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Aeroelastic Flight Test Techniques and Instrumentation

by

J.W.G. van Nuenen and G. Plazzoli

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by
J.W.G. van Nunen and G.Piazzoli
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PREFACE

Soon after its founding in 1952, the Advisory Group for Aerospace Research and Development recognized the need for a comprehensive publication on flight test techniques and the associated instrumentation. Under the direction of the AGARD Flight Test Panel (now the Flight Mechanics Panel), a Flight Test Manual was published in the years 1954 to 1956. The Manual was divided into four volumes: I. Performance, II. Stability and Control, III. Instrumentation Catalog, and IV. Instrumentation Systems.

Since then flight test instrumentation has developed rapidly in a broad field of sophisticated techniques. In view of this development the Flight Test Instrumentation Group of the Flight Mechanics Panel was asked in 1968 to update Volumes III and IV of the Flight Test Manual. Upon the advice of the Group, the Panel decided that Volume III would not be continued and that Volume IV would be replaced by a series of separately published monographs on selected subjects of flight test instrumentation: The AGARD Flight Test Instrumentation Series. The first volume of the Series gives a general introduction to the basic principles of flight test instrumentation engineering and is composed from contributions by several specialized authors. Each of the other volumes provides a more detailed treatise by a specialist on a selected instrumentation subject. Mr W.D. Mace and Mr A. Pool were willing to accept the responsibility of editing the Series, and Prof. D. Bcsman assisted them in editing the introductory volume. In 1975 Mr K.C. Sanderson succeeded Mr Mace as an editor. AGARD was fortunate in finding competent editors and authors willing to contribute their knowledge and to spend considerable time in the preparation of this Series.

It is hoped that this Series will satisfy the existing need for specialized documentation in the field of flight test instrumentation and as such may promote a better understanding between the flight test engineer and the instrumentation and data processing specialists. Such understanding is essential for the efficient design and execution of flight test programs.

The efforts of the Flight Test Instrumentation Group members and the assistance of the Flight Mechanics Panel in the preparation of this Series are greatly appreciated. In particular, credit is due to the late Mr N.O. Matthews. Mr Matthews was Chairman of the Flight Test Instrumentation Group from 1976 until 1978 during which period major portions of this volume were prepared.

F.N. STOLIKER
Member, Flight Mechanics Panel
Interim Chairman, Flight Test Instrumentation Group
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1.0 Introduction

An important problem during the design of a new aircraft is to make sure that the structure (proposed and actual) is free from flutter within the proposed flight envelope. This requirement is essential since flutter involves a structural instability and as such will cause damage to the structure when it occurs. Flutter can only be prevented by proper design of the structure and, as a consequence, the study on flutter must start at a very early stage of the development of a new aircraft. It will continue along with this development up to and into the early flight test stage. For military aircraft which are to be equipped with a large variety of external stores, the study of the flutter problem may even extend beyond the first stage of flight testing. This is caused by the fact that variations in the external loading of an aircraft may change its flutter characteristics considerably.

Flutter has been recognised as a problem even in the early days of flying and although quite some effort has been made in the mean time towards an understanding of the problem, flutter still is a major point to be considered in the design of an aircraft. This is due in part to the fact that the development of new materials and more sophisticated methods of construction has led to an ever decreasing relative thickness of the lifting surfaces of an aircraft. The stiffness of these structures has, however, hardly improved and, therefore, the sensitivity to flutter has increased accordingly. On the other hand flying speeds also have increased considerably over the years and, as a consequence, the aerodynamic loads related to the flutter problem have not only grown, but have also changed their character due to the appearance of shock waves.

In the context of a discussion of aeroelastic test techniques it is appropriate to reserve some space to clarify somewhat the mechanism of flutter. A complex structure like an aircraft will in general possess an infinite number of vibration modes. For reasons of simplicity it will, however, first be assumed that the wing, which contributes the larger part of the aerodynamic loading, possesses only two degrees of freedom: translation and rotation. This situation is often a good approximation to what is found in practice when dealing with a flutter investigation. To further simplify the problem it will be assumed that both vibration modes have the same frequency and execute a harmonic motion. When this system is placed in an airstream, unsteady aerodynamic forces will be generated by the oscillatory motions of the system. Let us suppose now that there exists no phase difference between the two motions, i.e. translation and rotation both reach their maximum at the same time (figure 1a). Considering then the net work done by the aerodynamic forces over one cycle it will be found that the work is positive during some parts of the cycle and negative during others but that, as a whole, zero net work is done over a complete cycle. This means that the stability of the system remains unaffected by the presence of the unsteady aerodynamic loads. When, however, the rotation is $90^\circ$ ahead of the translatory motion quite another situation will occur (figure 1b). Then the aerodynamic force generated by the rotation is positive during the whole translatory upstroke and negative during the complete downstroke and consequently positive net work is done over a complete cycle. This means that energy has to be dissipated by the structural damping forces in the system or, in other words, the stability of the system has been diminished by the presence of the unsteady aerodynamic loads. At low flying speeds the structure will be capable of dissipating the energy which is fed into it, but beyond a certain speed, the so-called flutter speed, it will become unstable and reach a flutter condition. Vibrations occurring under such conditions will increase in amplitude with every cycle until the structural limits of the element under concern are reached. Because of the relatively high frequencies involved, once an unstable flutter condition is reached, the time to destruction is often too short to take corrective action. It is for this reason that flutter should not occur within the flight envelope of an aircraft and flutter flight tests are the last element in a chain by which the safe boundaries of flight are established.

To a certain extent the situation sketched above is also encountered when considering the possible coupling between wing bending and the fundamental rotation of a control surface. An important parameter in
that case is the relative location of the centre of gravity and the axis of rotation of the control surface (Fig. 2). In case the c.g. is positioned aft of the axis of rotation, inertial effects will produce a tendency for the control surface to move to a positive deflection during the upward motion of the wing. The reverse will evidently happen during the downward stroke of the wing and as a result the control surface generates aerodynamic forces which try to sustain the bending motion of the wing. Completely the opposite occurs, of course, when the c.g. position is in front of the axis of rotation; in that case the aerodynamic force generated by the movement of the control surface tries to reduce the motion of the wing. When the c.g. lies in the axis of rotation, no coupling will exist between the two motions and inertial effects will not generate aerodynamic forces to sustain or to reduce the wing motion. It will be clear that the first situation can lead to flutter problems and that the remedy to the problem can in principle be found in a proper alignment of the relative position of the c.g. and the axis of rotation of the control surface.

Because of the direct impact the flutter problem may have on the design of a new aircraft these problems are considered right from the beginning of a new design. During the development stage this investigation into the flutter problem will pass various stages as is indicated in figure 3. As long as only limited information is available on the detailed design of the structure, flutter calculations are based on theoretically obtained data. However, as the design progresses, experimental data such as unsteady aerodynamic loads from windtunnel tests and stiffness data from scaled models may also be introduced to update the flutter calculations. Coming closer to the end of the prefabrication stage, a dynamically scaled windtunnel model may be tested to improve the predictions. Once the actual aircraft has become available, the actual structure can be used to measure the vibrational characteristics. Ground vibration tests and static stiffness tests on certain parts of the aircraft will provide information to refine the flutter calculations. Finally, flight flutter tests are carried out to establish the safe boundaries of the envelope. If at any stage the calculations or the measurements indicate that flutter problems exist, appropriate modifications will have to be made to the structure of the aircraft.

For the design of a flight test programme a "flutter diagram" as shown in figure 4 is extremely useful. In this diagram it is indicated how the stability of a system, in this case a system with two degrees of freedom, develops with respect to airspeed. Starting at zero airspeed the vibrational characteristics of the mechanical system are shown: two resonance frequencies and the corresponding structural damping coefficients. With increasing airspeed the damping of the two vibration modes will generally increase, but above a certain speed the damping of one of the modes may start to decrease. At the point where the damping becomes zero the flutter speed has been reached for this specific system. The rate of decrease of damping with increasing airspeed depends highly on the characteristics of the system, but in some cases this rate of decrease may be very fast. This means that even a small increase in airspeed could change the system from a stable to an unstable condition (see the dotted line in figure 4). In such cases the flutter diagram, which is based on the results of all calculations done before the flight testing and on the previously obtained flight test results, should warn the flight test engineer that he should proceed with very small speed increments. The frequencies which have to be considered may lie, depending on the type of aircraft, in the range from 1 to 100 Hz. Damping coefficients normally range from 0.03 to 0.20.

In the following paragraphs a discussion will be presented about the specific topics to be considered when carrying out flight flutter tests. To that end the following subjects will be dealt with in some detail:

- general principles of flutter measurement techniques
- means to excite the aircraft during flight
- measurement of the vibrational characteristics during flight
- analysis techniques
- flight test procedures.

2.0 General principles of flutter measurement techniques.

2.1 General aspects

As outlined in the preceding chapter, the main data to be obtained from a flutter test are: the frequency and the corresponding damping coefficient of those vibration modes that dominate the problem. As a consequence it will be necessary to excite the aircraft and to measure the response of the aircraft
to that excitation and, in many cases, also the exciting force. The number and the nature of the vibration modes to be excited are very much dependent on the specific aircraft being tested and may thus change considerably from one aircraft to another. Such variations may also occur as the loading conditions of the aircraft change, e.g. due to the consumption of fuel or the presence of external stores. Knowledge of the vibration modes to be excited is important since the location and magnitude of the exciting force has to be adapted to these modes in order to be effective. This kind of information can be obtained through flutter calculations. This is an additional reason for making and refining such computations with ground vibration test data before performing a flight flutter test.

Along with the development of the means to excite an aircraft during flight a more or less continuous adaptation of the measuring procedures has taken place throughout the years. The various means to excite an aircraft will be treated in some detail in the next chapter. This present chapter will cover the principles of the various excitation procedures together with their pros and cons.

The following test methods will be discussed in some detail:
- decaying signal
- pulse excitation
- harmonic excitation
- sweep excitation
- random excitation.

2.2 Decaying signal

Probably the most simple procedure to determine the resonance frequencies and corresponding dampings of an aircraft is by exciting the structure for some time at resonance and then abruptly stopping the excitation. This excitation can be generated by both aerodynamic and mechanical means. As a result, the amplitude of vibration of the aircraft will die out and a time history as shown in figure 5 will occur. From this time history the resonance frequency \( f \) can be easily determined by establishing the time \( T \) between two successive crossings from, for instance, negative to positive amplitude:

\[
T = \frac{1}{f} \quad \text{(in Hz)}.
\]

The corresponding damping coefficient can be computed by various procedures. The first one is to determine the natural logarithm of the amplitude ratio of two peaks of similar sign. The damping coefficient \( g \) is related to this ratio through the relationship:

\[
g = \frac{1}{2\pi N} \ln \frac{A_0}{A_{2N}}
\]

in which \( N \): number of cycles between \( A_0 \) and \( A_{2N} \) (for definition see fig. 5).

Another possibility is to produce a plot of the natural logarithm of the peak amplitudes versus the number of peaks. According to the previous relationship this plot should show a straight line, the slope of which is a direct measure for the value of the damping coefficient.

