Dynamics and Radiative Yields from Aluminum Multiple Wire Arrays

D.G. Colombant, M. Lampe, and J. Davis

Plasma Physics Division

and

H.W. Bloomberg

Science Applications, Inc.
McLean, Virginia 22101

August 1978

This research was sponsored by the Defense Nuclear Agency under Subtask T99QAXLB203, work unit 12 and work unit title, Advanced Concepts Theory.
**Title:** Dynamics and Radiative Yields from Aluminum Multiple Wire Arrays

**Authors:** D. G. Colombant, M. Lampe, J. Davis and H. W. Bloomberg

**Performing Organization:**
Naval Research Laboratory  
Washington, D.C. 20375

**Controlled Office:**
Defense Nuclear Agency  
Washington, D.C. 20305

**Report Date:** August 1978

**Number of Pages:** 53

**Distribution Statement:** Approved for public release; distribution unlimited.

**Supplementary Notes:**
This research was sponsored by the Defense Nuclear Agency under Subtask T99QAKL203; Work Unit Code and Title - 12 "Advanced Concepts Theory".

**Key Words:** Exploding wires, X-ray sources, Z-pinchs

**Abstract:**
Numerical results are presented for the dynamics and radiative yields from aluminum multiple wire arrays. In order to reach qualitative agreement with experimental results, it is found that the introduction of an enhanced anomalous resistivity is required. Physical origin of enhanced resistivity is briefly discussed.
## CONTENTS

I. INTRODUCTION ................................................. 1  
II. REVIEW OF EXPERIMENTAL RESULTS .................. 3  
III. THEORETICAL CONSIDERATIONS ..................... 5  
IV. NUMERICAL STUDIES ..................................... 12  
V. CONCLUSIONS ................................................. 19  
REFERENCES ..................................................... 22
DYNAMICS AND RADIATIVE YIELDS FROM ALUMINUM MULTIPLE WIRE ARRAYS

I. Introduction

In recent years, very impressive progress has been made in the development of multiple exploding wire arrays as x-ray sources using the new high-power generators, Pithon at Physics International (PI) and Blackjack IV at Maxwell Laboratories (MLI). These results also demonstrate the exceedingly complex nature of the physical processes characteristic of exploding wire radiation sources. Axial structure, including sausaging, flared and pinched regions, kinks and hot spots, is clearly seen, and may play an essential role in the radiation emission. Energy coupling from the generator to the wire plasma is sometimes poor. Energy may also be dissipated in field emitted electrons and/or ions, and in radiation-induced plasma short circuits. Non-fluid effects, e.g. runaway electrons, appear to occur in the wire plasma. Azimuthal asymmetries originating in the behavior of the separate wires at early times (i.e. before they assemble) may also play an important role. It appears that a complete theoretical analysis of wire phenomenology will eventually have to come to grips with all of these phenomena — i.e. variation in all three dimensions r, θ, z, as well as non-fluid and diode effects.

These different aspects of multiple wire phenomena are presently under study in our program and elsewhere; azimuthal asymmetries and

Note: Manuscript submitted July 10, 1978.
individual wire effects are discussed in a concurrent report\textsuperscript{1}, and other aspects of the problem will be reported on subsequently. The present report deals with a further elaboration of the azimuthally symmetric model of wire arrays\textsuperscript{2}, which avoids the complications enumerated above by averaging over axial and azimuthal structure, and including other effects such as anomalous transport coefficients, within a fluid model. The report stresses the significance of anomalous resistive heating in accounting for the recent experimental results. In support of this point, a series of numerical experiments on the new WHYRAC code\textsuperscript{3} are discussed. These extend our previous numerical studies\textsuperscript{2}, with the new code eliminating many shortcomings of the previous NRL wire code\textsuperscript{4} (and other numerical treatments), e.g. the circuit is now treated exactly, separate electron and ion temperatures $T_e$ and $T_i$ are calculated, various forms of anomalous transport can be modeled, and radiative energy transport, including opacity effects, is treated more accurately than in any previous code of this type.

The outline of the report is as follows. In Section II we discuss relevant features of the recent experiments. In Section III, theoretical considerations are reviewed, and the significance of resistive heating is pointed out. In Section IV, we briefly introduce the WHYRAC code, and report on a recent series of studies with it. In Section V, we summarize and discuss the results.
II. Review of Experimental Results

Without going into details of machine characteristics, we describe here the aspects of the experimental results that are of particular interest with respect to the computer simulation. A typical time plot of the voltage in the pulse forming region in front of the Blumlein is shown in Fig. 1. This voltage pulse is dependent on the generator characteristics alone, and not on the specifics of the wire load. In Fig. 2, the current through the machine is plotted as a function of time. Classically the wire plasma is expected to form a low resistance and initially low inductance channel so that the current trace should be largely independent* of the specific wire array properties. This conclusion seems to be borne out in the experiments where the properties of the wire array do not markedly affect the current pulse. However, it should be recognized that the current measurement is usually taken at a position removed from the vicinity of the wire cage. What is measured is, in effect, a sum of the currents through the wire cage as well as possible currents across the electrodes in the vacuum feed region of the generator. Thus the apparent independence of the total current trace from the wire array properties does not prove definitively that the current through the wire plasma shares this property.

