LEVEL

RADC-TR-78-165, Volume V (of five)
Final Technical Report
October 1978

BAYESTAN SOFTWARE PREDICTION MODELS
Summary of Technical Progress

Amee L. Walt
Syracuse University

Approved for public release; distribution unlimited.

NOAA AIR DEVELOPMENT CENTER
Air Force Systems Command
Caldwell Air Force Base, New York 13007

79 04 20 007
This report has been reviewed by the RADC Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS it will be releasable to the general public, including foreign nations.

RADC-TR-78-155, Volume V (of five) has been reviewed and is approved for publication.

APPROVED:

ALAN H. SUKERT
Project Engineer

APPROVED:

WENDALL C. BAUMAN, Colonel, USAF
Chief, Information Sciences Division

FOR THE COMMANDER:

JOHN P. HUSS
Acting Chief, Plans Office

If your address has changed or if you wish to be removed from the RADC mailing list, or if the address is no longer employed by your organization, please notify RADC (1S1B) Griffiss AFB NY 13441. This will assist us in maintaining a current mailing list.

Do not return this copy. Retain or destroy.
This report summarizes the technical activities pursued under Contract F30602-76-C-0097, Bayesian Software Prediction Models, with Rome Air Development Center. Research work, discussed in previous volumes, discussing imperfect debugging and imperfect maintenance software performance models, is summarized and some additional work in development of software reliability demonstration test plans is described.
# TABLE OF CONTENTS

1. INTRODUCTION .............................................. 2

2. A SOFTWARE PERFORMANCE MODEL UNDER IMPERFECT DEBUGGING . 4
   2.1 Model and Main Results. ............................... 4
      2.1.1 Distribution to time to a specified number of remaining errors. ................. 5
      2.1.2 Probability distribution of a given number of remaining errors at time \( t \) ......... 6
      2.1.3 Expected number of total and imperfect debugging errors .......................... 6
      2.1.4 Reliability function ................................. 7
      2.1.5 Gamma approximation ................................. 7
      2.1.6 Numerical examples ................................. 8
   2.2 Analysis of Total and Imperfect Debugging Errors in a Real Time Control System. .... 11

3. AVAILABILITY ANALYSIS OF SOFTWARE SYSTEMS UNDER IMPERFECT MAINTENANCE. .......... 12
   3.1 Model and Performance Measures. ........................ 12
   3.2 Numerical Example ..................................... 14
   3.3 A Nomogram for the Expected Time to a Specified Number of Errors and to Determine Manpower Requirements. ......................... 14

4. BAYESIAN AND CLASSICAL INFERENCE FOR THE IMPERFECT DEBUGGING AND MAINTENANCE MODELS .......... 20
   4.1 Maximum Likelihood Method ............................. 20
   4.2 Bayesian Inference ................................. 22
5. BAYESIAN SOFTWARE CORRECTION LIMIT POLICIES ........ 24
6. SOFTWARE RELIABILITY DEMONSTRATION TEST PLANS ....... 26
7. COMPUTER PROGRAMS ........................................... 27
SELECTED REFERENCES ........................................... 28

APPENDIX A AN ANALYSIS OF RECURRENT SOFTWARE ERRORS IN A REAL-TIME CONTROL SYSTEM ................. A-1

APPENDIX B SOFTWARE RELIABILITY DEMONSTRATION TEST PLANS ................................................. B-1

APPENDIX C COMPUTER PROGRAMS ................................ C-1

C.1 Programs for the Imperfect Debugging Model (Section 2) ........ C-1

C.2 Programs for Simulation of Imperfect Debugging Data ....... C-10

C.3 Programs for the Imperfect Maintenance Model of Section 3 ... C-17

C.4 Program for Bayesian Software Correction Limit Policies (Section 5) .... C-26
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Probability Distribution of Time to $n_0$ Remaining Errors</td>
<td>9</td>
</tr>
<tr>
<td>2.2</td>
<td>Expected Number of Remaining Errors versus Time</td>
<td>10</td>
</tr>
<tr>
<td>3.1</td>
<td>Plots of State Occupancy Probabilities and Software System Availability</td>
<td>15</td>
</tr>
<tr>
<td>3.2</td>
<td>Expected Number of Software Errors Detected and Corrected by Time $t$</td>
<td>16</td>
</tr>
<tr>
<td>3.3</td>
<td>A Nomogram for the Expected Time to a Specified Number of Errors</td>
<td>18</td>
</tr>
<tr>
<td>5.1</td>
<td>Sequence of Corrective Actions in Operational Phase</td>
<td>25</td>
</tr>
<tr>
<td>A.1</td>
<td>Joint confidence regions for $N$ and $\lambda$ for $p=p$</td>
<td>A-5</td>
</tr>
<tr>
<td>A.2</td>
<td>Actual and fitted SPRs by month</td>
<td>A-6</td>
</tr>
<tr>
<td>A.3</td>
<td>Plots of the actual and predicted number of remaining errors</td>
<td>A-7</td>
</tr>
<tr>
<td>B.1</td>
<td>Contours of $\tilde{\alpha}, \beta^*$ for Design of Test Plans</td>
<td>B-18</td>
</tr>
<tr>
<td>B.2</td>
<td>Contours of $\tilde{\alpha}, \beta^{**}$ for Design of Test Plans</td>
<td>B-19</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1 Error Data by Month</td>
<td>A-2</td>
</tr>
<tr>
<td>A.2 A Summary of Error Data Analyses</td>
<td>A-4</td>
</tr>
<tr>
<td>C.1 Subroutine FRSTPT</td>
<td>C-2</td>
</tr>
<tr>
<td>C.2 Subroutine FPT2</td>
<td>C-3</td>
</tr>
<tr>
<td>C.3 Subroutine State</td>
<td>C-4</td>
</tr>
<tr>
<td>C.4 Subroutine Mean</td>
<td>C-5</td>
</tr>
<tr>
<td>C.5 Subroutine TBF</td>
<td>C-6</td>
</tr>
<tr>
<td>C.6 Subroutine MDGAMM</td>
<td>C-7</td>
</tr>
<tr>
<td>C.7 Subroutine GAMMA</td>
<td>C-9</td>
</tr>
<tr>
<td>C.8 Subroutine SMLT</td>
<td>C-11</td>
</tr>
<tr>
<td>C.9 Subroutine MLE</td>
<td>C-12</td>
</tr>
<tr>
<td>C.10 Subroutine BAYES</td>
<td>C-14</td>
</tr>
<tr>
<td>C.11 Subroutine GGUB</td>
<td>C-16</td>
</tr>
<tr>
<td>C.12 Subroutine COMP</td>
<td>C-18</td>
</tr>
<tr>
<td>C.13 Subroutine FIRST</td>
<td>C-19</td>
</tr>
<tr>
<td>C.14 Subroutine STT</td>
<td>C-20</td>
</tr>
<tr>
<td>C.15 Subroutine AVAIL</td>
<td>C-21</td>
</tr>
<tr>
<td>C.16 Subroutine EXPCT</td>
<td>C-22</td>
</tr>
<tr>
<td>C.17 Subroutine MDGAMM</td>
<td>C-23</td>
</tr>
<tr>
<td>C.18 Subroutine GAMMA</td>
<td>C-25</td>
</tr>
<tr>
<td>C.19 Subroutine MDL1</td>
<td>C-27</td>
</tr>
<tr>
<td>C.20 Subroutine MDL2</td>
<td>C-28</td>
</tr>
<tr>
<td>C.21 Subroutine DATA1</td>
<td>C-30</td>
</tr>
<tr>
<td>C.22 Subroutine GGUB</td>
<td>C-31</td>
</tr>
<tr>
<td>C.23 Subroutine OPTMM</td>
<td>C-32</td>
</tr>
</tbody>
</table>
EVALUATION

The necessity for more complex software systems in such areas as command and control and intelligence has led to the desire for better methods for predicting software errors and reliability to insure that software produced is of higher quality and of lower cost. This desire has been expressed in numerous industry and Government sponsored conferences, as well as in documents such as the Joint Commanders' Software Reliability Working Group Report (November 1975). As a result, numerous efforts have been initiated to develop and validate mathematical models for predicting such quantities as the number of remaining errors in a software package and the time to achieve a desired level of reliability. In addition, efforts have been initiated to develop better methods for determining when a software package should be released to a potential user. However, these efforts have not produced measures with the desired accuracy or confidence for general applicability.

This effort was initiated in response to this need for developing better and more accurate software error prediction and demonstration tests and fits into the goals of RADC TPO No. 5, Software Cost Reduction in the subthrust of Software Quality (Software Modeling). This report summarizes the development of mathematical models for predicting quantities such as the expected number of errors during both software development and software maintenance. These models assume errors are not corrected with probability 1, i.e. imperfect development and maintenance. The report also describes the development of statistical tests for determining whether a software package should be accepted or rejected after completion of testing. The importance of these developments is that they represent the first attempt to develop both software error prediction models that incorporate imperfect debugging and thus more closely reflect the actual software error detection and correction process, and software demonstration tests that allow better statistical criteria for accepting a software package.

The theory and equations developed under this effort will lead to much needed predictive measures for use by software managers in more accurately tracking software development projects in terms of stated error and reliability objectives. In addition, the associated confidence limits and other related statistical quantities developed under this effort will insure more widespread use of these techniques. The acceptance criteria developed will permit better control of the release of software packages so that software is not given to a potential user before it is ready for operational usage. Finally, the measures developed under this effort will be applicable to current software development projects and thus help to produce the high quality, low cost software needed for today's systems.

ALAN N. SUKERT
Project Engineer
ABSTRACT

This report provides a summary of the technical activities pursued under Contract F30602-76-C-0097 with RADC during January 1976-April 1978. Research work discussed in previous reports under this contract is summarized and some additional work is described. Also included is a brief description of research in progress.
1. INTRODUCTION

During the past ten years the field of software engineering has grown considerably in importance and scope. A primary motivation for this growth has come from an ever increasing cost of developing and maintaining software systems. This is specially true for the DOD which needs high quality, low cost software for its operations. As a result of this increased importance, various fields within software engineering are maturing into disciplines of study and research, for example, software design techniques, structured programming and other improved programming methodologies, program testing and debugging techniques, software performance modelling, and techniques of program validation and verification. The ultimate objective of studies in all these fields is to develop tools that will be useful in the design, development and operational phases of the software life cycle. The objective of studies dealing with software error analysis and modelling has been to develop analytical tools which can be used for improving software performance. Such studies can be classified into one (or both) of two categories. In the first category the emphasis is on the analysis of software error data collected from small or large projects, during development and/or operational phases. Studies in the second category are primarily aimed at the development of analytical models which are then used to obtain the reliability and other quantitative measures of software performance.

Typical of the first category are the studies by Akiyama [1], Belady and Lehman [3], Fries [6], Endres [5], Baker [2], Motley et al [18], Miyamoto [16], Willman et al [35], Schneidewind [26],
Shooman et al [29], Suker [30,31], Rye et al [24], Thayer et al [32], and Wagoner [34]. These studies range in size from an analysis of small data sets (108 errors), e.g. Wagoner [34], to analysis of large sets (3500 errors), e.g. Thayer et al [32] and encompass data from an on-line system [16], an operating system [3], to that from the Apollo project [24].

In the second category of papers, several models have been proposed and studied during the last six years. These include 'exponential type' models of Shooman [28], Jelinski and Moranda [11,12], and Schick and Wolverton [25]; models based on the non-homogeneous Poisson process proposed by Goel and Okumoto [9] and Schneidewind [27], and a Bayesian model by Littlewood and Verrall [15]. Halstead [10] has developed a theory based on 'software physics' for various measures of the performance of a software system.

Musa [19] has introduced a model which is based on a large number of parameters derived from the software system being modelled. Trivedi and Shooman [33] consider a Markov model in which they incorporate the time spent for removal of errors.

Most of the above studies assume that errors are removed with certainty when detected. The purpose of the investigation summarized in this report is to develop and study models for software performance which account for the probabilistic nature of the programmer's action during debugging and operational phases of the software system, to provide a methodology for classical and Bayesian inference for various quantitative measures of performance, to develop optimum Bayesian software correctional limit policies, and to develop software reliability demonstration test plans. Results of these studies are summarized in Sections 2 through 6.
2. A SOFTWARE PERFORMANCE MODEL UNDER IMPERFECT DEBUGGING

The purpose of this modelling effort was to establish quantitative measures for software systems by incorporating the probabilistic nature of the programmer's actions during the debugging phase. Such occurrences have been termed as recurrent errors by Fries [6], Thayer [32] and Willman et al [35], erroneous debugging by Miyamoto [16], bad fixes by Jones [13] and accounted as an error reduction factor by Musa [19]. In this study we call them the imperfect debugging errors.

Even though the presence of the imperfect debugging phenomenon has been known, no published model, with the possible exception of Musa's error reduction factor, provides an explicit way to account for it in software performance prediction. The model and other related results for this topic are summarized below. Details of this work are given in [7]. These results are useful for software development personnel in establishing manpower requirements to achieve a desired quality in the system before it is released for operational use. Trade-off studies between cost of debugging and software quality can be undertaken using the results given below.

