOTE: REFERBACK FOLLOWED BY A NUMBER (1 THROUGH 9) IS FOR
USER REFERENCE WITHIN THE USER MANUAL AND SHOULD NOT BE
MISCONSTRUED AS PART OF THE INPUT OR OUTPUT OF THE PROGRAM.)

AFTER LOGIN PROCEDURES, THE USER MUST INITIATE EXECUTION OF
THE PROGRAM. TO DO THIS, THE USER MUST INPUT EACH OF THE
FOLLOWING WHEN A COMMAND IS REQUESTED:

```
ATTACH, FILE2, PCRC
ATTACH, LIB, IMSL, ID=LIBRARY, SN=ASD
LIBRARY, LIB
RETURN OUTPUT
CONNECT TAPE9
FTN, I=FILE 2, L=0
```

OUTPUT IS RETURNED (DISCONNECTED) TO BYPASS PRINTING OF
NUMEROUS TRIVIAL ERROR WARNINGS. AFTER PROCESSING OF PCRC
IS COMPLETED, THE USER MUST ENTER: CONNECT OUTPUT
AFTER A COMMAND REQUEST IF PROCESSING IS TO PROCEED ON OTHER
PROGRAMS.

IF THE USER ALSO WANTS TO SEE THE CRACK SIZE DENSITY FUNCTIONS
AS PART OF THE OUTPUT, THE USER WILL ADDITIONALLY INPUT AFTER A
COMMAND REQUEST:

```
CONNECT TAPE1
```

THE PROGRAM WILL BEGIN EXECUTION AT THIS POINT WHEN THE USER
ENTERS AFTER A COMMAND REQUEST:

```
LGO
```

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC
THE USER MUST BE VERY CAREFUL IN ENTERING RESPONSES. FOR INSTANCE, THE USER MUST ENTER YES, Y, NO, N, OR VARIABLE NAMES WITHOUT ANY BLANKS TO INSURE ACCURATE READING OF THE RESPONSE BY THE PROGRAM. THE PROGRAM BEGINS BY OUTPUTTING THE FOLLOWING:

(REFER BACK 1):

ENTER UT, UB, UM, US...:

THE USER ENTERS VALUES FOR EACH FIELD SEPARATED BY COMMAS, WHERE:

- UT IS INSPECTION INTERVAL,
- UB IS GROWTH RATE,
- UM IS USAGE BIAS, AND
- US IS FLT. BY FLT. VARIABILITY.

THE PROGRAM RESPONDS WITH:

ENTER FOALPH, FOBETA...:

THE USER ENTERS VALUES FOR EACH FIELD SEPARATED BY COMMAS, WHERE:

- FOALPH IS SHAPE PARAMETER AND
- FOBETA IS SCALE PARAMETER
FOR THE INITIAL FLAW SIZE DISTRIBUTION.

THE PROGRAM RESPONDS WITH:

ENTER FRALPH, FRBETA...:

THE USER ENTERS VALUES FOR EACH FIELD SEPARATED BY COMMAS, WHERE:

- FRALPH IS SHAPE PARAMETER AND
- FRBETA IS SCALE PARAMETER
FOR THE REPAIRED FLAW SIZE DISTRIBUTION.

THE PROGRAM RESPONDS WITH:

ENTER HALPH, HBETA...:

THE USER ENTERS VALUES FOR EACH FIELD SEPARATED BY COMMAS, WHERE:

- HALPH IS SHAPE PARAMETER AND
- HBETA IS SCALE PARAMETER
FOR THE PROBABILITY OF CRACK DETECTION.

THE PROGRAM RESPONDS WITH:

ENTER DELTA, ICYCLE, N...:

THE USER ENTERS VALUES FOR EACH FIELD SEPARATED BY COMMAS.
WHERE: DELTA IS INCREMENT FOR CRACK SIZE DENSITY FUNCTIONS
ICYCLE IS NUMBER OF INSPECTIONS AND
N IS NUMBER OF ITEMS.

THE PROGRAM Responds WITH:

(REFEREACK 2):

ENTER AA(I),I=1,4..:

THE USER ENTERS THE VALUES FOR THE FOUR ELEMENTS OF THE ARRAY
WHERE: AA(I) DEFINE LEVEL I CRACKS

THE PROGRAM Responds WITH:

(REFEREACK 3):

ENTER COST(J),J=1,4..:

THE USER ENTERS THE VALUES FOR THE FOUR ELEMENTS OF THE ARRAY
WHERE: COST(J) IS THE REPAIR COST FOR LEVEL J CRACKS.