*The wire plasma inductance $L_p$ increases as the wires implode, but typically does not dominate the circuit. However, the impedance due to $dL_p/dt$, is large enough to cause some modification of the current pulse at the time of assembly.
A time plot for radiated energy from an aluminum wire array, in the spectral region above 1 keV (Fig. 3), shows a rather simple structure. The density and temperature of an imploding plasma increases very rapidly just prior to the attainment of pressure equilibrium; an abrupt rise in the radiation flux is expected when this occurs. Subsequently, the radiating column is expected to expand and contract about its equilibrium position. The radiated power will fall during an expansion and rise during a contraction. This character is seen to occur in the decay part of the radiation pulse plotted in Fig. 3. The decay itself occurs when the coupling to the load decreases. The interesting feature of the radiation pulse from our point of view is that the value of the FWHM is 50-60 nsec. The effects of plasma column contractions and expansions of the radiated power are relatively minor. We feel that a major test of the validity of a numerical simulation is to yield the relatively broad radiation pulse found in the experiments.

The spectra observed in exploded wire experiments yield information on the average temperature and density of the wire plasma. Based on a particular opacity model, which is described in detail by Davis et al.\textsuperscript{5}, it has been inferred from the spectra of several aluminum wire arrays that the x-rays with $h\nu > 1$ keV are emitted from a plasma region with average temperature and density about 550 eV and $3 \times 10^{19}$ ions/cm\textsuperscript{3}, respectively. The opacity model was also used (in conjunction with a radiative transfer code) to calculate the radiated power from a uniform plasma with the inferred properties.
The calculated radiative output agreed with the experimental results when the diameter of the radiating column was taken to be 2mm, which is consistent with pinhole photographs estimates. This good agreement of both spectral properties and flux with observations confirms our confidence in the opacity model. The numerical simulations described below can thus be compared with the temperature and size of the radiation source region, as inferred by spectroscopic analysis of the experimental data.

III. Theoretical Considerations

The introduction of wire arrays instead of single wire in low-inductance machines was done originally to reduce the current rise time and to take advantage of the energy stored in the kinetic stage of implosion. A previous model\(^2\) was developed for multiple wire dynamics, according to which the pinched plasma is heated by thermalization of the kinetic energy of implosion, and the radiation pulse occurs during a transient stage which terminates when this thermal energy is radiated away. Resistive heating was neglected during the radiation pulse, since it could be shown that classical resistance is negligible. However, recent experiments indicate the need for revisions in this model and strongly suggest that anomalous resistive heating occurs. In present high power machines, the anomalous resistive heating, instead of the kinetic energy of the implosion, seems to be the source of the radiated energy.

In this section, we shall recall the main features of our previous model and outline those new features that seem to be
indicated by recent experiments. It is assumed throughout this report that the wire plasma can be represented as azimuthally symmetric, i.e. that the existence of separate and distinct wires at early times can be ignored. For our purpose, then, the initial condition is one in which the plasma is cool and is located in a thin shell centered on the initial array radius. Previous computer simulations\textsuperscript{6} indicated the following sequence of phenomena: (1) The plasma heats resistively to a few tens of eV (classical resistance is sufficient to do this). (2) The plasma properties (temperature, density profile) become essentially independent of the exact choice of initial conditions. The plasma shell thickness typically becomes larger than the final radius of the pinched plasma. (3) If the plasma resistivity is assumed to be classical, ohmic heating is found to be negligible at all later times, and the wire plasma resistance is negligible compared to other impedances in the generator/plasma circuit. (4) As the current increases, the magnetic pinch force increases and the plasma shell implodes. During this stage, energy input to the plasma is dominated by the Lorentz force, not by ohmic heating. In circuit terms, the rate of increase of plasma energy is

\[ \frac{\Delta E}{\Delta t} = \frac{1}{2} L_p I^2, \tag{1} \]

where $L_P$ is the plasma inductance. This energy input is to ion kinetic energy, but some of it is thermalized by shocks during the run-in. (5) When the plasma "assembles" - i.e. the hollow annulus
hits the axis, the plasma pressure is still too low to balance the magnetic pinch force. Pressure balance occurs when

$$\langle (1 + Z)T \rangle = \langle 1 + Z \rangle T_B \equiv \frac{I^2}{200k_B N_i},$$

(2)

where $T_B$ is the Bennett temperature in eV, $N_i$ is the number of ions per cm length, $I$ is the current in amps, and $k_B = 1.6 \times 10^{-12}$ ergs/eV is Boltzmann's constant. The imploding plasma overshoots pressure balance, and begins to "bounce" outward when

$$\langle T \rangle \approx 2 T_B.$$  

(3)

At the bounce, the plasma kinetic energy is, by definition, at a minimum; nearly all the kinetic energy of the run-in has been thermalized. (6) The plasma radius $r_p$ at the bounce (i.e. the minimum value of $r_p$), is determined by the requirement that the energy acquired during the run-in, from Eq. (1), equal the thermal energy $\frac{3}{2} N_i (1 + Z) T + \varepsilon_1 N_i$, where $\varepsilon_i$ is the mean energy invested in ionization and excitation (per ion). Use of Eq. (3) yields

$$\int dt \frac{1}{2} r_p^2 \approx \frac{3}{2} N_i (1 + Z) T_B + 2 \varepsilon_i N_i,$$

