X-RAY BUILD-UP FACTORS

THESIS

AFIT/GNE/PH/78D-18

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X-RAY BUILD-UP FACTORS

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by
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Preface

Although x-ray air build-up factors can be an extremely important tool in simplifying the x-ray air transport problem, no complete set of build-up factors exists in current literature. Many computer codes also exist to handle the x-ray air transport problem, but none have been used to calculate a complete set of build-up factors.

I have used the least expensive code which retains adequate accuracy to compile a complete set of x-ray air build-up factors. I must acknowledge the help of Dr. C. J. Bridgman for his help in guiding me in the correct direction for making this study a success. I also must acknowledge the help of Major Winfield S. Bigelow who was invaluable in getting the program to run successfully.
# Contents

Preface .......................................................... 11
List of Figures .................................................. v
Abstract .......................................................... ix

I. Introduction ..................................................... 1
   Purpose ..................................................... 1
   Theory of Build-up Factors .................................. 1
   Scope ...................................................... 2
   Plan of Development ........................................ 3

II. Computer Models ................................................ 4
   Introduction ................................................ 4
   Monte Carlo ................................................ 4
   Discrete Ordinates Method ................................ 6
   Moments Method ............................................ 7

III. Program Used .................................................. 8
   Literature Search .......................................... 8
   Monte Carlo ................................................ 8
   Discrete Ordinates Method ................................ 9
   Moments Method ............................................ 9
   Description of PHOTDIS ....................................
     Input .................................................... 10

IV. Results ........................................................ 11
   Choice of Options ......................................... 11
   Energies .................................................... 11
   Ranges ..................................................... 11
   Graphs of Build-up Factors ................................ 11
   Empirical Build-up Factor Equation ........................ 13

V. Discussion ......................................................
   Intrinsic Estimate of the Accuracy ........................ 121
   Comparison of Results to Others ......................... 121
     Introduction .............................................. 121
     Monte Carlo ............................................. 122
     Discrete Ordinates Method .............................. 137
     Bigelow ................................................. 137

VI. Conclusion ..................................................... 138
   Purpose and Scope ......................................... 138
   Results and Discussion .................................... 138
   Recommendations and Summary ............................ 138

Bibliography ..................................................... 142
Contents

Appendix A: Solution of the Boltzmann Transport Equation by the Moments Method ...................................... 144
Appendix B: Sample Input and Program Listing ....................... 150
**List of Figures**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Mean-free-paths for Various Altitudes</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td>Energy Build-up Factors for 12 keV.</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>&quot; &quot; &quot; &quot; 14 &quot;</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>&quot; &quot; &quot; &quot; 16 &quot;</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>&quot; &quot; &quot; &quot; 18 &quot;</td>
<td>19</td>
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<td>21</td>
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<td>7</td>
<td>&quot; &quot; &quot; &quot; 24 &quot;</td>
<td>22</td>
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<td>8</td>
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<td>Figure</td>
<td>Energy Build-up Factors for 78 keV</td>
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<td>68</td>
<td>Energy Build-up Factors for 230 keV</td>
<td>83</td>
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<tr>
<td>102</td>
<td>Energy Build-up Factors for 850 keV</td>
<td>117</td>
</tr>
<tr>
<td>103</td>
<td>&quot;        &quot;        &quot;        900 &quot;</td>
<td>118</td>
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<tr>
<td>105</td>
<td>&quot;        &quot;        &quot;        1000 &quot;</td>
<td>120</td>
</tr>
<tr>
<td>106</td>
<td>Comparison of Energy Build-up Factors at 10 Mean-free-paths</td>
<td>123</td>
</tr>
<tr>
<td>107</td>
<td>Comparison of Energy Build-up Factors for 60 keV</td>
<td>125</td>
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<tr>
<td>108</td>
<td>&quot;        &quot;        &quot;        &quot;        20 &quot;</td>
<td>126</td>
</tr>
<tr>
<td>109</td>
<td>&quot;        &quot;        &quot;        &quot;        40 &quot;</td>
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<td>128</td>
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<td>&quot;        &quot;        &quot;        &quot;        20 &quot;</td>
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<td>&quot;        &quot;        &quot;        &quot;        40 &quot;</td>
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<td>&quot;        &quot;        &quot;        &quot;        90 &quot;</td>
<td>132</td>
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<td>115</td>
<td>&quot;        &quot;        &quot;        &quot;        120 &quot;</td>
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<td>&quot;        &quot;        &quot;        &quot;        150 &quot;</td>
<td>134</td>
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<td>&quot;        &quot;        &quot;        &quot;        200 &quot;</td>
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<td>&quot;        &quot;        &quot;        &quot;        &quot;</td>
<td>139</td>
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<tr>
<td>120</td>
<td>Energy Build-up Factors for 12-120 keV</td>
<td>140</td>
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<tr>
<td>121</td>
<td>&quot;        &quot;        &quot;        150-1000 keV</td>
<td>141</td>
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Abstract

This report is a compilation of time integrated x-ray energy build-up factors from a monoenergetic point source in infinite homogeneous air. These factors were computed by the use of PHOTDIS, a moments method computer code, and performed on a CDC 6600. This code was chosen after a literature search and a review of many computer models. Energies from 12 keV to 1000 keV and ranges from 1 mean-free-path to 15 mean-free-paths are considered. All results are presented on semi-log graphs with each graph containing one energy. This program is estimated to have an error of at most 20%.

The results are compared to Monte Carlo and Discrete Ordinates calculations. Even though the moments calculations do not completely agree with any of the Monte Carlo calculations, the moments calculation agree with the average of the Monte Carlo calculations. A complete derivation of the moments method from the Boltzmann Transport Equation is also included.
X-RAY BUILD-UP FACTORS

I Introduction

Purpose

The purpose of this report is to provide one complete set of time integrated monoenergetic x-ray energy build-up factors in infinite homogeneous air. This study has used the moments method in the form of the program PHOTDIS (Ref 1). This program is written in Fortran and was run on the CDC 6600 of the Aeronautical Systems Division at Wright-Patterson Air Force Base, Ohio. This study was undertaken because of the lack of a complete set of these build-up factors.

Theory of Build-up Factors

The normal treatment of photon attenuation in an absorbing medium at range r from an isotropic monoenergetic point source is

\[
F(r) = \frac{3S}{4\pi r^2}
\]

(1)

where

\( F(r) \) is the energy fluence at distance \( r \) in Joules/m\(^2\) (or Cal/cm\(^2\))

\( S \) is the total energy emitted by the monoenergetic source in Joules (or Calories)

\( \mu \) is the total macroscopic cross section in \( m^{-1} \) (or cm\(^{-1}\))

\( r \) is the distance of interest in meters (or centimeters)

This equation accurately describes the fluence if the x-rays interact only by photoelectric absorption. So this equation is accurate for low energy x-rays, energy below about 12 keV where photoelectric absorption dominate.
For x-rays of energy above 12 keV, Compton scatter plays an increasingly important part. Therefore, Eq (1) must be modified to account for the Compton scatter. One possible modification is

$$F(r) = \frac{B \rho e^{-\mu_r}}{4\pi r^2}$$

(2)

where $B$ is the energy dependent build-up factor, or the correction factor to account for Compton scatter.

Eq (2) assumes a monoenergetic source which is not realistic in nuclear effects calculations. However, the real polyenergetic sources can be treated by using Eq (2) in a multienergy group calculation:

$$F(r) = \sum \frac{B_i S_i e^{-\mu_r}}{4\pi r^2}$$

(3)

where $S_i$ represents the fraction of the continuous source energy spectra emitted within the bounds of energy group $i$. Eq (2) also assumes that the x-ray fluence varies only with one spatial coordinate which becomes increasingly invalid at larger energies and higher altitudes. This assumption is made to prevent the calculational complication of a two-dimensional geometry. However, two-dimensional effects can be approximated by rearranging Eq (3) into a $4\pi r^2$ fluence and employing mass integral scaling (Ref 2):

$$4\pi r^2 F = \sum S_i B_i \exp(-\int_0^r \mu_r \, dr')$$

(4)

These equations also ignore the time variable of the x-ray because most targets respond to total x-ray dose not x-ray dose rate. Finally, the calculations for build-up factors are presented in terms of mean-free-path which makes them applicable to homogeneous air at any altitude.

**Scope**

Due to the limits of the convergence of the moments method program, the
ranges considered were from one to 15 mean-free-paths of the source photon energy. The build-up factor for energies below about 12 keV is nearly one, so energies below 12 keV were not investigated. This program does not include pair production, so energies used in the calculations are all below 1000 keV. To facilitate the users' evaluation of the accuracy of the build-up factors presented here, comparison to other calculations are also included in this report.

Plan of Development

This report starts with an explanation of computer models which are used in x-ray transport problems. A discussion of the program PHOTDIS follows. The graphs of the build-up factors for each energy are the main portion of this report. The discussion of the results contain a comparison to others who have made x-ray transport calculations. Finally a detailed description of the derivation of the moments equation and the reconstruction of the energy fluence can be found in the appendix.
II Computer Models

Introduction

Three numerical models have evolved which are applicable to compute total fluence of x-rays where energy is greater than 12 keV. The three models are Monte Carlo, Discrete Ordinates, and the Moments Method. Each model has advantages and disadvantages which will be discussed along with a brief description of each method. Since the moments method was selected as the model used in this project, it will be presented in depth in Appendix A as well as the brief overview in this section.

Monte Carlo

In the Monte Carlo method, a photon leaving the source is traced through the medium of interest. As the photon is traced, a record is kept of its energy, direction and position. Compton, pair production and photoelectric events are experienced by the photons in accordance with the cross sections for these events at a rate determined by statistical probability. The photon is followed until it is absorbed or until it passes out of the area of interest. Many photons are traced by this method. At a predetermined distance r, the energy of all the photons reaching that distance is added together and divided by \( 4\pi r^2 \). This is the total energy fluence at that distance. To minimize computations without sacrificing accuracy, statistical improving schemes are used. These schemes are used to sample only "important" photons, which are determined by a "weight". The weight of a photon is an artificial biasing which is introduced while tracing the photon. Several statistical improving schemes are statistical estimations, exponential transformations, Russian Roulette and non-absorptive weighting. (Ref 3:19-20)

Statistical estimation may be a last-flight estimator which calculates
the probability that a photon which has just undergone a Compton event will travel a predetermined distance before absorption. Another statistical estimation scheme is next-event-estimator. This scheme calculates the probability that a photon which has just scattered will reach a predetermined distance after it is involved in another scatter. (Ref 3:20)

Exponential transformation biases the distance to the next event to increase the probability of a long flight. The weight associated with a photon is adjusted to correct for the bias. (Ref 3:21)

If a photon has lost so much of its weight through adjustment for biasing, Russian Roulette may be used. A random number is generated and compared with a survival probability, which is also less than one. If the random number, which is also less than one, is less than the survival probability, the weight is multiplied by the reciprocal of the survival probability and the tracking is continued. If the random number is greater than the survival probability, the tracking is discontinued. (Ref 3:21)

In non-absorptive weighting, at each event the photon's weight is scaled by a probability that the photon will not be absorbed. The scaling factor is the scattering cross section divided by the total cross section. When weight correction reduces the photon's ability to contribute to a very small amount, Russian Roulette is used. (Ref 3:22)

The accuracy of the answer depends on many factors. The statistical fluctuation produced by the stochastic nature of the method along with the type of weighting used are two major factors. Other factors affecting the accuracy are the cross sections and the number of photon tracked (histories). The number of histories is limited by the size of the computer and the amount of computer time and money spent on the problem. But the Monte Carlo method can treat a complex geometry consisting of many different materials.
Discrete Ordinates Method

The Discrete Ordinates method deals with a numerical solution to the Boltzmann transport equation. The Boltzmann transport equation (which also describes time independent fluence) is

\[ \Omega \cdot V F(r, \Omega, E) + \mu^t F(r, \Omega, E) = S(r, \Omega, E) + \int \mu^s F(r, \Omega', E') \, d\Omega' \, dE' \]  

where

- \( F \) is the fluence
- \( \Omega \) is the direction vector
- \( r \) is the coordinates
- \( E \) is the energy
- \( S \) is the source function
- \( \mu^t \) is the total cross section
- \( \mu^s \) is the scatter cross section from direction \( \Omega' \) and energy \( E' \) to direction \( \Omega \) and energy \( E \)

The most popular Discrete Ordinates programs solve the Boltzmann transport equation numerically by a method known as the \( S_N \) method. The \( S_N \) method expands the scatter cross section in the integral term in Legendre polynomials. By using the addition theorem for the Legendre polynomial, that integral reduces to one in energy and one in angle. Numerical quadrature is employed to evaluate the angle integral. The equation is separated into a group of equations by replacing the energy integral with a summation of group to group scatter terms. Since the resulting equations are differential equations, finite difference approximations are used to reduce these equations to a group of coupled algebraic equations. The numerical quadrature and finite difference approximations introduce truncation error. On the other hand if more terms in the quadrature and a finer mesh are used, round-off error is increased and the answer may not converge. So some error is unavoidable. The computation takes less time than Monte Carlo computations.
but can not handle complex geometries.