A last procedure giving a quick indication about the damping coefficient is to establish the number of cycles required for the oscillation to decay to half of the original amplitude, say \( h \). The damping coefficient then is approximately:

\[
g = \frac{0.22}{h}
\]

The pros of this type of excitation can be listed as:
- The response signal can be easily analysed towards resonance frequency and corresponding damping coefficient;
- The excitation signal is very simple to generate. The frequency of the signal generator must however be variable, in order to follow variations in the resonance frequency;
- The testing time required to excite the structure and to analyse the response signals will in general be rather short.

In contrast with these points in favour of this method there also are some cons to be mentioned:
- Since the excitation applied to the aircraft is of a harmonic type, information on only one degree of freedom will be obtained each time an excitation is applied. So, although the method is quick in itself, it nevertheless may take some time to gather the required information when more degrees of freedom are involved in the problem;
- If spurious inputs occur in addition to the excitation, it may be difficult to obtain a sufficiently high signal-to-noise ratio in the response signal. Then especially the determination of the damping may become problematic, since the decay will consist of too small a number of usable peaks;
- A last problem may arise when two resonance frequencies come close together. Such a neighbourhood of frequencies causes a beating in the decaying signal and complicates the determination of the damping coefficient. To obtain the required information in such a situation filtering of the signal or the application of spectral methods may be considered, but in both cases the procedure has lost its primary attraction: simplicity. In case of filtering the decaying time signal should be reversed in order to ensure that the filter characteristics are not included in the analysed damping coefficient.

2.3 Pulse excitation

Another method of excitation is the pulse. This short duration excitation, which may be generated either by pilot induced control surface motions or by pyrotechnic means, leads to a response signal which often resembles very much the decaying signal discussed above because the structure responds in only one vibration mode. Then the same procedures as mentioned in the preceding paragraph can be applied to obtain the required information from the tests.

For this method of excitation the following pros can be mentioned:
- The excitation is of a very simple nature;
- If the response to the excitation occurs principally in one degree of freedom only, the calculation of the resonance frequency and damping can be done along very simple procedures. This situation often happens when only a limited number of degrees of freedom is involved, one of which is clearly much less stable than the others.

Some cons should also be put forward with respect to the pulse excitation:
- In principle, the pulse excitation contains power over a large range of frequencies and as a result the aircraft may respond in more than one degree of freedom. The response signal will then exhibit a beating which means that a determination of the damping coefficient cannot be done straight away, but only after a filtering has taken place;
- Because of the short duration of the pulse excitation, the energy stored in the excitation may not be sufficient to obtain a response signal with a reasonable signal-to-noise ratio. The result then is that determination of the damping coefficients and frequencies can only be done by applying more sophisticated analysis procedures.

2.4 Harmonic excitation

The next possibility in the sequence of excitation procedures is the harmonic excitation. As will be discussed in the next chapter, this excitation can be produced aerodynamically (by control deflections) or mechanically (by inertia exciters). A striking difference with the foregoing procedures is that in applying the harmonic excitation both the input force and the response of the structure have to be measured. The application of this method requires more analysis equipment than those previously discussed. This can, however, still be of a general nature, such as a transfer function analyser.

In essence the method works along the following lines. A harmonically oscillating excitation force is applied, the frequency of which is changed in small steps over a certain range. At each frequency the relationship between input, i.e. excitation force, and output, i.e. response of the structure, is determined in terms of amplitude ratio and phase shift between the two signals. This relationship can be presented in a so-called Kennedy-Pancu plot. The principle of such a plot for a system with one degree of freedom is shown in figure 6. The response vectors, normalized to unit input force, are drawn from the origin 0 in the direction of their phase angle. For a system with one degree of freedom the end points of these vectors will lie on a circle through 0 and the resonance frequency \( f_{\text{res}} \) will occur at a phase angle of 90 degrees. The resonance frequency can also be found as the point where the rate of change of frequency
along the circle is lowest. For the calculation of the damping coefficient a frequency range \( \delta f \) must be chosen, starting from the resonance frequency, which lies well within the high part of the amplitude curve, and the corresponding angle \( \theta \) at the centre \( M \) of the circle must be measured. Then the damping coefficient is

\[
g = \frac{\delta f}{\tau_{\text{res}}} \left( 2 + \cotg \frac{\theta}{2} \right).
\]

A description of this method and the derivation of the equation are given in Reference 1.

In Figure 7, which has been adapted from Reference 2, plots obtained with practical data are shown. The upper plot shows the results for a system which can be regarded as having one degree of freedom. It will be seen that the points closely follow the circle. The results for a system with 3 degrees of freedom are shown in the lower figure. It will be seen that three circles appear, the resonance frequencies of which must be determined by the second method described above. The points near to each resonance frequency are hardly affected by the other two resonances and can be used for determining three circles and the corresponding values of \( \theta \).

This harmonic excitation has certain pros of which should be mentioned:
- Because of the well-defined frequency of the excitation the procedure appears to be very selective to the occurrence of various resonance frequencies;
- In general a reasonable amount of energy can be produced by the excitation equipment, which means that a response signal with a fairly good signal-to-noise ratio can be established.

There however are also some cons to be mentioned when considering this method:
- The procedure works quite slowly, since the excitation has to dwell some time at each of the test frequencies selected. This dwell is required since the procedure assumes a steady state, which has to be established each time the excitation frequency is changed. This is especially true when the tests are carried out in a noisy environment. Moreover when dealing with a lightly damped system the spacing between the various frequencies has to be very small in order to enable a correct description of the circular plot from which resonance frequency and damping coefficient are to be computed;
- When two resonance frequencies come close together, it may be very difficult to separate them. There will always be some noise present and due to this unwanted excitation it may become problematic to obtain sufficiently stable conditions to determine the relationship between excitation and response. It should be noted however that this problem is not uniquely connected to this procedure, but that other excitation methods suffer from the same and that the solution to it has to be found by applying the most suitable analysis procedure;
- The establishment of the relationship between excitation and response requires more sophisticated measuring equipment than is generally needed for the previous two methods.

### 2.5 Sweep Excitation

Up to here the analysis of the response signals generated could be performed with commonly available electronic equipment. In recent years Fourier Analyzers have also become more widely available and with the advent of this specialised equipment it has become possible to apply certain excitation procedures which in earlier days could not be thought of because of the lack of appropriate analysis equipment.

A first type of excitation to be considered in this context is the sweep excitation. In this case a sinusoidal excitation with continuously varying frequency is applied by aerodynamic or mechanical means. The time elapsed between start and stop of the excitation and the rates of change of amplitude and frequency may be a point of consideration for each specific test. The response of a structure to such a type of excitation will be as shown in fig. 8; when passing a resonance frequency the amplitude of the response will increase above that measured at the off-resonance frequencies. To determine the resonance frequencies and corresponding damping coefficients from such signals, use has to be made of spectral techniques using the following relationship:

\[
H(k\omega) = \Phi_{xy}(k\omega) / \Phi_{xx}(k\omega)
\]

where:

- \( H(k\omega) \) : transfer function of system in both amplitude and phase
- \( \Phi_{xy}(k\omega) \) : cross power spectral density of excitation and response
\( \Phi_{XX}(\omega) \): auto power spectral density of excitation.

The transfer function contains complete information about the stability of the system, and actual values concerning the resonance frequencies and damping coefficients can be obtained by analyzing either the amplitude and phase characteristics of the transfer function or the power spectral density. Examples of both ways of presentation are given in fig. 9. In the complex notation the actual division into real and imaginary parts will depend on the type of response measured, e.g., acceleration or displacement. However, when dealing with the two parts individually the resonance frequencies can be determined from the location of a peak in the one part which corresponds to a zero crossing in the other part. For the damping coefficient the following relationship holds:

\[
g = \frac{\left(\frac{f_a/f_b}\right)^2 - 1}{\left(\frac{f_a/f_b}\right)^2 + 1}
\]

in which:  
- \( f_a \): the frequency above resonance where plot shows a peak  
- \( f_b \): the frequency below resonance where plot exhibits a peak.

If the real and imaginary part are combined into a vector plot, the same analysis procedures can be applied as with the harmonic excitation.

The lowest graph in figure 9 shows the modulus of the transfer function \( |H(\omega)| \). In this presentation all phase information has been lost and, as a consequence, the information can be given in one single plot. A vector plot resonance frequencies can be determined as those frequencies at which the plot exhibits distinct peaks, while the associated damping coefficient is found as:

\[
g = \frac{\Delta f}{f_{res}}
\]

in which:  
- \( \Delta f \): frequency difference at half the height of the peak  
- \( f_{res} \): resonance frequency.

In the foregoing it has been assumed, that all resonance conditions are sufficiently far apart to allow each of them to be regarded to act as a one degree of freedom system.

In case this assumption cannot be made, special attention will have to be given to the analysis procedure in order to obtain the required information. As far as the spectral density of the excitation is concerned, one should only take care that, when considering a system with several degrees of freedom, the sweep is adapted in such a way that all vibration modes of interest are excited properly. To that end various types of sweeps have been developed and applied throughout the past years, e.g., linearly varying frequency or exponentially varying frequency.

A very special case of sweep excitation is the so-called impulse sine wave. This input is described as 

\[
\sin\omega t
\]

and quite some attention has been paid to it in reference 3. The most interesting features of this sweep function appear to be a completely flat auto power spectral density from zero frequency up to \( \omega_1 \) and a response resembling the decaying oscillation (Fig.10). Especially because of this last property it is expected to be a promising procedure.

The pros of sweep excitation can be listed as follows:  
- In principle the procedure is able to detect the various resonance frequencies present in the system rather well;  
- Since the transfer function is determined by comparing the input and output signals of the system through the cross spectral density function, the procedure is reasonably insensitive to the effects of noise in the response signals;  
- In general the test duration can be rather short, although the presence of noise may have an adverse effect. On the other hand a proper choice of the type of the sweep, e.g., the impulse sine wave, may again shorten the test time.

Notwithstanding these attractive points some cons should also be mentioned:
In case some resonance frequencies come close together it may become difficult to distinguish between the different degrees of freedom;

- Presence of noise will cause a deterioration of the response signal. This problem can be reduced by applying a sweep of longer duration. Another possibility is to use sweeps of short duration and to collect response data from more than one sweep and average the results;

- Another problem arising when applying short duration sweeps may be the inability to inject sufficient energy into the system, which means that the response will remain too small to be analysed with sufficient accuracy. This problem may especially show up when using the impulse sine wave, thus adding a negative point to an otherwise attractive procedure.

2.6 Random excitation

A last method of excitation which also requires the use of spectral techniques for analysis of the results, is the random excitation. When this method is used, free air turbulence is usually the source of excitation, though random excitation using aerodynamic or mechanical means is also possible. The input will vary randomly and will, in general, contain information over a large range of frequencies. The response will, however, mainly contain information around the various resonance frequencies of the system and the corresponding time signal may look like figure 11. This time history represents the response to a random excitation of a system in which one of the resonance frequencies predominates.