(4)

and making the very simple estimate $2 \varepsilon_i = \frac{1}{2} (1 + Z) T_B$, this relation reduces to

$$r_p \approx 0.15 r_o,$$

(5)

where $r_o$ is the initial array radius. This gives $r_p \sim \frac{1}{2}$ mm to 2 mm for typical experimental parameters, and a mean ion density at peak compression $n_i \sim 3 \times 10^{20}$ cm$^{-3}$ to $2 \times 10^{19}$ cm$^{-3}$. (7) If radiative cooling is slow compared to the characteristic hydrodynamic time scale
\[ r_p/c_s, \] where \( c_s \) is the sound speed (hydrodynamics-dominated regime), the plasma then oscillates about pressure balance, maintaining a temperature that varies from about \( 2T_B \) to \( \frac{1}{2}T_B \). The plasma thermal energy \( \sim 2N_i(1+Z)T_B \), which derived originally from the kinetic energy of the run-in, is slowly radiated away. When this energy is used up, the plasma slowly collapses to higher density. Radiation peaks correspond to times when the plasma is most compressed and hottest. (8) If radiative cooling is rapid (radiation-dominated or refrigerative regime), then the plasma cannot maintain \( \langle T \rangle \geq T_B \), and therefore fails to sustain pressure balance. In this case, the plasma collapses rapidly to very high density, cooling further by emitting copious, but mainly very soft, radiation.

For further elaboration of this model, the reader is referred to Ref. 2. The key point for our consideration here is that the hydro-dominated regime of this model showed qualitative agreement e.g. in plasma temperature and density, radiation fluence and radiation source radius, with the more successful Al multiple wire experiments on Owl II' and Blackjack III. But if a wire system evolved into a refrigerative collapse, the code predicted much less K-line radiation than was observed in the experiments.

The above model can only be successful when the radiation yields do not exceed the kinetic energy in the implosion. However, in the case of the new high power machines, it is clear that the radiated output greatly exceeds the implosion energy. Furthermore, voltage-current traces in the recent experiments (see Fig. 4) indicate that
the plasma impedance remains well-matched to the generator after plasma assembly, i.e. much of the energy is coupled into the plasma after assembly.

As discussed in Section II, the radiation pulse width is much longer than the hydrodynamic time scale characterized by the oscillations of the plasma column. This implies that the plasma radiates while it is in a quasi-steady-state, in which the plasma is maintained close to the Bennett temperature. If \( I \) is roughly constant during this stage, a steady state at roughly constant radius and density can be sustained only if a steady energy input balances the radiative cooling, keeping the plasma at \( T_B \). This energy influx appears in the voltage-current traces as a resistive load, and within the limits of a one-fluid model can be interpreted as anomalous plasma resistance. Assuming that \( \langle T_i \rangle = \langle T_e \rangle = T_B \), and further simplifying matters by assuming that the plasma temperature, density and current density are uniform, it is convenient to express the radiation flux as a factor \( g_1(n, T_B) \) times the well-known Bremsstrahlung flux, and the resistivity \( \eta \) as a factor \( g_2(n, T_B) \) times the classical (Spitzer) value; it can then be shown that the ratio of radiative energy loss rate \( \dot{E}_{\text{rad}} \) to resistive heating rate \( \dot{E}_{\text{res}} \) is

\[
\frac{\dot{E}_{\text{rad}}}{\dot{E}_{\text{res}}} = \left( \frac{I}{I_{PB}(n,T_B)} \right)^2,
\]

where \( I_{PB} \) is the Pease-Braginskii current,
\[
I_{PB} = (1.6 \, MA) \left( \frac{Z + 1}{2Z} \right) \frac{g_2(n, T_B)}{g_1(n, T_B)} \left( \frac{\ell n \Lambda}{10} \right)^{\frac{1}{2}},
\]

and \( \ell n \Lambda \equiv 25.3 - 1.15 \ln_{10} n_e + 2.3 \ln_{10} T_e \) is the Coulomb logarithm. For our case, \((Z + 1)/2Z \approx 0.5\) and \(\ell n \Lambda \sim 8\).

We note that for classical resistivity and solely Bremstrahlung radiation,

\[
I_{PB} \approx 0.71 \, MA
\]

becomes a constant, and a steady state can exist only at this current; if the current is higher, the plasma must collapse. What is more, radiation losses in addition to Bremstrahlung further reduce \(I_{PB}\). For higher currents, it is clear that classical resistive heating is negligible — as assumed in our previous treatment — and cannot support a steady-state. This suggests the existence of a large anomalous resistance, which can support such a steady state.

For example, using our code results to determine \(g_1\) in a typical case, we find that in order for \(I_{PB}\) to be equal to the measured current, the anomalous resistivity factor must be \(g_2 \sim 20\). A higher value for \(g_1\) (higher radiation level) will require a higher value for \(g_2\) (anomalous resistivity) for a steady-state to be maintained.