Moments Method

The Moments method also produces a solution to the Boltzmann Transport Equation by numerical methods but involves more analytical reduction than does Discrete Ordinates. In this method, the scatter cross section and the fluence in all terms of Eq (5) are expanded in Legendre polynomials. Using the addition theorem, the scatter multiple integral reduces to a single integral in energy. By multiplying through the equation by the appropriate powerset directional angle factors and integrating each term over direction, the equation is reduced to a differential—integro equation set. By multiplying this set of equations by the spatial moments and integrating over all space, the equations reduce to the moments equations. The moments equations are recursive and because of the integration in energy, they must be evaluated by numerical quadrature in energy space. Once the moments have been calculated, the fluence is reconstructed by means of summing all the moments of a particular energy after they have been multiplied by a set of biorthogonal polynomials. A complete derivation of the moments equation and the reconstruction of the fluence is given in Appendix A. (Ref 117-14)

Since only a finite number of moments can be used, this method is also limited in accuracy. But depending on how many moments are used, the error can be limited. Thus a predetermined accuracy can be reached. This method is also limited to the time independent system and to a simple geometry and a homogeneous infinite medium. The time used to evaluate the fluence by this method is usually less than either Monte Carlo or Discrete Ordinates in such simple geometries.
Program Used

Literature Search

Monte Carlo. Many Monte Carlo programs were considered for this compilation of x-ray build-up factors. The MASTER program file (Ref 4) is a set of programs and cross section libraries used to solve radiation transport problems. The programs in this file are FASTER, BETA and TEMPER. All the programs can handle complex geometry, nonlinear and time dependent photon fluences. But the generality of this file makes it difficult for the user not acquainted with this file to provide data for a problem. The DART and DART II codes (Ref 5 and 6) are codes which handle time dependent photon transport in air. DART is a one dimensional code which assumes homogeneous air, while DART II assumes nonhomogeneous air and takes into account the curvature of the earth. DART II is especially suited for high altitude air transport. The THISTLE code (Ref 7) is a time dependent code which describes x-ray transport in exponential atmosphere.

The MORSE code (Ref 8) is a gamma ray transport code which uses multigroup cross sections. This code can handle three or one dimensional problems. It treats the atmosphere as a homogeneous medium and the answers are in a time dependent form. The HAN code (Ref 9) is a modified MORSE code which incorporates varying air density at high altitudes and to take into account the curvature of the earth. One of the most popular Monte Carlo codes used for x-ray radiation transport is the PHOTRAN code (Ref 10,11 and 12). This code handles time dependent radiation transport and calculates energy deposition, photon flux, electron flux or tissue equivalent dose. It can consider energies from zero to 100 MeV taking into account coherent and incoherent scattering, photoelectric effect, pair production, fluorescent and annihilation radiation. It can consider three dimensional geometry in
a homogeneous medium. AMDY is a series of transport codes (Ref 13) designed for time dependent photon transport which can handle three dimensional homogeneous atmosphere.

All the codes listed above were considered but rejected because of the amount of time and money involved in implementing and running these programs.

**Discrete Ordinates.** DTFXRAY is a one dimensional code (Ref 14) using Discrete Ordinates method of solution. This code does not treat time dependence and assumes a homogeneous atmosphere. ANISN is a Discrete Ordinates code (Ref 3) which treats a homogeneous atmosphere in one dimension. It does not handle time dependence.

**Moments Method.** PHOTDIS is a one dimensional code (Ref 1) for computing transport in a homogeneous atmosphere. It does not handle time dependence but it does compute the build-up factors. This program was selected because of the ease of input and the direct output of build-up factors. The speed of this program was another factor in its selection since time dependence and inhomogeneous atmosphere are not considered. Another factor for choosing a moments method program is the approximation of the error through convergence analysis which is not available from the other methods of solution.

**Description of PHOTDIS**

The program PHOTDIS consists of two phases, Phase I and Phase II. In Phase I, the moments for the scattered fluence are calculated. Most of Phase I is concerned with calculating the integral in Eq (28), which is found in Appendix A. Phase II reconstructs the fluence from the moments using Eqs (31), (32) and (33). The majority of Phase II is a numerical calculation of Eq (33), which is made difficult since only values of \( W_n \) are known. At the end of Phase II, the build-up factor is calculated using Eqs (34), (35) and (36). A more complete description of the program can be
found in Ref 1.

**Input**

The input to the program consists of 12 different sets of data. They are:

1) **Klein-Nishina, Photoelectric and Compton scatter cross section as a function of energy.**

2) Parameter determining if mesh parameters are to be input.

3) Altitude.

4) Density of the air at the altitude considered.

5) Energy above which no correction is to be made to the Klein-Nishina scattering equation.

6) Parameters determining which phases are to be run.

7) Number of space points, angular variables and a parameter determining if angular fluence is to be calculated.

8) Number of moments to be used.

9) Source energy and source strength.

10) Mesh parameters.

11) Angles at which the fluence is to be calculated.

12) Distance at which build-up factors are to be calculated.

The cross sections used for this study are from UCRL-50174 (Ref 13) and from AFWL-TR-67-11 (Ref 14). A listing of the code and a sample input are furnished in Appendix B.
IV Results

Choice of Options

Energies. As stated in Section I, the magnitude of the build-up factor is related to the amount of Compton scatter. For energies where Compton scatter is a major contributor to the total cross section, the build-up factor is greater than one. Therefore, this study begins at 12 keV since below that energy, Compton scatter is negligible. For energies below 12 keV, Eq (1) can be used since the build-up factor is approximately one. The Compton scatter cross section begins to decrease for energies above 100 keV. This decrease continues until the Compton scatter is very low, but is the only major contributor to the total cross section. Pair production, which is zero below 1020 keV, increases very rapidly just above 1020 keV and quickly becomes the major factor in the total cross section. Since the program does not handle pair production, this study terminates at 1000 keV.

Ranges. For each energy, calculations were made of the build-up factor for a distance from the source from one to 15 mean-free-paths in increments of one mean-free-path. The answers are not converged well beyond 15 mean-free-paths. The build-up factors at the various distances in mean-free-paths are valid at any altitude, but the mean-free-path varies with altitude. So at any altitude of interest, the build-up factor can be obtained for any distance in meters by converting the range in meters to mean-free-paths at any altitude. Fig 0 shows the variance of the mean-free-path with altitude for different energies.

Graphs of Build-up Factors

Fig 1 to Fig 105 are the graphs of the build-up factors for various energies ranging from 12 keV to 1000 keV. For energies between 12 and 100
keV, the increment is 2 keV. For energies between 100 and 200 keV, the increment is 5 keV. For energies between 200 and 500 keV, the increment is 10 keV and for energies higher than 500 keV, the increment is 50 keV. On each graph, the points shown are the calculated results with the curve fitted by spline-fitting technique performed by the computer and drawn by a Calcomp plotter Model 765.

**Empirical Build-up Factor Equation**

Taylor (Ref 19) has developed a simple equation to describe build-up factors previously calculated. This equation is

\[ B = A_1 e^{c_1 y} + A_2 e^{c_2 y} \]  \hspace{1cm} (6)

where

- \( y \) is number of mean-free-paths of source energy
- \( A_2 = 1 - A_1 \)

\( A_1, c_1 \) and \( c_2 \) are constants to be determined from calculated Build-up factors.

A calculation was performed using the build-up factors obtained in this study to obtain these constants. A list of these constants and the maximum percent difference is shown in Table I. (Text continues on page 121)
<table>
<thead>
<tr>
<th>Energy in keV</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>Maximum % difference below 8 KFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>-0.227</td>
<td>1.227</td>
<td>-0.400</td>
<td>0.000</td>
<td>0.35 %</td>
</tr>
<tr>
<td>14</td>
<td>-0.370</td>
<td>1.370</td>
<td>-0.400</td>
<td>0.000</td>
<td>0.89 %</td>
</tr>
<tr>
<td>16</td>
<td>-0.323</td>
<td>1.323</td>
<td>-0.680</td>
<td>0.020</td>
<td>1.78 %</td>
</tr>
<tr>
<td>18</td>
<td>-0.634</td>
<td>1.634</td>
<td>-0.460</td>
<td>0.020</td>
<td>0.91 %</td>
</tr>
<tr>
<td>20</td>
<td>-1.072</td>
<td>2.072</td>
<td>-0.360</td>
<td>0.020</td>
<td>2.71 %</td>
</tr>
<tr>
<td>22</td>
<td>-1.048</td>
<td>2.048</td>
<td>-0.480</td>
<td>0.040</td>
<td>2.90 %</td>
</tr>
<tr>
<td>24</td>
<td>-1.740</td>
<td>2.748</td>
<td>-0.340</td>
<td>0.040</td>
<td>0.75 %</td>
</tr>
<tr>
<td>26</td>
<td>-2.673</td>
<td>3.673</td>
<td>-0.260</td>
<td>0.040</td>
<td>1.85 %</td>
</tr>
<tr>
<td>28</td>
<td>-2.664</td>
<td>3.664</td>
<td>-0.300</td>
<td>0.060</td>
<td>3.87 %</td>
</tr>
<tr>
<td>30</td>
<td>-6.038</td>
<td>7.038</td>
<td>-0.140</td>
<td>0.040</td>
<td>2.90 %</td>
</tr>
<tr>
<td>32</td>
<td>-8.805</td>
<td>9.805</td>
<td>-0.100</td>
<td>0.040</td>
<td>1.45 %</td>
</tr>
<tr>
<td>34</td>
<td>-8.504</td>
<td>9.504</td>
<td>-0.100</td>
<td>0.060</td>
<td>1.63 %</td>
</tr>
<tr>
<td>36</td>
<td>-75.83</td>
<td>76.83</td>
<td>0.000</td>
<td>0.020</td>
<td>0.97 %</td>
</tr>
<tr>
<td>38</td>
<td>-20.03</td>
<td>21.03</td>
<td>-0.020</td>
<td>0.060</td>
<td>3.77 %</td>
</tr>
<tr>
<td>40</td>
<td>-16.94</td>
<td>17.94</td>
<td>-0.020</td>
<td>0.060</td>
<td>6.93 %</td>
</tr>
<tr>
<td>45</td>
<td>14.59</td>
<td>-13.59</td>
<td>0.120</td>
<td>-0.020</td>
<td>13.7 %</td>
</tr>
<tr>
<td>50</td>
<td>11.31</td>
<td>-10.31</td>
<td>0.160</td>
<td>-0.040</td>
<td>21.1 %</td>
</tr>
<tr>
<td>55</td>
<td>109.2</td>
<td>-108.2</td>
<td>0.120</td>
<td>0.100</td>
<td>18.0 %</td>
</tr>
<tr>
<td>60</td>
<td>-110.5</td>
<td>12.05</td>
<td>0.000</td>
<td>0.200</td>
<td>26.6 %</td>
</tr>
<tr>
<td>70</td>
<td>-114.1</td>
<td>115.1</td>
<td>0.140</td>
<td>0.160</td>
<td>25.7 %</td>
</tr>
<tr>
<td>80</td>
<td>-113.1</td>
<td>114.1</td>
<td>0.160</td>
<td>0.180</td>
<td>27.6 %</td>
</tr>
<tr>
<td>100</td>
<td>-10.93</td>
<td>11.93</td>
<td>0.060</td>
<td>0.260</td>
<td>36.0 %</td>
</tr>
<tr>
<td>120</td>
<td>-8.153</td>
<td>9.153</td>
<td>0.020</td>
<td>0.280</td>
<td>39.1 %</td>
</tr>
<tr>
<td>150</td>
<td>13.14</td>
<td>-12.14</td>
<td>0.260</td>
<td>0.100</td>
<td>36.3 %</td>
</tr>
<tr>
<td>200</td>
<td>-88.92</td>
<td>89.92</td>
<td>0.180</td>
<td>0.200</td>
<td>33.1 %</td>
</tr>
<tr>
<td>250</td>
<td>-6.308</td>
<td>7.308</td>
<td>0.000</td>
<td>0.260</td>
<td>39.9 %</td>
</tr>
<tr>
<td>300</td>
<td>19.89</td>
<td>-18.89</td>
<td>0.200</td>
<td>0.120</td>
<td>33.1 %</td>
</tr>
<tr>
<td>350</td>
<td>72.37</td>
<td>-71.37</td>
<td>0.160</td>
<td>0.140</td>
<td>31.2 %</td>
</tr>
<tr>
<td>400</td>
<td>-6.063</td>
<td>7.063</td>
<td>0.000</td>
<td>0.220</td>
<td>35.1 %</td>
</tr>
<tr>
<td>500</td>
<td>16.29</td>
<td>-15.29</td>
<td>0.160</td>
<td>0.080</td>
<td>28.8 %</td>
</tr>
<tr>
<td>600</td>
<td>-57.58</td>
<td>57.58</td>
<td>0.100</td>
<td>0.120</td>
<td>25.7 %</td>
</tr>
<tr>
<td>750</td>
<td>-17.20</td>
<td>18.20</td>
<td>0.060</td>
<td>0.120</td>
<td>22.6 %</td>
</tr>
</tbody>
</table>
FIG. 0 MEAN-FREE-PATH AT VARIOUS ALTITUDE
FIG. 1 ENERGY BUILD-UP FACTORS FOR 12 KEV
Due to the limits of the convergence of the moments method program, the

**FIG. 2 ENERGY BUILD-UP FACTORS FOR 14 KEV**
FIG. 3 ENERGY BUILD-UP FACTORS FOR 16 KEV
Statistical estimation may be a last-flight estimator which calculates

\[
\text{\textbf{FIG. 4 ENERGY BUILD-UP FACTORS FOR 18 KEV}}
\]
error is increased and the answer may not converge. By that time it is unavoidable. The computation takes less time than Monte Carlo computations.

