THEORETICAL STUDY OF
FINITE AMPLITUDE STANDING WAVES
IN
RECTANGULAR CAVITIES WITH PERTURBED BOUNDARIES

by

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Thesis Advisor: A. B. Coppens

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**Theoretical Study of Finite Amplitude Standing Waves in Rectangular Cavities with Perturbed Boundaries**

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**Abstract**

The effects of various geometrical boundary perturbations on finite-amplitude acoustical standing waves in a rectangular, rigid-walled cavity were investigated using non-linear theory. The standing waves that exist in an ideal cavity must be corrected when the boundaries are irregular. Three specific examples (stepped, linear and wedged perturbations) were worked out to demonstrate the corrections (in first order) near
degeneracies for small perturbations. Those specific examples were compared to the experiments. The present theoretical model qualitatively predicts the effect of the perturbations on the behavior of the nonlinearly generated second harmonic. However, there are unexplained quantitative discrepancies between experiment and theory for a couple of cases.
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Finite Amplitude Standing Waves in Rectangular Cavities with Perturbed Boundaries
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( \rho )</td>
<td>Instantaneous density of the fluid</td>
</tr>
<tr>
<td>( \rho_0 )</td>
<td>Equilibrium density of the fluid</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Velocity potential</td>
</tr>
<tr>
<td>( \nabla \phi )</td>
<td>Particle velocity</td>
</tr>
<tr>
<td>( s )</td>
<td>Condensation</td>
</tr>
<tr>
<td>( \nabla )</td>
<td>Gradient operator</td>
</tr>
<tr>
<td>( \nabla \cdot )</td>
<td>Divergence operator</td>
</tr>
<tr>
<td>( \nabla \times )</td>
<td>Curl operator</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Shear viscosity coefficient</td>
</tr>
<tr>
<td>( \eta_B )</td>
<td>Bulk viscosity coefficient</td>
</tr>
<tr>
<td>( b = \frac{(4/3) \eta}{\eta_B} )</td>
<td></td>
</tr>
<tr>
<td>( \gamma = \frac{C_p}{C_v} )</td>
<td>Ratio of specific heats</td>
</tr>
<tr>
<td>( C_0^2 = \frac{dP}{d\rho} )</td>
<td>At ( \rho = \rho_0 ) for acoustical processes</td>
</tr>
<tr>
<td>( C_0 )</td>
<td>Speed of sound in an unbounded volume of air</td>
</tr>
<tr>
<td>( P )</td>
<td>Instantaneous total pressure</td>
</tr>
<tr>
<td>( P_0 )</td>
<td>Equilibrium total pressure</td>
</tr>
<tr>
<td>( p = P - P_0 )</td>
<td>Acoustic pressure</td>
</tr>
<tr>
<td>( D = \nabla^2 - \frac{c^2}{\rho \omega^2} \frac{\partial^2}{\partial t^2} )</td>
<td>D'Lambertian operator</td>
</tr>
<tr>
<td>( D_L = \nabla^2 )</td>
<td>D'Lambertian operator with losses</td>
</tr>
<tr>
<td>( \nabla^2 )</td>
<td>Laplacian operator</td>
</tr>
<tr>
<td>( c_p )</td>
<td>The frequency dependent apparent phase speed for standing waves in cavity</td>
</tr>
<tr>
<td>RHS</td>
<td>Right hand side of equation</td>
</tr>
<tr>
<td>LHS</td>
<td>Left hand side of equation</td>
</tr>
<tr>
<td>( (n,m,l) )</td>
<td>A (time-independent) normal mode of a rectangular, rigid-walled cavity of dimensions ( L_x, L_y ) and ( L_z ) such that ( k_x = n \pi / L_x, k_y = m \pi / L_y, k_z = l \pi / L_z )</td>
</tr>
</tbody>
</table>
(n,m,l/w,0) A standing wave designation when the (n,m,l) mode is driven at angular frequency \( w \); \( \theta \) is the phase angle with respect to \( t=0 \).

\[ Q \]
Quality factor

\[ Q_n \]
Quality factor at resonance of the \( n \)th standing wave when driven

\[ \beta = (\gamma + 1)/2 \]
For a gas

\[ M_0 \]
Peak Mach number of the driven standing wave

\[ C_n \]
Effective phase speed associated with the \( n \)th normal mode

\[ w \]
(Angular) frequency at which the cavity is driven

\[ w_n \]
(Angular) resonance frequency of the \( n \)th standing wave when driven

\[ \Delta \]
Magnitude of perturbation on the boundary

\[ t \]
Time

\[ \epsilon \]
Perturbation parameter

\[ p_0 \]
Classical linear solution for pressure for ideal boundaries

\[ p' \]
First-order perturbation correction due to boundary irregularities

\[ 1(t) \]
Unit step function

\[ \delta(t) \]
Unit impulse function

\[ \text{Re}\{\} \]
Real part of \{ \}

\[ a_0 \]
0th order Fourier coefficient
ACKNOWLEDGEMENT

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1. INTRODUCTION

The purpose of this research was to investigate some of the effects of boundary wall perturbations on finite amplitude standing waves in a rigid-walled rectangular cavity. The investigation was prompted by an examination of the experimental results of Coppens and Sanders\cite{3}, the research of DeVall\cite{5} and of Kilmer\cite{4}, which suggested the existence of the excitation of modes other than those belonging to the family of the driven mode.
2. BACKGROUND

A plane elastic wave travelling in a non-dissipative fluid will change waveform as predicted by the relevant hydrodynamic equations [6],[7]. If the problem is extended to absorptive media, only waves of relatively high amplitude will change waveforms appreciably.

At the Naval Postgraduate School, Coppens and Sanders [3], Kilmer [4], and DeVall [5] have dealt with the study of finite amplitude waves in rigid-walled rectangular cavities.

One interesting result of these cavity experiments was the appearance of excitations of modes which were not family members of the driven mode. For example, assume a rigid cavity of dimensions $L_x, L_y, L_z$ is driven acoustically at frequency $w$, the resultant pressure standing wave is of the form

$$\cos k_x x \cos k_y y \cos k_z z \cos (w t + \theta)$$  \hspace{1cm} (2.1)

where $k_x = \frac{\pi}{L_x}$, $k_y = \frac{\pi}{L_y}$, $k_z = \frac{\pi}{L_z}$ \hspace{1cm} (2.2)

and $N, M, L$ are integers. Eq. (2.1) can be represented by the notational shorthand

$$(N, M, L/w, \theta)$$

If the cavity is driven to excite the $(0, M, 0)$ mode, then the family of standing waves consist of all of those of the form $(0, nM, 0/nw, \theta_n)$ when $nw \neq n_0 w, M, 0$.

As it is stated in [3], "The standing waves which can be excited in any real cavity deviate from the predictions of the linear wave equation with ideal boundary conditions.
"for the following reasons:

(a). The presence of boundary-layer losses at the cavity surfaces yields a dispersive contribution to the wave equation.

(b). Geometrical irregularities alter the effective dimensions of the cavity.

Both of these mechanisms can be treated as equivalent as long as the shift in frequency are so small that the actual resonances are close to the theoretical values resulting from the classical model." These are treated by assuming the dimensions are exact, and the apparent phase speed is determined on that basis.

The resonance frequency for each standing wave is defined as

\[ w_n = C_n \sqrt{\left( n_x k_x \right)^2 + \left( n_y k_y \right)^2 + \left( n_z k_z \right)^2} \]  

(2.3)

where \( k \)'s are given by Eq.(2.2) and \( n \) is a shorthand for the set \( (n_x, n_y, n_z) \) where \( n_x, n_y, n_z \) are integers, and \( C_n \) is the apparent phase speed appropriate for that frequency.

The non-linear wave equation applicable to this problem can be obtained as follows:

The continuity equation for wave propagation in Eulerian coordinates is

\[ \nabla \cdot (\rho \mathbf{u}) + \frac{\partial \rho}{\partial t} = 0 \]  

(2.4)

this equation can be written in terms of the condensation, \( s \equiv (\rho - \rho_0)/\rho_0 \) as

\[ \nabla \cdot \left[ (1 + s) \mathbf{u} \right] + \frac{\partial s}{\partial t} = 0 \]  

(2.5)

The equation of motion in Eulerian coordinates for a contained viscous fluid is
\[\rho \left[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla P + b \gamma \nabla (\nabla \cdot \mathbf{u}) - \gamma \nabla \times \nabla \times \mathbf{u} + \text{ODAT} \quad (2.6)\]

where
\[P = \frac{\rho c_p^2}{\delta} \left( \frac{\delta}{\rho} \right)^2 = \frac{\rho c_p^2}{\delta} \left[ 1 + \gamma s + \frac{\gamma (\gamma - 1)}{2} s^2 \right] \quad (2.7)\]

ODAT = Other dispersive and absorptive terms arising from boundary effects.

