TAILORED MODULATION TRANSFER FUNCTION AND THE
APPLICATION TO DUAL BEAM INTERFEROMETRY

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A technique for background suppression for use in Dual Beam Interferometers and other imaging systems is described. Background suppression is achieved by using pairs of optical systems whose modulation transfer functions match at low spatial frequencies but not at the higher frequencies which characterize a target. Various pairs of systems are discussed, and signal-to-RMS-clutter-leakages are calculated.
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1. INTRODUCTION

Visidyne has developed a unique approach to the detection of small sources using systems in which the detector is sized to the system diffraction limit. The technique, Tailored Modulation Transfer Function (TMTF), was developed for use in the Dual Beam Interferometric System (DBIS) background suppression system and also has application to other imaging systems. TMTF essentially consists of subtracting the outputs of two balanced optical systems with predetermined modulation transfer functions. The result is that the contribution to the measured signal from low spatial frequencies is suppressed.

Section 2 of this report analyzes various techniques for tailoring modulation transfer functions.

Section 3 compares the use of TMTF with a diffraction limited system for moving and fixed targets.

Section 4 discusses the application of this technique to the DBIS.

* Patent Pending
2. TAILORED MODULATION TRANSFER FUNCTION (TMTF)

A background suppression technique employing a spatial filter for use in surveillance applications has previously been described. In the original scheme, the lower spatial frequencies in the background were suppressed by subtracting, at each point in the detector array, the signal resulting from a slightly defocussed image of the scene being viewed from that of the corresponding sharply-focussed image. However, in addition to defocussing, there are several other degrees of freedom which can be used to optimize the optical system design. The degrees of freedom include the following.

A. Introducing controlled amounts of spherical aberration.
B. Using annular entrance apertures having central obstructions of various sizes.
C. Using circular entrance apertures of various diameters.
D. Varying the transmittance in the aperture as a function of radius by applying appropriate neutral density coatings to the optical surfaces.
E. Introducing controlled amounts of defocus.

This list has been confined to degrees of freedom which yield point spread functions whose form does not vary with azimuth or field angle. Thus, spherical aberration has been included, while astigmatism and coma have not.

 Ideally, the two modulation transfer functions of the spatial filter should match closely at the lower spatial frequencies, and then diverge sharply just below the spatial frequency range which characterizes a target.

Figures 1 through 5 show various combinations of MTFs in which this type of matching has been attempted.

In Figure 1, which combines the MTF of a system having 0.45 waves of spherical aberration with that of an annulus, it can be seen that the curves match reasonably well at low frequencies. However, the non-zero value of Curve B at the higher frequencies degrades the system contrast in the region which would characterize a target.

In Figure 2, the combination of an annulus with a smaller diameter circular aperture, yields a match that is essentially perfect at lower spatial frequencies, while Curve B does go to zero at the higher frequencies. Figure 3 is for a similar combination, showing how the system MTF (Curve A minus Curve B) can be modified in order to cover various ranges of spatial frequencies.

In Figure 4 two circular apertures of different diameters have been combined. The power within the central peak would be large; however, the low spatial frequencies are not effectively suppressed.

In Figure 5, the diameter of the smaller aperture has been increased and its transmittance has been varied linearly with radius from 1.00 at the center to 0.20 at the edge. The low spatial frequencies are more effectively suppressed than was the case for Figure 4.