THE PROGRAM Responds WITH:

(REFEREACK 4):

DO YOU WANT MULTIPLE VALUES FOR AIC..:

IF THE INSPECTION COSTS WILL VARY FOR EACH ICYCLE
THE USER ENTERS YES OR Y. THE PROGRAM Responds WITH:

ENTER VALUES FOR AIC--ONE VALUE PER ICYCLE..:

THE USER ENTERS AS MANY VALUES FOR AIC AS THE VALUE OF ICYCLE.
IF THE INSPECTION COSTS WILL REMAIN THE SAME FOR EACH ICYCLE
THE USER ENTERS NO OR N. THE PROGRAM Responds WITH:

ENTER VALUE FOR AIC..:

THE USER ENTERS ONE VALUE; THE PROGRAM USES THIS VALUE FOR
EACH ICYCLE.

(REFEREACK 5):

THE PROGRAM WILL RESPOND WITH A LISTING OF ALL INPUT VALUES,
THEIR VARIABLE NAMES, AND WHAT THE VARIABLE NAME REPRESENTS.
THE PROGRAM WILL THEN ASK:
IS CRACK SIZE DATA CORRECT...:

THE USER ENTERS YES OR Y IF CALCULATIONS ARE TO PROCEED.
IF THE USER WANTS TO CHANGE SOME INPUT DATA, THE USER ENTERS
NO OR N. THE PROGRAM WILL BYPASS EXECUTION OF CALCULATIONS
FOR THIS DATA AND ALLOWS THE USER TO CHANGE DATA AS IF THE
USER HAS REQUESTED ADDITIONAL PROCESSING (REFERBACK 7).
ASSUMING THE USER WANTS CALCULATIONS STEPS TO BE EXECUTED,
THE PROGRAM WILL PRINT OUT A SET OF CALCULATIONS FOR EACH
ICYCLE. IF, IN ADDITION, THE USER HAS REQUESTED THE DENSITY
FUNCTION, ITS VALUES WILL BE DISPLAYED BETWEEN SETS OF
CALCULATIONS. AT THE END OF THE SET OF CALCULATIONS, THE PROGRAM
WILL ASK:

(REFERBACK 6):

DO YOU WANT TO CONTINUE PROCESSING...:

IF NO OR N IS ENTERED THE PROCESSING OF PCRC STOPS.
IF YES OR Y IS ENTERED OR IF THE CALCULATIONS HAVE BEEN
BYPASSED THE PROGRAM ASKS:

(REFERBACK 7):

DO YOU WANT TO RE-INITIALIZE ALL VALUES...:

IF YES OR Y IS ENTERED, THE PROGRAM WILL ASK FOR INPUT DATA
LINE BY LINE AS IN THE INITIATION OF THE PROGRAM (REFERBACK 1)
AND BYPASS FURTHER QUESTIONS FOR RE-INITIATION.
IF NO OR N IS ENTERED, THE PROGRAM WILL ASK:

DO YOU WANT TO RE-INITIALIZE THE VALUES OF AA(I)...:

IF YES OR Y IS ENTERED, THE PROGRAM WILL ASK FOR VALUES AS IN
THE INITIAL READ FOR AA(I) (REFERBACK 2).
REGARDLESS OF THE PRECEDING ANSWER, THE PROGRAM WILL ASK:

DO YOU WANT TO RE-INITIALIZE THE VALUES OF COST(J)...:

IF YES OR Y IS ENTERED, THE PROGRAM WILL ASK FOR VALUES AS IN
THE INITIAL READ FOR COST(J) (REFERBACK 3).
REGARDLESS OF THE PRECEDING ANSWER, THE PROGRAM WILL ASK:

ENTER THE VARIABLE NAME Whose VALUE IS TO BE RE-INITIALIZED...

THE USER SHOULD BE AWARE THAT THE PROGRAM SEARCHES FOR THE
VARIABLE NAME ENTERED. THE USER SHOULD ENTER ONE OF THE
VARIABLE NAMES EXACTLY AS LISTED:
THE PROGRAM WILL LOOK FOR THE VARIABLE NAME ENTERED AND, IF THE VARIABLE NAME IS NOT FOUND, THE PROGRAM WILL RETURN ASKING IF ADDITIONAL RE-INITIALIZATIONS ARE WANTED (REFERBACK 8). IF THAT VARIABLE NAME IS FOUND, THE PROGRAM WILL RESPOND WITH:

ENTER THE NEW VALUE...:

THE USER WILL ENTER THE NEW VALUE, AND THAT VALUE WILL BE ASSIGNED TO THE VARIABLE NAME PREVIOUSLY ENTERED.