2.1 Model and Main Results

The following parameters are used for model development and analysis:

\[ N = \] the initial number of errors in the software system at the beginning of the debugging activity
\[ p = \] probability that the error causing a software failure is removed when detected
\( q = 1-p \), the probability of imperfect debugging
\( \lambda = \) software error occurrence rate

Let a random variable \( X(t) \) denote the number of errors remaining in the system at time \( t \). Then \( X(t) \) describes the state of the system at time \( t \). We consider the stochastic process \( \{X(t), t \geq 0\} \) to be a semi-Markov process with the one-step transition probability, \( Q_{ij}(t) \), the probability that the next failure resulting in \( j \) remaining errors will be by time \( t \) when a software system has \( i \) remaining errors at time zero. Then

\[
Q_{ij}(t) = \begin{cases} 
    p(1-e^{-\lambda t}) & \text{if } j = i - 1 \\
    q(1-e^{-\lambda t}) & \text{if } j = i
\end{cases}
\]

If we start with \( N \) errors at \( t=0 \), we are interested in the expressions for various quantities that describe the software system performance. These quantities are given below.

2.1.1 Distribution to time to a specified number of remaining errors

Let \( G_{N,n_0}(t) \equiv \Pr(T_{N,n_0} \leq t) \) be the cdf of the time \( T_{N,n_0} \), required to obtain a software system with \( n_0 \) remaining errors, \( n_0 = 0, 1, 2, \ldots, N-1 \). Then

\[
G_{N,n_0}(t) = \sum_{j=1}^{N-n_0} B_{N,j,n_0} \{1-e^{-\lambda t}\}^{-(n_0+j)\lambda t}
\]

where

\[
B_{N,j,n_0} = \frac{N!}{n_0!j!(N-n_0-j)!} (-1)^{j-1} \cdot \frac{j}{n_0+j}.
\]
The expected time required to obtain a software system with \( n_0 \) remaining errors is given by

\[
E[T_{N,n_0}] = \sum_{j=1}^{N-n_0} B_{N,j;n_0}/(n_0+j)p^j.
\]

2.1.2 Probability distribution of a given number of remaining errors at time \( t \)

Probability that there are \( n_0 \) remaining errors at time \( t \) is

\[
P_{N,n_0}(t) = \mathbb{P}\{X(t)=n_0|X(0)=N\} = G_{N,n_0}(t) - G_{N,n_0-1}(t), \quad n_0=0,1,2,\ldots,N,
\]

where

\[
G_{N,N}(t)=1
\]

and

\[
G_{N,-1}(t) = 0.
\]

Also, the expected number of errors remaining at time \( t \) is

\[
E[X(t)|X(0)=N] = N e^{-\lambda t}.
\]

2.1.3 Expected number of total and imperfect debugging errors

The expected total number of errors, \( M_N(t) \), and errors due to imperfect debugging, \( D_N(t) \), during a debugging time period \( t \) are given by
\[ M_N(t) = \frac{N}{P} (1-e^{-\lambda pt}) , \]

and

\[ D_N(t) = q \cdot M_N(t) . \]

2.1.4 Reliability function

The software system reliability between (k-1)st and kth failures is given by

\[ R_k(x) = \sum_{j=0}^{k-1} \binom{k-1}{j} p^{k-j-1} q^{j} e^{-\lambda x} . \]

2.1.5 Gamma approximation

The computation of the quantity \( B_{N,j,n_0} \) and hence \( G_{N,n_0}(t) \) becomes cumbersome when \( N \) is large. We have found that the following gamma approximation yields satisfactory results for large software systems with large values of \( N \).

\[ G_{N,n_0}(t) \approx \frac{t^\beta \cdot (\beta x)^{\alpha-1}}{\Gamma(\alpha)} \cdot e^{-\beta x} . \]

where the scale parameter \( \beta \) and the shape parameter \( \alpha \) are estimated as

\[ \beta = p \lambda \frac{\sum_{j=n_0+1}^{N} 1/j}{\sum_{j=n_0+1}^{N} 1/j^2} , \]

and

\[ \alpha = \frac{\sum_{j=n_0+1}^{N} \left( \frac{1}{j} \right)^2}{\sum_{j=n_0+1}^{N} \left( \frac{1}{j} \right)^2} . \]
2.1.6 Numerical examples

To illustrate the usefulness of the above results consider a software system with $N=100$, $\lambda=0.02$. The probability distributions of times to $n_0=0(1)10$ remaining errors are obtained as above and are shown in Figure 2.1 for $p=0.9$. We see that at $t=200$, say, the probability of having zero errors in the system is approximately 0.1, of one error about 0.15, of 2 about 0.23 and so on.

Plots of expected number of remaining errors at various times are given in Figure 2.2 for $p=0.8(0.05)1.00$. For $p=0.9$ and $t=100$, there will be about 18 errors left in the system. From a study of these plots, one can plan the resource requirements and also conduct trade-off studies between available resources and resulting product.
Figure 2.1 Probability Distribution of Time to $n_0$ Remaining Errors
Figure 2.2  Expected Number of Remaining Errors versus Time

\[ N = 100 \]
\[ \lambda = 0.02 \]
2.2 Analysis of Total and Imperfect Debugging Errors in a Real-Time Control System

The results described in Section 2.1 were used to analyze the software error data from a large-scale software project - a real-time control system for a land-based radar system developed by Raytheon Co. in a modular fashion, see Willman [35]. The data base was extracted from 2165 Software Problem Reports written against 109 operational software modules over the development phase. Details of this analysis are given in Appendix A.
3. AVAILABILITY ANALYSIS OF SOFTWARE SYSTEMS UNDER IMPERFECT MAINTENANCE

In this section we describe a model developed for the operational phase of the software system subject to imperfect error maintenance and also incorporate the time spent for error maintenance.

3.1 Model and Performance Measures

The following parameters are used in this model:

\( N \) = Initial number of errors in the software system at the beginning of software operation

\( p \) = Probability that the error causing a software failure is removed/maintained when detected.

\( q = 1-p \), the Probability of imperfect maintenance/removal

\( \lambda_i \) = Software error occurrence rate per unit time when there are \( i \) errors in the system

\( \nu_i \) = Software error correction/maintenance rate per unit time when \( i \) errors remain in the system

Consider a stochastic process \( \{X(t), t \geq 0\} \), whose states are defined as

\[
X(t) = \begin{cases} 
  i & \text{if the software system is operational while there are } i \text{ errors in the system (} i=0,1,2,\ldots,N \) \\
  D & \text{if the software system is down for error removal i.e., for maintenance.}
\end{cases}
\]

Then, the process \( \{X(t), t \geq 0\} \) forms a semi-Markov process with the one step transition probability (the probability that the next up-down cycle, resulting in \( j \) remaining errors, will be completed
by time $t$ when a software package has $i$ remaining errors at time zero) given by
\[
\begin{align*}
Q_{ij}^{(D)}(t) &= \begin{cases} 
-\lambda_i t -\mu_i t & \text{if } j = i-1 \\
q(1-e^{-\lambda_i t}) - p(1-e^{-\mu_i t}) & \text{if } j = i \\
0 & \text{otherwise}
\end{cases}
\end{align*}
\]

Based upon the above model, expressions for the following performance measures of the software system are derived by Okumoto and Goel [21]:

(i) Distribution of time from $N$ to a specified number of remaining errors $n_0$

(ii) Expected time required for the system to go from $N$ to $n_0$ errors, $E[T_{N,n_0}]$

(iii) Probability of the system being operational at some time with $n_0$ remaining errors, $P_{N,n_0}(t)$.

(iv) Software system availability, i.e., the probability of the system being operational at time $t$, $A(t) = \sum_{n_0=0}^{N} P_{N,n_0}(t)$

(v) Probability that the number of errors remaining in the system is $n$, $P(\bar{N}(t) = n)$ and the expected number of errors at $t$, $E(\bar{N}(t))$

(vi) Expected number of errors detected and corrected by $t$, denoted by $M_{N}^{D}(t)$ and $M_{N}^{C}(t)$, respectively.

The above quantities are useful for software managers for estimating the time and manpower requirements to achieve a desired level of performance during the operational phase of the software system.
3.2 Numerical Example

For illustration purposes consider the case when \( \lambda_i = i \lambda \), \( \mu_i = i \mu \), \( N = 100 \), \( p = 0.9 \), \( \lambda = 0.02 \) and \( \mu = 0.05 \). Using the expressions for various performance measures from (21), the plots of state occupancy probabilities and availability are given in Figure 3.1 and the plots of \( M_N^D(t) \) and \( M_N^C(t) \) are given in Figure 3.2. From a study of such plots for various values of \( \lambda, \mu \) and \( p \), one can obtain adequate information about the behavior of the software system as well as about the resource requirements to achieve a desired level of performance. Thus, if \( p \) is known to be 0.9, and the values of \( \lambda \) and \( \mu \) that can be provided for, then the system availability at \( t = 300 \) will be approximately 0.5. If this is not satisfactory, one has to provide additional or better resources that will yield better values for one or more parameters.

3.3 A Nomogram for the Expected Time to a Specified Number of Errors and to Determine Manpower Requirements

We present a simple nomogram to calculate the expected time required to remove a specified number of errors from a software system which will satisfy the desired performance requirements. We consider the case when \( \lambda_i = i \lambda \) and \( \mu_i = i \mu \), i.e., when the error detection and error correction rates are proportional to the number of remaining errors with \( \lambda \) and \( \mu \), respectively, the constants of proportionality. Letting the ratio \( \lambda/\mu = p \), we have

\[
E[T_{N,n_0}] = \frac{1+\theta}{p \lambda} \sum_{i=n_0+1}^{N} \frac{1}{i}.
\]
Figure 3.1 Plots of State Occupancy Probabilities and Software System Availability
Figure 3.2  Expected Number of Software Errors Detected and Corrected by Time t

\[ M_N^D(t) \]

\[ M_N^C(t) \]

- \( N = 100 \)
- \( \lambda = 0.02 \)
- \( \mu = 0.05 \)
- \( \rho = 0.9 \)
For a large software system, this can be approximated as

$$E[T_{N,n_0}] \approx \frac{(1+p)}{p\lambda} \log(P_{n_0+1}).$$

Letting $\phi(\phi,p) = (1+p)\phi$, where $\phi = \log\left(P_{n_0+1}\right)$, we get

$$E[T_{N,n_0}] = \frac{\phi(\phi,p)}{p\lambda}.$$

A nomogram which gives the contours of $\phi(\phi,p)$ in the $(\phi,p)$ phase is given in Figure 3.3. To illustrate the use of this nomogram, consider the case when $N=100$, $p=0.9$, $\lambda=0.02$ per day, $\mu=0.05$ per day and the desired value of $n_0$ is 10. We proceed as follows.

Step 1. Compute $\phi = \frac{\lambda}{\mu} = 0.02$ and $\phi = \log\left(P_{n_0+1}\right) = \log(100+1) = 0.963$. 

Step 2. Corresponding to $\phi = 0.963$ and $\rho = 0.4$, from Figure 3.3 the value of $\phi(0.963,0.4)$ is 3.2.

Step 3. Compute the value $E[T_{100,10}] = \frac{\phi(\phi,p)}{p\lambda} = \frac{3.2}{(0.9)(0.02)} = 177.8$

Thus, for the given conditions the expected time required to go from 100 to 10 errors is 177.8 days.

Now we consider another application of this nomogram to show how it can be used in determining the manpower requirements for a specified objective of remaining errors. Suppose we want to go from $N=1000$ to $n_0=10$ errors in $E[T_{1000,10}] = 500$ days when the error occurrence rate is $\lambda=0.01$ errors per day and the probability $p$ of perfect correction is 0.95. We are interested in determining the manpower requirement to accomplish this objective.

The first thing to determine is the value of $\mu$ that will satisfy these requirements. We know that
\[ \Phi(\phi, \rho) = 2.5(0.5)25 \]

Figure 3.3 A Nomogram for the Expected Time to a Specified Number of Errors
\[ \phi(\lambda, \rho) = p^\lambda E[T_{n_0}] \]

and hence

\[ \phi(\lambda, \rho) = (.95)(.01)(750) = 7.125. \]

From the nomogram in Figure 3.3, for \( \phi(\lambda, \rho) = 7.125 \) and 
\( \phi = \log\left(\frac{1001}{11}\right) = 1.96 \), the value of \( \rho = 0.58 \) and hence \( \mu = \lambda / \rho = 0.0172 \) errors/day. If the error removal rate per person per day is 0.01, 
then we will need 1.7 people to meet the desired objective.
4. BAYESIAN AND CLASSICAL INFERENCE FOR THE IMPERFECT DEBUGGING AND MAINTENANCE MODELS

In this section we describe two methods for statistical inference of the parameters of the models described in Sections 2 and 3. The first one is the classical approach based on maximum likelihood estimation and the second is a Bayesian approach based on the prior distributions of the unknown parameters. The parameters under consideration are the initial number of software errors \( N \), the error occurrence rate for each error \( \lambda \), and the probability of perfect debugging \( p \). An additional parameter for the imperfect maintenance model is \( \nu \). We give only the method for the model of Section 2. The same procedure can be used for the model of Section 3.

