Phase and Symbol Sequence Decoding on Random Walk Phase Channels

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Consider the problem of estimating phase and decoding data symbols from baseband data. Assume the phase sequence is a random walk on the circle and the symbols are drawn independently from an equiprobable alphabet for transmission over a perfectly equalized channel. A dynamic programming algorithm (Viterbi algorithm) is derived for decoding a maximum a posteriori (MAP) phase-symbol sequence on a finite dimensional phase-symbol trellis. Simulation results for binary and 8-ary phase modulated (PM) symbol sets are presented.
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Consider the problem of estimating phase and decoding data symbols from baseband data. Assume the phase sequence is a random walk on the circle and the symbols are drawn independently from an equiprobable alphabet for transmission over a perfectly equalized channel. A dynamic programming algorithm (Viterbi algorithm) is derived for decoding a maximum a posteriori (MAP) phase-symbol sequence on a finite dimensional phase-symbol trellis. Simulation results for binary and 8-ary phase modulated (PM) symbol sets are presented.

1. Introduction

Current techniques for decoding data symbols transmitted over random phase channels include the decision-directed phase-lock loop (DPDLL) of [1] and the decision-directed stochastic approximation procedure of [2]. The problem of maximum a posteriori (MAP) sequence estimation for improving on the DPLL has been recognized in [3], where the author derives a path metric and indicates its role in a forward dynamic programming algorithm for obtaining MAP phase-symbol sequences. However, because of the way random phase is modelled in [3], two simplifying assumptions must be made in order to implement a tractable, finite dimensional algorithm.

In this paper we observe that only phase-module 2π is of interest in data communication applications. This motivates us to wrap the phase around the circle, so to speak, and obtain a folded normal model for transition probabilities in random walk on the circle. It is then straightforward to pose a MAP phase-symbol sequence estimation problem as in [5] and [6]. The basic idea is to discretize the phase space [0, 2π) to a finite dimensional grid and to use a dynamic programming algorithm (Viterbi algorithm) to keep track of survivor phase-symbol sequences that can ultimately approximate the MAP sequence.

II. Model for Symbol Transmission over a Random Walk Phase Channel

As our model for symbol transmission over a perfectly equalized channel we assume

\[ z_k = i_k e^{j\theta_k} + n_k, \quad k = 1, 2, \ldots \]  

(1)

Here \( z_k \) is a sequence of baseband measurements, \( i_k \) is an independent sequence of M-ary complex symbols drawn from an equiprobable alphabet, \( \theta_k \) is a random walk phase sequence, and \( n_k \) is an additive noise sequence. The sequences \( i_k \), \( i_k \), and \( n_k \) are independent of each other.

The phase is modelled as a random walk on the circle taking values in \([0, 2\pi)\). This sequence is Markov with transition probabilities characterized by the following folded-normal law [4, 5]:

\[
f(\theta_k | \theta_{k-1}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(\theta_k - \theta_{k-1})^2}{2}}
\]

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sequence \{i_k\}_{k=1}^K
\max f((z_k)_{k=1}^K, (\psi_k)_{k=1}^K, (\delta_k)_{k=1}^K)
(5)
\text{Maximization of this joint density function is equivalent to maximization of the a posteriori density}
\max f((i_k)_{k=1}^K, (\psi_k)_{k=1}^K, (\delta_k)_{k=1}^K), \text{so we call the maximizing}
\{i_k\}_{k=1}^K, \{\psi_k\}_{k=1}^K, \{\delta_k\}_{k=1}^K \text{ the MAP phase-symbol sequences.}
\text{If instead we maximize the in } f(\cdot, \ldots) \text{ and ignore uninteresting constants we have the problem}
\max \Gamma_k
\{i_k\}_{k=1}^K, \{\psi_k\}_{k=1}^K
\text{where } \Gamma_k \text{ is defined recursively in terms of the path metric } p_k
\Gamma_k = \Gamma_{k-1} + p_k
p_k = -\frac{1}{2} \frac{1}{n} |z_{k-1} e^{-j\phi_{k-1}}|^2 + \ln f(\phi_k)
(7)
In this form the maximization problem may be efficiently solved by using a dynamic programming algorithm (or Viterbi Algorithm) on a finite-dimensional grid of phase-symbol pairs. The reader is referred to [5] and [6] for details.