In fact, these results point to the following trend for higher-\(Z\) materials. If a quasi-steady state is necessary to insure sustained radiation levels, and if the machines are operated at the same current level for various elements, the increase in anomalous resistivity will have to match the increase in radiated power as the atomic weight of the element increases. Mismatches may occur if the
anomalous resistivity does not increase, so as to match the particular radiative characteristics of a given high Z element, and in some cases a steady state, i.e. a sustained radiation pulse, may be attainable only by reducing the current. Unless anomalous resistivity is understood in physical terms, no scaling of the results presented here seems to be possible.

The assumption of anomalous resistivity also helps to explain the low density central pinch inferred experimentally. Such a soft pinch is possible if the pressure builds up in the center of the plasma annulus before assembly occurs. This can occur if the current penetrates the high-density plasma annulus, heating the annulus inner surface and causing it to expand toward the wire array axis. Continued current flow in this region heats it up significantly, and pressure balance is nearly achieved with the remainder of the plasma when it assembles. Current penetration to the central region will occur only if the characteristic magnetic diffusion time

$$\tau = \frac{4\pi \ell^2}{c^2 \eta},$$

where $\ell$ is the plasma annulus thickness, is no longer than the current rise time scale, i.e., a few tens of nsec. Based on Spitzer resistivity, current penetration in 20 nsec will occur, for $\ell = 1$ mm, only if $T \leq 20$ eV; but an anomalous resistivity factor $g_2 = 50$ allows current penetration to occur if $T = 260$ eV. Since this latter temperature is more typical of the temperature reached in the plasma during implosion, current penetration, central pressure build-up, and
consequent pinch softening is possible only if the resistivity is anomalously high.

In the next section, we shall discuss a series of numerical experiments, in which various forms of anomalous resistivity are introduced heuristically. We find that it is indeed possible in this way to generate code results that show qualitative agreement with the experiments. The possible sources of such anomalous resistivity are the subject of intense study, at present, and will be discussed in future reports.

IV. Numerical Studies

A. The WHYRAC Code

A one-dimensional code, WHYRAC\textsuperscript{3}, has been developed in order to study the dynamics of the multiple wire array. As discussed previously, the code provides a self-consistent treatment of the multiple wire plasma, based on the assumptions that: (a) The wire plasma can be adequately represented by an azimuthally symmetric model. This is well founded if the wire plasmas merge to form an annulus plasma early in the run-in stage\textsuperscript{1}, but could be qualitatively valid even if this does not occur. (b) Dependence on the axial variable \( z \) may also be ignored. (c) No current paths exist other than through the wire plasma.

WHYRAC is a one-fluid, two temperature code using FCT\textsuperscript{9} to solve the fluid equations. It is Eulerian, but has variable mesh spacing and regridding capabilities that insure good resolution over the wide range of spatial scales that occur as the pinch ensues.
Circuit equations are solved self-consistently with the plasma variables, and the plasma is treated in an exact way as a circuit element. To handle radiative energetics (energy sources and sinks), an atomic physics/radiation transport package is called at each time step and solved self-consistently with the fluid and circuit equations. This package is a great advance over anything previously used in for this purpose in exploding wire studies, but the extent of detail included in it is limited by the requirement that it be rapid-running. In addition, a much more detailed and accurate atomic/radiation package is used as a post-processor, to generate accurate spectra at less frequent intervals. Initially all the wire mass is taken to constitute a cool plasma distributed in an annulus near the original array radius - e.g. typically $T = 1 \text{ eV}$, and annulus thickness 0.5 mm, although it is found that the results are insensitive to the exact choice of initial temperature and density profile. The essential input parameters are the wire material, wire mass, array radius, return current cage radius, transmission line impedance, vacuum feed inductance and either the voltage or current waveform. (The generator voltage waveform, taken from experimental data, is used as the input in the simulations reported here; a typical voltage trace is shown in Fig. 5). Output diagnostics provided by the code include radial profiles of $n_i, n_e, T_e, T_i$, radial velocity $V$, current density $J$, magnetic field $B$, axial electric field $E$, as well as radiation powers in lines and continuum in any frequency bin of
interest; for really accurate spectra, however, the post-processor is used.

B. Numerical Experiments

We shall discuss, for specificity, a series of numerical simulations that were carried out for a single set of typical experimental parameters with $A$ plasma mass $1.14 \times 10^{-4}$ g/cm, and the array radius $r_0 = 1.5$ cm. In the different simulations, several of the transport coefficients were varied, as will be discussed.

In the first simulation, the resistivity was taken to be classical except that when the current flow speed $u = |J/n_e e|$ exceeded the ion sound speed, the resistivity was increased to 100 times classical (Spitzer). This was intended to represent, in a very crude way, anomalous resistance due to the well known streaming instabilities—particularly the modified two-stream and (if $T_e \gg T_i$) ion sound. It has been shown in a series of papers\textsuperscript{10} that in this type of situation, it is not the exact value of the anomalous transport coefficient that is crucial but only the fact that the instability turns on at a certain value of $u$ and that resistivity becomes very large for larger values of $u$; the system typically adjusts itself so that $u$ always remains at the critical—i.e. marginally stable—value.