The available data for estimation purposes is generally given as \( t=(t_1, t_2, \ldots, t_n) \), the times between software failures and \( y=(y_1, y_2, \ldots, y_n) \), an indicator variable for imperfect debugging such that \( y_i=1 \) if the ith failure is caused by an error due to imperfect debugging and \( y_i=0 \) if the error is not due to imperfect debugging.

The maximum likelihood and Bayesian approaches to the estimation of the above parameters are summarized below. The details of the procedure are given in [20].

4.1 Maximum Likelihood Method

The likelihood function for \( N, p \) and \( \lambda \) is

\[
L(N, p, \lambda | t, y) = \prod_{i=1}^{n} \left( N - p(i-1) \right)^{y_i} \left( 1 - p(i-1) \right)^{1-y_i} \left( N - p(i-1) \right)^{\lambda t_i} \cdot \left( \frac{q(i-1)}{N-p(i-1)} \right)^{y_i} \cdot \left( \frac{N-(i-1)}{N-p(i-1)} \right)^{1-y_i}
\]
The maximum likelihood estimates ($\hat{N}, \hat{p}$ and $\hat{\lambda}$) are obtained as solutions of the simultaneous non-linear equations

\[
\lambda \sum_{i=1}^{n} t_i = \sum_{i=1}^{n} \frac{l-y_i}{N-(i-1)}
\]

\[
\lambda \sum_{i=1}^{n} (i-1)t_i = \sum_{i=1}^{n} \frac{y_i}{q}
\]

\[
n/\lambda = \sum_{i=1}^{n} \{N-p(i-1)\}t_i
\]

The joint 100(1-\(\alpha\))% confidence regions for \(N\), \(p\) and \(\lambda\) are obtained from

\[
\ell(\hat{N}, \hat{p}, \hat{\lambda} | \xi, \gamma) - \ell(N, p, \lambda | \xi, \gamma) = \frac{1}{2} \chi^2_{3; \alpha},
\]

where

\[
\ell(\cdot | \xi, \gamma) = \log L(\cdot | t; \gamma).
\]

The estimated variance–covariance matrix for \(\hat{N}, \hat{p}\) and \(\hat{\lambda}\) is

\[
\hat{\Sigma}_{\text{cov}} = \begin{bmatrix}
  r_{NN} & r_{NP} & r_{NL} \\
  r_{PN} & r_{PP} & r_{PL} \\
  r_{LN} & r_{LP} & r_{LL}
\end{bmatrix}
\]

where

\[
r_{NN} = \sum_{i=1}^{n} \frac{1}{(N-(i-1))N-p(i-1)}/(N-(i-1))(N-p(i-1))
\]

\[
r_{NP} = r_{PN} = 0
\]
\[ r_{NN} = r_{\lambda N} = \frac{1}{\lambda} \sum_{i=1}^{n} \frac{1}{(N-p(i-1))} \]

\[ r_{pp} = \begin{cases} \frac{1}{q} \sum_{i=1}^{n} \frac{(i-1)}{(N-p(i-1))} & \text{if } q \neq 0 \\ 0 & \text{if } q = 0 \end{cases} \]

\[ r_{p\lambda} = r_{\lambda p} = -\frac{1}{\lambda} \sum_{i=1}^{n} \frac{(i-1)}{(N-p(i-1))} \]

\[ r_{\lambda \lambda} = n/\lambda^2 . \]

4.2 Bayesian Inference

Now we describe a Bayesian approach for obtaining posterior point estimates and the highest posterior density (HPD) region for parameters \( N, p \) and \( \lambda \).

The choice of the prior distribution for a parameter is governed by several factors. In our case we take the conjugate priors, which for \( N \) and \( \lambda \) are gamma distributions while for \( p \) it is a beta distribution, i.e.,

\[ P(N) = N^{a-1} e^{-\beta N}, \quad N > 0 \]

\[ P(p) = p^{\alpha-1}(1-p)^{\beta-1}, \quad 0 \leq p \leq 1 \]

\[ P(\lambda) = \lambda^{\mu-1} e^{-\gamma \lambda}, \quad \lambda > 0 . \]

By applying Bayes theorem the joint posterior distribution of \( N, p \) and \( \lambda \) for given priors and the data is obtained as

\[ p(N, p, \lambda | \xi, \gamma) \propto p(N, p, \lambda) L(N, p, \lambda | \xi, \gamma) . \]
Let \( \hat{N}, \hat{p}, \hat{\lambda} \) be the Bayesian point estimates for \( N, p \) and \( \lambda \), respectively. That is, the point \((\hat{N}, \hat{p}, \hat{\lambda})\) is the mode of the joint posterior distribution \( p(N, p, \lambda | t, y) \), i.e. it attains its maximum at \((\hat{N}, \hat{p}, \hat{\lambda})\). Then, \( \hat{N}, \hat{p}, \hat{\lambda} \) are the values that satisfy

\[
\begin{align*}
-\lambda \sum_{i=1}^{n} t_i + \sum_{i=1}^{n} \frac{1-y_i}{N-(i-1)} + \frac{\alpha-1}{N} - \beta &= 0, \\
\lambda \sum_{i=1}^{n} (i-1)t_i - \frac{1}{2}y_i/(1-p) + \frac{n-1}{p} - \frac{\rho-1}{1-p} &= 0,
\end{align*}
\]

and

\[
n/\lambda - \sum_{i=1}^{n} (N-p(i-1))t_i + \frac{\mu-1}{\lambda} - \gamma = 0.
\]

Finally, the \(100(1-\alpha)\%\) Bayesian confidence region is given by

\[
f(N, p, \lambda) = C,
\]

where

\[
f(N, p, \lambda) = n \log \lambda - \sum_{i=1}^{n} (N-p(i-1))t_i + \sum_{i=1}^{n} y_i \log (1-p) + \sum_{i=1}^{n} (1-y_i) \log (N-(i-1)) + (\alpha-1) \log N - 3N \\
+ (\mu-1) \log \lambda - \gamma \lambda \\
+ (\nu-1) \log p + (\rho-1) \log (1-p)
\]

and

\[
C = f(\hat{N}, \hat{p}, \hat{\lambda}) - \frac{1}{2} \chi^2_{3, \alpha}.
\]

23
5. BAYESIAN SOFTWARE CORRECTION LIMIT POLICIES

The objective of the investigation was to provide an optimum correction limit policy for a large-scale software system subject to random error occurrences and error removals in an operational phase. When an error occurs a corrective action is undertaken to remove it. Such an action can be scheduled at two levels, which we call Phase I and Phase II. By Phase I we mean that the corrective action will be undertaken by the programmer while Phase II action is undertaken by a system analyst or system designer. First, Phase I corrective action is scheduled for a specified time $T$. If the error is not corrected in this time, it is referred to Phase II. This sequence of corrective actions in an operational phase is shown in Figure 5.1. Our objective is to determine the optimum value $T^*$ of $T$ which minimizes the long run average cost. Two models are developed for this purpose. In the first model we assume that the cost of observations of error occurrence and correction time, prior to the implementation of the optimum policy, is negligible. The second model incorporates the cost of observations.

Details of the model and related results are given in reference [8].
Figure 5.1 Sequence of Corrective Actions in Operational Phase
6. SOFTWARE RELIABILITY DEMONSTRATION TEST PLANS

The purpose of this task was to describe the theory, methodology, and procedures for software reliability demonstration tests. Such tests are to be conducted to ensure that the software system being considered for acquisition has achieved desired reliability. The situation we have in mind for applying these tests is that the software system has gone through the usual development phases, including testing and debugging, and is being presented for testing to demonstrate its reliability. Software reliability for this purpose will be defined as the probability that a given software program operates for a given time period, without a software error, on the machine for which it was designed given that it is used within design limits.

A complete description of the results on this project is given in Appendix B.
7. COMPUTER PROGRAMS

The computer programs required for computations of the quantities described in Sections 2 through 6 are given in Appendix C. The programs are self-explanatory, give the list of input/output parameters and include listings of the needed subroutines.
SELECTED REFERENCES


APPENDIX A

AN ANALYSIS OF RECURRENT SOFTWARE ERRORS IN A REAL-TIME CONTROL SYSTEM

In this Appendix we present an analysis of software error data from a large-scale software project using the imperfect debugging model discussed in Section 2. The model parameters are estimated from the data and the values predicted from the model are compared with the observed values. Joint confidence regions for the parameters are also constructed which permit a study of the sensitivity of predictions.

A.1 Analysis of Error Data from a Real Time Control System

A real-time control system for a land-based radar system was developed by Raytheon Co. in a modular fashion. Nearly all of the modules were written in JOVIAL/J3. The error data base was extracted from 2165 Software Problem Reports (SPRs) written against 109 operational software modules over the development phases and is described in [35]. Table A.1 shows the distribution of the SPRs by month opened during a 22 month period of integration, acceptance and operational testing phases.

The available data give the number of total errors \( u_1 \) and the number of imperfect debugging or recurrent errors \( v_1 \) detected by time \( t_1 \). The parameters under consideration are the initial number of software errors \( N \), the error occurrence rate for each error \( \lambda \), and the probability \( p \) of perfect debugging. We first estimate the parameters \( N, p \) and \( \lambda \) from these data \( i, y, y \) and then compare the results obtained from IDM with the observed values.
### TABLE A.1

**ERROR DATA BY MONTH**

<table>
<thead>
<tr>
<th>Month $t_i$</th>
<th>Total Number of Errors $u_i - u_{i-1}$</th>
<th>$u_i$</th>
<th>Imperfect Debugging Errors $v_i - v_{i-1}$</th>
<th>$v_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>122</td>
<td>122</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>98</td>
<td>220</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>82</td>
<td>302</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>75</td>
<td>377</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>113</td>
<td>490</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>85</td>
<td>575</td>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>105</td>
<td>680</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>47</td>
<td>727</td>
<td>1</td>
<td>19</td>
</tr>
<tr>
<td>9</td>
<td>61</td>
<td>788</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>25</td>
<td>813</td>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>11</td>
<td>28</td>
<td>841</td>
<td>0</td>
<td>21</td>
</tr>
<tr>
<td>12</td>
<td>42</td>
<td>883</td>
<td>1</td>
<td>22</td>
</tr>
<tr>
<td>13</td>
<td>18</td>
<td>901</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>14</td>
<td>17</td>
<td>918</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>15</td>
<td>28</td>
<td>946</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>16</td>
<td>14</td>
<td>960</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>17</td>
<td>5</td>
<td>965</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>18</td>
<td>3</td>
<td>968</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>19</td>
<td>3</td>
<td>971</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>20</td>
<td>13</td>
<td>984</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>21</td>
<td>5</td>
<td>989</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>22</td>
<td>10</td>
<td>999</td>
<td>0</td>
<td>22</td>
</tr>
</tbody>
</table>
Using the data in Table A.1 and the results from Section 2, the results are obtained as shown in Table A.2.

The joint confidence regions for $N$ and $\lambda$ for $p=0.974$ are plotted in Figure A.1. The plots of the actual and fitted SPR's by month are shown in Figure A.2, and the plots of actual and predicted number of remaining errors are given in Figure A.3.
TABLE A.2
A SUMMARY OF ERROR DATA ANALYSES

<table>
<thead>
<tr>
<th>Quantities of Interest</th>
<th>Calculated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{N} )</td>
<td>1079 (Errors)</td>
</tr>
<tr>
<td>( \hat{p} )</td>
<td>0.974</td>
</tr>
<tr>
<td>( \hat{\lambda} )</td>
<td>0.1235 (per month)</td>
</tr>
<tr>
<td>( \hat{a} (= \frac{\hat{N}}{\hat{p}}) )</td>
<td>1108 (Errors)</td>
</tr>
<tr>
<td>( \hat{b} (= \hat{p}\lambda) )</td>
<td>0.1203 (per month)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>90% bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>( [\hat{N}, \bar{\hat{N}}] )</td>
</tr>
<tr>
<td>( [\hat{p}, \bar{\hat{p}}] )</td>
</tr>
<tr>
<td>( [\hat{\lambda}, \bar{\hat{\lambda}}] )</td>
</tr>
<tr>
<td>( [\hat{a}, \bar{\hat{a}}] )</td>
</tr>
<tr>
<td>( [\hat{b}, \bar{\hat{b}}] )</td>
</tr>
<tr>
<td>( \hat{p}_{N, \lambda} )</td>
</tr>
<tr>
<td>( \hat{p}_{a, b} )</td>
</tr>
</tbody>
</table>
Figure A.1 Joint confidence regions for $N$ and $\lambda$ for $p=\hat{p}$
Figure A.2 Actual and fitted SPRs by month

\( \hat{N} = 1079 \)

\( \hat{\lambda} = 0.974 \)

\( \bar{\lambda} = 0.123 \)

TOTAL

FITTED

ACTUAL
Figure A.3 Plots of the actual and predicted number of remaining errors.
APPENDIX B

SOFTWARE RELIABILITY DEMONSTRATION TEST PLANS

B.1. INTRODUCTION AND PROBLEM DEFINITION

The purpose of this report is to describe the theory, methodology, and procedures for software reliability demonstration tests. Such tests are to be conducted to ensure that the software system being considered for acquisition has achieved desired reliability. The situation we have in mind for applying these tests is that the software system has gone through the usual development phases, including testing and debugging, and is being presented for testing to demonstrate its reliability. Software reliability for this purpose will be defined as the probability that a given software program operates for a given time period, without a software error, on the machine for which it was designed, given that it is used within design limits.