Eq. (2.6) can be rearranged in the form of
\[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \frac{1}{\rho} \nabla P = \frac{1}{\rho} \mathbf{L} \mathbf{u} \quad (2.8)\]

where the operator \(\mathbf{L}\) describes those physical processes leading to absorption and dispersion.

One can write \(\nabla \times \mathbf{u} = 0\) and therefore \(\mathbf{u} = \nabla \phi\), where \(\phi\) is the velocity potential, based on the irrotational velocity assumption. Hence, \(\mathbf{L} \mathbf{u} = \mathbf{L} \nabla \phi\). Replacing \(\mathbf{u} = \nabla \phi\) and using the condensation, Eq. (2.8) can be written as
\[\frac{\partial}{\partial t} \nabla \phi + \frac{1}{2} \nabla (\nabla \phi)^2 + \frac{\rho c_p^2}{\delta} \nabla (1 + s)^{\frac{s-1}{2}} \nabla \phi = \nabla \mathbf{L} \phi \quad (2.9)\]

Now, with the help of Eq. (2.5) and (2.9), and after a good deal of manipulation the non-linear wave equation may be approximated in terms of velocity potential
\[c_p^2 \phi_t^2 \phi = \frac{\partial}{\partial t} \left[ (\nabla \phi)^2 + \frac{\gamma - 1}{\gamma} \frac{1}{c_p^2} (\frac{\partial \phi}{\partial t})^2 \right] \quad (2.10)\]

where
\[c_p^2 \phi_t^2 \phi \equiv c_0^2 \phi_t^2 + \frac{\partial}{\partial t} \mathbf{L}\]

It should be noted that if the fluid is lossless and \(c_p = c_0\) then (2.10) reduces to a previously known non-linear wave equation [9]

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\[ C_o^4 \Omega^2 \Phi = \frac{\partial}{\partial t} \left[ (\nabla \Phi)^2 + \frac{\gamma - 1}{2} \left( \frac{\partial \Phi}{\partial t} \right)^2 \right] \]

(2.10a)

In order to express the approximate non-linear wave equation in terms of acoustic pressure and particle velocity, one can rearrange the Eq.(2.9) in terms of \( p \) and \( \bar{u} \) and combine that equation with (2.10). The result is a quadratically non-linear wave equation \[ C_p \Omega^2 \left( \frac{p}{\rho c_o^2} \right) = -\frac{1}{2} \frac{\partial^2}{\partial t^2} \left[ \gamma \left( \frac{p}{\rho c_o^2} \right)^2 + \left( \frac{\bar{u}}{c_o} \right)^2 \right] \]

\[ + \frac{i}{2} C_o^4 \nabla^2 \left[ \left( \frac{p}{\rho c_o} \right)^2 - \left( \frac{\bar{u}}{c_o} \right)^2 \right] \]

(2.11)

If it chances to be that \( \left( \frac{p}{\rho c_o^2} \right)^2 \) and \( \left( \frac{\bar{u}}{c_o} \right)^2 \) nearly satisfy the wave equation, \( C_o^4 \Omega^2 ( ) = 0 \), then on the RHS of Eq. (2.11) \( C_o^4 \nabla^2 = \frac{\partial^2}{\partial t^2} \) and (2.11) becomes

\[ C_p \Omega^2 \left( \frac{p}{\rho c_o^2} \right) = -\frac{\partial^2}{\partial t^2} \left[ \frac{\gamma - 1}{2} \left( \frac{p}{\rho c_o^2} \right)^2 + \left( \frac{\bar{u}}{c_o} \right)^2 \right] \]

(2.12)

Further, if it happens that \( \frac{\partial^2}{\partial t^2} \left( \frac{p}{\rho c_o^2} \right)^2 \approx \frac{\partial^2}{\partial t^2} \left( \frac{\bar{u}}{c_o} \right)^2 \), as is true for solutions to the wave equation separated in cartesian coordinates, then \[ C_p \Omega^2 \left( \frac{p}{\rho c_o^2} \right) = -\frac{\partial^2}{\partial t^2} \left[ \frac{\gamma + 1}{2} \left( \frac{p}{\rho c_o^2} \right)^2 \right] \]

(2.13)

(Note that this is true only for cartesian coordinates.)

As it is stated in [3] "The LHS of Eq.(2.13) is the classical, linear wave equation pertinent to the system under study. The RHS can be interpreted as a forcing function consisting of a three-dimensional spatial distribution of phase-coherent sources. In a second-order perturbation theory,
"this forcing function is obtained from the classical (first-order) solution of the acoustic problem. The second-order perturbation solution describes the nonlinearities resulting from the self interaction of the classical solution. Higher-order perturbation solutions consider the interaction of the non-linear solution with itself, and the forcing function is composed of products of both classical and nonlinearly generated terms.

Thus, if a system is driven at frequency \( w \), the nonlinear term in equation (2.13) will force the existence of all integer multiples \( nw \) of the driving frequency and the full solution must contain all harmonics of the input frequency. In a closed cavity, each of those nonlinearly generated waves whose frequency lies close to the resonance frequency of a standing wave of the cavity and whose associated spatial function matches that of the standing wave can be strongly excited. Just how strongly will depend on the quality factor \( Q \) for the particular resonance and the difference between the resonance frequency of the standing wave and the harmonic \( nw \).

"Consider two limiting cases.

"(1) If the forcing function does not have its frequency \( nw \) close to \( w_n \), this standing wave is being forced at a frequency far removed from its resonance. Since this yields the inequality

\[
| C_0 \frac{\partial^2 \rho}{\partial t^2} | \gg | \frac{\partial}{\partial t} \rho | \tag{2.14}
\]

losses can be ignored in..."Eq.(2.13)."
"(2) If $nw \sim w_n$, then the standing wave is being forced near resonance, and losses must be retained in..." Eq. (2.13).

"The value of $C_n$ can be determined from the apparent dimensions of the cavity and the measured resonance frequency $w_n$.

"The losses are described by the measured $Q_n$ of the resonance. This means that the linear-wave equation operator for the system can be written as

$$C_0 \square^2 + \frac{2}{\lambda^2} \frac{\partial}{\partial t} = \frac{C_n \square^2 - nw}{Q_n} \frac{\partial}{\partial t}$$

(2.15)

"Comparison of cases (1) and (2) reveals that the response of the cavity when $nw \sim w_n$ is order of $1/Q$ compared to that when $nw \sim w_n$. Thus for the high-$Q$ resonances usually encountered in cavities with rigid walls, the components in the forcing function which excite standing waves far from resonance can be ignored compared to those components exciting standing waves near resonance....

"The non-linear, coupled, transcendental equation applicable to this problem can be expressed as

$$R_n \{ \cos \} (\theta_n - \phi_n) = N_0^0 M_0 Q_n \cos \theta_n \left[ \frac{1}{2} \sum_{j=1}^{n-1} R_j R_{n-j} \{ \cos \} (\phi_j + \phi_{n-j})$$

$$- \sum_{j=1}^{\infty} R_{n+j} \{ \cos \} (\theta_{n+j} - \phi_j) \right]$$

(2.16)

for all $n \geq 1$. The values of $Q_n$ and $w_n$ must be determined from the infinitesimal-amplitude behavior of the cavity. The Mach number $M_0$ and driving frequency $w$ are known and $N_0$ has the value

$N_0 = 1/2$ for a one-dimensional standing wave

$1/4$ for a two-dimensional standing wave

$1/8$ for a three-dimensional standing wave."
$R_n$ is the Fourier coefficient of $n$th harmonic component, normalized such that $R_1=1$. $\phi_n$ is the phase angle of the $n$th harmonic component, where $\phi_1=0$, and the phase angle $\theta_n$ is given by [3]

$$\tan \theta_n = -F_n$$

(2.17)

where

$$F_n = \frac{(nw)^2 - \omega_n^2}{(nw)^2 + \omega_n^2} \approx 2Q_n \left( \frac{n\omega - \omega_n}{\omega_n} \right) \text{ for } \frac{n\omega - \omega_n}{\omega_n} \ll 1$$

(2.18)

Equation (2.16) can be solved by a method of successive approximations on a digital computer. This has been done by [3] and [5] and both decided that the theoretical model can be used to identify the modes of a non-ideal, rigid-walled cavity provided quantities $e_n$ to be defined later are sufficiently small. The theoretical model in its present form fails to account for the excitation of modes other than those belonging to the family of the driven mode. This excitation was observed to occur only in the case of nearly degenerate modes. It is believed to be caused by some linear coupling mechanism within the cavity.

