AN AT-SEA SIMULATION OF ADAPTIVE BEAMFORMING

INTRODUCTION

The Least Mean Square (LMS) algorithm is a method of extracting a target signal in the presence of interfering noise sources. Widrow and Griffiths developed the theory and applied simulated tests using random numbers. These tests indicate that the method is feasible for Laboratory conditions. The purpose of the sea test is to evaluate the method for actual ocean conditions. In this section, the experiment of adaptive beamforming is described. This includes a brief description of adaptive beamforming, the computer program, and the results.

ADMINISTRATIVE INFORMATION

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EXPERIMENT

The geometry of the experiment is given in Figure 1. A four element vertical array is suspended from the FLIP. The depths of the elements are 2,500, 2,521, 2,563, and 2,601 ft. The USNS SANDS (AGOR-6) is at a distance of 10,000 ft from the FLIP. A 400 Hz projector is suspended from the USNS SANDS, at a depth of 1000 ft.

The Honeywell projector is driven by a pseudo-random noise (PRN) signal centered at 500 Hz in a 200 Hz band. The HX-90 has a maximum source level of 100.6 db (1μ bar at 1 yd). Driving the projector with a PRN signal reduces the level to 91.1 db and at a range of 10,000 ft the expected acoustic level at the receiving hydrophones is approximately +19 db. The frequency response of the HX-90 is shown in Figure 2.

A block diagram of the transmitting system is shown in Figure 3. A PRN signal that can either be continuous or gated is filtered through a Butterworth filter that has a 200 Hz bandwidth centered at 500 Hz. The signal is then amplified and transmitted. The CML amplifier has a maximum output power of 5 kW and with a 40% projector efficiency, the electrical power is more than adequate to drive the projector at full power if required.

A block diagram of the receiving equipment is shown in Figure 4. The signal received at the hydrophone is amplified and the information is telemetered back to the ship where it is filtered through identical
Butterworth filters and recorded on magnetic tape in analog form. The analog signals also go to a 12 bit A/D converter and these are processed on the 1230 Univac computer. The digitized signals are also recorded on Univac 1540 magnetic tape.

In order that the projector be in the Fraunhofer region of the array, the distance D between projector and array must be such that,

\[ D \geq \frac{5l^2}{\lambda} \]

where \( l \) is the length of the array and \( \lambda \) is the wavelength.

For 500 cps, \( \lambda \approx 10 \text{ ft.} \) Letting \( l = 100 \text{ ft.} \), then \( D \geq 5000 \text{ ft.} \)

Since \( D = 10,000 \text{ ft.} \), the signal from the projector may be approximated by a plane wave at the array. The beamwidth (bw) of a linear array for a conventional pattern is

\[ bw \approx \frac{\lambda}{L} \frac{180}{\pi} \]

Thus, for our experiment, the bw \( \approx 6 \) degrees.

**Adaptive Beamforming**

We will follow the theory developed by Widrow and Griffiths on the subject of adaptive beamforming. A tapped delay line is attached to each element of a K element array. There are L weights in each tapped delay line. (See Figure 5). The purpose of the algorithm is to adjust the weights such that there is a least mean square (LMS) difference between the output y and the expected signal. The problem was first solved by Wiener and the final answer was in terms of an inverse matrix. Widrow and Griffiths obtained an approximate iterative solution. The main attractive features of the solution is that the noise field need
not be known beforehand and that it is not necessary to invert a matrix. In our simulation, the power spectrum of the signal is assumed to be known. In obtaining the LMS difference, the noise tends to be minimized and the pattern of the array tends to have notches where the noise interferences are strongest.

Based on the thermal structure of the area and a ray tracing program, the angle from the array to the propagation path of the plane wave from the projector is found to be 80°. This is the direct path. In the computer program, the array is steered broadside, so that a signal coming from this direction is enhanced. A signal is not simulated in this experiment. A plane wave striking the array in any direction other than broadside is called an interfering noise in this article. Thus the plane wave from the projector is an interfering noise and one of the purposes of this test is the nulling out of this noise. It should be noted that the interfering directions are not inputs to the computer but rather are obtained during the adaptive process.