Note that, for the systems shown in Figures 2 through 5, the effective collecting areas of the pairs of apertures are not matched. In Figure 2, for instance, the collecting area of the annulus is about 3 times larger than that of the circular aperture. This mismatch would have to be corrected in an actual system. One way to do this would be to enlarge the smaller aperture to the required area, and, at the same time, divide the aperture into three separate circular areas whose corresponding images add incoherently in the image plane. (The three images would add incoherently if, for instance, optical path length changes of different amounts were introduced in each of the three areas of the aperture.) Thus, the
Curve B. MTF of a Circular Aperture of Diameter .465

Curve A. MTF of an Annulus with a 0.52 Central Obstruction. Diameter equals 1.0

A minus B
FIGURE 3

Curve B. MTF of a Circular Aperture of Diameter .621

Curve A. MTF of an Annulus with a 0.38 Central Obstruction. Diameter equals 1.0

A minus B
FIGURE 4

Curve A. MTF of a Circular Aperture of Diameter 1.0

Curve B. MTF of a Circular Aperture of Diameter 0.333

A minus B
collecting area would be increased without changing the MTF. Another way would be to decrease the transmittance of the annular aperture to .33 with a neutral density filter. In a large aperture system, however, this would be a wasteful use of the polished area.

Figures 1 through 5 demonstrate that there are sufficient degrees of freedom available to allow extensive manipulation in the optical design process. It is reasonable to expect that, with further work, a system which combines the effective low frequency suppression of the system shown in Figure 3 with the high efficiency (large fraction of power in the central peak of the diffraction pattern) of the system shown in Figure 5 can be found.

Additional Point Spread Functions

An effort has been made to devise an optical system whose MTF could quickly and conveniently be changed while maintaining a fixed collecting area. That is, pairs of systems were sought in which one member of the pair could be transformed into the other member, and then back again, with minimum mechanical disturbance to the instrument. Two additional degrees of freedom were found useful here.

A. Introducing controlled amounts of optical path difference in various zones of the aperture.

B. Producing small oscillatory movements of the image with respect to the detector face at frequencies high compared to the frame rate of the detector system.

The first degree of freedom can be implemented by mounting the desired zone of one of the optical components, such as a zone of the primary mirror, on a system of transducers. Since the desired optical path length

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S. D. Gupta and Mahipal Singh, Optica Acta 21, 1974, 737-753
changes are only a fraction of a wavelength, these transducers could be piezoelectric crystals. By varying the high voltage across the crystals, the magnitude of the path length change can readily be varied.

The effects of small, oscillatory motions of the image with respect to the detector array can be understood by realizing that the effective point spread function of the system is actually the instantaneous point spread function averaged over the frame time. Then, the effective MTF is the Fourier Transform of this effective point spread function.

A useful point spread function and MTF results from oscillating the instantaneous point spread function in a small circle whose radius is on the order of the radius of the first dark ring. It can be shown that the MTF of this oscillating system \((\text{MTF})_{\text{OSC}}\) is related to the MTF of the stationary system \((\text{MTF})_{\text{S}}\) as follows:

\[
(\text{MTF})_{\text{OSC}} = J_0(2\pi R \omega) (\text{MTF})_{\text{S}}
\]

where \(J_0\) is the zero order Bessel Function, \(R\) is the radius of the circular oscillation, and \(\omega\) is the spatial frequency in cycles per unit length.

The oscillatory motion of the image could be produced by wobbling the secondary mirror of the optical system in pitch and yaw such that its optical axis traces out a small cone. The half angle of the cone would be very small, typically an arc second or less. Thus, this motion could also be produced by piezoelectric drives. A reasonable frequency for this motion would be 1,000 cycles per second.

Figure 6 shows the MTF of an optical system having a 1/2 wave optical path length shift over the central 30% of its aperture matched to a diffraction limited system with a circular oscillation. The radius \(R\) of the oscillation is \(1.03/\omega_{\text{max}}\), where \(\omega_{\text{max}}\) is the cutoff frequency. Note that the MTF of the oscillated system is negative beyond 0.4 \(\omega_{\text{max}}\) so that the net MTF (A minus B) is actually enhanced for higher spatial...
frequencies. The match of the two curves, at low spatial frequencies, however, is not very good.

A modified system with improved matching at low spatial frequencies is shown in Figure 7. Here, the MTF of the oscillated system has been made more linear at low frequencies by introducing a 1/2 wave path length shift over the outer 2% of the radius of the aperture. Also, the radius of oscillation was changed to \( \omega_{0} \). Note that beyond 0.6 \( \omega_{0} \max \) the net MTF of the combined systems is actually higher than that of a diffraction limited system. The profile of the point spread function of the combined systems is shown in Figure 8.

