A FRAMEWORK FOR POLICY-LEVEL LOGISTICS MANAGEMENT

Volume III / A Macro Analysis of DoD Logistics Systems

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This report is the final volume of a three-volume study entitled "A Macro Analysis of DoD Logistics Systems", initiated in July 1976 by the Assistant Secretary of Defense (Installations and Logistics). The intent of the study was to examine DoD logistics from a macro point of view so as to understand OSD's policy role and provide useful tools for exercising it. Volume I, Logistics Systems in the Department of Defense, describes the Services' logistics systems, their organizational structures, management practices, and accounting, budgeting and reporting systems. Volume II, Structure and Analysis of the Air Force Logistics System, presents the results of a thorough search for aggregate indicators of performance, productivity and readiness that could provide a macro overview of the Air Force logistics system. This report, Volume III, addresses the problems of analyzing, interpreting and augmenting aggregate management indicators. The result is a conceptual framework that places those indicators in the context of the policy-level management role.

We use the word "framework" to refer to an abstract structure for thinking through policy-level management problems. This structure raises methodological questions about the information and analytical needs of policy management. Our intent is to stimulate discussion of some very complex conceptual issues. We believe that the work presented here represents the state of the art in quantitative analysis, modeling methodology, and management processes as applied to policy-making and long-range planning in DoD. Implementation of the framework could take numerous forms, each of which would require some further research.
Many individuals within LMI and DoD have offered useful comments and insights throughout this study. It would be impossible to list all of them, but we would like to acknowledge the constructive criticism and encouragement of four in particular: Mr. Everett Pyatt, now Principal Deputy Assistant Secretary of the Navy for Logistics; Mr. Oscar Goldfarb, Deputy for Supply and Maintenance in the Office of the Deputy Assistant Secretary of the Air Force for Logistics; Mr. Hugh McCullough and Dr. John Bennett, both former Principal Deputy Assistant Secretaries of Defense for Installations and Logistics, who have experienced the problems of policy-level management first-hand. However, we assume full responsibility for the content of this report.
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I. INTRODUCTION

OSD LOGISTICS MANAGEMENT ROLE

The Office of the Secretary of Defense (OSD) has overall management responsibility for the DoD. The magnitude of this responsibility can be gauged by such impressive statistics as an annual budget in excess of 125 billion dollars, 4.5 million military and civilian employees, 20,000 complex military aircraft, and over 500 major installations.

To facilitate the management task, such a large establishment must be broken down into smaller segments. Thus, the DoD is divided into three large Military Departments, each of which is responsible for an almost equal share of the defense budget. However, OSD retains full responsibility and authority for all resources assigned to DoD.

We have chosen to view the Secretary of Defense as the chairman or Chief Executive Officer of a large, integrated organization, and OSD as the Office of the Chairman. The Military Departments then represent three major operating divisions of the organization. In this sense, OSD is policy-level management, and the Military Departments are operational-level management. OSD's role is essentially policy formulation, resource allocation and performance monitoring for the entire Defense Department. This report analyzes the needs of DoD policy-level management and develops a framework for identifying management tools useful at this level.

The Need for Aggregate Information

Given the size and complexity of DoD, neither the Secretary of
Defense nor his staff can become involved in all, or even many, of its operational details. Still, the Secretary must have an informed perspective on what is occurring so that he can exercise his authority to ensure that the Government's defense objectives are being achieved. The OSD staff can provide such a perspective, but the basic information must come from the Military Departments.

OSD must therefore be able to specify the type of information most helpful to the Secretary. There is a particular need at the policy level for information related to the allocation of resources—to support budget requests submitted to the President and Congress, to apportion appropriated funds, to approve major resource or fund commitments, and to evaluate the defense capability being achieved. The Secretary also needs information to set DoD objectives and develop policy consistent with them.

At the same time, the Secretary is necessarily limited as to the kind and amount of information he can effectively use. He does not have the time to review and digest detailed reports. Because the defense structure is so complex, he cannot possibly consider all the ramifications of every issue. At the policy level, highly aggregate information is needed to maintain visibility over the operational system and to support policy and resource analysis and decision-making.

Such information cannot be a simple accumulation of information from the operational levels. What is provided to policy-level management must be tailored to its purposes, selectively aggregated, and carefully analyzed. The relationships among resources, performance, and policies must be clarified so that OSD management will be able to decide how best to achieve its objective of overall defense capability.
Relating Input to Output

Various efforts have been made to develop a better understanding of how to relate defense output, as represented by measurements of capability, to defense input, as represented by logistics resources and policies. At the policy level, output needs to be defined in highly aggregate terms. But the process of aggregation is complicated by problems in both definition and measurement.

There are numerous reporting systems in DoD that describe output at the aggregate level. For example, the FORSTAT reporting system is intended to describe the capability of individual units in highly aggregate and judgmental terms. Other reporting systems measure the readiness or availability status of individual weapon systems at the unit level and indicate to what extent shortages in logistics resources affect weapon system status. Many of the existing systems can be aggregated for OSD use, and they do depict defense capability. However, such aggregate reporting of output does not give policy-level managers all the information they need.

Input such as manpower, equipment, and spare parts is reported and managed within the Services; various reporting activities supply OSD with this kind of aggregate information. These reports show overall gains and losses, usually in terms of dollars. Such reports are useful for budget and cost analysis, and efforts to determine the costs of particular activities, such as base-level operations and depot maintenance production continue. But again, information on input alone cannot satisfy all the needs of policy-level management.

* The FORSTAT reporting system is described in JCS Publication 6, Volume II.
The policy-level manager's responsibility for resource allocation, policy analyses, and planning requires a reasonable understanding of the relationships between defense input and output. Techniques for understanding these relationships are already used at various levels of operational management. For example, there are techniques for determining spare parts requirements consistent with specified supply performance goals, as measured by expected backorder rates. Similarly, manpower requirements are being computed by simulation techniques relating aircraft operations to maintenance. Thus, the necessary relationships between input and output are being developed and used within specific functional or resource areas. Our concern is how to apply this knowledge at the OSD level.

The solution is not simply to aggregate details; the process would be too time-consuming and still would not produce sufficiently aggregate information. The relationships themselves must be specified in aggregate terms. Thus, output must consist of broad measures of defense capability reflecting the status of combat organizations and supporting activities, and must be expressed in terms appropriate for decision-making at the OSD level. This is because policy-level management has to work with approximations of current status in order to formulate broad operational guidance.

Policy-level management in DoD is affected not only by its approximate perception of the input-output relationships, but also by the complexity of the output itself, which is expressed in terms of "readiness", "availability", or "capability". These are general terms, but when specifically defined, they take on specialized meanings and are therefore no longer the measures of overall defense capability of concern to policy-level management.

One approach used by DoD to overcome this difficulty has been to use detailed measures as proxies for logistics system performance. Thus, supply
fill rates, maintenance man-hours per flying hour, or mean time between failures have been used as measures of logistics system capability. However, it is difficult to translate such detailed statistics into meaningful indicators of defense capability, because the relationships between these two sets of variables are imperfectly understood. Furthermore, volume II of this series demonstrated the difficulty of tracing such relationships and interpreting the measures used in the Air Force. Nonetheless, these detailed statistics reflect the current state of the art in capability assessment; a good example is the Air Force Quarterly World-wide Logistics Report. This report has been designed for the major Air Force commands to use in assessing their wartime support capability and to help Air Force headquarters to determine how it might assist its major commands.

In other instances, output measures such as aircraft availability, defined in specialized terms, have been used to evaluate the impact of aircraft spares expenditures and stockage policy. But because such applications usually reflect only parts of overall defense capability, the results can offer only a limited perspective to policy-level management. Considerable judgment is necessary to integrate such partial determinations of defense capability into a complete picture of input in relation to output.

The problems of defining output measures reflecting defense capability and relating them precisely to input measures representing resources and policies necessarily mean that any cause-and-effect structure will have to be based on approximations. Furthermore, such a structure must produce information consistent with our concept of the OSD policy management role. The tools described in this report are consistent with these views; they can be used on highly aggregate but carefully structured data to tie broad output measures to resource and policy input.
MANAGEMENT INDICATORS

This study was initially designed to investigate what indicators might be available for use at the policy level of logistics management. This section describes our early efforts to understand the use of management indicators within the DoD and our evaluation of the principal management indicator system developed for OSD. It also summarizes our prior work in the task.

The traditional role of indicators in DoD logistics has been to:

- Assess overall logistics systems performance against some set of goals and standards
- Provide early knowledge of significant changes in system performance, i.e., detect and correct errors
- Improve communications between OSD, OMB, and Congress; between OSD and the Military Services; and within OSD.

But despite the elaborate accounting and reporting systems that were developed to support such indicators, a systematic process for determining the degree to which logistics factors influence both logistics and operational performance has been lacking.

LPMES

The most recent and perhaps best example of an OSD management indicator system was the Logistics Performance Measurement and Evaluation System (LPMES). LPMES was a quarterly report of the status of key logistics functions relative to specified goals and standards. Initiated in 1969, LPMES was eventually suspended in 1976 because of general dissatisfaction with its results. Having reviewed the concept and performance of LPMES, we feel that there are three principal reasons that could account for dissatisfaction with its usefulness at the OSD level.

First, LPMES had a limited structure along functional lines. The indicators were selected by functional managers for relevance to a particular
function, and not necessarily to fit into an overall representation of logistics capability (see Table 1). The result was that LPMES lacked the kind of systemwide perspective on DoD logistics performance that would be desired by the Assistant Secretary of Defense (Manpower, Reserve Affairs and Logistics) (ASD(MRA&L)).

TABLE 1. INDICATORS USED IN LPMES

Materiel Obligations Outstanding
Minimize Wholesale Item Range
On-Time Pipeline Performance
Item Identification Improvement
Utilization of Long Supply, Excess and Surplus Property
Stock Availability
Letter Contracts
Undefinitized Change Orders
Competition
Small Business
Progress Payments

Containerization

Maintenance Capital Investment
Maintenance Manhours Per Flying Hour
Aircraft Engine Mean Time Between Overhaul
Modification Management
Depot Maintenance Production Cost and Performance
Maintenance Manpower
Depot Maintenance Workload Program

Class I Value Engineering Change Proposals (VECPs) Received
Delays in Ship Deliveries

Second, the imposed goals used to evaluate each indicator were also selected for pertinence to a particular function, and not necessarily for their relationship to other indicators. This suggests that the indicators were oriented to detailed operational management rather than to policy-level needs. The environment of military logistics is a dynamic one and calls for
flexible management response. The goals set by policy managers must be consistent with such an environment. The precise goals imposed by OSD or operational management through LPMES were not.

Third, the LPMES reports were not current by OSD standards. By the time the quarterly reports were published and distributed, the information was from three to six months old. OSD management was reluctant to work with operational data it considered out of date. For much of the data represented, this lack of timeliness was a serious deficiency in LPMES.

Although there was some staff satisfaction with LPMES, particularly from those segments concerned with stressing an operational management focus for OSD, ASD(MRA&L)'s general dissatisfaction with the system led eventually to the request that LMI investigate performance measures and indicators more appropriate to OSD's logistics management role.

An Air Force Example

Our initial response to that request was Phase 1 of this study, a description of significant management aspects of the current DoD logistics system. Phase 2 explored the Air Force logistics system as a test bed for examining and developing management indicators. The primary focus in Phase 2 was the interactions between aircraft operational readiness and logistics system performance, and we relied heavily on "structural" and trend analysis.

By structural analysis, we meant a formal method for defining the logistics system in graphic terms that could potentially be subjected to systematic and quantitative analysis. Our reliance on structural analysis was based on the conviction that it was an appropriate technique for depicting and understanding the basic relationships of functional activities (supply, maintenance, transportation, installations and housing, etc.), organizational levels (base, depot, etc.), and management responsibilities.
Figure 1 served as a departure point for our structural analysis. The figure represents the most macro structure we could devise, and it provided the investigation with a systemwide perspective. It also served the important function of tying resources (the logistics support cycle) to output (the operational cycle), in terms of number of sorties or operationally ready aircraft. When examined at a finer level of detail, this representation of the Air Force logistics system aided our initial investigation of relationships between variables. It was used to guide data collection, develop hypotheses about cause-and-effect relationships, and select indicators relating inputs like maintenance man-hours to outputs like flying hours.

The results of the structural analysis were combined with the techniques of trend analysis in our statistical examination of the data. Trend analysis was used first to identify changes over time in significant logistics variables, such as Operationally Ready (OR) rates, Not Operationally Ready, Supply (NORS) incidents, Not Repairable This Station (NRTS) rates, etc. Next, we examined and aggregated input and output data from various Air Force reporting systems, cutting across several functional areas, and attempted to relate observed changes in one variable over time to expected changes in related variables with or without time lags. The structural analysis helped to guide the selection of related variables.

There were two advantages to this approach. The combined use of structural and trend analysis produced a quick overview of the Air Force logistics system. It also provided revealing insights into the historical movements of major logistics system performance measures and resource trends. However, trends can be misinterpreted as a result of aggregation and oversimplification. Further, it was difficult to establish statistically significant cause-and-effect relationships with trend analysis.
FIGURE 1

AIR FORCE LOGISTICS STRUCTURE
We found that the Air Force data systems were generally quite satisfactory in the coverage and accuracy required by this type of aggregate analysis.

Straightforward identification of cause-and-effect relationships would help make the determination of an appropriate management indicator system much simpler. The dynamic and complex nature of logistic system relationships, however, requires a different perspective on the use of management indicators, specifically, that the indicators used will change depending on the issues being faced by policy-level management.

The problems associated with discerning cause and effect from trend analysis can be illustrated with an example from volume II. Figure 2 shows the OR, NORS, and Not Operationally Ready, Maintenance (NORM), rates for all Air Force aircraft from FY 1970 through FY 1976. Note that the OR rate in 1972 was about 70 percent. The most obvious trend is a steady decrease in the OR rate and a corresponding increase in the NORM rate over the whole period, while the NORS rate remains relatively unchanged. Are these trends truly indicative of a reduced operational capability? At least four factors may be at work.

The first is what we term the "masking" of the NORS rate by the NORM rate, i.e., an aircraft cannot be reported as NORS if it is awaiting or in maintenance, even though it may have a spare parts deficiency. Hence, whenever an aircraft requires both a replacement part and additional maintenance, it is reported as NORM, not NORS. The probable effect of this masking is to understate the true NORS rate.

The terms NORM, NORS, and OR are no longer used in reporting weapon system readiness. As of November 30, 1977, other terminology has been introduced, but the concepts involved are largely the same.
The second factor is a change in NORM reporting as of October 1973. At that time, the Air Force introduced two categories of NORM: NORM-F (flyable) and NORM-G (grounded). Although the definition of NORM did not change, the NORS plus NORM rate increased sharply in FY 1974. It is possible that this modification in reporting, which made the need to better identify NORM aircraft more critical, contributed significantly to that rise.

A third factor is that from FY 1970 through FY 1973 the Air Force phased out the Century series of fighters and some older cargo aircraft; these aircraft typically exhibited high OR rates. Concurrently, new aircraft, such as the F-111 and C-5, were introduced into the inventory. These aircraft have since experienced significantly lower OR rates.

Finally, the latter part of the FY 1970 through FY 1976 period coincided with the termination of U.S. participation in the Vietnam conflict. Thus, the amount of aircraft usage decreased.

It appears, then, that a conclusion about the tendency in operational capability based solely upon examination of OR, NORS, and NORM trends could be misleading. Additional knowledge is needed to sharpen our ability to examine relationships and trends.

The supply area offers another example of the problems of using and interpreting aggregate indicators. While the NORS rate remained fairly constant during FY 1970 through FY 1976, we also found that an associated measure of supply performance, the NORS incident-hour rate, was increasing significantly. Thus, we have inconsistent indications of supply performance from two relevant indicators from two different data reporting systems.

We sought to explore the reasons behind the roughly constant NORS rate and the rising NORS incident-hour rate, i.e., the ratio of NORS incident-hours to aircraft possessed hours. A NORS incident occurs whenever a
part fails and, at the same time, the aircraft is in a NORS condition. NORS incident-hours accrue from the submission of a NORS requisition until termination of the NORS incident condition. The two rates are displayed in Figure 3.

The figure shows that the NORS incident-hour rate has risen sharply, in contrast to the NORS rate. There are several possible reasons why these rates do not coincide. The most apparent is that multiple NORS incidents can and do occur on a NORS aircraft. Thus, the increasing NORS incident-hour rate relative to the NORS rate could be reflecting a tendency toward more NORS incidents per aircraft over time. Another reason could be the masking of NORS conditions by reporting the aircraft NORM. There is a tendency to reclassify an aircraft as NORM after it has been declared NORS. In such cases, the NORS incident-hours accrue until the incident is terminated. Another explanation could be the use of cannibalization to consolidate separate NORS incidents onto a single aircraft. This situation is akin to the multiple occurrence of NORS incidents on a single aircraft, since each NORS incident contributes to the NORS incident hours. The NORS incident data do show a slight increase in the use of cannibalization to terminate NORS incidents, which would support our interpretation of this trend effect.

We also observed other Air Force behavior on terminating NORS that would tend to restrain the NORS rate, specifically the greater use of War Reserve Materiel (WRM) to terminate NORS incidents (Figure 4). WRM usage terminates a NORS condition, thus stopping further NORS aircraft hours but not necessarily NORS incident hours. However, it is clear that such use of WRM does have some effect on emergency capability, and that, unless top management is aware of this practice, it can misinterpret overall capability. Here again, we see the complicated analysis necessary to interpret apparently obvious logistics system trends.
As a result of the Air Force analysis, we felt it necessary to build upon structural and trend analysis by developing a deeper understanding of the relationships between variables, not only drawing upon quantitative analysis of the type we first used, but also by expanding the opportunity to use less formal and more qualitative knowledge, what we will later refer to as "institutional knowledge." To accomplish this, the details of the management processes had to be explored and a conceptual model of management functions built. This model served to distinguish the needs of policy-level management from those of operational management. Those needs were translated into guidelines for selecting management tools.

GUIDELINES

The two criteria we used in developing policy-level management tools were that they should use aggregate information and maintain a systemwide perspective. In addition, the tools selected for discussion exhibit the following characteristics:

- While it would be desirable to quantify cause and effect, it may be impossible to do so directly; the setting of broad bounds on the system may well be the best we can do, utilizing approximations and estimates of variables. Even so, we believe such a viewpoint is compatible with the role of policy-level management in the setting of broad guidelines.

- All too frequently, top managers view analytical techniques as the special province of technical specialists, to be used in behalf of management. The tools we discuss in this report should be applied by policy-level management as part of the planning and policy-making process done at that level, if their use is to be effective.

- Finally, the management tools reviewed here require that their users take as much advantage as possible of the institutional knowledge of logistics managers throughout DoD.

Using these criteria, our objective is to formulate a framework that will identify specific management tools in a functional interrelationship so they may be readily recognized by top-level managers as adaptable to management processes involving heavy reliance on intuition and judgment. Our approach is
three-fold: first, to define the structural nature of the management process; second, to determine some concepts that explain the functioning of that process; and, finally, to investigate what tools are available and needed, and offer some examples of how such techniques can perform together coherently and methodically.

Our approach in this volume is inevitably more abstract than in the previous ones, since we are no longer addressing problems on a simple technical plane. We will therefore attempt to maintain as much consistency as possible in the definition and use of those abstractions.
2. MANAGEMENT SYSTEM CONCEPTS

In this chapter, we discuss the management system concepts and approaches used in Phase 3 of this task. We begin with a description of the different management processes that must be carried out at the various levels of DoD and then consider how our technical contribution can assist OSD in performing its management functions.

THE DOD MANAGEMENT PROCESS

Figure 5 identifies the basic activities of the DoD management process as we have visualized them for this study. There are three major management activities: top-level (institutional) management, operational (technical) management, and the management information system.

The major distinction between top-level and operational management is in the types of problems they face. Top management must usually deal with problems that are vaguely defined, imperfectly understood, and difficult to relate to criteria for identifying and evaluating alternative solutions. (In fact, the choice of the criteria themselves is a major part of the solution process.) We denote such vaguely defined problems as "issues" in Figure 5.