In order to develop an appropriate test plan, we must first choose a model which adequately describes the error occurrence phenomenon. Most software error models are based on the assumption that successive errors follow a decreasing failure rate because of the reduction in the number of remaining errors. However, in this appendix we assume that the times between errors during the demonstration phase follow an exponential distribution, i.e. have
a constant failure rate. As a justification for this assumption, we contend that the demonstration time and the number of errors encountered during demonstration will be relatively small and hence a constant failure rate model will be an adequate representation of the error phenomenon. At worst, this model will yield a somewhat conservative test from the viewpoint of DOD.

Let the distribution of error occurrence times be given by

\[ f(t|\lambda) = \lambda \exp(-t\lambda), \quad t \geq 0, \lambda > 0 \]  

(B.1-1)

Then the following measures of software reliability can be used interchangeably:

- Reliability, \( R(t) = P(t > \tau) = e^{-\tau \lambda} \)  
- Meantime Between Software Failure (MTBSF) = \( 1/\lambda \)  
- Software Failure Rate (SFR) = \( \lambda \)

In the following we shall take \( \lambda \) as the measure of software reliability.

In the classical set up, the demonstration tests are generally designed to distinguish between two values of \( \lambda \), namely the maximum acceptable SFR, \( \lambda_1 \) and the specified SFR, \( \lambda_0 \). The decision to accept or reject the software system is based upon test results which are subject to random fluctuation. A loss is incurred when either an accept or a reject decision is wrongly taken.

The loss corresponding to wrong decisions is quantified by two risks called the producer's risk, \( \alpha = P(R|\lambda_0) \), and the consumer's risk, \( \beta = P(A|\lambda_1) \), where \( A \) and \( B \) denote acceptance and rejection of the
software system, respectively.

In this context various types of demonstration plans can be designed to limit $\alpha$ and $\beta$ to desired values. For example, a truncated single sample plan for the system is employed as follows. The plan consists of using the software in an environment which is representative of the operational environment for a time period $T$. The number of errors $r$ encountered during this time period is recorded. (In this study we assume that all errors are of the same severity. When errors are classified according to the degree of severity, the problem becomes quite complicated and is beyond the scope of this investigation). If $r$ is less than or equal to a pre-specified number, $r^*$, the software is accepted. Otherwise, the system is rejected. The design of such a plan consists of obtaining the quantities $T$ and $r^*$ such that the desired risks $\alpha$ and $\beta$ are satisfied.

One disadvantage in the above approach is that $\lambda$ is assumed to be an unknown, fixed constant. In practice there may be sufficient reason to believe that knowledge about $\lambda$ is available in some quantifiable form and one would like to incorporate such information in the development of the demonstration test plan. Another important situation arises when the risks associated with the demonstration test are not adequately represented by the classical risks $(\alpha, \beta)$ and interest lies in associating risks with the posterior distribution of $\lambda$. In such cases one resorts to a Bayesian approach for test design.
The problem under consideration then, is the design of single sample software reliability demonstration plans when error occurrence times are assumed to follow the exponential distribution.

Before describing the development of the test plans, we first discuss the various risks that arise in the above situations in Section B.2. Fixed time, classical tests are then considered in Section B.3. In Sections B.4 and B.5 Bayesian tests are developed for two situations: (i) \( \lambda \) has a noninformative prior distribution, and (ii) information about \( \lambda \) can be quantified from the testing and debugging data.

Section B.6 presents the step-by-step procedure to be employed to demonstrate the properties and performance of the demonstration test plans.
**B.2. DEFINITIONS AND INTERPRETATION OF RISKS**

The following quantities are of interest

1. The probability $P(R|\lambda = \lambda_0) = \alpha$ that a software system with specified SFR is rejected.

2. The probability $P(A|\lambda = \lambda_1) = \beta$ that a software system with maximum acceptable SFR is accepted.

3. The probability $P(A|\lambda \geq \lambda_1) = \tilde{\beta}$ that a software system which is of unacceptable reliability is accepted.

4. The probability $P(R|\lambda \leq \lambda_0) = \tilde{\alpha}$ that a software system of acceptable reliability is rejected.

5. The probability $P(\lambda \geq \lambda_1|A) = \beta^*$ that the SFR of a system which has been accepted is more than the maximum acceptable SFR.

6. The probability $P(\lambda \geq \lambda_0|A) = \beta^{**}$ that the SFR of a system which has been accepted is more than the specified SFR.

7. The probability $P(\lambda \leq \lambda_0|R) = \alpha^*$ that the SFR of a system which has been rejected is less than the specified SFR.

8. The probability $P(R)$ that the software system is rejected.

9. The probability $P(A)$ that the software system is accepted.

10. The probability $P(\lambda \geq \lambda')$ that a-priori, the software system has SFR which is more than $\lambda'$.

These quantities will now be discussed in some detail.
B. 2.1, Classical Risks (\( \alpha, \beta \))

The classical producer's risk \( \alpha \) and consumer's risk \( \beta \) are defined as follows:

\[
\alpha = P(\text{R} | \lambda = \lambda_0), \text{ the probability of rejecting a software system whose SFR is equal to the specified value, } \lambda_0. \tag{B.2-1}
\]

\[
\beta = P(\text{A} | \lambda = \lambda_1), \text{ the probability of accepting a software system whose SFR is equal to the maximum acceptable value.} \tag{B.2-2}
\]

The \((\alpha, \beta)\) risks represent two points on the classical operating characteristic (OC) curve which is a plot of \( P(\text{A} | \lambda) \) versus \( \lambda \). These risks do not provide an explicit control of the probability of acceptance for values of \( \lambda \) other than \( \lambda_1 \) and \( \lambda_0 \). However, \( P(\text{A} | \lambda) \) decreases monotonically with \( \lambda \). Hence, if \( \lambda < \lambda_0 \), the probability of rejection is less than \( \alpha \). If \( \lambda > \lambda_1 \), the probability of acceptance is less than \( \beta \). The shape of the OC curve governs the degree of protection provided in the indifference zone between \( \lambda_1 \) and \( \lambda_0 \).

B. 2.2, Average Risks \((\overline{\alpha}, \overline{\beta})\)

The average risks are defined as follows

\[
\overline{\alpha} = P(\text{R} | \lambda \leq \lambda_0), \text{ the probability of rejecting a software system with a SFR less than or equal to the specified value, } \lambda_0. \tag{B.2-3}
\]

\[
\overline{\beta} = P(\text{A} | \lambda \geq \lambda_1), \text{ the probability of accepting a software system with a SFR greater than or equal to } \lambda_1. \tag{B.2-4}
\]
Mathematically, the risks may be expressed as:

\[
P(R|\lambda \leq \lambda_0) = \frac{\int_0^{\lambda_0} P(R|\lambda)p(\lambda)d\lambda}{\int_0^{\lambda_0} p(\lambda)d\lambda}
\]  \hspace{1cm} \text{(B.2-5)}

or

\[
\bar{\alpha} = \int_0^{\lambda_0} P(R|\lambda)p(\lambda|\lambda \leq \lambda_0)d\lambda
\]  \hspace{1cm} \text{(B.2-6)}

and, similarly

\[
\bar{\beta} = \int_{\lambda_1}^{\infty} P(A|\lambda)p(\lambda|\lambda \geq \lambda_1)d\lambda
\]  \hspace{1cm} \text{(B.2-7)}

The average risks provide the following protection. If the producer produces a large number of software systems, then, in the long run, less than \(\bar{\alpha}\) percent of the desired ones will be rejected. If the consumer buys a large number of software systems, then, in the long run, less than 100 \(\bar{\beta}\) percent of the bad systems will be accepted. No explicit control on the probability of acceptance is provided at any specific value of \(\lambda\) when we use the average risk criteria.
2.3 Posterior Risks ($\alpha^*$, $\beta^*$)

The ($\alpha^*$, $\beta^*$) risks are defined as follows

$$\alpha^* = P(\lambda \leq \lambda_0 | R)$$

This risk is the long run probability of a rejected software system being good.

$$\beta^* = P(\lambda \geq \lambda_1 | A)$$

This risk is the long run probability of an accepted system being bad.

Mathematically,

$$\alpha^* = P(\lambda \leq \lambda_0 | R) = \int_0^{\lambda_0} P(\lambda | R) d\lambda = \frac{\int_0^{\lambda_0} P(R | \lambda) p(\lambda) d\lambda}{\int_0^\infty P(R | \lambda) p(\lambda) d\lambda}$$

$$\beta^* = P(\lambda \geq \lambda_1 | A) = \int_{\lambda_1}^\infty P(\lambda | A) d\lambda = \frac{\int_{\lambda_1}^\infty P(A | \lambda) p(\lambda) d\lambda}{\int_0^\infty P(A | \lambda) p(\lambda) d\lambda}$$

These risks can also be interpreted in a "degree of belief" sense. Thus, $\alpha^*$ would represent a person's degree of belief that if a software system has been rejected, it has a SFR which is better than the specified value. Similarly, $\beta^*$ would be the degree of belief that the SFR of an accepted system is worse than the maximum acceptable value.
B. 2.4 Probability of Rejection $P(R)$

This is a single number given by

$$P(R) = \int_0^\infty P(R|\lambda)p(\lambda)d\lambda$$  \hspace{1cm} (B.2-12)

or

$$P(R) = 1 - P(A) = 1 - \int_0^\infty P(A|\lambda)p(\lambda)d\lambda$$  \hspace{1cm} (B.2-13)

Note that the integration is over the entire range of $\lambda$ and specification of $\lambda_0$, which is usually specified in conjunction with a producer's risk, is unnecessary.

In the frequency sense we have

$$P(R) = \frac{\text{Total number of systems rejected}}{\text{Total number of systems tested}}$$

For the producer this criterion implies that, in the long run, less than $(100)\cdot P(R)$ percent of the systems will be rejected.
B.2.5. Alternate Posterior Consumer's Risk $\beta^{**}$

We define a new risk associated with the posterior distribution of $\lambda$ as follows

$$\beta^{**} = P(\lambda \geq \lambda_0 | A) = \int_{\lambda_0}^{\infty} f(\lambda | A) d\lambda,$$  \hspace{1cm} (B.2-14)

Where $f(\lambda | A)$ is the pdf of $\lambda$ conditional on acceptance. This can be written as

$$\beta^{**} = \frac{\int_{\lambda_0}^{\infty} P(A | \lambda) p(\lambda) d\lambda}{\int_{0}^{\infty} P(A | \lambda) p(\lambda) d\lambda}. \hspace{1cm} (B.2-15)$$

This risk gives the long run probability of the accepted system having a $\lambda$ above the specified SFR $\lambda_0$. 
B. 3. DESIGN OF TEST PLAN FOR CLASSICAL RISKS

Now we consider the design of a single sample plan where the parameters are $T$ and $r^*$. Since the time to software failure is exponential, the observed number of software errors $r$ in fixed time $T$ has a Poisson distribution, i.e.

$$f(r|\lambda) = \frac{e^{-T\lambda}(T\lambda)^r}{r!} \quad (B.3-1)$$

The two risks can be written as:

$$\sum_{r=0}^{r^*} \frac{e^{-T\lambda_0}(T\lambda_0)^r}{r!} = 1 - \alpha \quad (B.3-2)$$

and

$$\sum_{r=0}^{r^*} \frac{e^{-T\lambda_1}(T\lambda_1)^r}{r!} = \beta \quad (B.3-3)$$

Given $\lambda_1$, $\lambda_0$, $\alpha$ and $\beta$, the above equations can be simultaneously solved to obtain software test time $T$ and allowable number of errors $r^*$.

Individual specification of $\lambda_1$ and $\lambda_0$ is not necessary. Let $K = \lambda_1/\lambda_0$, and let $T^* = T \lambda_0$. Then, to satisfy the stated risks we can write
Given \((K, \alpha, \beta)\) the above two equations can be solved to obtain \((T^*, r^*)\). These equations can be solved numerically or by using tables of cumulative Poisson probabilities. Clearly, test plans with identical \(\lambda_0/\lambda_1\) have the same \(T^*\) and \(r^*\) values. Given the specified value \(\lambda_0\), the actual test time is obtained as \(T = T^*/\lambda_0\).
4. BAYESIAN TEST PLANS FOR NONINFORMATIVE PRIOR DISTRIBUTION

In this section we consider the case when it is possible to express the unknown parameter \( \lambda \) in terms of a metric \( \phi(\lambda) \) so that the corresponding likelihood is data translated. This means that the likelihood function for \( \phi(\lambda) \) is completely determined a-priori except for its location which depends on the software failure data yet to be observed. This state of indifference can be expressed by taking \( \phi(\lambda) \) to be locally uniform, and the resulting prior distribution is called noninformative for \( \phi(\lambda) \) with respect to data. A more detailed discussion on noninformative prior distribution can be found in Box and Tiao [1]*. In our case, a noninformative prior for \( \lambda \) is

\[
P(\lambda) = \lambda^{-1/2}
\]  \hspace{1cm} (B.4-1)

Now, if \( T \) is the test time for a software system, the number of errors in \( T \) will be given by a Poisson distribution with parameter \( T \lambda \) as mentioned earlier. Letting \( \Lambda = T \lambda \), the prior for \( \Lambda \) can be written as

\[
P(\Lambda) = \Lambda^{-1/2}
\]  \hspace{1cm} (B.4-2)

*References at the end of this Appendix
The joint probability distribution of \( A \) and \( r \) is:

\[
p(r, A) = p(r|A) \cdot p(A)
\]

\[
\propto \frac{(A)^r \cdot e^{-A}}{r!} \cdot A^{-1/2}
\]

or

\[
p(r, A) = b' \cdot \frac{A^{r-1/2} \cdot e^{-A}}{r!}
\]

(B.4-3)

where \( b' \) is the normalizing constant.

