RESEARCH ON THE NATURE OF GROUND TILTS IN THE PERIOD RANGE ONE

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RESEARCH ON THE NATURE OF GROUND TILTS IN THE PERIOD RANGE $10^3$ TO $10^7$ SECONDS

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The apparent tidal tilt anomaly first identified at Bedford, Massachusetts is shown to extend to Maynard approximately 17 miles WSW and to be attributable to ocean tide loading effects. The analysis technique used has been demonstrated to be capable of determining the M_2 tide with a standard deviation of 2 nanoradians using a single month's data. Evidence is presented that the observed atmospheric pressure effects are site rather than instrumental in nature. While stability of individual deep borehole installations is sub-
stantially better than for shallow installations local effects still mask true secular ground movement. The nature of the tilt records is such that simple averaging will not significantly improve the secular tilt estimates. It appears that new techniques capable of recognizing only those effects which act simultaneously on all tiltmeters are required.
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1. INTRODUCTION

The ability to make precise measurements of tilt has become increasingly important in the testing and use of gyros, precise surveying, and monitoring slow earth and structural deformations. In addition, measurement of variation in the horizontal (tilt) and vertical components of gravity have long been recognized as potentially powerful methods of measuring the low frequency elastic properties of the earth. In general, the vertical component tends to respond to the global properties of the earth while the horizontal component (tilt) is more sensitive to local structural discontinuities.

Most secular deformation of the earth's crust, including deformation preceding earthquakes, will be accompanied by tilting. Tilt anomalies caused by artificially induced fracturing (massive hydrofractures) have been measured in the field by Wood (M.D. Wood, personal communication).

In the tidal frequency band, analysis of ocean tide loading provides a direct measurement of the elastic properties of the underlying crust (see, for example, Lambert, 1970). Calculations by Beaumont and Berger (1974) show that significant changes of the tidal tilt signal should result from dilatancy which might precede earthquakes in many regions.

An extensive network of near-surface tiltmeters has been deployed by the U.S. Geological Survey and cooperating universities to attempt to measure the secular tilt as part of the nation's earthquake prediction program (Johnson and Mortenson, 1975). The U.S.G.S. instruments and installation techniques do not however appear to be designed for the resolution of tidal tilt anomalies and the results of the secular measurements do not yet seem to have lived up to expectations. Hopefully, the disappointment over the results of that program will not result in abandonment of the concept of borehole tiltometry as an earthquake prediction tool.

In contrast to the U.S.G.S. approach, the AFGL tilt program is based on the premise that reliable methods for
separating true ground motion from extraneous site noise should be developed and tested in areas of known tilt before any attempt is made to extend the measurements to areas of unknown tilt.

From the point of view of tiltmeter testing eastern Massachusetts is ideal. Bedrock in the site areas is relatively competent and stable. The only secular deformation is that thought to be caused by post-glacial uplift. Results from first-order leveling as well as theoretical considerations indicate that this should be less than 20 nanoradians per year in the site area. Because of this stability we do not anticipate that our records will be contaminated by transients caused by earthquake precursors, real or imagined.

At the high frequency end of the spectrum the tidal signal is roughly equally partitioned between the direct body tide and the indirect ocean loading effects. While focusing our attention on the problem of accurate measurement of the tilt in two narrow frequency bands, near DC and in the tidal band, where the signal is expected to be extremely consistent, we have nevertheless uncovered evidence for a number of contaminating effects which may be expected to spread across a much broader portion of the tilt spectrum. These include: ocean storm loading, atmospheric pressure, pore pressure, groundwater convection, thermoelastic coupling, and rainfall.

Some of these effects have been shown to be dependent on the depth and method of installation while others are installation-independent. The most obvious positive result of this study has been the improvement of techniques for separation of body tide and ocean tide loading effects, and hence to the structural properties of the crust underlying New England. However, we believe that the demonstration of the complications which may arise from other transient phenomena has an even broader significance. In particular the failure to recognize and remove contamination due to atmospheric pressure and rainfall, sometimes acting through near surface discontinuities,
may lead to false conclusions regarding the true geophysically significant ground motion. These effects can affect any tilt array whether it is designed to study elastic property variations, ocean tide signals, site stability, man-made deformations or the search for earthquake precursors. (See, for example, Herbst, 1976, Wood 1977).

In 1970, the first tripartite tiltmeter cluster was installed by AFGL at Bedford, Massachusetts. Analysis of the data (Cabaniss 1974, McConnell, et al. 1973) from the Bedford tiltmeter cluster had shown that short baselength tiltmeters could be emplaced at the center of cylindrical boreholes in such a manner that they yield consistent results at tidal and higher frequencies. More specifically the studies concluded:

1. Borehole tilt measurements could eliminate most of the extraneous strain induced tilts associated with other types of underground installation.

2. Measurements of the tilt spectrum at tidal frequencies can be obtained with a precision of 5 nanoradians or better when records over several months are analyzed.

3. The orientation of the tilt ellipse can be obtained within $1 \pm 0.5^\circ$ or better when a biaxial instrument of appropriate sensitivity was used.

4. Theoretical studies using simple earth models allow prediction of ocean tide induced tilts within 20% in amplitude in the direction of the semi-major axis.

5. Measurement of the ocean tide induced tilts provides a powerful method of studying the elastic properties of the earth and the transmission of stresses through it.

Of particular interest was the observation that the $M_2$ tidal components recorded by separate instruments the Bedford array, although in agreement with one another, differed from those predicted from theoretical considerations. Whether this discrepancy was due to local or regional geological effects, uncertainty in the ocean tide loading function, or fundamental
errors in the theory of tidal tilts was not yet known.

In spite of the demonstrated coherence at tidal frequencies, there was virtually no observed agreement at periods longer than a few days. These long period incoherent tilts are normally lumped in the category "drift." Such drift can be attributed to a number of factors, including changes in pore pressure in the neighborhood of the borehole, effects of the borehole/tiltmeter coupling and instrumental effects. At the time this study began no definitive study had yet been carried out, nor had a suitable technique been developed to separate these various effects. Development of techniques for separation and correction of "drift" is a prerequisite to determining the true low frequency ground movement using tiltmeters.

We proposed to utilize the data presently being collected from the Bedford and Maynard tiltmeter arrays to attempt to resolve the problems of anomalous tidal tilts and to investigate apparent secular tilts.

With respect to the problem of tidal tilts our plan was to compare tidal tilts at Maynard with those at Bedford to determine whether the anomalous tilts were purely a local phenomenon or were more regional in nature. If they were found to be regional an attempt would be made to find an explanation, in light of geological structure of New England, ocean tidal effects or invalid assumptions in the theory of solid earth tides.

A second area of emphasis of the project study was to develop experimental and analytical procedures for separating true ground movement from instrumental and installation effects, particularly for the Maynard array.

