PROCEDURES FOR COMPUTING THE FREEBOARD REQUIREMENTS
OF DISPLACEMENT MONOHULLS

by

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NOTATION

A  Wave spectral parameter
a  Area
B  Wave spectral parameter
c  Composite (subscript)
C_5 Slamming parameter
F_\text{p}  Froude Number
f  Probability density function
(h_\text{w})_\text{OBS}  Observed wave height
i  Integer (subscript)
I  Maximum value of subscript i
j  Integer (subscript)
J  Maximum value of subscript j
k  Kinematic (subscript)
L_w  Wavelength
N  Total number of observations
n  Number of observations in a specified cell
P  Probability
P_s  Probability of slamming
R  Relative sea state
r_a  Amplitude of relative motion in regular waves
r_{1/3}  Significant single amplitude of relative motion in random waves
\dot{r}_{1/3}  Significant single amplitude of relative velocity in random waves
\dot{r}_t  Threshold relative velocity for slamming
s_a  Amplitude of absolute vertical motion in regular waves
<table>
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<tr>
<td>$\tilde{\sigma}_{1/3}$</td>
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<td>$T$</td>
<td>Ship draft</td>
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<tr>
<td>$(T_{w})_0$</td>
<td>Modal wave period</td>
</tr>
<tr>
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<td>Observed wave period</td>
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ABSTRACT

Methodologies for empirical assessment of the non-kinematic components of ship-to-wave relative motion, change of level, and bow wave profile are presented. Computation of wave contours from observed wave data is described. Linear superposition computations in the wavelength/ship length domain are outlined.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

This document provides supporting details for the 1979 Society of Naval Architects and Marine Engineers STAR Symposium paper "Minimum Freeboard Requirements for Dry Foredecks: A Design Procedure," Reference 1. The topics involved are:

1. Methodology for empirical estimation of nonkinematic relative motion, change of level, and bow wave profile;
2. Derivation of wave environment characteristics from observed wave data; and
3. Linear superposition and related computations.

Each of these topics is treated hereinafter.

NONKINEMATIC RELATIVE MOTION, CHANGE OF LEVEL AND BOW WAVE PROFILE

Here the empirical formulations used to account for the nonkinematic components of relative motion, for change of level, and for bow wave profile are described.

*A complete listing of references is given on page 11.
RELATIVE MOTION

Data reported in References 2 and 3 and data from other experiments which have not been reported were used. Various plottings of the ratio of measured to kinematic relative motion transfer function were tried. (Since undistorted wave dimensions were used on both sides of the comparison, this is equivalent to a direct comparison of measured to kinematic relative motion.) Strong trends were found with $L_w/L_{pp}$ and with Ship Station, but the effect of $F_n$ was hard to discern. In this context, it should be pointed out that most of the available data was for $F_n$ in the 0.20 to 0.40 range. Some data applicable to $F_n = 0.05$ was, however, located. It indicated that the ratio of measured to kinematic relative motion transfer functions was close to unity.

The available data also indicated that the ratio tended to unity as $L_w/L_{pp}$ increased to the order of 4.0 and as Ship Station 6 was approached. Aft of Ship Station 6, the ratio again diverged from unity, but this location was considered to be a reasonable stopping point for above-water bow considerations. Fortunately, the relative motions associated with short waves are small (the transfer function converges to unity in the kinematic case), so it was felt that the short-wave end of the ratio could be handled in the course of spectral closure considerations.

The available data were ultimately fairied in coordinates of $(r_a/r_a')/c$ versus $L_w/L_{pp}$ with due attention to the various limiting trends involved. A sample plot and fairing is shown in Figure 1. Similar plots and fairings were made for all ship locations at which data were available. Cross fairings over Ship Station were also performed. The family of curves presented in Figure 2 resulted.

CHANGE OF LEVEL

Change of level was evaluated on the basis of calm water measurements for a number of ships. It was found that these measurements collapsed rather well at fixed $F_n$ if ships with and without bow domes were separated. Hence, it was decided that a viable approximation could be obtained by Froude scaling from a similar hull. It is obviously important that the presence or absence of a bow dome be taken as a criterion for similarity.
BOW WAVE PROFILE

Calm water measurements for a number of ships were again employed. As a rule, these measurements were available only for design speed. In those cases for which such measurements were available at two or more speeds, it was evident that the crest of the bow wave moved forward as well as decreasing in amplitude with decreasing speed. It was also evident that the presence of a bow dome tended to "sharpen" and increase the height of the bow wave's crest.