To get the marginal distribution of \( r \), we let \( A \) be defined over the range \((0,d)\) where \( d \) is sufficiently large and \( d < \infty \). Then we have

\[
p(r) = \int_0^\infty p(r, A) \cdot dA = \int_0^\infty b' \cdot \frac{A^{r-1/2} \cdot e^{-A}}{r!} \cdot dA
\]

(B.4-4)

Now we choose \( d \) such that

\[
\int_0^\infty \frac{b' \cdot A^{r-1/2} \cdot e^{-A}}{r!} \cdot dA < \epsilon
\]

(B.4-5)

for some given sufficiently small \( \epsilon > 0 \): Then

\[
p(r) = \frac{b'}{r!} \cdot \Gamma(r + \frac{1}{2})
\]

(B.4-6)

and

\[
P(A) = \sum_{r=0}^{r*} p(r) = \sum_{r=0}^{r*} \frac{b' \cdot \Gamma(r + \frac{1}{2})}{r!}
\]

(B.4-7)
In order to get expressions for various risks, we proceed as follows.

Let \( \Lambda_0 = T\lambda_0 \). Then from (B.4-3) we get,

\[
P(\lambda \leq \lambda_0, \text{accept}) = P(\Lambda \leq \Lambda_0, \text{accept})
\]

\[
= \sum_{r=0}^{r^*} \int_{b'}^{\Lambda_0} \frac{\Lambda^{r-1/2} \cdot e^{-\Lambda}}{r!} d\Lambda
\]

Substituting the above expressions into the appropriate formulae in Section 2, we get the equations for desired risks.

For example, if we are interested in the risk combination \((\bar{\alpha}, \beta^*)\), we get from (B.2-3) and (B.2-9), respectively:

\[
\bar{\alpha} = 1 - P(\Lambda | \lambda \leq \lambda_0)
\]

\[
= 1 - \frac{P(\Lambda, \lambda \leq \lambda_0)}{P(\lambda \leq \lambda_0)}
\]

\[
= 1 - \frac{\sum_{r=0}^{r^*} \int_{b'}^{\Lambda_0} \frac{\Lambda^{r-1/2} \cdot e^{-\Lambda}}{r!} d\Lambda}{\int_{b'}^{\Lambda_0} \Lambda^{-1/2} d\Lambda}
\]

or

\[
\bar{\alpha} = 1 - \frac{\sum_{r=0}^{r^*} \int_{0}^{\Lambda_0} \frac{\Lambda^{r-1/2} \cdot e^{-\Lambda}}{r!} d\Lambda}{\int_{0}^{\Lambda_0} \Lambda^{-1/2} d\Lambda}
\]
and

\[ \beta^* = P(\lambda \geq \lambda_1 | A) \]

\[ = 1 - P(\lambda \leq \lambda_1 | A) \]

\[ = 1 - \sum_{r=0}^{r^*} \frac{\Lambda_1 \Lambda^{r-1/2} \cdot e^{-\Lambda}}{\Gamma(r + \frac{1}{2})} \frac{1}{r!} \]

or

\[ \beta^* = 1 - \frac{\sum_{r=0}^{r^*} \frac{\Gamma(r + \frac{1}{2})}{r!}}{\sum_{r=0}^{r^*} \frac{\Lambda^{r-1/2} \cdot e^{-\Lambda}}{\Gamma(r + \frac{1}{2})}} \]

(B.4-10)

If the consumer's risk of interest is \( \beta^{**} \), then from (B.2-14) we have

\[ \beta^{**} = P(\lambda \geq \lambda_0 | A) \]

\[ = 1 - P(\lambda \leq \lambda_0 | A) \]

\[ = 1 - \sum_{r=0}^{r^*} \frac{\Lambda_0 \Lambda^{r-1/2} \cdot e^{-\Lambda}}{\Gamma(r + \frac{1}{2})} \frac{1}{r!} \]

(B.4-11)

\[ B-16 \]
The design plan $T$ and $r^*$ is obtained by simultaneously solving two equations, one each for the producer's and the consumer's risk. The proper choice of risk combinations will be governed by the protection desired and agreed upon by the vendor and the buyer of the software system or systems.

Two nomograms for the design of plans for $(\bar{\alpha}, \beta^*)$ and $(\bar{\alpha}, \beta^{**})$ are given on the following pages.
FIG. B-2 CONTOURS OF $\bar{a}, \beta^*$ FOR DESIGN OF TEST PLANS
B. 5. BAYESIAN TEST PLANS FOR CONJUGATE PRIOR

In some situations, data on failures and times between failures during the testing and debugging phase may be available. Such data can be used to obtain an appropriate prior distribution for \( \lambda \), the software failure rate. A flexible prior and one which is mathematically tractable in this case is a two parameter gamma given by

\[
p(\lambda) = \frac{\tau^p}{\Gamma(p)} \cdot \lambda^{pT} \cdot e^{-\tau\lambda}
\]  \hspace{1cm} (B5-1)

Estimates of the parameters \( p \) and \( \tau \) can be obtained from the available data. Using this prior, the expressions for the various producer's and consumer's risks are obtained as follows:

Expressions for Producer's Risks:

Classical:

\[
1 - \alpha = \sum_{r=0}^{r^*} \frac{e^{-T\lambda_0} \cdot (T\lambda_0)^r}{r!}
\]  \hspace{1cm} (B.5-2)

Average:

\[
1 - \tilde{\alpha} = \int_0^{\lambda_0} \frac{\tau^p}{\Gamma(p)} \cdot \lambda^{p-1} \cdot e^{-\tau\lambda} \cdot (T\lambda)^2 \, d\lambda
\]  \hspace{1cm} (B.5-3)

\[
= \int_0^{\lambda_0} \frac{\tau^p}{\Gamma(p)} \cdot \lambda^{p-1} \cdot e^{-\tau\lambda} \cdot (T\lambda_0)^2 \, d\lambda
\]
Posterior:

\[
\begin{align*}
\lambda_0 \left( \int \frac{e^{-T\lambda} \cdot (T\lambda)^r}{r!} \cdot \frac{\lambda^p}{\Gamma_p} \cdot \lambda^{p-1} \cdot e^{-\tau\lambda} \cdot d\lambda \right) & = 0 \quad \sum_{r=0}^{r^*} \\
1 - \alpha^* & = \frac{1 - \int \frac{e^{-T\lambda} \cdot (T\lambda)^r}{r!} \cdot \frac{\lambda^p}{\Gamma_p} \cdot \lambda^{p-1} \cdot e^{-\tau\lambda} \cdot d\lambda}{1 - \sum_{r=0}^{\infty} \frac{e^{-T\lambda} \cdot (T\lambda)^r}{r!} \cdot \frac{\lambda^p}{\Gamma_p} \cdot \lambda^{p-1} \cdot e^{-\tau\lambda} \cdot d\lambda} \quad (B.5-4)
\end{align*}
\]

Probability of Rejection:

\[
P(R) = 1 - \int \sum_{r=0}^{\infty} \frac{e^{-T\lambda} \cdot (T\lambda)^r}{r!} \cdot \frac{\lambda^p}{\Gamma_p} \cdot \lambda^{p-1} \cdot e^{-\tau\lambda} \cdot d\lambda \quad (B.5-5)
\]

Expressions for Consumer's Risks:

Classical:

\[
\beta = \sum_{r=0}^{r^*} \frac{e^{-T\lambda_0} \cdot (T\lambda_0)^r}{r!} 
\]

Average:

\[
\bar{\beta} = \frac{1}{\lambda_1} \int \left( \sum_{r=0}^{\infty} \frac{e^{-T\lambda} \cdot (T\lambda)^r}{r!} \cdot \frac{\lambda^p}{\Gamma_p} \cdot \lambda^{p-1} \cdot e^{-\tau\lambda} \cdot d\lambda \right) = \frac{1}{\lambda_1} \int \frac{\lambda^p}{\Gamma_p} \cdot \lambda^{p-1} \cdot e^{-\tau\lambda} \cdot d\lambda 
\]

\[B - 21\]
Posterior:

\[
\beta^* = \frac{\int_0^\infty \left\{ \sum_{r=0}^\infty \frac{e^{-T \lambda r} \cdot (T \lambda)^r}{r!} \right\} \frac{\tau^p}{\Gamma p} \cdot \lambda^{p-1} \cdot e^{-T \lambda} \cdot d\lambda}{\int_0^\infty \left\{ \sum_{r=0}^\infty \frac{e^{-T \lambda r} \cdot (T \lambda)^r}{r!} \right\} \frac{\tau^p}{\Gamma p} \cdot \lambda^{p-1} \cdot e^{-T \lambda} \cdot d\lambda}
\]  

(B.5-8)

Alternate Posterior:

\[
\beta^{**} = \frac{\int_0^\infty \left\{ \sum_{r=0}^\infty \frac{e^{-T \lambda r} \cdot (T \lambda)^r}{r!} \right\} \frac{\tau^p}{\Gamma p} \cdot \lambda^{p-1} \cdot e^{-T \lambda} \cdot d\lambda}{\int_0^\infty \left\{ \sum_{r=0}^\infty \frac{e^{-T \lambda r} \cdot (T \lambda)^r}{r!} \right\} \frac{\tau^p}{\Gamma p} \cdot \lambda^{p-1} \cdot e^{-T \lambda} \cdot d\lambda}
\]  

(B.5-9)
B. 6. PROCEDURE FOR DEMONSTRATING THE PERFORMANCE OF THE TEST PLANS

This section provides a step-by-step procedure that will be used to demonstrate the properties and performance of the demonstration test plans for a software system as developed in the previous sections. These procedures will be followed in conducting the demonstration on site.

Prior to conducting the demonstration test in practice, the DOD must make sure that the software system has been developed according to specifications and all the stated development phases have been carried out to meet the objectives.

The steps to be followed for a demonstration test are as follows.

1. Decide the risk criteria (producer's and consumer's risks) for demonstration.

2. Choose the values of the two risks.

3. Design a test plan $T, r^*$ which meets the risks in Step 2 as closely as possible. (Designs will be obtained using computer programs developed for this purpose.)

4. Simulate software errors using the software error simulator.

5. Apply the demonstration test of Step (3) to simulated errors and make accept/reject decisions.

6. Compare the recorded results with theoretical results.
B.7. REFERENCES


Bayesian Inference in Statistical Analysis. Addison-Wesley


"Reliability Acceptance Sampling Plans Based Upon Prior
Distribution: Risk Criteria and Their Interpretation,"
RADC-TR-76-294, Volume II. (A033516)
APPENDIX C

COMPUTER PROGRAMS

C.1 Programs for the Imperfect Debugging Model (Section 2)

Computer programs to compute the following quantities, for given N, p and \( \lambda \) are given in Tables C.1 to C.8.