The purpose of this research is to see if the presence of wall irregularities can explain how non-family members may be strongly excited, and to present an example to support this theory.
3. DEFINITIONS AND NOTATIONS

A. FREQUENCY PARAMETER

The frequency parameter is a quantity which indicates the position of the driving frequency relative to the resonance frequency, $f_1$, of driven mode in terms of the $Q_1$ of the driven mode. The frequency parameter is defined by

$$ F_1 = 2Q_1(f_f_1)/f_1 $$

(3.1)

B. STRENGTH PARAMETER

The investigation of the pressure waveform in the cavity required the calculation of the strength parameter from the observable quantities. The strength parameter is defined as

$$ \text{STRPM} = M_0 Q_1 $$

(3.2)

where

$$ M_0 = p_1/(\rho_0 C_0^2) $$

(3.3)

and $p_1$ is the peak amplitude of $p_1$, the pressure distribution of the driven mode.

In terms of observable or calculable quantities it is reformulated as

$$ \text{STRPM} = \sqrt{2} V \Phi Q_1 / (S_m \rho C_0)^2 $$

(3.4)

where $V$ and $S_m$ are the rms voltage reading and microphone sensitivity respectively of the receiver used to sense the standing wave.
C. $e_n$ is defined to indicate the position of $w_n$ relative to the classical harmonic frequencies, $nw_1$.

$$e_n = \frac{w_n - nw_1}{nw_1}$$

(3.5)

and one can relate $e_n$ with $F_n$ from (2.18) such as

$$F_n = 2Q_n e_n$$

(3.6)

D. A pictorial representation of $F_n$ which will be useful throughout the development is given in Fig. 1. From now on three subscripts will be used for convenience, i.e. $F_n$ becomes $F_{nm}$. 

\[ \frac{1}{Q_{nm}} \]

\[ 1 - \left( \frac{w_{nml}}{nw} \right)^2 \]

\[ Q_{nml} \]

\[ \cot \theta_{nml} \]

\[ \sin \theta_{nml} = \frac{Q_{nml}}{(1 + f_{nml}^2)^{1/2}} \]

(3.7)

(3.8)
4. THEORETICAL DEVELOPMENT

A. CAVITY DESCRIPTION

Assume a perfectly rigid-walled rectangular cavity which has one wall perturbed such that the cavity dimensions are $L_x[1+\epsilon f(y,z)], L_y$ and $L_z$ as shown in Fig. 2 below. Also assume the perturbation on the boundary is very small compared to the cavity dimensions, $|\epsilon f(y,z)| \ll 1$.

![Diagram of a perturbed cavity](image)

FIGURE 2

The cavity is to be excited by a source near the origin in such a way that the $(N,M,L)$ mode is driven at a frequency close to its resonance frequency.
B. THE PERTURBED BOUNDARY

For a rigid-walled rectangular cavity with ideal boundaries ($\varepsilon = 0$), the pressure $p_0$ obtained from the linear wave equation with losses

$$\Box_L p_0 = 0 \quad (4.1)$$

is subject to the following conditions,

$$\nabla p_0 \cdot \hat{n} = 0 \quad \text{at} \quad x=0, L_x \quad y=0, L_y \quad z=0, L_z \quad (4.2)$$

where $\hat{n}$ is the local normal to the ideal boundary. The solution for $p_0$ in terms of Mach number is given by\cite{4}

$$\frac{p_0}{\rho_0 c_0^2} = M_0 \cos k_x x \cos k_y y \cos k_z z \cos (\omega t + \theta) \quad (4.3)$$

$$= M_0 (N, M, L/w, \theta)$$

where $k$'s are given in Eq. (2.2), and

$$w = c_p \left( k_x^2 + k_y^2 + k_z^2 \right)^{1/2} \quad (4.4)$$

If the cavity has perturbed walls, the solution will be in terms of a summation of the classical linear solution for ideal boundaries plus perturbation correction terms due to the irregular boundary:

$$p = p_0 + \varepsilon p' + \varepsilon^2 p'' + \ldots. \quad (4.5)$$

Since the magnitude of the boundary perturbation is kept small, second and higher terms in $\varepsilon$ can be considered insignificant, so that

$$p = p_0 + \varepsilon p' \quad \text{(to first order)} \quad (4.6)$$

and $p$ must satisfy the following conditions,

$$\Box^2 p = 0 \quad (4.7)$$
and
\[ \nabla p \cdot \hat{n} = 0 \quad \text{at } x=0, L_x [1+\epsilon f(y,z)] \]  
\[ y=0, L_y \]  
\[ z=0, L_z \]

where \( \hat{n} \), the local normal to the real surface, is obtained by taking the gradient of the equation for the boundary, given by [2]
\[ \hat{n} = \nabla \{ x-L_x [1+\epsilon f(y,z)] \} \]  
\[ (4.10) \]
Thus, to the first order in \( \epsilon \),
\[ \hat{n} = x-L_x \frac{\partial f(y,z)}{\partial y} \hat{y} - L_x \frac{\partial f(y,z)}{\partial z} \hat{z} \]  
\[ (4.11) \]
and when Eq. (4.11) is used in (4.8) the result is
\[ \left[ \frac{\partial p}{\partial x} - L_x \frac{\partial f(y,z)}{\partial y} \frac{\partial p}{\partial y} - L_x \frac{\partial f(y,z)}{\partial z} \frac{\partial p}{\partial z} \right]_{x=L_x [1+\epsilon f(y,z)]} = 0 \]  
\[ (4.12) \]
A Taylor series expansion [4] for \( p \) evaluated at the real boundary \( L_x [1+\epsilon f(y,z)] \) produces
\[ p|_{x=L_x [1+\epsilon f(y,z)]} = p|_{x=L_x} + \frac{\partial p}{\partial x} \bigg|_{x=L_x} L_x f(y,z) + \frac{1}{2} \left[ \frac{\partial^2 p}{\partial x^2} \right]_{x=L_x} [L_x f(y,z)]^2 + \cdots \]  
\[ (4.13) \]
Substituting Eq. (4.6) into RHS of (4.13), taking the partial derivative with respect to \( x \) on both sides and keeping the first-order terms in \( \epsilon \), yields
\[ \frac{\partial p}{\partial x} \bigg|_{x=L_x [1+\epsilon f(y,z)]} = \frac{\partial p}{\partial x} \bigg|_{x=L_x} + \epsilon \frac{\partial^2 p}{\partial x^2} \bigg|_{x=L_x} L_x f(y,z) + \cdots \]  
\[ (4.14) \]
Taking the partial derivatives with respect to \( y \) and \( z \) and using exactly the same procedure gives
\[ \frac{\partial p}{\partial y} \bigg|_{x=L_x [1+\epsilon f(y,z)]} = \frac{\partial p}{\partial y} \bigg|_{x=L_x} + \epsilon \frac{\partial^2 p}{\partial y^2} \bigg|_{x=L_x} L_x f(y,z) + \cdots \]  
\[ (4.15) \]
\[
\frac{\partial p'}{\partial x} \bigg|_{x=L_x} = \frac{\partial p}{\partial z} \bigg|_{x=L_x} + \epsilon \frac{\partial p'}{\partial z} \bigg|_{x=L_x} + \frac{\partial^2 p}{\partial z^2} \bigg|_{x=L_x} \epsilon L_x f(y,z) + \ldots
\]  

(4.16)