The analysis towards the determination of the various resonance frequencies and damping coefficients makes use of the relationship:

$$H(k\omega) = \frac{\Phi_{xy}(\omega)}{\Phi_{xx}(\omega/\omega)}$$

or

$$|H(k\omega)|^2 = \frac{\Phi_{yy}(\omega)}{\Phi_{xx}(\omega/\omega)}$$

In which:

- $H(k\omega)$: complete transfer function of system
- $\Phi_{xy}(\omega)$: cross power spectral density of excitation and response
- $\Phi_{xx}(\omega)$: auto power spectral density of excitation
- $\Phi_{yy}(\omega)$: auto power spectral density of response.

Application of the first expression requires the measurement of both input and output and the analysis can be performed along the same lines as indicated for the sweep function. The second expression however opens the possibility to obtain meaningful results from the tests even without having the input signal available. For in case the auto power spectral density of the input can be assumed to be sufficiently flat in the frequency range of interest, i.e. in general around the resonance frequencies of the system, the relationship can be reduced to:

$$|H(k\omega)|^2 = \frac{\Phi_{yy}(\omega)}{\Phi_{xx}(\omega/\omega)} \times \text{constant}$$

As is discussed in section 3.1.2, the spectrum of free air turbulence generally is sufficiently flat. This means that the shape of the auto power spectral density (PSD) of the response signal due to free air turbulence resembles the shape of the transfer function of the system and, as a consequence, information about resonance frequencies and corresponding dampings can be obtained by analysing the PSD-functions of the response by the methods outlined in fig. 9.

A few years ago a different method for analysing response signals from randomly excited systems was developed. This procedure is called the randomdec analysis and has been reported in ref. 4. The principles of this method are outlined in figure 12. In short the procedure works as follows: Within the response signal a selection level is chosen and each time the signal crosses this level it is shifted towards a common reference time. Adding all the successive partial time signals finally results in a signal resembling the free decay signal, from which the resonance frequency and damping coefficient can be analysed by simple computations.

A pro of the random excitation is that excitation may be very simple, especially when considering that free-air turbulence has a random character over a broad enough spectrum of frequencies with no distinct peaks in its spectrum within the frequency band of interest for flight flutter tests.
Among the cons there should be listed:
- A basic feature of a random signal is that its frequency content changes with time and that one can only speak about a characteristic power spectral density function after having analysed the signal for some time. Especially for signals from lightly damped systems the averaging time required to obtain a sufficiently high statistical confidence may become quite long. Measuring times in the order of 4 minutes per test point should not be considered unusual in this respect;
- The previous negative point can be improved by applying the randomdec method, since there the required information can be obtained in a much shorter time;
- When using free-air turbulence as the excitation source it may be difficult to have the more complex high frequency vibration modes excited. This is due to the fact, that the energy content as well as the correlation of the turbulence decreases with increasing frequency.

2.7 Choice of an excitation method

In the preceding pages several excitation procedures have been dealt with each of them having their own specific pros and cons. It is realized, that a choice between the various possibilities remains very difficult, for in general such a choice will not be determined by one single reason, but by a combination of considerations such as:
- what sort of analysis equipment is available
- what is the level of experience of the test team
- what is the time of preparation for the test
- how many test points have to be collected in one test flight
- within what time frame do the test results have to be produced
- what is the complexity of the flutter problem.

Finally it should be noted that in prototype flutter flight testing it is common practice not to rely on only one excitation procedure, but to apply more than one method in order to safeguard against overlooking certain problems, for instance by not being able to excite certain vibration modes.

3.0 Means to excite an aircraft

As pointed out previously, it will be necessary to excite an aircraft in order to obtain the information, i.e. resonance frequencies and corresponding damping coefficients, required to establish the flutter stability of the aircraft. Throughout the years several means and methods of excitation have been developed and applied. Although the various methods may at first sight create the impression of being quite different from each other, a more thorough inspection will reveal that they are all based on a limited number of basic principles. The principal methods for producing the required excitation force are:
- excitation by an aerodynamic force
- excitation through a moving mass
- excitation by pyrotechnic means.

In the following chapters these principles will be discussed in some detail.

3.1 Aerodynamic means

3.1.1 Control surfaces

Probably the most directly available and simplest way to apply an aerodynamic force to an aircraft is by means of the control surfaces. Applying abrupt displacements to the control surfaces will result in aerodynamic forces that are of short duration and as such will cause the aircraft to respond in its lower frequency vibration modes. The response will be of the decaying oscillation type and the analysis towards resonance frequency and corresponding damping coefficient will in general be simple. To excite the aircraft in the antisymmetrical vibration modes, abrupt aerodynamic forces have to be generated through the ailerons, while symmetrical vibration modes can be excited through the stabilizer or elevator. In principle antisymmetrical vibration modes may also be excited by rudder pulses, but the efficiency of this kind of excitation will often be rather low due to the response characteristics of the fuselage.

It will be evident that this type of excitation is very simple indeed, since use is made of standard available tools. This simplicity has, however, also its drawbacks. First of all the limits of the
frequency range should be noted. This is especially the case when the stick and the control surfaces are linked by rods, but also on aircraft with powered controls this type of excitation has proved to be limited in frequency. From various experiences it can be stated that impulsive excitation through the normal flight controls is limited to about 10 Hz; excitation through the stabilizer may fall off at even lower frequencies because of the dynamic transfer characteristics of the fuselage. Another point to be considered is the poor repeatability of the applied force. This is mainly due to the fact that the impulsive force is in fact generated by the pilot. The time history of the force will, therefore, change from test point to test point and, as a consequence, the frequency distribution will also be different for the various test points. In that respect it has been observed that sharply applied control inputs of small amplitude are to be preferred above inputs of large amplitude, since these latter tend to be of longer duration and consequently more limited in frequency range. A further point may be the fact that the location of the applied force is fixed by the position of the controls. Certain vibration modes of the aircraft will, therefore, not be excited as well as one would like. For instance, when a nodal line of a certain vibration mode lies in the aileron, the excitation generated by this surface will be very ineffective for this specific mode. In the worst case the response will even contain no information at all on the mode under concern. An advantage of this method of excitation is, however, that there is in principle no restriction as to the number of pulses.

Because of its simplicity the method of stick pulse excitation is still widely used, though in many cases only in an exploratory phase of the flight testing. However, on small, lightweight aircraft such as sailplanes or small private airplanes the method is often applied as the only source of excitation. Another application may be when the flight flutter testing concerns the investigation of the effects of small modifications to the aircraft, as is often the case in external store certification programmes.

An extension of the possibilities of the control surfaces as a means to excite an aircraft has become available by the introduction of electrically and/or hydraulically operated controls. In this case it is no longer necessary for the pilot to generate the aerodynamic force, since his ask can be taken over by introducing an electrical signal directly to the servos of the control surface under concern. In this way the input is made directly to the control surface, which means that the frequency content of the force applied can be increased to a higher limit. Generally the electrical signal fed to the servos is of a sinusoidally varying type with frequency sweeping over a certain range. Often the pilot also has the option to select fixed frequencies in order to have the force acting at resonance with one of the vibration modes. Applications of this type of excitation have been reported for flight flutter investigations of several recent aircraft, such as F-15, Concorde and Jaguar.

As with the pilot-induced aerodynamic excitation, the excitation through the control of the servos is rather simple. An attractive point in both procedures is that hardly any additional weight is added to the aircraft, which means that the vibrational behaviour of the aircraft will not be disturbed. An advantage of the method using electric or hydraulic inputs is the good control one has over the applied force. It is also easier to measure the actual input force, which means that a great improvement can be obtained with respect to the data reduction of the response signals.

Having both input (force) and output (response) signals available, one can use, for instance, cross-spectral techniques for calculating the resonant frequencies and corresponding damping coefficients. In addition, a direct activation of the servos may also lead to larger aerodynamic forces and thus to responses that are less affected by noise. As far as the frequency content of the generated forces is concerned, frequencies as high as 35 Hz have been reported for forces generated by the ailerons.

The only remaining point that can be considered as negative is that the location of the control surfaces is fixed and that, as a result thereof, certain vibration modes may be excited only very weakly. When considering the use of this method, one should investigate the vibrational behaviour of the aircraft very thoroughly in order to see whether all important vibration modes will be excited. When this appears not to be the case, one might have to decide to use a different excitation procedure notwithstanding the simplicity of the one discussed above.

### Oscillating vanes

A few years ago another excitation method was developed, in which additional vanes are applied to generate the aerodynamic excitation force. Examples of such vane installations can be found in figure 13.
In principle, such a vane consists of a small airfoil protruding from the wing or the tail section. Since the vane is not a standard part of the aircraft structure, its location can be chosen in accordance with the vibration modes of the aircraft. The most favourable locations in this respect are the wing tips and the tips of fin and stabilizer. Other locations have been used, as well, such as the aft fuselage or the trailing edge of the wing. An example of the installation of vanes on an aircraft is shown in figure 14. The vanes are generally positioned on a shaft which is oscillated by means of an electric or hydraulic motor. This oscillation of the vane will result in a varying aerodynamic force acting on the aircraft structure. The amplitude of the force generated in this manner is, of course, directly related to the dimensions of the vane, the dynamic pressure of the airstream and the amplitude of rotation imposed on the vane. The time history of the motion and thus of the force can easily be controlled by a function generator; normal practice is, however, to have the system perform sinusoidally varying motions with a frequency which can either be constant or sweeping through a certain range.

This method has already been used in flight flutter investigations of the DC-10, L-1011, S-3A, F-14, C-5A and A-10. Better than anything else this rather long list of applications may demonstrate the attractiveness of this method. This means that it should be possible to mention several advantages that are peculiar to this procedure of producing an excitation. The most outstanding point is that the control of the exciting force, i.e. amplitude and time history, is rather simple. The frequency range of the oscillatory force can be extended to rather high frequencies; in several cases it has been stated that the excitation went as high as 40-50 Hz, which covers almost all of the important resonance frequencies for most aircraft. Since the vanes are additions to the normal aircraft structure, their positions can be selected in such a manner that the oscillatory force will act as effectively as possible. In other words, when the vibration modes of the aircraft are known, the vanes can be located as far as possible from the nodal lines of the important vibration modes. The force produced in this manner is very well repeatable and can in principle be applied as many times during one flight as is required. Usually the amplitude of the force can be made large enough to produce a response that is very well discernable from the response due to turbulence. This means that the analysis towards resonance frequencies and damping coefficients can in general be made rather simply by exciting the aircraft at resonance and then abruptly stopping the excitation. The resulting decaying oscillation can be analysed straightforwardly. A further improvement can be obtained by also measuring the exciting force imposed on the structure. Than analysis procedures can be applied in which input and output signals are correlated with each other. A simple means to determine this force is to attach a strain gauge bridge to the shaft on which the vane is mounted. In order to measure only shaft bending, special combinations of strain gauge bridges should be made. Further it should be realized that, due to local airflow effects, steady forces may also act on the vane and that it may be necessary to eliminate this non-oscillating part from the measured signal.