- Mean and variance of the first passage time from N to \( n_0 \) (SUBROUTINE FRSTRT)
- Pdf and cdf of the first passage time from N to \( n_0 \) (SUBROUTINE FPT2)
- Probability of the software system having \( n_0 \) errors remaining at time \( t \) (SUBROUTINE STATE)
- Expected number of errors remaining at time \( t \) and expected number of total errors and imperfect debugging errors detected by time \( t \) (SUBROUTINE MEAN)
- MTTF and reliability for given \( k \) at time \( x \) (SUBROUTINE TBF)

The programs are self-explanatory and include the required subroutines.
TABLE C.1 SUBROUTINE FRSTPT

---
c subroutine frstpt(n,p,r,n0,et,vt,a,b)-----------------------------
c function - compute mean and variance of first passage time
   - parameters of Gamma distribution
     - from n to n0, and estimate shape and scale
   - usage - call frstpt (n,p,r,n0,et,vt,a,b)
   - parameters
     - n - (input.) initial no. of errors
     - p - (input.) prob. of perfect debugging
     - r - (input.) detection rate / error
     - n0 - (input.) desired no. of errors
     - et - (output.) mean time from n to n0
     - vt - (output.) variance of time from n to n0
     - a - (output.) shape parameter of Gamma distribution
     - b - (output.) scale parameter of Gamma distribution

---
c subroutine frstpt (n,p,r,n0,et,vt,a,b)
x1=0.0
x2=0.0
n=n0+1
do 55 j=n1,n
   x1=x1+1.0/float(j)
x2=x2+1.0/float(j)**2
55 continue
   et=x1/p/r
   vt=x2/p/p/r/r
   b=et/vt
   a=et*b
   return
;end

---
TABLE C.2  SUBROUTINE FPT2

c subroutine fpt2 (n,p,r,n0,t,pdf,cdf)-------------------------------------
c function   - compute p.d.f. and c.d.f. of first passage time from
c usage  n to n0
c parameters n - (input.) initial no. of errors
               p - (input.) prob. of perfect debugging
               r - (input.) detection rate / error
               n0 - (input.) desired no. of errors
               t - (input.) time
               pdf - (output.) p.d.f. of first passage time from n to n0
               cdf - (output.) c.d.f. of first passage time from n to n0
read. subroutines  - frstpt, ndsamm
-------------------------------------
c subroutine fpt2 (n,p,r,n0,t,pdf,cdf)
call frstpt (n,p,r,n0,ret,vt,a,b)
x=b*t
x1=(a-1.0)*acos(x)
x2=algsam(a)
x3=x1-x*x2
if (x3 <= -88.0) go to 33
pdf=exp(x3)
go to 44
33 pdf=0.0
44 call ndsamm (x,a,cdf)
return
22 cdf=0.0
pdf=0.0
return
end
subroutine state (n,p,r,n0,t,pt)

function - compute probability of being n0 errors remaining at time t

useage - call state (n,p,r,n0,t,pt)

parameters
n - (input.) initial no. of errors
p - (input.) prob. of perfect debugging
r - (input.) detection rate / error
n0 - (input.) desired no. of errors
t - (input.) time
pt - (output.) prob. of being n0 errors at time t

read. subroutines - f*t2

subroutine state (n,p,r,n0,t,pt)
if (n0.eq.n) go to 33
    call f*t2(n,p,r,n0,t,pt,pdf,cdf)
    pt1=cdf
    go to 44
33 pt1=1.0
44 nl=n0-1
    if (nl) 11,22,22
11 pt2=0.0
    go to 99
22 call f*t2 (n,p,r,n1,t,pt,pdf,cdf)
    pt2=cdf
99 pt=pt1-pt2
return
end
TABLE C.4 SUBROUTINE MEAN

```c
subroutine mean (n,p,r,t,er,ed0,ed1) ------------------------------

function - compute expected no. of errors remaining at time t,
detected, and detected due to imperfect debugging by time t

usage - call mean (n,p,r,t,er,ed0,ed1)

parameters n - (input.) initial no. of errors
p - (input.) prob. of perfect debugging
r - (input.) detection rate/ error
f - (input.) time

er - (output.) expected no. of errors remaining at time t
ed0 - (output.) expected no. of errors detected by time t
ed1 - (output.) expected no. of imperfect debugging errors detected by time t

subroutine mean (n,p,r,t,er,ed0,ed1)
er=float(n)*exp(-p*r*t)
ed0=(float(n)-er)/p
ed1=ed0*(1.0-p)
return
end
```

TABLE C.5 SUBROUTINE TBF

```
c subroutine tbf (n,p,r,k,x,rel,xmttf)-----------------------------
c function            - compute reliability and mttf
 c usage                - call tbf (n,p,r,k,x,rel,xmttf)
c c parameters          n - (input.) initial no. of errors
 c r - (input.) prob. of perfect debugging
 c p - (input.) decision rate / error
 c k - (input.) k-th failure
 c x - (input.) time
 c rel - (output.) reliability at time x after (k-1)st failure
 c xmttf - (output.) mean time between (k-1)st and
 c k-th failures
 c subroutine tbf (n,p,r,k,x,rel,xmttf)
xmttf=1.0/(float(n)-p*float(k-1))/r
rel=exp(-x/xmttf)
return
end
```
TABLE C.6  SUBROUTINE MDGAMM

subroutine mdgamm (x,p,prob)          --from imsl--

  function  -- compute incomplete gamma distribution
  usage     -- call mdgamm (x,p,prob)
  parameters x - (input.) value to which gamma is to be integrated
                 p - (input.) gamma parameter
                 prob - (output.) prob = integral of $\Gamma(p)$ to $x$
  read. subroutines - gamma

  subroutine mdgamm (x,p,prob)
  dimension v(6),v1(6)
  equivalence (v(3),v1(1))
  prob=0.0
  if (x .le. 0.0) go to 5
    go to 9000
  5 if (p .le. 0.0) go to 10
    go to 9000
  10 if (x .eq. 0.0) go to 9005
     fn1=1gama(p)
     cnt=p*alog(x)
     wcnt=x+fn1
     if ((cnt-wcnt) .lt. -88.0) go to 15
     ax=0.0
     go to 20
  15 ax=exp(cnt-wcnt)
   20 bis=1.e35
     cut=1.e-8
     if ((x .le. 1.0) .or. (x .lt. p)) go to 40
     w=1.0-p
     z=x+w1.0
     cnt=0.0
     v(1)=1.0
     v(2)=x
     v(3)=x+1.0
     v(4)=z*x
     prob=v(3)/v(4)
   25 cnt=cnt+1.0
     w=1.0
     z=z+2.0
     wcnt=wcnt
     v(5)=v(1)*z-v(1)*wcnt
     v(6)=v(2)*z-v(2)*wcnt
     if (v(6) .eq. 0.0) go to 50
     ratio=v(5)/v(6)
     reduc=abs(prob-ratio)
     if (reduc .lt. cut) go to 30
     if (reduc .le. ratio*cut) go to 35
TABLE C.6 (Continued)

30 prob=ratio
   so to 50
35 prob=1.0-prob*ax
   so to 9005
40 ratio=r
   cnt=1.0
   prob=1.0
45 ratio=ratio+1.0
   cnt=cnt+ratio
   prob=prob+cnt
   if (cnt .lt. cut) go to 45
      prob=prob*ax/p
   so to 9005
50 do 55 i=1,4
   v(i)=v1(i)
55 continue
   if (abs(v(5)) .lt. bis) go to 2
   do 60 i=1,4
      v(i)=v(i)/bis
   60 continue
   so to 25
9000 continue
9005 return
end

function alamma(p)
   if (p .lt. 3.1) go to 15
   call samma (p,sp)
   alamma=alos(sp)
   return
15 z1=(p-0.5)*alos(p)-p+0.5*alos(2.0*3.1415)
z2=1.0/12.0/p
z3=1.0/360.0/p/p/p
z4=1.0/1260.0/p/p/p/p/p
z5=1.0/1680.0/p/p/p/p/p/p/p/p/p/p
alamma=z1+z2-z3+z4-z5
return
end
TABLE C.7
SUBROUTINE GAMMA

Subroutine samma (xx, sx) -----------------------------from ims1-----

Function: Compute a samma function of parameter xx

Usage: Call samma (xx, sx)

Parameters:
xx - (input.) Parameter of samma function
sx - (output.) Value of samma function

Subroutine samma (xx, sx)
if (xx = 57.) 6, 6, 4
4 sx = 1.0e30
return
6 x = xx
err = 1.e-6
sx = 1.
if (x < 2.) 50, 50, 15
10 if (x < 2.) 110, 110, 15
15 x = x - 1.
sx = sx * x
50 to 10
50 if (x < 1.) 60, 120, 110
60 if (x - err) 62, 62, 80
62 w = float (int (x)) - x
64 if (abs (w) - err) 120, 120, 64
70 if (1. - w - err) 120, 120, 70
70 if (x - err) 80, 80, 110
80 sx = sx / x
w = w + 1.
sx = sx * w
110 to 70
110 w = x - 1.
w = 1. + w * (-0.57710174 + w * (0.9858540 + w * (-0.8764218 + w * (0.8328212 + w * (-0.5684729
\ct w (0.2548205 + w * (-0.0514993)))))))))
120 return
end
C.2 Programs for Simulation of Imperfect Debugging Data

Computer programs to simulate the data required to estimate the parameters \( N, p \) and \( \lambda \), are given in Tables C.8 to C.11. These programs perform the following functions:

- Simulate data \((t, \chi)\) for given \( N, p \) and \( \lambda \) (SUBROUTINE SMLT)
- Compute the mle's of \( N, p \) and \( \lambda \) given the data \((t, \chi)\) and also obtain the estimate of variance-covariance matrix (SUBROUTINE MLE)
- Compute the Bayesian estimates of \( N, p \) and \( \lambda \) for given data \((t, \chi)\) (SUBROUTINE BAYES)

The programs are self-explanatory.
TABLE C.8 SUBROUTINE SMLT

```fortran
subroutine smlt (n,p,r,nn,iseed,t,iw)                                  
  function simulate time between s/w failures for an imperfect debbuging model
  usage call smlt (n,p,r,nn,iseed,t,iw)
  parameters n - (input) initial no. of s/w errors
                        p - (input) prob. of perfect debbuging
                        r - (input) detection rate / error
                        nn - (input) no. of observations for s/w failure time
                        iseed - (input) an integer value in the exclusive range (1,2147483647), iseed is replaced by a new iseed to be used in subsequent calls.
                         t - output vector of length nn, containing time between s/w failures
                         iw - output vector of length nn, indicating the type of error which is 1 if the i-th failure is caused by an error due to imperfect debbuging, or 0 otherwise.
  read, subroutine gsub
  dimension t(nn),iw(nn),rr(2)                                      
  nr=n
  i=0
  do 5 i=1,nn
      if (nr .eq. 0) go to 99
      xa=1.0/r/float(nr)
      call gsub (iseed,1,rr)
      t(i)=-xa ALOG(rr(1))
      call gsub (iseed,1,rr)
      if (rr(1)-(1.0-p)) 22,22,33
          22 nr=nr-1
          go to 44
      44 call gsub (iseed,1,rr)
      if (rr(1)-float(ie)/float(nr)) 55,55,66
      66 iw(i)=0
      go to 77
      55 iw(i)=1
      77 print 100,i,t(i),iw(i),nr,ie
  100 format(i5,f15.5,i5,i5)
  5 continue
99 return
end
```

C-11
TABLE C.9  SUBROUTINE MLE

```fortran
SUBROUTINE MLE (t, iv, nn, en, ep, er, ecov)
  USES
  CALL MLE (t, iv, nn, en, ep, er, ecov)

PARAMETERS  
  t - (Input.) a vector of length nn, containing time between s/w failures
  iv - (Input.) a vector of length nn, indicating the type of error which is 1 if the i-th failure is caused by an error due to imperfect debugging, or 0 otherwise
  n - (Input.) no. of observations for s/w failure time
  en - (Output.) an estimate of parameter n
  ep - (Output.) an estimate of parameter p
  er - (Output.) an estimate of parameter lambda
  ecov - (Output.) an estimate of variance-covariance matrix (3x3)
END
```

```fortran
SUBROUTINE MLE (t, iv, nn, en, ep, er, ecov)
  DIMENSION t(nn), iv(nn), ecov(3,3)
  x1=0.0
  x2=0.0
  y=0.0
  DO 5 i=1, nn
    x1=x1+t(i)
    x2=x2+t(i)*FLOAT(i-1)
    iv(i)=0.0
  ENDDO
  DO 15 j=1, 20
    x3=x1*EN0-EP0*X2
    x4=0.0
    x5=0.0
    DO 25 i=1, nn
      x4=x4+FLOAT(1-IV(i))/(EN0-FLOAT(i-1))**2
      x5=x5+FLOAT(1-IV(i))/(EN0-FLOAT(i-1))
    ENDDO
    F0=x5-x1*FLOAT(nn)/x3
    FN=x4+X1*X1*FLOAT(nn)/X3/X3
    FP=-X1*X2*FLOAT(nn)/X3/X3
    PH0=FLOAT(nn)*(1.0-EP0)**2/X3-Y
    PHN=-(1.0-EP0)*FLOAT(nn)**2/X1/X3/X3
    PHP=FLOAT(nn)**2*(-1.0+(1.0-EP0)**2/X3)/X3
    HK=FN*PHP-PHN*FP
    HH=-(F0*PHP-PHN*FP)/HK
    XK=(FN*PH0-PHN*FP)/HK
    EN0=EN0+HH
    EP0=EP0+XK
```