Transforming one of the systems of Figure 7 into the other can be done by minute motions of the optical components, using the systems of transducers. First, the 1/2 wave optical path difference across the central 30% of the primary mirror would be removed by translating this section of the primary by 1/4 wave. Then, the path difference on the outer 2% of the primary would be introduced by translating this zone by 1/4 wave. Finally, the oscillatory motion of the secondary mirror would be begun. The transformation could probably be done in a few milliseconds.

A secondary advantage of pairs of systems of this type is that the effective collecting areas are automatically matched to each other.
FIGURE 6

CURVE A. HALF WAVE PATH DIFFERENCE OVER CENTRAL 30%

CURVE B. OSCILLATED SYSTEM

A MINUS B

\[ \frac{\omega}{\omega_{\text{max}}} \]
FIGURE 7.

CURVE A. HALF WAVE PATH DIFFERENCE OVER CENTRAL 30%

CURVE B. OSCILLATED SYSTEM WITH HALF WAVE PATH DIFFERENCE ON OUTER 2%

MTF

A MINUS B

$\omega / \omega_{max}$
FIGURE 8: PROFILE OF NET POINT SPREAD FUNCTION
3. ANALYSIS OF SPATIAL BACKGROUND SUPPRESSION WITH TMTF

We have evaluated the RMS spatial clutter leakage for the newly de-
vised Visidyne spatial filter Figures 7 and 8 and have compared it to
the spatial clutter leakage for a conventional lens (of the same aperture)
with and without frame-to-frame subtraction. To facilitate comparison,
our analysis follows that of D. Fried (Optical Sciences Company). The
mean square clutter leakage through a $N^{th}$-order frame-to-frame differencing
processor is given by the expression

$$\langle |X_N|^2 \rangle = (2\pi)^{-2} (\Delta x \Delta y)^2 \int \beta(\hat{k}) \left| \tau_{opt}(\hat{k}) \right|^2 \times \left| \tau_s(\hat{k}) \right|^2 \left| \tau_d(\hat{k}) \right|^2 \left| \tau_N(\hat{k}) \right|^2 d\hat{k}$$

Here $\hat{k}$ is spatial frequency (radians/km) as projected on the ground; $\beta(\hat{k})$
is the background clutter power spectrum; $\tau_{opt}(\hat{k})$ is the optical system
transfer function; $\tau_s(\hat{k})$ is a smear transfer function, the value of which
is determined by the line-of-sight motion during an exposure time; $\tau_d(\hat{k})$
is the focal plane transfer function, the exact nature of which is deter-
mined by the detector size $\Delta x \Delta y$; and $\tau_N(\hat{k})$ is the transfer function of a
$N^{th}$-difference processor. These functions are defined by the expressions

$$\beta(\hat{k}) = C_1 \hat{k}^{-(n+1)}$$

where the measured one-dimensional power spectral density is

$$\text{PSD}(k) = C_2 \hat{k}^{-n}$$

$$\tau_s(\hat{k}) = \frac{\sin(\hat{k} \cdot \hat{X})}{\frac{1}{4} \hat{k} \cdot \hat{X}}$$

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where $\mathbf{x}$ is the sensor line-of-sight motion during a single frame-time,

$$
\tau_d(\mathbf{k}) = \begin{bmatrix}
\sin(\frac{1}{2}K_x \Delta_x) \\
\frac{1}{2}K_x \Delta_x \\
\frac{1}{2}K_y \Delta_y \\
\sin(\frac{1}{2}K_y \Delta_y)
\end{bmatrix}
$$

where $\Delta_x$ and $\Delta_y$ are the detector size, x- and y-components, as projected on the ground; and

$$
|\tau_N(\mathbf{k})| = \{\sin(\frac{1}{2}K \cdot \mathbf{x})\}^N
$$

The RMS clutter leakage vs. the line-of-sight motion during a frame-time is then calculated by numerically integrating equation (1). This has been evaluated for four cases: (1) the Visidyne spatial filter and frame-to-frame subtraction; (2) the Visidyne spatial filter alone; (3) a conventional optical system with frame-to-frame subtraction; and (4) a conventional optical system. The frame-to-frame subtraction process, in general, becomes a less effective filter as the line-of-sight motion increases.