As a result of the initial work, several areas where further improvement in tilt measuring technology was required became apparent. These included: improvement in shallow borehole field installation techniques; improvement in techniques for bridging data discontinuities; in-depth analysis of the transients observed over the entire time history of the Bedford-Maynard tilt network.
In this report our major emphasis is on the deep borehole measurements from the Maynard cluster, in particular on the measurement of the total semi-diurnal tide and the direct determination of the relative contributions of the body and ocean tide effects. The problems of shallow hole measurements, and secular tilts are discussed more briefly.
2. THE AFGL EASTERN MASSACHUSETTS TILT NETWORK

At present the AFGL eastern Massachusetts tilt network includes a total of eight permanent bedrock boreholes (Figure 2-1); three at Bedford (42°28.6'N, 71°17.6'W), three at Maynard (42°25'N, 71°29'W), and two at AFGL's Haskell observatory (42°27.1'N, 71°16.3'W). An additional three biaxial remote levelling tiltmeters suitable for installation in shallow holes have been installed in the overburden within the limits of the Maynard bedrock cluster.

As the work described in this report is based primarily on measurements made at the Bedford and Maynard clusters, a more detailed description of these is appropriate.

2.1 Bedford Cluster

Each Bedford tiltmeter houses a pair of diamagnetic type sensors based on a cylindrical mass suspended in a magnetic field (Simon, 1971). The motion of the mass is detected electro-optically, amplified and low pass filtered, and recorded digitally at half minute intervals on magnetic tape. The resolution of the digitizer was such that the least count ranges from 2 - 10 nanoradians.

To minimize the effects of tilt strain coupling and thermal effects the tiltmeters are installed in vertical boreholes drilled through 0.5 to 1 m of overburden and 15 to 20 meters into a foliated granitic gneiss.

Below the overburden the holes were uncased thus allowing free flow of water into them. The instruments were placed at the bottom of the hole and held by wedges against the sides (Figure 2-2).

The instruments were oriented to ± 1° (optimistically), approximately NW-SE and NE-SW so that the signal amplitude would be approximately equal on both components (i.e. M2 amplitude about 100 nanoradians).

2.2 Maynard Deep Cluster

In the winter and early spring of 1975 a second cluster
containing three biaxial instruments of the servoed flat bubble type, (Earth Sciences Research, Inc. 1975) was installed at Maynard, Massachusetts, about 17 km WSW of Bedford (Figure 2-1). As in the case of the Bedford cluster three holes were drilled about 100 meters apart into a foliated granitic gneiss. Several changes were made in the installation technique for comparison with the approach used at Bedford. The Maynard holes were drilled through about 20 meters of overburden and 100 meters into the bedrock. The increased depth was intended primarily to eliminate the annual temperature variations which had been shown to be significant at depths of 20 meters at Bedford. The holes were cased to the bottom, which was sealed, and grouted along the outside to insure good contact with the rock.

Casing both facilitated installation above the bottom of the hole and reduced the danger of loose rock fragments permanently jamming the instrument in place. The sealing kept the holes dry eliminating the effects of convecting groundwater which had been shown to cause seasonal high frequency noise at Bedford.

Unlike the Bedford tiltmeters, the Maynard instruments were supported well off the bottom of the hole by spring loaded platforms as shown in Figure 2-2. Two of the instruments were placed 3 meters above the bottom of the hole, the third was placed about 70 m above the bottom.

As at Bedford, the data was amplified, lowpass filtered and recorded digitally on magnetic tape. The resolution of the digitizer was such that at the highest operating gain the least count was 0.4 nanoradians. Actual recording was made with least counts varying from 2.0 to 0.4 nanoradians in order to maintain as high a gain as possible without mechanically releveling the instruments.

2.3 Maynard Shallow Cluster

In addition to the deep hole installations at Bedford and Maynard which are capable of measuring tidal tilts to a high
FIGURE 2-2 Bedford (A) and Maynard (B) installation techniques.
degree of accuracy a shallow hole cluster (approximately 3 meters depth of burial) was installed at the Maynard site. Our previous experience had indicated that shallow installations were generally seriously disturbed by the effects of rainfall and temperature. However, the success claimed by the U.S. Geological survey in operating shallow installations, together with their much lower cost, prompted us to undertake a reexamination of the adequacy of shallow installations for our progress. Details of the shallow installation technique together with preliminary tidal analysis results are discussed in more detail in Lewkowicz and McConnell (1977).

2.4 Instrument Calibration

Although originally calibrations were believed to be accurate to no better than 5 percent the procedure, now applied to all instruments prior to installation, has been improved to the extent that calibrations of better than 1 percent can be made routinely. The present technique uses a precision tilt table designed by L. Burris of Lacoste and Romberg Inc. and manufactured by Radian Corporation. The table has a range of approximately 100 microradians and a least count on the levelling screw readout of 0.1 nanoradians. The scale factor of the table varies uniformly over the range with total variation demonstrated to be less than .4%. Repeatability during a run is usually better than 0.1 microradians.

One or more tiltmeters are placed on the table and adjusted until the null reading corresponds to the mid-range position of the table. The table is then tilted to one end of its range using the calibrated levelling screw. The output of the tiltmeter is measured at a series of table positions approximately 5 microradians apart, extending from one end to the other of the range. A complete traverse from one end of the table to the other requires approximately 25 minutes. Particular care is taken to avoid backlash in the levelling screw. Tiltmeter output voltages are recorded at each step. The above process is then repeated in the opposite direction.
A least squares fit of a straight line is then made to the upward and downward calibration results separately. Three plots are prepared using an HP9830A calculator equipped with a digital plotter. The first shows the actual data together with the best least-squares fitting straight lines. The second shows the residual as a function of the tilt of the table. The third plot gives the residual as a function of output voltage. By comparing the two sets of residual plots one can usually determine whether the residuals result from nonlinearities in the instrument, the tilt table, or simple random noise.

Figures 2-3 and 2-4 show actual calibration plots for one of the tiltmeters used.
**FIGURE 2-3** Calibration curve for one of the tiltmeters used.
FIGURE 2-4  Tiltmeter calibration residual.
3. DATA REDUCTION AND ANALYSIS

Figure 3-1 presents an overview of the collection, reduction and analysis procedure. The major steps are as follows:

1. Raw data, consisting of tiltmeter signals together with such environmental parameters as atmospheric pressure, temperature, and standard cell voltage are sampled at 1/2 minute intervals and recorded on magnetic tape.

2. The original data tape is then used to generate a new tape containing 20 minute triangularly weighted means at 1/2 hour intervals.

3. The resulting reduced time series is edited with linear interpolation over short (1.5 hour or less) sections where data is absent or apparently erroneous. Gaps are introduced over longer sections of unreliable data.