C-12
TABLE C.9 (Continued)

```
print 100, j, en0, ep0
100 format(i5, 2e15.5)
if (amax1(abs(hh/en0), abs(xk/ep0)) .lt. 0.00001) go to 11
15 continue
11 en=en0
ep=ep0
er=float(nn)/x3
rrn=0.0
rpr=0.0
rnr=0.0
do 35 i=1, nn
rrn=rrn+1.0/(en-float(i-1))/(en-en0-float(i-1))
rpr=rpr+float(i-1)/(en-ep-float(i-1))
rnr=rnr+1.0/(en-en0-float(i-1))
35 continue
rrp=rrnr/er
rpp=rpr/er
rpr=float(nn)/er/er
rx=rrn&rpr*rnr-rrn*rpp*rpr
ecov(1,1)=(rpp&rpr-rrn&rpr)/rx
ecov(1,2)=rpr*rnr/rx
ecov(1,3)=rrn*rpp/rx
ecov(2,2)=(rrn&rr-rrn&rpr)/rx
ecov(2,3)=rnr*rpr/rx
ecov(3,3)=rrn*rpp/rx
ecov(2,1)=ecov(1,2)
ecov(3,1)=ecov(1,3)
ecov(3,2)=ecov(2,3)
return
end
```
TABLE C.10 SUBROUTINE BAYES

```fortran
subroutine baves (t,iv,nn,alpha,beta,pi,rho,xmu,damma,bn,bp,br)
  
  function - obtain bayesian estimates of unknown parameters n, p,
  and lambda for idm
  
  usage - call baves (t,iv,nn,alpha,beta,pi,rho,xmu,damma,bn,bp,br)
  
  parameters t - (input.) a vector of length nn, containing time
  between s/w failures
  
  iv - (input.) a vector of length nn, indicating the
  type of error which is 1 if the i-th
  failure is caused by an error due to
  imperfect debugging, or 0 otherwise
  
  nn - (input.) no. of observations for s/w failure time
  
  alpha - (input.) shape parameter of gamma prior for n
  
  beta - (input.) scale parameter of gamma prior for n
  
  pi - (input.) first parameter of beta prior for p
  
  rho - (input.) second parameter of beta prior for p
  
  xmu - (input.) first parameter of beta prior for lambda
  
  damma - (input.) scale parameter of gamma prior for lambda
  
  bn - (output.) bayesian estimate of n
  
  bp - (output.) bayesian estimate of p
  
  br - (output.) bayesian estimate of lambda
  
end subroutine baves
```

```fortran
dimension t(nn), iv(nn)
x1=0.0
x2=0.0
w=0.0

do 5 i=1,nn
  x1=x1+t(i)*float(i)
  x2=x2+t(i)*float(i-1)
  w=w+float(iv(i))
5 continue

bn0=float(nn+1)
bp0=1.0

do 15 j=1,20
  x3=x1*bn0-bp0*x2+damma
  x4=0.0
  x5=0.0
15 continue
```

C-14
TABLE C.10 (Continued)

do 25 i=1,nn
x4=x4+float(1-i)/(bn0-float(i-1))**2
x5=x5+float(1-i)/(bn0-float(i-1))
25 continue
f0=x5-x1*(xmu+float(nn-1))/x3+(alpha-1.0)/bn0-beta
fn=-x4+x1*(xmu+float(nn-1))/x3-x3-(alpha-1.0)/bn0/bn0
fp=-x4*x2*(xmu+float(nn-1))/x3/x3
ph0=(xmu+float(nn-1))*(1.0-br0)*x2/x3+y1*(pi-1.0)*(1.0-br0)/br0-(rho-1.0)
phn=-((1.0-br0)*(xmu+float(nn-1))*x2*x1/x3/x3
php=(xmu+float(nn-1))*x2*1.01.0-br0)*x2/x3/x3-(pi-1.0)/br0/br0
hk=fn*php-phn*fp
hk=-((f0*php-ph0*fp)/hk
xk=-(fn*ph0-phn*fp)/hk
bn0=bn0+thh
br=br+2*xk
print 100,j,brn0,brp0
100 format(i5,2e15.5)
if (amax1(abs(hh/bn0),abs(xk/bp0)) .le. 0.00001) go to 11
15 continue
11 bn=bn0
br=br0
br=(xmu+float(nn-1))/x3
return
end
TABLE C.11  SUBROUTINE GGUB

subroutine ggub (iseed,n,r) from imsl--------
function - basic uniform (0,1) pseudo-random number
generator
usage - call ggub (iseed,n,r)
parameters iseed - (input,) an integer value in the exclusive
range (1,2147483647), iseed is replaced by
a new iseed to be used in subsequent calls.
n - (input,) no. of deviates to be generated
r(n) - (output vector of length n, containing the
deviates in (0,1))--------------------------------------

subroutine ggub (iseed,n,r)
dimension r(1)
double precision z,dp,dm
dm=dble(float(2**31-1))
dp=dble(float(1/2**31))
z=iseed
do 5 i=1,n
  z=mod(16807.d0*z,dm)
  r(i)=z/dp
  iseed=z
5 return
end
C.3 Programs for the Imperfect Maintenance Model of Section 3

Computer programs to compute the following quantities of interest for given \( N, p, \lambda \) and \( \mu \) are given in Tables C.12 to C.18.

- Mean and variance of the first passage time from \( N \) to \( n_0 \) (SUBROUTINE COMP)
- Pdf and cdf of the first passage time from \( N \) to \( n_0 \) (SUBROUTINE FIRST)
- Probability of the software system being operational with \( n_0 \) remaining errors at time \( t \) (SUBROUTINE STT)
- Software system availability at time \( t \) (SUBROUTINE AVAIL)
- Expected number of errors detected and corrected by time \( t \) (SUBROUTINE EXPC)

The programs are self-explanatory and include the required subroutines (MDGAMM and GAMMA).
TABLE C.12  SUBROUTINE COMP

SUBROUTINE COMP (n,p,r,xm,k1,k2,et,vt,a,b,r1,r2)

FUNCTION
compute mean and variance of first passage

time; estimate the gamma parameters, and

obtain the constants r1 and r2

USAGE
- call COMP (n,p,r,xm,k1,k2,et,vt,a,b,r1,r2)

PARAMETERS
n - (input.) initial no. of errors
p - (input.) prob. of perfect debugging
r - (input.) detection rate / error
xm - (input.) correction rate / error
k1 - (input.) first destination
k2 - (input.) second destination
et - (output.) mean first passage time
vt - (output.) variance of first passage time
a - (output.) shape parameter of gamma distribution
b - (output.) scale parameter of gamma distribution
r1 - (output.) smaller constant
r2 - (output.) larger constant

SUBROUTINE COMP (n,p,r,xm,k1,k2,et,vt,a,b,r1,r2)

kk1=k1+1
kk2=k2+1
rr=sort((r+ xm)**2-4.0*r*xm)
ri=(r+ xm-rr)/2.0
r2=(r+ xm+rr)/2.0
er1=0.0
er2=0.0
vr1=0.0
vr2=0.0
if (kk1 .GT. n) go to 11
do 5 i=kk1,n
er1=er1+1.0/float(i)
vr1=vr1+1.0/float(i)/float(i)
5 continue
11 er1=er1/r1
vr1=vr1/r1/r1
if (kk2 .GT. n) go to 22
do 15 i=kk2,n
er2=er2+1.0/float(i)
vr2=vr2+1.0/float(i)/float(i)
15 continue
22 er2=er2/r2
vr2=vr2/r2/r2
et=er1+er2
vt=vr1+vr2
if (vt .EQ. 0.0) go to 33
b=et/vt
89 goto 99
33 b=0.0
99 a=et*b
return
end.
TABLE C.13 SUBROUTINE FIRST

subroutine first (n,p,r,xm,k1,k2,t,pdf,cdf,r1,r2) ----
  function - compute pdf and cdf of first passage time
  and also obtain the constants r1 and r2
  usage - call first (n,p,r,xm,k1,k2,t,pdf,cdf,r1,r2)
  parameters
    n - (input.) initial no. of errors
    r - (input.) prob. of perfect debugging
    p - (input.) detection rate / error
    k1 - (input.) first destination
    k2 - (input.) second destination
    xm - (input.) correction rate / error
    pdf - (output.) pdf of first passage time
    cdf - (output.) cdf of first passage time
    r1 - (output.) smaller constant
    r2 - (output.) larger constant
read, subroutines - comp, mdsamm, sgamma

        subroutine first (n,p,r,xm,k1,k2,t,pdf,cdf,r1,r2)
         if (min0(k1,k2) .lt. 0) go to 22
            call comp (n,p,r,xm,k1,k2,t,vtr,vta,r1,r2)
         if (b .eq. 0.0) go to 11
            x=b*t
            x1=(a-1.0)*alog(x)
            x2=mdsamm(a)
            x3=x1-x2
            if (x3 .lt. -88.0) go to 33
            pdf=exp(x3)
            go to 44
        33 pdf=0.0
        44 call mdsamm (x,a,cdf)
        return
         11 cdf=1.0
         pdf=1.0
         return
         22 cdf=0.0
         pdf=0.0
         return
        end
TABLE C.14  SUBROUTINE STT

subroutine stt (n,p,r,xm,n0,t,prob)-----------------------------
c function  -- compute the probability of being n0 s/w
c usage    -- errors remaining at time t
c parameters
    n  -- (input.) initial no. of errors
    p  -- (input.) prob. of perfect debugging
    r  -- (input.) detection rate / error
    xm -- (input.) correction rate / error
    n0 -- (input.) specified no. of errors
    t  -- (input.) time
    prob -- (output.) prob. of being n0 errors remaining at time t
read subroutine  -- first
-----------------------------

subroutine stt (n,p,r,xm,n0,t,prob)
call first (n,p,r,xm,n0,n0,t,pdf,cdf,r1,r2)
prob=cdf
k=n0-1
cl=(r-r2)/(r1-r2)
c2=(r1-r)/(r1-r2)
call first (n,p,r,xm,k,n0,t,pdf,cdf,r1,r2)
prob=prob-cl*cdf
call first (n,p,r,xm,n0,k,t,pdf,cdf,r1,r2)
prob=prob-c2*cdf
return
end
TABLE C.15    SUBROUTINE AVAIL

subroutine avail (n,p,r,xm,t,at)-------------------------------
c function  -- compute s/w system availability at time t
c usage     -- call avail (n,p,r,xm,t,at)
c parameters n - (input.) initial no. of errors
                p - (input.) prob. of perfect debugging
                r - (input.) detection rate / error
                xm - (input.) correction rate / error
                t - (input.) time
                at - (output.) availability at time t
read. subroutine  -- stt

subroutine avail (n,p,r,xm,t,at)
at=0.0
n1=n+1
do 5 i=1,n1
   n0=i-1
   call stt (n,p,r,xm,n0,t,prob)
   at=at+prob
5 continue
return
end
TABLE C.16  SUBROUTINE EXPCT

```c
subroutine expct (n,p,r,xm,t,xmd,xmc)
  ! function - compute mean no. of errors detected and
  ! corrected by time t
  ! usage - call expct (n,p,r,xm,t,xmd,xmc)
  ! parameters
  ! n - (input.) initial no. of errors
  ! p - (input.) prob. of perfect debugging
  ! r - (input.) detection rate / error
  ! xm - (input.) correction rate / error
  ! t - (input.) time
  ! xmd - (output.) mean no. of errors detected by
       !       time t
  ! xmc - (output.) mean no. of errors corrected by
       !       time t
  ! read. subroutine - first
  !---------------------------------------------------------------------
  h1=0.0
  h2=0.0
  h=0.0
  do 5 i=1,n
    k=i-1
    call first (n,p,r,xm,k,k,t,pdf,cdf,rl,r2)
    h1=h1+cdf
    call first (n,p,r,xm,k,k,t,pdf,cdf,rl,r2)
    h2=n2+cdf
    call first (n,p,r,xm,k,k,t,pdf,cdf,rl,r2)
    h=h+cdf
  5 continue
  xmc=h/r
  xmd=(h1*(1.0-xm/r1)+h2*(xm/r2-1.0))*r/(r1-r2)
  return
end
```
TABLE C.17  SUBROUTINE MDGAMM

subroutine mdgamm (x,p,prob) from imsl

c function - compute incomplete gamma distribution
c usage - call mdgamm (x,p,prob)
c parameters x - (input.) value to which gamma is to be integrated
c p - (input.) gamma parameter
c prob - (output.) prob. integral of gamma(p) to x

c read, subroutines - gamma

dimension v(6),v1(6)
equivalence (v(3),v1(1))
prob=0.0
if (x.le.0.0) go to 5
   go to 9000
5 if (p.st.0.0) go to 10
   go to 9000
10 if (x,oe.,0.0) go to 9005
   fnlg=algamma(p)
cnt=p*alog(x)
   vcnt=x+fnlg
   if ((cnt-vcnt) .st. -88.0) go to 15
   ax=0.0
   go to 20
   15 ax=exp(cnt-vcnt)
20 big=1.0e35
cut=1.e-8
   if ((x,le.,1.0) .or. (x,lt.,p)) go to 40
   y=1.0-p
   z=x+u1.0
   cnt=0.0
   v(1)=1.0
   v(2)=x
   v(3)=x+1.0
   v(4)=z*x
   prob=v(3)/v(4)
25 cnt=cnt+1.0
   w=y*v1.0
   z=z+2.0
   wcnt=w*cnt
   v(5)=v1(1)*z-v(1)*wcnt
   v(6)=v1(2)*z-v(2)*wcnt
   if (v(6),oe.,0.0) go to 50

C-23
TABLE C.17 (Continued)

\[
\text{ratio} = \frac{v(5)}{v(6)}
\]

\[
\text{reduc} = \text{abs}(\text{prob} - \text{ratio})
\]

if \((\text{reduc} \leq \text{cut})\) go to 30

if \((\text{reduc} \leq \text{ratio} \times \text{cut})\) go to 35

30 \(\text{prob} = \text{ratio}\)

so to 50

35 \(\text{prob} = 1.0 - \text{prob} \times \text{ax}\)

so to 9005

40 \(\text{ratio} = \text{prob}\)

