FINITE ELEMENT ANALYSIS OF A STEEL CANISTER PERFORATING A STEEL PLATE

by

Jan C. Schulz
Detonation Physics Division
Research Department

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NAVAL WEAPONS CENTER
China Lake, California 93555

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Finite element analysis of the transient response of thin-walled structures subjected to impact loadings becomes more complicated when a lack of axisymmetry requires a three-dimensional treatment. In such cases careful modeling is needed to provide sufficient solution accuracy while at the same time keeping computer costs down. In this report the analysis of one such structure, a cylindrical steel canister with a large circular hole in its side, is described. The problem of the canister perforating a steel plate from either normal or oblique angles is considered. SAP IV, a general purpose linear elastic structural analysis code was used. Both static and dynamic analyses were conducted.

This problem was solved as a demonstration of analysis capabilities in this area. The stress and deformation results obtained are interpreted in terms of one-dimensional theory. An assessment is made of the weakening effects of the hole and of the effectiveness of a reinforcing ring and cover plate in reducing these effects. In the course of this analysis, some shortcomings were found in the model used. These are described and suggestions for improvements and further work are given.
INTRODUCTION

Finite element analysis of the transient response of thin-walled structures subjected to impact loadings becomes more complicated when a lack of axisymmetry requires a three-dimensional treatment. In such cases careful modeling is needed to provide sufficient solution accuracy while at the same time keeping computer costs down. In this report the analysis of one such structure, a cylindrical steel canister with a large circular hole in its side, is described. The problem of the canister perforating a steel plate from either normal or oblique angles is considered. SAP IV, a general purpose linear elastic structural analysis code was used. Both static and dynamic analyses were conducted.

This problem was solved as a demonstration of analysis capabilities in this area. The stress and deformation results obtained are interpreted in terms of one-dimensional theory. An assessment is made of the weakening effects of the hole and of the effectiveness of a reinforcing ring and cover plate in reducing these effects. In the course of this analysis, some shortcomings were found in the model used. These are described and suggestions for improvements and further work are given.

DESCRIPTION OF PROBLEM

The canister shown in Figure 1 is a steel cylinder 13.5 inches (0.343 m) in diameter, 37.6 inches (0.955 m) long, and 0.5 inch (0.0127 m) thick. The front end is closed by an end plate 1.5 inches (0.0381 m) thick, while the rear end is closed by an end plate 0.5 inch (0.0127 m) thick. An 8.25-inch-(0.210 m) inside diameter hole with its center 15.75 inches (0.400 m) from the front end is cut in the side of the canister. The edge of this hole is reinforced by a steel reinforcing ring, a cross-sectional view of which is shown in Section B-B. The hole is closed by a 1-inch-(0.0254 m) thick steel cover plate (not shown on the drawing), which is bolted to the reinforcing ring.

This canister, traveling at a speed of 1,000 ft/sec (305 m/s), impacts and perforates a 0.5-inch-(0.0127 m) thick steel plate. Impact may occur at either zero or 10 degrees obliquity. In the latter case, it is assumed that the edge on the hole side impacts first. The elastic properties of both the plate and the canister are taken as: Young's modulus $E = 30 \times 10^6$ psi ($2 \times 10^{11}$ N/m²) and Poisson's ratio $\nu = 0.3$. 
FIGURE 1. Canister Geometry.
The problem is to determine the stresses and deformations in the canister as a function of time after initial impact. The time period of interest is taken as the time required for a disturbance to propagate from the front end of the canister to the rear end, and back to the vicinity of the hole (about 250 microseconds). This fairly short time period was dictated primarily by run cost considerations. In some practical situations a longer time solution might be desired.