FIG. 6 ENERGY BUILD-UP FACTORS FOR 22 KEV
FIG. 7 ENERGY BUILD-UP FACTORS FOR 24 KEV
Eqs (34), (35) and (36). A more complete description of the program can be

FIG. 9 ENERGY BUILD-UP FACTORS FOR 28 KEV
FIG. 10 ENERGY BUILD-UP FACTORS FOR 30 KEV
FIG. 11 ENERGY BUILD-UP FACTORS FOR 32 KEV
FIG. 12 ENERGY BUILD-UP FACTORS FOR 34 KEV
FIG. 13 ENERGY BUILD-UP FACTORS FOR 36 KEV
FIG. 14 ENERGY BUILD-UP FACTORS FOR 38 KEV
FIG. 15 ENERGY BUILD-UP FACTORS FOR 40 REV

DISTANCE FROM SOURCE IN MEAN-FREE-PATHS

BUILD-UP FACTOR

$10^0$ $10^1$ $10^2$

$0.00$ $3.00$ $6.00$ $9.00$ $12.00$ $15.00$
FIG. 16 ENERGY BUILD-UP FACTORS FOR 42 KEV
FIG. 18 ENERGY BUILD-UP FACTORS FOR 46 KEV
FIG. 19 ENERGY BUILD-UP FACTORS FOR 48 KEV
FIG. 20 ENERGY BUILD-UP FACTORS FOR 50 KEV
FIG. 21 ENERGY BUILD-UP FACTORS FOR 52 KEV
FIG. 22 ENERGY BUILD-UP FACTORS FOR 54 KEV
FIG. 23 ENERGY BUILD-UP FACTORS FOR 56 KEV
FIG. 24 ENERGY BUILD-UP FACTORS FOR 58 KEV
FIG. 25 ENERGY BUILD-UP FACTORS FOR 60 KEV
FIG. 26 ENERGY BUILD-UP FACTORS FOR 62 KEV
FIG. 27 ENERGY BUILD-UP FACTORS FOR 64 KEV
FIG. 28 ENERGY BUILD-UP FACTORS FOR 66 KEV
FIG. 29 ENERGY BUILD-UP FACTORS FOR 68 KEV
FIG. 30 ENERGY BUILD-UP FACTORS FOR 70 KEV
FIG. 31 ENERGY BUILD-UP FACTORS FOR 72 KEV
FIG. 32 ENERGY BUILD-UP FACTORS FOR 74 KEV
FIG. 33 ENERGY BUILD-UP FACTORS FOR 76 KEV
FIG. 34 ENERGY BUILD-UP FACTORS FOR 76 KEV
FIG. 35 ENERGY BUILD-UP FACTORS FOR 80 KEV
FIG. 36 ENERGY BUILD-UP FACTORS FOR 82 KEV
FIG. 37 ENERGY BUILD-UP FACTORS FOR 84 KEV
FIG. 38 ENERGY BUILD-UP FACTORS FOR 86 KEV
Fig. 39 Energy Build-Up Factors for 88 KeV
FIG. 40 ENERGY BUILD-UP FACTORS FOR 90 KEV
FIG. 42 ENERGY BUILD-UP FACTORS FOR 94 KEV
FIG. 43 ENERGY BUILD-UP FACTORS FOR 96 KEV
Fig. 44 Energy Build-Up Factors for 98 keV
FIG. 45 ENERGY BUILD-UP FACTORS FOR 100 KEV
FIG. 46 ENERGY BUILD-UP FACTORS FOR 105 KEV
FIG. 47 ENERGY BUILD-UP FACTORS FOR 110 KEV
FIG. 48 ENERGY BUILD-UP FACTORS FOR 115 KEV
FIG. 49 ENERGY BUILD-UP FACTORS FOR 120 KEV
FIG. 50  
ENERGY BUILD-UP FACTORS FOR 125 KEV  

DISTANCE FROM SOURCE IN MEAN-FREE-PATHS  

BUILD-UP FACTOR  

10^{-6} 2 3 4 5 6 7 8 9 10^1 2 3 4 5 6  

0.00 3.00 6.00 9.00 12.00 15.00
FIG. 51 ENERGY BUILD-UP FACTORS FOR 130 KEV
FIG. 52 ENERGY BUILD-UP FACTORS FOR 135 KEV
FIG. 53 ENERGY BUILD-UP FACTORS FOR 140 KEV
FIG. 54 ENERGY BUILD-UP FACTORS FOR 145 KEV
FIG. 55 ENERGY BUILD-UP FACTORS FOR 150 KEV
FIG. 56 ENERGY BUILD-UP FACTORS FOR 155 KEV
FIG. 57 ENERGY BUILD-UP FACTORS FOR 160 KEV
FIG. 58 ENERGY BUILD-UP FACTORS FOR 165 KEV
FIG. 59 ENERGY BUILD-UP FACTORS FOR 170 KEV
FIG. 60: ENERGY BUILD-UP FACTORS FOR 175 KEV
FIG. 61 ENERGY BUILD-UP FACTORS FOR 180 KEV
FIG. 62 ENERGY BUILD-UP FACTORS FOR 185 KEV
FIG. 63 ENERGY BUILD-UP FACTORS FOR 190 KEV
FIG. 64  ENERGY BUILD-UP FACTORS FOR 195 KEV
FIG. 65 ENERGY BUILD-UP FACTORS FOR 200 KEV
FIG. 66 ENERGY BUILD-UP FACTORS FOR 210 KEV
FIG. 67 ENERGY BUILD-UP FACTORS FOR 220 KEV
FIG. 68 ENERGY BUILD-UP FACTORS FOR 230 KEV
FIG. 69 ENERGY BUILD-UP FACTORS FOR 240 KEV
FIG. 70 ENERGY BUILD-UP FACTORS FOR 250 KEV
FIG. 71 ENERGY BUILD-UP FACTORS FOR 260 KEV
FIG. 72 ENERGY BUILD-UP FACTORS FOR 270 KEV
FIG. 73 ENERGY BUILD-UP FACTORS FOR 280 KEV
FIG. 74 ENERGY BUILD-UP FACTORS FOR 290 KEV
FIG. 75 ENERGY BUILD-UP FACTORS FOR 300 KEV
FIG. 76 ENERGY BUILD-UP FACTORS FOR 310 KEV
FIG. 77 ENERGY BUILD-UP FACTORS FOR 320 KEV
FIG. 78 ENERGY BUILD-UP FACTORS FOR 330 KEV
FIG. 79 ENERGY BUILD-UP FACTORS FOR 340 KEV
FIG. 80 ENERGY BUILD-UP FACTORS FOR 350 KEV
FIG. 81 ENERGY BUILD-UP FACTORS FOR 360 KEV
FIG. 82 ENERGY BUILD-UP FACTORS FOR 370 KEV
FIG. 83 ENERGY BUILD-UP FACTORS FOR 380 KEV
FIG. 84 ENERGY BUILD-UP FACTORS FOR 390 KEV
FIG. 85 ENERGY BUILD-UP FACTORS FOR 400 KEV
FIG. 86 ENERGY BUILD-UP FACTORS FOR 410 KEV
FIG. 87 ENERGY BUILD-UP FACTORS FOR 420 KEV
Fig. 88 Energy build-up factors for 430 keV
FIG. 89 ENERGY BUILD-UP FACTORS FOR 440 KEV
FIG. 90 ENERG Y BUILD-UP FACTORS FOR 450 KEV
FIG. 91 ENERGY BUILD-UP FACTORS FOR 460 KEV
FIG. 92 ENERGY BUILD-UP FACTORS FOR 470 KEV

DISTANCE FROM SOURCE IN MEAN-FREE-PATHS

BUILD-UP FACTOR

$10^0$ $2$ $10^1$ $2$ $10^2$ $2$ $10^3$ $2$ $10^4$ $2$ $10^5$ $2$ $10^6$
FIG. 93 ENERGY BUILD-UP FACTORS FOR 480 KEV
FIG. 94 ENERGY BUILD-UP FACTORS FOR 490 KEV
FIG. 95 ENERGY BUILD-UP FACTORS FOR 500 KEV
FIG. 96 ENERGY BUILD-UP FACTORS FOR 550 KEV
FIG. 97  ENERGY BUILD-UP FACTORS FOR 600 KEV
FIG. 98 Energy Build-Up Factors for 650 KeV
Fig. 99 Energy Build-up Factors for 700 keV
FIG. 100 ENERGY BUILD-UP FACTORS FOR 750 KEV
FIG. 101 ENERGY BUILD-UP FACTORS FOR 800 KEV
FIG. 102  ENERGY BUILD-UP FACTORS FOR 850 KEV
Fig. 103 Energy Build-up Factors for 900 KeV
FIG. 104 ENERGY BUILD-UP FACTORS FOR 950 KEV
FIG. 105 ENERGY BUILD-UP FACTORS FOR 1000 KEV


V Discussion

Intrinsic Estimate of the Accuracy

To examine the accuracy of this program, the convergence of a finite number of moments must be examined first. If all the moments could be calculated from Eq (28), found in Appendix A, no error from this source would be introduced. To estimate the error injected by the truncation of terms, examination of the change of the values of the expansion coefficient $F^m(r,E)$ as more moments are used in Eqs (31) and (32) would be used. This analysis was performed by Bigelow (Ref 1:116-119) on this program and his findings indicate that the total error produced by the truncation of all but the first nine terms results in a 5% error in the expansion coefficients.

The error introduced by calculating the scattered fluence by using the polynomial reconstruction, Eq (34), is about 1% The numerical error introduced by the calculations performed in Eq (1), (29), (35) and (36) are from 1% at small distances for small energies to 15% at large distances for large energies. The figures used above were also obtained by analysis performed by Bigelow (Ref 1:126-127).

Therefore, the total maximum error for the build-up factors introduced from all sources is 20%, which only occurs at large distances for large energies.

Comparison of Results to Others

Introduction. Because of the limited amount of data for build-up factors of the energy range considered in this study for air, some modification of the data obtained from other sources had to be made. In some cases, the data obtained from other sources were in the form of energy fluence instead of build-up factors. To obtain the build-up factors in this case, Eqs (1)
and (36) were used. In other cases, the results were presented as a fraction of vacuum fluence that reached a certain distance. Then, the following calculations were performed. The vacuum fluence can be defined as

$$F^V = S/k_{mf}^2$$  \hspace{1cm} (7)

By dividing Eq (2) by Eq (7) the fraction of the vacuum fluence $V$ is obtained:

$$V = e^{-\mu r}$$  \hspace{1cm} (8)

$\mu r$ is equal to the number of mean-free-paths $y$. Solving Eq (8) for $B$

$$B = V/e^{-y}$$  \hspace{1cm} (9)

So for any mean-free-path of interest, the build-up factor can be extracted from data giving only fraction of the vacuum fluence to that distance.

All of the comparisons made below are in terms of percent difference which is defined by the following equation:

$$\text{(Percent difference)} = \frac{(\text{Moment-method result}) - (\text{Result used in comparison})}{\text{(Moment-method result)}} \times 100\% = \text{PD}$$  \hspace{1cm} (10)

where PD is the percent difference.

Monte Carlo. Banks (Ref 15), using a modified CGRE Monte Carlo code, performed photon transport calculations for energies from 20 to 300 keV. He presented some of the results in the form of a graph of build-up factors at 10 mean-free-paths. This graph as well as a similar graph of the build-up factors calculated by the moments method are superimposed and presented in Fig 106. The percent difference for the energies below 50 keV are as low as -47%. But above 50 keV, the percent differences range up to 26% with the average being 23%. Taking into account the error of the build-up factor calculation associated with the moments method, the rest of the error can be attributed to the use of different cross sections.

Coleman did a study computing x-ray transport and the results are
FIG. 106 COMPARISON OF ENERGY BUILD-UP FACTORS AT 10 MEAN-FREE-PATHS
presented in Banks (Ref 15). Since the results are presented as a fraction of vacuum fluence in mean-free-paths, the conversion to build-up factors must be performed. A comparison to the build-up factors calculated by the moments method is presented in Fig 107. It can be seen that the percent differences for 60 keV, which is the only energy range given, are from 17% to 52%.

Krumbein (Ref 16) using SAGE performed calculations of x-ray air transport. SAGE is United Nuclear’s spherical Monte Carlo photon code. The calculations were carried out at various energies from 12 keV to 100 keV and out to 10 mean-free-paths. Even though the calculations for the fluence used to calculate the build-up factors were performed using the density of air at an altitude of 22,860 meters, this had no effect on the build-up factors since the energy build-up factor is independent of altitude when it is reported as a function of mean-free-path. Comparison to Krumbein’s calculations is shown in Figs 108, 109 and 110. For 20 keV, the percent differences range from 7.5% to 17%. For 40 keV, the percent differences range from 18% to 32%. For 100 keV, the percent differences range from 12% to 27%. These differences can be accounted for from the different cross sections used and by taking into account the error produced in the calculation by the moments method.

One of the most extensive calculations using a Monte Carlo code to solve the x-ray air transport problem in air was performed by Shelton and Keith (Ref 17). This calculation was performed by the HAT code. The results are presented in comparison to the moments method calculation in Figs 111-118. For 20 keV, the percent differences are from -8% to -69%. For 40 keV, the percent differences are from -2% to -413%. For 60 keV, the percent differences are from -8% to -76%. For 90 keV, the percent differences are from 3% to -70%. For 120 keV, the percent differences are from 7% to
FIG. 107 COMPARISON OF ENERGY BUILD-UP FACTORS FOR 60 KEV
FIG. 108 COMPARISON OF ENERGY BUILD-UP FACTORS FOR 20 KEV
FIG. 109 COMPARISON OF ENERGY BUILD-UP FACTORS FOR 40 KEV
FIG. 110 COMPARISON OF ENERGY BUILD-UP FACTORS FOR 100 KEV
FIG. 111 COMPARISON OF ENERGY BUILD-UP FACTORS FOR 20 KEV
Figure 112: Comparison of energy build-up factors for 40 keV
FIG. 113  COMPARISON OF ENERGY BUILD-UP FACTORS FOR 60 KEV
FIG. 114 COMPARISON OF ENERGY BUILD-UP FACTORS FOR 50 KEV
FIG. 115 COMPARISON OF ENERGY BUILD-UP FACTORS FOR 120 KEV
FIG. 116. COMPARISON OF ENERGY BUILD-UP FACTORS FOR 150 KEV
FIG. 117 COMPARISON OF ENERGY BUILD-UP FACTORS FOR 200 keV
FIG. 118 COMPARISON OF ENERGY BUILD-UP FACTORS FOR 300 KEV
-27%. For 150 keV, the percent differences are from 5% to -9%. For 200 keV, the percent differences are from 1% to -63%. For 300 keV, the percent differences are from -2% to -96%. The differences go up as range increases in all cases except 150 keV. The absence of smooth curves in the range verses build-up factor semi-log graphs indicates sizable statistical variation in the build-up factors calculated by this Monte Carlo code. This can account for the large amount of disagreement at some ranges and energies.