Substituting (4.14), (4.15) and (4.16) into (4.12) and keeping the first-order terms in \( \epsilon \) results in

\[
\frac{\partial p'}{\partial x} \bigg|_{x=L_x} = L_x \left[ - f(y,z) \frac{\partial^2 p}{\partial x^2} + \frac{\partial f(y,z)}{\partial y} \frac{\partial p}{\partial y} + \frac{\partial f(y,z)}{\partial z} \frac{\partial p}{\partial z} \right] \bigg|_{x=L_x}
\]

(4.17)

The RHS of Eq. (4.17) can be represented as a Fourier series in cosines, so that \( p' \) can be expressed as a summation of normal modes. Hence,

\[
\frac{\partial p'}{\partial x} \bigg|_{x=L_x} = \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} a_{ml} \cos \frac{m\pi}{L_y} y \cos \frac{l\pi}{L_z} z \cos (wt + \theta)
\]  

or

(4.18)

where

\[
a_{00} = \frac{1}{L_y L_z} \iint_{0}^{L_y} \int_{0}^{L_z} [G] dy dz
\]  

(4.19a)

\[
a_{m0} = \frac{2}{L_y L_z} \iint_{0}^{L_y} \int_{0}^{L_z} [G] \cos \frac{m\pi}{L_y} y dy dz
\]  

(4.19b)

\[
a_{0l} = \frac{2}{L_y L_z} \iint_{0}^{L_y} \int_{0}^{L_z} [G] \cos \frac{l\pi}{L_z} z dy dz
\]  

(4.19c)

\[
a_{ml} = \frac{4}{L_y L_z} \iint_{0}^{L_y} \int_{0}^{L_z} [G] \cos \frac{m\pi}{L_y} y \cos \frac{l\pi}{L_z} z dy dz , \quad m \neq 0, l \neq 0
\]  

(4.19d)

and \( G \) is defined as

\[
G = \frac{\partial p'}{\partial x} \bigg|_{x=L_x}
\]  

(4.19e)
In order to find the contribution to \( p' \) from each of the terms, it is stated [2] that for a cavity forced by a dynamic boundary condition at a boundary

\[
c_p^2 \frac{\partial^2}{\partial t^2} p' = 0 \tag{4.20}
\]

and the dynamic boundary condition from the \((m, l)\)th term is

\[
\frac{\partial p_m}{\partial x}
\bigg|_{x=L_x} = A \cos \frac{m\pi}{L_y} y \cos \frac{n\pi}{L_z} z \ e^{i(wt+\theta)} \tag{4.21}
\]

then

\[
p_m' = -A \frac{1}{\left( \frac{w}{c_0} \right)^2 L_x} \sum_{n=0}^{\infty} \Delta_n (-1)^n S_{nm} \cos \frac{m\pi}{L_y} x \cos \frac{n\pi}{L_z} z \ e^{i(wt+\theta+\sigma_{nm})} \tag{4.22}
\]

where

\[
\Delta_n = \begin{cases} 
1 & \text{if } n=0 \\
2 & \text{if } n=1, 2, 3, \ldots 
\end{cases} \tag{4.23}
\]

and \( S_{nm} \) is given by Eq. (3.8).

Applying this solution to Eq. (4.18) gives the complete solution for the first-order perturbation

\[
p' = \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} p_m' \tag{4.24}
\]

and combining (4.24) with (4.6) yields the total acoustic pressure in the cavity.

\[
p = (n, m, l / w, \theta) - \frac{\varepsilon}{(\frac{w}{c_0})^2 L_x} \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} d_m \Delta_n (-1)^n S_{nm} (n, m, l / w, \theta+\sigma_{nm}) \tag{4.25}
\]

If this is near a resonance, \( \omega \approx \omega_{nm} \), then this term will dominate the summation and all other non-degenerate terms can be omitted. Consequently, the Eq. (4.25) becomes

\[
p = (n, m, l / w, \theta) - \frac{\varepsilon}{(\frac{w}{c_0})^2 L_x} d_m \Delta_n (-1)^n S_{nm} (n, m, l / w, \theta+\sigma_{nm}) \tag{4.26}
\]
5. SPECIFIC EXAMPLES

A. CAVITY WITH STEPPED PERTURBATION

Assume that the rigid-walled rectangular cavity given in Fig. 2 is perturbed as shown in Fig. 3 below, and also assume that the cavity is driven in the \((0,1,0)\) mode resulting \((0,1,0/w, \Theta)\) standing wave and that the \((0,2,0)\) and \((1,0,0)\) modes are (nearly) degenerate.

From Fig. 3 the equation for the boundary at \(L_x\) can be found,

\[
x = L_x \left\{ 1 - \frac{\Delta}{L_x} \left[ I(y-L') - I(y-L) \right] \right\}
\]  
(5A.1)

By means of Eq. (4.8) \(\xi\) and \(f(y,z)\) can be written as

\[
\xi = \frac{\Delta}{L_x}
\]

\[
f(y,z) = - \left[ I(y-L') - I(y-L) \right] \quad \text{at} \quad L' < y < L
\]  
(5A.2)

(5A.3)

Since the \((0,2,0)\) and \((1,0,0)\) modes are degenerate the emphasis of this development will be on these particular modes. The pressure distribution of \((0,2,0)\) mode is

\[
\mathcal{P}_{020} = P_2 \cos \frac{2\pi}{L_y} y \cos (2\pi t + \Theta_2)
\]

or

\[
\mathcal{P}_{020} = P_2 \left( 0, 2, 0 / 2w, \Theta_2 \right)
\]  
(5A.4)

where \(P_2\) is the amplitude of \((0,2,0)\) mode.
Utilizing the theory developed in the preceding sections and using the equations (4.17) through (4.26), the first-order perturbation correction can be found

\[
\frac{\partial p'}{\partial x}\bigg|_{x=L_x} = L_x \left[ \frac{\partial f(y,z)}{\partial y} \frac{\partial P_{20}}{\partial y} \right]_{x=L_x}
\]

\[
= \frac{2\pi P_2 L_x}{L_y} \left\{ \sin \frac{2\pi y}{L_y} \cos (2\omega t + \Theta_2) \right\} \left[ \delta (y-L) - \delta (y-L') \right]_{x=L_x}
\]

Eq. (5A.5) can be written as a Fourier series

\[
\frac{\partial p'}{\partial x}\bigg|_{x=L_x} = \frac{2\pi P_2 L_x}{L_y} \sum_{m=0}^{\infty} \partial_m \cos \left( \frac{2\pi m y}{L_y} \cos (2\omega t + \Theta_2) \right) (5A.6)
\]

Inversion of the Eq. (5A.5) and (5A.6) yields the Fourier coefficients

\[
\partial_m = \frac{2}{L_y} \left[ \sin \left( \frac{2\pi m L}{L_y} \right) \cos \left( \frac{2\pi L'}{L_y} \right) - \sin \left( \frac{2\pi m L}{L_y} \right) \cos \left( \frac{2\pi L}{L_y} \right) \right] (5A.7)
\]

and

\[
\partial_0 = \frac{1}{L_y} \left[ \sin \left( \frac{2\pi L'}{L_y} \right) - \sin \left( \frac{2\pi L}{L_y} \right) \right] (5A.8)
\]

Recalling Eq. (4.23) and (4.24), first order perturbation correction \( p' \) is found as

\[
p' = -\frac{2\pi P_2 L_x}{L_y} \frac{\partial_0}{\left( \frac{2\omega}{C_0} \right)^2 L_x} (2)(-1)^{\frac{1}{2}} Q_{100} \sin \sigma_{100} \cos \frac{\pi x}{L_x} \frac{1}{e} \sin (2\omega t + \Theta_2 + \sigma_{100}) (5A.9)
\]

and

\[
\epsilon p' = \frac{4\pi P_2}{L_y} \epsilon \frac{\partial_0}{\left( \frac{2\omega}{C_0} \right)^2} Q_{100} \sin \sigma_{100} \cos \frac{\pi x}{L_x} \frac{1}{e} \sin (2\omega t + \Theta_2 + \sigma_{100}) (5A.10)
\]

where

\[
\left( \frac{2\omega}{C_0} \right)^2 = (2k_{100})^2 = \left( \frac{4\pi}{L_y} \right)^2
\]