4. The time series is then analyzed using the standard Vanicek least squares approach to determine the coefficients of the selected components of the times series. These include linear trend, individual tidal constituents, entire tide species, pressure effects and datum shifts.

5. Where datum shifts occur in the original series an optional second pass is made during which the residual series from step 4 is adjusted to minimize distortion across the shifts and then reprocessed to give final coefficients.

Most of the problems associated with steps 1 and 2 had been satisfactorily resolved by AFGL prior to commencement of the present contract. The only major exception was locating sporadic unpredictable sign reversals which was ultimately
FIGURE 3-1 Data flow.
traced to a design error in the NLS data acquisition system.

The bulk of our data reduction effort was devoted to steps 3, 4 and 5.

Because of the long time required to obtain enough data for the analysis of tides and secular tilts, data acquisition must often be interrupted for such reasons as power failure, instrument servicing, tape changes or the necessity of releveling. These interruptions often result in transient effects, unrelated to true ground movement, which must be eliminated prior to analysis. Removal of these unwanted effects takes place in steps 3 and 5 and is discussed in more detail below. Step 4 is the major subject of chapter 4 and Appendix 1.

3.1 Data Editing

A system of computer programs was written for the AFGL CDC6600 computer to assist in analysis of the digital data from the tilt network. These programs initially provided for:

1. Automatic scanning of decimated data tapes to detect and correct for relevels, datum shifts and gaps.
2. Introduction of gain and/or calibration changes within the data set.
3. Provision for merging, ordering, and elimination of redundant discontinuity information from manually entered and automatically recognized transitions in the data.
4. Interfacing to the Vanicek least squares data analysis programs.

When reduction of the Maynard tiltmeter results was attempted on a routine basis it was found that malfunctions of the NLS digital data acquisition system had introduced sporadic spurious polarity reversals. While most of these could be removed by the AFGL preprocessing programs some reversals of data polarity still remained. This caused erroneous values to be introduced in some of the .5 hour averages which were to be used for the actual tilt analysis. These, together with
occasional data gaps and extraneous data resulting from periodic tests, required the development of an editing capability in the main processing programs.

We therefore added three new features to the data reduction programs. These included the ability to flip the sign of one or more channels of data over any prescribed interval; the ability to linearly interpolate data over a specified number of points before beginning the tidal processing and a major rearrangement of the in-core storage of data.

3.2 Datum Shifts

Our original analysis program used only the standard Vanicek method to eliminate drift effects. This fits a linear trend to the entire time series and imposes datum shifts which force the residual series, with the trend and fitting functions removed, to have a mean of zero over each segment between discontinuities. This approach has the unfortunate characteristic that when the trend is not linear, noise may be introduced into the resulting spectrum at all frequencies of interest from secular to tidal. In order to eliminate this problem we first concentrated on fitting a separate trend to each interval between discontinuities. It was soon recognized however that both the single trend and the multiple trend approaches will give rise to spurious offsets in the presence of transients resulting from interruption of normal tiltmeter operation. These spurious effects will in turn induce secular tilts where none in fact exist. We therefore sought improved techniques for bridging the discontinuities.

The procedure eventually adopted utilized the assumption that the best estimate of the behavior of the time series over a data gap which includes a datum shift is the behavior of the detided series over intervals on either side of the gap. For purposes of automatic reduction the length of the interval to be examined was chosen to be equal to the length of the gap.
4. THE SEMI-DIURNAL TIDES

Tidal tilt may be produced by the gravitational field of the moon and sun interacting directly with the earth and indirectly through the ocean tides. Similarly, direct and indirect strains of the same order of magnitude as the tilts are produced. These strains in turn may interact with cavities, inhomogeneities, and topography to produce secondary tilts which can be comparable with the direct effects. Measured tilt may be further contaminated by periodic thermoelastic effects, temperature, barometric pressure, and ground water fluctuations.

As discussed in the introduction, prior to commencement of this study, the feasibility of consistent measurement of tidal tilt had been established by measurements at the Bedford site. Since one of the major purposes of the present study was to determine the extent of the apparent anomaly detected at Bedford, a brief review of these results is in order.

4.1 Background: Bedford Tides

The Bedford data, after removal of offsets, power failure recoveries, etc., were detrended, digitally filtered, decimated to 0.5 hour interval and analyzed for 12 constituents using least squares over an interval from Dec. 10, 1972 - March 3, 1973. Two partially overlapping intervals of 55 days were separately treated. For instruments No. 1 and No. 3 the $M_2$ tidal ellipse was computed and compared. The most notable results of the analysis were as follows:

1. The phase of the semi-major axis of the $M_2$ tidal ellipse from the two instruments agreed within $2^\circ$.
2. The orientation of the semi-major axes of the $M_2$ tidal ellipses agreed to within $1.3^\circ$ in azimuth, well within the uncertainty of the instrument orientation.
3. The amplitude of the semi-major axes of the $M_2$ tidal ellipse agreed to within 2.1 nanoradians or 2 percent, well within the uncertainty in the calibration that had been obtained at that time.
The above agreement held to 2 nanoradians in semi-major amplitude and 1° in phase and azimuth when the data were analyzed for two partially overlapping intervals of 55 days, thus demonstrating that stationarity prevails between the first and second half of the time period at this frequency.

Although the ellipse for instrument no. 2 could not be calculated because of a malfunction of one of the two orthogonal sensors, the tilts in the direction of the operational sensor were consistent in amplitude and phase with the results from the other two instruments.

The excellent agreement between the Bedford tiltmeters, more than 100 meters apart and at different depths established that results could be obtained which are representative of tidal effects at the particular site. Since the instruments were all oriented in approximately the same direction it provides no information on possible end effects through strain tilt coupling at the bottom of the borehole.

By comparison, Lennon and Baker (1973) reported Bidston observed ellipse differences of 15% in amplitude, 2° in azimuth and 4° in phase for different instruments in the same vault. Such variations have been ascribed by King and Bilham (1973) and Harrison (1976) to strain induced tilting through cavity effects.

While the results obtained were internally consistent they were otherwise somewhat unexpected. In particular, the $M_2$ ellipse (Figure 4-1) is more like a straight line. The amplitude of the semi-major axis is approximately 135 nanoradians while the semi-minor axis is less than 1 nanoradian. As the corresponding ellipse for the theoretical body tide has a semi-major axis of 37 nanoradians in the E-W direction and a semi-minor axis of 26 nonradians, there was an unexplained effect larger than the theoretical body tide in the E-W direction and almost exactly equal and opposite to the body tide in the N-S direction.