Obviously there are also some negative points when applying the vane technique. First of all the addition of the extra mass may cause the vibration modes of the aircraft to change; this effect will, in general, be more pronounced for a small and light-weight aircraft than for a large and heavy plane. Another point to be considered is that the addition of an extra lifting surface may change the flutter behaviour of the aircraft. In the most pessimistic case it may even turn out that the flutter characteristics have deteriorated because of the presence of the vane. To a large extent this may be caused by the contradictory requirements imposed on the selection of the position of the vanes: they should be located such that they produce an effective exciting force, but at the same time they are not allowed to be so effective that they adversely affect the flutter characteristics.

Because this vane excitation is very powerful, certain precautions have to be taken. These include:

- Extra hydraulic tubing directly connecting the vanes with the hydraulic power source (or with a separate power source). If it should be connected to the existing tubing of, for instance, the aircraft control surfaces, the large oscillating flow in the common parts of the tubing may affect the controllability, especially in case of flutter vane failure;
- Mass balancing of the vane so that the vane will trail in case of hydraulic pressure failure;
- Automatic or manual tripping of the excitation in the event of excessive response.

Finally it should be noted that any vane system should be designed and manufactured to its own specific requirements. An off-the-shelf purchase is usually not possible, since too many requirements have to be met which change from aircraft to aircraft.
3.1.3 Turbulence

A very simple method of excitation is to make use of free air turbulence. In essence, turbulence can be regarded as random variations in wind speed and wind direction which are caused by local variations in temperature. When flying through turbulent air the aircraft will be excited in a very nearly random manner; in other methods this excitation is considered as parasitic and measures are taken to avoid it. It is, however, also possible to use it deliberately as a specific source of excitation for the determination of the flutter characteristics of an aircraft.

From various studies which have been performed over the last decades it has been found that the free air or atmospheric turbulence can be described as a random, continuous, homogeneous and isotropic process in space which, when considered locally, appears to be stationary and to obey the Gaussian distribution law (Ref. 18). The energy contained in this atmospheric turbulence has been found to be as indicated in fig.15: the power is concentrated at the low side of the frequency range and decreases exponentially at the higher frequencies. It should be noted that the power of the turbulence is distributed very smoothly over the range of frequencies.

The response of an aircraft to such excitation will in principle take place in several of its resonance modes. It will be evident, however, that the lower frequency modes will be excited better than the higher frequency m-dps because of the concentration of the available power at these frequencies. As the higher frequency modes are in general of a more complex nature, also the effectiveness of the forces due to turbulence tends to become rather small. Generally, the response will be of a complex nature as is shown in fig. 11 and an analysis towards resonance frequencies and corresponding damping coefficients can only be achieved through the use of sophisticated analysis equipment. Because of the smooth character of the excitation, however, the analysis can in principle be done by using only the response signal.

Although this method is very simple to use, there are only two cases known as yet in which it has been applied: the flight flutter tests on the P61 "Seamaster" (as early as 1958) and on the YF-16. The reason why the procedure has not been applied more often is probably the fact that sophisticated analysis equipment is needed to extract the required information from the response signal. Besides, the random nature of the input causes the response of the aircraft to be random too and as a result response signals of quite long time length are generally required to obtain results with a sufficiently high statistical confidence level. An improvement to this method can be obtained by applying the randomdec analysis procedure, but even then remarkably long time histories are still required. A problem connected with this is that within this time lapse the flight conditions have to be maintained constant, which puts some strain upon the test pilot. This also excludes flights in which dives are required to attain the desired speed.

An improvement in the use of this excitation source can be obtained by measuring not only the response of the aircraft making certain assumptions as to the character of the excitation, but by determining the exciting force as well as the response. In that case cross correlations can be made between the input and output signals, which will result in the suppression of unwanted information in the response and in a decrease in the length of the measuring time required to obtain sufficiently reliable test data. In fact, it is not even necessary to measure the actual force exerted by the turbulence; one may confine oneself to determine a signal which is proportional to this force.

A way to obtain this type of signal during flight is to use a measuring vane as sketched in fig. 16. This vane is allowed to rotate freely about an axis that is oriented perpendicularly to the flow. The vane axis is located in front of the centre of pressure of the vane. Oscillations of the vane about its axis are measured by a potentiometer. The vane is mounted on a shaft in order to bring it outside the boundary layer of the aircraft. Favourable locations for this measuring device are near the front of the fuselage or on a boom close to the leading edge of the wing. To determine the various components of the field of turbulence, use is made of the equations which govern the flight mechanics of an aircraft in stabilized flight. For the vertical plane:

\[ l = \theta - \frac{\dot{z}}{V} + \frac{\dot{w}}{V} \]

In which:
- \( l \) : aerodynamic angle of incidence
- \( \theta \) : pitch angle of the aircraft
- \( \dot{z} \) : vertical velocity of the aircraft
vertical velocity component of the turbulence
V : airspeed

and for the horizontal plane:

\[ j = \phi - \frac{j}{V} + \frac{w_j}{V} \]

In which:

\[ j \] : aerodynamic angle of sideslip
\[ \phi \] : yaw angle of the aircraft
\[ \dot{y} \] : transverse velocity of the aircraft
\[ w_j \] : horizontal velocity component of the turbulence
\[ V \] : airspeed.

Clearly the aim now is to calculate the quantities \( w_i \) and \( w_j \). This can be achieved by measuring at the mounting position of the vane:
- the vertical acceleration \( \ddot{z} \) or the lateral acceleration \( \ddot{y} \)
- the angular velocity of the aircraft about the vane axis direction (\( \dot{\phi} \) or \( \dot{\psi} \))
- the vane deflection (\( i \) or \( j \)).

In combination with the airspeed of the aircraft, these quantities can be combined in real time by means of an on-board analog or digital computer to give the vertical velocity component of the turbulence

\[ w_i = V \cdot i - V \int \dot{\phi} \, dt + \int \ddot{z} \, dt \]

or the horizontal velocity component of the turbulence

\[ w_j = V \cdot j - V \int \dot{\psi} \, dt + \int \ddot{y} \, dt. \]

Before using this method as a means to determine the turbulence, certain calibrations have to be carried out to establish the sensitivity of the vane for translations and/or rotations of the aircraft. This calibration can be performed in special test flights in calm air: the aircraft is made to perform sinusoidal oscillations about the chosen axis and readings are taken from the three transducers in the system. By adjusting the signal from the potentiometer which measures the rotation of the vane, the calculated turbulence velocity can be made zero, which should be the case under turbulence-free conditions. If this setting is maintained during the actual flight tests, the variable calculated during flight through turbulent air then will represent the turbulence velocity component under concern.

In establishing the calibration of the vane system it should be realized that this calibration may depend on Mach number, since the local flow conditions at the measuring vane may change with a variation of that parameter. It should further be noted that all three transducers involved should be located in close proximity if they are to produce signals that are compatible with each other and do not exhibit unwanted phase shifts. This latter problem may, however, not be completely avoidable because the stiffness and the damping of the vane are directly governed by the unsteady aerodynamic forces acting upon it. This problem should be solved by introducing a proper filtering procedure into the electronic system.

Although this system possesses certain attractive points (rather simple procedure for the flight tests, measuring system reasonably small and thus of small mass, modifications to the aircraft limited) it has not yet been applied during flight flutter tests. It is, however, used on the B-1 aircraft to determine the turbulence field and to use this information as an input to an active control system set up to minimize the response of the aircraft to turbulence.

3.2 Moving mass

3.2.1 Inertia exciters

The inertia exciter usually consists of a rotating out-of-balance weight attached to a shaft which can be driven through the frequency range of interest. Quite often, however, two identical weights are applied and combined in such a way, that the resulting sinusoidal excitation force acts in one direction only rather than rotates as is the case if only one weight is used. The rotation of the main shaft may be produced by a variable speed electric or hydraulic motor. Often frequency sweeps are applied but
frequency dwell followed by an abrupt stop can also be used to produce the required response signals of the aircraft.

The use of this type of exciter has certain attractive points. First, the simplicity of the system should be noted. The design is rather simple and the exciter can be built without requiring too large an effort. Several exciters, each producing different excitation forces, can be distributed throughout the aircraft without much trouble and can be controlled individually to generate specific types of excitation, e.g. symmetrical or antisymmetrical. The frequency range can be extended to rather high frequencies; frequencies as high as 65 Hz have been mentioned. In principle these exciters can be located at those positions in the aircraft where they will produce the most effective excitation of the vibration modes of interest. To a certain degree the amplitude of the force can be controlled by changing either the weight of the mass or the amplitude of the motion of the mass. The dimensions of such exciters are rather small: an electrically driven device measuring roughly $10 \times 10 \times 20 \text{ cm}^3$ can produce a force of about 1600 N at a frequency of 40 Hz. Servo-controlled hydraulic exciters using oscillating masses have similar dimensions and characteristics. As already mentioned earlier, the frequency control of the exciters is rather simple: frequency sweeps as well as frequency dwelling followed by abrupt stopping has been applied in several flight flutter tests. It is reasonably simple to determine the force exerted on the aircraft structure or at least to obtain a signal which is proportional to that force. The actual force can be measured by installing a strain gauge bridge system between the inertia exciter and the structure, while a signal proportional to the force can be obtained by measuring the displacement of the mass and the frequency of oscillation.

Quite naturally, there are also a few negative aspects to the use of these inertia shakers. First of all there is the fact that the weight of the complete installation is added to the weight of the aircraft structure. How this may affect the vibrational characteristics of the aircraft will of course depend on the ratio of the shaker mass to the local aircraft mass. A further problem may be the generation of a sufficiently large exciting force at low frequencies. This is due to the fact, that the force produced is proportional to the moving mass and its acceleration which means that at low frequencies either a large mass or a large amplitude of oscillation is required to obtain a reasonable exciting force. Both may, however, be limited by the available space or other restrictions imposed by the surrounding aircraft structure.

Examples of flight flutter tests in which this technique has been applied, are those of the F-111, F-14, F-5E, B-1 and the MRCA Tornado.

3.2.2 Electrodynamic exciters

A means to overcome the problem of adding parasitic mass to the vibrating aircraft is the installation of an electrodynamic exciter. The basic principle of such an exciter is sketched in fig. 17. A permanent magnet is elastically suspended from the aircraft structure by means of a set of springs whose stiffness is such that the resonance frequency of the magnets on the springs is appreciably lower than the lowest excitation frequency. Usually a frequency ratio of about 3 can be taken as a good compromise. In the air gap between the poles of the magnets a light-weight coil is mounted. This coil is rigidly attached to the structure, which means that its mass has to be added to the aircraft weight. Feeding an oscillating electric signal to the coil now will cause the exciter to produce an oscillating force equal to

$$F = HL \cdot I = ma \alpha f^2$$

where:

- $H$ : magnetic field strength in the air gap
- $L$ : length of the windings in the coil
- $I$ : electric current in the coil
- $m$ : mass of the magnets
- $\alpha$ : amplitude of oscillation of the magnets
- $f$ : frequency of the oscillation.