Operational management, on the other hand, is concerned with solving more definite problems for which criteria are more readily available, particularly if top management is performing its policy development role. For our purposes, policy represents a constraint, provided by top management, on the range of solutions available to operational management. In reality, there are numerous gradations between these two extremes, but emphasizing them supplies a context for our concept of DoD management roles.
The management information system is a major connection between top-level and operational management. The term "management information" includes the full range of information flow between these two levels, not only the formal flow provided by the computer-based reporting systems and similar routine communications, but also the informal exchange of information characterized in this study as "institutional knowledge". The issues confronting top management are such that both the description and resolution of them must often depend on informal communication with operational managers.
Top-Level (Institutional) Management

Thus, we see top-level DoD management as being primarily concerned with long-range planning and with identifying policy variables that should guide such planning. The level of defense capability would be one such class of policy variables. Knowledge about these variables comes from the management information system. Changes in them serve to inform top management of potential issues and lead to evaluations and, if necessary, to policy changes. To generate policy alternatives, top management uses relationships between variables that relate input such as resources to output such as defense capability obtained through analysis of information.

Top management's knowledge about such relationships is necessarily limited because of the complexity of the management structure, its constrained ability to absorb detailed information, and the difficulty of establishing cause-and-effect relationships. However, within the limits of its perception and the difficulties of defining appropriate criteria, top management selects policy that serves to constrain operational management. Such policy must often be broad and somewhat vague in its guidance due to the uncertainties that characterize the decision-making environment.

Operational (Technical) Management

Operational management, in contrast to top-level management, deals with problems that are better defined and constrained by guidance from the top level. Such guidance includes goals, policy, and resource limitations. Operational management identifies problems that arise in its efforts to comply with top-level goals, given the policy and resource constraints reflected in relationships between input and output variables.

In some cases, problems can be solved by operational management
without intervention by top-level management. Thus, a redistribution of resources within permissible limits might be one type of operational management action for dealing with an existing or anticipated problem. In other cases, information must be supplied to top-level management to change established guidance for solving identified problems.

Such interaction between levels of management often revolves around reaching a mutual understanding of the status of the operational system, or of the relationships between input and output variables that should be accepted by top-level management. This form of interaction necessarily requires an information flow tailored to facilitate top-level management understanding of its role in effecting solutions. Such understanding might be conveyed through observed changes in management indicators or through improvement in the determination of relationships between variables contained in analyses performed by operational management.

**Management Information System**

The management information system represents all the information flow between top-level and operational management. It includes communications from top management, in the form of guidance and questions, specifically tailored information sent to the policy level by operational management, and data prepared by operational management that the policy manager has asked to see. Thus, the management information system is a composite of many information and communications systems and sources.

The management information system, then, links top-level and operational management. The system contains both formal and informal types of reporting and communication. It receives raw information from the operational level, which it processes to serve both operational and top management uses, and performs analyses directed at defining possible problem areas and at
quantifying relationships among variables. Structuring the management information system to serve the needs of top-level management represents an important aspect of the framework developed in this study.

Management indicators are one major category of information shared by top and operational management and used to identify potential problems. Another category is knowledge of the relationships among indicators. Top management uses such relationships to generate policy and resource alternatives for dealing with likely causes of problems and evaluating the impact of alternative courses of action. These relationships are also used by operational management (possibly in more detail) to identify problems that can be associated with the policy constraints imposed by top management.

It is useful at this point to stress the importance of institutional knowledge to top management decision-making. We visualize the main sources of such knowledge as located at the operational level. Our concept of reducing the need for a heavy formal flow of information between the levels of management depends upon how skillfully top management can draw upon informal, institutionally available information. This means that policy management and operational management must have shared goals and objectives. It also means that access to institutional knowledge must be an established and accepted mode of procedure. We believe the role we have assumed for top-level management as a broad policy-maker will both necessitate and facilitate the effective use of institutional knowledge.

In this section we have differentiated between the two levels represented in our concept of DoD management, and have identified the character of their interactions through the medium of the management information system. In the next section, we examine the use of this information system, particularly in the context of the kinds of management tools represented in our proposed framework for policy-level management.
USE OF INFORMATION IN A MANAGEMENT SYSTEM

Here we view management systems from the perspective of the tools required for transforming raw data into useful information. We set this examination in the context of our investigation of military logistics management systems.

The use of data in a management system reflects the evolution of management tools that has accompanied the maturation of management technology. We illustrate this evolution in Figure 6. At the core of this technology is the raw data, both formal operational data and informal judgmental inputs, supplied to the management information system by operational management. Those data are normally subjected to some type of analysis, the requirements for which are determined by particular management practices. The whole of this technology is embedded in certain management concepts. We will look at the stages in Figure 6 and indicate how the management tools developed by LMI can contribute to each aspect of the management process represented.

In this representation, however, no thorough evaluation of currently accepted management concepts was attempted; instead, certain broad assumptions about organizational roles were made. The principal assumption was that top DoD management should be concerned with broad policy and long-range planning, not specific operational problems.

At the core of the system represented in Figure 6 are the data from the operational systems; if left untreated, the data represent only potential information. The next stage in the evolution of management technology is the application of analytical techniques. Exemplifying that application are the tools we used, such as structural analysis and trend analysis.

Structural analysis represents systematic examination of activities, functions, resources, and material reflecting organizational and hierarchical
Figure 6

Representation of a Management System

1. Assumptions about Organizational Roles
2. Hierarchical Analysis
3. Flow Analysis Resource Analysis
4. Data Base
5. Structural Analysis
6. Event Analysis

Management Concepts
Management Practices
Analytical Techniques
Management Information
relationships. In volume II of this study, we addressed the organization of Air Force logistics activities (Base Supply, Base Maintenance, Central Supply, etc.), functions (operations, supply, maintenance, and transportation), aggregate resources (funding appropriations and manpower), and materials (aircraft, engines, exchangeables, and consumables). From structural analysis, we seek a better understanding of the interrelationships between these elements and their contribution to operational capability.

Trend analysis examines the variations in state or system variables, such as OR rates, exchangeable demands, etc., and attempts to evaluate the causes of such variations by relating them to changes in resource or policy variables. The interpretation of trends requires the judgment of both analysts and managers.

Both structural and trend analysis contribute to the same objective: to develop a quantitative, predictive management tool for relating logistics input to operational output. The analytic knowledge thereby gained should help tailor operational information for use at the policy level.

Volume II also addressed management indicators as a tool, but stopped short of specifying criteria for selecting them. Given the current state of the art, and the complex and dynamic nature of the logistics system, such criteria must be subjective. Hence, managers at all levels must rely on their own judgment as to which indicators are most important. As a tool, management indicators span the first two stages represented in Figure 6, with aspects of both basic management information and moderately advanced analytical techniques.

Our latest analytical efforts include two relatively new applications of available techniques, flow analysis and resource analysis. The reasons for
selecting them are discussed below under "Candidate Management Tools"; chapters 3 and 4 describe them in detail. In broad terms, flow analysis is a graph-theoretic method of describing quantitatively material flows between activities of an operational system. Application of network theory permits extension of the flow analysis to treatment of capacities in the operational system, including maximum flow through the system. Flow analysis allows tradeoffs across the state variables (the flows or capacities) both within and between different activities in the network (e.g., base maintenance or depot maintenance); it can also be extended to clarify the impact of time delays and queues in the system. Resource analysis is akin to, and draws upon, flow analysis; it examines the impact of physical resources (such as spare parts inventories), financial resources, manpower, and management policy on flows and capacities, i.e., the impact of policy and resource variables on state variables.

These analytical techniques approach the limit of strictly quantitative applications within our representation of a management system. Proceeding outward to management practices, we require more qualitative and subjective tools, because of the need to consider management objectives explicitly. Here, the actions of managers are the most far-reaching: long-range planning, policy development, and resource allocation. To accomplish these actions, managers must negotiate and compromise, trading off goals with conflicts. Their decisions cannot be based solely on rigorous optimization tools and formal criteria; they must, instead, reflect shadings of personal priorities and preferences for alternative courses of action. Consequently, top managers need a reliable procedure for evaluating alternative courses of action, a technique that enables them to combine subjective assessments with more concrete analytic techniques. The technique we propose is called hierarchical analysis and will be discussed in detail in chapter 5.
REQUIREMENTS FOR MANAGEMENT TOOLS

Thus far, management activity within DoD has been examined from two perspectives. In the first section of this chapter, we outlined the differences between policy-level and operational-level management, particularly in terms of information requirements. The second section emphasized the different ways management uses information, particularly the role of management tools. One way of relating these two perspectives is to realize that policy-level management necessarily involves more subjective judgment because of its concern with broad issues and longer-range planning, while operational management can apply more analytic approaches, since it functions in a more constrained and directed environment.

In volume II of this study we attempted to bridge policy and operational management with management indicators. On the basis of our understanding of the Air Force logistics structure, we identified significant indicators potentially of interest to OSD. Trend analysis was also used to evaluate the indicators, but its limitations prevented the establishment of cause-and-effect relationships. Furthermore, without adequate criteria for selecting and using indicators it was impossible actually to recommend any to OSD.

The development of such criteria had been originally planned for the third phase of this task. However, as this report demonstrates, our viewpoint has changed. We now believe that because of the dynamic and complex nature of the DoD logistics system, it will be necessary to track many different indicators, not only those pertaining to recognized problems or issues, but also those helpful in identifying issues for policy-level management. The policy-level staff will be responsible for selecting indicators and telling policy managers what the information itself means.
Thus, we recognize that indicators are only one aspect of the necessary interaction between policy and operational management. Our viewpoint has become more comprehensive. First, a management system must have some way of examining relationships between variables that makes use of aggregate data and takes into account interactions in the logistics decision-making process. Second, processes are needed for specifying goals or objectives so as to associate them with resource allocation and policy selection, and for identifying issues that may affect goals, resource allocation, or policy, through monitoring of the operational system.

Thus, management tools should be capable of organizing objectives systematically and quantitatively and make explicit use of judgment at the policy level. These objectives must be represented in ways that then can be used in analytic models. Management tools must also be able to deal with several variables concurrently, in view of the interactive nature of a logistics system. Finally, because policy-level management cannot deal with operational details, management tools have to operate with aggregate information, handle relatively fuzzy types of relationships, and provide outputs reflecting bounds for decision-making rather than specific point estimates or detailed policy solutions. The next section assesses some candidate management tools in terms of these requirements.

CANDIDATE MANAGEMENT TOOLS

In this section, the candidate management tools are briefly reviewed in terms of how well they incorporate multiple objectives, deal with many interacting variables, use aggregate information, and exploit individual judgment within the context of the planning, programming and budgeting process. None of the techniques surveyed meets our needs completely. Instead, we have had to modify and adapt some of them to fit our concept of a DoD policy-level management system.
The techniques surveyed have all been applied with some degree of success in real-world management analysis. Detailed descriptions of these tools are not included; however, when possible, we do cite specific references. More details are included for techniques that are closest to those chosen for our framework.

**Regression Analysis**

Regression analysis is a method for estimating the functional relationship of dependent and independent variables, taking into account statistical variation. Normally, one dependent variable is considered at a time, and certain restrictive assumptions are made about the character of statistical variation. However, regression analysis is a useful method for developing hypotheses about relationships.

**Input-Output Analysis**

Input-output analysis treats simultaneous and interactive relationships among many interdependent variables. However, it usually assumes fixed and proportionate relationships between input and output variables.²

**Goal Programming**

Goal programming is an adaptation of mathematical programming that allows consideration of multiple objectives. Judgment is used in weighting the objectives, but goal programming has the usual limitations of linearity in variables required by linear programming, and fixed and proportionate relationships among variables are assumed.²

**System Dynamics**

System dynamics represents a systems approach to analyzing relationships among variables and allows for the effects of feedback. Differential equations are used, and the technique becomes quite complicated if applied to problems of more than a few variables. The mathematical conditions limit the
flexibility of representing relationships, and the required parameters are often difficult to estimate.³

Utility Theory

Utility theory assumes heavy use of judgment in developing relationships between variables defining the benefits and costs being assessed. Allowances for risk due to uncertainty are incorporated into the relationships. However, utility theory is a highly theoretical technique that has been rigorously developed primarily for relatively well-structured situations. Its adaptation to complex problems necessarily requires relaxation of underlying theoretical assumptions.⁴

Risk Analysis

Risk analysis is related to utility theory and depends heavily on the specification of subjective probabilities and utilities. It has been successfully used in specific situations, but at present is not easily adaptable to the complex decision-making process of planning, programming, and budgeting.⁵

Digraph Analysis

Digraph analysis graphically portrays relationships among variables represented in qualitative and quantitative dimensions. It has been used to analyze complex policy situations similar to those experienced by DoD management on an aggregate level. In some formulations, strict mathematical conditions are imposed to solve for the steady state behavior of the system being modeled, but digraph analysis seems to have primary value to us as a graphical device for representing aggregate relationships and variables in macro analysis.⁶
Hierarchical Analysis

Hierarchical analysis involves the specification of objectives, activities to fulfill them, and policies to guide the activities. Within this hierarchical structure, priorities are assigned to the elements at each level, according to their relative importance in affecting the elements at the next higher level. The priorities are established through pairwise comparisons at each level, and the underlying mathematical theory guarantees preservation of the priorities throughout the hierarchy. Hierarchical analysis is compatible with the decision-making structure developed in this study and has been successfully applied to significant policy studies.

The above descriptions suggest our preferences for policy-level management tools. None of them exactly suits our framework, but we have developed ways to adapt and use them. We need a tool for identifying and displaying relationships among many aggregate variables and for treating inter- and intra-system feedback. Graphical techniques like network and digraph analysis can therefore be useful. The basic concepts of system dynamics, especially the treatment of feedback, are also applicable.

A tool for measuring the effects of policies and resources on the capacities of logistics and operational activities is also necessary. Here again, graphical techniques like digraph analysis provide a point of departure for discovering the relevant relationships and identifying the potential impact. Finally, we visualize the DoD decision-making framework as having a hierarchical structure, so we have selected a form of hierarchical analysis to fulfill the role of organizing and prioritizing logistics concepts, goals, objectives, and policies.
TOOLS SELECTED BY LMI

On the basis of our survey of the above techniques, we have selected three basic tools for use in our analytical framework. They are: flow analysis, resource analysis, and hierarchical analysis. The next three chapters describe these tools in detail; here, we simply introduce them and place them in context.

Flow Analysis

Flow analysis is used to display the relationships between logistics activities, which are nodes on a flow graph, and between the flows and capacities on the arcs connecting these activities. This representation is similar to that in Figure 1. Two activities in an aircraft flow graph might be "Flight Operations" and "Base Aircraft Maintenance". The arc between them would then represent the flow "Number of Aircraft With Unscheduled Failures" per unit time. Flow graphs incorporate feedback considerations and can take advantage of some of the concepts used in system dynamics. We also make use of concepts from network theory such as capacity and maximal flow. Thus, our version of flow analysis is derived from graph analysis, uses elements of system dynamics, and draws upon the algorithms for maximal flow in a network.

Resource Analysis

Resource analysis is also a form of graph analysis used to identify impacts of resources and policies on the state variables (i.e., flow and capacities). Used in this way, resource analysis provides connections between the individual flow graphs and between the flow graphs and hierarchical analysis. The connections are made by having the arcs (flows and capacities) of the flow graphs correspond to one type of node on the impact graphs. Resource and policy variables are connected to these nodes through directed arcs.
These directed arcs, then, represent the relationships between a state variable and resources/policy (control) variables. The priorities developed in hierarchical analysis should to some extent reflect the relationships exposed by the impact graphs.

Hierarchical Analysis

Hierarchical analysis is a direct application of the work of Professor Saaty to our representation of the DoD planning process. Hierarchical analysis is used in our framework to provide an explicit procedure for relating defense objectives to broad logistics concepts and policies. This approach enables policy managers to establish priorities on systemwide objectives and policies. A connection between hierarchical analysis and flow analysis is possible because nodes on the flow graphs (logistics activities) have been included as explicit elements in the hierarchy. We consider this application of hierarchical analysis to DoD policy-level management to be unique.

As can be seen from these brief descriptions, flow and resource analysis fall largely into the category of "analytical techniques" according to our representation of a management system in Figure 6. Both techniques involve the identification and development of relationships between system variables, with the possibility of quantitative expressions representing these relationships. In chapters 3 and 4 we discuss in detail what kinds of quantitative relationships seem appropriate to the highly aggregate and dynamic type of analysis useful to policy-level management. Chapter 4 also presents an algorithmic approach to performing analytic calculations with the flow and resource graphs.

We view hierarchical analysis as falling into the category of "management practices", because development of the hierarchy involves highly informed
judgments about both its structure and the priorities established for the elements within it. Although hierarchical analysis includes explicit algorithms for deriving priorities, the structure itself is more important. Hierarchical analysis is described in detail in chapter 5.

**OVERVIEW OF LMI APPROACH**

Figure 7 is another representation of the management processes described at the beginning of this chapter. Top or institutional management is represented by the two boxes on the right; the logistics system and its operational management are at the bottom; and the management information system is at the left.

![Diagram](image-url)
The top management process thus has two stages. The first is the evaluation of defense strategies for their logistics implications, which leads to the formulation of logistics issues. From this stage comes some minimum set of logistics goals and objectives. The second stage is the evaluation of alternative resource allocations and logistics policies in support of these goals and objectives. Thus, at the end of the second stage, top management will have obtained the resource allocations, policies, and plans within which the logistics system must operate.

The operational system functions within the policy and resource restrictions thus established. Information is then fed back to top management where it is collected and analyzed.

Management indicators are a primary means of providing policy-level management with information on the value of system variables. Flow and resource analyses are designed to expose and clarify the relationships between these variables. Hierarchical analysis assists in the structuring of logistics goals and objectives by providing a mechanism for interactive analysis of logistics issues, taking into account objectives, policies, and resources.
3. FLOW ANALYSIS

STRUCTURAL DESCRIPTION

Chapter 1 presented a macro structure of Air Force logistics (Figure 1); volume II of this study depicted more detailed structures. The choice of these particular structures was dictated to a great extent by their compatibility with existing reporting systems. In this chapter, we expand the Air Force logistics structure to provide a better accounting of system flows and inventories. By developing individual flow graphs for different categories of materiel, we can take advantage of the algorithmic potential of that structure.

Initial Flow Graph Development

The categories of materiel selected for initial flow graph development were aircraft, engines, exchangeable items, and consumable items. Candidates for future development include support equipment, missiles and ammunition, and manpower. We emphasize that the structure developed for Air Force logistics is not readily transferable to Army and Navy operations. Service data reporting systems are constructed very differently, and the data elements are interpreted and measured differently. The missions of each Service, their logistics support structures, and their corresponding management philosophies are sufficiently diverse to defy direct comparison.

We chose the flow graph as the structural representation of Air Force logistics because it focuses attention on the logistics processing of materiel. Examples of flow graphs are presented later in Figures 8 through 11. With a complete flow graph, rates of materiel flow, potential bottlenecks
in the logistics system, and limitations on support capability can be identified. The steady state inventories of aircraft and support materiel can be identified, as well as the transient behavior of these inventories when the system is subjected to perturbations. By linking resources and policies to the flows and capacities of the logistics system or to the states of aircraft in that system, we can evaluate resource and policy tradeoffs for their impact on logistics capability. These impacts are the subject of chapter 4, "Resource Analysis."

**Logistics Activities**

There are two structural elements in a flow graph: nodes and arcs. Nodes are the circles or static points on the flow graph; arcs are the arrows connecting the nodes and indicating movement of materiel from one node to another.

What the nodes and arcs represent depends on the analytical mode in which the flow graph is used. In the *state transition* mode, nodes represent states of the materiel being processed, and arcs represent the transition from one state to another. This mode requires state transition rules or probabilities. In the *flow network* mode, nodes represent logistics or operational activities/locations; arcs represent the flow of materiel from one activity to another. Logistics activities are those places where materiel is physically processed, i.e., inspected, tested, repaired, stored, or distributed.

We have already shown that aggregation is necessary to reduce both the complexity of the flow graph and the quantity of information required to support it at a manageable level. Our selection of aggregate logistics activities (or materiel states) is based on compatibility with existing reporting systems. We identify two functions - maintenance and supply - and two echelons - depot (or central) and organizational (or base). The transportation
function is associated with the flow of materiel from one activity to another (i.e., the transition from one state to another). Other delineations and levels of detail are possible and may be necessary for certain management responsibilities.