Computer Program

The signal is assumed to be flat with bandwidth W and centered at frequency fo. For this type of a signal, the autocorrelation function

\[ P_s(\tau) = \frac{\sin \frac{\pi W \tau}{T}}{\frac{\pi W \tau}{T}} \cos (2\pi f_o \tau) \]

It is convenient to change notation at this point. Referring back to Figure 5, let \( x_1, x_2, \ldots x_k \) be the input values to the first taps of the receivers response. Let \( x_{k+1}, x_{k+2}, \ldots x_{2k} \) be the inputs to the
second taps of the k receivers etc. Thus at any iteration there is a set of $Z_i$, where $i = 1, 2, \ldots, KL$. Let the center point of each tapped delay line be the reference point for the correlation values. For an even number of taps, the correlation of the signal between the adjacent tap and the reference point is $P_s\left(\frac{\Delta}{2}\right)$, where $\Delta$ is the delay shown in Figure 5. Using the symmetry properties of time, the auto-correlation of the input to the first tap with that of the reference point is $P_s\left(\frac{\Delta - 1}{2}\right)$. Thus a table of KL values of $P_s(\tau)$ is computed and stored in the computer.

The computer program used in this test includes two equations. The first equation updates the weights for each iteration. After each 1000 iterations, the beam pattern subroutine is called and these weights are used to compute the beam pattern. The iteration equation is

$$w(i+1) = w(i) + \nu \left[ P_s(i) - y \cdot \text{sgn}(x(i)) \right]$$

where

$$y = \sum_{i=1}^{KL} w(i) x(i)$$

and

$$x(i)$$ is the input at the $i^{th}$ tap

$$w(i)$$ is the weight at the $i^{th}$ tap

$$y$$ is the output of the array

$$P_s(i)$$ is the signal correlation between the $i^{th}$ tap and the reference point

$$\text{sgn}(x) = -1, \quad x < 0$$

$$= +1, \quad x > 0$$

$\nu$ is an arbitrary constant.
Equation (1) by itself is not a difficult equation to program. The inclusion of fixed point arithmetic and subroutines for real-time processing increase the difficulties.

For an array steered in the \( \theta_d \) direction and for a plane wave striking at the \( \theta \) direction, the output of the array is

\[
y(t, \theta) = \sum_{m=1}^{K} \sum_{l=0}^{L-1} W(l, m) \chi(t - \Delta + T_m(\theta_d) - T_n(\theta))
\]

where \( T_m \) is the delay time to reach the \( m \)th element.

The power output of the array is

\[
P_m(\theta) = \overline{y(t, \theta)} = \sum_{m=1}^{K} \sum_{l=0}^{L-1} \sum_{i=1}^{s} \sum_{j=1}^{s} W(l, m) W(i, n) 
\]

\[
R[\chi(t - \Delta + T_m(\theta_d) - T_n(\theta)) + T_m(\theta) - T_n(\theta)]
\]

where \( R(\chi(t)) \) is the autocorrelation function of the stationary random process \( \chi(t) \). There are \( K^{2L} \) terms to compute for each value of \( \theta \).

The time required to compute equation (2) can easily be in terms of minutes for the case of 180 values of \( \theta \). In order to overcome this difficulty, terms of identical arguments are regrouped. A complete description of this regrouping and taking advantage of symmetry properties is given in reference (3). The resulting equation is

\[
P(\theta) = \sum_{m=1}^{K} \sum_{n=1}^{n+1} \sum_{p=1}^{n+1} \sum_{q=1}^{K} C(m, n, p) T(m, n, o; \theta_d, \theta)
\]

\[
+ 2 \sum_{n=1}^{K} \sum_{m=1}^{n+1} C(m, n, 0) T(m, n, 0; \theta_d, \theta)
\]

\[
+ 2 \sum_{m=1}^{K} \sum_{n=1}^{K} C(m, n, p) T(m, n, p; \theta_d, \theta)
\]

where \( \epsilon_p = 1, f = 0 \)

\( \epsilon = 2, f > 0 \)
\[ C(m, n, \beta) = \sum_{i=0}^{L-1} W(i+\beta, m) W(i, n) \]

\[ T(m, n, \beta; \Theta_d, \Theta) = R\left[ k_t + T_\Theta(\Theta_d) - T_\Theta(\Theta_d) + T_\Theta(\Theta) - T_\Theta(\Theta) \right] \]

Whereas equation (2) requires \( k^2 L^2 \) terms, equation (3) requires \( (k^2 - k + 1) - \frac{k^2 - k}{L} \) terms of \( T(m, n, \beta; \Theta_d, \Theta) \).