Hence, the total pressure associated with the angular frequency \( 2\omega \), in the cavity is

\[
p = p_{20} + \epsilon p'
\]

\[
= P_2 (0,2,0/2\omega,\Theta_2) + P_2 \frac{L_y}{4\pi} \epsilon \partial_0 Q_{100} \sin \sigma_{100} (1,0,0/2\omega,\Theta_2 + \sigma_{100}) (5A.11)
\]
The total pressure at the microphone position, $x=0$ and $y=L_y$, is
\[
\left. P \right|_{\text{mic. position}} = P_2 \Re \left\{ e^{i(2\omega t + \theta_2)} + \left[ \frac{B}{\omega} \varepsilon \delta_0 Q_{100} \sin \sigma_{100} \right] e^{i(2\omega t + \theta_2 + \sigma_{100})} \right\} \tag{5A.12}
\]

Define $B = \left[ \text{ } \right]$ and after a little manipulation and use of trigonometric identities, (5A.12) becomes
\[
\left. P \right|_{\text{mic. position}} = P_2 \left\{ (1 + B \cos \sigma_{100}) \cos(2\omega t + \theta_2) - (B \sin \sigma_{100}) \sin(2\omega t + \theta_2) \right\} \tag{5A.13}
\]
and the amplitude of the total pressure in the cavity is
\[
\left. P \right|_{\text{mic. position}} = P_2 \sqrt{(1 + B \cos \sigma_{100})^2 + (B \sin \sigma_{100})^2} \tag{5A.14}
\]

Eq. (5A.14) is the corrected value of the amplitude of the second harmonic of the driving mode, obtained by Eq. (2.16), because of the boundary irregularity given in Fig. 3.

Now, it is desired to express $\sin \sigma_{100}$ in terms of the frequency parameter of the driving mode, $(0,1,0)$. With the help of Fig. 1, $\sin \sigma_{100}$ can be written as
\[
\sin \sigma_{100} = \frac{1}{(1 + Q_{100} \left[ 1 - \left( \frac{\bar{\theta}_{100}}{2f} \right)^2 \right]^2)^{1/2}} \tag{5A.15}
\]

If $f$ approaches to zero then $\sigma_{100}$ approaches to $\pi$, and if $f$ approaches to infinity then $\sigma_{100}$ is close to zero. In these same limits $\cos \sigma_{100}$ goes to $-1$ and $+1$ respectively. Hence
\[
\cos \sigma_{100} = \pm \sqrt{1 - \sin^2 \sigma_{100}} \tag{5A.16}
\]
For \( 2f \approx f_{100} \), (5A.15) becomes

\[
\sin \theta_{100} = \frac{1}{\sqrt{1 + \left(\frac{2f - f_{100}}{f_{100}}\right)^2}} \quad (5A.17)
\]

Recalling Eq. (3.1) and (3.5), \( F_{100} \) and \( e_{100} \) can be written in the form of

\[
F_{010} = 2Q_{010} \frac{f - f_{010}}{f_{010}} \quad (5A.18)
\]

\[
e_{100} = \frac{f_{100} - 2f_{010}}{2f_{010}} \quad (5A.19)
\]

Eq. (5A.19) can be solved for \( f_{100} \) and this substituted into (5A.17)

\[
2Q_{100} \frac{2f - f_{100}}{f_{100}} = 2Q_{010} \frac{Q_{100}}{Q_{010}} \frac{f - f_{010} (1 + e_{100})}{f_{010} (1 + e_{100})} \quad (5A.20)
\]

Use of \( 1/(1+e_{100}) = 1-e_{100} \) and little manipulation reveals

\[
\sin \theta_{100} = \frac{1}{\sqrt{1 + \left(\frac{Q_{010}}{Q_{010}} \left[F_{010} - 2Q_{010} e_{100} \right] (1-e_{100})\right)^2}} \quad (5A.21)
\]

As a result, equations (5A.14), (5A.16), (5A.21) are the final amplitude correction of the second harmonic of the driving mode obtained by Eq. (2.16).

A computer program for this was developed by author and is given in appendix A.
B. LINEARLY PERTURBED CAVITY

Using the same assumptions in section A, assume that the rigid-walled rectangular cavity is perturbed linearly as shown in Fig. 4 below.

\[ x = L_x \left[ 1 + \frac{\Delta}{L_x} \left( 1 - \frac{2}{L_y} y \right) \right] \]  

(5B.1)

Hence,

\[ \epsilon = \frac{\Delta}{L_x} \]  

(5B.2)

and

\[ f(y,z) = \left( 1 - \frac{2}{L_y} y \right) \]  

(5B.3)

Applying the same procedure as in section A, the first-order perturbation correction and the total acoustic pressure associated with angular frequency \( 2\omega \) in that particular cavity can be found

\[ \frac{\partial P'}{\partial x} \bigg|_{x=L_x} = L_x \left\{ \left[ -\frac{2\pi P_0}{L_y} \sin \frac{2\pi y}{L_y} \cos (2\omega t + \theta) \right] \left( -\frac{2}{L_y} \right) \right\} \]  

(5B.4)

\[ = \frac{4\pi P_0}{L_y^2} L_x \sin \frac{2\pi y}{L_y} \cos (2\omega t + \theta) \]

Eq. (5B.4) can be written as a Fourier series and the Fourier coefficient, \( a_m \), is found by an integration procedure evaluated in the interval 0 to \( L_y \). The result is

\[ a_m = -\frac{1}{M} \left\{ \frac{\cos(2\pi M)}{(2-m)} + \frac{\cos(2\pi M)}{(2+m)} - \frac{1}{(2-m)} - \frac{1}{(2+m)} \right\}, \ m \neq 2 \]

\[ a_0 = 0.0 \]

\[ a_2 = 0.0 \]  

(5B.5)
Recalling the Eq. (4.24), the first-order perturbation correction for (0,2,0) mode is

\[
\rho' = -\frac{4\pi R^2 L_x}{L_y^2} \frac{2}{(2\omega)^2} L_x (2)(-1)^{1} S_{100} (1,0,0/2\omega,\theta_2 + \sigma_{100})
\] (5B.6)

and the total acoustic pressure associated with angular frequency 2\omega in the cavity becomes

\[
\rho = (0.2,0/2\omega,\theta_2) - \frac{4\pi R^2}{L_y^2} \frac{G\rho_e}{(2\omega)^2} (2)(-1)^{1} S_{100} (1,0,0/2\omega,\theta_2 + \sigma_{100})
\] (5B.7)

since \(a_0 = 0.0\)

\[
\rho = (0.2,0/2\omega,\theta_2)
\] (5B.8)

According to the calculation developed above there is no need to make a first-order perturbation correction to the (0,2,0) mode in the cavity shown in Fig. 4. As a result, the pressure distribution is equal to the second harmonic of the driven mode, since \(a_0 = 0.0\) and this yields \(\rho' = 0.0\)
C. CAVITY WITH WEDGED PERTURBATION

Under the same assumptions made in sections A and B, assume that the cavity is perturbed as shown in Fig. 5 below.

The equation for this perturbation is

\[ x = L_x \left[ 1 - \frac{A}{L_x} \left( \frac{2y}{L_y} - 1 \right) \right] \]  

(5C.1)

By means of Eq. (4.8), \( \varepsilon \) and \( f(y, z) \) can be written as

\[ \varepsilon = \frac{A}{L_x} \]  

(5C.2)

and

\[ f(y, z) = \begin{cases} 0.0 & , \ y \leq L_y/2 \\ -( \frac{2y}{L_y} - 1) & , \ y > L_y/2 \end{cases} \]  

(5C.3)

Applying exactly the same procedure followed in section A, the total pressure amplitude in the cavity (in first-order perturbation) is

\[ P_{\text{mic. position}} = P_2 \sqrt{(1 + B \cos \sigma_{100})^2 + (B \sin \sigma_{100})^2} \]  

(5C.4)

where

\[ B = \frac{1}{2\pi} \partial_e Q_{100} \sin \sigma_{100} \] 

\( \sin \sigma_{100} \) and \( \cos \sigma_{100} \) are given by Eq. (5A.21) and (5A.16) respectively.

The theoretical predictions of these specific examples were examined with series of experiments developed by [8]. The further discussions about these will be given in the next section.
6. RESULTS

In this section the theoretical predictions performed in sections 5A, 5B and 5C will be compared to the experimental results obtained by [8].