Time dependence of the total thermal, kinetic, and radiated energies, as well as of the maximum density at a given time, are shown in Fig. 6. We note that the behavior is that of the "refrigerative" mode, where the plasma collapses abruptly to very high
density, because of radiative losses. The temperature falls well below $T_B$ as this happens. As shown in Fig. 7, line radiation, particularly the K-line radiation at $h\nu \sim 1.7$ keV, falls rapidly after an initial peak; thus the K-line radiation pulse of interest is very short ($\sim 5$ nsec). Continuum radiation, whose rate scales as density, since the plasma is optically thin to it, dominates the energy loss in the dense cool state. The main effect of anomalous resistance, of the type introduced here, is simply to prevent current flow speeds $u \gg c_s$, and thereby to prevent the current from flowing heavily in low density regions on the outside of the plasma. Since the current then flows predominantly in dense plasma where $u < c_s$, anomalous resistive heating is unimportant, and fails to prevent the refrigerative collapse.

Density and temperature profiles at $t = 110$ nsec (when the radiation spike ends) are shown in Fig. 8. The central region is at high density ($n_1 \sim 10^{21}$ cm$^{-3}$) and is already quite cold. Radiation originates predominantly in this region, and is dominated by continuum. A surrounding region of plasma is at very low density ($n_1 \sim 5 \times 10^{17}$ cm$^{-3}$) and $T \sim 4.5$ keV, while the very low density outer corona attains still higher temperatures. This compares poorly with the experimental results, which show: (1) a radiation pulse duration $\sim$ tens of nano-seconds, (2) a radiating region of moderate density $n_1 \sim 3 \times 10^{19}$ cm$^3$ and fairly high temperature, $T_e \sim 500$ eV, as inferred by spectroscopy; (3) harder radiation emanating mainly from a central region and softer radiation from the outer region; (4) much more K-line radiation than
is predicted by the code; (5) little or no dip in the current trace at the time of assembly, as compared to a strong dip in the computer simulation (due to a very large rate of change of plasma inductance \( L_p \) during the plasma implosion).

From the discussion of Sec. III, it seems likely that the poor agreement of the computer simulation with experiment is due to the inadequacy of resistive heating to support a steady state. The central core plasma, in particular, fails to heat sufficiently to achieve pressure balance. We have proceeded on the hypothesis that the fundamental framework of the calculational model (fluid, with only radial dependence) is reasonable, but that other effects which are not presently well understood can result in changes in the effective transport coefficients. Thus we modify these coefficients in a sequence of well defined ways to study their effect on the results:

(1) In order to enhance penetration of heat into the core, and thus possibly soften the assembly, thermal conductivity was increased by up to a factor of 80 over the classical value. No significant change in the results occurred. (2) For \( u > c_s \), the anomalous resistance was increased to 1000 times the classical value. No significant change occurred, thus supporting the marginal stability picture that the numerical value of the anomalous transport coefficients values does not play a crucial role, once the instability threshold is computed accurately. (3) Although we believe that radiative emission is well modeled in WHYRAC, we tested the sensitivity of the results to the line radiation rate (due to the uncertainty in opacity values) by
decreasing this rate by a factor up to 40 for all the lines. No essential change in the refrigerative collapse occurred. Continuum radiation was sufficient to cause the collapse. (4) A large anomalous resistivity was imposed everywhere, even when the current drift speed was small. For simplicity, this anomalous resistivity was assumed to be of the form of a constant $g_2$ times the classical resistivity $\eta(T)$. For a wide range of choices of $g_2$, this assumption completely changes the results, leading to evolution that shows qualitative agreement with the experimental results, i.e. the plasma persists in quasi-steady equilibrium after assembly, with $\langle T \rangle$ oscillating gently about $T_B$, and with much of the radiation emitted in lines at $\hbar\nu > 1$ keV. The time evolution of one such simulation, with the choice $g_2 = 50$, is shown in Fig. 9. In Fig. 10, we note that the radiation pulse has been broadened significantly. Furthermore, the temperature and density profiles of Fig. 11, at $t = 113$ nsec (near the time of peak radiation rate), show a plasma radius $\sim 1.5$ mm, in reasonable agreement with Eq. (5) and with the experiments, and a detailed structure with a hot central core ($n_1 \sim 5 \times 10^{18}$ cm$^{-3}$, $T_e \sim 3$ keV, radius $\sim 1$ mm) surrounded by a warm, somewhat denser outer ring ($n_1 \sim 3 \times 10^{19}$ cm$^{-3}$, $T_e \sim 500$ eV). This type of structure agrees with the experimental observations that the harder radiation is emitted from the core. Also, the K-line radiation in the simulation is emitted principally from the outer region, whose temperature and
density agree reasonably well* with the values inferred spectro-
scopically from the experimental data. The dip in the current trace
I(t) at assembly, in Fig. 13, is seen to be much weaker than the
corresponding dip for \( g_2 = 1 \) (in Fig. 12), since the rapid refrig-
erative collapse does not occur. Thus, this simulation shows good
general agreement with experiment.