The latter factor simply implied that it would again be necessary to segregate ships with and without bow domes. The shift in crest location with speed, though, introduced a considerable difficulty. It did not appear that this phenomenon could be accounted for by any simple scaling or nondimensionalization scheme.

To accommodate the shift in crest location (and simultaneously the change in crest height for a given ship) with speed, simplified versions of the formulas presented in Reference 4 were used. Specifically, it was found that a point on the bow wave profile at ship speed \( V_0 \), say \( \zeta_{p0}(x_0) \) where \( \zeta_{p0} \) is the wave profile height measured positive upward from the waterline and \( x_0 \) is the ship location measured positive aft of the forward perpendicular, could be transformed to approximate the bow wave profile point \( \zeta_p(x) \) at ship speed \( V \) using

\[
\zeta_p(x) = \left( \frac{V}{V_0} \right)^{5/4} \zeta_{p0}(x_0) \quad (1)
\]

and

\[
x = \left( \frac{V}{V_0} \right)^{3/2} x_0 \quad (2)
\]

By applying equations (1) and (2) for a sufficient number of \( x_0 \)'s, then, a bow wave profile at speed \( V \) can be constructed.

It was found that preceding the transformation defined by (1) and (2) with Froude scaling to a common ship length caused the available bow wave profile to collapse rather well for subsets with and without bow domes. Ultimately then, the procedure for bow wave profile estimation involved both Froude scaling and transforming the data for a similar hull. Figure 3 illustrates the procedure for a typical case.
DERIVATION OF WAVE ENVIRONMENT CHARACTERISTICS FROM HOGBEN AND LUMB DATA

The Hogben and Lumb atlas presents, for certain ocean areas and seasons, the number of observations reported for coded combinations of observed wave height, \( h_{w \text{OB}} \), and observed wave period, \( T_{w \text{OB}} \). It is assumed that areas and seasons can be combined by direct summation of the relevant tabulations. Let the result of such a summation be represented by \( h_{w \text{OB}} \), \( T_{w \text{OB}} \) and \( n_{ij} \) where \( i = 1, 2, \ldots, I \) and \( j = 1, 2, \ldots, J \). To each combination of the subscripts \( i \) and \( j \) there corresponds an area, \( a_{ij} \) in \( h_{w \text{OB}}, T_{w \text{OB}} \) space. The joint probability density function of \( h_{w \text{OB}} \) and \( T_{w \text{OB}} \), say \( f\{[(h_{w \text{OB}})]_i, [(T_{w \text{OB}})]_j\} \) can thus be empirically approximated for each \( (i, j) \) combination by

\[
f\{[(h_{w \text{OB}})]_i, [(T_{w \text{OB}})]_j\} = \frac{n_{ij}}{N a_{ij}} \tag{3}
\]

where \( N \) is the total number of observations, i.e.,

\[
N = \sum_{j=1}^{J} \sum_{i=1}^{I} n_{ij} \tag{4}
\]

Now, let \( f_0\{[(h_{w \text{OB}})]_i, [(T_{w \text{OB}})]_j\} \) be an arbitrarily specified value of \( f\{[(h_{w \text{OB}})]_i, [(T_{w \text{OB}})]_j\} \). Compute

\[
P_0 = \sum_{j=1}^{J} \sum_{i=1}^{I} a_{ij} [f\{[(h_{w \text{OB}})]_i, [(T_{w \text{OB}})]_j\}] - f_0\{[(h_{w \text{OB}})]_i, [(T_{w \text{OB}})]_j\} \tag{5}
\]

setting \( f\{[(h_{w \text{OB}})]_i, [(T_{w \text{OB}})]_j\} - f_0\{[(h_{w \text{OB}})]_i, [(T_{w \text{OB}})]_j\} \) equal to zero for all combinations of \( i \) and \( j \) such that \( f_0\{[(h_{w \text{OB}})]_i, [(T_{w \text{OB}})]_j\} \geq f\{[(h_{w \text{OB}})]_i, [(T_{w \text{OB}})]_j\} \). Then the boundaries of the \( [(h_{w \text{OB}}), (T_{w \text{OB}})] \) cells for which \( f\{[(h_{w \text{OB}})]_i, [(T_{w \text{OB}})]_j\} - f_0\{[(h_{w \text{OB}})]_i, [(T_{w \text{OB}})]_j\} \) is nonzero roughly define a contour along which \( f\{[(h_{w \text{OB}})]_i, [(T_{w \text{OB}})]_j\} \) is constant at the \( f_0\{[(h_{w \text{OB}})]_i, [(T_{w \text{OB}})]_j\} \) level. Further, \( P_0 \) approximates the probability that a given \( [(h_{w \text{OB}}), (T_{w \text{OB}})] \) observation will fall within this contour.