**FINITE ELEMENT MODEL**

The canister is symmetrical about a plane containing the cylinder axis and the center of the hole. Hence, it is necessary to consider only one of the halves formed by this plane in the analysis. One such half, rolled out flat and with the finite element grid indicated on it, is shown in Figure 2. The important features of the model are summarized below:

1. The cylindrical part of the canister is modeled with 138 quadrilateral thin shell elements.
2. The reinforcing ring is modeled with 12 three-dimensional beam elements connecting adjacent nodes around the periphery of the hole.
3. The cover plate is modeled with three long slender thin shell elements. This model cover plate can carry considerable load in the longitudinal direction where the loading is primarily compressive. However, because it is not attached to the lateral edge of the hole, it cannot transmit load in the transverse direction. This rather crude representation of the cover plate was intended to take into consideration uncertainties as to the ability of the attachment bolts to transmit tensile stresses between the ring and cover plate thereby preventing a gap from opening between them in the presence of tensile loads.
4. The thick end plate at the front of the canister is assumed to be rigid. This condition is enforced by means of 9 three-dimensional beam elements connecting adjacent nodes around the front end of the canister. These beam elements are an order of magnitude stiffer than the rest of the structure and prevent any change in shape of the front circumference.
5. The mass of the end plate is accounted for by means of 10 concentrated masses attached to the nodes around the front circumference.
6. During perforation of the plate, impact loads are applied to the front end of the canister. These loads are included as concentrated time-varying forces in the longitudinal direction applied to the nodes around the front circumference. The determination of the magnitude and duration of these forces will be discussed later.
7. The rear end of the canister is assumed to be fixed (built-in). This boundary condition is reasonable in view of the fairly short time period for which a solution is desired, since the fact that the rear
FIGURE 2. Finite Element Model of Canister.
end is not free to move cannot be perceived in the vicinity of the hole during this time. If a longer time solution were desired, a modeling of the rear end plate accounting for its inertial and elastic behavior would be required.

STATIC ANALYSIS

To gain confidence in the model and also to obtain results for later comparison with dynamic results, several static runs were made. A load of sufficient magnitude to produce an average longitudinal stress of 100,000 psi (6.9 x 10^8 N/m^2) in an undisturbed cylinder (a cylinder with no hole) was applied to the end of the canister. Three different configurations were considered:

1. Canister with a hole and no reinforcement
2. Canister with a reinforcing ring.
3. Canister with a ring and cover plate

The results are summarized in Figure 3, which is a plot of longitudinal stress versus circumferential distance from the center of the hole at Section A-A on the drawing. The two vertical lines drawn at 5.2 and 6.2 inches (0.132 and 0.157 m) correspond to the inner edge of the ring and the edge of the shell, respectively.

For the case of a canister with a hole and no reinforcement there is a stress concentration at the lateral edge of the hole (point B on the drawing) of about 6.7. That a stress concentration should exist at this point is not unexpected. It may be surprising, however, that its magnitude is so high. For an infinite flat plate with a hole, the stress concentration is only 3. The higher value in the present case results partly from the fact that the plate (actually a cylinder) is of finite width and partly from the presence of the rigid end plate which rotates under load and alters the stress distribution in the cylinder.

Addition of a reinforcing ring reduces the stress concentration substantially, to a value of 4. However, high stresses are produced in the ring itself leading to a stress concentration at the inner edge of the ring of 6.2. A large part of this stress is due to bending. In this static case, therefore, the effect of the reinforcing ring is to shift the maximum stress from the shell to the ring without lowering the overall stress level very much.

The addition of a cover plate completely removes the stress concentration at the lateral edge of the hole. The maximum stress now occurs at point C at the front edge of the hole where the concentration is only about 1.1. This behavior is reasonable since the addition of a
FIGURE 3. Canister Problem, Static Solution, 100,000 psi ($6.9 \times 10^8$ N/m$^2$) Average Longitudinal Stress.
cover plate amounts virtually to filling in the hole, in which case there would be no stress concentration at all.

DYNAMIC LOADING

During perforation of the plate, the front end of the canister is subjected to a complex system of forces which cannot be determined precisely. However, by making some simplifying assumptions a reasonable approximate loading can be derived. The nature of this loading depends on whether the impact is at normal or nonnormal obliquity. These two situations are illustrated in Figure 4.

In the case of impact at normal obliquity, the entire front end of the canister contacts the surface of the plate at one instant of time, and the loading is axisymmetric. The forces exerted on the canister are of two types—forces required to shear out a disk of plate material approximately the diameter of the canister, and forces required to accelerate this disk up to the speed of the canister. These latter forces can be shown to dominate. The impulse applied to the canister is \( I = 626 \text{ lb} \cdot \text{sec} \) (2780 N⋅sec). The time during which this impulse is applied can be taken as twice the transit time for a stress disturbance to propagate across the thickness of the plate, or \( T = 5 \) microseconds.