**Discrete Ordinates.** Because of the limited amount of published results of discrete ordinates calculations, only one comparison of the moments method calculation to discrete ordinates will be presented. Dupree (Ref 18) used DTFXRAY to perform a limited calculation at 100 keV. Since energy fluence was obtained for only three distances, a table of results along with the results of the moments method and the percent differences are presented as Table II. The results are in great disparity. The erratic agreement is due to either the limited and truncated discrete ordinates calculation, which was a P3S8 or opposite maximum error in the discrete ordinates and moments method.

**Bigelow.** Because the program used by Bigelow was the same one that was used for this study, comparison would not be in order. But using the same input, the exact results were obtained, showing that the program was working as programmed and any error calculations performed by Bigelow are applicable to this study. When recent cross sections for energy range from 1 keV to 100 keV were used, deviation in results occurred at small energies.

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VI Conclusion

Purpose and Scope

The purpose of this report is to provide a complete set of monoenergetic build-up factors which are used to simplify the x-ray air transport problem. To compute these build-up factors, a moments method program was used. The program PHOTDIS was chosen and run on a CDC 6600. The energies considered were from 14 keV to 1000 keV. The ranges considered were from one to 15 mean-free-paths.

Results and Discussion

The results were presented in a set of graphs for each energy. The intrinsic accuracy of the program estimated by convergence analysis performed by Bigelow was found to be a maximum of 20%. Comparison to others found the results of the moments method calculation to be higher than some Monte Carlo calculations and lower than others. For 10 mean-free-paths, this is shown in Fig 119. To facilitate use of the data presented, selected energies are presented together in Figs 120 and 121.

Recommendations and Summary

Because of the limited time and the nature of the program, experimentation with different number of moments and the number of polynomials used in the reconstruction could not be performed. It is recommended that any other extensive study that computes build-up factors using the moments method should vary the number of moments and reconstruction polynomials.
FIG. 119 COMPARISON OF ENERGY BUILD-UP FACTORS AT 10 MEAN-FREE-PATHS
FIG. 120 ENERGY BUILD-UP FACTORS FOR 12-120 KEV
FIG. 121 ENERGY BUILD-UP FACTORS FOR 150-1000 KEV
Bibliography


APPENDIX A

Solution of the Boltzmann Transport Equation by the Moments Method

Reduction of the Boltzmann Transport Equation

From Eq (5), the Boltzmann Transport Equation integrated over all time for spherical coordinates can be extracted:

\[ \frac{\partial F(r,E,\mu)}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial F(r,E,\mu)}{\partial r} \right) + \mu \frac{\partial F(r,E,\mu)}{\partial \mu} = \int \int F(r,E,\mu') \lambda_{\mu} \lambda_{\mu'} \sigma_{\mu} (E' \rightarrow E) dE' d\mu' \]

(11)

where

\[ \mu \] is the cosine of the angle \( \theta \) in spherical coordinates.

Since the virgin fluence can be calculated without resort to computers by Eq (1), the fluence in Eq (11) is the scattered fluence.

To expand the functions in Eq (11), a set of polynomials is needed. Legendre polynomials conform to the requirements for expansion polynomials. These polynomials have the following properties:

\[ \int_{-1}^{+1} P_n(x) P_m(x) dx = \frac{2n+1}{4\pi} \delta_{mn} \]  \hspace{2cm} (12)

\[ f(x) = \sum_{n=0}^{\infty} \frac{2n+1}{4\pi} f_n P_n(x) \]  \hspace{2cm} (13)

\[ f_n = \int_{-1}^{+1} f(x) P_n(x) dx \]  \hspace{2cm} (14)

Therefore, the expansion of the fluence \( F \), the Scatter cross section \( \mu^S \), and the source \( S \) in Legendre polynomials is

\[ F(r,E,\mu) = \sum_{n=0}^{\infty} \frac{2n+1}{4\pi} f_n P_n(r,E) P_n(\mu) \]  \hspace{2cm} (15)

\[ \mu^S(\mu_o,E' \rightarrow E) = \sum_{n=0}^{\infty} \frac{2n+1}{4\pi} s_n(\mu_o,E' \rightarrow E) P_n(\mu) \]  \hspace{2cm} (16)
Substituting Eqs (15), (16) and (17) into Eq (11) yields

\[
S(r, E, \mu) = \sum_{m=0}^{\infty} \frac{2m+1}{4\pi} S_m(r, E) P_{m}(\mu) \tag{17}
\]

By rearranging the scattering integral (the second term on the right side), it becomes:

\[
\int \left[ \left( \sum_{n=0}^{\infty} F_n(r, E) \frac{2n+1}{4\pi} \right) \left( \sum_{m=0}^{\infty} \frac{\mu_m(E' \rightarrow E) P_{m}(\mu')}{4\pi} \right) \int_0^2 \int^{-1}_1 P_{n}(\mu') P_{m}(\mu) \, \mu' \, d\mu' \, dE' \right] \delta_{mn}
\]

By using the addition theorem on the double integral above, the integral becomes

\[
\int_0^{2\pi} \int^{-1}_1 \left\{ P_{m}(\mu) P_{m}(\mu') + 2 \sum_{k=1}^{m} \frac{(\mu - k - 1) k}{2} P_{m}(\mu') \cos[k(\phi - \phi')] \right\} P_{m}(\mu) \, d\mu \, d\phi
\]

Upon performing the integrals

\[
\frac{4\pi}{2m+1} P_{m}(\mu) \delta_{mn}
\]

So the scattering integral in Eq (18) becomes

\[
\int_0^{E} \left[ \left( \sum_{n=0}^{\infty} F_n(r, E) \frac{2n+1}{4\pi} \right) \left( \sum_{m=0}^{\infty} \frac{\mu_m(E' \rightarrow E) P_{m}(\mu)}{4\pi} \right) \frac{4\pi}{2m+1} P_{m}(\mu) \delta_{mn} \right] \, dE'
\]

which equals

\[
\int_0^{E} \sum_{n=0}^{\infty} F_n(r, E) \frac{2n+1}{4\pi} \mu_n(E' \rightarrow E) P_{n}(\mu) \, dE'
\]

Putting the last term into Eq (18), the full equation becomes
\[
\sum_{n=0}^{\infty} \frac{\partial F_n(r,E)}{\partial r} \frac{2n+1}{4\pi} P_n(\mu) + \sum_{n=0}^{\infty} \frac{1-n^2}{r} \frac{\partial F_n(r,E)}{\partial \mu} \frac{2n+1}{4\pi} P_n(\mu)
\]

+ \sum_{n=0}^{\infty} \mu_n F_n(r,E)P_n(\mu) 2n+1 \frac{2n+1}{4\pi} P_n(\mu)
\int_{0}^{E} \sum_{n=0}^{\infty} P_n(r,E')S_n(E'-E) 2n+1 \frac{2n+1}{4\pi} P_n(\mu) dE'

(19)

+ \sum_{n=0}^{\infty} \frac{2n+1}{4\pi} S_n(E)\delta(r)P_n(\mu)

By multiplying each term of Eq (19) by 2n\mu F_n(\mu) d\mu and integrating each term from -1 to +1, the following equation is obtained:

\[
\sum_{n=0}^{\infty} \frac{2n+1}{2} \frac{\partial F_n(r,E)}{\partial r} \int_{-1}^{1} P_n(\mu)P_n(\mu) d\mu
\]

+ \sum_{n=0}^{\infty} \frac{2n+1}{2} \frac{\partial F_n(r,E)}{\partial r} \int_{-1}^{1} (1-\mu^2) \frac{\partial F_n(r,E)}{\partial \mu} P_n(\mu) d\mu + \mu F_n(r,E) S_n(E)\delta(r) = S_n(E)\delta(r)

(20)

To simplify Eq (20), the recursion relations for Legendre polynomials must be employed:

\[
P_m(\mu) = \frac{1}{2m+1} \left[ (m+1)P_{m+1}(\mu) + mP_{m-1}(\mu) \right]
\]

(21)

\[
(1-\mu^2) \frac{\partial P_m(\mu)}{\partial \mu} = m \left[ P_{m-1}(\mu) - \frac{1}{2m+1} \left[ (m+1)P_{m+1}(\mu) + mP_{m-1}(\mu) \right] \right]
\]

(22)

By substituting Eq (21) into the first term of Eq (20) and by substituting Eq (22) into the second term of Eq (20) and performing the integration, the final \( P_2 \) equation is obtained:

\[
\frac{P_{l+1}(r,E)}{2l+1} \frac{\partial F_{l+1}(r,E)}{\partial r} + \frac{l}{2l+1} \frac{\partial F_{l-1}(r,E)}{\partial r} + \frac{(l+1)(l+2)}{2l+1} \frac{F_{l+1}(r,E)}{r}
\]

\[
- \frac{F_{l-1}(r,E)}{2l+1} + \mu F_{l}(r,E) = S_{l}(E)\delta(r) + \int_{0}^{E} F_{l}(r,E')S_{l}(E'-E) dE'
\]

(23)

It is necessary to introduce the definition of the moments of a function \( f \):

\[
M_n = \int_{0}^{\infty} r^n f(r) 4\pi r^2 dr
\]

(24)
where $M_n$ is the $n$th moment of the function $f$. To obtain the moments of the fluence in Eq (23), it is essential to multiply each term in Eq (23) by $r^n 4\pi r^2 dr$ and integrate from zero to infinity:

$$
\frac{(l+1)(l+2)}{2l+1} \int_0^\infty r^n \frac{dF_{l+1}(r,E)}{dr} 4\pi r^2 dr + \frac{l}{2l+1} \int_0^\infty r^n \frac{dF_{l-1}(r,E)}{dr} 4\pi r^2 dr
$$

$$
+ \frac{(l+1)(l+2)}{2l+1} \int_0^\infty r^n \frac{F_{l+1}(r,E)}{r} 4\pi r^2 dr - \frac{(l-1)l}{2l+1} \int_0^\infty r^n \frac{F_{l-1}(r,E)}{r} 4\pi r^2 dr
$$

$$
+ \mu_t \int_0^\infty r^n F_{l}(r,E) 4\pi r^2 dr - \int_0^E \int_0^\infty r^n \mu^2 F_{l}(r,E) 4\pi r^2 dr dE + \int_0^\infty r^n S_{l}(E) S(r) 4\pi r^2 dr
$$

Using the moments definition, Eq (24), the following equation is extracted:

$$
\frac{(l+1)(l+2)}{2l+1} M_{l+1,n-1} - \frac{(l-1)l}{2l+1} M_{l-1,n-1} + \mu_t N_{ln} = S_{l}(E) + \int_0^E M_{ln} S dE
$$

where $M_{ln}$ is the $n$th moment of the $l$th expansion coefficient. By applying integration by parts to the first two terms of Eq (26), they become

$$
- \frac{(l+1)(l+2)}{2l+1} M_{l+1,n-1} - \frac{l}{2l+1} M_{l-1,n-1}
$$

So the full moments equation is

$$
\begin{bmatrix}
- \frac{(l+1)(n+2)}{2l+1} & \frac{l}{2l+1}
\end{bmatrix}
\begin{bmatrix}
M_{l+1,n-1}
\end{bmatrix}
+ \begin{bmatrix}
- \frac{(l+1)(l+2)}{2l+1}
- \frac{(l-1)l}{2l+1}
\end{bmatrix}
\begin{bmatrix}
M_{l-1,n-1}
\end{bmatrix}
+ \mu_t N_{ln} = S_{l}(E) + \int_0^E M_{ln} S dE
$$

Rearranging and combining terms, the recursion equation for the moments is

$$
\mu_t N_{ln} = \int_0^E \mu^2 M_{ln} dE + S_{l}(E) + \frac{l^2 + 4l + 1 - n}{2l+1} M_{l+1,n-1} + \frac{l^2 + 4l - 2 - n}{2l+1} M_{l-1,n-1}
$$

The integral is evaluated by numerical quadrature. With Eq (28), all the moments can be calculated. But only moments where $l-n$ is even or $l-n$ is odd are related. Also moments where $l-n$ are negative have no physical significance.
Reconstruction of the Fluence

Even though Eq (28) is for the scattered contribution to the fluence, the once scattered fluence can also be calculated directly using (Ref 112)

\[
F^1(r,E,\mu) = \frac{S_n u(t \cdot \cos \alpha) t^S(E \cdot E)}{8n^2r(\sin \alpha)(\sin \theta)} \left\{ \exp \left[ -r \left( \mu^t(E \cdot E \cdot \sin \alpha + \mu^t(E \cdot \sin (\alpha - \theta)) \right) \right] \right\} \tag{29}
\]

where

- \( F^1 \) is the once scattered fluence
- \( \alpha = \cos^{-1} \left[ 1 + \left( m_o c^2 / E_o \right) - \left( m_o c^2 / E \right) \right] \)
- \( u(x) \) is the unit step function = \[
\begin{cases} 
0 & \text{for } x < 0 \\
1 & \text{for } x > 0 
\end{cases}
\]
- \( E_o \) is the source energy
- \( S_T \) is the total source strength
- \( m_o \) is the mass of an electron

Therefore, all that is needed to find the total fluence is the multiscattered fluence. This fluence can be calculated from the multiscattered moments given by

\[
\frac{m^m_{ln}}{m^1_{ln}} = m_{ln}^m - m_{ln}^1 \tag{30}
\]

where

- \( m^m_{ln} \) is the multiscattered moments
- \( m^1_{ln} \) is the once scattered moment calculated from the once scattered fluence

To find the multiscattered fluence from its moments, a moments reconstruction must be performed. The first step is to define the expansion function of the moments, \( G \), which is

\[
G_{ln} = S_k(E) \sum_{j=0}^{N} (-1)^j \frac{(2j+\ell+1)ln_1}{(2j+2\ell+1)(n-j+1)} m_{\ell,\ell+2j}^m \tag{31}
\]

Using the available moments, the expansion function's values can be used to calculate the expansion coefficients, \( m^m_{\ell}(r, \theta) \) for the multiscattered fluence.
\[ F^m_\ell(x,E) = \frac{y^{2\ell}e^{-y}}{4\pi y^2} \sum_{n=0}^{N} G_{\ell n} W_n(y) \]  

(32)

where

\[ W_{n+1}^\ell = \frac{1}{2(n+1)} \left[ \left( 2n + 2\ell + 1 - y \right) W_n^\ell + y \frac{dW_n^\ell}{dy} \right] \]  

(33)

and \( W_0^\ell = 1 \). It should be noted that in Eq (31), only the moments where \( l-n \) are positive and even are needed to calculate the expansion function.