The selection of specific activities should ideally be accomplished with the participation of the logistics managers themselves. The flow graph must reflect their conception of the appropriate level of managerial control.

Logistics Flows and Capacities

In our framework, we concentrate on using the flow graph in the flow network mode, as opposed to the state transition mode. In the flow network mode, both a flow, measured in units of materiel per unit time, and a capacity, or a maximum flow, are associated with each arc. The notion of capacity is extremely important in developing a concept of system capability, i.e., readiness. If we could specify capacities for all the arcs, we could calculate a maximum flow in the system. This maximum flow would correspond to a surge or mobilization capability.

In chapter 4, flow and capacity variables represent the "state variables" of the logistics system. They are the variables management wishes to keep within some limits so that the system can adequately perform its function. The performance of the system may then be measured as a rate of flow or a capacity utilization.

Resource and policy variables, on the other hand, represent the "control variables" of the system. They are the variables management manipulates to maintain the state variables within prescribed limits. Measures of productivity and policy compliance are usually expressed in terms of the control variables.
We can also use flows and capacities to identify the logistics objectives corresponding to specified operational objectives. Macro objectives like "increase the capacity of base maintenance to process aircraft," however, tend to be too aggregate and abstract to be a basis for the formulation of logistics policy and resource guidance. A slightly more detailed breakout of objectives can provide management with the degree of concreteness needed to assess the more "macro" objectives properly. These objectives are structured by means of hierarchical analysis, described in chapter 5.

While flows of materiel may exhibit substantial variability at different points in time, they are relatively easy to manage. Capacities, on the other hand, are extremely difficult to measure. In the industrial sector, this does not present the problem it does in the military sector. Economic incentives impel an industrial organization to utilize its capacity to the fullest possible extent, or to reduce or inactivate excess capacity. Because day-to-day operations are very close to capacity, operational measures such as materiel flows are reasonable indicators of the organization's health.

In DoD, on the other hand, the incentives and even the military role itself require maintaining excess capacity. Hence, tools comparable to operating statements, balance sheets, and financial ratios simply are not as useful to top-level DoD management as they are to the board of directors of an industrial organization.

Queues, which can build up in a system, provide a source of measures which can serve as "proxies" for capacities. Backlogs of repair are examples of such proxies. These measures are only proxies because they do not necessarily indicate full capacity utilization or overload. A repair backlog, for example, is sometimes allowed to build up simply because there is no immediate
demand for the item. Also, capacities are dependent on the particular conditions existing at any point in time. Hence, the capability of a logistics system to support a wartime effort will vary from scenario to scenario, and any attempt to measure that capability must consider a variety of possible conditions.

**Aircraft**

A flow graph for Air Force aircraft is shown in Figure 8. Because the structure of the flow graph serves to filter information and hence to facilitate communication between levels of management, it should reflect a reasonable consensus of what constitutes the "macro" logistics system. Again, the selection of a structure should ideally be left to the managers who will be using it. The assessment of alternative structural concepts for DoD logistics is, in fact, an important role of institutional management. Such assessments may establish the need for new policies to change logistics system structures. For example, the establishment of more centralized intermediate maintenance activities, each serving a number of Air Force bases and field organizations, might require a change in the flow graphs. There is nothing unique about the flow graph in Figure 8; it simply represents our perception of the current logistics system.

Each circle, or node, in Figure 8 represents an activity or location at which aircraft are maintained, parked, or flown. The loops at each node are for use in the state transition mode. Each loop represents a pool of aircraft at that location and corresponds to an aircraft state (i.e., in flight, operationally ready, not operationally ready, and in depot maintenance). In the flow network mode, aircraft are on an arc being processed from one location to another. In either mode, all Air Force aircraft are somewhere
on the diagram at any point in time. The dotted arcs represent ways in which the total number of aircraft in the system can change. Hence, the industry node serves as a "source" of aircraft and the attrition node as a "sink."

We intentionally omitted the transportation activity as a separate node or set of nodes, primarily because the length of time an aircraft is in transit is usually small relative to the times it is in other states. In all our current flow graphs, a problem in transportation might be reflected as an increased time between transitions in the state transition mode, or as a decreased flow and capacity in the flow network mode. It may become necessary to account more explicitly for transportation activity if major problems develop there, particularly when the flow graph is used in the state transition mode.

While we have been discussing this flow graph in terms of total Air Force aircraft, it can also be used for aircraft classes (e.g., strategic, tactical, transport, tankers, etc.) and individual aircraft types (e.g., F-4, F-15, A-7, B-52, etc.). The more specialized flow graphs would be appropriate in many cases. When aircraft types and classes are aggregated in a single flow graph, information on deviations within a specific aircraft class or type may be lost.

Engines

Figure 9 is a flow graph for aircraft engines. Again, the loops provide an accounting of engines, that is, engines are either installed on an aircraft, being worked on in maintenance (base or depot), or being stored in a rotatable engine pool (base or central). Engines can also be subdivided into classes, e.g., jet and reciprocating.
Exchangeables

Exchangeable flows are depicted in Figure 10. War Reserve Materiel (WRM) is an important addition to this graph, for WRM can also be used to satisfy peacetime requirements if certain policy criteria are met.

The squares in Figure 10 distinguish aircraft maintenance activities from reparable maintenance. It is at these square nodes that components are removed from and replaced on individual aircraft.

Consumables

The consumable flow graph in Figure 11 is much simpler than the others, primarily due to the absence of component repair.

What is not considered in any of the flow graphs is the impact of flows and capacities in one graph on those in another. Consumable flows affect the maintenance activities of all the other flow graphs. Exchangeable flows affect engine and aircraft maintenance, and engine flows affect aircraft maintenance. These effects will be discussed more explicitly in chapter 4.

ANALYTICAL CAPABILITY OF FLOW GRAPHS

Network Analysis

A primary technique for exploiting the algorithmic potential of the flow graph involves the calculation of the maximum flow in the system. This calculation identifies activities that will be operating at capacity when the system is producing maximum output. Thus, potential bottlenecks can be singled out. For example, suppose analysis showed that the factors limiting aircraft sortie capability were the availability of fuel and the number of trained pilots, rather than the capability of the logistics activities to produce operationally ready aircraft. Management should then concentrate on improving fuel and pilot availability and not on increasing the capacity of
FIGURE II
CONSUMABLE (EOQ) FLOWS

BASE MAINTENANCE

# CONSUMABLES DELIVERED (BASE)

# CONSUMABLES DELIVERED (CENTRAL TO BASE)

CENTRAL SUPPLY

AVERAGE CONSUMABLE INVENTORY (BASE)

# CONSUMABLES REPLENISHED

BASE SUPPLY

AVERAGE CONSUMABLE INVENTORY (CENTRAL)

# CONSUMABLES UTILIZED

WRM CONSUMABLES

INDUSTRY

CONSUMABLES PURCHASED

CONSUMABLES DELIVERED (DEPOT)

DEPOT MAINTENANCE

CONSUMABLE FAILURES (BASE)

CONSUMABLE FAILURES (DEPOT)

AVERAGE CONSUMABLES IN WRM
logistics activities. Naturally other considerations enter into such a decision, for example, the lead time needed to implement capital improvements. These tradeoffs are discussed in the following subsection.

The classic work on network analysis is Ford and Fulkerson. Since then, there have been many other books and articles on the subject, and a number of applications have been attempted. One of the more recent examples is a "Dynamic Flight Student Flow Model" developed by Caves and Wilkinson. The model produces a minimum time-to-train solution. If the costs associated with particular flows are identified, a minimum cost solution for a given level of output can also be calculated.

At a higher level of sophistication, a tool developed by Forrester - system dynamics - goes beyond network analysis. System dynamics exposes the transient and cyclical behavior of a system's variables rather than simply the static, steady-state behavior. These more complex, dynamic characteristics result from the multiple and non-linear feedback occurring among the variables of complex systems. System dynamics generally involves the development of differential equations to describe the system. These equations can then be incorporated into a computer simulation to identify time leads and lags, determine sensitivity to perturbations, and project long-range system stability.

Unfortunately, the development of a reasonable set of differential equations is both difficult and time-consuming. As a result, most applications have been limited to consideration of only a small number of variables. However, some recent applications of system dynamics have simplified the model development by relying on subjective parameters for variables difficult to measure. An example is the work of Killingsworth and Cummings. They develop a relatively simple model and adapt it to the system dynamics simulation language and software package, DYNAMO.
Simulation models can also be developed from the flow graphs in the state transition mode. Such a simulation requires either the specification of transition probabilities at each discrete time unit or the specification of mean transition times. Mission cycles are often modeled in this way.

**Tradeoff Analysis**

Flow graphs can help clarify certain tradeoffs in logistics systems, including those between alternative support concepts and between capital investment and operational expenditures. Additionally, the association of different degrees of uncertainty with various support concepts and with capital investment allows consideration for risk tradeoffs.

Based on our flow graphs, support concept tradeoffs would correspond either to a redesign of the logistics system (i.e., a change in the structure of the flow graph) or a shift in emphasis from one part of the system to another. For example, greater reliance on base maintenance or the elimination of the depot level altogether would represent a more decentralized support concept. A reliability-centered maintenance concept might have an effect on scheduled maintenance and/or overhauls.

Tradeoffs between capital investment and operational expenditures correspond to tradeoffs between increasing system capacities and increasing peacetime flows. These tradeoffs must take into consideration probable wartime scenarios, time perceptions and preferences, and the impact of peacetime operations on mobility and surge capability. Some level of peacetime operations, for example, is necessary to train pilots, develop logistics skills, and simply keep the system "in tune."

The most important considerations in these tradeoffs are, of course, the relative costs and risks of the alternatives. These will be discussed further in chapter 4. Risks arise as a result of difficulties in measuring
capacities and in estimating changes in them. These difficulties stem from
the fact that a capacity represents a system’s potential and is not directly
observable unless the system is operating at capacity.

A system’s potential depends on the conditions under which it will
be asked to respond. The development of scenarios is a typical method used to
establish these conditions. Even when the worst case scenario is used, how-
ever, some assumptions about the conditions (sometimes referred to as exo-
geneous variables) must be made when assessing capacity. One of the roles of
the institutional management of DoD, in fact, ought to be the structuring and
evaluation of these assumptions. Uncertainty with respect to these assump-
tions may make it desirable to select alternatives with less expected benefit
(or utility) simply to avoid the risk associated with other alternatives. The
treatment of risk has been accomplished with a number of techniques. They
range from the simple treatment of Hertz\textsuperscript{6} to the multiple objective, utility
theory approach of Keeney and Raiffa.\textsuperscript{7}

APPLICATION OF FLOW GRAPHS

Measurement of Flows

We can derive a reasonable approximation of the actual flows in the
flow graphs over a given period of time (e.g., a year) from the manipulation
of logistics data. The aircraft flows in Figure 8, for example, could be
calculated with a knowledge of average aircraft inventory, average OR rates,
NOR (Unscheduled) rates, and NOR (Scheduled) rates, aircraft sorties, aircraft
flight hours, aircraft depot hours, average depot processing time per aircraft
(i.e., depot flow time), and average aircraft failures per flight hour.

Other data could be substituted for these measures. We could, for
example, replace the last three data items by the number of aircraft sent to
the depot and the average time an aircraft occupied a NOR (Unscheduled) (i.e., unscheduled maintenance flow time) and NOR (Scheduled) (i.e., scheduled maintenance flow time) status. If significant changes occurred in these measures over the period of concern, the beginning-of-period and end-of-period data would also be valuable. A change in the backlog of aircraft at the depot, for example, would indicate a difference between the number of aircraft sent to the depot and the number returned to operational units.

To demonstrate the type of calculation needed to assign an average flow to each arc of Figure 2, we extracted the following FY 1976 data from our volume II report:

- Aircraft Inventory = 9,289 aircraft
- Sorties = 1,520,000 aircraft
- NORS-G Hours = 4,010,000 hours
- NORM-G (Scheduled) Hours = 5,750,000 hours
- NORM-G (Unscheduled) Hours = 11,490,000 hours
- Depot Hours = 9,430,000 hours

To perform the calculations, we need to assume a percentage rate for the number of sorties that result in a not operationally ready condition. We also need average lengths of time that aircraft remain in scheduled maintenance and in depot maintenance. For our example, we assume:

- Failure Rate = 65% of sorties
- Average Scheduled Maintenance Flow Time = 7 days x 24 hrs./day = 168 hrs./aircraft
- Average Depot Flow Time = 150 days x 24 hrs./day = 3600 hrs./aircraft

The calculations, then, are:

Aircraft Returning from Flight with Failures =

\[0.65 \times 1,520,000 = 980,000 \text{ aircraft/yr.}\]
Aircraft Processed through Scheduled Maintenance = 
\[ 5,750,000 \div 168 = 34,226 \text{ aircraft/yr.} \]

Aircraft Processed by Depot Maintenance = 
\[ 9,430,000 \div 3600 = 2619 \text{ aircraft/yr.} \]

Average Unscheduled Maintenance Flow Time 
\[ 11,490,000 \div 988,000 = 11.63 \text{ hrs./aircraft} \]

Measurement of Capacities

If a significant change in flow occurs between the beginning and the end of a period, the indications are that: (1) a change in capacity occurred at some point in the system; (2) the utilization of that capacity changed; or (3) a change in policy had the effect of reducing capacity. An increase in the backlog of aircraft at the depot, for example, might indicate that the capacity of the depot to process aircraft had been reached, or alternatively, that depot personnel were being utilized for other functions. A backlog, however, should only be regarded as a "proxy" for capacity. A reported backlog often does not reflect a repair workload requirement. This could be a result of the retention of items at that location for insurance purposes, for example.

When logistics operations at an activity reach capacity, a further increase in the rate of aircraft sent to that activity will result in an increase in the average time to process an aircraft there. This increase should be reflected as a difference between the aircraft sent to that activity and the aircraft processed by that activity over the period of concern. When operations are not at capacity, measurements that would indicate a change in capacity are very difficult to obtain.

Capacity measurements are easier to obtain in relatively simple and repetitive operations than in the multi-faceted operations that characterize
logistics activities. But even in these simple operations, attainment of a meaningful measure often means stressing the system to its limit. Operational readiness inspections, for example, are periodically performed to test the capability of selected activities; but as a source of data, inspections are extremely expensive and limited to what can be measured in short periods of time. This is often not satisfactory for determining a sustained capability. Recent war data, when available, can be used in conjunction with a simulation model such as a mission cycle model or perhaps the Air Force Logistics Composite (LCOM) model.

Since approximations are usually sufficient for performing policy functions at the institutional level of management, simple calculations are often adequate for estimating a capacity. Stoller has identified four system parameters that can be used to estimate capacity:

- **Production Rate** - the average output of the system
- **Throughput Time** - the average time required per unit output to process through the system
- **Utilization Rate** - the average fraction of available production time during which productive facilities are actively processing units through the system
- **Waiting Rate** - the average fraction of throughput time during which units are not actively being processed by the system.

Often, estimates of parameters like these are available from models operated by commands within the Military Departments. More sophisticated calculations than those used by Stoller are based on queuing theory and have been developed to estimate sortie capability as a function of maintenance personnel, maintenance team size, and number of teams. A simple example of capacity estimation was used in a 1975 GAO report.
THE SIGNIFICANCE OF INSTITUTIONAL KNOWLEDGE

The point to be made is that, while capacities are very difficult to measure directly, there are proxies, simple calculations, and simulations, which address capacity indirectly. Although the resulting estimates are approximations, they often provide sufficient information for top-level DoD management to perform its policy-making functions. Those functions include the management of excess capacity (sometimes referred to as "slack"). Only by addressing capacity can resources and policies be linked to an overall defense capability.

Because of the problems associated with measuring capacity, greater reliance must be placed on expert managerial judgment - what we have been referring to as institutional knowledge. This is knowledge acquired through individual experience of, and exposure to, the intricacies of logistics systems. A major challenge in developing a policy-level analytical capability is how to exploit the institutional knowledge available. We discuss this challenge further in succeeding chapters.
4. RESOURCE ANALYSIS

STRUCTURAL DESCRIPTION

This chapter describes the impacts of resources and policies on the system flows and capacities represented by the flow graphs in chapter 3. "Resource Analysis" is our term for a tool designed to expose, display, and analyze these impacts. We explain the structure of this tool, the problems in quantifying it, and its potential uses. Specific uses will largely depend on future methodological development, but the basic use of resource analysis is to organize and integrate information about relationships between control variables (resources and policies) and state variables (flows and capacities). This type of information, even if approximate, is essential to top-level DoD involvement in planning, programming, and budgeting, and other policy-making activities.

The Use of Impact Graphs

Traditional methods of treating control variables, particularly resources, include the development of cost-estimating relationships from historical data and the construction of resource allocation models (e.g., mathematical programming), which either minimize cost for a specific goal or maximize benefit for a specific cost. There are two problems in applying this approach to policy-level concerns.

First, the relationships between highly aggregate variables tend to be vague and ill-structured, both intuitively and empirically. This is because the scope of policy-level management is much larger than that addressed by the optimization models just mentioned. Not only does the number of significant variables increase by orders of magnitude, but the variables themselves
are also complexly interrelated. Top-level management, unlike operational management, deals with the whole logistics system and the many management levels within it. Hence, policy-level managers are interested in sociotechnical relationships, rather than simply technical ones. Historical data, however, are not very useful in developing sociotechnical relationships, because human beings can exhibit innovative and adaptive behavior that cannot be inferred directly from historical performance.

The second problem with traditional methods of treating control variables is that the relevant criteria for policy-making are never as simplistic as "minimizing cost" or "maximizing benefit." Resolution of conflicting criteria is a major responsibility of top management. A criterion like "readiness" is extremely vague and involves many interrelated factors. Measures like the OR rates discussed in chapter 1 simply do not capture the total concept. The notion of capacity introduced in chapter 3 was an attempt to address readiness, but capacities are also very difficult to measure directly.

Recognition of these problems led us to a more descriptive approach to the treatment of control variables. This approach focuses on providing a structure that links the control variables to the state variables so as to accommodate approximate, discontinuous, and other ill-structured relationships. Management can use this structure normatively by experimenting interactively with alternative parameter values and individually assessing the results. This interaction can range from using the structure simply to organize one's ideas about a particular policy issue to experimenting with alternative mixes of resources and policies in a computerized version (assuming that sufficient quantification has been introduced).
The impact graph in Figure 12 depicts the structure of resource analysis. In an impact graph, an arrow represents the relationship between a resource/policy variable and a capacity from a flow graph. There may also be impacts from the flows and capacities from other flow graphs. The central circle in Figure 12 represents a capacity; the variables surrounding the capacity are those over which management can exercise some control. We chose, for example, to represent resource variables by broad budget categories. Small shifts in funds allocated to these categories can have a significant effect on the performance and capability of the logistics system.

It is necessary to distinguish policy variables from resource variables, since some policies are difficult to relate directly to resources.
Yet, because they have a significant impact on the system, some policy variables must be included. Examples of such variables are scheduled maintenance, frequency of programmed depot maintenance, and design reliability and maintainability guidelines for newly acquired aircraft. A policy variable, however, is not itself a policy; it only reflects a policy. (We are using the word, "policy," in a very broad context. The distinguishing characteristic of a "policy" is that it has the effect of limiting the alternatives available to operational management.)

There are other variables affecting capacity, such as flows or capacities from other flow graphs, but there are also variables that might best be classified as exogenous. These variables are not directly controllable, but provide the context within which the logistics system and its management operates. Exogenous variables are not included in the impact graphs presented in this chapter. But relationships between control variables and state variables do reflect the impact of exogenous variables. In some policy situations, exogenous variables may be so constraining and critical that they should be explicitly included in the structural representation. This type of variable (e.g., economic stability, cultural change, energy availability, etc.), however, is frequently very difficult to measure.