One could, of course, define any number of surfaces in \( [(h_{w \text{OB}}), (T_{w \text{OB}})] \) that would in some manner bound the fraction \( P_0 \) of all
observations. Establishing the surface at a constant \( f_0[(h_w)_{OBS},(T_w)_{OBS}] \) is a rather arbitrary expedient. The author finds this definition to be intuitively appealing. It would, however, be of interest to explore alternative definitions.

We can, in a crude sense, identify the largest value of \( f{[(h_w)_{OBS}]_j,[(T_w)_{OBS}]_j} \) for a given value of \( j \), say \( \max f{[(h_w)_{OBS}]_j,[(T_w)_{OBS}]_j=m} \), with the most probable value of \( (h_w)_{OBS} \) given the specified \( (T_w)_{OBS} \). Similarly, the most probable value of \( (T_w)_{OBS} \) for a given \( (h_w)_{OBS} \) can be identified by \( \max f{[(T_w)_{OBS}]_j,[(h_w)_{OBS}]_j=m} \). These "most probable" values, together with the constant \( f_0[(h_w)_{OBS},(T_w)_{OBS}] \) contours and associated probabilities \( P_0 \) constitute the basic results needed here. By manual fairing and then a transformation to significant wave height versus modal wave period coordinates using the Nordenström calibrations⁶, wave environment characterizations such as that presented in Figure 3 of Reference 1.

LINEAR SUPERPOSITION AND RELATED COMPUTATIONS

Here several formulas presented in the course of the discussion of ship response statistic computations given in Reference 1 are derived.

WAVE SPECTRA

The Bretschneider⁷ wave spectral family can be written

\[
S_\zeta(\omega) = A\omega^{-5}e^{-B\omega^{-4}}
\]  

(6)

for \( \omega \) from 0 to infinity. The area under this spectrum is the variance of wave elevation, i.e.,

\[
\sigma_\zeta^2 = \int_0^\infty S_\zeta(\omega)d\omega = \frac{A}{4B}
\]  

(7)

Further, the spectrum has a unique mode where

\[
\frac{d}{d\omega}[S_\zeta(\omega)] = 4B\omega^{-4} - 5 = 0
\]  

(8)
Letting \( \omega_0 \) represent the mode, (8) shows it to be

\[
\omega_0 = \left(\frac{4B}{5}\right)^{1/4}
\]  

(9)

The wave period corresponding to \( \omega_0 \) is taken to be the modal period, \((T_w)_0\). Thus,

\[
(T_w)_0 = \frac{2\pi}{\left(\frac{4B}{5}\right)^{1/4}}
\]  

(10)

Now consider transformation to the \( L_w \)-domain in accord with the gravity wave relationship

\[
\omega = \left(\frac{2\pi g}{L_w}\right)^{1/2}
\]  

(11)

Under this transformation, (6) becomes

\[
S_c(L_w) = \frac{AL_w}{2(2\pi g)^2} - \frac{BL_w^2}{(2\pi g)^2}
\]  

(12)

Integrating over \( S_c(L_w) \) from 0 to infinity reproduces the right-hand side of (7). Differentiating \( S_c(L_w) \) with respect to \( L_w \) and equating the result to zero yields the modal wavelength, \((L_w)_0\), as

\[
(L_w)_0 = \frac{2\pi g}{\sqrt{2B}}
\]  

(13)

The unit \((\xi_w)^{1/3}\) condition can be imposed via (7):

\[
(\xi_w)^{1/3} = 4\sqrt{\frac{A}{g}} = 2\sqrt{\frac{A}{B}} = 1
\]  

(14)

Substituting \( A \) and \( B \) from (13) and (14) into (12) yields

\[
S_c(L_w) = \frac{L_w}{16[(L_w)_0]^2} e^{-\frac{1}{2}\left[\frac{L_w}{(L_w)_0}\right]^2}
\]  

(15)
Nondimensionalizing the range of (15) by

\[ \lambda = \frac{L_w}{L_{pp}} \]  

yields

\[ S_\zeta(\lambda) = \frac{\lambda L_{pp}}{16(L_w)_0^2} e^{-\frac{1}{2 \left( \frac{\lambda L_{pp}}{(L_w)_0} \right)^2}} \]  

Now, by defining the relative sea state parameter, R, to be

\[ R = \frac{(L_w)_0}{L_{pp}} \]

equation 17 can be written

\[ S_\zeta(\lambda) = \frac{\lambda}{16R^2} e^{-\frac{1}{2 \left( \frac{\lambda}{R} \right)^2}} \]  

which is the form given in Reference 1.