If it is assumed that the impulse is applied as a rectangular pulse, then the force will be \( F = 1.25 \times 10^8 \text{ lb} \) (5.56 \( \times 10^8 \) N). This same force value can be calculated for the impact of two cylindrical rods having the same diameter as the canister and colliding at a relative velocity of 1,000 ft/sec (305 m/s), thus indicating that this force level (and corresponding time scale) are appropriate.

In the case of impact at 10 degrees obliquity, the end of the canister does not contact the plate all at one time. Instead, one edge (the edge nearest the hole) touches first. As the canister continues to move forward, more and more of its end contacts the plate. The result is a band of loading that sweeps across the end of the canister starting on the hole side and ending on the opposite side.

In this case a slightly larger, elliptical-shaped disk of material is punched out. If it is again assumed that the forces required to accelerate this disk up to the speed of the canister are dominant, the total impulse applied to the canister is \( I = 635 \text{ lb} \cdot \text{sec} \) (2820 N⋅sec).

Although this impulse is virtually the same as in the normal obliquity case, the time during which it is applied increases by an order of magnitude. This time, which is the time required for the canister to move forward from its initial contact point to a point where the entire disk has been punched out, is about \( T = 195 \) microseconds.
NWC TM 3345

LOADING

1. NORMAL OBLIQUITY
   I = 626 LB-SEC (2790 N-SEC)
   T = 5 MICROSECONDS
   LOAD APPLIED OVER ENTIRE END PLATE

2. 10 DEGREES OBLIQUITY
   I = 595 LB-SEC (2680 N-SEC)
   T = 166 MICROSECONDS
   LOAD SWEEPS ACROSS END PLATE
   STARTING FROM HOLE SIDE

FIGURE 4. Loads due to impact at zero and 10 degrees obliquity.
The total impulse applied to the canister as a result of the band of loading sweeping across the end plate is represented in the model by a series of concentrated forces applied consecutively to the nodes around the circumference starting on the hole side. If it is assumed that each force acts for 5 microseconds and is proportional to a band of area associated with the node where it is applied, then these forces can be calculated. The magnitudes and arrival times of the forces at each of the nodes are given in Table 1.

**TABLE 1. Nodal Loads for Impact at 10 Degrees Obliquity.**

<table>
<thead>
<tr>
<th>Node</th>
<th>$F$, lb (N) x $10^{-6}$</th>
<th>$t$, sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.07 (0.32)</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>1.74 (7.73)</td>
<td>5.8</td>
</tr>
<tr>
<td>3</td>
<td>5.78 (25.7)</td>
<td>22.3</td>
</tr>
<tr>
<td>4</td>
<td>10.37 (46.2)</td>
<td>47.7</td>
</tr>
<tr>
<td>5</td>
<td>13.37 (59.5)</td>
<td>78.9</td>
</tr>
<tr>
<td>6</td>
<td>13.37 (59.5)</td>
<td>112.1</td>
</tr>
<tr>
<td>7</td>
<td>10.37 (46.2)</td>
<td>143.3</td>
</tr>
<tr>
<td>8</td>
<td>5.78 (25.7)</td>
<td>168.7</td>
</tr>
<tr>
<td>9</td>
<td>1.74 (7.73)</td>
<td>185.3</td>
</tr>
<tr>
<td>10</td>
<td>0.07 (0.32)</td>
<td>191.0</td>
</tr>
</tbody>
</table>

**DYNAMIC ANALYSIS**

Dynamic runs were made for three different configurations. An integration time step of 5 microseconds was used in all cases. This rather large time step, which is equal in magnitude to the loading duration for the normal obliquity case, might be expected to result in inaccurate input of the impulsive loading applied to the end of the canister. However, supplementary runs made with a one-dimensional model indicated that approximately the correct loading was produced.

**CANISTER WITH REINFORCING RING, NORMAL OBLIQUITY**

The first dynamic case considered was a canister with a reinforcing ring impacting at normal obliquity. The results are summarized in Figures 5 through 10.