The multiscattered fluence is

\[ F^m_\ell(x,E,\mu) = \sum_{\ell=0}^{L} \frac{2^{\ell+1}}{4\pi} F^m_\ell(x,E)P_\ell(\mu) \]  

(34)

So the total fluence is

\[ F(x,E,\mu) = F^0(x,E,\mu) + F^1(x,E,\mu) + F^m(x,E,\mu) \]  

(35)

where \( F^0(x,E,\mu) \) is the virgin fluence calculated from Eq (1), \( F^1(x,E,\mu) \) calculated from Eq (29) and \( F^m(x,E,\mu) \) calculated from Eq (34). The build-up factor is given by the following equation:

\[ B = \frac{F(x,E,\mu)}{F^0(x,E,\mu)} \]  

(36)
APPENDIX B

Sample Input and Program Listing

Sample Input

The sample input listed on pages 151-154 is the input used in this study. This listing is the data input into unit 5. Unit 4 inputs the number of energies to run, the energies and the ranges in mean-free-paths.

Program Listing

The program listed on pages 155-177 is the version of PHCTDIS used in this study. This program is exactly the same as the program used by Bigelow (Ref 1) except for format of input and output. Output is on units 2, 3, and 6. Unit 2 is output for input of a plotting program used to generate the plots on pages 16-120. Units 3 and 6 are printed output. Much intermediate output is eliminated for this study, since fluence, moments and convergence analysis are not desired. This program, run on a CDC 6600 takes 48 central processor seconds and 260 input-output seconds per energy using five space points.
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0.1500 E-03 | 0.8000 E-05 | 0.2625 E-00 |
0.2000 E-03 | 0.5000 E-05 | 0.2099 E-00 |
0.3000 E-03 | 0.1000 E-05 | 0.1521 E-00 |
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**ARGON**

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1.500  | 7.63400E+04 | 9.33000E-01 |
2.000  | 3.40000E+04 | 1.45000E+00 |
3.000  | 1.11000E+04 | 2.52000E+00 |
3.201  | 9.22700E+03 | 2.02000E+00 |
3.202  | 9.16500E+04 | 2.17000E+00 |
4.000  | 5.22300E+04 | 3.22000E+00 |
5.000  | 2.30900E+04 | 4.63000E+00 |
8.000  | 7.37200E+03 | 5.11000E+00 |
10.000 | 4.20300E+03 | 6.18000E+00 |
15.000 | 1.27000E+03 | 7.25000E+00 |
20.000 | 5.71300E+02 | 8.07000E+00 |
30.000 | 1.72200E+02 | 9.74000E+00 |
40.000 | 5.20700E+01 | 8.56000E+00 |
50.000 | 3.04000E+01 | 9.40000E+00 |
60.000 | 1.74100E+01 | 8.59000E+00 |
80.000 | 7.67700E+00 | 6.78000E+00 |
100.000 | 3.52300E+00 | 8.10000E+00 |
150.000 | 1.01200E+00 | 7.43000E+00 |
200.000 | 4.27000E+01 | 7.28000E+00 |
300.000 | 1.28300E+01 | 6.31000E+00 |
400.000 | 5.48700E+02 | 5.71000E+00 |
500.000 | 3.01200E+02 | 4.63000E+00 |
600.000 | 1.90000E+02 | 4.57000E+00 |
700.000 | 8.40000E+03 | 4.22000E+10 |
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- Values in the table represent some dimensionless quantities.
- The table shows a progression of values that increase with each step.
PROGRAM PHOTDIS(*INPUT,OUTPUT,TAPE5,TAPE6,TAPE8,TAPE3,TAPE4,TAPE3,
1TAPE2)

SSSSSSSSSS SUBROUTINES USED BY THIS PROGRAM SSSSSSSSS SS

SIG.....CALCULATES TOTAL MACROSCOPIC CROSS SECTION
AKERN.....CALCULATES DIFFERENTIAL SCATTERING CROSS SECTION
(SCATTERING KERNEL)
POLKER.....CALCULATES PRODUCT OF LEGENDRE POLYNOMIA. AND SCAT-
TERING KERNEL
ONEC.....EVALUATES PART OF THE ONE-SCATTER FORMULA, EQ (21)
WXPN.....USES RECIPROCAL RELATION TO CALCULATE COEFFICIENTS FOR
THE EXPANSION POLYNOMIALS ( EQ (54) )
QUA.....CALCULATES EXPANSION COEFFICIENT OF EQ (27)
FIRST.....CALCULATES ANGULAR DISTRIBUTION OF ONCE-SCATTERED PHO-
TONS, ALSO ALL-ANGLE FIRST SCATTER, AND FIRST SCATTER
ENERGY DENSITY
SPACE.....INTERFLUXKAGE OF THE MOMENTS
INC.....CONTROLS INDEXING OF ARRAY
TORIAL.....CALCULATES FACTORIALS

COMMON PHOT(3,40),ENERG(3,40),INC(3,40),ETA(3,17),CAPK(3,17),DN(3
1),DNZ(3),NXS,NFL,TOLE,TLCK,PIE,ZERO,MAXX,XMU,XMUT,TEMP,YX,DM
2U,SIGMA,SINH,L,FIGO,ONE,YLNC,LY,LP,YX,XLANG,BETA,ZERO,
3NTRANS,XMU,XMUT,GBP,NYP,NCUT,FACTOR,SCAT(10,19),
5SIGMA(31),LAMDA(31),ONESCT(5,33,21),ALANG(5,33),ALENG(5),XMU(621),Y(5),PMU(2,21),VIRGIN(5),ALENG(5),A(10,19)

DIMENSION AP(10,19,25),DELTA(10,19),ONEP(5,33,21),SUMN(10,5,33
1),PC(10),PD(10),PLNK(10),PLKM1(10),PLKM2(10),PLK(10),7(3),NAT
3NS(3),HORD(3,7),NC(9),DP(9),ONE(10,19),INTEG(10,19),NS(10,19),BMS
41(10,19),RS2(10,19),BMS3(10,19),BONEP(10,19),
5NSHND(510),ICOf(16)

REAL LAMDA,INC,NATNS,INTEG
INTEGER 6,61,62
DATA CLITE/2.997925E10/,FORPIE/12.56537/

RECURSION RELATION FOR LEGENDRE POLYNOMIALS
ALEGRC(X,ARG,POLY1,POLY0)=((2.*X+1.)*ARG*POLY1-X*POLY0)/(X+1.)

********************************************************************

READ(4,1007)J09C
WRITE(3,2013)
WRITE(6,2013)
DO 999 JOR=1, J09C
FACTOR=511.006
DO 996 NSTUF=1,10

155
NSUMNO (NSTUF) = 1
ALPHA = 1.
NPOINT = 1

INPUT CROSS SECTIONS FOR NITROGEN, OXYGEN AND ARGON, ETA AND
CAPK ARE PARAMETERS USED TO DETERMINE THE SHERE-ELON CORRERCTIO
TO THE KLEIN-NISHINA DIFFERENTIAL CROSS SECTION FORMULA

DO 180 I = 1, 3
READ (5, 1010) (NOR (I, J), J = 1, 2), 7 (I), NATOMS (I), DN (I)
READ (5, 1000) (ETA (I, K), CAPK (I, K), K = 1, 17)
READ (5, 1002) (ENERG (I, K), PHOT (I, K), INJO (I, K), K = 1, 40)
CONTINUE

IPASS = 2 ...... WAVELENGTH MESH PARAMETERS TO BE INPUT
= 1 OTHERWISE
READ (5, 1007) IPASS

ALTITUDE AND PARTICLE DENSITY
READ (5, 1005) ALT
WRITE (3, 1105) ALT
READ (5, 1005) DENSITY

ENERGY ABOVE WHICH NO CORRECTION IS TO BE MADE TO THE KLEIN-
NISHINA SCATTERING LAW
READ (5, 1005) ENKN
XLCK = FACTO / ENKN + .0001
JND = 2 FOR PHASE II (RECONSTRUCTION) ONLY
= 1 OTHERWISE
READ (5, 1001) NFL, NXS, JND
NYP = 1, 5 ...... NUMBER OF SPACE POINTS
LEN = 1 ...... ANGULAR PHOTON DENSITY IS TO BE CALCULATED
= 2 CALCULATE THE ALL-ANGLE DENSITY ONLY
READ (5, 1001) NYP, NXS, LEN

NSTOP/2 = NUMBER OF MOMENTS TO BE USED INITIALLY IN RECONSTRUCTION
OF THE ALL-ANGLE DENSITY ... NA0/2 = NUMBER OF MOMENTS TO BE ADDED
TO THE RECONSTRUCTION EACH TIME THROUGH UNTIL NMAX/2 IS REACHED...
READ (5, 1001) NSTOP, NA0D, NMAX

EZERO = SOURCE ENERGY **** QZERO = SOURCE STRENGTH
READ (5, 1000) QZERO
READ (4, 1100) EZERO
IF (EOF (4) .NE. 0.) GO TO 500

******************************************************************************

$$$$$ UNITS ... IN GENERAL, CGS UNITS ... ENERGIES IN KEV

629 INO = 1
LPJ = 1
PIE = 3.14159
RZERO = 7.9398E-26
TOLE = .00001
TOLK = .0001

181 LMAX = NMAX/2
183 LMXP = LMAX + 1
NMAXP = NMAX + 1

156
HST0P=HST0P+1
DO 72 I=1,NEL
DN(I)=DENS*DN(I)*NATOMS(I)
DN7(I)=DN(I)*7(I)
DO 72 K=1,17
CAP=1.0*CAPK(I,K)/7(I)
IF(CAP).LT.131,131,132
131 CAPK(I,K)=0.1E-30
GO TO 72
132 CAPK(I,K)=CAP
72 CONTINUE
XLAMO=FACTOR/EPERO
G=1

C DETERMINE THE WAVELENGTH MESH TO BE USED IN CALCULATION OF MOMENTS
C NTRANS .... STEP SIZE SHIFTS FROM .02 TO .04
C NDL .... SHIFT FROM .04 TO .08
C NDL24 .... ALL MOMENTS REQUIRED FOR INTEGRATION CAN BE STORED IN
C ARRAY (MOMENTS FOR 25 VALUES OF THE WAVELENGTH)
C NCTOT .... NUMBER OF INTEGRATION STEP SIZE CHANGES
C NCUT .... NUMBER OF MESH POINT AT THE FIRST SCATTER CUTS OFF
C NCUT .... NUMBER OF MESH POINT WHERE SECOND SCATTER CUTS OFF
C NBIGGP .... NUMBER OF POINTS IN THE WAVELENGTH MESH
C NC .... LOCATIONS OF STEP SIZE CHANGES

GO TO(403,791),IPASS

403 D(1)=.02
NC(1)=1
NC(2)=21
NTRANS=21
NC(3)=71
NC(4)=331
NCTOT=4
NDL=71
NDLP24=95
NCUT=61
NCUT=91
NBIGGP=331
404 ID=1
NCG=NC(1)
DELTA=D(1)
NBIGG = NBIGG - 1
NGMAX = NBIGG / 10
LAMDA (1) = XLAMO
NGG = 1

71 ENERGY = FACTOR / LAMDA (NGG)
CALL SIG (ENERGY, SIGMAT (NGG))
IF (NGG - NBIGG) 70, 69, 69
70 NGG = NBIGG + 1
IF (NGG - NCC) 406, 406, 405
405 DELTA = DELTA + DELTA
ID = ID + 1
D(ID) = DELTA
NCC = NC(ID + 1)
406 LAMDA (NGG) = LAMDA (NGG - 1) + DELTA
GO TO 71

69 CONTINUE
CALL POLKER (PC, XLAMO, LAMDA (NCUT))
SIG0 = SIGMAT (1)
SIGOSQ = SIG0 * SIG0 / FORPIE
SIG0IN = 0.1 * SIG0
WRITE (3, 1102) ETF0, SIG0IN
C IF (JND .EQ. 2) GO TO 126
C WRITE (6) NBIGG, LMAXP, NMAXP
WRITE (9) NBIGG, LMAXP, NMAXP
C
C $$$$$ BEGIN PHASE 1 $$$$$
C
C $$$$$
C
C NGG = 1
MCTR = 2
NOUT = 11
ASSIGN 784 TO IGO
ASSIGN 783 TO ITAP
ASSIGN 259 TO IFOR
GO TO 17
C RETURN POINT FOR BEGINNING OF CALCULATION OF MOMENTS FOR THE NEXT C MESH POINT
C
C 361 J = 1
LAST = 1
DO 352 LP = 1, LMAXP
DO 352 NP = 1, NMAXP
352 INTEG (LP, NP) = 0.
XLG = LAMDA (MCTR)
NGG = 4CTR
GO TO I30, (761, 784)
784 JUMP = 1
ASSIGN 267 TO NSK
NDISCO = 1
LSEG = 0
NSKIP=1