The crucial, and as yet still unresolved, issue in developing impact graphs is the degree to which the relationships between control variables and state variables can and should be quantified. Even without any quantification, the impact graph does provide a visual representation, which can assist management in a qualitative assessment of the potential impacts of changes in control variables. Impact graphs can also serve as a frame of reference for raising questions and otherwise communicating about logistics management. We discuss quantification problems and the potential uses of quantified impact graphs in the "Analytical Capability" section below.
Resources

In organizations like DoD, which consume large quantities of resources, resource allocation represents a primary constraint on the capacity of the system being managed. The actual allocation is a complex process, involving extensive bargaining and compromising among many parties. The Planning, Programming, and Budgeting System (PPBS) provides the schedule for this process. The role of the Assistant Secretary of Defense and his staff in resource allocation is one of guidance and review. Resource or fiscal guidance as established in documents like the Planning and Programming Guidance Memorandum (PPGM), while not necessarily binding on the Services, reflects the general direction in which the current administration would like to see the Defense Establishment move. The review of Program Objective Memoranda (POMs) and budget submissions is designed to ensure that the proposed distribution of resources is in consonance with the interests of DoD as a whole and with other Federal Government priorities.

Changes in resource allocation also represent managerial potential for changing the operations and structure of a system. Resources, however, are seldom dramatically reallocated from year to year. The bulk of DoD resources is needed to meet recurring requirements established in previous years. Those requirements develop a momentum difficult to stop without creating turmoil and disruption in the system. Hence, the use of resources as a means of control tends to be limited to incremental changes in the allocation of the small portion of the budget that can be considered discretionary. Even small changes can have significant impacts on the logistics systems, however.

In addition to resource allocation, management also has some discretionary power as to the rate at which those resources are used over the budget period. In many instances, it may be desirable to retain some excess
resources. This gives management a certain flexibility in responding to unexpected developments like an energy crisis. However, there are cost incentives favoring the early disposition of allocated resources. One of management's dilemmas is to find a balance between these conflicting aspects of resource utilization. The resolution of this dilemma is generally communicated through statements of policy. In the impact graphs, we treat such policies with selected policy variables.

Policies

If resource variables represent a means of exerting direct control on the system, policy variables represent a more subtle means. The policy variables selected for impact graphs should have a potentially strong impact on the logistics system, but be difficult to relate directly to a resource. This does not preclude converting policy variables to resource units of measurement.

In this context, policies represent a wide range of managerial prerogatives. In fact, resource allocation guidance can be viewed as a form of policy; we have simply chosen to treat resource variables separately in the impact graphs. Other forms of policy include plans, strategies, rules-of-thumb, guidelines, procedures, decision criteria, and performance standards.

The common assumption that effective management control must involve performance standards is a very narrow view. Indeed, we believe that at the institutional level of management, performance standards are a relatively ineffective means of control. Strict performance standards can, for example, lead to distortions in the performance measures being monitored. The use of performance standards should, for the most part, be left to the discretion of operational-level management.
IMPACT GRAPHS

Aircraft

An impact graph for the capacities in the aircraft flow graph (Figure 8) is depicted in Figure 13. While this diagram appears complicated, it represents as macro a level of detail as we could accept without losing major aspects of the Air Force logistics system. Greater levels of detail are possible, of course. Base maintenance (B/M) resources (MilPers), for example, which are treated here as a single resource variable, could conceivably be subdivided into skill levels if necessary. We have tried to conform to current budget categories in identifying resource variables. This may not be the most revealing criterion possible, but it can simplify the task of developing relationships.

For some resource variables, entire flow graphs could be developed for better understanding of the management of these resources (e.g., flows of support equipment and manpower). Similarly, the impacts from other flow graphs could conceivably be collapsed into a single resource variable (e.g., the consumable flow graph could be treated as a single budgeted resource). The point at which development of flow and impact graphs should stop must be determined by the managers who are going to use them, subject to the following guidelines:

1. The level of detail should be compatible with appropriate managerial roles. At the institutional level, this role should focus on broad management and support concepts and philosophies.
2. One additional layer of detail, beyond that over which institutional level management should exercise control, can assist in linking that level with the levels below (e.g., the Office of the ASD(MRA&L) and the Offices of the Military Department assistant secretaries responsible for logistics).

So, although the level of detail in these flow and impact graphs may exceed
the managerial responsibilities of the ASD(MRA&L), it provides some consistency between management levels.

We have chosen, for the time being, not to identify specific policy variables. Rather, we have depicted these impacts by naming the policies involved. The choice of policy variables will, to a large degree, depend on the ease with which relationships between them and system capacities can be quantitatively developed. Also, only a few policies are included, those having direct and significant impacts. Some flows, in fact, are almost entirely determined by policy (e.g., aircraft sent to scheduled maintenance). Finally, the conversion of policy variables to comparable resource units of measurement may be advantageous in rendering the impact graph more analytically manageable. This possibility is discussed further in the "Analytical Capability" section below.

Engines

The impacts of budgeted resources and selected policy parameters on the capacities in the engine flow graph are depicted in Figure 14. This impact graph differs from the others in one respect: the base engine maintenance activity involves numerous flows of engines (enclosed in the dotted segment of the diagram). Because these flows actually occur in the same maintenance shop, and the maintenance functions are performed by the same personnel, it is difficult to identify impacts on specific flow capacities. It is much easier, for the purpose of developing relationships, to treat these four functions (base returns, engine installations, engines for buildup and removals) as a whole. The reason why these functions are often separated is to allow a complete accounting of the engine inventory, i.e., whether they are installed, built-up and ready to be installed, failed but still on an aircraft, failed and being repaired, or repaired and ready for buildup when necessary.
Exchangeables

An impact graph for exchangeable items appears in Figure 15. This impact graph exhibits a different structure from the previous two in that there are now separate subgraphs. This is somewhat misleading for two reasons.

First, all control variables indirectly affect all flows through the networks in the flow graphs. The impact graphs should not therefore be manipulated independently of their effects on the flow graphs.

Second, some control variables may affect flow capacities in ways not depicted on an impact graph. For example, in Figure 15, Military Construction (MilCon) funds are shown as affecting only the exchangeable inventory capacity (warehousing and storage facilities). Of course, maintenance facilities will also have an impact on the reparable maintenance capability. We have chosen, however, not to depict the impact of "MilCon $" on component repair capacity. The assumption is that maintenance facilities used to repair aircraft are also used to repair components. So, as long as facilities are available for aircraft maintenance, there will be no capacity constraint on reparable maintenance. If aircraft maintenance facilities are not available, there will be no demand for component repair. If the validity of this assumption were in doubt, a new arrow could be added connecting "MilCon $" to "base repairs" and "depot repairs."

Consumables

The impact graph for consumables is depicted in Figure 16. Notice that the capacity to provide consumable items affects the capacities of the other three classes of materiel.

As an example of a policy variable, the percentage of NORS incidents terminated by WRM could reflect WRM policy. Policy for using and relating WRM
FIGURE 16
CONSUMABLE IMPACT GRAPH

CONSUMABLE $ (O and M)

TRANSPORTATION $

PRODUCE $ CONSUMABLE PARTS DELIVERED (CENTRAL TO BASE)

# MILCON $ C/S $ AND MILPERS

# A/C WITH FAILURES

# CONSUMABLE PART FAILURES

# AVERAGE CONSUMABLE INVENTORY (BASE)

# AVERAGE CONSUMABLE INVENTORY (CENTRAL)

# STOCKAGE POLICIES

# B/S MILPERS

# EXCHANGEABLE REPAIRS (DEPOT)

# ENGINE RETURNS (DEPOT)

# A/C PROCESSED D/M

# CONSUMABLE PARTS DELIVERIES (CENTRAL TO DEPOT)

# EXCHANGEABLE REPAIRS (BASE)

# ENGINE RETURNS (BASE)

# A/C PROCESSED B/M

# CONSUMABLE PARTS DELIVERIES (BASE)

# EXCHANGEABLE REPAIRS (DEPOT)

# ENGINE RETURNS (DEPOT)

# A/C PROCESSED D/M

# CONSUMABLE PARTS DELIVERIES (CENTRAL TO DEPOT)

# EXCHANGEABLE REPAIRS (BASE)

# ENGINE RETURNS (BASE)

# A/C PROCESSED B/M

# CONSUMABLE PARTS DELIVERIES (BASE)

# EXCHANGEABLE REPAIRS (DEPOT)

# ENGINE RETURNS (DEPOT)

# A/C PROCESSED D/M

# CONSUMABLE PARTS DELIVERIES (CENTRAL TO DEPOT)

# EXCHANGEABLE REPAIRS (BASE)

# ENGINE RETURNS (BASE)

# A/C PROCESSED B/M

# CONSUMABLE PARTS DELIVERIES (BASE)

# EXCHANGEABLE REPAIRS (DEPOT)

# ENGINE RETURNS (DEPOT)

# A/C PROCESSED D/M

# CONSUMABLE PARTS DELIVERIES (CENTRAL TO DEPOT)

# EXCHANGEABLE REPAIRS (BASE)

# ENGINE RETURNS (BASE)

# A/C PROCESSED B/M

# CONSUMABLE PARTS DELIVERIES (BASE)

# EXCHANGEABLE REPAIRS (DEPOT)

# ENGINE RETURNS (DEPOT)

# A/C PROCESSED D/M

# CONSUMABLE PARTS DELIVERIES (CENTRAL TO DEPOT)

# EXCHANGEABLE REPAIRS (BASE)

# ENGINE RETURNS (BASE)

# A/C PROCESSED B/M

# CONSUMABLE PARTS DELIVERIES (BASE)

# EXCHANGEABLE REPAIRS (DEPOT)

# ENGINE RETURNS (DEPOT)

# A/C PROCESSED D/M

# CONSUMABLE PARTS DELIVERIES (CENTRAL TO DEPOT)

# EXCHANGEABLE REPAIRS (BASE)

# ENGINE RETURNS (BASE)

# A/C PROCESSED B/M
in peacetime definitely affects the capacity of the logistics system to provide spare parts. Of course, this policy would change in time of war. In fact, as the name implies, WRM represents a reserve capacity, the utilization of which is limited in peacetime by policy.

ANALYTICAL CAPABILITY OF IMPACT GRAPHS

Tradeoff Analysis

The primary role of "resource analysis" is to assist top-level management in making qualitative or quantitative tradeoffs between alternative mixes of resources and between alternative strategies for using them (policies). The impact graph can be used qualitatively to organize one's ideas about a particular tradeoff issue and communicate them to others. Quantification makes algorithmic assessments possible, perhaps with the assistance of a computer model that links the impact graphs to the flow graphs. In the quantitative mode, alternatives can be assessed for their impact on capability (maximum flow).

Impact graphs can be misleading, however. They do not, for example, make explicit any interdependence among the control variables. If the effect of one resource or policy change is influenced by other such changes introduced concurrently, the resulting impact on the system may be very different from that expected for the original change. In other words, impacts from simultaneous changes in control variables may cancel or compound each other. Hence, control variables may be complementary; changes made together may have a greater impact than the sum of the effects of the same changes made separately. Likewise, there may be some substitution between control variables; an impact may be realizable by change in one of a number of variables, but changing more than one may not have any greater impact than changing only one.
Impact graphs also do not explicitly reflect the time lag between when a change is implemented and when its effect is observed in the system. The issue of time lags and associated time preferences is extremely important at the policy level of management. It is at this level that the difficult tradeoffs between capital investment and current operations must be evaluated. Current performance is very often sacrificed to permit the buildup of capacity in the long term. These are strategic tradeoffs with very significant consequences for both defense technology and defense organization.

The quantification of impact graphs should, therefore, address the interrelationships of control variables and time lags. Such quantification must rely, to a great extent, on "institutional knowledge," the experience and judgment of people who understand the intricacies of DoD logistics. Our experience in Phase 2 of this study demonstrated that historical data, while valuable for many applications, are not by themselves sufficient to expose the relationships needed to interpret the significance of trends.

Institutional knowledge is an invaluable source of such information because the human mind has a unique capability to sort out complex interrelationships. This capability ought to be used to interpret information derived from formal data collection systems. The key issue to be addressed is the development of methodological instruments for generating data from institutional knowledge. The remainder of this subsection discusses briefly the state of the art in this area and the problems associated with existing methods.

Collecting and Processing Subjective Data: A large number of methodological tools that use subjective data have been tested. These tools include questionnaire methods (marketing research), content analysis (communication research), Delphi techniques (futures research), and interactive
computer techniques (operations research). None of these methods is satisfactory for our needs.

However, there is some promising research on the deficiencies of existing methods. The majority of this research focuses on weighting techniques and subjective probabilities. Some of the weighting techniques include cross-impact analysis, digraph analysis, multidimensional scaling, interpretive structural modeling, and hierarchical analysis and prioritization. (We have chosen to use a variation of Saaty's method in our hierarchical analysis, discussed in chapter 5, but for a purpose different from that in the impact graphs.) System dynamics modeling, discussed in chapter 3, must also often rely on expert knowledge for the estimation of its dynamic parameters.

One problem with weighting techniques is that an assumption of linearity must be made, at least over some incremental range of the relationships being considered. A number of the techniques involve pairwise comparisons of the elements in the structure selected. While this provides the contributor with a way of organizing the relationships relative to each other, it also limits the consideration to binary relationships. If a set of variables is, in fact, highly interrelated, binary relationships may not be adequate. The primary means for addressing nonlinearity and higher order relationships in weighting techniques has been the introduction of subjective probabilities.

Substantial discussion of subjective probabilities is included in the literature on multiple criteria decision-making. Unlike some multiple criteria methods, such as goal programming, compromise programming, and various interactive programming techniques, probabilistic techniques introduce considerations for uncertainty into the possible outcomes of a decision. Subjective probabilities have been introduced into cross-impact analysis, but
perhaps the most significant application has been multiple objective utility theory.10

Utility theory requires the identification of a utility scale or metric (usually dollars) upon which alternatives can be assessed. In our case, this implies the conversion of policy variables into some equivalent resource value. This is a reasonable manipulation, as policy generally has substantial implications for resource management. In fact, institutional management often exercises influence through policy, which affects the way resources are managed at the operational level, as opposed to changing the resources allocated per se.

The problem with subjective probability approaches to complex policy situations is that the uncertainty experienced may not be of the type that can be treated with probability assessments. Rather, it may fall into the category of "primary" uncertainty, where one is ignorant not only of the probability distributions of possible outcomes, but also of the dimensions (i.e., significant variables) of possible outcomes.11 More generalized methods that do not restrict the specification of uncertainty to subjective probabilities have been suggested as a means of dealing with such complex situations. In this category are fuzzy set theory12 and constraint analysis.13 Empirical applications of these theoretical developments, however, are not extensive.

Again, one of the most challenging management research needs is the development of methodological instruments to generate empirical data compatible with these more generalized methods. Some promising efforts in this direction are based on interactive simulation and gaming, and the use of technologies like computer conferencing.14

A final note on the topic of generalized methods is appropriate. Traditional analytical and modeling techniques have been very successfully
applied at the operational levels of management. By taking an engineering approach to modeling, the developers of these techniques have been able to attain high degrees of accuracy in parameter estimation and in empirical validation. But the engineering approach sacrifices theoretical scope and generality. At the policy level, a broad theoretical scope that allows the reevaluation of model (and system) structures is needed. To achieve this kind of scope requires an acceptance of limitations on parameter estimation and on model validation. Very approximate, even qualitative (if necessary), information must be satisfactory. Model validation, in this context, includes demonstration, managerial participation, and, eventually, implementation. This broader approach is much more compatible with the role of top-level management than is the traditional engineering approach.

Risk Analysis

A by-product of tradeoff analysis is the opportunity for risk analysis. When a relationship cannot be developed as a one-to-one functional correspondence between a state variable and the control variables, the impacts of the control variables are uncertain. If this uncertainty can be expressed as a probability distribution, utility theory can be used to discount the alternative impacts with respect to their relative "riskiness." When the probability distribution is not known, a uniform distribution can be assumed, and the ratio of the variance to the mean used as a risk discount factor.15

When neither the probability distribution nor the precise boundaries on the distribution of possible impacts are known, fuzzy set theory can sometimes be helpful. Clark and Pipino have demonstrated that, with respect to uncertainty considerations, utility theory is a restricted form of the more generalized fuzzy set theory.16 Hence, the principles of utility theory can be applied to fuzzy uncertainty. The use of risk discounting may not be the
best way to treat risk, but at present it is the state of the art. When risk
discounting is combined with time discounting, the methodological problems
become more severe. Some work in this area, however, appears promising.17

The application of risk analysis to the impact graphs, in whatever
form, allows management to hedge in situations where a decision based solely
on expected (or most probable) outcomes could result in undesirable or even
disastrous consequences, even though the perceived probabilities of such are
low. When very low probabilities are associated with highly undesirable
consequences, the situation is often referred to as one with significant
"downside risk." Management's avoidance of downside risk has often been
observed, but there has been little analytical treatment of it. This is
another research issue that needs to be addressed.

Sensitivity Analysis

As previously mentioned, fully quantified flow and impact graphs
could be adopted to a computer model. Such a model could enable a manager to
evaluate the sensitivity of logistics system parameters to changes (inten-
tional or otherwise) in resources and policies. For this type of micro model
to be effective, managers themselves must participate in supplying input and
in assessing output.

Model development efforts should emphasize a sound, yet tractable,
structure, rather than precise parameter estimation. Insights into the basic
structure of a logistics system are possible even though the output estimates
are rough. The power of a computer model is its ability to specify the impact
of a given set of resource and policy variables on support capability, by
calculating the maximum flows in the flow graphs. High degrees of sensitivity
might suggest a need for closer control of the control variables or for moni-
toring of system parameters exhibiting high variability.
APPLICATION OF IMPACT GRAPHS

Two simple examples of the development of relationships in an impact graph are presented here. The first involves a weighting scheme, and the second an incremental estimation scheme. These techniques should not be regarded as recommended methods. They serve only to illustrate the problems of using institutional knowledge as an information source and to emphasize the need for further research.

An Example Using a Weighting Technique

For this example, we assume a simple impact graph as shown in Figure 17. C is a flow capacity on an arc in a flow graph (not depicted). R₁, R₂, and R₃ are the control variables that management can manipulate to effect a change in the state variable, C. For this example, we assume that R₁, R₂, and R₃ are resources and are constrained only by a budget limit, B. In practice, of course, the amount of flexibility to change resource levels is very small. Hence, a more accurate formulation might consider only minor incremental changes from current funding levels.

![A SIMPLE IMPACT GRAPH](image-url)
The mix of $R_1$, $R_2$, and $R_3$ produces a $C$, which is not necessarily unique. The reason that $C$ is not unique is that there may be other variables (particularly exogeneous variables) not included in the analysis, but having an impact on $C$. These variables are not included either because they are not controllable, or because we have not observed them to have a significant impact, or because they are difficult to measure. In addition, the precise relationship between resources (and policies) and capacities may be very difficult to derive if the variables are non-linearly interrelated.

Due to this complexity, we need ways to simplify the quantitative specification of the relationship. In general terms, the relationship is:

$$C = \phi (R_1, R_2, R_3)$$

where $\phi$ is the relation. The simplification we will discuss here involves the assignment of weights to each resource to indicate the relative impact of that resource over some relevant range of values. The formulation of this is:

Maximize $C = w_1 R_1 + w_2 R_2 + w_3 R_3$

subject to:

$$R_1 \leq r_1$$
$$R_2 \leq r_2$$
$$R_3 \leq r_3$$

$$R_1 + R_2 + R_3 \leq B$$

where, for $i = 1, 2, 3$, the $w_i$ are the weights and $r_i$ are the points at which another resource becomes more effective. $B$ is the total budget constraint.

In this example, $w_1 > w_2 > w_3$ we would allocate money first to $R_1$ (until we reached $r_1$), then to $R_2$, and finally to $R_3$. Again, this is a very simplistic formulation; it assumes that interactions between variables are negligible.
An alternative approach, which introduces some nonlinearity into the analysis, involves the division of B into small intervals. Weights, in this case, are established not only for each resource but also for each respective interval. One way to do this is to assign weights, $w_{ik}$, for each resource $R_i$, where $k$ represents the number of applications of $R_i$. So, if $R_1$ is selected for the first interval, it is because $w_{11}$ is greater than any other $w_{i1}$. If $R_1$ is also selected for the second interval, it is because $w_{12}$ is greater than the weight of the first application of any other resource, i.e., any $w_{i1}, i \neq 1$.