From the above expression it may be seen that it is easy to control the force produced by the exciter, since that force is directly proportional to the current through the coil. This also means that this quantity can serve as a means for the measurement of the force. Looking at the other side of
the exciter, i.e. the moving mass of the magnets, it will be clear that a certain exciter force is
equivalent to a certain combination of mass, displacement and frequency. Especially for the lower
frequency range this may again cause problems, since quite large magnets or large movements of the magnets
are required to produce excitation forces of any importance. A mechanical stop within the exciter prevents
the magnets from travelling too long distances.

This type of exciters has been widely used, especially in France. Applications have been reported for
the flight flutter tests of various aircraft, such as C160-A03 "Transall", Nord 2.502 and Concorde.
Excitation force levels up to about 700 N have been mentioned.

The fact that the electrodynamic shaker only adds a small mass to the vibrating aircraft structure
can be considered as an advantage of this equipment. Further, several exciters can be operated simultaneously
at any required relative phase, since they can all be controlled from one common power source. In principle,
the excitation force generated by the exciter can be maintained constant and independent of driving
frequency or flight speed simply because the force is proportional to the current supplied to the coil.
This also means that during a frequency sweep the force can be kept constant, which makes it possible
to restrict the analysis to only the response signals since the input conditions are constant. Finally,
the electrodynamic shaker offers the possibility of adding damping to the structure to which it is attached.
For that purpose a feed-back control loop should be installed in which a signal proportional to the
relative velocity between the magnets and the structure (which is almost equal to the velocity of the
structure) is fed back to the coil.

Naturally, this system of excitation also has some drawbacks. First of all the exciters have to be
built into the aircraft, often at locations where mounting is difficult. This means that one has to take
account of these items at an early stage of the development of the aircraft. Obviously, this applies to
the installation space, the corresponding wiring, etc. Excitation at low frequencies will, in general,
be difficult because the resonance frequency of the magnets on their suspension cannot be made low
enough. Moreover, in case it has been possible to produce an exciter that can provide reasonably low
frequencies, parasitic excitation sources such as turbulence may act on the magnets and will adversely
affect the efficiency of the exciter.

3.3 Pyrotechnic means

A last method to produce an excitation force during flight is by making use of pyrotechnic means.
In principle these exciters contain a charge of black powder which is ignited and causes an excitation
force of short duration (in the order of 20 - 30 milliseconds, for instance). The actual time history of this
force and thus the frequency content can be controlled by the design of the exciter, also called bonker.
When considering the use of bonkers for a certain flight flutter test, one should first establish the
characteristics which the time history of the pulse force should obey. In this respect it should be
realized that the response of a point \( P_m \) in a structure to a pulse of rectangular (or trapezoidal) form
can be written as (Ref. 9):

\[
\Gamma(P_m, t) = \sum_k \frac{L_k(P_m) L_k(P_m)}{\mu_k} H_k e^{-\lambda_k t} \sin (2 \pi f_k t + \phi_k)
\]

In which:

\( \Gamma \) : acceleration of the response at measuring point \( P_m \)
\( L_k(P_m) \) : normalised displacement in vibration mode \( k \) at point \( P_m \)
\( P_0 \) : point of application of pulse force
\( \mu_k \) : generalised, i.e. effective, mass of structure in vibration mode \( k \)
\( H_k \) : adaptation function; calculated for each type of pulse and dependent on the actual duration
and the intensity of the pulse
\( \lambda_k \) : damping of the structure in vibration mode \( k \)
\( f_k \) : frequency of vibration mode \( k \).

An example of an adaptation function \( H_k \) is given in fig. 18. It will be seen that the frequency
spectrum of this function comprises several lobes. This means, that the pulse is selective to a certain
extents, since only that part of the spectrum between the two half-power points of the first lobe should be chosen as the useful frequency range of the pulse. The width of this frequency range and its location are directly related to the characteristics of the bonker. From the above expression it may also be learned that the acceleration level reached at a certain point $P_m$ at the end of the combustion time of the bonker depends only on the generalised mass of the structure in the specific vibration mode, $M_h$, and not on the damping value $\alpha$. This is due to the fact that the combustion time is very short, which means that the damping related part of the expression, $e^{-\alpha t}$, does not change very much during the short duration of the pulse. This makes it possible to check, during ground trials, the response level that will be obtained during flight, provided the vibration modes do not change drastically.

Especially in France quite some effort has been expended in the development of a wide range of bonkers (Ref.9). An example of such a bonker is presented in fig.18. Basically, the system consists of two hollowed steel plates which are screwed together. At one end a divergent nozzle is located with its axis perpendicular to the plane of the bonker. Close to it is an extra chamber that serves as a safety device in case an unwanted overpressure occurs. The bonker is sealed to prevent the charge from becoming humid. The charge consists of two layers glued to the internal side of the plates. By choice of the thickness of these layers the combustion time, and thus the frequency range which will be excited, can be controlled. The igniter is located at the other end of the bonker. The weight of the specimen shown is about half a kilo. The thrust provided reaches to a maximum of about 600 N.

The use of this type of exciter has several advantages. First, the short duration of the excitation means that it can also be applied during tests in which the test conditions change quickly, for instance during a dive. Since both the mass and the dimensions of the exciter are very small, they can be mounted practically anywhere in the aircraft without disturbing the vibrational characteristics of the aircraft. As such they may also be applied near the trailing edge of a wing or in an aileron. The installation of a bonker requires only very few extra measures. No high energy supply is required to get the charge activated. Simultaneous activation of several exciters is in principle possible, although small differences in the instant of firing may occur.

Certain disadvantages also go along with the application of these exciter. First, the pulse excitation is not completely selective; however, with a proper choice of the combustion time one has the ability to concentrate the force on a selected frequency range. It will be clear that repetitive use of the bonkers during one test flight is out of the question. Reloading the exciters with a new charge can only be done between flights, provided the exciter has been built for that purpose. Before installing a pulse exciter at a certain place it should also be considered whether the shear force introduced into the structure can be taken. Finally, it should be noted that the material from which the charges are manufactured may be affected by the environmental temperature. This may result in quite different pulse characteristics and may, in a bad case, produce an excitation of questionable quality.

Because of its small mass and dimensions these exciters lend themselves very much for application in light aircraft and gliders. Due to the short duration of the force they can serve in high speed aircraft during tests flights in which the test conditions can be maintained for only a short time. Among the aircraft on which exciters of this type have been used during flight flutter tests are the HR 100-210, Concorde, A-300B Airbus, Jaguar, MRCA Tornado, F-28 and VFW-614.

### Concluding remarks

In this chapter several means to excite an aircraft during flight flutter testing have been presented, each with their own specific pros and cons. It may be clear that the choice as to which excitation system should be applied will depend on several considerations. Some aspects which may decide the choice are:

- the experience already present in the test team
- the stage of development at which the flutter engineer is engaged: an early stage such as occurs in a prototype development or a very late stage such as store certification programmes
- the sort of analysis equipment available.

Frequently several excitation systems are used to increase the chance of success. Examples of such combined excitation installations are: F-111 in which vanes and inertia shakers were applied, A-300B which used electrodynamic shakers, control surfaces and bonkers and the MRCA Tornado in which inertia shakers were utilized.
Possibly more than for other types of flight tests the specialists, i.e. the flutter engineers, will have to have a considerable input in the decision about the type of test instrumentation to be used such as the type of exciters and the locations at which they should be positioned. Good cooperation between the flight test and flutter departments has to be established in order to guarantee a worthwhile and sensible flight flutter test programme.

4.0 Measurement of the vibrational characteristics in flight

4.1 General

From the previous chapters it will have become evident that the data to be collected during flight flutter tests can be divided in two groups: those for measuring the response of the aircraft and those for measuring the excitation force.

In the first group the most commonly applied transducers are accelerometers and strain gauge bridges. In principle there is no clear preference for one over the other. With both types of transducers, translational and rotational vibrations can be measured, such as will be required, for example, for a wing bending/rudder rotation flutter case. The decision as to what type of transducer will be used in a specific flight flutter test will depend to a certain extent on the stage of the construction of the aircraft at the time such tests are discussed. If the execution of flight flutter tests is decided during the design of the prototype, a good many strain gauge bridges will already be available because of other requirements such as stress analysis. Many of these bridges can also be used for flight flutter testing. Additional bridges can easily be put in at this stage. So in prototype flight testing strain gauge bridges will form an important part of the flight test instrumentation, though accelerometers will often be used also. If, however, flight flutter tests have to be performed on an already existing aircraft, as often occurs in case of an external store certification programme, one may want to use accelerometers rather than strain gauge bridges. In such a case the installation of strain gauge bridges will often be much more complicated than the installation of accelerometers. As the strain gauge bridge is sensitive to strain in the structure and the accelerometers to displacements of the structure, the locations at which the two types of transducers are applied, will in principle be different. In a wing, for instance, a strain gauge bridge will in general be applied near the root, while an accelerometer will be located at the wing tip.

In general no specific requirements will be made concerning the amplitude accuracy of these transducers. For the determination of the damping coefficient no absolute amplitudes are required but only amplitude ratios (see for instance the procedures applied in the decaying oscillation case, section 2.2). A good linearity is, however, a primary requirement and it is also essential that the phase relation between the different transducers is of good quality. This is the more important when a rotational mode has to be determined from two linear transducers, or when the symmetry or antisymmetry of a vibration mode has to be established.

Since the transducers are meant to measure the overall vibrational behaviour of the aircraft, care should be taken to mount the transducers so that local vibrations of the structure do not significantly affect the measuring signal.

Regarding the frequency characteristics of these transducers it should be realized that the frequency range to be covered in a flight flutter test normally ranges up to about 50 Hz. This requirement should play a role in selecting the particular type of transducer (especially when selecting accelerometers).

A final point to be taken into account is the fact that the relatively large variations in environmental conditions, such as pressure and temperature, should not influence the response characteristics of the transducers. Large variations may occur in high-speed flutter tests, where large temperature increases may occur locally due to aerodynamic heating.

The second group of measuring signals concerns the determination of an externally applied excitation force. It will be evident that the transducer to be used in this case is completely determined by the type of excitation force applied. It can range from the measurement of a current, as in case of an electrodynamic exciter, to a strain gauge bridge for measuring a vane excitation force. The requirements as to

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a) The authors are indebted to Mr. A. M. Mackenzie of Rockwell International Corporation for his valuable contribution in preparing this chapter.
amplitude accuracy, frequency response and sensitivity for environmental conditions for these transducers will in general be the same as for the type of transducers measuring the response of the structure.

4.2 Transducers

4.2.1 Accelerometers

The accelerometers normally applied in flight flutter testing can be divided into two groups: those in which the measuring signal is generated by a strain gauge bridge and those in which a piezoelectric element generates the signal. Each of these two types has its own specific properties.

Strain gauge accelerometers are usually rather bulky. A power supply is required for the excitation of the strain gauge bridge. The frequency response of these sensors ranges down to d.c., which makes them very convenient for low-frequency flutter measurements, but it also means that rigid body motions such as short period oscillations and manoeuvres of the aircraft are measured also. The resonance frequency generally is relatively low (order of 200-300 Hz), but if they have internal fluid damping, they are usually sufficiently insensitive to vibrations from engines, etc.