C FIND WHETHER DISCONTINUITY AT FIRST SCATTER CUT OFF IS WITHIN THE
C RANGE OF INTEGRATION AND LOCATE THE LOWER LIMIT OF THE INTEGRAL
C
IF(MCTR .GE. NCUT) GO TO 300
301 NGS=1
300 JUMP=2
XLAMG2=LAMDA(MCTR)-2.*
DO 304 NGS=NCUT,MCTR.
TEST=XLAMG2-LAMDA(NGS)
IF(ABS(TEST)-1.E-05)305,305,610
610 IF(NGS)305,305,304
304 CONTINUE
303 NDISCO=2
XLAMG2=LAMDA(MCTR)-2.*
NGS=1
313 DO 307 NGS=NCUT,MCTR.
TEST=XLAMG2-LAMDA(NGS)
IF(ABS(TEST)-1.E-05)305,305,313
312 IF(NGS)305,305,307
307 CONTINUE
305 LIMLOW=NGS
GO TO 302
306 LIMLOW=NGS-1
302 NBKSP=NGS-LIMLOW
C POSITION TAPE 9 TO READ IN MOMENTS ... LOOKING FOR THE MOMENTS
C CORRESPONDING TO THE LOWER LIMIT OF THE INTEGRAL
C
IF(NBKSP .GT. LIMLOW) GO TO 750
751 DO 752 I=1,NBKSP
752 BACKSPACE 9
GO TO 753
750 REWIND 9
READ(9)NRRIGG,LMAXP,NMAXP
753 READ(9)NSS,((NS(LP,NP),LP=1,LMAXP),NP=1,NMAXP)
IF(NSS,LIMLOW) GO TO 753
CALL POLKER(PO,LAMDA,XLG)
IF(LIMLOW .NE. 1) GO TO 354
DO 355 LP=1,LMAXP
355 PLNK(LP)=PO(LP)
GO TO 310
354 CALL POLKER(PLNK,LAMDA(LIMLOW),XLG)
C C FIND WHERE DELTA LAMDA CHANGE IS THAT IS CLOSEST TO MCTR
C
310 DO 502 ID=1,NCID
IF(MCTR-NCID)317,317,502
502 CONTINUE

159
SUBDIVIDE THE INTEGRAL SO THAT SEGMENTS ARE FORMED WHICH HAVE
NO CHANGE IN STEP SIZE WITHIN THEM AND NO DISCONTINUITIES
WHERESEGS IS THE MESH POINT AT THE END OF SUCH A SEGMENT

317
c
336
319
326
261
263
264
572
562
563
570
600
571
576
566
565
567
568
572
569
539
GO TO 555
LNDSEG=NC
IF (LAMBDA(NGG-1)-LAMBDA(NCC)) > 264,261,252
IF (LAMBDA(NGG-2)-LAMBDA(NCC)) > 264,263,254
NSkip=3
ASSIGN 266 TO NSK
GO TO 572
LSEG=NC
ASSIGN 266 TO NSK
GO TO 572
NSkip=1
ASSIGN 267 TO NSK
LSEG=NC
ASSIGN 266 TO NSK
GO TO 572
IF(NC(ID) - LIMLOW) = 562,562,563
CONTINUE
IF(MCTR- NC) = 571, 70, 570
NCC=NC,ID
GO TO 564
LNDSEG=NC
IF(LSEG-NC) = 561, 560, 560
LNDSEG=NC
GO TO 564
LNDSEG=NC
GO TO 565
CONTINUE
IF(LIMLOW-NCUT) = 311, 314, 311
DO 315 LP=1, LMAXP
DO 315 NP=1, NMAXP
BS(LP, NP) = BS(LP, NP) - DELTAB(LP, NP)
IF(LIMLOW-LSEG) = 569, 568, 569
J=2
NPOINT=4
LNDSEG=MCTR
LIML=LIMLOW+J
DELTA=LAMBDA(LIML)-LAMBDA(LIMLOW)
KLL=LAMBDA(LIMLOW)
GO TO (339, 340), J
NPOINT=LNDSEG-LIMLOW+1
CALCULATE THE CONTRIBUTION TO INTEGRAL FROM THIS SEGMENT
THIS CONTROL SECTION CONTROLS THE COURSE OF THE INTEGRATION OF THIS SEGMENT. IT SETS UP THE PARTICULAR COMBINATION OF INTEGRATION SCHEMES TO BE USED.

340 IF(9-NPOINT) < 209, 209, 210
210 GO TO(250,202,207,204,205,206,207,208),NPOINT
202 ASSIGN 260 TO IST
   GO TO 222
203 ASSIGN 260 TO IST
   GO TO 225
204 ASSIGN 260 TO IST
   GO TO 228
205 ASSIGN 204 TO IST
   GO TO 222
206 ASSIGN 204 TO IST
   GO TO 225
207 ASSIGN 204 TO IST
   GO TO 228
208 ASSIGN 206 TO IST
   GO TO 225
209 LSIMP=NPOINT-3
   IF((NPOINT/2)-2-NPOINT) < 211, 212, 212
211 LSIMP=4
   GO TO 213
212 LSIMP=7
   ASSIGN 213 TO IST
   GO TO 228
213 ASSIGN 259 TO IST
   GO TO 228
259 IF(LSIMP-LLSIMP) < 15, 204, 204
215 ASSIGN 259 TO IST
   LSIMP=LSIMP+2
   GO TO 225

END OF THE CONTROL SECTION FOR A SPECIFIC SEGMENT

THIS SECTION IS USED ONLY AFTER NDLP24 HAS BEEN REACHED.

DELTA=.08
XLL=LAM3DA(NGG-2)
ASSIGN 228 TO IST
ASSIGN 761 TO ITRAP
ASSIGN 764 TO IFR
CALL POKER(PO,XLAMO,XLG)
CALL POKER(PLNK,XLL,XLG)
DO 767 LP=1,LMAXP
DO 769 NP=1,NMAXP
767 9S(LP,NP)=ARRAY(LP,NP,G)

TRAPEZOIDAL RULE

XLM1=XLL
XLL=XLM1+DELTA
IF(ABS(XLL-XLG)-.00001)217,219,218

217 LAST=2
218 DO 271 LP=1,LMAXP
PLNKM1(LP)=PLNK(LP)
DO 271 NP=1,NMAXP

271 BS41(LP,NP)=BS(LP,NP)
CALL POLKER(PLNK,XLL,XLG)
GO TO ITREP, (791,783)

781 G=INC(G)
DO 782 LP=1,LMAXP
DO 782 NP=1,NMAXP

782 BS(LP,NP)=ARRAY(LP,NP,G)
GO TO 250

783 GO TO(220,221),LAST

220 READ(9)NGEF,((BS(LP,NP),LP=1,LMAXP),NP=1,NMAXP)
GO TO 250

221 DO 272 LP=1,LMAXP

C

C AVOID TRUNCATION AT UPPER LIMIT OF INTEGRAL
C (NOTICE THAT THIS DOES NOT APPLY WHEN THE KLEIN-NISHINA FORMULA IS
C BEING USED)
C
IF(XLG.GT.XLNCK) PLNK(LP)=PLNK1(LP)

DO 272 NP=1,NMAXP

272 BS(LP,NP)=0.

250 H=DELTAXLG/2.
DO 349 LP=1,LMAXP
DO 349 NP=1,NMAXP

349 LPNP=LP-NP

350 LPNP=LP-NP

351 INTEG(LP,NP)=INTEG(LP,NP)+H*(PLNKM1(LP)*BS1(LP,NP)/XLM1+PLNK(LP)*
BS(LP,NP)/XLL)

349 CONTINUE
GO TO IST, (204,206,213,259,260,228,342)

C

C SIMPSON'S RULE
C
225 XLM2=XLL
XL41=XLM2+DELTAXLL
XL2=XLM1+DELTAXLL
IF(AABS(XLL-XLG)-.00001)542,543,543

542 LAST=2
543 DO 273 LP=1,LMAXP
PLNKM2(LP)=PLNK(LP)
DO 273 NP=1,NMAXP

273 BS42(LP,NP)=BS(LP,NP)
READ(9)NGEF,((BS41(LP,NP),LP=1,LMAXP),NP=1,NMAXP)
CALL POLKER(PLNK41,XLM1,XLG)
CALL POLKER(PLNK,XLL,XLG)
GO TO(223,274),LAST

223 READ(9)NGEF,((BS(LP,NP),LP=1,LMAXP),NP=1,NMAXP)
GO TO 251
DO 274 LP=1,LMAXP
   IF(XLG .GT. XLG') PLNK(LP)=PLNKM1(LP)
DO 274 NP=1,NMAXP
274 BS(LP,NP)=0.
251 I=-DELTALXLG/3.
DO 346 LP=1,LMAXP
DO 346 NP=1,NMAXP
IF(LP-NP)347,348,346
347 LPNP=LP+NP
IF((LPNP/2) .GT. LP+NP)346,348,348
348 INTEGR1(LP, NP)=ANF(PLNKM2(LP)*BSM2(LP,NP)/XLH2+4.*PLNKM1(LP)*BSM1(LP,NP)/XLH1+PLNK(LP)*BSLP(LP,NP)/XLH)
CONTINUE
GO TO IST, (204,206,213,253,260,228,342)

FOUR POINT RULE

XLH3=XLH
XLH2=XLH3+DELTAL
XLH1=XLH2+DELTAL
XLH=XLH1+DELTAL
IF(ABS(XLH-XLG)-.00001)544,545,545
544 LAST=2
545 DO 275 LP=1,LMAXP
   PLNKM3(LP)=PLNK(LP)
DO 275 NP=1,NMAXP
275 BS3(LP,NP)=BS(LP,NP)
   CALL POLKER(PLNKM3(LP),XLH2,XLG)
   CALL POLKER(PLNKM2(LP),XLH1,XLG)
   CALL POLKER(PLNK(LP),XLH,XLG)
GO TO IFOR, (764,269)
764 G=INC(G)
   G1=INC(G)
   G2=INC(G1)
DO 768 LP=1,LMAXP
DO 768 NP=1,NMAXP
BSM2(LP,NP)=ARRAY(LP,NP,G)
BSM1(LP,NP)=ARRAY(LP,NP,G1)
768 BS(LP,NP)=ARRAY(LP,NP,G2)
G=G2
IF(LAST .EQ. 2) ASSIGN 342 TO IST
GO TO (257,270),LAST
269 READ(9)XGEF,((BSM1(LP,NP),LP=1,LMAXP),NP=1,NMAXP)
GO TO (257,573),LAST
573 GO TO NSK, (266,257)
266 NSKIP=NSKIP-1
   ASSIGN 267 TO NSK
GO TO 259
267 READ(9)XGEF,((BSM1(LP,NP),LP=1,LMAXP),NP=1,NMAXP)
GO TO (258,574),LAST
574 GO TO (258,270),NSKIP
0 NSKIP=NSKIP-1
GO TO 257

163
263 GO TO (226, 227), LAST
226 READ(9) NGF, ((RS(LP, NP), LP=1, LMAXP), NP=1, NMAXP)
     GO TO 252
227 CONTINUE
     DO 276 LP=1, LMAXP
     IF (XLG .GT. XLNCF) PLNK(LP) = PLNM1(LP)
     DO 276 NP=1, NMAXP
276 GS(LP, NP) = 0.
252 H = DELTA * XLG * 3 / 9.
     DO 343 LP=1, LMAXP
     DO 343 NP=1, NMAXP
     IF (LP-NP) 344, 345, 343
344 LPNP = LP+NP
     IF (((LPNP/2) * 2- LPNP) ) 343, 345, 345
345 INTEG(LP, NP) = INTEG(LP, NP) + *(PLNK(LP) * RS3(LP, NP)) / XLM2 + 3. * PLNM2
1(LP) * RS3(LP, NP) / XLM1 + PLNK(LP) * RS(L2P, NP) / XLM1
343 CONTINUE
     GO TO IST, (204, 206, 213, 259, 260, 228, 343)
C IS THIS THE LAST SEGMENT IN THE INTEGRAL...
C
260 GO TO (341, 342), LAST
342 MCTR=MCTR+1
     H = H / XLG
     GO TO 501
341 LIMLOW = LNDSEG
C RETURN TO CALCULATE THE NEXT SEGMENT
C
GO TO 572
C CALCULATE MOMENTS IN THE DESIRED PORTION OF THE L,N PLANE
C
501 LL=0
     NN=0
     L=0
     N=0
20 NP=N+1
     LP=L+1
651 IF(NGG=2) 365, 365, 365
366 DETER = D(LP) * LNA3DA(NGCUT)
     DELTAB(LP, NP) = SPACE(N, L, SIGO, DETER, DELTAB(L, N), DELTB(LP+1, N), SIGMAT(NGCUT))
365 SIGMA = SIGMAT(NGC)
     OTERM = XLG*PO(LP)
     STERM = OTERM + INTEG(LP, NP)
     BSCAT(LP, NP) = SPACE(N, L, SIGO, STERM, BSCAT(L, N), BSCAT(LP+1, N), SIGMA)
1/(1-H*PLNK(LP)/SIGMA)
     BONE(LP, NP) = SPACE(N, L, SIGO, STERM, BONE(L, N), BONE(LP+1, N), SIGMA)
     BONEP(LP, NP) = BSCAT(LP, NP) - BONE(LP, NP)
26 IF(N-NMAX) 16, 17, 17
16 IF(L) 19, 19, 18
18 L=L-1
N=N+1
GO TO 20
LL=LL+1
NN=NN+1
L=LL
N=NN
GO TO 20