As an example, divide B into five equal intervals:

\[ \text{0} \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad B \]

Assume the following weights:

\[
\begin{align*}
    w_{11} &= .52 \\
    w_{12} &= .37 \\
    w_{13} &= .20 \\
    w_{21} &= .43 \\
    w_{22} &= .32 \\
    w_{23} &= .17 \\
    w_{31} &= .35 \\
    w_{32} &= .23 \\
    w_{33} &= .19
\end{align*}
\]

Let $B = $5 billion and assume upper constraints on each resource of:

\[
\begin{align*}
    r_1 &= $1.5 billion \\
    r_2 &= $3.5 billion \\
    r_3 &= $2 billion
\end{align*}
\]

For interval 1, the first $1 billion, the allocation goes to $R_1$ because $w_{11} = .52$ is greater than any other $w_{i1}$. For interval 2, the allocation, $1 billion, goes to $R_2$ for its $w_{21} = .43$. In interval 3, the first $1.5 billion goes to $R_1 (w_{12} = .37)$, and the second $1.5 billion goes to $R_3 (w_{31} = .35)$. This is because $R_1$ has reached its maximum $r_1 = 1.5$. The first $1.5 billion of interval 4 goes to $R_3 (w_{31} = .35)$ and the second $1.5 billion
goes to \( R_2 \) \((w_{22} = .32)\). The first $5 billion of interval 5 goes to \( R_2 \) \((w_{22} = .32)\) and the second $5 billion goes to \( R_2 \) \((w_{32} = .23)\). The calculation for the state variable, \( C \), is:

\[
C = .52(1) + .43(1) + .35(.5) + .35(1) + .32(1) + .23(.5) = 1.91
\]

If the weights are established on a scale of zero to one, the 1.91 would have to be multiplied by some unit conversion factor to get a measure of \( C \). Graphically, the resulting relationship looks like that in Figure 18.

**Figure 18**

**Resource Allocation by Budget Intervals**

While this method accounts for more complexity than did the simple weighting method, it still does not address the following:

- two equal weights in a particular interval
- the dependence of a weight on the allocations of all resources which preceded it
- complementarity and substitutability between resources.

The first problem can be handled by making the intervals sufficiently small so that the choice between the two equally weighted variables is inconsequential. Human judgment might also be used. The second problem could be handled by allocating resources to an interval prior to establishing weights for the next interval. The third problem requires the identification of those variables.
which are complementary and should, therefore, be allocated together, and
those variables which are substitutes, and should not be allocated together.
Again, some human judgment is required.

The real problem with weighting methods arises when we add more
state and control variables. The linkages evident in Figures 13 through 16
complicate the situation considerably. If, in addition, we introduce time
lags to indicate that some variables take longer to have an effect than
others, the number of weight combinations we would have to establish becomes
astronomical.

The use of minimum and maximum estimates of marginal changes in a
state variable from its current state, for given changes in control variables,
provides an alternative approach, which requires substantially less infor-
mation than the above weighting technique. The resulting relationships,
however, will be much more approximate. If this method is used, the question
that arises is: What degree of approximation is acceptable for broad
policy-making and resource guidance?

An Example Using an Incremental Estimating Technique

The following example is similar to the weighting techniques; but
rather than assign weights to represent the strength of relationships, this
time we will deduce the relationship by assigning resources or groups of
resources (resource sets) for allocation and then estimate the corresponding
change in the state variable. Figure 19 is the simple impact graph used in
this example.

To use this method, a number of assumptions must be made. First,
for each set of estimates, a time horizon must be specified. The estimates
must then be solicited for a range of time horizons. The significance of the
time horizon is that in selecting resource sets, no effects beyond one year
are considered. Second, an upper limit on the funding for a particular state variable is assumed. Third, it is assumed that Defense managers and operators will conform to Defense policy guidance with respect to resource utilization and management.

This incremental estimating technique allows for interdependence among resources (control variables), including complementarity and substitutability. But it also requires (1) greater exercise of judgment with respect to identifying those resources that are most critical to improving a state variable, and (2) the knowledge needed to estimate corresponding changes in the state variable. For the state variable depicted in Figure 18, Base Aircraft Maintenance, Table 2 provides hypothetical numbers for a one-year time horizon and an upper funding limit of $6 billion.
TABLE 2. HYPOTHETICAL RELATIONSHIPS BETWEEN BASE AIRCRAFT MAINTENANCE FUNDING LEVELS AND CAPACITY

Time Horizon: 1 Year
Upper Limit: $6 Billion

<table>
<thead>
<tr>
<th>Increment No.</th>
<th>Resource Sets</th>
<th>Increment Size</th>
<th>Resource Ratio</th>
<th>Estimate of Change in State Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>$1,2,3,4,5$</td>
<td>$4B$</td>
<td>$8:1:4:2:1$</td>
<td>600,000 aircraft/yr.</td>
</tr>
<tr>
<td>1</td>
<td>$R_3$</td>
<td>$.5B$</td>
<td>1</td>
<td>17,000</td>
</tr>
<tr>
<td>2</td>
<td>$R_3R_4$</td>
<td>$.3B$</td>
<td>2:1</td>
<td>8,000</td>
</tr>
<tr>
<td>3</td>
<td>$R_1$</td>
<td>$.6B$</td>
<td>1</td>
<td>10,000</td>
</tr>
<tr>
<td>4</td>
<td>$R_1R_2$</td>
<td>$.4B$</td>
<td>3:1</td>
<td>6,000</td>
</tr>
<tr>
<td>5</td>
<td>$R_3$</td>
<td>$.2B$</td>
<td>1</td>
<td>2,000</td>
</tr>
<tr>
<td>Upper Limit</td>
<td>$6B$</td>
<td></td>
<td></td>
<td>643,000</td>
</tr>
</tbody>
</table>

In this case, any resource that takes more than a year to have an impact will be excluded from selection. When tables for three or four time horizons have been developed, time discount factors (or some other method for treating time preferences) can be applied. A composite resource allocation and corresponding state variable estimates can then be calculated.

To interpret this table, we begin with the left-hand column. The first row represents a funding "Base" over which there is no control. These funds have been committed by requirements of irrevocable decisions already made. In this example, $4 billion is considered uncontrollable. The next five rows represent funding increments to be added to the base in order of decreasing priority. In this example, $R_3$, exchangeable items, was determined to be the most critical until $0.5 billion has been allocated at which time $R_4$, consumable items, becomes equally important. The resource set, $R_3R_4$, should be allocated together to get the full impact of available funds. This may be a result of complementarity between resources or of equally important resources.
The column labeled "Resource Ratio" is a specification of how much of each resource is allocated in a particular increment. In increment 2, for example, two-thirds of the $0.3 billion goes to $R_3$ and one-third to $R_4$. The right-hand column is the estimate of the expected change in the state variable, the capacity of Base Aircraft Maintenance to process aircraft, measured in aircraft per year. When these point estimates are connected and put into graphical form, the relationship appears as in Figure 20.

**FIGURE 20**

**GRAPHICAL RELATIONSHIP GENERATED BY TABLE 2**

![Graphical relationship](image)

The major problem with this incremental estimating technique is the difficulty of generating the estimates. Such estimates should come from individuals with both a macro perspective and a detailed knowledge of where current problems, bottlenecks, and opportunities for improvement lie. On the
other hand, the types of estimates needed are not unlike many of those re-quired as supporting material in annual budget submissions.

Modifications to this method might alleviate the estimation problem to some extent. Rather than requiring direct estimates of changes in a state variable, it might be easier to estimate changes in other parameters, which can then be used to calculate a change in the state variable. The simple equations suggested in chapter 3 might serve this purpose. It might also be possible to use existing models to calculate changes in a state variable.

Another modification would be to reduce the requirement for point estimates from individual respondents. A range of possible values (i.e., thresholds) would be easier to arrive at. This would introduce uncertainty into the analysis and allow for consideration of risk tradeoffs.

Finally, rather than beginning with a "Base" funding level, it might be reasonable to begin at the current funding level and then search in both directions for possible incremental changes. The method used in the example is more compatible with the concept of Zero Base Budgeting in that it requires a justification of the base. On the other hand, starting with the current situation is probably more compatible with the way resource allocation is actually practiced, and may also be easier to treat analytically.

**INSTITUTIONAL KNOWLEDGE AND RESOURCE ANALYSIS**

There is a major need for research in the area of policy analysis to develop methodological tools that can elicit responses from individuals concerning relationships between state and control variables. We believe that significant information about these relationships is part of the existing institutional knowledge within DoD. What is needed is not only appropriate means for collecting and analyzing this information, but also creative ways of displaying it. Improved channels of communication also need to be identified.
In chapter 5, "Hierarchical Analysis," we present a technique for collecting and processing judgmental data with respect to logistics goals and objectives. We do not, however, believe that the same technique can be used to adequately quantify the relationships in resource analysis. The method of pairwise comparisons relied on in hierarchical analysis is not sufficient to treat the high degree of interdependency common among very aggregate resource and policy variables.

At its present state of development, as described in this chapter, resource analysis provides a structure, in the form of impact graphs, which can assist in the process of assessing alternative resource allocations and policy decisions. This structure specifically allows consideration for the interdependencies, i.e., the higher-order relationships, among control variables, something that traditional economic, optimization, and graphical structures do not allow. Thus, resource analysis can be a valuable tool for facilitating discussion and debate on very complex issues of policy and on budget and policy formulation. It can also serve as an information filter by providing a context for interpreting logistics data.
5. HIERARCHICAL ANALYSIS

STRUCTURAL DESCRIPTION

Flow analysis (chapter 3) and resource analysis (chapter 4) are tools for structuring the operational aspects of a logistics system and for evaluating potential changes in its structure. They provide individual managers and administrators with a way to organize both the information they receive and their own ideas about the system they are managing. But these tools do not address differences between individual managers, e.g., how they perceive and assess the system, or where they believe the most critical problems and the greatest potential for improvement lie. These differences represent management issues. Hierarchical analysis is a tool designed to assist in resolving such differences and, more specifically, in structuring the goals and objectives to be imposed on the operational logistics system.

Many opportunities exist for creative research in the development of techniques to carry out the function of hierarchical analysis. The existing techniques all have certain methodological limitations. However, we believe that a hierarchical structure provides an extremely powerful means of reducing the apparent complexity of large systems like DoD logistics, regardless of the specific technique used to analyze that structure. The technique we selected was developed and applied to issues of policy by Saaty\(^1\) and involves pairwise comparisons of elements at each level of a hierarchy, using a scale of relative importance. Through simple calculations, priorities for logistics goals and objectives can be generated. Despite certain limitations, discussed later in the chapter,
we believe this technique represents the state of the art in the structur-
ing of multiple and conflicting goals and objectives. It also provides
an essential element of our overall framework, namely, a mechanism for
facilitating discussion and debate on logistics management issues.

This chapter describes the technique and offers a numerical application
for illustrative purposes. The mathematical development and theoretical
support for this technique is presented in the appendix.

Hierarchical Development

The information processing limitations imposed on individual
managers and on organizations provide the rationale for using a hier-
archical structure. By reducing complex problems and phenomena into
subproblems and subphenomena, and those into further subdivisions,
complexity can be reduced to a manageable level.\textsuperscript{2} The development of
a hierarchy requires the identification of subsystems (i.e., clusters
of variables), which are relatively (or conditionally) independent of
each other. These subsystems (or clusters) form the elements of a level
of the hierarchy and are related to each other only through the elements
of the next higher level (aggregations of subsystems). A system which
can be hierarchically structured is said to be "nearly decomposable."

When a system that is not nearly decomposable is hierarchically
structured, some of the interrelationships between subsystems (i.e.,
the elements of a particular level of the hierarchy) may be ignored.
Some inconsistency may result from the technique presented here if certain
interrelationships are ignored. A certain amount of inconsistency
is acceptable, but too much can render the priorities calculated very
sensitive to changes (or perturbations) in importance factors. This
problem is discussed in the appendix.
For our purposes, the levels of a hierarchy correspond to distinct areas of managerial influence and control. There are no strict procedures for developing a hierarchy, only guidelines. But if hierarchical analysis is to be effective, DoD administrators and logistics managers themselves must participate in the development of an appropriate structure and in the assignment of priorities within it. Only if the structure is agreeable to all the parties involved can the tool fulfill its purpose of facilitating discussion, debate, and communications on policy issues.

In the hierarchy we offer as an illustration, the apex corresponds to an overall logistics purpose, which can be stated simply as "the logistics support of national defense systems." At the second level of the hierarchy are global logistics objectives. These global objectives reflect aspects of "readiness," and, as such, are broad concepts. They are not directly measurable, but they are distinct objectives that managers can associate with logistics. The third level of the hierarchy corresponds to the structural aspects of the logistics system. The elements of this level are logistics activities. The fourth and final level consists of policy objectives, which each activity pursues in making its contribution to the global objectives. This level corresponds to the desired performance of the logistics system. Policy objectives are generally measurable.

Many hierarchies are possible, and many levels of detail can be added if necessary. One of the criteria for a hierarchical structure is that it should be no more detailed than is necessary for its purpose.

The appropriate level of detail for a particular level of management is a difficult question. On the one hand (as in resource analysis), the level of detail must be sufficient to provide continuity and ensure consistency between management levels. On the other hand, excessive detail interferes
with the organization and simplification of complex policy issues. A hierarchy with more than four or five levels of detail becomes very time-consuming to quantify, as administrative personnel must argue out each importance factor individually. Further, for an element at a particular level to have more than six or seven elements associated with it at the next level presents some methodological problems when the scale of relative importance has only five operationally defined factors. (The total of nine factors does, however, allow some flexibility.)

In the following section, we present an example of a logistics hierarchy for the Air Force. The level of detail goes beyond that necessary for the OSD institutional role as we have described it. In fact, it is probably more appropriate to the top management role in the Air Force itself. The OSD role is more one to assess all three Military Departments and examine inter-Service possibilities for alternative logistics support concepts. However, the additional level of detail provides the scope necessary to understand the structure and operation of the logistics systems within each Service. That understanding is necessary to facilitate communications and coordination between OSD and the Services.

**Priority Scaling**

When a hierarchy has been constructed and agreed upon, the next step is quantitative comparison of the elements in that hierarchy, level by level. To do this, a scaling technique must be selected. The technique used by Saaty involves pairwise comparisons of elements at one level of the hierarchy with respect to their importance for elements at the next higher level. The importance scale used is shown in Table 3.
The choice of a scale with nine factors is based on psychological research that has demonstrated that the human mind has difficulty in discriminating among elements on scales with any more than seven, plus or minus two, degrees of discrimination (see appendix.) Only five of the nine factors, however, are associated with a verbal description. The other four are used for compromising between any two adjacent verbal descriptions. Reciprocals are used as the reverse importance factors. That is, if element A is given an importance of 9 over element B, element B has an importance factor of 1/9 of element A.

### TABLE 3. IMPORTANCE SCALE

<table>
<thead>
<tr>
<th>Scaling Factor</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal Importance; Indifference</td>
</tr>
<tr>
<td>3</td>
<td>Weak Importance of One over Another</td>
</tr>
<tr>
<td>5</td>
<td>Essential or Strong Importance</td>
</tr>
<tr>
<td>7</td>
<td>Demonstrated Importance</td>
</tr>
<tr>
<td>9</td>
<td>Absolute Importance</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Intermediate Values between the Two Adjacent Factors; Used for Compromise</td>
</tr>
</tbody>
</table>

**Reciprocals**

If Element I has One of the above Numbers Assigned When Compared to J, Then J Has the Reciprocal Value When Compared to I

After importance factors are established, priorities are calculated using simple matrix manipulations. We present an example of these calculations in the application section of this chapter.

Two procedures can be used to establish importance factors. The first is for each manager to establish individual factors. These individual
results would then be compared for discrepancies, which in turn would suggest issues for debate. Group discussion should be aimed at resolving the differences and arriving at a consensus on these factors. The second procedure is for managers to arrive at importance factors through a group process. Discussion and debate would precede the establishment of each individual factor.

Many reasons for individual differences can be postulated. Among them are: differences in definitions of the elements of the hierarchy, and the context within which they are viewed; differences in individual perceptions of the purpose of the exercise; and differences in the meaning individuals associate with the importance scale. These sources of variability should be addressed by clarifying ambiguities prior to assigning importance factors. Other sources of variability include: different perceptions of the logistics system, its problems, bottlenecks, and opportunities for improvement; different experiences with, and exposure to, logistics operations; and different functional roles and personal interests. The airing and debating of these differences leads to mutual learning by the participants, a systemwide perspective, and possibly the identification of innovative alternatives.

AN AIR FORCE LOGISTICS HIERARCHY

Figure 21 is an example of a hierarchy for Air Force logistics. While National Defense is shown at the top of the hierarchy, for the purposes of our example, the hierarchy begins with Logistics. Ideally, the hierarchy should be used for all defense functions, and even by top-level administrative officials for all Federal Government activities. But because the scope of our study is limited to DoD logistics management, we have assumed the existence of some higher-level process for determining the relative importance of logistics to national defense, and of national defense to the Federal Government.
FIGURE 21
AN AIR FORCE LOGISTICS HIERARCHY

NATIONAL DEFENSE

DEFENSE FUNCTIONS
R and D
OPERATIONS
LOGISTICS
INSTALLATIONS

GLOBAL LOGISTICS OBJECTIVES
PEACETIME MATERIEL READINESS
SUSTAINABILITY
FACILITIES SUPPORT

AIR FORCE LOGISTICS ACTIVITIES
BASE MAINTENANCE
DEPOT MAINTENANCE
BASE SUPPLY
CENTRAL SUPPLY
TRANSPORTATION
INDUSTRIAL BASE

AIR FORCE POLICY OBJECTIVES
1. REDUCE NORM RATE
2. IMPROVE SKILLS
3. INCREASE PERSONNEL UTILIZATION

1. INCREASE AIRCRAFT MODS
2. REDUCE REPAIRABLE BACKLOG
3. INCREASE PERSONNEL UTILIZATION

1. INCREASE FILL RATE
2. ACHIEVE WARM LEVELS
3. REDUCE MURS INCIDENTS

1. INCREASE FILL RATE
2. IMPROVE SUPPLY RESPONSIVENESS
3. IMPROVE INVENTORY MANAGEMENT

1. REDUCE TRANSPORTATION DELAYS
2. INCREASE SURGE CAPABILITY
3. IMPROVE MATERIEL RELIABILITY

1. REDUCE PROCUREMENT LEAD TIME
2. IMPROVE MATERIEL MAINTAINABILITY

...
At the next level of the hierarchy, below Logistics are Global Logistics Objectives. The terms chosen to represent these objectives—Peacetime Materiel Readiness and Sustainability—correspond to the "omnibus issues" appearing in the FY 1978 logistics issue paper. They represent a subdivision of the contribution of logistics to National defense.

Air Force Logistics Activities are identified at the next level. In our example, these activities correspond to nodes on the flow graphs. The importance placed on different logistics activities is one way of highlighting alternative support concepts. Evaluation of support concepts is an important role of top-level management in the Military Departments and is also relevant to the OSD role in establishing a balance among all DoD logistics activities.

The bottom level of our example are Air Force Policy Objectives, which each logistics activity pursues or attempts to satisfy in making its contribution to the total logistics effort. The priorities that management places on these objectives reflect their perceptions of the importance of each policy objective to the global objectives. Such perceptions should indicate where current problems exist, where the most cost-effective opportunities are located, and also where managerial preferences for change lie. This level of the hierarchy is probably a level of detail below that appropriate to the top level of OSD, where the focus should be more on the structure of the logistics system than on its performance. The performance level is included to provide the continuity and scope necessary to ensure coordination between institutional and operational management.

ANALYTICAL CAPABILITY OF HIERARCHICAL ANALYSIS

Issue Analysis

Policy issues generally evolve from situations of conflict. Conflict may represent disagreements not only over the best means of achieving
prescribed objectives, but also over the objectives themselves. The advantage of structuring objectives and policies and associating them with logistics activities is that points of conflict can be identified. With the prioritization procedure, some of the conflicts may be resolved without debate. Others will require considerable discussion, bargaining, and compromise.