The response of these optical systems to the signal from both moving and stationary targets has also been evaluated. For this procedure we convolved a detector element, the point spread function of the optical system, the target shape, the target motion and the line-of-sight detector motion by means of a fast Fourier transform algorithm. The result is expressed mathematically as follows:

$$
S(\mathbf{r}) = (2\pi)^{-2} \Delta_x \Delta_y \int \tau_d(\mathbf{k}) \cdot \tau_{opt}(\mathbf{K}) \cdot \theta(\mathbf{k}) \cdot \tau_v(\mathbf{k}) \cdot \tau_e(\mathbf{k}) \cdot e^{i\mathbf{k} \cdot \mathbf{r}} \, d\mathbf{k}
$$

In this expression $\theta(\mathbf{k})$ and $\tau_v$ are the transforms of the target shape and target motion, respectively; $\mathbf{r}$ is the position of the target at the middle of a frame-time; and all other notations are the same as defined above.
Sample results from this calculation are shown in Figures 9, 10, 11, and 12. In all cases, the target is assumed to be small compared to the footprint size, which, in turn, is chosen to be a square whose dimension is \(0.9 \lambda/D\). (The diffraction "limit" is \(1.2 \lambda/D\).) The target motion and the line-of-sight motion due to pointing jitter were, arbitrarily, made perpendicular to each other.

Each point on the convolution map (Figures 9 through 12) represents the detector response (integrated over a frame time) for various locations of the target at the center of a frame-time. When frame-to-frame subtraction is not involved, the value at the center of the map is the maximum detector response. For frame-to-frame subtraction, it is assumed that the target moves from the center of one pixel to the edge of that pixel in one frame-time. The detector would then have the responses indicated by the squares in Figures 9 through 12. During the subsequent frame, the target moves to an adjacent pixel but still yields a signal in the first pixel equal to the value indicated by the small circles in the figures. Upon subtraction, in the frame-to-frame differencing process, the net signal detected equals the difference in values indicated by the squares and circles. We have carried out this analysis for a variety of detector sizes, a range of line-of-sight motions, and the two optical systems under consideration.

The signal to RMS clutter leakage was then evaluated from Equations (1) and (2) for the four cases: (1) the Visidyne spatial filter and frame-to-frame subtraction; (2) the Visidyne spatial filter alone; (3) a conventional optical system with frame-to-frame subtraction; and (4) a conventional optical system. The results are presented in Figure 13. The ordinate is labeled with relative units only.

Consider first a conventional optical system (4) and the Visidyne spatial filter without frame-to-frame subtraction (2). The RMS clutter leakage and target signal are nearly independent of the line-of-sight motion provided that this motion is small compared to the footprint size. The
Figure 13: Footprint Size

- **I** VISIDyne Spatial Filter w/Subtraction
- **II** VISIDyne Spatial Filter
- **III** Conventional Optical System with Subtraction
- **IV** Conventional Optical System

Footprint = \(0.9 \frac{\lambda}{D}\)

**Target Signal/RMS Clutter Leakage**

- \(10^{-1}\)
- \(10^{0}\)
- \(10^{1}\)
- \(10^{2}\)

**Line-of-Sight Motion**

**Footprint Size**

\(0.01\) to \(1\)
Visidyne spatial filter, which significantly reduces the RMS clutter leakage is shown to be nearly an order of magnitude more sensitive than the conventional optical system (cf. curves II and IV of Figure 13). For both systems a full field-of-view of 2000 x 2000 pixels was assumed to allow the integral (1) to converge. (Our calculations depend only logarithmically on this value.) For line-of-sight motions comparable to or larger than the footprint size, the target signal is degraded because it spends only a part of the frame-time within a pixel.