\(\text{cnt} = 1.0\)

\(\text{prob} = 1.0\)

45 \(\text{ratio} = \text{ratio} + 1.0\)

\(\text{cnt} = \text{cnt} \times \text{ratio}\)

\(\text{prob} = \text{prob} \times \text{cnt}\)

if \((\text{cnt} \geq \text{cut})\) go to 45

\(\text{prob} = \text{prob} \times \text{ax} / \text{p}\)

so to 9005

50 \(\text{do 55 i=1,4}\)

\(v(i) = v(i)\)

55 \(\text{continue}\)

if \((\text{abs}(v(5)) > \text{lt. bis})\) go to 25

\(\text{do 60 i=1,4}\)

\(v(i) = v(i) / \text{bis}\)

60 \(\text{continue}\)

so to 25

9000 \(\text{continue}\)

9005 \(\text{return}\)

end

function \(\text{alsamma}(\text{p})\)

if \((\text{p} \leq 31.0)\) go to 15

call \(\text{gamma}(\text{p}, \text{sp})\)

\(\text{alsamma} = \text{alsynn}(\text{sp})\)

\(\text{return}\)

15 \(\text{z1} = (\text{p}-0.5) \times \text{alog} - \text{p} + 0.5 \times \text{alog}(2.0 \times 3.1415)\)

\(\text{z2} = 1.0 / 12.0 / \text{p}\)

\(\text{z3} = 1.0 / 360.0 / \text{p} / \text{p}\)

\(\text{z4} = 1.0 / 1260.0 / \text{p} / \text{p} / \text{p} / \text{p}\)

\(\text{z5} = 1.0 / 1680.0 / \text{p} / \text{p} / \text{p} / \text{p} / \text{p} / \text{p}\)

\(\text{alsamma} = \text{z1} + \text{z2} - \text{z3} + \text{z4} - \text{z5}\)

\(\text{return}\)

end
TABLE C.18 SUBROUTINE GAMMA

```c
C subroutine samma (xx,gx) -------------------------------from imsl------
C function compute a samma function of parameter xx
C usage call samma (xx,gx)
C parameters xx (input.) parameter of samma function
C gx (output.) value of samma function
C
C subroutine samma(xx,gx)
  if(xx<57.) 6,6,4
  4 gx=1.0e30
  return
  6 x=xx
  err=1.e-6
  gx=1.
  if(xx<2.) 50,50,15
  10 if (xx<2.) 110,110,15
  15 x=x-1.
  gx=gx*x
  go to 10
  50 if (xx<1.) 60,120,110
  60 if (xx<err) 62,62,80
  62 y=real(int(xx))
  if (abs(y-err) 120,120,64
  64 if (1.-y<err) 120,120,70
  70 if(xx<1.) 80,80,110
  80 gx=gx/x
  x=xx+1.
  go to 70
  110 y=x-1.
  vx1=1.+y*(-0.5771017+y*(0.9856540+y*(-0.8764218+y*(0.6328212+y*(-0.5684729
\ctw=(0.2548205+y*(-0.0514993))))))
  gx=gx*vx1
  120 return
end
```
C.4 Program for Bayesian Software Correction Limit Policies

(Section 5)

Computer programs to compute the optimum policies for model 1 and model 2 are given in Tables C.19 to C.23. These programs perform the following functions:

• For model 1, simulate error occurrence time and correction time and then compute Bayesian estimates of mean correction time, optimum correction limit time, and its minimum cost per unit time (SUBROUTINE MDL1)

• For model 2, simulate error occurrence time and correction time and then compute Bayesian estimate of mean correction time, optimum correction limit time, and its minimum cost per unit times, and also provide the optimum sample size (SUBROUTINE MDL2)

The programs are self-explanatory and include the required subroutines (eg. DATA 1, GGUB and OPTMM).
TABLE C.19  SUBROUTINE MDLI

Subroutine MDLI (iseed, nn, r, xmu1, xmu2, c, alpha, beta, xn, yn, tt, ct)

Function
- simulate error occurrence time and correction time
- and then compute bayesian estimate of mean correction time, optimum correction limit time, and its minimum cost per unit time

Usage
- call MDLI (iseed, nn, r, xmu1, xmu2, c, alpha, beta, xn, yn, tt, ct)

Parameters
iseed - (input.) an integer value in the exclusive range (1, 2147483647), iseed is replaced by a new iseed to be used in subsequent calls.

nn - (input.) no. of observations
r - (input.) error occurrence rate
xmu1 - (input.) mean correction time in phase 1
xmu2 - (input.) mean correction time in phase 2
c - (input.) a vector of length 3
  c(1) - cost per unit time of error correction in phase 1
  c(2) - cost per unit time of error correction in phase 2
  c(3) - sampling cost per sample size
alpha - (input.) a vector of length 2
  alpha(1) - shape parameter of inverted gamma prior for mean correction time in phase 1
  alpha(2) - shape parameter of inverted gamma prior for error occurrence rate
beta - (input.) a vector of length 2
  beta(1) - scale parameter of inverted gamma prior for mean correction time in phase 1
  beta(2) - scale parameter of inverted gamma prior for error occurrence rate
xn - (output.) bayesian estimate of error occurrence rate
yn - (output.) bayesian estimate of mean correction time in phase 1
ct - (output.) optimum correction limit time
ct - (output.) minimum cost per unit time

end

reed. subroutines - datal, opitm

******************************************************************************

subroutine MDLI (iseed, nn, r, xmu1, xmu2, c, alpha, beta, xn, yn, tt, ct)
dimension c(3), alpha(2), beta(2), xy(2)
call datal (iseed, nn, r, xmu1, xmu2)
xn=(xx+b)/alpha(2)
xn=xn/(alpha(2)+float(nn-1))
a=0.0
b=xn
call opitm (nn, y, c, alpha, beta, xmu2, a, b, yn, tt, ct)
return
end
TABLE C.20  SUBROUTINE MDL2

subroutine mdl2 (iseed,r,xmu1,xmu2,c,alpha,beta,nn,xn,wn,tt,ct)

c c function - simulate error occurrence time and correction time

c c and then compute bayesian estimate of mean

c c correction time, optimum correction limit time,
c c and its minimum cost per unit time, and also

c c provide optimum sample size

c c usage - call mdl2 (iseed,r,xmu1,xmu2,c,alpha,beta,nn,xn,wn,tt,ct)

c c parameters iseed - (input.) an integer value in the exclusive range

c c (1,2147483647). iseed is replaced by a new iseed

c c to be used in subsequent calls.

c c r - (input.) error occurrence rate

c c xmu1 - (input.) mean correction time in phase 1

c c xmu2 - (input.) mean correction time in phase 2

c c c - (input.) a vector of length 3

c c c(1) - cost per unit time of error correction

c c in phase 1

c c c(2) - cost per unit time of error correction

c c in phase 2

c c c(3) - sampling cost per sample size

c c alpha - (input.) a vector of length 2

c c alpha(1) - shape parameter of inverted gamma

c c prior for mean correction time

c c in phase 1

c c alpha(2) - shape parameter of inverted gamma

c c prior for error occurrence rate

c c beta - (input.) a vector of length 2

c c beta(1) - scale parameter of inverted gamma

c c prior for mean correction time

c c in phase 1

c c beta(2) - scale parameter of inverted gamma

c c prior for error occurrence rate

c c nn - (output.) optimum sample size

c c xn - (output.) bayesian estimate of error occurrence rate

c c wn - (output.) bayesian estimate of mean correction
c c time in phase 1

c c tt - (output.) optimum correction limit time

c c ct - (output.) minimum cost per unit time.

read, subroutines - ssb, optam

-----------------------------------------------

---

subroutine mdl2 (iseed,r,xmu1,xmu2,c,alpha,beta,nn,xn,wn,tt,ct)
TABLE C.20  (Continued)

dimension c(3), alpha(2), beta(2), xy(2)
x=0.0
y=0.0
do 5 nn=1,100
call dsub (iseed,2,xy)
x=x-t*alog(xy(1))
y=y-mu*talog(xy(2))
if (nn .eq. 1) so to 5
nn=(x+beta(2))/(alpha(2)+float(nn-1))
a=c(3)*float(nn)
b=x+y
print 100,nn,y+nu,c+alpha,b+beta,x+mu2,v+nu+w+nu+nu
5 continue
100 format(i5,6e15.5)
if (tt-tt2) 11,11,5
22 if (tt-tt2) 11,11,5
5 continue
11 tt=tt
ct=ct
return
end
TABLE C.21  SUBROUTINE DATAL

subroutine datal (iseed,nn,r,xmul,xx,vv)-----------------------------------------
c  function - simulate error occurrence time and correction
c  usage   - call datal (iseed,nn,r,xmul,xx,vv)
c  parameters iseed - (input.) an integer value in the exclusive
                  - range (1,2147483647). iseed is replaced
                  - by a new iseed to be used in subsequent calls.
c nn  - (input.) sample size
c r   - (input.) error occurrence rate
c xmul - (input.) mean correction time in phase 1
c xx  - (output.) total amount of error occurrence time
                  - in phase 1
c vv  - (output.) total amount of error correction time
read, subroutine - ssub

--------------------
subroutine datal (iseed,nn,r,xmul,xx,vv)
dimension xv(2)
x=0.0
yy=0.0
do 5 i=1,nn
call ssub (iseed,2,xv)
xx=xx-r*alos(xv(1))
vv=vv-xmul*alos(xv(2))
5 continue
return
end
TABLE C.22 SUBROUTINE GGUB

```fortran
C subroutine SSub (iseed, n, r)---from IMSL------
C function  -- basic uniform (0,1) pseudo-random number generator
C usage  -- call SSub (iseed, n, r)
C parameters iseed  -- (input.) an integer value in the exclusive range (1,2147483647). iseed is replaced by a new iseed to be used in subsequent calls.
n  -- (input.) no. of deviates to be generated
r  -- output vector of length n, containing the deviates in (0,1)
C
C subroutine SSub(iseed, n, r)
dimension r(1)
double precision z, dpm, dpn, dp
data dpm = 2147483647.d0, 1.d0/
dp = dpm/dpm+dpn
z = iseed
do 5 i = 1, n
   z = mod(16807.d0*z, dpm),
5   r(i) = z*dp
   iseed = z
return
end
```
TABLE C.23  SUBROUTINE OPTMM

subroutine optmm (nn,wy,c,alpha,beta,xmu2,a,b,vn,tt,ct)---------
c  function - compute bayesian estimate of mean correction
time, optimum correction limit time, and its minimum cost per unit time
c  usage - call optmm (nn,wy,c,alpha,beta,xmu2,a,b,vn,tt,ct)
c  parameters
  nn - (input.) no. of observations
  vw - (input.) total amount of observed correction
time
c  c - (input.) a vector of length 3
  c(1) - cost per unit time of error correction
         in case 1
  c(2) - cost per unit time of error correction
         in case 2
  c(3) - sampling cost per sample size
alpha - (input.) a vector of length 2
  alpha(1) - shape parameter of inverted samma
              prior for mean correction time
              in case 1
  alpha(2) - shape parameter of inverted samma
              prior for error occurrence rate
beta - (input.) a vector of length 2
  beta(1) - scale parameter of inverted samma
            prior for mean correction time
            in case 1
  beta(2) - scale parameter of inverted samma
            prior for error occurrence rate
xmu2 - (input.) mean correction time in case 2
a - (input.) constant of a in equation (a-1)
b - (input.) constant of b in equation (a-1)
vw - (output.) bayesian estimate of mean correction
time in case 1
tt - (output.) optimum correction limit time
tc - (output.) minimum cost per unit time

(Continued on the next page)

subroutine optmm (nn,wy,c,alpha,beta,xmu2,a,b,vn,tt,ct)
dimension c(3),alpha(2),beta(2)
s=a=c(2)**b-a
bb=(c(1)*b-a)/xmu2
alpha=alpha(1)+float(nn-1)
vw=alpha(2)/alpha(1)+beta(2)/alpha(2)
d1=xmu2/beta(2)
d2=d1*(d1+beta(1))/xmu2
if (tt .lt. 0.0) go to 22
do 5 i=1,20
rt=s3/(1.0+t/s1)
st=s3/(1.0-(1.0+t/s1))**(-a0)
s=t**(1.0+t/s1)**(-a0-1.0)
x1=alpha(2)-c(1)*s1
f0=rt*x1+c(2)-c(1)*s1
r1=-s3/s1/(1.0+t/s1)**2
r=r1*r1
h=-f0/rf
t=th
print 100,s,i,t
100 format(i5,es15.5)
if (abs(h/t) .le. 0.00001) go to 11
5 continue
11 tt=t
   ct=(c(1)-c(2)*xmu2*rt)/(1.0-xmu2*rt)
      return
22 tt=0.0
   ct=(a+c(2)*xmu2)/(b+xmu2)
      return
end
MISSION
of
American Development Group