THE USER WILL BE ABLE TO CHANGE THE VALUE OF ONLY ONE VARIABLE (BY ENTERING VARIABLE NAME AND VALUE IN TWO ENTRIES) AND THE PROGRAM WILL ASK:

(REFERBACK 8):

DO YOU WANT TO RE-INITIALIZE ADDITIONAL VALUES...:

IF THE USER REQUESTS ADDITIONAL CHANGES BY ENTERING YES OR Y, THE PROGRAM WILL RESPOND AS IN RE-INITIALIZATION OF VALUES

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY PURCHASED TO DD0
(REFERBACK 8). If the user enters NO or N, the program will respond with a listing of input data for user approval (REFERBACK 5).

(Note: After terminating processing of PCRC, if the user is processing additional programs before logout, the user must enter: CONNECT OUTPUT after a command request. Output was returned (disconnected) initially to bypass printing of numerous trivial error warnings.)
COMMAND- ATTACH,FILE2,PCRC

PF CYCLE NO. = 001
COMMAND- ATTACH,LIB,IMSL,LD=LIBRARY,SN=ASD

PF CYCLE NO. = 999
COMMAND- LIBRARY,LIB

COMMAND- RETURN OUTPUT
COMMAND- CONNECT TAPE9
COMMAND- FTN,I=FILE2,L=0

2.173 CP SECONDS COMPILATION TIME
COMMAND- CONNECT TAPE1

COMMAND- LGO

ENTER UT,UB,UM,US ..: 2000,.2614E-03,1.00,3.00
ENTER FOALPH,F0BETA ..: 2,.0190
ENTER FRALPH,FRBETA ..: 2,.0190
ENTER HALPH,HBETA ..: 1,.1
ENTER DELTA,ICYCLE,N ..: .004,6,1000
ENTER AA(I),I=1,4 ..: 0,.016,.032,.064
ENTER COST(J),J=1,4 ..: 3,3,3,100
DO YOU WANT MULTIPLE VALUES FOR AIC.: N
ENTER VALUE FOR AIC.: 5000
CRACK SIZE DATA

INITIAL FLAW SIZE DISTRIBUTION—WEIBULL
SHAPE PARAMETER = FOALPH = 2.0
SCALE PARAMETER = FOBETA = .0190

CRACK GROWTH PARAMETERS
INSPECTION INTERVAL = UT = 2000.
GROWTH RATE = UB = .2614E-03
USAGE BIAS = UM = 1.00
FLT BY FLT VARIABILITY = US = 3.00

PROBABILITY OF CRACK DETECTION—WEIBULL
SHAPE PARAMETER = HALPH = 1.0
SCALE PARAMETER = HBETA = .1000

REPAIRED FLAW SIZE DISTRIBUTION—WEIBULL
SHAPE PARAMETER = FRALPH = 2.0
SCALE PARAMETER = FRBETA = .0190

PROGRAM PARAMETERS
CRACK LENGTH INCREMENTS = DELTA = .004
NUMBER OF INSPECTIONS = ICYLE = 6

INSPECTION AND REPAIR PARAMETERS
NUMBER OF ITEMS = N = 1000
INSPECTION FLAW SIZE = AA(I),I=1,4 = 0.000, .016, .032, .064,
REPAIR COSTS = COST(J),J=1,4 = 3.00, 3.00, 3.00, 100.0
INSPECTION COST = AIC(II),II=1,ICYLE = 5000., 5000., 5000.,
                  5000., 5000., 5000.,

IS CRACK SIZE DATA CORRECT...: Y
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<th>F+(A)</th>
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MIN. A 0.000 MAX. A .128

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<td>0.086637</td>
</tr>
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<td>3</td>
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<tr>
<td>4</td>
<td>0.064 - 1.28</td>
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K(1) = 0.238658

E(C) S(C)
6655. 313.