The rationale behind hierarchical analysis is not that it will resolve policy issues. Rather, it can provide a mutually agreeable framework for communication between conflicting parties. Conflicts arising from misunderstanding or ignorance are generally easier to resolve than those resulting from different interests (political, bureaucratic, or otherwise) and different perspectives on appropriate defense strategies.

In these latter conflicts, however, the institutional level of management can and should make a major contribution. The operational levels manage technological systems, like inventories, maintenance, and transportation of materiel. The policy levels, on the other hand, manage socio-technical systems, including management itself. This institutional function can best be performed by implementing more effective channels of communication and encouraging constructive debate among conflicting interests. "More effective channels of communication" does not, however, translate into a larger quantity of information flowing to OSD. Rather, it refers to a more meaningful structure for interpreting that information.

An example of an issue appropriate for OSD consideration would be the degree of centralization or decentralization of logistics activities. Centralization would be reflected by greater emphasis on depot maintenance and central supply, decentralization by greater emphasis on base maintenance and base supply. Another such issue would be the degree of component repairability designed into new weapon systems. Procurement of repairable items
implies a more labor-intensive organization, requiring emphasis on maintenance activities. Procurement of consumable items implies a more capital-intensive organization, requiring emphasis on supply and industrial base activities. These types of issues impinge directly on national defense strategy and have both inter-Service and intra-Service implications. They affect resource allocation, policy formulation, and the effectiveness of the logistics system in performing its support role.

**Priority Analysis**

The successful use of hierarchical analysis (i.e., the assignment of importance factors to all elements in the hierarchy) results in a priority ordering of the elements at each level of the hierarchy. Furthermore, the method ensures the preservation of priorities throughout the hierarchy. The mathematical foundations of the method are presented in the appendix and are not discussed here. A brief example, however, is presented in the applications section.

The numbers generated by hierarchical analysis represent the priorities placed on each element relative to other elements at the same level, as collectively determined by the individuals participating in the process. The numbers should, then, reflect where the group perceives the problems and opportunities to lie. The use of the pairwise comparisons to establish importance factors, however, limits the use of the technique to elements that are not strongly interrelated. This limitation, nonetheless, is not all that relevant, because the value of hierarchical analysis is as much in the process involved in establishing the importance factors, as it is in the numbers generated. Discussion of priorities serve as a vehicle for new insights, which would not be possible with formal reporting systems alone.
The priorities generated by hierarchical analysis can, however, serve as a frame of reference for other, more qualitative policy-making activities. New policies, plans, and resource allocations would reflect the priorities established. The priorities could also be used to identify those aspects of logistics where management indicators need to be monitored. If gaps in currently reported information are apparent, new indicators should be developed. The point is that hierarchical analysis is a flexible tool that can be used in many ways and for many purposes. It does not make decisions or formulate policy but is a means of facilitating communication, stimulating discussion, and generating insights.

APPLICATION OF HIERARCHICAL ANALYSIS

In this section, we used the Air Force logistics hierarchy of Figure 21 to show how importance factors are established and priorities calculated. We established the importance factors in this example; they are not meant to reflect any recommendations for policy change. Our numbers are illustrative only.

After a hierarchy has been agreed upon, the first step is to establish importance factors on the global logistics objectives. In our example, we chose to place overriding importance on 'Peacetime Materiel Readiness' as opposed to Sustainability, realizing that the two are not independent. When put in matrix form, the importance factors appear as in Table 4. The first matrix in Table 4 is interpreted as follows: with respect to the Logistics Support of National Defense, Peacetime Materiel Readiness (PMR) is given absolute (9 on the importance scale of Table 3) importance over Sustainability (SUS). The diagonal of these matrices consists of elements whose values are always 1, since anything compared to itself will be of equal
### TABLE 4. ESTABLISHING RELATIVE IMPORTANCE VALUES

<table>
<thead>
<tr>
<th>Logistics Support of National Defense</th>
<th>PMR</th>
<th>SUS</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMR</td>
<td>1</td>
<td>9</td>
<td>(0.9)</td>
</tr>
<tr>
<td>SUS</td>
<td>1/9</td>
<td>1</td>
<td>(0.1)</td>
</tr>
</tbody>
</table>

Eigenvalue $= \lambda = 2$

<table>
<thead>
<tr>
<th>Peacetime Materiel Readiness</th>
<th>BM</th>
<th>DM</th>
<th>BS</th>
<th>CS</th>
<th>TR</th>
<th>IB</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>BM</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>7</td>
<td>(0.44)</td>
</tr>
<tr>
<td>DM</td>
<td>1/5</td>
<td>1</td>
<td>1/5</td>
<td>1/4</td>
<td>1/3</td>
<td>1</td>
<td>(0.04)</td>
</tr>
<tr>
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Eigenvalue $= \lambda = 6.51$

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</tr>
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<td>(0.29)</td>
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</tbody>
</table>

Eigenvalue $= \lambda = 6.43$
importance. Also, the matrices are always reciprocal. That means that SUS will be given an importance factor of 1/9 (the reciprocal of 9) over PMR.

The λ shown below each matrix is called an eigenvalue of the matrix and is used to calculate the eigenvector (or priority vector), which appears to the right of each matrix. The calculation of eigenvalues and eigenvectors for large matrices can quickly become too complicated to do by hand, but computer programs are readily available for this purpose. The eigenvector represents the relative priorities of the elements compared in the matrix.
For comparative purposes, the eigenvectors are always normalized so that their elements sum to one. So, at the level of global objectives in our example, PMR is given nine-tenths of the available priority and SUS, one-tenth.

At the next level of the hierarchy, logistics activities, two matrices are needed: one for pairwise comparisons with respect to Peacetime Materiel Readiness and the other with respect to Sustainability. Notice that the importance factors are different depending on which global objective they are being compared to. For example, Base Maintenance (BM) is given an importance factor of 3 over Base Supply (BS) with respect to Peacetime Materiel Readiness; however, they are judged of equal importance with respect to Sustainability. Likewise, the priority vectors for each matrix will be different. With respect to Peacetime Materiel Readiness, Base Maintenance (BM) is given top priority (.44), with Base Supply (BS) second (.30). With respect to Sustainability, Base Maintenance (BM) and the Industrial Base (IB) are given equal priority (.29).

At the final level of our hierarchy are the policy objectives associated with each logistics activity (Figure 21). For simplicity, we considered only two or three policy objectives for each activity. As mentioned previously, the detail at the policy objectives level of the hierarchy is perhaps greater than is appropriate for the OSD management role. The topics of interest at this level include logistics support concepts and corresponding aspects of logistics structure, for example, centralization/decentralization and labor-intensive/capital-intensive tradeoffs. Therefore, institutional management's participation at this level is appropriate. To be sure, the major concern at the policy objectives level is with implementation, which falls more under the purview of operational management. Nevertheless, including policy objectives
in the hierarchy gives it the concreteness it needs to be meaningful to the administrators using it.

The priorities for policy objectives might best be passed on to operational management as institutional management's perceptions of where the problems are. Operational management would then be able to correct any misunderstandings that may exist at the institutional level. Another possibility would be to use input from operational managers themselves to assist in establishing importance factors.

The final step in the procedure is the calculation of composite priority vectors from the individual priority vectors. These calculations involve simple matrix multiplication and are shown in Table 5. The first calculation multiplies the priority vector for the two global objectives by the two respective vectors for logistics activities. The result is a new priority vector for logistics activities, with Base Maintenance (BM) getting the largest portion of the available priority (.425), followed by Base Supply (BS) (.287) and Central Supply (CS) (.109). This new vector is then multiplied by the matrix of policy objectives.* The result is a composite vector over all policy objectives that preserves priorities at every level of the hierarchy. In our example, improving maintenance skills received top priority (.27); reducing NORS incidents was second (.21); and reducing the NORM rate was third (.13). (See the appendix for a discussion of matrix multiplication and of the justification for the composite vector.)

PROSPECTS FOR HIERARCHICAL ANALYSIS

While the flow and resource analyses discussed in chapter 3 and 4 still need substantial technical development, hierarchical analysis is fully

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* In Table 5, the 17 x 6 matrix of policy objectives is not shown. Because there are many zeros in that matrix, it can be partitioned into the six individual vectors shown.
### Table 5. Calculating Priorities

<table>
<thead>
<tr>
<th></th>
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<th>SUS (0.1)</th>
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<td>NORM</td>
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<td>SKILLS</td>
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<td>MAINT</td>
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</table>
developed and could be implemented with minimal effort. The essential ingredient for successful implementation is the cooperation of administrators and logistics managers.

Numerous applications of hierarchical analysis have been successfully demonstrated and documented. They cover a wide range of policy situations including:

- The design of a transportation system for the Sudan\(^4\)
- Projections of higher education demands\(^5\)
- Long-range corporation planning\(^6\)
- An assessment of world influence\(^7\)
- Energy policy analysis.\(^8\)

One should not, however, interpret the success of hierarchical analysis as an indication that no further extensions are possible. On the contrary, its success has stimulated interest in extending it well beyond its current capability. Further research should remove some of the limitations mentioned in this chapter.

In summary, hierarchical analysis adds to our framework the capability for analyzing complex policy issues, where objectives and other decision criteria may be vaguely defined, and where there may be conflicting views on how to resolve the problem. The technique does not replace current management practices but augments them with a structured exercise aimed at clarifying the issues and generating new insights.

Hierarchical analysis does require a certain amount of management time. It could be scheduled to coincide with milestones in the PPB process or other relevant recurring activities. No one administrator should be expected to devote more than a day or two a year to the exercise, but at least a half a day should be allotted for each group session. Some time could be saved if
each participant were required to prepare his own hierarchy and assign his own priorities prior to a group session.

We do not recommend that the numbers generated by hierarchical analysis become hard-and-fast DoD priorities. The numbers should be viewed primarily as an historical record of the exercise, i.e., as a reminder of the issues discussed. Some insights may also be obtained by comparing exercises conducted at different times or by different groups.

Problems with hierarchical analysis may arise with respect to the following: (1) highly interrelated elements at a particular level of the hierarchy, (2) arrival at a consensus on all importance factors, and (3) full and realistic participation by all members of the group. (Motivation to participate may be hindered by the somewhat unrealistic setting of the exercise; top management interest and enthusiasm can help the problem.) Still, hierarchical analysis represents the state of the art in structured group decision-making processes aimed at facilitating effective communications.
SUMMARY OF PREVIOUS CHAPTERS

The purpose of this chapter is to bring together the management system concepts and selected management tools discussed in the preceding chapters, and to show how they fit into our proposed framework for policy-level logistics management.

In Chapter 2 we identified the basic activities of the DoD management process, distinguishing between top-level (institutional) and operational-level (technical) management responsibilities. Next, we illustrated the evolution of management tools in the context of our representation of a management system. Finally, we examined available management tools and selected three as most appropriate for our purposes: flow, resource, and hierarchical analysis. Chapters 3 through 5 examined the application of these tools in a military logistics environment. In this chapter, we present an analytical framework that outlines a systematic procedure for relating aggregate system input (resources and policies) to system output (defense capability and performance).

THE DEVELOPMENT OF APPROPRIATE FEEDBACK

The key to this framework is a thorough understanding of the use of feedback in the management cycle. (See Figure 22.) With judicious use of information from the operational logistics system, the appropriate management tools can assist in all major management functions. Feedback meeting the requirements for aggregate information and a systemwide perspective will help managers to:

- Improve and expand their capability to filter information flows
- Comprehend what the operational system is doing
- Become more effective and efficient in the decision-making process.

We will return to and enlarge upon Figure 22 throughout this section.

Components of a Management Information Structure

Our management information structure has two purposes: (1) to provide specific information that relates system input and output to goals and objectives—on a routine basis; and (2) to respond to nonroutine queries. Thus, the structure needs two components: a formal data base and institutional knowledge.
The formal data base is a processing system which:
- Receives, manipulates, and stores data from the operational system
- Produces output information on the values of state, resource, and policy variables in a form acceptable to top management.

The functions of data storage and process control are vital for two reasons. First, an adequate data base enhances the statistical reliability of the output and, hence, the validity of subsequent analysis and conclusions. Second, an historical data base is necessary to perform trend analysis for such purposes as tracking indicators selected for monitoring.

The other component, institutional knowledge, should be thought of as an unstructured repository of information. It is the sum total of the technical background and experience possessed by managers at all levels of the operational system. It can answer many different types of questions pertaining to the evaluation of data base output, questions that cannot be answered by quantitative analysis. In many respects, institutional knowledge transcends the purely quantitative data base, enriches its description, and frequently alters its implications.

Unfortunately, institutional knowledge is neither catalogued nor indexed. Not all top-level OSD managers are equally skilled at applying it. In addition, the managers themselves have different degrees of operational experience, and thus different opinions about the utility of institutional knowledge. These differences suggest a need for developing some measure of the value of institutional knowledge. Adequately calibrated, this knowledge can provide judgmental estimates of policy and resource parameters affecting top management decisions. As pointed out in chapter 2, it is possibly the best source of information about what will work and what will not.
Management Indicators

Management indicators are the next step in the development of appropriate feedback. The purpose of indicators is to improve the visibility of the logistics system with respect to policies and objectives. In most organizations, objectives are hierarchical, i.e., attainment of an objective at a high level results from attainment of numerous objectives at a lower level. Hence, measures of attainment are likewise hierarchical.

This relationship between objectives and measures is two-edged, however. The process of selecting measures may suggest to managers that the objectives need to be revised. Hence, establishing objectives and selecting measures are interactive processes, which means that the measures should be designed with as much care as is expended in establishing the objectives.

Based on our structure of the management process, indicators can be categorized in terms of the following types:

- Resource utilization
- Policy compliance
- System performance.

A resource utilization indicator might provide information on whether resources are being used where they were allocated. A policy compliance indicator might be one designed to determine if reliability and maintainability goals are being achieved. System performance indicators may be designed to provide measures of materiel flows and of proxies for flow capacities (e.g. repair backlogs, changes in the NORM rate, etc.) Performance indicators are related to the other categories because resources and policies are inputs to system performance.
In Phase 2 of this study, we collected, analyzed, and displayed indicators, but we did not organize them into the above groups. We have now attempted to develop a framework that uses these indicators and that incorporates tools to be used at the top management level.

The major problem is what criteria, if any, can be used for selecting indicators. We believe that absolute criteria cannot be determined; the selection process must be iterative, evolutionary, and dynamic. A comprehensive set of indicators can be developed only through the interactions of the operational system, the information structure, and top management. These groups must work in concert to test and revise indicators, refine reporting rules and regulations, and add or delete indicators dictated by managerial requirements.

Having addressed the management information structure, the management functions which it supports, and the management tools to be applied, we are now prepared to specify their roles in an analytical framework.

AN ANALYTICAL FRAMEWORK FOR POLICY-LEVEL MANAGEMENT

We can describe this framework most readily in terms of information flows, such as those portrayed in Figure 23, which converts the functional diagram of Figure 22 into an information flow chart. In the following section, we explain how the management information components and management tools should interact to provide a coherent management system.

The operational logistics system provides feedback of data to the management information structure, constantly adding to the logistics data base and expanding the scope of institutional knowledge. The logistics data base provides top management with either management indicators or with data that
FIGURE 23
INFORMATION FLOWS IN A MANAGEMENT SYSTEM
can be processed by trend or structural analysis into indicators. The body of information represented by management indicators flows three ways:

- Indicators pose specific questions on the operational system's, performance, questions that can best be answered through institutional knowledge.

- Indicators provide trend and structural data, which are the basis for flow and resource analysis.

- Indicators provide information on resource utilization, system performance, and policy compliance needed for hierarchical analysis.

The information provided by our framework exposes and clarifies the relationships between system variables treated in flow and resource analysis, and assists in the reevaluation of the priorities derived from hierarchical analysis. The relationships reveal constraints on the policy, planning, and resource allocation alternatives available to top management; the priorities help to define logistics goals and objectives. Together, the relationships and priorities contribute to the evaluation of policy alternatives by providing a structured view of the system and of the direction top management judges it best proceed.

APPLICATION TO OSD

We believe the approach presented in this report provides a fresh perspective on how policy management at the OSD level can function in relation to the Services. Further, the technical proposals made can make many beneficial contributions to such policy management.

Top-Level Visibility

OSD does not lack information, but the information provided is in piecemeal fashion and has been produced primarily for operational management purposes. The purpose of our framework is to provide policy-level management with directly relevant information. This means that the information must have
a systemwide perspective, and be processed in an appropriately aggregate form.
The proposed framework has been designed to enhance the visibility of information at the OSD level by:

- Providing a systematic, methodical structure for use of management tools;
- Linking resources and policies to material flows and capacities;
- Addressing management objectives through priority assignments;
- Operating on aggregate information;
- Using multiple forms of feedback; and
- Assisting management in evaluating alternatives.

**Issue Identification and Analysis**

The framework for policy-level logistics management presented in this report is directly applicable to OSD needs. The techniques described here can expose relationships between system performance, resources, and policies sufficiently aggregate for OSD purposes. With these tools, OSD can make tradeoffs on system variables that take into account its assignment of priorities and assessment of objectives. Such a capability will enhance the quality of its decision-making, and permit the exercise of a level of control appropriate to its role.

When fully developed, this framework should be applicable to the identification and analysis of logistics issues.

For example, management might wish to know:

Is the increasing NORM rate a reflection of manpower limitations, or is it a reflection of a shifting aircraft mix, or is it even indicative of a reduced logistics capability?
Use of these management tools should also assist ASD (MRA&L) participation in the PPBS. One question that could be addressed is:

What tradeoffs between base and depot maintenance resources are possible, given some specified readiness and sustainability objectives?

Finally, the tools should, if properly implemented, have significant application to long-range or strategic planning. Here an example of an appropriate question might be:

What logistics support concepts are compatible with long-term defense goals and projected defense technologies?

Structured Interaction

The one word that best describes the contribution of this framework is "structure". With a meaningful structure, interaction between institutional and operational management should be improved. The quantity of extraneous data flowing to top management should be reduced, and the quality of that information still provided to OSD should be improved. The management tools in the framework cannot and will not replace policy-making as we know it. That process is intrinsically one of bargain and compromise and is strongly influenced by political considerations. The framework provides OSD management with a method for processing relevant information and thereby enriches the policy-making process.
7. RECOMMENDATIONS

STATUS OF THE FRAMEWORK

This report provides a framework for policy-level logistics management, focusing on broad policy guidance, resource allocation, and strategic planning. The framework contains several unique elements:

- It distinguishes policy management from operational management, focusing on the needs of the policy level.
- It proposes an explicit and tested method for structuring and evaluating goals, objectives, and policies.
- It contains quantitative tools for policy analysis, resource allocation, and monitoring the operational system.
- It incorporates management indicators within policy management.
- It provides for the use of institutional knowledge as a major part of the information exchange between the policy and operational management.

The benefits and advantages to be gained from these elements justify further research on the framework, the techniques contained in it, and its application. The following describes the further work that should be supported.

REQUIRED RESEARCH

The major piece of required research is quantification of the impact graphs. Chapter 4 discussed some possible approaches to this problem. We have pointed out the difficulties in measuring capacities of logistics activities, which are the state variables in flow and resource analysis. The selection of appropriate proxies for the capacities could be a satisfactory way to quantify them for the impact graphs. Both historical data and institutional knowledge could contribute to this effort.
Existing detailed models could also be used for quantification of the impact graphs. The METRIC-type models developed within the Air Force can be used to generate tradeoff curves that relate an appropriate measure of supply system performance with exchangeable items to dollar investment in such items. The measures of effectiveness used with these models are typically expected number of backorders and in the case of LMI Availability (METRIC), supply availability rates. Tradeoffs are calculated over a wide range of aggregate investment dollars, so that depending on the investment interval being analyzed, the relevant slope of the tradeoff curve can be calculated to provide the coefficient for the impact graph.

Maintenance-operations simulations could also be used to help develop the impact graphs. The Air Force's LCOM (Logistics Composite Model) is a detailed simulation of aircraft and maintenance operations that emphasizes the manpower resource impact on aircraft squadron sortie capacity. Manpower is treated in terms of both skill category and skill level. The intent would be to establish how the results of such simulations should be used to determine the structure and parameters of the impact graph.