The information on the empirical losses and resonance frequencies is contained in the Q's and e's. These are the values used in the computer program to predict the harmonic distortion on the basis of Eq. (2.16) and is plotted as thin solid curves. The results of including the first-order perturbation correction are plotted as thick solid curves for each specific example. The theoretical curves in figures 6 through 16 were plotted along with the experimentally-measured values for the cavity configurations shown on top of each figure. The theoretically-predicted values were generated for frequency-parameter intervals of 0.2, and the experimentally-measured values were plotted as square-blocks. Data were taken, and theoretical predictions made, for different strength parameters for the (0,n,0) mode associated with different cavity configurations. It is important to note that the n=2 distortion peaks when the system is driven at this frequency. That is, when the driving frequency \( w \) is equal to \( (1/2) w_2 \), there is maximum content of \( P_2 \). The point where this occurs for each \( P_n/P_1 \) curve is indicated by an arrow with the label of \( F_{020} \). At that point the value of frequency parameter is

\[
F_{020} = 2^{0_{010}} e_{020}
\]

(The same thing could be done, of course, for any member...
of the (0,n,0) family). The arrow labelled as \( F_{100} \) indicates the position where the nearly degenerate (1,0,0) mode is resonant, and the value of \( F_{100} \) is

\[ F_{100} = 2 Q_{010} e_{100} \]

The theoretical predictions made in section 5B were compared to the experimental results as seen in Fig.7. When Fig.7 is compared to Fig.6, the unperturbed cavity, it is clearly seen that the theory and experiment are excellently in agreement.

For a wedged perturbation, the theory predicts the frequency of the second harmonic at which the effect of the perturbation occurs as seen in Fig.8. The predicted magnitude of the perturbation effect for this configuration is in good agreement with the experiment. The anomalous behavior of the third harmonic in Fig.8 is unexplained.

For stepped perturbation, when the cavity is perturbed, but the geometry leads to no predicted correction as seen in Fig.9 or leads to predicted correction less than about 0.02 as in Fig.12 or less than about 0.05 as in Fig.14, then it was observed that there was very little or no effect from the (1,0,0) mode. Agreement for these cases is good except for the region lying between frequency parameter 4 and 9 in Fig.9. What happened in that region is also unexplained, but it was observed one time only. When the amount of perturbation correction is increased the theory predicts effects larger than experimentally observed. However, the effect of the perturbation appears at the right frequency

32
parameter as is seen in Fig.'s 10,11,13,15 and 16.

Choosing the shim position, length and magnitude is very important as well as is choice of the strength parameter. For the shims, $\Delta = 0.04$ and 0.25 inches for stepped and wedged perturbations respectively. The effect of strength parameter can be seen in Fig.'s 15 and 16. The experimental data associated with the third harmonic in Fig.15 were believed to come from harmonic distortion in the piston motion.
<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1 (Q)</td>
<td>2.57</td>
</tr>
<tr>
<td>Mode 2 (Q)</td>
<td>3.77</td>
</tr>
<tr>
<td>Mode 3 (Q)</td>
<td>4.50</td>
</tr>
</tbody>
</table>

**Figure 11**

Graph showing frequency parameter against frequency with data points and curves for different modes.
Figure 12
Figure 13

Frequency Parameter

Table:

<table>
<thead>
<tr>
<th>Mode</th>
<th>q</th>
<th>Frequency</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.2753</td>
<td>100.3927</td>
<td>100.552x10^2</td>
</tr>
<tr>
<td>2.0</td>
<td>0.3235</td>
<td>E(2)d</td>
<td>E(2)d</td>
</tr>
<tr>
<td>3.0</td>
<td>0.47155</td>
<td>E(3)5.52x10^7</td>
<td>E(3)5.52x10^7</td>
</tr>
</tbody>
</table>

Legend:

- Ideal
- Perturbed
- Experiment
FIGURE 14

FREQUENCY PARAMETER

n=2

n=3

n=4

F_20

F_100

P_n/P_1

0.1

0.01

0.001

-2

-1

0

1

2

3

4

5

MONE D. O1 OD
Q(1): 167.4
Q(2): 372
Q(10): 48.16
Q(11): 0.0
E(1): 0.0
E(2): 8.9 x 10^5
E(3): 12.4 x 10^5

NTOP: 20
NMAX: 10
NULS 6

IDEAL
PERTURBED
EXPERIMENT

42
FIGURE 15

MODE: 010
ST RPM = 20.5
NTOP = 20
NMAX = 10

\( Q(0) = 278.01 \)
\( Q(1) = 369.96 \)
\( E(1) = 7.4 \times 10^4 \)

\( Q(2) = 387.2 \)
\( E(2) = 3.6 \times 10^4 \)

\( Q(3) = 468.39 \)
\( E(3) = 20.8 \times 10^4 \)

\( k = 1.82 \)

FREQUENCY PARAMETER

FIGURE 15
7. CONCLUSION

Non-linear theory has been applied to standing waves in a rigid-walled rectangular cavity with a perturbed boundary in order to find one possible mechanism for the excitation of a standing wave other than those belonging the family of the driven mode. It was observed that such an excitation exists if the boundary perturbation and the dimensions of the cavity are favorably chosen.

It appears that the present theoretical model successfully predicts the major features of harmonic content for finite-amplitude standing waves in the cavity when the geometry leads to no perturbation correction (Fig. 7 and 9). When the magnitude of the perturbation is increased the predicted features were larger than experimentally observed. Second-order perturbation corrections may be needed to account for these discrepancies.
APPENDIX A

The original computer program for Eq. (2.16) was prepared by Coppens in 1973, and author made some extensions to that program so that it would (1) calculate the perturbation correction and (2) present the results graphically. The program calculates the relative amplitudes and phase angles of standing waves in cavities keeping the strength parameter constant and changing the frequency parameter to generate response curves showing the amplitudes of the nonlinearly excited standing waves as function of the frequency parameter $F$. It also calculates the perturbation correction according to Eq. (4.24) and then finds the total relative pressure amplitude using Eq. (4.26). The program also draws the graph of the relative pressure amplitudes of the ideal cavity, total relative pressure amplitude of the perturbed cavity and the experimental data on a 3 cycle semilog paper. The Versatec Graphics Plotting Manual [10] was used for the graphical processes on the IBM 360 of the Randolph Church Computer Center, Naval Postgraduate School.
SOME USEFUL INFORMATION ABOUT COMPUTER PROGRAM

* Quantities marked with (*) must be controlled or changed for each run.

*KON The number of iterations throughout the region of interest. For this program the iterations are performed with 0.2 intervals.

*NCUR The number of the experimental curves to be drawn + 1

*NDAT The number of experimental data in the region of interest

BUR(I,J) The array that stores the experimental data

*X L The length of the cavity in the x-direction

*YL The length of the cavity in the y-direction

*DELT A The magnitude of the perturbation

*STRPM Strength parameter

*FREQ Frequency parameter stored in ATA(I,1) and ZER(I,1). Input as the maximum value of FREQ in the region of interest

XDAT(JET) The x coordinate

YDAT(JET) Value of the curve f(x) for XDAT

ATA(I,N),N=1 The array that stores the logarithmic value of the pressure amplitudes of the harmonics of the driving mode

ZER(I,N),N=1 The array that stores the linear value of the pressure amplitudes of the harmonics of the driving mode

*Q(I) Quality factors of driving mode and harmonics of it

*E(I) e's value of driving mode and harmonics of it
*Q100 Quality factor of \((1,0,0)\) mode

*E100 e-value of \((1,0,0)\) mode

*XDAT(JET+1) Integer value of left-hand corner on the \(x\)-axis. It must have the same value as the 7th argument of subroutine CALL AXIS for \(x\)-axis.

*YDAT(JET+1) Integer value of left-hand corner on the \(y\)-axis. It must have the same value as the 7th argument of subroutine CALL AXIS for \(y\)-axis.

\[ \text{XDAT(JET+2)} \] Increment value of \(x\) and \(y\) for scaling purposes

\[ \text{YDAT(JET+2)} \]

HUM The linear value of the total acoustic pressure amplitude associated with angular frequency \(2\omega\).