For frame-to-frame subtraction, the RMS clutter leakage generally increases (linearly) as the line-of-sight motion increases from zero to the footprint size (curves I and III of Figure 13). For still larger line-of-sight motion, the clutter leakage begins to decrease because of the spatial filtering action of the term \( \tau_s \) in Equation (1). However, for large line-of-sight motions, the signal decreases faster than the RMS clutter leakage, and hence, the signal-to-noise ratio continues to decline. The Visidyne filter with frame-to-frame subtraction is generally found to be about a factor of 2 more sensitive than the conventional optical system with subtraction. For larger line-of-sight motions the advantage becomes even greater.

The above calculations were also repeated for a variety of footprint sizes ranging from 0.9 \( \lambda/D \) to 3 \( \lambda/D \). The basic advantages of the proposed TMTF system are maintained throughout this range.
4. ADVANTAGES OF TMTF AND ITS APPLICATION TO DUAL BEAM INTERFEROMETRY

TMTF is a technique for real time background suppression in a diffraction limited optical system. More conventional techniques (e.g., chopping) cannot be used in a diffraction limited system since, by definition, the image of the target is comparable in size to the detector element. Secondly, TMTF does not use the transit time of the target through the footprint as the initial target discriminant. In many cases of interest, the principal contribution to the background is clutter leakage produced by the motion of the optical system line of sight. For these cases, the noise or clutter leakage can be reduced by reducing the measurement time. Post detection algorithms can then be used to determine target velocity.

It should be emphasized that TMTF is an imaging concept and, therefore, applicable to multi-element focal plane detector systems.

As discussed previously, TMTF is designed to provide optical suppression in real time of background radiation. This is particularly important when spectroscopic measurements are required. Since sequential subtraction techniques (e.g., frame-to-frame differencing) requires the storage of an amplitude for each frequency interval or the equivalent and the number of frequency bins for a given system could easily range from $10^2$ to $10^3$, real time background suppression provides significant reduction in complexity and cost in a spectroscopic or interferometric system.

An additional advantage of real time background suppression in DBIS is that it reduces the dynamic range required for a specified measurement accuracy, since the suppressed background does not contribute to the signal at zero retardation.

When used in a dual output DBIS, TMTF suppresses both spatial structure and temporal structure associated with large scale spatial frequencies. As shown in Section 2, the range of spatial frequencies to be
suppressed is one of the parameters that can be varied in the optical design. Figures 4.1 and 4.2 summarize the advantages of TMTF when used with an imaging system and with DBIS.
ADVANTAGES OF TMTF IN IMAGING SYSTEM

Frame Time is Independent of Target Velocity and Can be Selected to Reduce Clutter Leakage.

Technique is Independent of Target Velocity and Can be Used for Both Stationary and Moving Targets.

Technique is Applicable to Diffraction Limited Optical Systems.
ADVANTAGES OF TMTF IN DUAL BEAM INTERFEROMETRY (DBI)

Frame Time is Independent of Target Velocity and Can be Selected to Reduce Clutter Leakage.

Technique is Independent of Target Velocity and Can be Used for Both Stationary and Moving Targets.

Background Optical Suppression Reduces the Dynamic Range Requirements of the Detection Electronics.

Background Suppression is Done Optically and in Real Time, Thus Minimizing the System Post Detection Data Processing Requirements.

DBI Output Signals Contain Target Spectral, Temporal, and Positional Information.

Technique is Applicable to Diffraction Limited Optical Systems.

DBI is an Imaging System; Therefore, the Technique is Applicable to Multi-Element Focal Plane Detection Systems.

FIGURE 4.2