Our survey of existing detailed models used by the Services indicates that they can be very useful in the research phase. These models are receiving greater acceptance within DoD, and their quality is constantly being improved. The linking of these detailed models to the aggregate impact graphs will facilitate consistency between the Service use of the models and OSD use of the techniques proposed in this report.

Another topic discussed in chapter 4 as important for successful development of the impact graphs is the use of subjective judgment to obtain estimates of the impact of changing budget and program situations on logistics capability. Here, we are referring to the use of institutional knowledge to
obtain subjective information. Research is needed to determine effective ways of eliciting responses from individuals and groups concerning relationships between state and control variables. We strongly believe that sufficient information about these relationships exists within DoD. Questionnaires are one way of getting at this information, but other methods, including the use of interactive computer techniques, may be more effective. Another possibility is to use a modified form of hierarchical analysis to quantify the values on an impact graph, which would quantify subjective judgement in an explicit and consistent manner.

**OTHER IMPROVEMENTS**

A few other topics deserve attention because they will improve the quality of the tools used within our framework. One such subject is the introduction of time lags into the flow and impact graphs. These time lags need to be incorporated without unduly complicating the evaluation process. Recognizing that time lags can be used to represent the policy as well as physical constraints, we would prefer to depict time lags as single valued variables rather than distributed variables since the former are simpler to handle. Research in this area would involve using institutional knowledge to obtain estimates of time lags, as well as sensitivity testing with the flow and impact graphs to assess the effects of time lags on the types of guidance compatible with policy-level management.

Risk analysis is another possible area for improvement. Recent work on the subject was described in Chapter 4. Our purpose would be to adapt the available knowledge to our needs, and thereby expand the quantitative capability of our tools.
VALIDATION

Given continued management sponsorship of the development of this framework, we should give early attention to what is required to validate our approach. By validation we mean an objective, and preferably quantitative, assessment of the output provided by the management tools proposed in this report. We suggest applying them to realistic problems, computing the outputs, and comparing them against results obtained by other available methods. Bases of comparison would include agreement of the numerical results, the relative costs of producing the findings, and the timeliness with which they could be provided.

The DoD problem used in chapter 5 might serve as a means of validation. It involves a policy-level problem, and our work to date has developed a structure and context for analyzing it. Also, it is the type of logistics problem that has to be addressed each year, since it focuses on issues like readiness and sustainability. DoD has made decisions on resource allocation in this area for FY 1979, and their results either for that year or the following year could serve as validating numbers with appropriate interpretation.

Thus, our effort would involve quantification of the flow and impact graphs, and their use in conjunction with the hierarchical analysis would provide the estimates needed for validation. Because we would have to rely on institutional knowledge for certain relationships and estimates, we would require some access to informed DoD people, as well as to some official documents. Such a validation exercise could be completed within six months, following the necessary development of a data collection instrument for resource analysis.
IMPLEMENTATION

The validation test would go far toward achieving implementation of the proposed framework. Initial contacts at the policy and operational levels during the validation phase could be augmented, as necessary, to prepare for subsequent implementation.

When implemented, the system should provide for computer operation of the flow and impact graphs. Much of the computer programming can be done during the validation phase. In its final form, the system should include interactive computer processing of the flow and impact graphs, designed for use by management personnel.

The process of hierarchical analysis should also be computerized during the validation phase. Again, provision for interactive computation would be valuable, so that users could receive immediate feedback on the weights assigned to the hierarchical elements.

Thus, the validation and implementation phases would probably overlap to a considerable extent, with the implementation phase serving largely to add greater computer capability to the operation of the tools. Implementation time, therefore, should not be much longer than the six months estimated for the validation phase.

ADDITIONAL APPLICATIONS

Service Logistics Systems

Our development effort used the Air Force as a testing ground for our proposed policy-level management framework. We believe that the system could be used by the Air Force for its own policy management, and by OSD to encourage similar systems in the other Services. As with the Air Force, the Army and Navy systems could be operated independently as they are implemented,
so there would be no need to wait for one system before the others could be put into operation.

The role of OSD in the initial stages should be to encourage and monitor the Service efforts. This experience should help OSD to define its own needs more clearly. Very likely, OSD's needs will ultimately generate special requirements for information and policy that will transcend those of the individual Services. We believe that much of this information could be drawn from existing institutional knowledge within the Services, so that the formal information needs would not be expanded.

**Manpower**

The concept of policy management developed in this report for logistics should apply equally well to manpower. In fact, objectives and policies established in either management area, without due regard for their impact in the other, could have adverse implications. From an overall budget standpoint, the ties between logistics and manpower are very strong, since about 40-50 percent of DoD manpower is assigned to logistics specialties, and over 65 percent of logistics costs can be attributed to personnel.

Manpower is shown explicitly as elements in the impact graphs in chapter 4, in terms of appropriate budget funding categories that constrain the flow and capacities of the logistics activities. Manpower is also implicitly reflected in most logistics activities and in the state variables of logistics performance contained in the flow graphs. Use of the tools described in this report for manpower management could mean a more comprehensive representation of manpower impacts by explicitly identifying resource variables according to major skills, specialties, and grades. In turn, demands for manpower skills could be translated into recruiting and training workloads for manpower management.
The mention of manpower is to emphasize the generality of the tools we have developed and the significance of the interactions between manpower and logistics management; and to indicate that more detail on manpower can be included if desired. A complete analysis for manpower and personnel would be comparable to that required for logistics. Such an effort could be done independently of the logistics work, but at the same time take advantage of the experience gained in the prior research.

We believe this framework for policy-level logistics management has a potential for significant contribution to DoD. Much of the conceptual and empirical effort has been done. Relatively little work remains to be done to make this framework useable.
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INTRODUCTION

One of the most difficult, yet important, problems for decision-makers is how to rank or prioritize multiple activities, elements, criteria, etc., logically and rationally. Such ranking is often vital to allocate resources among activities or to implement activities by priority. Without such a ranking, decisions may be made and actions taken that produce no progress toward stated objectives or, what is worse, results contrary to those objectives. Yet, too often the method used is an intuitive, qualitative technique, which provides no sound foundation for the process other than the intelligence and experience of the decision-maker.

Dr. Thomas L. Saaty has developed a quantitative algorithmic method that permits the decision-maker not only to structure the decision environment, but also to solve the ranking or prioritization problem. This appendix is an abbreviated discussion of that method and its supporting theory, as originally presented in "A Scaling Method for Priorities in Hierarchical Structures." Except where otherwise noted, all of the procedures and theory herein are derived from this source.

In the following section, we explore eigenvalue theory and show how it can be applied to quantitative judgmental assignments, using paired comparisons to produce a vector of numbers which express the relative preferences for, or importance of, a set of related objects. Problems of consistency in preference matrices are also discussed.
The final section discusses the development of hierarchies and their use in conjunction with the eigenvalue analysis presented in the previous section. It concludes with a mathematical justification for the use of matrix and eigenvalue theory in that development.

Hierarchical analysis is not merely an intellectual exercise. Saaty has applied this method to the development of a master transportation plan for the Sudan which has received widespread critical acclaim.²

**QUANTIFICATION OF JUDGMENTS**

**Eigenvalue Analysis**

The eigenvalue theory described below has a number of applications in many areas of mathematics. We start by stating a problem: given a square $n \times n$ matrix $A$, determine the values of the scalar, $\lambda$, and the non-zero vectors, $w$, which satisfy the equation:

$$Aw = \lambda w.$$  

(1)

This matrix equation may also be written as:

$$a_{11}w_1 + a_{12}w_2 + \ldots + a_{1n}w_n = \lambda w_1$$

$$a_{21}w_1 + a_{22}w_2 + \ldots + a_{2n}w_n = \lambda w_2$$

$$\cdots$$

$$a_{n1}w_1 + a_{n2}w_2 + \ldots + a_{nn}w_n = \lambda w_n$$

(2)

Subtracting the right side of (2) from both sides, we obtain:

$$(a_{11} - \lambda)w_1 + a_{12}w_2 + \ldots + a_{1n}w_n = 0$$

(3)

$$a_{21}w_1 + (a_{22} - \lambda)w_2 + \ldots + a_{2n}w_n = 0$$

$$\cdots$$

$$a_{n1}w_1 + a_{n2}w_2 + \ldots + (a_{nn} - \lambda)w_n = 0$$
This can be written in matrix notation as:
\[(A - \lambda I_n)w = 0,\]  \((4)\)
where \(I_n\) is the \(n \times n\) identity matrix. Obviously, if \(w\) is a zero vector, 
(4) is satisfied. If \(w\) is non-zero, however, solutions to (4) exist only if the determinant of \((A - \lambda I_n)\) is zero, written as:
\[|A - \lambda I_n| = 0,\]  \((5)\)
Values of \(\lambda\) that satisfy those solutions are called the eigenvalues, characteristic values, proper values, or latent roots of (5). The corresponding solutions \(w \neq 0\) of (4) are called eigenvectors or characteristic vectors of \(A\). A matrix of order \(n\) will have \(n\) eigenvalues, not necessarily real or distinct.

To illustrate, we will find the eigenvalues and eigenvectors of the matrix:

\[A = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \]

Solution:
\[A - \lambda I = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-\lambda & 3 \\ 3 & 1-\lambda \end{pmatrix} \]
\((6)\)
\[0 = (A - \lambda I) = (1 - \lambda)^2 - 9 = \lambda^2 - 2\lambda - 8 = (\lambda - 4)(\lambda + 2) \]
\((7)\)
The eigenvalues of \(A\), then, are 4 and -2.

With these values of \(\lambda\) in the format of (1), we obtain
\[\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}w = 4w \]
\((8)\)
and
\[\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}w = -2w \]
\((9)\)
Rearranging these equations into the format of (3), we obtain:

\[(I - 4)w_1 + 3w_2 = 0\]  
\[3w_1 + (1 - 4)w_2 = 0\]  \hspace{1cm} (10)

and

\[(1 + 2)w_1 + 3w_2 = 0\]  
\[3w_1 + (1 + 2)w_2 = 0\]  \hspace{1cm} (11)

From (10), \(w_1 = w_2\); from (11), \(w_1 = -w_2\). Hence, there is no unique \(w\) for each of the corresponding eigenvalues of \(A\), since any set of real or complex numbers satisfying either (10) or (11) is an eigenvector of \(A\), corresponding to the eigenvalues \(\lambda = 4\) and \(\lambda = -2\), respectively.

If we consider only matrices whose elements are all real and positive, then it can be shown that such a matrix has a real positive \(\lambda\), which is at least as large as any of the remaining \((n - 1)\lambda\)'s. Let us call that value \(\lambda_{\text{max}}\). Also, the eigenvector \(w\) associated with \(\lambda_{\text{max}}\) can be chosen real and with all positive components.

Further, let us consider only reciprocal matrices, ones in which \(a_{ij} = 1/a_{ji}\) for all \(i\) and \(j\). Obviously, \(a_{ii} = 1\) for all \(i\) and, equally obviously, there can be no zero entries if the \(a_{ij}\) are all \(> 0\). Also, we impose the constraint that

\[\sum_{i=1}^{n} w_i = 1,\]

which merely ensures a unique value for the eigenvector associated with \(\lambda_{\text{max}}\).

To determine the eigenvalues of an \(n \times n\) matrix requires the solution of a polynomial equation of degree \(n\). As \(n\) increases, it becomes
more and more inefficient to obtain that polynomial by a direct expansion of
the determinants defining it. Numerical techniques are preferable and are
frequently available in computer library routines.

The remainder of this section describes Saaty's application of
eigenvalue theory described above to the pairwise assignment of quantitative
judgments and his treatment of the inconsistency problem.

Pairwise Comparisons

One of the basic problems in the management process is the inte-
gration of human judgments into decisions; that is, how can opinions be
systematically interjected into, and accommodated by, the process so that the
manager may formulate alternatives and select decisions? To limit the scope
of the problem, we will consider only the quantification of judgments. In
other words, we will attempt to design a procedure by which a real number can
be assigned to represent the expression of human judgment or preferences
between and among a variety of "objects." Here, "objects" may refer to
specific entities, alternative choices, policies, activities, etc.

The first question is: how many objects should be considered at one
time? Miller showed that an individual cannot simultaneously compare more
than seven (plus or minus two) objects without confusion. Hence, we could
choose as many as nine objects for simultaneous comparison. To make the
problem as simple as possible, however, we choose to limit our objects to two,
i.e., pairwise comparisons. With this simplification we exclude all but
two objects, compare them with respect to some criterion, and assign to that
comparison a unique real number. Hence, with n objects \(O_i, i = 1, 2 \ldots, n\),
we compare \(O_1\) with \(O_2\), excluding the remaining \(n-2\) \(O_i\), and assign a number \(a_{12}\)
to the comparison. Next, we compare \(O_1\) with \(O_3\), excluding the remaining \(O_i\)
(and, in fact, attempting to exclude the results of our comparison of \(O_1\) with
and assign a number $a_{13}$ to that comparison, and so forth, until we have generated $a_{12}$ through $a_{1n}$. Next, we compare $O_2$ with $O_3$, etc., generating $a_{23}$ through $a_{2n}$, and continue until we have produced all the values of $a_{ij}$ for $j > i$.

The next question is: what scale of real numbers should we use for the $a_{ij}$? Whatever the scale, it should satisfy four criteria:

- It should represent people's differences in feelings when they make comparisons and, insofar as possible, all distinct shades of feeling from indifference to absolute preference.

- The principal gradations of the scale should be unit values, i.e., one.

- The people making the comparisons must be aware of all scale gradations at the same time without confusion.

- The gradations of the scale should be positive, real numbers (to permit us to employ the eigenvalue theory discussed earlier).

The first issue to settle is: what number should be used to measure "equality" or "indifference" between two objects? Clearly, comparing an object with itself produces equality, so the issue reduces to what value to assign to $a_{ii}$, i.e., the real number generated by comparing $O_i$ with itself. But since we are considering only reciprocal matrices, in which $a_{ii} = 1$ for all $i$, we assign unity as the measure of equality or indifference.

Next, what should be the range of the scale? Miller's findings lead us to choose a scale of $7 + 2 = 9$ units. With unity as our benchmark, the scale values will range from 1 to 9. This selection results in satisfying all of the criteria mentioned above. Since we consider only reciprocal matrices (fortunately, maintaining the constraint of positive numbers), the scale for comparing $O_j$ with $O_i$ ($a_{ij}$) will range from $1/9$ to 1. In other words, when our comparison of $O_i$ with $O_j$ results in $a_{ij}$, our comparison of $O_j$ with $O_i$ automatically results in $a_{ji} = 1/a_{ij}$. Table A-1 reproduced from chapter 5, defines this scale and explains it in subjective terms. We do not rule out
### TABLE A-1. IMPORTANCE SCALE

<table>
<thead>
<tr>
<th>Intensity of Importance</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance</td>
<td>Two activities contribute equally to the objective.</td>
</tr>
<tr>
<td>3</td>
<td>Weak importance of one over another</td>
<td>Experience and judgement slightly favor one activity over another</td>
</tr>
<tr>
<td>5</td>
<td>Essential or strong importance</td>
<td>Experience and judgment strongly favor one activity over another</td>
</tr>
<tr>
<td>7</td>
<td>Demonstrated importance</td>
<td>An activity is strongly favored and its dominance is demonstrated in practice</td>
</tr>
<tr>
<td>9</td>
<td>Absolute importance</td>
<td>The evidence favoring one activity over another is of the highest possible order of affirmation</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Intermediate values between two adjacent judgments</td>
<td>When compromise is needed</td>
</tr>
</tbody>
</table>

**Reciprocals**

If activity i has one of the above numbers assigned to it compared to activity j, then j has the reciprocal value compared to i.

**Rationals**

Ratios arising from the scale.

If consistency is to be forced by obtaining n numerical values to span the matrix.

Two significant questions have to be answered:

- What is the justification for using that scale to evaluate importance or preference?

- Assuming that scale can be justified, how can we translate the individual comparisons into a priority-ordered set? We will
illustrate the answer with an analogy and subsequently will introduce a theorem that provides rigorous justification.

Suppose we wish to compare the physical weights $w_i$ of $n$ objects $A_i$, $i = 1, 2, \ldots, n$, but we have only a balance scale so that we are unable to measure their absolute weights. With a balance scale, however, we can compare $A_i$ with $A_j$; record the value of the ratio $w_i/w_j$, denoted as $a_{ij}$; and display the results as matrix $A$:

$$
A = \begin{bmatrix}
  w_1 / w_1 & w_1 / w_2 & \cdots & w_1 / w_n \\
  w_2 / w_1 & w_2 / w_2 & \cdots & w_2 / w_n \\
  \vdots & \vdots & \ddots & \vdots \\
  w_n / w_1 & w_n / w_2 & \cdots & w_n / w_n
\end{bmatrix} = (a_{ij})
$$

How can we determine the relative physical weights of all the $A_i$? Let $w^T = (w_1, w_2, \ldots, w_n)$ be the vector of the absolute physical weights. Then, if we multiply the matrix $A$ by $w$, we obtain $Aw = nw$; i.e., $(w_i/w_1) \cdot w_1 + (w_i/w_2) \cdot w_2 + \cdots + (w_i/w_n) \cdot w_n = n \cdot w_i$ for $i = 1, 2, \ldots, n$. Now we solve this equation for $w$, i.e.:

$$(A - nI) w = 0. \quad (12)$$

But this is precisely the same format as (4), which leads to the solution of the eigenvalues of $A$, with $\lambda = n$.

Once we have solved for $\lambda_{\text{max}}$, we can produce a non-unique solution for $w^*$. If we again impose the arbitrary normalizing constraint that

$$\sum_{i=1}^{n} w_i = 1,$$

*We use $\lambda_{\text{max}}$ because there is no assurance that any of the remaining eigenvalues are real and positive.*
we have produced a unique value for $w$ that displays the relative physical weights of all the $n$ objects.

For example, suppose we have four objects whose weights (unknown to us) are: $w_1 = 5$, $w_2 = 4$, $w_3 = 2$, and $w_4 = 1$. Comparing these objects pairwise on a balance scale (i.e., comparing object 1 with object 2, object 1 with object 3, etc.), we derive the following results:

$$A = \begin{pmatrix}
\frac{5}{5} & \frac{5}{4} & \frac{5}{2} & \frac{5}{1} \\
\frac{4}{5} & \frac{4}{4} & \frac{4}{2} & \frac{4}{1} \\
\frac{2}{5} & \frac{2}{4} & \frac{2}{2} & \frac{2}{1} \\
\frac{1}{5} & \frac{1}{4} & \frac{1}{2} & \frac{1}{1}
\end{pmatrix}$$

(13)

As it happens, $\lambda_{\text{max}} = 4$. Using this value in the format of (4), we produce:

$$\begin{pmatrix}
1 & -4 & 1.25 & 2.5 & 5 \\
.8 & 1 & -4 & 2 & 4 \\
.4 & .5 & 1 & -4 & 2 \\
.2 & .25 & .5 & 1 & -4
\end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix} = 0$$

(14)

Again, since (14) does not produce a unique value for $2$, we add the normalizing constraint that

$$\frac{4}{4} \sum_{i=1}^{4} w_i = 1.$$

The resulting value of $w^T$ is $(5/12, 4/12, 2/12, 1/12)$ and reproduces the relative physical weights of the four objects.

Hence, we have shown how the use of pairwise comparisons on a numerical scale, coupled with eigenvalue analysis, can generate an ordered set of numbers, which reflect precisely the relative weights of all the objects compared. But there is no reason why we must restrict our interpretation of the ratios $w_i/w_j$ to those of physical weights; we can interpret them to
mean size, importance, or personal preference. Indeed, Saaty shows that, regardless of the interpretation, if there exists some absolute ordinal scale by which objects are measured, then the ordered values of the elements of \( w \) generated by the eigenvalue theory preserve the ordering of the ordinal scale. This preservation of ordinal consistency is of central importance in justifying the use of the preference scale.