C 17 CONTINUE
C USE OF ARRAY FOR MOMENTS STORAGE REDUCES RUN TIME
C IF(NGG .LT. NDLP) GO TO 765
DO 762 P=1,LMAXP
DO 762 NP=1,NMAXP
762 ARRAY(LP,NP,5)=3SCAT(LP,NP)
G=INC(G)
IF(NGG .LT. NDLP?4) GO TO 765
ASSIGN 751 TO IGO
GO TO 766
765 WRITE(9)NGG,((3SCAT(LP,NP),LP=1,LMAXP),NP=1,NMAXP)
766 IF(NGG .LT. NDLP) GO TO 754
NOUT=NOJT+10
WRITE(8)NGG,((0UNEP(LP,NP),LP=1,LMAXP),NP=1,NMAXP)
ENERGY=ACTOR/ENBDATA(NGG)
CONTINUE
754 IF(NGG .LT. NRGEP) GO TO 361
REWIND 8
126 DO 438 NG=1,33
DO 438 IY=1,5
DO 438 LP=1,10
DO 780 NP=1,19
780 A(LP,NP)=0.
438 SUMN(LP,IY,NG)=0.
C $$$$$ BEGIN PHASE II $$$$$
C $$$$$ $$$$$
C RECONSTRUCTION OF THE PHOTON DENSITY DISTRIBUT FROM ITS
C MOMENTS BY THE POLYNOMIAL METHOD
C 624 NRG=NGCUT=(NCUT-1)/10
C $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C NMU= ANGULAR VARIABLES (COS(THETA) )
C NYP= NUMBER OF SPACE POINTS (MEAN FREE PATHS)
C READ(5,1003)(XMU(NU),NU=1,NMU)
READ(5,1004)(Y(IY),IY=1,NYP)
IF(EOF(5), NF, 0.) GO TO 500
C $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
DO 44 MJ=1,NMU
PMU(1,MU)=1.
44 PMU(2,MJ)=X*MU(MU)
412 IND=2
NUNIT=8
C
C UNCOLLIDED PHOTON DENSITY

630 DO 48 IY=1,NYP
   Y1Y=Y(IY)
   VIRGINIY=ZERO*EXP(-Y1Y)*(SIGO/Y1Y)**2/(CLITE*FORPIE)
48 CONTINUE
C
C ONCE-SCATTERED PHOTON DENSITY

C GARB= ZERO*SICO/(CLITE*FORPIE )
CALL FIRST
NGST=NGCUT
ASSIGN 531 TO KST
GO TO 680
631 CONTINUE
DO 710 NG=1,NGCUT
NGG=1+10*NG
ENERGY=FACTOR/LAMBDA(NGG)
710 CONTINUE
C
DO 414 IY=1,NYP
DO 414 NG=1,NGMAX
DO 424 MU=1,NMU
424 ONESCT(IY,NG,MU)=0.
414 SUMN(1,IY,NG)=0.
DO 415 NP=1,NMAXP
415 A(1,NP)=0.
C
408 LSTOP=NSTOP/2
IF(LEND .EQ. 2) LSTOP=0
105 LSTOP=LSTOP+1
NEXT=(NSTOP+2)/2
433 CONTINUE
436 CONTINUE
C
C RECONSTRUCTION OF THE LEGENDRE EXPANSION COEFFICIENTS

DO 140 LP=1,LSTOP
L=L+1
THOL=L+.
XL=L
101 HEND=NEXT-L
105 NSTARP=NSUMNO(LP)
IF(HEND-NSTARP)9,138,188
188 DO 621 NP=NSTARP,HEND
READ(NUNIT) NATGGP,LMAXP,NMAXP
N=NP-1
CALCULATE COEFFICIENTS OF THE EXPANSION POLYNOMIALS

CALL WXPAN

DO 39 NG=1,NGMAX
NGG=1+10*NG

BRING MULTIPLE-SCATTER MOMENTS IN FROM OFF LINE STORAGE

READ(NUNIT)NGS,((BSCAT(LPP,NPP),LPP=1,LMAXP),NPP=1,NMAXP)
IF(NGS-NGG)410,409,409

XLAMG=LAMBDA(NGS)
CALL BAITA
DO 39 IY=1,NYP
YIY=Y(IY)
WSUM=0.
DO 30 I=1,NP
WSUM=WSUM+X(LP,I)*YIY*(I-1)
WPR=BETA*WSUM
30 CONTINUE
SUMN(LP,IY,NG)=SUMN(LP,IY,NG)+WPR
REWIND NUNIT
621 CONTINUE
NSUMNO(LP,NENO)=1

OUTPUT SUMN FOR USE IN CONVERGENCE ANALYSIS

DO 119 NGGG=1,NGMAX
NGG=1+10*NGG
119. CONTINUE
189 CONTINUE
IF(NSTOP-NGMAX)140,120,120
120 CONTINUE
IF(L-1)6,46,45
45 LPJ=2

USING RECURSION RELATION FOR LEGENDRE POLYNOMIALS

DO 47 MJ=1,NMU
TEMP=PMU(2,MU)
PMU(2,MJ)=ALEGRA(XL,XMU(MU),TEMP,PMU(1,MU))
47 PMU(1,MJ)=TEMP
GO TO 117

46 LPJ=LP
117 CONTINUE
DO 742 IY=1,NYP
YIY=Y(IY)

ALPHA=1..... CALCULATING THE WEIGHT FUNCTION

FACT=YIY*ALPHA
WT=SIGOSQ*FACT**(L-2)*ALPHA**2
WT=WT*EXP(-FACT)
DO 411 NG=1,NGMAX
NGG=1+10*NG
WTSUM=WT*SUMN(LP,IY,NG)

* RECONSTRUCTION OF THE ANGULAR DEPENDENCE ... THE FIRST TERM IN
* THIS EXPANSION IS THE ALL-ANGLE DENSITY/(L*PIE)

DO 40 MJ=1,NMU
ONEP(IY,NG,MU)=ONEP(IY,NG,MU)+(TH0L*1.)*WTSUM*PMU(LP,J,MU)/FORPIE
40 CONTINUE

IF(L.EQ.0) SJ4N(IY,NG)=WTSUM
411 CONTINUE
IF(L.NE.0) GO TO 742
TSUM=(LAMBDA(11)-X/LAMO)*SUMN(IY,NG,1)/2.
NG1=1
NG2=2
NGG1=11
NGG2=21

740 DLM=LAMBDA(NGG2)-LAMBDA(NGG1)
TSUM=TSUM+DLM*(SMN(IY,NG,1)/LAMBDA(NGG1)+SUMN(IY,NG,2)/LAMBDA(NGG2))/2.
IF(NG2.EQ.NGMAX) GO TO 741
NG1=NG2
NG2=NG2+1
NGG1=NGG2
NGG2=NGG2+10
GO TO 740

C
C MULTIPLE-SCATTERED PHOTON ENERGY DENSITY

741 ALENGM(IY)=TSM*FACTOR
742 CONTINUE
140 CONTINUE
IF(NSTOP=NMAX) 117,116,116
115 NSTOP=NSTOP+NA0D
NSTOPP=NSTOP+1
GO TO 408
116 CONTINUE

DO 748 IY=1,NYP

C
C COMBINE MULTIPLE-SCATTERED AND ONCE-SCATTERED PHOTON ENERGY
C DENSITIES TO OBTAIN THE ENERGY DENSITY DUE TO ALL SCATTERED PHOTONS

C

748 ALENG(IY)=ALENG(IY)+ALENGM(IY)
C
C CALCULATE UNCOLLIDED ENERGY DENSITY AND ENERGY DENSITY BUILD-UP
C FACTOR

C

DO 760 IY=1,NYP
VIRGE=VIRGIN(IY)+EZERO
760 ALENGM(IY)=(ALENG(IY)+VIRGE)/VIRGE
CONTINUE

989 WRITE(6,1103) (Y(IY),IND=1,NYP)
WRITE(2,1101) (Y(IY),IND=1,NYP)

168
988 WRITE(6,1101)E,F,GM,IND,IND=1,NYP)
989 WRITE(2,1101)E,F
989 WRITE(2,1101)E,F
990 JO=1
744 DO 145 NG=1,NGMAX
745 NGS=1+10*NG
746 XLAGM=LAGM0+1(NG)
747 ENERGY=FACTOR/XLAGM
748 DO 145 J=1,NYP
749 IF(NG LE. NGCUT) SUMN(1,IY,NG)=SUMN(1,IY,NG)+ALANN(IY,NG)
750 CONTINUE
745 GO TO 745
751 DO 145 IY=1,NYP
752 SUMN(1,IY,NG)=SUMN(1,IY,NG)*XLAGM**2/FAC'TOR
753 CONTINUE
754 IF(JO (NE. 1) GO TO 747
755 JO=2
756 GO TO 744
757 CONTINUE
758 ASSIGN 990 TO KST
759 NGST=NGMAX
680 DO 124 NG=1,NGST
681 NGS=1+10*NG
682 ENERGY=FACTOR/LAGM0+1(NG)
683 DO 124 MU=1,NMU
684 ONEP(IY,NG,MU)=ONEP(IY,NG,MU)*FACTOR/(ENERGY**2)
685 CONTINUE
686 GO TO KST,(990,631)
990 REWIND 5
999 CONTINUE
999 WRITE(3,2013)
1000 FORMAT(6,2013)
500 STOP
C
1100 FORMAT(F5.1)
1101 FORMAT(F6.4)
1102 FORMAT(14 FOR,F7.1,12H KEV,1 MFP= ,F7.3,7H METERS)
1103 FORMAT(3X,15HLOAD UP FACTORS,1,10X,11HENERGY(KEV),14X,27H'1
3REE-PATHS FROM SOURCE,1/21X,5(5X,F6.3))
1104 FORMAT(13X,F6.1,4X,5(2X,F9.4))
1105 FORMAT(15H THIS RUN IS AT,F7.0,7H METERS)
C
1000 FORMAT(2(E14.7,2Y))
1001 FORMAT(3(I2,5,2X))
1002 FORMAT(3(E12.5,1X))
1003 FORMAT(12(F5.2,1X))
1004 FORMAT(10F7.3)
1005 FORMAT(E12.5)
1007 FORMAT(I2)
1010 FORMAT(2A6,4X,F4.0,3X,F3.0,3X,E13.7)
1011 FORMAT(9I4)
1012 FORMAT(6,4,7I4)
SUBROUTINE SIG(ENERGY, SIGMA)
COMMON PHOT(2,40), ENERG(3,40), INCO(3,40), FTA(3,17), CPK(3,17), ON(3),
ONZ(3), NXS, NEL, TOLE, TOLK, PIE, ZERO, LMAX, XMUMU, XMULAM, TEMP, Y1Y, DH
2U, SIGMA, SINTHL, SGO, ONF, XLNCK, LNP, LP, YLANG, BETA, ZERO,
3TRANS, NNMU, XLAMO, CARB, NYP, NCUT, FACTOR, 3SCAT(10,19),
5SIGMAT(331), LAMDA(331), ONESCT(5,33,21), ALANG1(5,33), ALEN5(5), XMU(621), Y(5), PMU(2,21), VIRGIN(5), ALEN5(5), A(10,19)
REAL INCO, INCOH

C ARITHMETIC STATEMENT FUNCTION FOR LOG-LOG INTERPOLATION
C
ALINTP(X, X1, Y1, X2, Y2) = EXP(ALOG(Y1) + ALOG(X/X1) * ALOG(Y2/Y1) / ALOG(X2/X1))
C
SIGMA = 0.
DO 50 I = 1, NPH
CONVF = DN(I) * 1.0
IF (ENERGY - TOLF) GT 52, 52, 53
52 PH0TO = PH0T(I, 1) * CONVF
INCOH = 0.
GO TO 56
53 DO 66 K = 2, NXS
IF (ABS(ENERGY - ENERG(I, K)) - TOLR) GT 65, 65, 66
65 PH0TO = PH0T(I, K) * CONVF
INCOH = INCOH + CONVF
GO TO 56
66 CONTINUE
67 PH0TO = CONVF * ALINTP(ENERGY, ENERG(I, K - 1), PHOT(I, K - 1), ENERG(I, K), PHOT(I, K))
INCOH = CONVF * ALINTP(ENERGY, ENERG(I, K - 1), INCO(I, K - 1), ENERG(I, K), INCO(I, K))
68 SIGMA = SIGMA + PH0TO + INCOH
RETURN
END