Since the Bedford tiltmeter cluster is located within 25
FIGURE 4-1 Bedford M2 ellipses.
km of the Atlantic coast the most obvious source of the apparent anomalous results is ocean tide loading. As a first test of this hypothesis a theoretical body tide time series was generated directly from the astronomical arguments, assuming a radially stratified earth and a value of the diminishing factor of 0.7 (Cabaniss 1974). The theoretical body tide was subtracted from the observed time series. The residual series which should approximate the "observed load tide" was then analyzed in the same manner as the original time series had been. Resulting ellipses are shown in Figure 4-2. Several theoretical ocean tide loads based on different models of the Atlantic $M_2$ distribution were then tested against the residual (McConnell et al. 1973) and were found incapable of explaining all the anomaly. Discrepancies of approximately 25 nanoradians remained in both the North-South and East-West directions.

4.2 Initial Maynard Results

Our initial approach to the analysis of Maynard data was the same as that used for the Bedford data with the exception that a different least squares analysis technique was utilized. The Maynard tidal analysis was carried out using a technique and programs developed by Vanicek (1971) which makes simultaneous fits of specified frequencies, linear trends, datum shifts and user defined time series to the observations. This technique eliminates the necessity of manual removal of trends and datum shifts. Figure 4-3 shows the $M_2$ ellipse resulting from a least squares fit of the 12 tidal components used in the analysis of the Bedford data over approximately 120 days between December 1977 and May 1977 for each of the Maynard instruments. Characteristic features observed in the Bedford $M_2$ signal are also obvious at Maynard. Again there is an extremely flat ellipse oriented approximately east-west with amplitude of approximately 100 nanoradians. About 25% of the 35 nanoradian decrease may be attributed to an increase in distance from the coast. We conclude that the flattening of
FIGURE 4-2  Comparison of computed $M_2$ load tide with difference between observed and theoretical body tide.
FIGURE 4-3 Maynard M₂ ellipses for 120 days of data.
the M₂ ellipse is neither a peculiarity of the Bedford site nor a result of tilt-strain coupling caused by proximity to the ends of the boreholes. We are therefore forced to choose between a regional structural effect extending over at least 20 km and the possibility that the uncertainty in our knowledge of the offshore ocean tide load is even poorer than we had previously suspected.

4.3 Stationarity Problems

One possible way to separate ocean loading effects from body effects is to make use of the differences in the ratios amongst the principal tides. This concept, first used by Doodson and Corkan in 1934 was extended by Melchior (1966) and Lambert (1970). Since the ratios of the amplitudes of the principal components of tilt caused by body tides will have the theoretical body tide ratio while those caused by ocean tides will have the ratios characteristic of the latter in the region causing the tilts, the two should be separable. Our first attempts to make such a separation were frustrated by recording difficulties which left us with insufficient Maynard data for stable estimates of even the M₂ amplitude. S₂ and N₂ which would also be required, fluctuated by factors of 2 or more. Typical results for M₂ are shown below in Table I for a 5 component fit (K₁, O₁, M₂, S₂, N₂).

<table>
<thead>
<tr>
<th></th>
<th>#1</th>
<th>#2</th>
<th>#3*</th>
<th>#1</th>
<th>#3</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAY021, 022, 024 (30 days)</td>
<td>108</td>
<td>96</td>
<td>92</td>
<td>1.13</td>
<td>.96</td>
</tr>
<tr>
<td>MAY031, 032, 033</td>
<td>153</td>
<td>97</td>
<td>86</td>
<td>1.58</td>
<td>.89</td>
</tr>
<tr>
<td>MAY058, 059 (20 days)</td>
<td>118</td>
<td>110</td>
<td>108</td>
<td>1.07</td>
<td>.98</td>
</tr>
<tr>
<td>MAY060, 061</td>
<td>122</td>
<td>106</td>
<td>113</td>
<td>1.15</td>
<td>1.07</td>
</tr>
</tbody>
</table>

*For the tiltmeter in Hole #3 a scale factor of 10.0 micro-radians per volt was assumed as it was decided not to remove this instrument for calibration after its "temporary" installation in May 1975.
It is clear from the table that the $M_2$ amplitude computed using the above approach does not give consistent results over a short series. If the ratios of the amplitude between different instruments remained constant then the difficulties could be attributed to neglect of other principal components. Unfortunately this is not the case and some other explanation for the fluctuations must be found.

A second conclusion to note is that the $M_2$ amplitude fluctuates with time. This is to be expected and might be attributed to the neglect of other major components. One possible approach to removing the discrepancy would be to solve for more components. Unfortunately this will also introduce more degrees of freedom into the solution and may increase the uncertainty in each individual component. Clearly an approach must be found which allows for the minor components without introducing additional uncertainty.

The tabulated results also show that the discrepancies between instruments cannot be entirely explained by the neglect of the minor components or by calibration uncertainties. If this were the case then the ratios of the computed $M_2$ amplitudes between instruments would remain constant even if the absolute values fluctuate in time. We therefore sought a perturbing influence which acts differently on each instrument and which has components near tidal frequencies.

Additional evidence for this type of perturbing effect came from our attempts to determine relative instrument orientation which at that time had not been measured. As discussed in a previous report, we expected that an improved estimate of the relative orientation might be obtained by applying a simple filter which passes diurnal and higher frequencies. A program was written to do this but the orientation results obtained were less consistent than those based on the $M_2$ signals alone.

We concluded therefore that if the consistency of the tidal estimates was to be improved:

1. A method would have to be found to introduce the influence of the minor components without increasing the
number of unknown variables.

2. And the cause should be determined for the non-coherent energy near tidal frequencies.

4.4 Separation of Ocean and Body Effects

Fortunately the solutions to the separation problem and the short time series problem turned out to be one and the same. Recent earth tide analysis methods (Melchior, 1978) rely on separation of each theoretical tide species into groups containing several constituents of closely related frequency. A least squares procedure is then used to determine the contribution of each group. The approach ultimately selected as being most appropriate for our requirements is described in more detail in Appendix I. It differs from the above in that we divide each species into an ocean-like group and a body-like group each group containing all the frequencies in the species. Each species is represented as the complex product of its dominant tide and a modulation function. We show in Appendix I that as long as:

1. The various constituents of the ocean tide have relative phase lags and/or amplitudes different from the body tide the ocean and body contributions may be separated using this method.

2. The ocean loading modulation functions are homogeneous over the region influencing the tilt the entire ocean loading function may be represented by its modulation at a single point within the region.

Figures 4-4 and 4-5 show the difference in phase lag between the various constituents as a function of frequency for the Boston ocean tides. As the body tide effects may be safely assumed to be frequency dependent, the first condition is clearly satisfied.

Figure 4-6 illustrates the homogeneity of the $S_2/M_2$ ratio of the Atlantic coast ports and the relative inhomogeneity of the $O_1/K_1$ ratio.