Figure 5 is a plot of longitudinal stress versus time at four points on the side of the cylinder diametrically opposite the hole.
FIGURE 7. Canister With Reinforcing Ring, Normal Obliquity.
FIGURE 9. Canister with Reinforcing Ring, Normal Obliquity.
FIGURE 10. Canister with Reinforcing Ring, Normal Obliquity.
The curves show a stress pulse traveling down the length of the cylinder. The shape of the pulse at location 9 near the front end is similar to that predicted by one-dimensional rod theory. At locations further down the cylinder, the pulse widens out and its magnitude decreases slightly. This dispersion of the pulse is attributed to transverse inertia effects.

The peak stress in the pulse of about 520,000 psi (3.9 x 10^9 N/m^2) is far above the elastic limit for steel. This means that the actual canister will undergo considerable plastic deformation and that the stress distribution will differ from that given by an elastic analysis. Hence, the results given here are not valid for the problem as originally stated. They are, however, of interest in showing the kinds of results that can be obtained for a problem such as this. Moreover, a scaled-down version of them would be applicable to a canister perforating a thinner plate at a lower impact velocity. They will be discussed in this light.

The stresses in Figure 5 are roughly what one would expect for a canister with no hole. In other words, the effect of the hole is not apparent on this side. On the opposite side, however, the effect of the hole is quite significant. Figure 6 is a plot of longitudinal stress versus time at three points on the side of the cylinder containing the hole. The stress pulse at location 1 (adjacent to the front end) is virtually identical to the stress at location 9 in the previous figure up to a time of 100 microseconds. After this time, the curve takes a sudden dip and the stress changes from compression to tension. The stresses at locations 19 and 37 show similar dips.

This behavior can be explained as follows: The stress pulse due to impact travels down the cylinder to the edge of the hole. This edge is similar to a free edge, and a pulse opposite in sign to the original pulse (i.e., tensile) is reflected off of it. When this reflected pulse reaches location 1, it combines with the original pulse to produce the dip shown. The time required for a stress pulse to travel from location 1 to the edge of the hole and back can be calculated and comes out to be about 100 microseconds. At locations 19 and 37 the time differences between the arrival of the original pulse and the return of the reflected pulse are less because these locations are closer to the edge of the hole.

In the static loading case, high stresses were produced in the shell and ring at the lateral edge of the hole. Hence, it is logical to examine the stresses at these locations. Figure 7 shows the stress at location 61, which is in the shell near the lateral edge of the hole. The contributions of in-plane and bending components (\(\sigma_\phi\) and \(\sigma_M\)), as well as the total stress (\(\sigma_T\)), are shown. Bending makes only a small contribution to the peak stress at this location. Moreover, the peak stress is only slightly greater than the peak stress for the original pulse. In contrast to the static case, there is essentially no stress concentration in the shell.
Figure 8 shows the stress at location 7, which is in the reinforcing ring at the lateral edge of the hole. Again, the in-plane and bending contributions are shown. In this case, bending makes an important contribution to the maximum stress. Moreover, the bending stress increases at later times so as to keep the total stress in the ring high. The stress concentration at this location turns out to be about 1.5, which is considerably lower than for the static loading case. This is not surprising since the hole doesn't see the pressure pulse for very long and probably doesn't have time to fully react to it. Also, the original stress pulse is uniform around the circumference. This is in contrast to the static case where the stress at the front end of the cylinder was significantly higher beneath the hole edge.

In addition to stresses, deformations of the structure have also been determined. Figure 9 is a plot of rotation of the front end versus time. No rotation occurs until about 100 microseconds after impact, the time required for a stress wave to propagate from the end to the edge of the hole and back to the end. Up to this time the axisymmetric loading produces only axial displacement of the end. When the reflected pulse returns, however, axisymmetry of the stress field near the end plate is destroyed and it begins to rotate in the sense indicated on the figure.

Figure 10 shows the changes in dimensions of the hole. The longitudinal diameter begins to decrease after about 50 microseconds, the time for a disturbance to reach the hole. At later times the rate of decrease slows down. The transverse diameter begins to increase after about 75 microseconds and continues to increase nearly linearly for the duration of the solution. These changes in diameter are consistent with the fact that a compressive loading in the longitudinal direction is being applied. Such a loading would be expected to close up the hole in the longitudinal direction and open it up in the transverse direction.