*\(B\) \[
\begin{align*}
\frac{e}{n} Q_{100} \sin(100/4) L_y a_0 & \text{ for stepped perturbation} \\
\frac{e}{n} Q_{100} \sin(100(a_0/2)) & \text{ for wedged perturbation}
\end{align*}
\]

and \(a_0\) is the 0th Fourier coefficient.
// EXEC FORTCLGW
// FORT.SYSIN DD *

STANDING WAVES IN CAVITIES
NO DATA READIN--TO START OFF ON SKIRTS
AND GENERATE A STARTING DECK
THIS PROGRAM CALCULATES RAMP(N) AND PHI(N) FOR
STANDING WAVES IN THREE DIMENSIONAL CAVITIES
THE PROGRAM KEEPS STRPN THE SAME BUT
CHANGES FREQ TO GENERATE A RESPONSE CURVE

DIMENSION RAMP(50), PHI(50), THETA(50), FAC(50), S(50), C(50),
2 RATIO(50), DPHI(50), F(50), E(50), Q(50), CCRP(50), CCRR(50),
3 ZER(62,10), BUR(62,10), V(100), VN(100), XDAT(62), YDAT(62), ATA(62,10)
DATA LMENK1/-30584/, LMENK2/-21846/
DATA ZER, BUR, ATA/1860+0./
DATA Y, V/300*0./
DATA XDAT, YDAT/124+0./
R=10.73.
L=0
K=0
KON=36
1 FORMAT(//33X,'STRENGTH PARAMETER =',F5.3,1/
2 'FREQUENCY PARAMETER =',F6.3,1/
3 'NTOP =',I5,1/
4 'NMAX =',I5,1/
5 'NT =',I5/)
2 FORMAT(//15X,'AMPLITUDE',8X,'PHI',1/
3 'RATIO',9X, 'DPHI',1/)
4 FORMAT(//22X,'12F15.4,1F14.5)
5 FORMAT(//12X,'15F15.4,1F14.5)
6 FORMAT(//10X,'12F10.3,110,1F10.3)
7 FORMAT(//8X,'10F10.3')
8 FORMAT(//20X,'1F15.4')
9 FORMAT(//11X,'1F14.5')
10 FORMAT(//11X, '1F14.5')
11 FORMAT(//11X, '1F14.5')

DO 475 READ THE EXPERIMENTAL DATAWS WITH FORMAT 1001
ACUR=NUMBER OF CURVES TO BE DRAWN
NCUR=3
NDAT=24
DE 475 I=1,NDAT
475 READ(5,1001)(BUR(I,J),J=1,NCUR)
1301 FORMAT (4F8.6)
DO 50 N=1.50
FIX(N)=0.0
RAMP(N)=0.0
PHI(N)=0.0
RATIO(N)=0.0
DPHI(N)=0.0
THETA(N)=0.0
FAC(N)=3.0
CCRP(N)=0.0
CORR(N)=0.0
S(N)=0.0
C(N)=0.0
Q(N)=0.3
50
N=1
PI=3.14159
RAMP(1)=0.0
PHI(1)=0.0
S(1)=0.0
C(1)=1.0

----------------------------------------
C INPUT VALUES FOR PERTURBATION CORRECTION
C
0100=339.6
E100=5.43E-2
XL=5.96
YL=1.2
DELTA=0.04

----------------------------------------
C INPUT PARAMETERS
C RECTANGULAR CAVITY
C FAMILY CF MODES GIVEN BY VALUE OF RELABS
C KEEP NTOP, NMAX LE 50
C
RELABS=1.0
STRPM=0.205
FREQ=5.2
MAP=10
N=20
FXP=.0.75
RMIN=1.0 E-4

C HERE THRU 90 CALCULATES INFINITESIMAL-AMPLITUDE
C RESONANCE PARAMETERS FOR NON-R-K CASE
C
O(1)=256.25
O(2)=323.85
O(3)=450.8
E(1)=0.0
E(2)=6.85E-4
E(3)=4.085E-4
CC 89 N=4,NMAX
XA=FLOAT(N)
Q(N)=0.1*SCRT(XN)
E(N)=0.301*COS(XN-5.0)

85 CONTINUE
WRITE(6,11)(N,Q(N),E(N),N=1,NMAX)
DC 4001 LS=1,KON
FREQ=FREQ-0.1
DC 50 N=1,NMAX
TEMP=2.0*E(N)*Q(1)
XNUM=-FRUP*TEMP
XABF=-FREQ*TEMP
XDEN=Q(1)/Q(N)
XD2=XDEN*XDEN
TEMP=SQR(T(XNUM**2+XD2))
TBF=SQR(T(XABF**2+XD2))
THETA(N)=ATAN2(XNUM,XDEN)
ARGN=XNUM-XNB
ARGD=XDEN+(XNUM*XNB/XDEN)
CORR(N)=ATAN2(ARGN,ARGD)
CCRN(N)=TMBF/TEMP
50 FAC(N)=0.5*STRPM/TEMP
FREQ=FRUP

HERE THROUGH 383 INITIALIZES RATIO(N) AND DPHI(N)
DO 380 CALCULATES INITIAL VALUES FOR RATIO(N) AND DPHI(N)
AND OBTAINS THE VALUE OF N=NT AFTER WHICH RATIO =0.0 AND
RAMP LE RMIN
NT=NMAX
T1=1.0
T2=0.0
DC 380 N=2,NMAX
TF(RAMP(N)-RMIN)365,369,370
369 NT=N-1
DC 382 J=N,NMAX
RATIO(J)=0.0
382 DFF(J)=0.0
GO TO 383
370 RATIO(N)=(RAMP(N)/RAMP(N-11)*0.5*(1.0+CORR(N))
DIFF=PHI(N)-PHI(N-1)
TF(ABS(DIFF)-PI)374,371,371
371 TF(DIFF-PI)=373,372,372
372 DIFF=DIFF-2.0*PI
GO TO 374
373 DIFF=DIFF+2.0*PI
374 CRF(N)=DIFF+CORR(N)
T1=T1*RATIO(N)
T2=T2+DPHI(N)
IF(ABS(T2-PI)474, 471, 471
471 IF(T2-PI)473, 472, 472
472 T2=T2-2.0*PI
GO TO 474
473 CONTINUE
WRITE(6,6)LN=1,7
WRITE(6,5)LN
C HERE THRU 81 DETERMINES EXTRAPOLATION PARAMETERS
C OVER N=2, NT AND THEN CALCULATES RATIO(N), PHI(N)
C FOR N=NT, NTOP. IF (NT LE 2), PROGRAM SKIPS TO 81

185 TR=0.0
TD=0.0
TN=0.0
TN=0.0
TN=0.0
X=0.0
DO 190 N=2, NT
XN=FLOAT(N)
X=1.0/XN
TR=TR+RATIO(N)
TC=TD+PHI(N)
TA=TN+X
TN=TN+RATIO(N)*X
TA=TN+PX+X
TN=TN+X
X=1D+1.0
190 CONTINUE
ROE=1.0/(TNN*XD-TN*TN)
XMR=TNN*TR-TN*TRN
XMD=TNN*TD-TN*TDN
RINF=XUM*ROE
DPINF=XUM*ROE
RSLP=(DPINF*XD-TR)/TN
DSL=(DPINF*XD-TR)/TN
C IF RINF G 1.0, REEALCULATION BASED UPON
C RINF=1.0 AND RATIO(2)
C IF(RINF=1.0)192,152,191
191 RINF=1.0
RSLP=2.0*(1.0-RATIO(2))
152 CONTINUE
WRITE(6,2)N=NT, RINF, RSLP, DPINF, DSLP
XL=FLOAT(N)
TEMP=1.0/XN
RATIO(N)=RMIN-RSLP*TEMP
DPH(N)=DPINF-DSSL*TEMP

80 CONTINUE
81 CONTINUE

DC 3003 LM=2.5)
HERE THRU 278 CALCULATES RAMP(N) AND PHI1(N) FROM
N=LP LE NMAX), (NT-1) AND EXTRAPOLATES THEM FROM ALL RAMP AND PHI BEYOND RAMP(N)=RMIN
ARE SET TO 0.0. THE ASSOCIATE S(N) AND C(N) ARE FOUND.