Thus both conditions are well satisfied along the coast.
FIGURE 4-4  Phase lag differences as a function of frequency for Boston diurnal ocean tides.
FIGURE 4-5
Phase lag differences as a function of frequency for Boston semi-diurnal ocean tides.
FIGURE 4-6 Complex amplitude and phase lag ratios for Atlantic coast semi-diurnal and diurnal tides.
by the semi-diurnal tides, but only the first is satisfied by the diurnal tides. If we make the assumption that: the most significant constituents of each species have the same ratios and relative phase lags for several thousand kilometers off shore as they have along the coast where they are well known; and that all constituents of a given species interact with the earth in the same way, we may then proceed to attempt a separation of ocean and body effects.

Before proceeding with such a separation it is instructive to examine the way in which a combination of body and ocean loads opposing one another can produce a series which is strikingly different than from either of the original series. This is well illustrated by the problem of the very small observed N—S $M_2$ tide.

The theoretical body tide in this direction at Maynard is approximately 26 nanoradians while the net observed amplitude is about -8 nanoradians (8 nanoradians 180° out of phase with the tidal potential). If the small observed tide were the result of some type of strain-tilt coupling effect attenuating the body tide the modulation would be that of the body tide. The semi-diurnal portion of the observed tide would be expected to look like the upper series of Figure 4-7. In the unlikely case that the small tide was due to ocean tide loading effects alone, the series should look like the middle example in the same figure. The lowest example in Figure 4-7 shows what happens when 26 nanoradians of body tide is combined with -34 nanoradians of Boston ocean tide. For all three examples, analysis for $M_2$ alone would indicate an amplitude of 8 nanoradians and identical phases. The most casual comparison of the three series however, shows distinct differences in the modulation which should be easily resolvable by an appropriate least squares technique.

The results of one separation attempt are shown in Figure 4-8. The "unbiased" separation yields a best least squares fit consisting of two counter rotating ellipses, a "body-like"
FIGURE 4-8  Typical separation of ocean and body effects at Maynard.
ellipse, similar in amplitude and phase to the theoretical body tide, and a larger "ocean-like" ellipse which reinforces the body tide in the east-west direction and cancels it in the north-south. The sum of the two is the observed $M_2$ tide. Although the entire separation technique relies on small differences in amplitudes and phase lags between minor components it yields a solution much like that obtained by simply subtracting the theoretical body tide from the original $M_2$ series (Figure 4-2). The solution however, is independent of any assumptions about whether the anomaly is primarily caused by ocean loading effects.

We must therefore, conclude that the major source of the difference between the observed $M_2$ tilt and the theoretical body tide in eastern Massachusetts is attributable to ocean loading effects.

One of the major features of the separation of the anomaly into body-like and ocean-like components is that the technique requires no assumptions regarding the manner in which the two forcing functions interact with the earth to produce the required tilt, except that the solid earth transfer function is frequency independent over the band encompassing the particular tidal species under investigation. The separation then provides no information on the extent to which the measured anomaly is influenced by tilt-strain coupling through local departures of the earth from an idealized isotropic elastic material. The evaluation of the contributions of topographic effects, cavity, and structural effects must be determined by other means. Although preliminary analysis of the Maynard data indicates that the individual boreholes may be subjected to such local perturbations the predominant pattern is one of an east-west tilt with reinforcing body and ocean tide and a N-S tilt where the two effects cancel to within 8 nanoradians or about 30 percent of the body tide at Maynard, and to within about 5 percent of the body tide at Bedford.
FIGURE 4-9 Comparison of results from 3 Maynard instruments for combined ocean and body series.
The shape of the ocean load ellipse is similar to that deduced by Cabaniss (1974) for the Bedford cluster and therefore is also inconsistent with that computed by McConnell et al. (1973) on the basis of published ocean tide models. Clearly improved ocean models are required.

4.5 Stability of Tidal Estimates; Earthquake Prediction

Beaumont and Berger (1974) have shown that in regions where dilatency is an earthquake precursor, this dilatency should be easily detectable through tilt and strain measurements. They show that for a typical model in mid-latitudes the ability to detect anomalies of the order of 2.5 nanoradians would permit the recognition of a 15 percent change in \( V_p/V_s \) at a distance of 35 km.

Four consecutive non-overlapping series, each containing 30 days of data, were analyzed for diurnal, semi-diurnal and long-period tides. The semi-diurnal terms were then examined for stability.

Figure 4-10a compares the computed amplitudes in terms of percent change in the computed \( M_2 \) semi-major axis with respect to the mean value for the instrument over the interval. Since the mean value is approximately 100 nanoradians each percentage point change corresponds to about 1 nanoradian.

Assuming that errors resulting from incorrect estimates of the ocean and body functions, as well as regional tide anomalies, act on each instrument in the same way, the scatter of the individual estimates about the mean at any time provide an estimate of the reliability of an individual measurement. On this assumption the standard deviation of an individual measurement is about 2 nanoradians. The standard deviation of the mean of 3 instruments is estimated to be .6 nanoradians.

Thus it is clear that even without further improvements in data processing techniques or instrumentation, improvements which we believe are possible, the Maynard cluster is a suitable prototype for a dilatency detection array.
FIGURE 4-10 Stability of $M_2$ over a four month interval at Maynard
Figure 4-10a shows some slight evidence for month to month fluctuations in the amplitude of $M_2$ of the order of 1 percent at the 60 percent confidence level. Whether these are in fact real remains to be investigated.

Figure 4-10b shows the azimuth of the $M_2$ principal axis with respect to the mean azimuth for each instrument. There is an obvious systematic clockwise rotation of about 3° per month. As the standard deviation of an individual measurement is about 2° and the standard deviation of the mean is about 1° we cannot attribute the slope to random errors in the data. The effect therefore requires further investigation.

Figure 4-10c shows the computed phase lag of the $M_2$ semi-major axis relative to the tidal potential. It is interesting to note that although instruments 2 and 3 are very stable there appears to be a systematic difference between them of 2°. This may be due to the fact that instrument 3 was not well calibrated prior to installation. With the exception of the third month value for instrument 1, for which at present we have no explanation, all the computed phase angles remain very constant in time.
5. PRESSURE EFFECTS ON THE MAYNARD CLUSTER

We have also located a possible source of the non-coherent signal near tidal frequencies. Analysis of the tapes showed a strong correlation of the detided residual signal with the barometric pressure monitor, particularly with hole #3 where almost the entire residual signal may be explained by this effect. Much of the pressure signal effect involves periods of the order of days and coefficients of the order of several nanoradians per millibar. Cross correlation of the residuals with the pressure showed lags varying from 0 to 2 hours between the pressure and the resulting tilt. As a result the tidal analysis program was modified to extract and remove the pressure effects.

In addition to the above effects we have evidence for anomalous tilts, greater than the normal pressure effect alone associated with winter storms. The cause of these remains to be investigated.

5.1 Instrumental Effects

Although the question of whether the pressure coefficients of the tiltmeter installations are due to the instruments themselves or to site effects remains unsettled at the present time, several lines of evidence indicate that it is not the former.