CANISTER WITH RING AND COVER PLATE, NORMAL OBLIQUITY

The second dynamic case considered was a canister with a reinforcing ring and a cover plate impacting at normal obliquity. The results are shown in Figures 11 through 15. These plots are similar to those in the previous section and will not be discussed in as much detail.

Figure 11 is a plot of longitudinal stress versus time at four locations down the side of the cylinder away from the hole. These curves are almost identical with those of Figure 5. There are only minor differences at large times. This confirms the observation made earlier that the stresses on the side away from the hole are not significantly affected by the presence of the hole.
FIGURE 13. Canister With Reinforcing Ring and Cover Plate, Normal Obliquity.
Figure 12 is a plot of stress versus time at three locations on the side of the cylinder containing the hole. The addition of a cover plate produces pronounced changes in the stress patterns on this side of the cylinder. At location 1 the stress is again almost identical to that at location 9 up until a time of 100 microseconds. Then, however, instead of a dip in stress occurring, a second peak of stress occurs.

This can be explained as follows: The cover plate, which is twice as thick as the shell, creates additional stiffness and mass on the far side of the hole. This makes the edge of the hole behave like a fixed edge, and a stress pulse of the same 'sign' (i.e., compressive) is reflected off of it. When this reflected pulse arrives back at location 1, a second compressive peak is created. The curve for location 19 also shows a second peak of stress. These two peaks are closer together because the time required for a pulse to travel from location 19 to the edge of the hole and back is less. At location 37 the original and reflected peaks occur almost simultaneously resulting in a single very high peak.

The cover plate effectively shields the lateral edge of the hole so that the stresses in the shell and ring in this area are lower than elsewhere. Consequently, stresses at these locations are not plotted. The worst stress condition occurs in the shell at location 37. The curve for this location is redrawn in Figure 13 with the in-plane and bending contributions shown. The stress concentration is again about 1.5. As in the static loading case, the addition of a cover plate shifts the worst stress condition from the lateral edge of the hole to the front edge of the hole. In the dynamic case, however, the cover plate does not reduce the magnitude of the stress concentration.

This result may be misleading. The model of the cover plate is rather coarse (especially in the longitudinal direction). The lack of degrees of freedom in this direction will render the model cover plate stiffer than the actual one. This additional stiffness may contribute to the high stress concentration at the front edge of the hole. It is possible that a more detailed model of the cover plate with more elements in the longitudinal direction would yield significantly lower stresses at this point.

Figure 14 shows the rotation of the front end. Again, no rotation occurs until about 100 microseconds. The addition of a cover plate makes the hole side of the cylinder stiffer and more massive so that the end now rotates in the opposite direction to the previous case.

Figure 15 shows the changes in diameter of the hole. The overall behavior is similar to the previous case; however, the magnitude of the dimensional changes are much less (about one-third as much). This is consistent with the presence of the cover plate, which should act to stiffen the hole.
The third dynamic case considered was a canister with a reinforcing ring impacting at 10 degrees obliquity. As previously mentioned, the impact loading at 10 degrees obliquity is considerably different from that at normal obliquity. The time of application is an order of magnitude larger, and the load is not applied over the entire end at once, but sweeps across it starting from the hole side. This produces results that are considerably different in character. These are shown in Figures 16 through 21.

Figure 16 shows the stresses at locations on the hole side of the cylinder. There is no sharp stress pulse in this case; rather, the stress builds up gradually during the more gradual application of the load. The maximum stress is about 260,000 psi (1.79 x 10^9 N/m^2), or about half the peak stress for the normal obliquity cases. This lower stress level is reasonable since the loading at 10 degrees obliquity has virtually the same impulse as at normal obliquity, but is applied over a much longer time period. The oscillations in the curves are probably due to the discontinuous nature of the loading applied to the model. They would not be present in the actual structure where the loading is smoother.

The stresses at locations on the side of the cylinder away from the hole are shown in Figure 17. Because the load is applied only on the hole side at the solution start, the stresses on the other side begin to build up at a later time.