**Consistency**

Before we show how the use of pairwise comparisons and eigenvalue analysis can be applied to hierarchies, we must address one problem: matrix consistency and the effect of departures from consistency upon the preservation of ordinal consistency just discussed. In the previous example, the \( A \) matrix was, by its very construction, consistent, i.e., every row of \( A \) was some multiple of the first row. For instance, in equation (13), row 2 equalled \( 4/5 \) times row 1, row 3 equalled \( 2/5 \) times row 1, etc. In such a case, the rank of \( A \) is one (the rank of a matrix \( A \) is the order of the largest square sub-matrix of \( A \) whose determinant does not equal zero). Let us now determine what are the eigenvalues of a reciprocal matrix \( A' \) of order \( n \) and rank one.

It is well known that the expansion of \((A - \lambda I)\) yields the characteristic polynomial \( f(\lambda) \) of the form:

\[
f(\lambda) = \lambda^n + s_1 \lambda^{n-1} + s_2 \lambda^{n-2} + \ldots + s_{n-1} \lambda + (-1)^n |A|,
\]

where

\[
s_m = (-1)^m \text{ times the sum of the determinants of all the } m \text{-square principal minors of } A.
\]

But, since the rank of \( A \) is one, the determinants of all the principal minors of \( A \) are zero for \( m > 1 \), as well as the determinant \( |A| \). The only non-zero...
determinants are those of order one, i.e., the diagonal elements of $A$ which, in a reciprocal matrix, are all one. Hence, $s_1 = -n$, and:

$$f(\lambda) = \lambda^{n-n} = \lambda^{n-1}(\lambda-n).$$

The eigenvalues of $A$ are obtained as usual by setting $f(\lambda) = 0$, or $\lambda = n$ and 0 (with multiplicity $n-1$). Then $\lambda_{\text{max}} = n$, and all others are zero.

It is also well known that the trace of $A$ (the sum of the diagonal elements of $A$) equals the sum of the eigenvalues. But, again, these diagonal elements are all one; hence, $\lambda_{\text{max}} = n$. The important thing to note is that, if $A$ is not consistent (i.e., if the rank of $A$ is greater than one), $\lambda_{\text{max}} > n$, always.

How can inconsistency arise? Suppose that, in our example, the balance scale was not precise, thus producing inexact values for the $a_{ij}$. If the imprecision were severe enough, then the transitivity relationships:

$$A_i > A_j \text{ and } A_j > A_k \text{ imply } A_i > A_k,$$

may not hold (where $A_i$ represents the $i$th row of $A$). That imprecision in our weight comparison process might result, say, from trying to estimate the relative weights of the objects by balancing them in our hands and recording our subjective (but imprecise) judgments as to which is heavier and in what ratio. We can show that small perturbations in the $a_{ij}$ result in only small perturbations in $\lambda_{\text{max}}$. The next questions are: what is the impact of inconsistency upon ordinal ranking; and how can we measure departures from consistency?

The answer to the first question is that ordinal consistency is preserved. Saaty shows that if $a_{ik} > a_{jk}$ for all $k$, then $w_i \geq w_j$ even if $\lambda_{\text{max}} > n$.

In answer to the second question, Saaty states that $\mu = (\lambda_{\text{max}} - n)/ (n-1)$ is a measure of inconsistency and is related to the statistical root.
mean square error. He observes that this statistic appears to follow a probability distribution whose variance is twice its mean, and which is quite similar to the $\chi^2$ distribution. Without knowing the distribution, he suggests using the ratio, $(x - \mu_0)/\sigma$ with $\mu_0 = 0$, i.e., $x/(2x)^{1/2}$ or $(x/2)^{1/2}$. This quantitative test may be used to confirm the hypothesis that $\mu = 0$ when the statistic is, say, $\leq 1$. For example, in a 6 x 6 reciprocal matrix suppose that $\lambda_{\text{max}} = 6.45$. Then $x = (6.45 - 6)/5 = 0.09$. The test statistic is $(x/2)^{1/2} = 0.21 < 1$. Hence, we may conclude in this case that the departures from technical consistency are not sufficient to invalidate the ordinal ranking implied by the resulting eigenvector $w$.

What are the implications if the test statistic $\geq 1$? First, Saaty emphasizes that preference or importance judgements need not be transitive. He illustrates by tournaments: team $C_1$ may lose to team $C_2$, which has lost to team $C_3$; yet $C_1$ may have won against $C_3$. In this sense, team performance may be inconsistent. Hence, we must be prepared to accept some inconsistency. In fact, the arithmetical properties of our preference scale are conducive to generating some degree of inconsistency, especially with large preference matrices. But, in general, we may say that, whenever the test statistic $\geq 1$, it should be regarded as a signal to reexamine and reevaluate the paired comparisons.

With this understanding of pairwise comparisons and eigenvalue analysis, we are now prepared to examine the concept of hierarchies and the application of eigenvalue analysis of preference at successive hierarchical levels.

*Based upon experimental results derived from randomly generated reciprocal matrices.
Hierarchies are difficult to discuss conceptually without resorting to examples or formalizing their properties mathematically. Examples, however, fail to provide an adequate guide for construction and definition of other hierarchies, and presentation of the mathematical formulation is best deferred until we have an intuitive grasp of the concept of hierarchy. That formulation shows ordinal preferences to be preserved throughout the levels of a hierarchy with the eigenvalue technique. This result means we can use the eigenvalue technique at each level of a hierarchy and, by weighting each successive hierarchical level by the preference vector of the previous level, generate an overall preference vector at the lowest level. That final vector will reflect all higher level ordinal preferences.

First of all, a hierarchy is a structure of various elements, partitioned into levels. These levels preserve some sense of order or distinction, as perceived by the person performing the partitioning. Each level consists of elements having some perceived property that reflects that order or distinction. Conventionally, the first, or top, level is one element, sometimes representing what in set theory is referred to as the universe of discourse. Successive levels may represent both structural and functional relations, and their elements may be physical entities or activities, possible future scenarios, objectives, policies, etc.

Saaty sets forth three substantive properties of hierarchical structures:

- They usually consist of a few kinds of subsystems in various combinations and arrangements.
- They are nearly decomposable, i.e., connections between levels are far simpler and more distinctive than the connections between
elements in a level. For example, one level might consist of objectives, while another might consist of activities. The distinguishing characteristics between objectives and activities are much more obvious than, say, the distinctions between the elements at the activity level. Hence, the aggregate properties, which first defined the partitioning of the levels, determine the interactions between levels, and not the properties of the individual elements.

- Regularities in the interactions between levels may themselves be classified and coded, taking advantage of redundancy to obtain greater simplicity. For example, the change in a system over time may be described by a differential equation which specifies the amount of change at any instant of time.

Saaty also describes five advantages of hierarchies:

- They provide a meaningful integration of systems. Thus, the integrated behavior or function of a hierarchical organization accounts for the fact that complicated changes in a large system can result in a single component, contrary to what we generally expect.

- They use aggregated elements in the form of levels to accomplish tasks.

- Greater detail occurs at the lower levels of the hierarchy, while greater understanding of its purpose occurs at the higher levels.

- Hierarchies are efficient and will evolve in natural systems much more rapidly than non-hierarchic systems with the same number of elements.

- Hierarchies are reliable and flexible; localized perturbations do not perturb the entire hierarchy. The overall purpose of the hierarchy is partitioned among the levels; each level solves a partial problem, and the totality meets the overall purpose.

At this point, having presented a generalized concept of hierarchies, we seek to outline a method for constructing them. Unfortunately, there seems to be no clear-cut, detailed method. However, we can derive some insights by examining two fundamental ways in which the idea of a hierarchy can be used.

The first way is nothing more than a hierarchical modeling of the real world. The second way is probably even more fundamental and points up the real power of hierarchies; breaking things down into large groupings or
clusters, and then breaking each of these into small clusters, and so on. The object would then be to obtain the priorities of all the elements by clustering, a far more efficient process than treating all the elements together.

While not describing a formal approach to constructing hierarchies, Saaty does outline two major application areas: conflict and planning problems. In the first, he describes the hierarchical levels as representing the actors who influence or control the conflict outcome, their objectives, policies, strategies, and the set of plausible outcomes that may result from their actions. Eigenvalue analysis then produces weights or priorities for the outcomes. This method thus provides a basis for approaching the actors (or the parties who control them) as to what may work best when their combined interests are considered, or to show them where to modify their positions to achieve a more desirable outcome.

In the second problem, planning, an outcome is often referred to as a scenario. Saaty asserts that these scenarios should represent "extremes" to ensure sufficient richness to the range of judgmental preferences for those scenarios. The set of scenario outcomes is then hierarchically weighted by the weights of the actors combined with the weights of their objectives, and finally with those of their strategies. The result of all influences on the set of outcomes is a composite outcome, the likely or composite future.

This composite outcome is characterized by a set of state variables, which should be selected so that their values describe adequately the real world modeled by the hierarchy. The values of the state variables are calibrated by weighting the corresponding values of the variables for each scenario considered. The values assigned to the state variables are usually determined by a numerical scale, with the present outcome (or status quo) taken as the zero reference point. The purpose of an analysis such as this is
to examine the attitudes of the actors about the future within a hierarchical framework.

The application of eigenvalue analysis to priorities within hierarchies draws its strength from the decomposable nature of hierarchies themselves. If it were not possible to conceive of and generate hierarchical levels, each consisting of a set of elements, then any attempt to form and quantify judgmental assessments between and among all elements would face enormous difficulties.

For example, consider a two-level hierarchical system consisting of activities and objectives, where a subset of objectives is associated with each activity. There should be no conceptual problem in generating a priority ordering of the objectives within each subset with eigenvalue analysis. Subsequently, each of the priority ordering vectors can be weighted by the relative priorities assigned to each activity, resulting in a composite weighting of the importance of each objective.

But suppose we were required to generate a priority ordering of all objectives without regard to the activities with which they were associated. We would be forced to juggle mentally a multitude of factors and relationships to arrive at each pairwise comparison. It would be well nigh impossible to arrive at a consensus as to the impact of those factors and relationships. The use of hierarchies, then, permits us to simplify and condense those impacts in a piecemeal fashion, thus restricting them to those necessary to identify direct factors and relationships.

Mathematical Formulation

The purpose of the following discussion is to derive a formal justification for the matrix multiplication technique; produce a composite priority or preference vector at any hierarchical level; and show that the ordering of
the elements of that vector preserves ordinal preference for the objects represented.

We start with a number of definitions:

1. An ordered set is any set \( S \) with a binary relation \( \preceq \) which satisfies these laws:
   - Reflexive: For all \( x \in S \), \( x \preceq x \)
   - Antisymmetric: If \( x \preceq y \) and \( y \preceq x \), then \( x = y \)
   - Transitive: If \( x \preceq y \) and \( y \preceq z \). Then \( x \preceq z \).

2. A simple or totally ordered set (also called a chain) is an ordered set such that if \( x, y \in S \), then either \( x \preceq y \) or \( y \preceq x \).

Saaty uses the notation \( x^- = \{ y \mid x \text{ covers } y \} \) and \( x^+ = \{ y \mid y \text{ covers } x \} \) for any element \( x \) in an ordered set \( S \). The element \( x \) is said to cover (or dominate) the element \( y \) if \( x > y \) and there exists no \( t \) such that \( x > t > y \).

3. Let \( H \) be a finite partially ordered set with largest element \( b \).

   \( H \) is a hierarchy if:
   - a) a partition of \( H \) into sets \( L_k, \ k=1,\ldots,h \), where \( L_1 = \{ b \} \);
   - b) \( x \in L_k \) implies \( x^- \subseteq L_{k+1}, \ k=1,\ldots,h-1; \) and
   - c) \( x \in L_k \) implies \( x^+ \subseteq L_{k-1}, \ k=2,\ldots,h \).

Saaty next asserts that, for each \( x \in H \), there is a suitable weighting (or priority) function \( w_x \), whose nature depends upon the phenomenon being hierarchically structured, and which maps \( x^- \) into the interval \([0,1]\) such that

\[
\sum_{y \in x^-} w_x(y) = 1
\]

This summation reflects the normalization procedure discussed earlier.

We may think of the sets \( L_k \) as the levels of the hierarchy, and \( w_x \) as the priority function of the elements in one level with respect to some objective \( x \). Note that, even if \( x^- \not\in L_k \) (for some \( k \)), we may define \( w_x \) for all of \( L_k \) by setting it equal to zero for all elements in \( L_k \) not in \( x^- \). It is
this priority function \( w_x \) that permits us to develop this important application of hierarchy theory. Another definition:

4. \( H \) is complete if, \( \forall x \in L_k, x^+ = L_{k-1} \) for \( k = 2, \ldots, h \).

Now we state the basic problem. Given any element \( x \in L_0 \), and subsets \( S \subseteq L_\beta \), with \( \alpha < \beta \), how do we define a function \( w_{x,S} \) that maps \( S \) into the interval \([0,1]\) and which reflects the properties of the priority functions \( w_y \) on the levels \( L_k, \ k = \alpha, \ldots, \beta-1 \)? Less technically, suppose we have some structure system with one major objective or goal \( b \), and a set \( L_h \) of basic activities within that system such that the whole of it can be modeled as a hierarchy (i.e., with a largest element \( b \) and lowest level \( L_h \)). Then our present structure and definitions permit us to determine the priorities of the elements at any level \( L_i \) with respect to some element (objective) at level \( L_{i-1} \). How, then, do we determine the priorities of the elements of \( L_h \) with respect to \( b \)? Saaty's method for solving this basic problem follows.

Assume that \( y = \{y_1, \ldots, y_{n_k}\} \subseteq L_k \) and that \( x = \{x_1, \ldots, x_{n_{k+1}}\} \subseteq L_{k+1} \); in fact, we may assume that \( Y = L_k \) and \( X = L_{k+1} \) by setting the priority functions at each level equal to zero for those elements of \( L_k \) not in \( Y \) and of \( L_{k+1} \) not in \( X \). Note that \( n_i \) denotes the number of elements in \( L_i \). Next, we assume there is an element \( z \in L_{k-1} \) such that \( y \subseteq z \). Now consider the priority functions \( w_z \), which maps \( Y \) into the interval \([0,1]\), and \( w_y \), which maps \( X \) into \([0,1]\). Next, construct the "priority function of the elements in \( X \) with respect to \( z \)," denoted \( w \), such that \( X \) is mapped into \([0,1]\), as follows:

\[
   w(x_i) = \sum_{j=1}^{n_k} w_y(x_j) \cdot w_z(y_j), \quad 1 = 1, \ldots, n_{k+1}
\]

This process simply weights the importance (or priority) of \( x_i \) with respect to the element \( y_j \) by multiplying it by the importance of \( y_i \) with respect to the element \( z \).
We can simplify the algorithmic process if we combine the $w_i(x_i)$ into a matrix $A$ by setting $a_{ij} = w_j(x_i)$.

Also, set $W_i = w(x_i)$ and $W_j = w_z(y_j)$, with $i=1,...,n_{k+1}$ and $j=1,...,n_k$.

Then (17) becomes:

$$W_i = \sum_{j=1}^{n_k} a_{ij} W_j', \quad i=1,...,n_{k+1}. \tag{18}$$

Now, we may conceive of the priority vector

$$W = (W_1,...,W_{n_{k+1}})$$

and of the priority matrix $A$, and formulate (18) as:

$$W = AW'. \tag{18}$$

Saaty summarizes all of the above into a principle of hierarchical composition. Given two finite sets $S$ and $T$, let $S$ be a set of properties and $T$, a set of objects having those properties as characteristics. Assume a priority function $w_j > 0 (j=1,...,n)$ for each $s_j \in S$ such that

$$\sum_{j=1}^{n} w_j = 1,$$

and assume also a priority function $w_{ij} > 0 (i=1,...,m)$ for each $t_i \in T$, relative to $s_j$. Then the convex combination of $w_{ij}$:

$$\sum_{j=1}^{n} w_{ij} w_j'$$

gives the numerical priority or relative importance of $t_i$ with respect to $S$.

This principle can, of course, be generalized to a chain of sets.

Now, one final definition:

5. Suppose that for each subgoal or activity $C_j$ in $L_k$ there is an ordinal scale $o_j$ over the activities $C_\alpha (\alpha=1,...,n_{k+1})$
in $L_{k+1}$. We define a partial order over $L_{k+1}$ as one in which

$C_{\alpha} \geq C_{\beta}$ if and only if $a_{\alpha j} \geq a_{\beta j}$ for $j=1, \ldots, n_k$.

With the following two theorems, which Saaty presents without proof, we validate the principle of hierarchical composition by showing that the ordinal preferences are preserved under composition:

**Theorem 1.** Let $w_{ij} = (w_{1j}, \ldots, w_{k+1j})$ be the eigenvector for $L_{k+1}$ with respect to $C_j$, and assume it preserves the order of the $a_{ij}$. Let $W_1, \ldots, W_n$ be the composite priority vector for $L_{k+1}$. Then $C_{\alpha} \geq C_{\beta}$ implies that $W_{\alpha} \geq W_{\beta}$.

**Theorem 2.** Let $H$ be a complete hierarchy with largest element $b$ and with $h$ levels. Let $A_k$ be the priority matrix of the $k$th level, $k=2, \ldots, h$. If $W'$ is the priority vector of the $p$th level with respect to some element $z$ in the $(p-1)$st level, then the priority vector $W$ of the $q$th level ($p<q$) with respect to $z$ is given by:

$$W = A_q \cdot A_{q-1} \ldots A_{p+1} W'.$$

Thus, the priority vector of the lowest level with respect to the element $b$ is given by:

$$W = A_h \cdot A_{h-1} \ldots A_2 W'.$$

If $L_1$ has but a single element, as has been our convention, then $W'$ is just a scalar; otherwise, it is a vector.

The consequence of these two theorems is to justify the matrix multiplication technique for weighting eigenvectors by showing that it preserves ordinal preferences for the elements at the lowest hierarchical level. We believe it important to mention that nothing in this mathematical formulation
imputes meaning to the difference in magnitude between the numerical priorities of any two elements in a hierarchical level; i.e., $w_{ij} - w_{i+1,j} = \gamma > 0$ implies only that the priority of element $x_i$ is greater than that of element $x_{i+1}$. The magnitude of $\gamma$ does not acquire any additional ordinal significance. However, we cannot be blamed for intuitively attaching more significance to the priority differences of three objects represented by a priority vector such as $(.75, .15, .10)$ than to a vector such as $(.35, .33, .32)$.

Likewise, this mathematical formulation contains nothing that might rule out its application to inconsistent priority matrices. Nonetheless, we still need to address the problem of measuring the impact of such inconsistency in hierarchical composition. Saaty asserts that such a measure has the same format as $(\lambda_{\text{max}} - n)/(n-1)$, discussed earlier. If $H$ is a hierarchy of $h$ levels $L_i$, let $n_{ij}$ be the dimensionality of the priority matrix $A_{ij}$ with respect to the $j$th element in $L_{i-1}$, and let $\lambda_{ij}$ be the corresponding eigenvalue. Then the consistency index $I$ is:

$$I = \frac{h \prod_{i=3}^{n_i-1} \lambda_{ij} - \prod_{i=3}^{n_i-1} n_{ij}}{(n_2 - 1) \prod_{i=3}^{n_i-1} \sum_{j=1}^{n_i} (n_{ij} - 1)} \geq 0. \tag{22}$$

Saaty does not indicate the probability distribution of $I$; most likely, that distribution would have to be determined experimentally.

One specific advantage of Saaty's preference scale not addressed before is its self-correcting capability relative to consistency. In matrix theory it is known that the eigenvalues are continuous functions of the matrix coefficients $(a_{ij})$. If we perturb the coefficients of a consistent matrix, $\lambda_{\text{max}}$ will remain close to $n$, while the $n-1$ other eigenvalues will remain close.
to zero. It is also known that $\lambda_{\text{max}}$ is a monotonic increasing function of $a_{ij}$. Hence, in a reciprocal matrix engendered by the use of that preference scale, a deviation (increase) in $\lambda_{\text{max}}$ induced by a departure from consistency in $a_{ij}$, is compensated for by a reduction induced by $1/a_{ij}$.
REFERENCES


## A Framework for Policy-Level Logistics Management (Vol. III, A Macro Analysis of DoD Logistics Systems)

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This report, the third and final volume of a series on Macro Analysis of DoD Logistics Systems, provides a framework for policy-level logistics management. The framework includes a decision structure, a set of techniques for its use, and specification of information requirements, including management indicators. The underlining research was sponsored by the Office of the Secretary of Defense.