The tilt sensor is suspended from a thick symmetrical piece of stainless steel which is coupled directly to the instrument feet. There is no contact between the lower portion of the pressure case and the tiltmeter assembly except through the symmetric contact at the joint between the case and the tiltmeter base. Thus any effects would be second order in the departures from circular symmetry of the system. During the design of the instrument this was carefully studied and no potential sources of pressure effects was identified.

The instruments are identical in all respects and thus any pressure coefficient related to the instruments themselves should be the same on all three instruments. Yet the coefficients varied in both amplitude and direction.
The apparent lag between the changes in pressure and the instrument response to these pressure changes would be consistent with an instrumental effect only if there was a pressure leakage between the inside and the outside of the case, causing the optical flat to bow under pressure differences between the inside of the bubble chamber and the interior of the tiltmeter case. If such a bowing took place it would result in an apparent spurious tilt which we estimate to be of the order of 156 nanoradians/millibar, where $\delta$ is the displacement of the center of the bubble from the center of the flat in centimeters. Examination of the typical bubble position during bench operations indicate that $\delta$ is normally less than 0.1 cm. This would correspond to a pressure coefficient less than 1.5 nanoradians per millibar. Worst case displacements will be about 15 nanoradians per millibar.

The calculations above refer to the changes in pressure within the case. Because the tiltmeter case is designed as a pressure vessel the ratio of pressure changes inside to changes outside is of the order of 0.5x10^{-4}. Thus in the absence of leaks in the case the resulting pressure induced tilt due to bending of the bubble will be less than 1 nanoradian/bar.

We must therefore conclude that direct pressure effects on the sensor cannot be responsible for the observed relationship between pressure and tilt.

Another possible source of the pressure effect, and one that seems more likely, would involve some type of oil can effect on the pressure case which in turn moves the mule shoe. If this mule shoe is in contact with the alignment pin or other part of the hole lock then a change in tilt of the instrument would be induced.

5.2 Site Effects

Site effects must also be considered. Such site effects could be related either to pressure differences between the borehole and the rock or the external pore pressure acting on
non-uniform rock in the neighborhood of the tiltmeter. Site effects dependent on pore pressure are commonly observed in both tilt and strain installations. Both of these are probably adequate to explain the observed effects. Probably the simplest explanation is the direct pressure effect on a non-homogeneous, non-isotropic porous rock. Linear compressibilities of typical materials associated with a tiltmeter site are shown in Table II.

TABLE II
Compressibility

<table>
<thead>
<tr>
<th>Material</th>
<th>Bulk</th>
<th>Linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>.6</td>
<td>.2</td>
</tr>
<tr>
<td>Silica Sand</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Water</td>
<td>50</td>
<td>17</td>
</tr>
<tr>
<td>24% water - 75% sand</td>
<td>15</td>
<td>5</td>
</tr>
</tbody>
</table>

Units are nanostrains/millibar

As the values shown in the table are of the order of nanostrains per millibar, it would be reasonable to assume that the installation would have a coefficient of the order of nanoradians per millibar unless the ground is exceptionally homogeneous, isotropic and symmetrical about the drill hole. We should therefore not be surprised to observe coefficients of the same order of magnitude as actually observed.

If it were important to separate site effects from instrument effects the instrument could be removed, reoriented, and replaced in a different orientation however, this would seem to be inappropriate at the present time as the effects may be removed by standard regression analysis techniques. Rotation of the instruments would introduce the possibility of a long settling in period in a hole which is far dirtier than at the original time of installation.
6. SECULAR TILTS

Because of the difficulties with the malfunctioning data acquisition system, our secular tilt investigations focused on detided spot readings taken at approximately 10 day intervals during tape changes. These were supplemented by occasional readings taken other times the site was visited. Vector plots of the Maynard tilts were prepared for an interval of approximately 1-1/2 years. These plots were visually examined in a search for patterns which might lead to identification of a particular direction characteristic of the instrument construction. No such correlations were found, and we conclude that the long term tilts are not caused by one single portion of the instrument or holelock. Similar comparisons of the secular tilts when rotated to their true relative orientation showed no common trends. Thus we conclude that the bulk of the observed tilt must be related to the individual installations or sites.

While failing to recognize a common pattern of secular tilt among the 3 deep borehole instruments some generalizations can be made regarding their behavior. The major conclusions, most of which were presented at the 1977 AGU Annual Meeting in Washington are:

1. Deep hole tilt installations have much lower levels of extraneous noise than shallow hole installations for tilts with periods less than $10^6$ seconds. Most of the tilts in this period range are attributable to thermoelastic and meteorological effects.

2. At periods near $10^7$ seconds the measured tilts of all individual deep tiltmeters are comparable with the best results published for shallow instruments. The stability is substantially better than the values we have been able to obtain with shallow installations.

3. As much as a year was required for the Maynard deep tilt meters to settle down after initial installation. Whether this was an instrumental characteristic or some
installation effect such as aging of the cement around the borehole is not yet clear.

4. Once the instruments settled in all followed a pattern of tilt from which departures could be easily recognized. While the scatter in the tilt from an individual hole ranged from about 2 microradians per year to as much as 10 microradians per year, comparison of the directions, rates and times of the various tilt events between the three instruments reduced the uncertainty on the regional tilt of the site to the order of 0.5 microradins per year.

Although visual inspection and correlation (or lack of correlation) between the time of onset and direction of various tilt events enabled us to discard most of these as being confined to the particular installation, we were unable to arrive at a satisfactory method of automatic, or semi-automatic removal of tilts which did not appear on all three instruments. We believe that development of such a method is possible and should be a major objective of any further research in this area.
7. SUMMARY AND CONCLUSIONS

We summarize below the major conclusions reported above as well as those to be drawn from the previous scientific report (Lewkowicz and McConnell, 1977).

7.1 Short Period Tilts

1. There is a consistent regional apparent tidal tilt anomaly, caused by ocean tide leading effects, which cannot be explained by the ocean tide maps which we have tried.

2. The Maynard cluster is sufficiently stable that it would be able to detect dilatancy related anomalies at several tens of kilometers from the boundary of a dilatant region.

3. There is evidence for unexplainable systematic rotations in the direction of the apparent total $M_2$ tide at Maynard which continue over 4 months or more.

4. There are occasional perturbations in the tidal tilt as measured in each borehole which do not appear in other boreholes. Our present tidal analysis techniques do not have sufficient resolution over short intervals of time to resolve the time of onset, duration, and amplitude of these perturbations.

5. Residual tilts of the Maynard cluster are strongly correlated with atmospheric pressure fluctuations. They do not appear to be coherent in either direction or amplitude across the array.