Piezoelectric accelerometers are normally very small and rugged. No power supply is required. Their output is very small and does not go down to zero frequency. A charge amplifier or a voltage amplifier with a very high impedance (several hundreds of Megohms) is required for reasonable low-frequency response. In many transducers a charge amplifier is built-in in the transducer. If this is not the case, the coaxial cable between the transducer and the amplifier should be as short as possible. Their natural frequency is usually very high (several ten-thousands of a Hz). This means that electrical filtering of high-frequency signals will be necessary, especially when digital recording or telemetry is to be used.

4.2.2 Strain gauge bridges

The use of these devices is to a certain extent restricted to prototype aircraft since the installation of a strain gauge bridge will usually be very difficult in an existing aircraft. The sensors are very small, but a power supply and amplifiers are required. The frequency response goes down to d.c., which means that static loads are also measured. In certain cases such d.c. offsets may cause problems during the data analysis and appropriate measures have to be taken to eliminate this d.c. shift. Finally the output of a strain gauge bridge may be prone to non-linearity, which effect has to be considered in the analysis procedures.

4.3 Number and location of transducers

In normal flutter testing it is necessary to obtain information on the behaviour of many vibration modes. The limitations of on-board recording and/or telemetry capacity preclude the use of sufficient transducers to completely define all mode shapes in flight. This is, however, not an important limitation since for the purpose of flight flutter testing it is sufficient that all important vibration modes can be recognized. This means that the locations of the transducers should be selected in accordance with these vibration modes and that a positioning of transducers on the nodal line of a certain vibration mode will be acceptable only in case this offers the possibility to differentiate between various modes. The probability that different modes possess coinciding nodal lines can be neglected.

In principle the total number of transducers will vary from one aircraft to another, but a good average is somewhere between fifteen and twenty transducers. The locations appearing in almost every transducer lay-out are: wing tip (both leading and trailing edge), tip of fin and stabiliser, centre of gravity, nose and tail of fuselage (both vertical and horizontal), front end of underwing stores or engines (both vertical and lateral) and control surfaces. A few examples of instrumentation schemes are presented in fig. 19 and 20. The latter figure is more complete because both the transducers measuring the response and the excitors are indicated. As may be observed from that drawing, the excitors are distributed throughout the aircraft in about the same manner as the transducers are, i.e. they are situated at those locations where they can produce as effective an excitation as possible.
4.4 Processing of the signals

4.4.1 Filtering

The signals obtained from accelerometers, strain gauges and other transducers usually contain components which are of no interest for the flutter investigation. These components can be either lower than the frequency range of interest (e.g. aircraft manoeuvres) or higher (e.g. local resonances not connected with flutter). The high-frequency components are usually filtered out before recording, telemetry or digitization in order to reduce the bandwidth or sampling rate required. This is usually done by passing the signal through a low-pass filter with a cut-off frequency above the highest frequency which is of interest. If the transducer is an accelerometer, filtering can also be done by a single or double integrator, which is in essence a low-pass filter with a natural frequency below the lowest frequency of interest. Then the amplitudes at the higher frequencies of interest are reduced with respect to the lower frequencies, which may be an advantage in specific cases.

The main requirements for such filters are that they reduce the amplitudes of spurious signals to insignificant values and that they retain the phase relationships in the frequency range which is of interest. In order to meet the former of these requirements, it is necessary to have some knowledge of the amplitudes and frequencies of the spurious signals. This can often be obtained from theoretical studies, but it will be necessary to verify these in flight. In the high-pass filters used to eliminate the effect of aircraft manoeuvres it is generally more difficult to avoid phase distortion. This filtering is therefore usually done later in the processing chain, when more elaborate equipment such as digital computers are available. It must also be kept in mind that the sample and hold circuits in digitizers can introduce some phase jitter.

4.4.2 Storage and telemetry

The data are always stored on on-board magnetic tape recorders. In the past FM recording was often used, but nowadays most data are recorded in digital form. In addition, it is common practice to telemeter all or part of the flutter data to the ground. These data can then be observed in real time by suitable specialists. This not only reduces the risk involved in flutter flying, but also allows greater increments of airspeed from test to test, and more test points per flight. The ground equipment used with this telemetry can range from simple strip recording of a few selected channels to a very complex setup involving a large computer with associated peripheral equipment. A disadvantage of using telemetry is that the flight must be within line of sight, unless repeater stations are used.

Telemetry for flutter testing can be either analog or digital, but both act as a constraint to the data that can be transmitted to the ground, and the dynamic range of that data.

Analog telemetry is usually FM/FM and although flutter data does not usually require a great bandwidth (typical ability to record 200 Hz is more than adequate), it is often limited to a maximum of about 20 parameters. It has the advantage that (apart from errors causing 180° phase inversion) it is not prone to introduce any serious phase shift in the data. Slight telemetry "dropout" is usually less disastrous than it would be in digital transmission. It is the most commonly used telemetry system for flutter testing (L-1011, B-1, YF-16 etc.).

Digital telemetry increases the number of parameters that can be transmitted, but requires a very high bit rate to provide adequate frequency response (typically on a medium sized military airplane 800 k bits/sec, giving 100 parameters at 10 bit accuracy, 800 samples/sec). Even small amounts of TM dropout can wreak havoc with the analysis.

5.0 Analysis techniques

Until recently the measured signals were handled all the way down to the analysis in an analogue form. This meant that only simple procedures could be applied unless time delay did not play an important role. In fact the situation was such that trace recordings of the time histories, particularly those of decaying oscillations, were processed manually and that further evaluations in the frequency domain hardly came into the picture.
With the advent of the digital computers the attitude has changed, however, and especially the development of the Fast Fourier Transform (FFT) techniques has presented great opportunities for more complex analysis procedures involving two signals simultaneously both in the time domain and in the frequency domain. It has thus become possible to produce in real time quantities such as (in the frequency domain) auto spectra of input or output, cross spectra and transfer functions and (in the time domain) auto correlations of input or output, cross correlations and impulse responses.

This change towards digital techniques has brought along several new possibilities and problems. Several of these will be touched upon in the following part of this chapter.

5.1 Sampling of signal

The first point to be considered when dealing with digitized values originating from analog signals is the problem of data sampling. In fact, digitization involves the definition of instantaneous values, whereas the original signal is continuous. The consequence of taking instantaneous values of a fluctuating signal at too low a rate can be that high frequency signal components are interpreted as low frequency signal components. This is called aliasing and to avoid this problem the sample frequency should be at least two times greater than the maximum frequency present in the original signal, i.e. \( f_{\text{samp}} \geq 2 f_{\text{max}} \).

Especially when applying accelerometers this requirement may ask for a high sampling rate because of the higher frequency components due to noise in the signal. A practical means to minimize this sampling rate thus is the use of low pass filters that suppress all frequencies above the maximum frequency of interest.

5.2 Resolution

Another point of interest when dealing with digital spectral techniques during flutter trials is the resolution of the spectrum. This resolution can be defined as the frequency increment between two individual lines in the spectrum and its value can be determined as:

\[
 f_{\text{resol}} = \frac{f_{\text{samp}}}{N}
\]

in which:

- \( f_{\text{samp}} \) = sampling frequency in Hz
- \( N \) = number of samples used in the calculation of the spectrum.

This resolution plays an important role in the estimation of resonance frequencies and the corresponding damping coefficients from a spectrum. As is indicated in fig. 9, the damping can be obtained from a spectral plot by determining the frequency difference across a peak at the half-power point of the peak. This assumes the ability to reasonably describe this peak and for this at least some five frequency points are required including the two boundary frequencies. This means that a high frequency resolution of the spectrum is needed to meet these requirements, particularly in situations in which both the damping and the frequency of the system are low. For example: when measuring a damping of 0.02 at 5 Hz, the width of the peak at the half-power point will be 0.10 Hz. If this peak is to be determined from five points, the frequency resolution should be \( f_{\text{resol}} = 0.025 \) Hz. At a sampling rate \( f_{\text{samp}} = 20 \) Hz, the above formula shows that the number of samples required is \( N = \frac{20}{0.025} = 800 \), which is equivalent to a measuring time length of \( \frac{800}{20} = 40 \) seconds. This signal length is required at each test point, thus in many cases a measuring duration will be required which is longer than the time in which significant information can be measured. Measures can be considered to overcome this problem. In the case of a decaying signal this can for instance be done by generating zeros in the analysis equipment during the non-relevant part of the measuring time, thereby artificially extending this measuring time.

The analysis equipment may also have a limitation in its frequency resolution. When selecting a specific equipment this requirement should be carefully considered.
5.3 Statistical reliability

When considering spectral densities, an important point is the statistical reliability with which a certain value is determined. In case the signal consists of a pure sine wave, the measurement of one single period will in principle be sufficient to obtain a complete description of this signal. As some distortion will always be present, the signal will have to be averaged over a longer period. If the distortion is not too great, a few periods, e.g. five, may be sufficient. If the signal is of a random nature it will not be possible to determine its characteristics from one single or a very limited number of periods. Instead one has to regard each measurement of the signal as an estimate of the true value. As such, each measured value is affected by a statistical error called $\epsilon$, the value of which can be determined from:

$$\epsilon = \frac{1}{\sqrt{BT}}$$

in which:

- $\epsilon$: the probability that 68% of the individual measurements will have a relative statistical error $\epsilon$ with respect to the true value
- $B$: bandwidth in Hz of the filter through which the signal has passed
- $T$: observation time in sec.

This relationship is valid when considering mean square (MS) values and holds for each of the individual frequency lines of which the spectrum consists. It may further be clear that an improvement of the estimate is proportional to the inverse of the square root of the observation time, i.e. doubling the reliability requires four times the observation time. To give an idea of the effect which this averaging has on power spectral density, fig. 22 shows PSD-plots made with different lengths of averaging time. Looking in more detail into the consequences of the above relationship it may be clear that the observation time may become very long, especially in situations where small bandwidths are required as is often the case with flutter experiments. As an illustration: the system shown in fig. 22 has a resonance frequency at about 46 Hz and a damping coefficient of 0.0435. As indicated earlier, this requires an $f_{res} = 0.46$ Hz or in other words: $B = 0.46$ Hz. Demanding a reliability of $\epsilon = 0.1$ would mean that the averaging time should be $T = \frac{100}{0.46} = 217$ sec. However, looking at the results of fig. 22, it can be observed that $T = 100$ sec already presents usable results and that even $T = 30$ sec leads to a plot from which the peak width at half power can be deduced without too much trouble. This is caused by the fact that the reliability calculated from the above formula holds for each individual frequency line, but that in a power spectral density plot there is a certain relation between the various frequency lines. Because of this interrelationship there is an apparent improvement of the reliability. It is, therefore, possible to use shorter time lengths than would appear from the above formula. Experience has been shown that a good rule-of-thumb in this respect is:

$$T = \frac{50}{g_{res}}$$

in which:

- $T$: observation or averaging time in sec
- $g$: damping coefficient
- $f_{res}$: resonance frequency in Hz.