CUMULATIVE TOTALS
E(C) S(C)
6655. 313.
APPENDIX B

PROGRAM LISTING
```
100  PROGRAM PCRC( INPUT, OUTPUT, TAPE1=100, TAPE9=100 )
110  COMMON/INT/ A, NN, AK, XX, (503), FM(503), FPL(503)
120  COMMON/CON/ C(500,3), NIP
130  COMMON/UCOM/ UT, UB, UM, US
140  COMMON/FCOM/ FOALPH, FOBETA
150  COMMON/HCOM/ HALPH, HBETA
160  COMMON/FRCOM/ FRALPH, FRBETA
170  DIMENSION BPAR(4), AA(5), HH(503), PO(4), COST(4), ICYCLE(20), XYZ(5)
180  EXTERNAL FLMINUS
190  DATA YES/3HYES/, V/1H'V /
200  DATA BPAR/40.0/
210  WRITE (9,1000) WRITE (9,1000)
220  FORMAT(4X, ENTER UT, UB, UM, US ..., 10, 4X)
230  READ(1,10, UB, UM, US
240  WRITE (9,1010)
250  FORMAT(4X, ENTER FOALPH, FOBETA ..., 10, 4X)
260  READ(1,10, FOALPH, FOBETA
270  WRITE (9,1020)
280  FORMAT(4X, ENTER FRALPH, FRBETA ..., 10, 4X)
290  READ(1,10, FRALPH, FRBETA
300  WRITE (9,1030)
310  FORMAT(4X, ENTER HALPH, HBETA ..., 10, 4X)
320  READ(1,10, HALPH, HBETA
330  WRITE (9,1040)
340  FORMAT(4X, ENTER DELTA, ICYCLE, N ..., 10, 4X)
350  READ(1,10, DELTA, ICYCLE, N
360  WRITE (9,1050)
370  FORMAT(4X, ENTER AA(I), I=1,4 ..., 10, 4X)
380  READ(1,10, AA(I), I=1,4
390  WRITE (9,1060)
400  FORMAT(4X, ENTER COST(J), J=1,4 ..., 10, 4X)
410  READ(1,10, COST(J), J=1,4
420  WRITE (9,1865)
430  FORMAT(4X, DO YOU WANT MULTIPLE VALUES FOR AIC ..., 10, 4X)
440  READ 9891, REPLY
450  IF (REPLY .EQ. YES, OR, REPLY .EQ. Y) GO TO 235
460  WRITE (9,7654)
470  READ(AICCLU)
480  DO 101 II=1, ICYCLE
490  AIC(II)=AICCLU
500  CONTINUE
510  GO TO 1500
520 写道 (9,8543)
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690 写道 (9,8543)
700 写道 (9,8543)
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710 FORMAT (8X,*SHAPE PARAMETER = FRALPH = #,F4.1)
720 FORMAT (8X,*SCALE PARAMETER = FRBETA = #,F6.4)
730 FORMAT (8X,*INSPECTION AND REPAIR PARAMETERS*)
740 FORMAT (8X,*INSPECTION FLAW SIZE = AA(I),I=1,4 = #)
750 FORMAT (8X,*REPAIR COSTS = COST(J),J=1,4 = #)
760 FORMAT (8X,*INSPECTION COST = AIC(I),I=1,ICYCLE = #)
770 FORMAT (8X,*NUMBER OF ITEMS N5.16)
780 FORMAT (8X,*PROGRAM PARAMETERS*)
790 FORMAT (8X,*CRACK LENGTH INCREMENTS DELTA = #,F5.3)
800 FORMAT (8X,*IS CRACK SIZE DATA CORRECT..#*#X*)
CALL DCSGUD1XX,HH,NIP,CC-NIP-1,XYZ(I),XYZ(I+1),FG(I),IER)
1320  A=AM+FQ(I)
1330  CONTINUE
1340  DO 60 I=1,NIP
1350  A=XY(I)
1360  FFL(I)=FPLUS(I)
1370  CONTINUE
1380  WRITE (9,2000)
1390  FORMAT(11//4X,20(1H8),12.9 TH CYCLE #20(IH8)))
1400  WRITE(1,2005)
1410  FORMAT(///8X,1HA,12X#F-(A#12X#F(A)12#)
1420  DO 65 I=1,NIP
1430  WRITE(1,2010)XX(I)+FMI(I)+FPL(I)
1440  WRITE(1X,3(E15.7,1X))
1450  CONTINUE
1460  WRITE (9,2012)AA(5)
1470  FORMAT(4X,*MIN. 0.000 MAX. A #5.3)
1480  WRITE (9,2015)
1490  FORMAT(///1X,1HJ,LOW UP6X#F(I,J))
1500  DO 70 J=1,4
1510  WRITE (9,2020)I,XY(I),XY(I+1),FG(I)
1520  FORMAT(11X,12X,F5.3,8#F5.3,2X,F12.6)
1530  CONTINUE
1540  WRITE (9,2025)AA(5)
1550  FORMAT(///12X,#K#5.3)
1560  EC=AIC(I)
1570  SGC=0.0
1580  DO 120 J=1,4
1590  EC=FC1FLOAT(N)#PO(J)*COST(J)
1600  SGC=SGC+COST(J))#2#FLOAT(N)#PO(J))#1.0-PO(J))
1610  CONTINUE
1620  WRITE (9,2030)EC,SGC
1630  FORMAT(///13X,#E(C)#9X,4HS(C))//6X,2(3X,F12.6))
1640  ELTOT=ECOTOT1
1650  SSOTOT=SSQTOT+SSGC
1660  WRITE (9,2040)ELTOT,SSOTOT,EC
1670  FORMAT(///1X,#CUMULATIVE TOTALS#13X,#E(C)#9X,4HS(C))