6. There may be a loading effect associated with major storms off the New England coast which cannot be attributed to pressure effects alone and which may require correlation with tide gage records from the New England area.
7.2 Long Period Tilts

The major conclusions of the study of longer period tilts are as follows:

1. Deep hole tilt installations have much lower levels of extraneous noise than shallow hole installations for tilts at periods less than $10^6$ seconds.

2. At periods in near $10^7$ seconds the measured tilts of individual deep tiltmeters are comparable with the best results published for shallow instruments.

3. As much as a year was required for the Maynard tiltmeters to settle down after initial installation. Whether this was an instrumental characteristic or some installation effect such as aging of the cement around the borehole is not yet clear.

4. Once the instruments settled in all followed a pattern of tilt from which departures could be easily recognized. While the uncertainty in the tilt measurements from an individual hole ranged from about 2 microradians per year to as much as 10 microradians per year, comparison of the directions, rates and times of the various tilt events reduced the uncertainty on the regional tilt of the site to the order of 0.5 microradians per year. We believe that with the present array and improvement in the analysis techniques this can be further reduced to as little as 0.2 microradians per year.

5. Present techniques for automatic identification of changes in the tilt rate are inadequate, and no estimate is available for the length of time required before a small change in tilt can be recognized. Techniques for the detection of such small changes, comparable with those to be expected from earthquake precursors and other regional crustal block movements, should be developed.
7.3 Shallow Hole Measurements

Preliminary conclusions from our shallow hole tilt experiments: Lewkowicz and McConnell (1977) are:

1. The technique presently employed by the USGS U.S. Geological Survey, Office of Earthquake Research which we are using for the deployment of shallow tiltmeters is inadequate for the measurement of crustal tilts in the period range from tidal to periods out to periods of several weeks.

2. At periods of weeks to years the adequacy of the USGS method is not clear but it appears from the data that for soil installations in regions affected by seasonal freezing and thawing they are not likely to be adequate.
REFERENCES


APPENDIX I.

1.1 Tide Series Representation

Any tide series may be represented by a function of the form

\[ y(t) = \sum_{n=1}^{N} Y_n \exp[i(2\pi f_n t + \phi_n)] \]  

(1)

where:
- \(Y_n\) = the amplitude of the nth component
- \(f_n\) = the frequency of the nth component
- \(t\) = time
- \(\phi_n\) = phase of the nth component
- \(N\) = number of constituents

The major distinguishing characteristic of a tide series is that the frequencies are very precisely known from astronomical considerations. Tidal analysis therefore consists only of determining the amplitude and phase of each component.

If we rearrange the series (1) in such a way that the first term \((n=1)\) is the dominant constituent of the series and rewrite it as

\[ y(t) = Y_1 \exp[i(2\pi f_1 t + \phi_1)] \left[1 + \sum_{n=2}^{N} \frac{Y_n}{Y_1} \exp[i(2\pi \Delta f_n t + \Delta \phi_n)]\right] \]  

(2)

where
- \(\Delta f_n = f_n - f_1\)
- \(\Delta \phi_n = \phi_n - \phi_1\)

then the term in \([\) defines the modulation of the dominant tide. (see for example Godin, 1972, p. 165ff). Having reduced the series to the product of the principal tide and the modulation function we may make the following observations:

1. If the modulation function is known for any tidal series then the series can be completely described
by the amplitude $Y_1$ and phase $\phi_1$ of the dominant constituent.

2. If a series is the sum of one or more subseries or groups of constituents each of which has a known modulation function, then the series may be completely described by the amplitude and phase of the dominant constituent of each subseries together with its modulation function.

If we limit our definition of tidal analysis to be the determination of the amplitudes and phases of the constituents of an observed series then it is safe to say that all tidal analysis involves the following steps:

1. Breaking the total series down into one or more subseries each containing a group of similar tides.

2. Explicitly or implicitly assuming a form for the modulation function for each subseries.

3. Determining the amplitude and phase of the dominant component of each subseries from the tidal observations.

To illustrate the generality of the above statement let us consider the traditional analysis of a series containing $N$ discrete constituents of unknown amplitude and phase ranked in order of decreasing amplitude. Let us furthermore assume that $M$ of these constituents are suspected of being of sufficient amplitude that they are measurable. $N-M$ are small and are assumed to be negligible. Then the series (2) may be rewritten as the sum of $N$ subseries

$$y(t) = \sum_{n=1}^{M} Y_n \exp[i(2\pi f_n t + \phi_n)] + \sum_{n=M+1}^{N} Y_n \exp[i(2\pi f_n t + \phi_n)]$$
If we choose to analyze for only the first $M$ components this is equivalent to assuming that the modulation function is 1 for $1 \leq n \leq M$ and 0 for $n > M$. In other words we have implicitly assumed values for the modulation functions of the neglected components. Any improvement in the modulation functions over outright neglect must inevitably lead to improvements in the estimates of the amplitudes and phases of the components which are being sought.

1.2 The Ocean Loading Function

Let us represent the water tide height a point whose position vector is given by $\mathbf{r}$ as $w(\mathbf{r}, t)$ where $\mathbf{r}$ is the position vector from the tilt observation point to the point where the water tide is measured. That portion of the tilt due to the combined gravitational attraction and earth deformation acting over an area $A$ may then be written as

$$\hat{\alpha}_w(t) = \int_{A} G(\mathbf{r}) w(t, \mathbf{r}) \, dA \quad (3)$$

where $G(\mathbf{r})$ is a Green's function.

Expanding $w(t, \mathbf{r})$ in the form of (2) we get

$$w(t, \mathbf{r}) = \sum_{n=1}^{N} W_n(\mathbf{r}) \exp[i(2\pi f_n t + \phi_w(\mathbf{r}))] \quad (4)$$

$$= \left[ 1 + \sum_{n=2}^{N} \beta_{wn}(\mathbf{r}) \exp[i2\pi f_n t] \right]$$

where $\beta_{wn}(\mathbf{r}) = \left[ W_n(\mathbf{r})/W_1(\mathbf{r}) \right] \exp[i\Delta \phi_{wn}(\mathbf{r})] \quad (5)$
If we consider a set of constituents and an area such that the phase and amplitude of each constituent stays relatively constant with respect to the dominant constituent then we may set

$$\beta_{wn}(\vec{r}) = \left[ W_n(\vec{r}_o)/W_1(\vec{r}_o) \right] \exp[i\Delta \phi_{wn}(\vec{r}_o)]$$

(6)

where $\vec{r}_o$ is the position vector of some representative point in area A.

The validity of assumption (6) must be verified for each area under consideration before one can justify using a single tide group to represent the ocean loading function over the area.