IV. An Optimum Decoding Algorithm for Phase-Coded Symbols
Suppose \{i_k\} is the set of \{e^{j2\pi(m-1)/M}\}_{m=1}^M \text{ corresponding to equally-spaced PM symbols with unit energy. Write}
\tau_k = e^{j\phi_k} + n_k, \quad k = 1, 2, \ldots
\psi_k = \delta_k + \phi_k, \quad \phi_k \in \{0, 1, \ldots, M-1\}
(8)
and consider the following MAP sequence estimation problem:
\max f((z_k)_{k=1}^K, (\psi_k)_{k=1}^K, (\delta_k)_{k=1}^K)
\{i_k\}_{k=1}^K, \{\psi_k\}_{k=1}^K
(9)
Assuming \{\psi_k\} to be defined on \{\tau, \delta, \nu\} one can show that the joint density in (9) may be written
\kappa = \sum_{k=1}^K \frac{1}{2\pi} |z_k|^2 \exp \left( 2\pi i \frac{\delta_k^2}{n} \right) \Gamma_{\psi_k}(\psi_k e^{j\phi_k-1} - (\psi_k-\delta_k-1))
(10)
\text{Call } \{\psi_k\}_1^K \text{ and } \{\delta_k\}_1^K \text{ the MAP sequences that jointly maximize } \kappa. \text{ These sequences enter jointly only in the } \Gamma_{\psi_k}(\cdot) \text{ term on the right hand side of (10). It follows then that } \kappa \text{ is minimized by choosing}
\delta_k = \phi_k - \delta_k - 1
(11)
where \{\cdot\} denotes the closest value of \{m-1\} 2s/M to \phi_k - \delta_k - 1 \text{ and } \delta_k \text{ is interpreted modulo } 2s. \text{ It follows}
\kappa \text{ may be rewritten as}
\kappa = \sum_{k=1}^K \frac{1}{2\pi} |z_k|^2 \exp \left( 2\pi i \frac{\delta_k^2}{n} \right) \Gamma_{\psi_k}(\psi_k e^{j\phi_k-1} - (\psi_k-\delta_k-1))
(12)
where R(x) is the difference between k and \{x\}. Thus we may maximize (12) with respect to \{\psi_k\}_1^K \text{ and then find } \{\delta_k\}_1^K \text{ from (11). In a sense, we are decoding }
\{\psi_k\}_1^K \text{, as if it had a "density" given by } \Gamma_{\psi_k}(\cdot) \text{ and simply using the algorithm of [5] and [6]. When}
\text{the overlap between } 2s/M \text{ translates of } g_1(\tau) \text{ is negligible, then}
\Gamma_{\psi_k}(\psi_k e^{j\phi_k-1}) = \sum_{k=1}^K \Gamma_{\psi_k}(\psi_k e^{j\phi_k-1})
\text{and we may write the problem of maximizing in } \kappa \text{ as in (6) and (7) with } p_k \text{ replaced by } p_k:
\kappa = \frac{1}{2\pi} |z_k|^2 \exp \left( 2\pi i \frac{\delta_k^2}{n} \right) \Gamma_{\psi_k}(\psi_k e^{j\phi_k-1})
(13)
The function \Gamma_{\psi_k} \text{ is the convolution of the folded phase density with the impulsive density of the data.}

V. Results
The phase space \{\tau, \delta, \nu\} has been discretized to 48 phase values and a Viterbi algorithm has been programmed to solve (9) along the lines outlined in (11)-(13). Initial phase acquisition has been achieved by sending a 100-symbol preamble followed by 900 data symbols. This procedure has been repeated from 10 to 20 times to obtain the performance results of Figs. 1 and 2. When counting symbol errors, bursts have been ignored. Shown in Fig. 1 are binary symbol error results for \sigma^2 = 0.01 \text{ rad}^2 (\sigma = 0.30) \text{ and SNR } 10 \log \frac{1}{15} \text{ ranging from 4 to 10 dB. The results for binary orthogonal symboling are of no inherent interest in their own right. They are presented simply to validate the simulation. The results for binary orthogonal symboling are interesting because they indicate performance nearly equivalent to a receiver that has perfect phase coherence. The circles of Fig. 2 represent error probabilities for 8-ARY PM symboling when SNR ranges from 16-19dB and \frac{1}{2} \text{ remains fixed at } 4.4 \times 10^{-3} \text{ rad}^2. \text{ The triangles represent error probabilities for the markedly simpler decision-directed algorithm of [2]. For pure PM and moderate values of the phase variance parameter } \sigma^2, \text{ there seems to be little to recommend the algorithm of Sections III and IV over the algorithm of [2]. A similar conclusion with regard to the DDPLL was reached in [3]. This conclusion is changed for 2M-PM symbol constellation such as 100-QASK, 4A-48, etc.}

References


Fig. 1. Symbol Error Probabilities for Binary Symboling.

Fig. 2. Symbol Error Probabilities for M-ary PM.

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