The worst stress in the shell occurs at location 54 at the lateral edge of the hole. This stress is plotted in Figure 18. The worst stress condition in the entire structure occurs in the reinforcing ring at the lateral edge of the hole. This stress is shown in Figure 19. The contribution of bending is substantial. The stress concentration is about 3. (Actually, the peak stress had not been reached at the end of the solution period.) This stress concentration is higher than for the normal obliquity cases. This is not surprising. Because the load is applied over a longer time period, the loading situation can be thought of as approaching the static loading situation where very high stress concentrations were produced.

Figure 20 shows the rotation of the front end. Rotation starts immediately because the loading is no longer axisymmetric. Since load is applied on the hole side first, the end rotates in the sense shown. At later times when load is being applied to the side away from the hole, the front end begins to rotate in the opposite direction.

Figure 21 shows the changes in dimensions of the hole. The curves are qualitatively similar to the curves obtained for the normal obliquity cases. However, the changes are stretched out in time.
FIGURE 10. Canister With Reinforcing Ring, 10 Degrees Obliquity.
FIGURE 19. Cylinder With Reinforcing Ring, 10 Degrees Obliquity.
FIGURE 20. Canister With Reinforcing Ring, 10 Degrees Obliquity.
FIGURE 21. Canister with Reinforcing Ring, 10 Degrees Obliquity.
SUGGESTIONS FOR FURTHER WORK

The finite element analysis described in this report involves a large number of assumptions and approximations. Some of these, as has already been pointed out, are not entirely reasonable or satisfactory. Among areas where improvement is needed are the following:

1. The use of a linear elastic program such as SAP IV for this problem cannot yield reasonable results because the maximum stresses predicted are far above the elastic limit. If the material were allowed to yield, the stress and deformation patterns within the structure would be significantly altered. What is needed is a nonlinear (elastic-plastic) analysis using a program such as ADINA, which is a nonlinear extension of SAP IV. Unfortunately, the nonlinear dynamic analysis results obtained using the version of ADINA implemented at NWC have not been satisfactory. The nonlinear capability to handle this problem is not presently available here.

2. The rather coarse representation of the cover plate was probably not accurate for dynamic analysis. A cover plate model with more elements would probably yield better results. Also, a model taking into better account the method of attachment of the plate would be desirable.

3. Since the rear end of the canister was built in, the present solution was only good for short times (up to 250 microseconds). In the case of oblique impact, where the time of application of the load is almost as long as the solution time, the structure did not have sufficient time to completely react to the loadings. A more realistic end condition (which would not be difficult to impose) would permit longer time solutions.

4. The loading applied to the front end of the model canister was only approximate. Because the actual loading is not known, it is difficult to assess the accuracy of the approximations used. The primary purpose of this analysis was to demonstrate the ability to use finite element techniques to determine the stresses and deformations in the canister resulting from impact. The exact form of the loading applied was not critical for this purpose. In a practical design situation, however, greater attention would have to be paid to the loading. It seems likely that the loading used at 10 degrees obliquity is less reliable than the loading at normal obliquity. In the latter case, the loading is almost a pure impulse (the time of application is almost zero) and the assumptions made in obtaining this impulse seems reasonable. In the 10-degree obliquity case, however, the loading is applied over an appreciable period of time permitting the development of secondary effects, such as bending of the plate, rotation and change in velocity of the canister, and deformation of the canister end plate. These considerations are not included in the loading used here.
CONCLUSIONS

The results of a finite element analysis of a steel canister with a hole in its side perforating a steel plate have been described. SAP IV, a linear elastic structural analysis code, was used and both static and dynamic runs were made. This analysis was undertaken as a demonstration of capability in this area. No experimental or analytical results exist with which to compare the solution. However, the stresses and deformations obtained seem reasonable in terms of one-dimensional theory.

The analysis predicted peak stresses near the hole that far exceed the elastic limit. The effectiveness of a reinforcing ring and cover plate in reducing the stress concentration due to the presence of the hole has been examined. It was shown that the stress concentration for a canister with reinforcing ring is less in the dynamic case than in the static case. However, the addition of a cover plate does not eliminate the stress concentration in the dynamic case as it does in the static case.