The results described above do not change drastically for
choices of \( g_2 \) up to 80; on the other hand \( g_2 = 10 \) results in a refrig-
erative collapse similar to that for \( g_2 = 1 \). The reason for this
would appear to be that the radiation rate at \( \langle T \rangle = T_B \) exceeds the
classical resistive heating rate by a large factor of order 20. If
the resistive heating rate is enhanced by exactly this factor, a
genuine steady state occurs; but if the resistive heating rate is
merely close to the radiative cooling rate, a quasi-steady state with
\( \langle T \rangle = T_B \) occurs, in which the plasma slowly expands (or contracts)
in such a way that the adiabatic cooling (or heating) is just
sufficient to maintain \( \langle T \rangle = T_B \). On the other hand, if the
radiative cooling rate far exceeds the resistive heating rate, the
collapse is sudden and refrigerative, rather than quasi-steady.

As a further test on the significance of anomalous resistance,
some additional computer simulations were run, in which the anomalous
resistivity factor \( g_2 = 50 \) was turned on at a time \( t_1 \neq 0 \). We found

*Some spectral details indicate that the experimental plasma is
moderately hotter than the temperature seen in the simulations.
that for $t_1 \leq 70$ nsec the results were the same as for $t_1 = 0$, i.e. the anomalous resistance simulations just described. On the other hand, anomalous resistance turned on at $t_1 > 90$ nsec failed to prevent the refrigerative collapse (although it is likely that the plasma would eventually have heated up again and come to pressure balance, if it were run long enough). We conclude that the effects of anomalous resistance at early times (e.g. increased current diffusion and heating of the interior plasma) are unimportant to the phenomena under discussion, but the effects of anomalous resistance during the implosion and afterwards are all-important.

V. Conclusions

It has been shown that one-dimensional (radial) fluid models with classical transport coefficients agree poorly with the results of recent high-power multiple wire experiments. The introduction of anomalous resistance only when the current flow velocity $u$ is high, based on the marginal stability picture of current-driven instabilities, fails to correct the inadequacies of the model. However qualitative agreement with the experiments is found when a large anomalous resistivity is introduced, which persists at low current flow velocity. This situation is similar to that in post-implosion theta pinches, where it is believed that lower hybrid instabilities are responsible for anomalous resistance at low $u$, and in plasma focuses, where anomalous resistance also appears to occur, for reasons which are not understood. Wire arrays on the present generation of low-inductance machines do not operate as they were
initially thought to (conversion of kinetic energy of implosion into radiative energy). In order to calculate the essential plasma heating process, and generate useful scaling laws, we believe that the anomalous resistance must be understood in detail. In particular, a sustained radiation pulse seems to require a balance between radiation flux and resistive heating; thus, prediction of optimum currents for various wire materials requires an understanding of how both of these quantities scale with Z and (through the condition $T = T_B$) with I.

The introduction of anomalous resistance into the transport simulates qualitatively the performance of the present machines within the limits of a one fluid model, in that

a) it provides an energy flow into the plasma after assembly, driving the radiation pulse, and

b) a high pressure forms at an early time on the axis of the wire array, preventing a total collapse of the wire plasma and softening the pinch.

Since the results of this paper point to the importance of anomalous transport, the detailed theory of various non-fluid transport mechanisms will be studied in future work. We conclude this paper with a few comments along these lines. In addition to the micro-instabilities which are traditionally regarded as the source of anomalous resistance, such as lower hybrid instability\(^{10}\) (which continues to operate at low current flow speed), a number of non-fluid phenomena are probably occurring in high-power wire plasmas, which may eventually be modeled as anomalous transport coefficients
or energy sources and sinks within the framework of a code like WHYRAC. The deposition of energy in the wire plasma by field emitted electrons or high-energy ions could be included in this way (although a separate diode code would be needed to calculate the evolution of such high-energy particles). Runaway electrons are likely to occur in the plasma, since the Dreicer critical field

\[ E_{cr} = 2.69 \Lambda Z e^3 n_e / T_e \]

is only \( E_{cr} \approx 1.2 \times 10^5 \) V/cm in the hot, low-density core shown in Fig. 11; the axial electric field easily exceeds \( E_{cr} \) in this case. The generation of suprathermal electrons would have to be treated self-consistently with the instabilities responsible for anomalous resistance. The ion Larmor radius approaches (and the mean free path exceeds) the characteristic dimensions of the pinched plasma in some high temperature regions; correct treatment may require the use of a hybrid kinetic-ion/fluid-electron picture, as has proven necessary in recent theta pinch work.\(^ {13} \) These and several other approaches to non-classical-resistive heating are presently being studied, because of the strong indication from these studies that they play a crucial role.
References