For the case of fig. 22 this leads to $T = \frac{50}{0.0435 \times 46} = 25$ sec which, as can be noticed from the plots, indeed gives reasonable results.

Another rule-of-thumb, often referred to in the literature requires that $BT \gg 10$. The results obtained with this relation agree very well with the previous procedure, as can be seen from a comparison for the specific case mentioned above: the first method leads to $T = 25$ sec, the second method to $T = 22$ sec.

Because of the impact it has on a flight flutter test it may be worthwhile to spend some time to establish the most suitable averaging time for each specific case. The use of non-optimal time lengths will cause either a waste of test time or unreliable results. In this context other analysis procedures, such as smoothing, should also be taken into account. These other procedures will be discussed in the following sections.
5.4 Coherence

When the analysis towards resonance frequencies and corresponding damping coefficients is made by means of the transfer function $H(i\omega)$, both the input to the system and the response of the system have to be measured. The reliability of the analysis will depend on the degree to which the measured input is accompanied by a parasitic excitation which cannot be determined. A good measure to establish the degree of reliability is the coherence function which is defined as:

$$\gamma_{xy}(\omega) = \frac{|\Phi_{xy}(\omega)|^2}{\Phi_{xx}(\omega) \Phi_{yy}(\omega)}$$

In which:
- $\Phi_{xy}(\omega)$: cross spectral density between measured input and response
- $\Phi_{xx}(\omega)$: power spectral density of measured input
- $\Phi_{yy}(\omega)$: power spectral density of response.

The value of this quantity will lie between zero and one; if $\gamma_{xy} = 0$ there is no correlation at all between measured input and output, whereas $\gamma_{xy} = 1$ is found when a perfect correlation exists between the measured input and the response. In other words: $\gamma_{xy} = 1$ means that the response is completely due to the measured input and no parasitic excitation is present.

In practice the coherence function will in general be smaller than unity, but if $\gamma_{xy} > 0.5$ reliable results may be expected from the procedures involving the transfer function. If $\gamma_{xy} < 0.5$ the influence of the parasitic excitation becomes too large. In that case one may as well confine oneself to an analysis of the output spectrum only. The excitation frequency will probably appear in that spectrum, but as it is known it can be ignored.

5.5 Identification of flutter parameters

In a previous chapter several procedures have been mentioned by which the flutter parameters, i.e. resonance frequencies and damping coefficients, can be determined from the measured signals. In principle these procedures lead to the required information in a straightforward manner: in case of a decaying signal the analysis can even be performed by hand, the spectral techniques require somewhat more sophisticated equipment. In the latter case, however, the analysis is in the end also performed by hand since the final assessment of resonance frequencies and damping coefficients is obtained by personal judgment and interpretation of the calculated power spectral density plots. These procedures will in principle result in well-defined data if the measuring signals do not contain too much noise. In practice, however, noise is always present and this may cause the interpretation of the final data, obtained either directly as from the decaying signal or indirectly through application of a spectral technique, to be rather sensitive to personal interpretation or, in the worst case, to be even completely impossible. The introduction of the digital computer has opened new possibilities to deal with these sorts of problems and to obtain uniquely defined data even from measuring signals which could not be handled hitherto. This is achieved by applying special analysis techniques. Even in cases in which there is hardly any noise present in the signal it may still be advantageous to obtain the required information through the application of these more sophisticated techniques.

Some of these techniques will be discussed in somewhat more length below. In the context of this paper this discussion does not claim to give a complete review of all methods available, but rather to give some idea about the possibilities. A complete survey can hardly be established, since there is still quite some development going on in this specific field.

Most of the techniques are, however, based on the same basic principles and it is considered to be more worthwhile to present here some information on these principles rather than on the specific procedures themselves.

5.5.1 Curve fitting/matching

A very common procedure to eliminate the effect of noise on measuring signals is the application of curve fitting to the signal under concern. The advent of the digital computers has greatly enlarged the
possibilities of using such methods in an automated and objective way and nowadays these procedures are applied in many fields of science and engineering.

Essentially the technique consists of describing the signal that has to be analysed as well as possible by an analytical expression. This expression will only be able to describe the main feature of the signal and certain, as yet unknown, coefficients have to be introduced to deal with the more specific characteristics of the signal. To illustrate this point the decaying signal case may be helpful. The decay of a one degree of freedom structure after an abrupt excitation can be described by:

\[ y(t) = a_0 + e^{-\eta t} \cdot \sin 2\pi f_{\text{res}} t \]

in which the, as yet unknown, coefficients are:
- \( a_0 \): static offset
- \( \eta \): damping of the structure
- \( f_{\text{res}} \): natural frequency of structure
- \( a \): amplitude of the oscillation.

This expression can be applied to every one degree of freedom system, and the description of a specific system can be obtained by introducing the proper values for the coefficients. Once the decay is known as a time history, these coefficients can be determined by applying the least-squares method which means that those values of the coefficients will be obtained for which the sum of the squares of the differences between the measured and the approximated decay becomes a minimum. By this least-squares curve fitting the effect of noise will be materially reduced.

An example of the application of this technique is presented in fig. 23, which was taken from ref. 13. These graphs relate to an analytical case for which the resonance frequency and the damping coefficient were known. The upper figure presents a comparison between the actual results and the decay obtained on a one degree of freedom structure after an abrupt excitation can be described by:

\[ y(t) = a_0 + e^{-\eta t} \cdot \sin 2\pi f_{\text{res}} t \]

Apart from the fact that such a curve fitting procedure smoothes the original data in an objective manner (no personal judgment is involved), it also provides the possibility to obtain frequencies and damping coefficients in an automated way. These quantities are directly related to the coefficients which determine the actual shape of the fitted curve.

Quite evidently the application of this curve fitting technique needs not be restricted to the reconstruction of a decaying signal as described above. The same technique can also be applied with advantage in other ways. As a simple example the following procedure for determining the damping of a decaying signal could be quoted: Read the values of a number of successive peaks of a decaying signal with some noise and calculate the logarithms of these values. These points should, for a system with one degree of freedom, lie on a straight line, the slope of which is a measure for the damping. This straight line can be determined by the least squares process. If the requirement that the system should have only one degree of freedom is not met, the measured signal should be adapted to this by proper filtering.

When the results are available in the form of a transfer function, similar curve fitting procedures are possible. If the transfer function is given in complex notation, it may be presented in a vector plot as indicated in fig. 7 or in two plots, giving the real and imaginary parts, as indicated in the upper part of fig. 9. In both cases curve fitting is very well feasible if the analytical expression to describe the basic behavior of the curve is known. In fact the lines drawn through the test points of fig. 7 were obtained by such a curve fitting technique. As can be seen from the lower plot, this technique can even be used for a system with more than one degree of freedom. Another way to present the transfer function is by
means of a plot of its power density (see the bottom part of fig. 9). With the use of the analytical description of the transfer function a least squares procedure can also be applied to these power density data and the required information can be calculated in a straightforward automated manner. An example is shown in fig. 24. The data relate to a wind tunnel experiment on a flutter model and the characteristics of the system under the specific conditions were not known beforehand. However, the information obtained by the curve fitting technique have been compared with results using other smoothing procedures and the agreement appeared to be very good. These tests also showed another important advantage of the application of this curve fitting technique. During the wind tunnel experiment the transfer function plots were also analysed manually and afterwards it was found that most of the damping values obtained in this manner were smaller than the actual values. Presumably this is due to the fact that it is difficult to determine the exact height of the peak from a few points. In that event the test engineer will tend to remain on the safe side.

An analysis procedure which is quite similar to the previous ones is outlined in ref. 14. In that method the damping is determined by integrating the transfer function or its power density data in the neighbourhood of the resonance frequency, which in fact is another way of averaging the data. For a single mode system this leads to the following expression for the damping coefficient:

\[ g = \frac{2}{\pi f \text{res}} \int_0^{\infty} \Phi_{yy}(f) f^2 df \]

when using the power spectral density data for the integration. The same expression also holds for multimode systems; in that case, however, the integration has to be limited to narrow ranges of frequencies about each resonance frequency. The truncation effects that are introduced in that manner, can be corrected. Procedures to perform such corrections are also presented in reference 14.

Still another technique has been developed and applied by Grumman (Ref. 15). In this technique, which requires the availability of a large computer system, an analytical model of the flutter behaviour of the aircraft is used. As in the previous methods, the analytical expression contains certain coefficients whose values must be determined for each specific case in order to bring the model in agreement with the measured data. As these coefficients are directly related to the vibrational characteristics of the aircraft under concern, they can be used as the basis for the determination of the frequencies and damping coefficients of the system. In contrast with the previous methods it has been found that for this matching procedure the noise content in the response signal should not be too high. It was further found to be of advantage to cover only a limited frequency range during each analysis. Under those circumstances the method gave very reasonable results, as can be seen from fig. 25 where data are shown for a two degree of freedom system. The matched results have been compared with data from a theoretical model. The agreement between actual and matched results is very good in this rather simple case, and for more complicated cases a similar good agreement has been found.

5.5.2 Successive Correlation

A different technique for reducing the effect of noise has been initiated by Skingle (ref. 16) and has later been extended by Houbolt (Ref. 17). This method can be applied to either the autocorrelation function or the power spectral density. By inverse Fourier transform of the power spectral density, which is a standard routine on all Fourier analyzers, the associated autocorrelation function can always be obtained. This autocorrelation function is symmetrical about zero. In the present method the left hand side of this function, i.e. the part for negative time shifts, is omitted and the remaining right hand side is then considered as a new time history resembling a decaying oscillation. This signal can be multiplied by an exponential function with a decay of about half that of the original signal. The resulting data are then transformed to a power spectral density and this plot will in general be much smoother than the original spectral density plot. If the improvement is not sufficient, the procedure can be repeated in order to obtain an even better quality. It will be evident that in the final assessment of the amount of damping, allowance should be made for the artificial damping introduced by the exponential functions. Results obtained with this procedure are presented in fig. 26 in successive diagrams. Another example of the application of this method is shown in fig. 27; in this specific case no use was made of the exponential weighting of the single-sided autocorrelation function.
It has been found that the application of this technique will not always improve the data. To illustrate this, fig. 28 shows the results obtained after several applications of this method (without exponential weighting). After one step the power spectral density plot is not yet as clean as one would like it to be, but after a second application the plot is worse and after some six steps the power spectral density shows two distinct peaks in close proximity, though the structure only possessed one resonance frequency in that region. When applying this technique one should, therefore, carefully consider the results each time the data have gone through the routine and be prepared to accept results which are not completely smooth.

If the left hand side of the autocorrelation function is not eliminated the successive correlation tends to concentrate on the resonance frequency with the highest peak in the power spectral density plot. This property can be used to locate resonance frequencies which are so close together that they cannot be found by other methods. An example of the application of this procedure is shown in fig. 29. It should be realized, that the power spectral density plots obtained in this manner can no longer be used for the determination of the damping coefficient.