Thus, we compute \( \frac{1}{L} \) terms for each \( \Theta \), instead of 1600 terms. This is the minimum number of values of \( T \) required for an arbitrary array geometry. Although equation (3) looks more formidable than equation (2), the programming difficulty was on the same order. The subroutines in CS-1 language are given in reference (4), for equations (1) and (3).

The inputs at each receiver is sampled at 1,024 samples/sec. The total sampling time is 4 secs. Thus the total number of samples for the 4 receivers is \( 1024 \times 4 \times 4 = 16,384 \) samples. The projector emitted the PRN noise for a period of 6 secs. The computer obtained 4 secs of this digitized data and this data was stored in memory. The A-D conversion is 12 bits. Each word of memory can store 30 bits. Since half words may be used in the Univac 1230, the total input data is stored in 8,192 words of memory. The 1230 has 32,000 words of memory, so that there is ample storage for the program, after the data is stored. In the program, the number of taps in each delay line is 10, i.e. \( L = 10 \).

Results

The computer program was first simulated on ambient noise alone, i.e.
the projector, called the interfering noise, was not turned on. The ratio of interfering noise to ambient noise was 10db. The results of this experiment for one run, is given in Figures (6) and (7). In this case \( u \) was set to .005. The pattern was drawn at the end of each 1000 iterations. There is no strong interfering plane wave and no large notch is obtained. All patterns are similar to one another. For the next case, the projector was turned on. The results of this case are shown in Figure (8) and (9). For this run, \( u = .01 \). At the end of 1000 iterations, there is no definite notch. At the end of 2000 iterations there is a notch of 14.5 db at 82° and at the end of 3000 iterations it is 17.5 db at 82°. At the end of 4000 iterations it is 15 db at 83°. Each run for the case of the turned on projector resulted in notches similar to the case shown.

A notch corresponding to the surface-reflected path was not obtained, probably because the test was conducted during very rough weather and the surface reflected signal may have been severely attenuated.

**Discussion and Conclusions**

The primary conclusion is that adaptive beamforming does work in an ocean environment. The conditions in the medium during 4 secs of adaptation are evidently stable enough for the adaptive iterative process to converge. A notch of about 17 db, an improvement of approximately 7 db over that obtained in the absence of the interfering source is good, realizing that there were only 4 receivers in this experiment.

Since there was only 1,024 samples/sec and since the highest fre-
The frequency passed through the filters was 600 Hz, there was aliasing and the distortion of the power spectrum and in turn the correlation values. The sampling rate should have been greater than or equal to 1200 samples. Of course, we could have sampled at twice the bandwidth to obtain all the information in the input, but this would require changing the algorithm. Since there was distortion in the power spectrum, then the notch obtained was not at a minimum. Future work is planned to find the numerical effect of aliasing in this case, on the notch. It is also planned to write a minimum variance distortionless look (MVDL) on the data. This will result in a lower signal-to-noise gain but no distortion in the signal. It is also planned to carry out the adaptive beamforming method on other fixed arrays under different ocean conditions.
REFERENCES


TRANSMITTING RESPONSE

- HX-90 WITH 3000 FT CABLE
- UNTUNED
- DEPTH 18 FT

FIGURE 2
IDENTICAL SETUP FOR OTHER HYDS

BANDLIMITED TX SIG.

IRIG B

FM

AM

VOICE

FM 3/4 SEC

FR 1300

AM

HYD 31

FM

DEMOD

BUTTERWORTH FILTER

BW = 200 Hz

= 500 Hz

A/D CON

1230 COMPUTER

DIGITAL TAPE

±10V MAX
PROJECTOR NOISE

\( f_s = 500 \text{ Hz} \)

\( \text{BW} = 200 \text{ Hz} \)

\( \mu = 0.1 \)

FIGURE 9