260 IF(LM=NMAX)260,260,261
260 LP=LM
261 LF=LM
262 DO 276 N=LP,NTOP
270 DC 277 J=N,NTOP
270 RAMP(N)=RAMP(N-1)*RATIO(N)
271 PHI(N)=PHI(N-1)+DPH(N)
272 IF(ABS(PHI(N))-PI)274,272,272
274 PHI(N)=PHI(N)-2.0+PI
275 S(N)=RAMP(N)*SIN(PHI(N))
275 C(N)=RAMP(N)*COS(PHI(N))
276 CONTINUE
278 CONTINUE
HERE THRU 300 CALCULATES NEW VALUES
OF FIX(N) FOR N=1,LP

300 CONTINUE
HERE THRU 131 CALCULATES NEW VALUES FOR RAMP(N),
PHI1(N), S(N), AND C(N) FOR N=2,LP. WHEN
(RAMP(N) L RMIN), THEN FOR (IN LE N LE LP) ALL
ARE SET TO ZERO
DC 100 N=2,LP
SLMC1=3.0
SLMC2=3.0
SLMS1 = 0.0
SLMS2 = 0.0
M = N - 1
101 IF(M) 105, 105, 102
102 DO 104 J = 1, N
K = N - J
SLMS1 = SLMS1 + S(J) * C(K) + C(J) * S(K)
104 SUMC1 = SUMC1 + C(J) * C(K) - S(J) * S(K)
105 M = M + 1 - N
106 DO 108 J = 1, N
K = N + J
SLMS2 = SLMS2 + S(K) * C(J) - C(K) * S(J)
108 SUMC2 = SUMC2 + C(K) * C(J) + S(K) * S(J)
109 F = 0.5 * SLMS1 - SUMS2
G = 0.5 * SUMC1 - SUMC2
110 TEST = F**2 + G**2
RAMP(N) = RAMP(N) + FIX(N) * (FAC(N) * SCRT(TEST) - RAMP(N))
111 IF(RAMP(N) < RMIN) 110, 111, 111
112 DC 131 J = N
RAMP(J) = 0.0
PHI(J) = 0.0
S(J) = 0.0
GC TO 58
113 TEST = ATAN2(F, G)
114 TEST = TEST + THETA(N) - PHI(N)
115 IF(ABS(TEST) < PI) 1113, 1113, 1113
116 IF(TEST - PI) 124, 123, 123
117 TEST = TEST - 2.0 * PI
GC TO 113
118 TEST = TEST + 2.0 * PI
119 PHI(N) = PHI(N) + FIX(N) * TEST
120 IF(ABS(PHI(N)) < PI) 1125, 129, 127
121 IF(PHI(N) > PI) 29, 28, 28
122 PHI(N) = PHI(N) - 2.0 * PI
GO TO 129
123 PHI(N) = PHI(N) + 2.0 * PI
124 S(N) = RAMP(N) * SIN(PHI(N))
125 C(N) = RAMP(N) * COS(PHI(N))
126 CONTINUE
58 CONTINUE
HERE THRU 183 CALCULATES NEW RATIO(N) AND DPHI(N)
FOR ALL NONZERO RAMP(N). RAMP(N) IS HIGHEST NONZERO RAMP, BUT NT IS NEVER GREATER THAN NMAX.
SLMS1 = RATIO(N) AND DPHI(N) ARE SET TO ZERO FOR (NT L N LE NMAX)
NT = NMAX
CG 180 N=2,NMAX 165 NT=N-1 182 DC 182 J=N,NMAX RATIO(J)=0.0 182 D Phi(J)=0.0 GC TO 183 170 RATIO(N)=RAMP(N)/RAMP(N-1) 170 DIFF=Phi(N)-Phi(N-1) 171 IF (DIFF=0) 173,174,171 172 DIFF=DIFF+2.0*PI 174 GC TO 174 173 DIFF=DIFF+2.0*PI 180 CONTINUE 183 CONTINUE WRITE(6,11)(N,RAMP(N),Phi(N),N=1,NT) 1000 CONTINUE WRITE(6,10) WRITE(6,11)STRPM,FREQ,NTOP,NMAX,NT WRITE(6,3) N=1 WRITE(6,4)IN,RAMP(N),Phi(N) WRITE(6,6)(N,RAMP(N),Phi(N),RATIO(N),D Phi(N),A=2,NMAX) ZER(ILS)=FREQ 1789 MEH=2,10 DC 1789 MEH=.001 GO TO 1788 1788 ZER(ILS,MEH)=R*LOG10(RAMP(MEH)*1000.0) 4000 CONTINUE C STARTING TO DRAW A SEMI-LOG(3*70) GRAPH AS A BACKGROUND CALL PLOTS(0,0,0) DC 1111 I=1,3 Z=1.0 2222 DO J=1,60 K=K+1 IF (Z+J.0) GO TO 3333 Y(K)=R*LOG10(Z+.1)-LOG10(Z) Z=Z+.1 GC TO 2222 3333 Y(K)=R*LOG10(Z+.2)-LOG10(Z)
2222 CONTINUE
1111 CONTINUE
DC 4444 I=1,3
  Z=1.0
DC 5555 J=1,9
L=L+1
V(I)=R*(ALOG10(Z+1.)-ALOG10(Z))
5555 Z=Z+1
4444 CONTINUE
CALL GRID(1.,0.,70.,3.,1.,118.),V(L,MASK1)
CALL GRID(0.,0.,7.,1.0,1027.),V(L,MASK2)
C STARTING TO SCALE THE AXES
C LABELING THE AXES
CALL AXIS(0.0,0.,'FREQUENCY PARAMETER',-20.7,0.,-2.,1.)
CALL AXIS(0.0,0.,'P(N)/P(1)',+9.,10.,90.,0.,1.)
C STARTING TO DRAW THEORETICAL CURVES
DO 1787 I=2,10
  JET=0
  KAT=0
  DC 1786 J=1,KON
  IF(ZER(J,1).NE.0.) GO TO 1785
  KAT=KAT+1
  GO TO 1786
1785 JET=JET+1
  XCAT(JET)=ZER(J,1)
  YCAT(JET)=ZER(J,1)
1786 CONTINUE
  IF(KAT.EQ.KCN) GO TO 1002
  XDAT(JET+1)=-2
  XCAT(JET+1)=1.
  YDAT(JET+1)=0.0
  YCAT(JET+2)=1.0
  CALL LINE(XCAT,YDAT,JET,1,0,0)
1787 CONTINUE
C STARTING TO DRAW EXPERIMENTAL CURVES
DO 1784 I=2,NCUR
  JET=0
  DC 1783 J=1,NDAT
  IF(BUR(J,1).EQ.0.) GO TO 1783
  JET=JET+1
  XCAT(JET)=BUR(J,1)
  YDAT(JET)=R*ALOG10(BUR(J,1)*1000.)
1783 CONTINUE
  XCAT(JET+1)=-2
  XCAT(JET+2)=1.
  YDAT(JET+1)=3.0
  YDAT(JET+2)=1.0
CALL LINE(XCAT,YDAT,JET,1,-1,0)
CONTINUE
1764 STARTING TO DRAW PERTURBATION CORRECTION
ZIR=2.*(G(1)+E100)
ZOR=1.-E100
ZAR=0.572(1)
EPSILON=DELTA/XL
JET=0
IF(A(ATA(I-1),EQ.0).GO TO 2950
JET=JET+1
SINSIG=1./SORT(1.+ZAR*(ATA(I,1)-ZIR)*ZOR)*2)
COSSIG=SORT(1.-SINSIG**2)
IF(A(ATA(I,1),JE,ZIR) COSSIG=-COSSIG
B=0.037+EPSILON*Q100*SIGN/G)/PI
HUM=ATA(I,2)*SORT((1.+B*COSSIG)**2+(B*SINSIG)**2)
YDAT(1)=R*ALOG10(HUM*1000.)
XDAT(1)=ATA(I,1)
2950 CONTINUE
XCAT(JET+1)=-2
YDAT(JET+2)=0.
YCAT(JET+2)=0.
CALL NEWPEN(3)
CALL LINE(XDAT,YDAT,JET,1,0,0)
CALL PLOT(0.,0.,+999)
WRITE(6,2951)
2951 FORMAT(11.2X,FREQ.PRAM.,P2/P1,P3/P1,P4/P1)
111 P5/P1,P6/P1,77*,20X,611(-1),3X,7)
DC 2952 J=1,KON
WRITE(6,2953)(ATA(I,J),J=1,6)
2953 FORMAT(2X,F5.2,9X,5(F8.3,6X),/)
2952 CONTINUE
STOP
END
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