Another variant of the procedures outlined above can be used to obtain information on closely proximated resonance frequencies. In this case the original power spectral density plot is modified by eliminating all information except that close to one resonance frequency. This modified power spectral density is transformed into the associated autocorrelation function. Then the power density of the right-hand part is again calculated. Already after a limited number of iterative applications of this procedure the data may have converted into a plot from which both the resonance frequency and the damping coefficient can be determined (Fig.30). It will be clear, that this property is very attractive for use in the course of a flutter experiment. For, in most flutter cases certain resonance frequencies tend to come very close together and in such a situation it is very convenient to be able to separate these frequencies while making use of standard equipment rather than very sophisticated tools.

Applying the above procedures one however should always be prepared for problems (see for instance fig. 28). In principle these methods are able to diminish the effect of noise present in a measuring signal, but since noise may also exhibit itself as more or less distinct peaks in a power spectral density plot the successive correlation may accentuate this false information rather than diminish it. Such problems may also occur when dealing with limited time length signals. For then the truncation of the signal will cause the appearance of rather distinct frequencies as side lobes in the power spectral density plot and it may be difficult to suppress this unwanted information completely.

6.0 Organization of a flight flutter test

From the previous chapters it may have become clear that the planning and execution of a flight flutter test requires the collaboration of several specialists from quite different disciplines. Each of these specialists will have to contribute to the programme accordingly his own peculiar field of interest. As with all test programmes, the organization can be split up in three distinct parts: preparation of the project, actual execution of the tests and post flight analysis. In the following parts of this chapter the various aspects to be discussed in some length.

6.1 Preparation of a flight test

The basic plan for a flight flutter test programme is always made by the flutter engineer. This specialist can judge what is required from such an expensive and time consuming test programme. This knowledge is derived from the flutter calculations which, in case of a prototype aircraft, have already been started in an early stage of the development of the aircraft. On the basis of these calculations answers can be found on many questions such as:

- Is the aircraft prone to flutter in the required flight envelope?
- What is the type of flutter to be expected (e.g. wing bending/wing torsion or wing bending/rudder rotation) and will this flutter have a mild or explosive character?
- Which are the most dangerous vibration modes and what is their shape? When this question has been answered, decisions can be taken about the number of vibration sensors (accelerometers or strain gauge bridges) and their location in the aircraft. The same information is also used to determine the type

...
of excitation and the locations at which the exciters will be mounted.

Finally, the results of the flutter calculations will play an important role in establishing a safe flight test programme. The basic requirements are that all important flight conditions are covered and that the sequence of test points (i.e. measurement at different combinations of airspeed, altitude, airplane external configuration and airplane loading) is selected in such a way, that the probability of encountering flutter becomes greater as the flight testing proceeds. Then the critical cases will occur when the experience of the test team is greatest and when the verification of the flutter calculations is advanced as far as possible.

At this stage the flight test and instrumentation departments are drawn in and the detailed design of the instrumentation and excitation system is made. The particulars about this design will depend very much on the type of aircraft and the available time. For flutter testing of prototype aircraft the instrumentation will have to be integrated with the system already planned for other purposes. In the case of ad hoc tests optimal use will have to be made of available instrumentation systems and components.

In parallel the ground station is instrumented. In its simplest form it is only a radio communication channel with the pilot (and sometimes a test observer in the aircraft) at the one end and the flight test engineer and the flutter engineer at the other end. In most cases, however, a telemetry connection will be specified so that the flutter engineer can observe the critical measuring channels in real time. Often some data processing is done in real time on these data. The data processing equipment may vary from a simple Fourier analyzer to a large computer complex.

Before the actual flying starts a very detailed flight plan must be made. The first stage is to decide on the sequence of the flight programme. In principle there are three different types of flight test plan: for aircraft in the low subsonic speed regime, in the high subsonic regime and in the transonic and supersonic speed regime.

A sequence of test points for an aircraft in the low subsonic speed regime may look like Fig. 31. The basic variable in this case is the dynamic pressure only. At each successive test point the speed and thus the dynamic pressure is increased until its maximum value is obtained. Decisive for the selection of the altitude is the requirement of a safe altitude, i.e. an altitude from which the aircraft can safely recover in case of an accident; at this altitude it should also be possible to attain the maximum diving speed $V_D$ in an accelerated dive.

In case of a high subsonic test programme a slightly different test plan has to be adopted (Fig. 32). In this case one has to distinguish between the influence of Mach number and of dynamic pressure on the flutter characteristics. In general the influence of Mach number will be less predictable than the effect of dynamic pressure and for that reason this type of flight test is usually started at a high altitude, where a sequence of Mach numbers can be flown at a reasonably low dynamic pressure. Then a second set of test points is flown at a lower altitude to determine the flutter behaviour at the higher dynamic pressures. Finally some test points may be flown at which both parameters reach their maximum values.

For a supersonic test flight the test sequence will again be different. As an example, Fig. 33 is based on the test flights with the Concorde. In the first part of the test programme both altitude and Mach number were increased after each test point. In this manner the dynamic pressure remained almost constant and the Mach number was the only variable. To also cover the maximum dynamic pressure, a few test points were flown at a lower altitude; in this part of the test programme special attention was given to the transonic speed regime at high dynamic pressure. In this region, quite drastic changes may occur in the characteristics of an aircraft.

Once the test sequence has been established, a detailed flight programme is made in which the test sequence in each flight is given and all procedures on board and between the pilot and the ground station are carefully prepared.

6.2 The execution of the flight test

During the actual execution of the flight test the flight test engineer and the flutter engineer will be close together in the ground station. The flutter engineer will have in front of him the results of the processed telemetry data and comparable information from the earlier flutter calculations. If significant differences occur, the flutter engineer may advise the flight test engineer to propose a smaller step in dynamic pressure and/or Mach number, or may even advise to terminate the flight and await a detailed
analysis before the flight testing is continued. It should be stressed that possible changes in the flight plan should, as far as possible, be discussed between the flutter expert, the flight test engineer and the pilot before the flight, so that the pilot will receive unexpected requests only in extreme cases.

6.3 Post flight analysis

During the execution of the flight test a quick-look analysis of the telemetered signals will already have been performed in most cases. The extent to which it will be necessary to perform a more extensive analysis of all available test results after each test flight will depend on the type of flight flutter test performed.

For prototype testing the post flight procedures may be quite extensive. If during the actual test flight only a limited number of measuring signals have been examined, the information extracted from other sensors will have to be examined to obtain a complete view of the flutter characteristics of the aircraft.

In case the flight testing forms part of a store certification programme, a more limited post flight analysis may be required since the flutter characteristics which are of interest may be restricted to a few well defined areas of the aircraft such as the wing.

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    Military Specification-Flying Quality of Piloted Airplanes,
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Fig. 1a Two degrees of freedom system with no phase difference; net work zero over one cycle.

Fig. 1b Two degrees of freedom system with 90° phase difference; positive net work over one cycle.

Fig. 2 Sketch of possible aeroelastic cases when considering wing bending/control surface rotation.
Fig. 3 Block diagram of flutter investigation during developmental stage of an aircraft

Fig. 4 Flutter diagram for a two degree of freedom system
AMPLITUDE

TIME (SECONDS)

**Resonance Frequency**: 
\[ f = \frac{1}{2\pi} \left( \frac{A_0}{A_0} \right) \]

**Damping**: 
\[ \zeta = \frac{1}{2\pi} \ln \left( \frac{A_0}{A_{2n}} \right) \]

**Also**: 
\[ \zeta = 0.63 \]

WITH \( n \) NUMBER OF CYCLES

WITH \( n \) NUMBER OF CYCLES

TO HALF-AMPLITUDE

**Damping Related to Slope of Straight Line**

**Fig. 5** Frequency and damping computation from decaying signal

**Fig. 6** The principle of the Kennedy-Pennew plot, applied to a system with one degree of freedom
Fig. 7 Examples of Kennedy–Pencu plots obtained from flutter tests (taken from ref. 2)

Fig. 8 Example of sweep excitation and corresponding response
Fig. 9 Example of complex and absolute presentation of transfer function obtained from sweep excitation

Fig. 10 Time signals obtained when applying an impulse sine wave as excitation
Fig. 11 Time signals of response of two degree of freedom system to random excitation; responses filtered by position.

Fig. 12 Sketch showing the principles of the random exc procedure (taken from ref. 4)
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A. AFT-FUSELAGE MOUNTED ON SS-3-A (TAKEN FROM REF. 5)

B. WING-TIP MOUNTED ON A-10 (TAKEN FROM REF. 6)

Fig. 13 Examples of aerodynamic vanes used to excite an aircraft

Fig. 14 Sketch of installation of aerodynamic vanes and control equipment on A-10 aircraft (taken from ref. 6)
Fig. 15 Power spectral density of free air turbulence accordingly the Dryden-model

$$Q_u(n) = \frac{2 \mu \nu}{2n^2 - \nu^2} \left[ 1 + \frac{\nu^2}{n^2} \right]$$

$$n = 1 \text{ FT/SEC}$$

Frequency range of interest for flight flutter testing

Fig. 16 Sketch of vane-system to measure turbulence velocity components (taken from ref. 7)
Fig. 17 Sketch showing the principles of an electrodynamic exciter (taken from ref. 8)

Fig. 18 Sketch of flat pulse exciter with corresponding adaptation function (adapted from ref. 10)
A. F-15 INSTRUMENTATION (TAKEN FROM REF. 11)

B. YF-16 INSTRUMENTATION (TAKEN FROM REF. 12)

Fig. 19 Examples of location of transducer during flutter testing

Fig. 20 Location of transducers and exciters on "Transall" (adapted from ref. 10)
A. F-15 Flutter Trials (Taken from Ref. 11)

B. TF-16 Flutter Trials (Taken from Ref. 12)

Fig. 21 Instrumentation setup in ground-based station
Fig. 22 Influence of averaging time on a power spectral density
Fig. 23 Curve fitting procedure applied on a decaying signal (taken from ref. 13)

Fig. 24 Least-squares procedure applied on a PSD-plot
Fig. 25 Flutter parameters obtained by matching experimental results to a mathematical model (taken from ref. 15)

Fig. 26 Lay-out of the successive single-sided autocorrelation procedure (adapted from ref. 17)
**Fig. 27** Smoothing effect as obtained by successive autocorrelation

**Fig. 28** Showing the deteriorating effect of applying the successive autocorrelation too many times
Fig. 29 Lay-out of the successive double-sided autocorrelation procedure (adapted from ref. 17)

Fig. 30 Selecting on closely spaced frequencies by erasing in frequency domain and successive autocorrelation (adapted from ref. 17)
Fig. 31 Flight test programme for a low subsonic speed regime

Fig. 32 Sketch of a flight test programme for the high subsonic speed regime

Fig. 33 Sketch of a flight test programme for transonic and supersonic speed regime
This AGARDograph is the 9th of the AGARD Flight Test Instrumentation Series and concentrates on the flight test instrumentation for determining the flutter behaviour of an aircraft. After a short review of the mechanism of flutter the following items are discussed in some detail:

- requirements which the type of excitation should obey in order to enable the determination of the flutter characteristics;
- possible means of excitation;
- appropriate instrumentation;
- data analysis procedures.

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