1680  = 0 /6X,2(3X,F12.0))
1690  CONTINUE
1700  C
1710  WRITE (9,2050)ECOTOT,SSOTOT
1720  FORMAT(11//4X,10(1H8),# TOTALS #10(1H8)///,10X,#E(C)#TOTAL#)
1730  9880  # 10X,10HS(C)-TOTAL//6X,2(3X,F12.0))
1740  9890  AIC(A)
1750  9795  WRITE(4X,*ENTER THE NEW VALUE...#4X)
1760  9750  WRITE (9,6000)
1770  6000  FORMAT(///#DO YOU WANT TO CONTINUE PROCESSING...#4X)
1780  6001  READ 9881,REFLY
1790  IF (REPLY .NE. YES .AND. REPLY .NE. Y ) GO TO 200
1790  6001  WRITE (9,7000)
1800  7000  FORMAT (4X,#DO YOU WANT TO RE-INITIALIZE ALL VALUES...#4X)
1810  7001  READ 9881,REFLY
1820  IF (REPLY .NE. YES .OR. REPLY .EQ. Y ) GO TO 1001
1830  7001  WRITE (9,7777)
1840  7777  WRITE (9,1000)
1850  1000  READ (AA(I),I=1,4)
1860  7777  WRITE (9,7777)
1870  7777  FORMAT (4X,#DO YOU WANT TO RE-INITIALIZE THE VALUE OF COST(J)...#)
1880  7777  #4X)
1910 READ Y881,KEY1
1920 IF (KEY1.GE.6 AND. KEY1.GE.9) GO TO 7001
1930 WRITE (9,1060)
1940 READ*((COST(J),J=1,4)
1950-7001 WRITE (9,8000)
1960 IF NN (4X,#ENTER THE VARIABLE NAME WHOSE VALUE IS TO BE RE—INITIA
1970 .LIZED. (1*4X)
1980 READ Y881,KEY1
1990 IF (KEY1.GE.2HUT) GO TO 8001
2000 WRITE (9,9876)
2010 READ*18
2020 GO TO 99999
2030-8001 IF (KEY1.GE.2HUT) GO TO 9001
2040 WRITE (9,9876)
2050 READ*18
2060 GO TO 99999
2070 IF NN (4X.#DO YOU WANT TO RE-INITIALIZE THE VALUE OF A (I). .18
2080 .4X)
2090-9001 IF (KEY1.GE.2HUM) GO TO 10001
2100 WRITE (9,9876)
2110 READ*18
2120 GO TO 99999
2130-10001 IF (KEY1.GE.2HUS) GO TO 11001
2140 WRITE (9,9876)
2150 READ*18
2160 GO TO 99999
2170-11001 IF (KEY1.GE.6HFOALPH) GO TO 12001
2180 WRITE (9,9876)
2190 READ*FALPH
2200 GO TO 99999
2210-12001 IF (KEY1.GE.6HFOBETA) GO TO 13001
2220 WRITE (9,9876)
2230 READ*FOBETA
2240 GO TO 99999
2250-13001 IF (KEY1.GE.6HFRALPH) GO TO 14001
2260 WRITE (9,9876)
2270 READ*FRALPH
2280 GO TO 99999
2290-14001 IF (KEY1.GE.6HFRBETA) GO TO 15001
2300 WRITE (9,9876)
2310 READ*FRBETA
2320 GO TO 99999
2330-15001 IF (KEY1.GE.5HHALPH) GO TO 16001
2340 WRITE (9,9876)
2350 READ*HALPH
2360 GO TO 99999
2370-16001 IF (KEY1.GE.5HHBETA) GO TO 17001
2380 WRITE (9,9876)
2390 READ*HBETA
2400 GO TO 99999
2410-17001 IF (KEY1.GE.5HDELT A) GO TO 18001
2420 WRITE (9,9876)
2430 READ*DELTA
2440 GO TO 99999
2450-18001 IF (KEY1.GE.6HICYCLE) GO TO 19001
2460 WRITE (9,9876)
2470 READ*ICYCLE
2480 WRITE (9,8745)
2490 READ Y881,KEY1
2500 IF (KEY1.GE.6 AND. KEY1.GE.9) GO TO 8642
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:510-  GO TO 236
:520-19001 IF (REPLY .NE. 'Y') GO TO 18003
:530-  WRITE (9,9876)
:540-  READ N
:550-  GO TO 99999
:560-18003 IF (REPLY .NE. 'NO') GO TO 99999
:570-  WRITE (9,9745)
:580-  READ Y001+REPLY
:590-  IF (REPLY .EQ. 'YES' .OR. REPLY .EQ. 'Y') GO TO 236
:600-7654 FORMAT ('XENTER VALUE FOR AIC...184X')
:610-8642 WRITE (9,7654)
:620-  READ AICVLU
:630-  GO 202 II+1ICYCLE
:640-  AIC(II)=AICVLU
:650-202 CONTINUE
:660-  GO TO 99999
:670-236 WRITE (9,6543)
:680-  READ+AIC(I)),II+1ICYCLE)
:690-99999 CONTINUE
:700-  WRITE (9,1234)
:710-1234 FORMAT ('XYOU WANT TO RE-INITIALIZE ADDITIONAL VALUES...184X')
:720-  READ Y001+REPLY
:730-  IF (REPLY .EQ. 'YES' .OR. REPLY .EQ. 'Y') GO TO 7001
:740-  GO TO 1500
:750-200 CONTINUE
:760-  STOP
:770-  END
```