**MODAL PERIOD TO LENGTH RELATIONSHIP**

It follows from (10) and (11) that

\[ (T_w)_0 = \frac{1}{2} \sqrt[4]{\frac{2\pi(L_w)_0}{g}} \]

as given in Reference 1.

**NUMERICAL INTEGRATION**

The integrals included in Equations (7) through (1) of Reference 1 must be numerically approximated. For those points defined by the strip theory computations, this is accomplished by a straightforward Lagrangian integration procedure. The difficult part of the problem is approximating the "tails" of the response spectra which lie beyond the computed points, i.e., obtaining spectral closure. In the cases of relative motion and velocity, which are of major concern here, the problem is emphasized by the
limiting behavior of the associated response amplitude operators. As \( \omega_e \) becomes large, \( [(r_a/\zeta_a)_k]^2 \) approaches unity and \( [(\ddot{r}_a/\zeta_a)_k]^2 = \omega_e^2 [(r_a/\zeta_a)_k]^2 \) becomes very large.

Working in the \( L_{wpp}/L \) domain minimizes the problems associated with the high frequency behavior of relative motion and velocity. In the \( \omega_e \) domain this behavior theoretically continues to infinity. In the \( L_{wpp}/L \) domain, it is confined to a small range near zero.

With \( L_{wpp}/L \) identified by \( \lambda \), let \( \min(\lambda) \) be the smallest value of \( \lambda \) for which strip theory computations are performed. Then \( [(r_a/\zeta_a)_k]^2 \) and \( [(\ddot{r}_a/\zeta_a)_k]^2 \) are extrapolated from their computed values at \( \min(\lambda) \) to unity at \( \lambda = 0 \). The particular values of \( \lambda \) for which extrapolated values are computed are 0.01, \( \min(\lambda)/4 \), \( \min(\lambda)/2 \) and \( 3[\min(\lambda)]/4 \). The extrapolated points are included in the Lagrangian integration.

Low \( \lambda \) closure for the acceleration integral is handled by extrapolation at the response spectrum level. It is assumed that the acceleration spectrum varies in a linear manner from its computed value at \( \min(\lambda) \) to zero at \( \lambda = 0 \). The resultant triangular area is included in the spectral area.

High \( \lambda \) closure is handled the same for all responses. This is done at the spectral level. A least squares line is fitted to the response spectrum ordinates at the three largest values of \( \lambda \) for which PHM computations are performed. The area bounded by this line, by the \( \lambda \) axis, and by the spectral ordinate at the largest value \( \lambda \) is then included in the spectral area. If the slope of the fitted line is positive, indicating that closure is not attained, a flag is set. This has not occurred in any computations performed to date. In fact, the contribution of the high \( \lambda \) closure area to response per unit wave height was generally insignificant.

LIMITING WAVE HEIGHT FOR SLAMMING

The probability of bottom slamming from Reference 8 can be written

\[
P_s = e^{-2\left[\frac{T}{T_{1/3}}\right]^2 + \frac{r_t}{T_{1/3}}^2}\]

In terms of response per unit wave height, (21) can be rewritten in the form
\[ P_s = e^{-2C_s / [(\tilde{\tau}_w)_{1/3}]^2} \]  

(22)

where \( C_s \) is a "slamming parameter" defined by

\[ C_s = \left[ \frac{T}{r_{1/3} / (\tilde{\tau}_w)_{1/3}} \right]^2 + \left[ \frac{\tilde{\tau}_w}{r_{1/3} / (\tilde{\tau}_w)_{1/3}} \right]^2 \]  

(23)

(This equation differs from Equation (13) of Reference 1 only in that the notation of the latter specializes it to Ship Station 3.) By specifying a numerical value for \( P_s \), say \( \lim(P_s) \), a corresponding value of \( (\tilde{\tau}_w)_{1/3} \), say \( \lim[(\tilde{\tau}_w)_{1/3}]_{\text{SLAM}} \), can be derived from (22). It is

\[ \lim[(\tilde{\tau}_w)_{1/3}]_{\text{SLAM}} = \left\{ \frac{-2C_s}{\ln[\lim(P_s)]} \right\}^{1/2} \]  

(24)

which, again except for specialized notation, is identical to Equation (12) of Reference 1.
REFERENCES


Figure 1 - Relative Motion Transfer Function Ratios in Way of Ship Station 0
Figure 2 - Composite to Kinematic Relative Motion Transfer Function Ratios

Figure 3 - Bow Wave Profile Scaling and Transformation
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