5. J. David, P. C. Kepple and J. P. Apruzese, to be published.

6. See Ref. 2; also Sec. IV of the present report.


Figure 1. Typical tube voltage (in arbitrary units) as a function of time.
Figure 2. Typical experimental current trace (in arbitrary units) as a function of time.
Figure 3. Total experimental radiation pulse as a function of time.
Figure 4. Input power corresponding to same shot.
Figure 5. Generator voltage used in the numerical simulation runs.
Figure 6. Kinetic, thermal and radiated energies as a function of time for classical resistivity run. Also, maximum density curve, showing strong pinching occurring at assembly.
Figure 7. Radiated powers as a function of time for classical resistivity run. (Arbitrary units -- each curve is normalized to its own maximum value.)
Figure 8. Electron temperature and ion density profiles as a function of radius at time near assembly for classical resistivity run. Note high axial ion density and low axial electron temperature.
Figure 9. Same plot as Fig. 6 but for an anomalously enhanced resistivity everywhere in the plasma. (50 times classical resistivity).
Figure 10. Radiated powers as a function of time for anomalous resistivity run. (Arbitrary units—each curve is normalized to its own maximum value.)
Figure 11. Electron temperature and ion density profile as a function of radius near assembly for anomalous resistivity run. Note structure and magnitude of ion density near axis as well as electron temperature.
Figure 12. Current trace for classical resistivity run as a function of time. Dip indicates pinching. (Arbitrary units).
Figure 13. Current trace for anomalous resistivity run as a function of time. (Arbitrary units). Dip in current is smoother than for classical resistivity run.
DISTRIBUTION LIST

DIRECTOR
Defense Advanced Rsch Proj Agency
Architect Building
1400 Wilson Blvd.
Arlington, Va. 22209
ATTN: Strategic Tech Office

Defense Communication Engineer Center
1860 Wiehle Avenue
Reston, Va. 22090
ATTN: CODE R820 R. L. Crawford
ATTN: Code R410 W. D. Dehert

DIRECTOR
Defense Communications Agency
Washington, D. C. 20305
ATTN: CODE 960
ATTN: CODE 480

Defense Documentation Center
Cameron Station
Alexandria, Va. 22314
ATTN: TC

DIRECTOR
Defense Intelligence Agency
Washington, D. C. 20301
ATTN: W. Wittig DC - 7D
ATTN: DT-1B

DIRECTOR
Defense Nuclear Agency
Washington, D. C. 20305
ATTN: STSI Archives
ATTN: STVL
ATTN: STTL Tech Library
ATTN: DUST
ATTN: RAAE

12 copies (if open publication)
2 copies (if otherwise)

2 copies
OJCS/J-6
The Pentagon
Washington, D. C. 20301
ATTN: J-6

DIRECTOR
Telecommunications & Com & Con Sys
Washington, D. C. 20301
ATTN: ASST DIR Info & Space Sys
ATTN: DEP ASST. SEC Sys

Weapons Systems Evaluation Group
400 Army-Navy Drive
Arlington, Va. 22202
ATTN: DOCUMENT CONTROL

COMMANDER
Harry Diamond Laboratories
2800 Powder Mill Road
Adelphi, Md. 20783
ATTN: ARXDO-NP

COMMANDER
TRASACA
White Sands Missile Range, NM 88002
ATTN: EAB

DIRECTOR
U. S. Army Ballistic Research Labs
Aberdeen Proving Ground, Md. 21003
ATTN: AM-CA Franklin E. Niles

U. S. Army Communications CMD
C-B Services Division
Pentagon Rm. 2D513
Washington, D. C. 20310
ATTN: CEBAD

COMMANDER
U. S. Army Electronics Command
Fort Monmouth, N. J. 07703
ATTN: AMSRL-TL-ENV Hans A. Bontke

COMMANDER
U. S. Army Material Command
5001 Eisenhower Avenue
Alexandria, Va. 22333
ATTN: AMCRD-UN-RE John F. Corrigan
COMMANDING OFFICER
Naval Intelligence Support CTR
1301 Suitland Road, Bldg. 5
Washington, D.C. 20390
ATTN: Mr. Dubbin Stic 12

DIRECTOR
Naval Research Laboratory
Washington, D.C. 20375
ATTN: HDQ COMM DIR Bruce Wald
ATTN: CODE 5460 Radio Propagation BR
ATTN: CODE 6701 Jack D. Brown
ATTN: CODE 6700 Division Superintendent 25 copies (if unclass)
ATTN: CODE 6790 Branch Head 1 copy (if classified)
ATTN: CODE 7127, Chas. Y. Johnson

COMMANDING OFFICER
Naval Space Surveillance System
Dahlgren, Va. 22448
ATTN: CAPT. J. H. Burton

COMMANDER
Naval Surface Weapons Center
White Oak, Silver Spring, Md. 20910
ATTN: CODE 1224 Navy Nuc Prgms Off
ATTN: CODE 730 Tech. Lib.

DIRECTOR
Strategic Systems Project Office
Navy Department
Washington, D.C. 20376
ATTN: NSP-2141

COMMANDER
ADC/AD
ENT AFB, Co., 80912
ATTN: ADDA

Headquarters
U.S. Army Elect Warfare Lab (ECOM)
White Sands Missile Range, NM 88002
ATTN: E. Butterfield

AF Cambridge Resch Labs, AFSC
L.C. Hanscom Field
Bedford, Ma 01730
ATTN: LKB Kenneth S. W. Champion
ATTN: OPR James C. Ulwick
ATTN: OPR Hervey P. Gauvin
NASA
600 Independence Ave., S. W.
Washington, D. C. 20546
ATTN: M. Dubin

Aerodyne Research, Inc.
Tech/Ops Building
20 South Avenue
Burlington, MA 01803
ATTN: M. Camac
ATTN: F. Bien