FUNCTION FMINUS(X)
COMMON/INIT/ A,II;AK,XX(503),FMI(503),FPL(503)
UP2=UP2-U2(X)
IF(II.GT.1) GO TO 10
TEMP=FQ(UF2)
GO TO 20
TEMP=FLNUM(UP2)
TEMP2=U2(X)
IF(FMINUS.LT.0.0) FMINUS=0.0
RETURN
END

FUNCTION U20(X)
COMMON/UCOM/ UT,UB,UM,US
U20=EXP(-UB*(UM+UT+SORT(UT)+US*X))
RETURN
END

FUNCTION F0(X)
COMMON/F0COM/ FOALPH,FOBETA
IF(X.LE.0.) GO TO 10
TERM2=(X/FOBETA)*FOALPH
10 IERM=0.
10 IF(XLE.0.)GO TO 10
H=1.0 EXP(TERM)
RETURN
END

FUNCTION H(X)
COMMON/HCOM/ HALPH,HBETA
IF(X.LE.0.) GO TO 10
TERM=(X/HBETA)*HALPH
GO TO 20
TERM=0.
H=1.0 EXP(TERM)
RETURN
END
4150- FUNCTION FPLUS(X, I)
4160- COMMON/INT/ A+I1;AK,XX(503),FMI(503),FPL(503)
4170- FPLUS=(AK+RI(X))+((1.0-H(X))#FMI(I))
4180- RETURN
4190- END

3200- FUNCTION FR(X)
3210- COMMON/FROM/ FRALPH,FRBETA
3220- IF (X .LE. 0.) GO TO 10
3230- TERM1=(X#FRBETA)##(FRALPH-1.)
3240- TERM2=(X#FRBETA)##FRALPH
3250- GO TO 20
3260- 10 TERM1=TERM1=0.
3270- 20 FR=FRALPH#FRBETA#TERM1#EXP(TERM2)
3280- RETURN
3290- END

3300- FUNCTION GI(X)
3310- GI=1.0/SQRT(2.0*3.141592654)*EXP(-X##2/2.0)
3320- RETURN
3330- END

3400- FUNCTION FPLIN(X)
3410- COMMON/INT/ A+I1;AK,XX(503),FMI(503),FPL(503)
3420- COMMON/COM/ CC(500,3);NIP
3430- DIMENSION Z(1);SOL(1)
3440- Z(I)=X
3450- CALL IC9EVU XX=FPLIN,NIP,CC,NIP_1,Z;SOL,1;IER)
3460- FPLIN=SOL(1)
3470- RETURN
3480- END
LIST OF REFERENCES