Substituting (4) into (3) and rearranging yields

$$\alpha(t, \theta) = H_{aw}(\theta) \left| \exp[i\phi_{aw}(\theta)] \frac{W_1(\vec{r}_o)}{W_1(\vec{r}_o)} \exp[i(2\pi f_1 t + \phi_{wl}(\vec{r}_o))] \right|$$

$$\left[ 1 + \sum_{n=2}^{N} \beta_{wn}(\vec{r}_o) \exp[i2\pi \Delta f_n t] \right]$$

(7)

where $\theta$ represents the azimuth along which the tilt is being measured, $H_{aw}$, the ocean load to tilt (frequency independent) response function is given by

$$H_{aw} = \int G(\vec{r}) \left[ \frac{W_1(\vec{r})}{W_1(\vec{r}_o)} \right] \exp[ (\phi_{wl}(\vec{r}) - \phi_{wl}(\vec{r}_o)) ] dA$$

(8)

Now let us note that under assumption (6) the component of tilt measured in the direction $\theta$, which is caused by the ocean tide loading may be written as

$$\alpha(\theta) = A_1(\theta) \exp[i(2\pi f_1 t + \phi_{aw}(\theta))] \left| 1 + \sum_{n=2}^{N} \beta_n(\vec{r}_o) \exp[i2\pi \Delta f_n t] \right|$$

(9)

Equating (7) and (9) we obtain

$$A_1(\theta) = \left| H_{aw}(\theta) \right| W_1(\vec{r}_o)$$

(10)

$$\phi_{aw}(\theta) = \phi_{wl}(\vec{r}_o)$$

(11)
1.3 Analysis for Amplitude and Phase

Let us now turn to the problem of determining the amplitude and phase of the principal (usually the dominant) component of one or more groups of tides assumed present within an observed time series.

The meaning of amplitude is clear, however in the discussion above no assumption has been made about the phase angles $\phi_n$ except that they are chosen to produce the observed time series for the particular time base chosen. It is obviously most appropriate for our purposes to use Greenwith time (UT) but the choice of a phase representation still requires some sort of compromise between analytic convenience and geophysical meaning. We have chosen to utilize as a standard the phase lag of the observed tide relative to the maximum of the tidal potential at the tiltmeter site. This has the advantage that on a perfectly elastic earth with no oceans the phase of the north-upward semi-diurnal tilts would be 0° everywhere in the northern hemisphere.

To recast (9) in an appropriate form we define

$$-k_{qn} = \phi_{qn} - \phi_{pn}$$

where $\phi_{qn}$ is the phase of the nth constituent of the qth series at $t=0$.

$\phi_{pn}$ is the phase of the nth constituent of the tidal potential at $t=0$.

$k_{qn}$ is the lag of the nth constituent of the qth series with respect to the nth constituent of the tidal potential.
Substituting and rearranging we obtain

\[ a(t) = (A_1(\theta) \cos(k)) F_c(t) + (A_1(\theta) \sin(k)) F_s(t) \]  \hspace{1cm} (10)

where

\[ F_c(t) = \sum_{n=1}^{N} \left( \frac{W_n}{W_1} \right) \cos(\zeta_n) \]

\[ F_s(t) = \sum_{n=1}^{N} \left( \frac{W_n}{W_1} \right) \sin(\zeta_n) \]

\[ \zeta_n = 2\pi f_n(t) + \phi_n - \Delta_{kn} \]

\[ \Delta_{kn} \] is the phase lag of the nth constituent with respect to the first (principal) constituent.

By making a least squares fit of the function \( F_c(t) \) and \( F_s(t) \) to the observed time series \( a(t) \) and setting the coefficients \( \{A_1(\theta)\cos(k_1)\} \) and \( \{A_1(\theta)\sin(k_1)\} \) respectively, the values of the amplitude \( A_1(\theta) \) and phase lag \( k_1 \) of the principal constituent in the direction \( \theta \), may be determined. If both body and ocean loading terms are believed to be present then a pair of fitting functions \( F_c \) and \( F_s \) for each should be used.
APPENDIX II: CONTRACT CHRONOLOGICAL SUMMARY

First quarter: 9/23/75-12/31/75

Work was begun to improve techniques for separating real ground motion from installation and instrument related drift in deep boreholes. Development started on a comprehensive system of computer programs to interface data from the tiltmeter arrays with commonly used spectral analysis techniques such as the Van-icek procedure for piecewise continuous data. Experiment for direct measurement of instrumental drift in the laboratory using USGS hanging technique begun.

Second quarter: 1/1/76 - 3/31/76

A program for tidal analysis for the Bedford and Maynard tilt tapes was completed and selected short tapes analysed. Some difficulties were encountered as a result of intermittent errors in the data acquisition system. Efforts continued on the search for suitable approaches to secular tilt analysis.

Third quarter: 4/1/76 - 6/30/76

Experimental work continued on drift studies, tiltmeter calibrations and installation techniques. Increasing difficulties were encountered in reducing data from the NLS data acquisition system and much of the effort during this quarter was devoted to developing techniques for recovery of good data from poor data tapes. Initial steps were taken to attempt to determine the causes of the NLS malfunctions. Hanging experiment was suspended as a result of unsatisfactory drift rates and thermal effects in the experimental apparatus.

Fourth quarter: 7/1/76 - 9/30/76

Theoretical analysis focused on attempting to determine the relative orientations of the instruments and comparing the stability of scale factors. Evidence to date indicated that there was no correlation between the drift rates in the Maynard array,
thus suggesting that the major effects were installation rather than regional in nature. Trip to West Virginia to study the USGS installation techniques and determine potential sources of difficulties. Attempts continued to determine the source of random errors in the NLS data acquisition system. An improved mount was designed for the hanging experiment. A regulated power supply was installed at Maynard to eliminate large fluctuations which had prevented proper functioning of the technical equipment. Orders were placed for pipe and other necessary equipment for the shallow instrument cluster.

Fifth quarter: 10/1/76 - 12/31/76

The cause of the problems with the NLS data acquisition system was isolated and corrected. Three shallow borehole tiltmeters were installed at Maynard.

Sixth quarter: 1/1/77 - 3/31/77

Modifications began on tidal analysis programs to incorporate the tide group approach. Pressure effects were recognized as potential source of error. The character of the secular tilts of the three Maynard deephole instruments began to emerge.

Seventh quarter: 4/1/77 - 6/30/77

During this quarter the modifications to the computer programs for the tide group analysis were completed and tested. Evidence for poor ocean tide models being the cause of the apparent anomaly were clearly demonstrated. Papers presented at the Annual Meeting of the American Geophysical Union in Washington. Hanging experiment abandoned as a result of inconclusive results. Two borehole tiltmeters were retrieved from Maynard for improved calibration and other tests.

Eighth quarter: 7/1/77 - 9/30/77

Calibration was begun on the deep borehole tiltmeters with particular emphasis on determining residual tilts at the approximately 0.5-1.0 microradian level. Analysis and writeup of results to date begun.