SUBROUTINE AKERN(AKERNL, XLP, XL)
COMMON PHOT(3,40), ENERG(3,40), INCO(3,40), FTA(3,17), CPK(3,17), ON(3),
ONZ(3), NXS, NEL, TOLE, TOLK, PIE, ZERO, LMAX, XMUMU, XMULAM, TEMP, Y1Y, DH
2U, SIGMA, SINTHL, SGO, ONF, XLNCK, LNP, LP, YLANG, BETA, ZERO,
3TRANS, NNMU, XLAMO, CARB, NYP, NCUT, FACTOR, 3SCAT(10,19),
5SIGMAT(331), LAMDA(331), ONESCT(5,33,21), ALANG1(5,33), ALEN5(5), XMU(621), Y(5), PMU(2,21), VIRGIN(5), ALEN5(5), A(10,19)
ALINTP(X, X1, Y1, X2, Y2) = EXP(ALOG(Y1) + ALOG(X/X1) * ALOG(Y2/Y1) / ALOG(X2/X1))
IF(XL-XLP>=0.0001) 85, 65, 6
85 IF(XL!=XLNL) GO TO 5
853 R=XLP/XL
  D=XL-XLP
  DNZK=0.
  DO 854 I=1, NEL
  854 DNZK=DNZK+DN7(I)
  GO TO 53
  5 IF(ABS(XL-XLP)<1.0-05) 6, 6, 53
  AKERNL=0.
  RETURN
  53 R=XLP/XL
  D=XL-XLP
  DO 53(1=NEL
  53 CONTINUE
  53 SUMMY=.137.*SRT(1.+R*(R+2.*D+D))/XLP
  IF(DUMMY-TOLK) 55, 6, 55
  55 DNZK=0.
  DO 54 I=1, NEL
  54 CONTINUE
  54 INTERPOLATION TO FIND THE CORRECTION FACTOR (CAPKAY)
  54 DO K=2,17
  54 IF(ABS(DUMMY-ETA(I,K))<TOLK) 58, 58, 59
  59 CONTINUE
  59 CAP1=CAPK(I,K-1)
  59 CAP2=CAPK(I,K)
  59 IF(CAP1-CAP2)<.9, 61
  61 CAPKAY=ALINTP(SUMMY,ETA(I,K-1),CAP1,ETA(I,K),CAP2)
  61 GO TO 54
  58 CAPKAY=CAPK(I,K)
  58 DNZK=DNZK+DN7(I)*CAPKAY
  58 AKERNL=IE*RZERO R**2*(F+XL/XLP-0.0001)*DNZK
  58 RETURN
  58 END
  58 SUBROUTINE DOLKFP (P, XLAMP, XLAMG)
  58 COMMON PHOT(7,40), ENERG(3,40), INCO(3,40), ETA(3,17), CAPK(3,17), DN(3,1), ONZ(3), NXS, NEL, TOLE, TOLK, PIE, ZERO, LMAX, XMULM, XMULY, TEMP, YTY, DM, 2U, SIGMA, SINHDL, IGO, OFN, XLNCK, L, NLP, NP, XXXXX, BETA, ZERO, 3NTRANS, NHU, XL1, GMU, GMV, RNU, NČUT, FACTOR, 3SCAT(10,19), 5SIGMAT(331), LAMDA(331), ONESCT(5,33,21), ALANG1(5,37), ALEN(5), XMU(621), Y(5), PHI(2,21), VIRGIN(5), ALEN(5), A(10,19)
  58 DIMENSION P(20)
  58 CALL AKERNL(P(1), XLAMP, XLAMG)
  58 ARG=1.+XLAMP-XLAMG
  58 P(2)=P(1)*ARG
  58 DO 184 L=2, LMAX
  184 XL=L
  184 P(L+1)=((2.*XL+1.)*ARG*F(L)-XL*P(L-1))/((XL+1.)**2)
  RETURN
SUBROUTINE ONCE
COMMON PHOT(3,40), ENERG(3,40), INCO(3,40), FTA(3,17), CAPK(3,17), DN(3,1), ONZ(3), NXS, NEL, TOL, TOLK, PIE, RZERO, LMAX, XMUMU, XMULAM, TEMP, YIY, DM 2U, SIGMA5, SINTHL, SIG0, ONE, XLMCK, L, N_P, NP, XLMAG, BETA, ZERO, 3NTRANS, XMU, XLAMO, GAR, NYP, NCUT, FACTOR, BSCT(10,19), 5SIGMAT(331), LAMDA(331), ONESCT(5,33,21), ALANG(5,33), ALENG(5), XMU(621), Y(5), PMU(2,21), VIRGIN(5), ALENGM(5), A(10,19)
XM=XUMU
IF(XMUMU .EQ. 1.) XMUMU=0.9999
IF(XMUMU.LT.XMULAM) GO TO 488
SINTHR=SORT(AM,1,-XMUMU**2)
ONE=TEMP*EXP(-YIY) (SINTHR+SIGMA5*(XMUMU*SINTHL-XMULAM*SINTHR)/SIG0)
1)/(SINTHL)*SINTHR
XMUMU=XM
RETURN
488 ONE=0.
RETURN
END

SUBROUTINE FIRST
COMMON PHOT(3,40), ENERG(3,40), INCO(3,40), FTA(3,17), CAPK(3,17), DN(3,1), ONZ(3), NXS, NEL, TOL, TOLK, PIE, RZERO, LMAX, XMUMU, XMULAM, TEMP, YIY, DM 2U, SIGMA5, SINTHL, SIG0, ONE, XLMCK, L, N_P, NP, XLMAG, BETA, ZERO, 3NTRANS, XMU, XLAMO, GAR, NYP, NCUT, FACTOR, BSCT(10,19), 5SIGMAT(331), LAMDA(331), ONESCT(5,33,21), ALANG(5,33), ALENG(5), XMU(621), Y(5), PMU(2,21), VIRGIN(5), ALENGM(5), A(10,19)
REAL LAMDA
DO 488 IY=1,NYP
YIY=Y(IY)
ALEN=0.
NG=1
NGG=1
NGO=1
ASSIGN 731 TO JON
730 XLAMG=LAMDA(NGG)
SIGMA5=SIGMAT(NGG)
IF(NGG .EQ. 1) XLAMG=XLAMO+.0001
IF(NGG .EQ. NGU) XLAMG=LAMDA(NCUT)-.001
CALL AKERN(XKER, XLAMO, XLAMG)
XMULAM=1.-XLAMG+XLAMO
SINTHL=SORT(LAMDA(1.-XMULAM**2))
TEMP=GBP**XKER/SINTHL
TEMP1=TEMP/YIY
ASSIGN 703 TO JON
ONETOT=0.
MU=1
C CALCULATION OF ANGLE-DEPENDENT ONCE-SCATTERED AND INTEGRATION
C TO FIND ALL-ANGLE DENSITY

CALL ONCE
HON=ONE
701 XMUMU=XHMUMU+DMU
IF(XMUMU.LE.XMULAM) GO TO ION,(703,702)
705 CALL ONCE
ONETOT=ONETOT+DMU*(ONE+HON)/2.
IF(XMUMU.LE.0.99001) DMU=0.001
IF(XMUMU.LE.0.99001) DMU=0.01
IF(XMUMU.LE.0.99001) DMU=0.02
IF(XMUMU.LE.0.99001) DMU=0.10
707 IF(NGG .NE. NGG) GO TO 700
IF(ABS(XMUMU-XMU1) .GT. 0.0001) GO TO 700
ONESCT(IY,NG,MM)=HON/(2.*PIF)
IF(MU.EQ.0. NMU) GO TO 706
MU=MU+1
700 HON=ONE
XMU1=XHMUMU
GO TO 701
703 XMUMU=XHMUMU+DMU
IF(DMU.GT.0.01) DMU=0.01
CHECK=ABS(XMUMU-XMULAM)
IF(CHECK.LE.0.01001) DMU=0.001
IF(CHECK.LE.0.00101) DMU=CHECK
XMUMU=XHMUMU-DMU
IF(DMU.GE.CHECK) ASSIGN 702 TO ION
GO TO 705
702 IF(NGG .NE. NGG) GO TO 498
XMUMU=XMU(MU)
CALL ONCE
GO TO 707
706 ALANG1(IY,NG)=ONETOT*XLAMG**2/FACTOR
C C FIND ALL-ANGLE ENERGY DENSITY DUE TO ONCE-SCATTERED PHOTONS
C
NG=NG+1
NGC=NGC+10
498 CONTINUE
GO TO ION,(731,732,734,735,736,737)
731 ASSIGN 732 TO ION
NGG=NGG+1
ONET=ONETOT/XLAMG
DLAM=LAMDA(NGG)-XLAMG
GO TO 730
732 ALEN=ALEN+DLAM*(ONET+ONETOT/XLAMG)/2.
IF(NGG.LT.NTRANS) GO TO 731
733 NGG=NGG+10
ONET1=ONETOT/XLAMG
ASSIGN 734 TO JOY
GO TO 730

734 ASSIGN 735 TO JOY
ONET2=ONETOT/XLAMG
NGG=NGG+10
GO TO 730

735 ASSIGN 735 TO JOY
ONET3=ONETOT/XLAMG
NGG=NGG+10
DLH=LAMDA(NGG)-XLAMG
GO TO 730

736 ASSIGN 737 TO JOY
ALEN=ALEN+ (ONET1+ONET2+ONET3)*ONETOT/XLAMG)*3.*DLM/3.

737 ALEN=ALEN+ (ONET+ONETOT/XLAMG)*DLM/2.
IF(NGG .LT. NCUT) GO TO 738
ALEN(IY)=ALEN*FACTOR
488 CONTINUE
RETURN

2025 FORMAT (1X,E12.5,3X,F6.3)
END

C
C
C
SUBROUTINE WXPN
COMMON PHOT(3,40),ENERG(3,40),INCO(3,40),FTA(3,17),CP(3,17),DN(3,1),DNZ(3),NXS:NEL,TOLE,TOLK,PIE,RZERO,LMAX,XMUMU,XMULAM,TEMP,YIY,OM2U,SIGMA,SINTHL,SGO,ONE,XLNC,LNP,NP,XLAMG,BETA,ZERO,
TRANS,WMU,XLAMO,GARR,NYP,NCUT,FACTOR,RSCAT(10,19),
SIGMAT(331),LAMDA(331),ONESCT(5,33,1),ALANG1(5,33),ALENG(5),XMU(621,Y(5),PHI(7,21),VIRGIN(5),ALENGM(5),A(10,19))
IF(N)41,41,42
41 A(LP,1)=1.
RETURN
42 NIP=N
N=N-1
TWONP=2*NIP
CON=2*(L+N)+1
A(LP,NIP+1)=-A(LP,NIP)/TWONP
IF(N)183,183,185
186 DO 37 I=1,N
IM=I-1
XI=N-IM
II=NIP-IM
37 A(LP,II)=(CON+XI)*A(LP,II)-A(LP,II-1))/TWONP
185 A(LP,1)=CON*A(LP,1)/TWONP
N=N+1
RETURN
2005 FORMAT (1,I3,2X,10E11.4,/) (5X,10E11.4)
SUBROUTINE RITA
COMMON PHOT(3,40), ENERG(3,40), INCO(3,40), EFTA(3,17), CIPX(3,17), DN(3)
1), DNZ(3), NXR, NL, TOL, TC, L, XHUMU, XMULM, TEM, YIY, CM
2), SIGMAS, SINTHL, SIGO, ONE, XLM, 1, 1, NP, XLAMG, BETA, DZERO,
3NTRANS, XMU, XLAMG, CARB, NYP, NCUT, FACTOR, RSCAT(10,19),
5SIGMAT(31), LAMN(31), ONESCT(5,33,21), ALANG(5,33), ALENG(5), XMU(
621), Y(5), PMU(2,21), VIRG(5), ALENG(5), A(10,19)
CLIT=2.997925E+10
SUMJN=0.
DO 36 J=1, NP
J=JP-1
LP2J=LP+J
TERMJ=TRIAL(LP2J-1)*RSCAT(LP,LP2J)
TERMJ=TERMJ/TERMJ(L,J+J+1)
TERMJ=TERMJ/TERMJ(N-J)
TERMJ=TERMJ/TERMJ(J)
JJ=J**2
IF(JJ* 2 .LT. J) GO TO 36
TERMJ=TERMJ
SUMJN=SUMJN+TERMJ
BETA=DZERO*TRIAL(N)*SUMJN/(CLIT*XLAMG)
RETURN
END

FUNCTION SPACE(N,L,SIGO,DETERM,DE1,DE2,SIGH)
XL=L
XN=N
LP=LP+1
XLP=LP
LEND=1
LLND=1
TERM=0.
IF(L-N)550,551, FF1
551 LEND=2
552 IF(L)650,650,651
550 IF(L)650,650,651
650 LLND=2
651 IF(N)133,133,133
133 GO TO (652,653), LLND
652 TERM=XL*(XN+XLP)*DE1
653 GO TO (552,553), LEND
552 TERM=TERM+XL*(XN-XL)*DE2
553 DETERM=DETERM+TERM*SIGO/(XN*(XL+XLP))
134 SPACE=DETERM/SIGH
RETURN
END
FUNCTION INC(K)
K=K+1
IF(K .GT. 25)  K=1
INC=K
RETURN
END

FUNCTION T0RIAL(K)
IF(K)=0,3?,33
32 T0RIAL=1.
RETURN
33 KTOR=1
DO 34 IT=1,K
34 KTOR=KTOR*IT
T0RIAL=KTO1.
RETURN
END
VITA

Gary Michael Kalansky was born on 14 September 1955 in Pittsburgh, Pennsylvania, the son of Michael Kalansky and Helen Jastzemska Kalansky. He graduated with honors from Steel Valley High School, Munhall Pennsylvania in June 1973. He graduated Cum Laude from Duquesne University, Pittsburgh, Pennsylvania in May 1977 receiving a Bachelor of Science in Physics. He received a reserve commission as a Second Lieutenant in June 1977 and entered the Air Force Institute of Technology in August 1977.

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**Title:** X-RAY BUILD-UP FACTORS

**Authors:** Gary M. Kalansky

2nd Lt.

**Abstract:**

This report is a compilation of time integrated x-ray energy build-up factors from a monoenergetic point source in infinite homogeneous air. These factors were computed by the use of PHOTDIS, a moments method computer code, and performed on a CDC 6600. This code was chosen after a literature search and a review of many computer models. Energies from 12 keV to 1000 keV and ranges from 1 mean-free-path to 15 mean-free-paths are considered. All results are presented on semi-log graphs with each graph containing one energy. This program is estimated to have an error of at most 20%. The results are...
compared to Monte Carlo and Discrete Ordinates calculations. Even though the moments calculations do not completely agree with any of the Monte Carlo calculations, the moments calculation agree with the average of the Monte Carlo calculations. A complete derivation of the moments method from the Boltzmann Transport Equation is also included.