Aerospace Corporation
P. O. Box 92957
Los Angeles, CA 90009
ATTN: T. M. Salmi
ATTN: S. P. Bower
ATTN: V. Josephson
ATTN: SMFA for PW
ATTN: R. Grove
ATTN: R. D. Rawcliffe
ATTN: T. Taylor
ATTN: Harris Mayer
ATTN: D. C. Cartwright

Analytical Systems Corporation
25 Ray Avenue
Burlington, MA 01803
ATTN: Radio Sciences

Avco-Everett Research Laboratory, Inc.
2585 Revere Beach Parkway
Everett, MA 02149
ATTN: Richard M. Patrick

Boeing Company, The
P. O. Box 3707
Seattle, WA 98124
ATTN: D. Murray
ATTN: Glen Keister

Brown Engineering Company, Inc.
Cummings Research Park
Huntsville, AL 35807
ATTN: David Lambert MS 18

California at San Diego, Univ. of
Building 500 Mather Campus
3172 Miramar Road
La Jolla, CA 92037
ATTN: Henry G. Booker
Calspan
P. O. Box 235
Buffalo, N. Y. 14221
ATTN: Romeo A. Deliberis

Computer Sciences Corporation
P. O. Box 530
6655 Arlington Blvd.
Falls Church, VA 22046
ATTN: H. Blank
ATTN: Barbara F. Adams

Comsat Laboratories
P. O. Box 115
Clarksburg, Md. 20734
ATTN: R. R. Taur

Cornell University
Department of Electrical Engineering
Ithaca, N. Y. 14850
ATTN: D. T. Farley, Jr.

ESL, Inc.
495 Java Drive
Sunnyvale, CA 93102
ATTN: J. Roberts
ATTN: V. L. Mower
ATTN: James Marshall
ATTN: R. K. Stevens

General Electric Company
Tempo—Center for Advanced Studies
816 State Street
Santa Barbara, CA 93102
ATTN: Don Chandler
ATTN: DASIAC
ATTN: Tim Stephens

General Electric Company
P. O. Box 1122
Syracuse, N. Y. 13201
ATTN: F. A. Reibert

General Research Corporation
P. O. Box 3587
Santa Barbara, CA 93105
ATTN: John Ise, Jr.
Geophysical Institute
University of Alaska
Fairbanks, AK 99701
ATTN: Technical Library
ATTN: Neil Brown
ATTN: T. N. Davis

GTE Sylvania, Inc.
189 B Street
Needham Heights, MA 02194
ATTN: Marshall Cross

HRB-SINGER, Inc.
Science Park, Science Park Road
P. O. Box 60
State College, PA 16801
ATTN: Larry Feathers

Honeywell Incorporated
Radiation Center
2 Forbes Road
Lexington, MA 02173
ATTN: W. Williamson

Illinois, University of
Department of Electrical Engineering
Urbana, IL 61801
ATTN: K. C. Yeh

Institute for Defense Analyses
400 Army-Navy Drive
Arlington, VA 22202
ATTN: Ernest Bauer
ATTN: Hans Wolfhard
ATTN: J. M. Aein
ATTN: Joel Bengston

Intl Tel & Telegraph Corporation
500 Washington Avenue
Nutley, N. J. 07110
ATTN: Technical Library

ITT Electro-Physics Laboratories, Inc.
9140 Old Annapolis Road
Columbus, Md. 21043
ATTN: John M. Kelso
Mission Research Corporation
735 State Street
Santa Barbara, CA 93101
ATTN: R. Hendrick
ATTN: Conrad L. Longmire
ATTN: Ralph Kilb
ATTN: R. E. Rosenthal
ATTN: R. Bogusch
ATTN: David Sowle
ATTN: M. Scheibe
ATTN: P. Fischer

Mitre Corporation, The
Route 62 and Middlesex Turnpike
P. O. Box 208
Bedford, MA 01730
ATTN: Chief Scientist W. Sen
ATTN: S. A. Morin M/S
ATTN: C. Hirding

North Carolina State Univ At Raleigh
Raleigh, N. C. 27507
ATTN: SEC Officer for Walter A. Flood

Pacific-Sierra Research Corp.
1456 Cloverfield Blvd.
Santa Monica, CA 90404
ATTN: E. C. Field, Jr.

Philco-Ford Corporation
Western Development Laboratories Div
3939 Fabian Way
Palo Alto, CA 94303
ATTN: J. T. Mattingley MS X22

Photometrics, Inc.
442 Marrett Road
Lexington, MA 02173
ATTN: Irving J. Kofsky

Mitre Corporation, The
Westgate Research Park
1820 Dolley Madison Blvd.
McLean, VA 22101
ATTN: Allen Schneider
Physical Dynamics, Inc.
P. O. Box 1069
Berkeley, CA 94701
   ATTN: Joseph B. Workman

Physical Sciences, Inc.
607 North Avenue, Door 18
Wakefield, MA 01880
   ATTN: Kurt Wray

R & D Associates
P. O. Box 3580
Santa Monica, CA 90403
   ATTN: Robert E. Lelevier
   ATTN: Forest Gilmore
   ATTN: Richard Latter
   ATTN: William B. Wright, Jr.

R & D Associates
1815 N. Ft. Myer Drive
11th Floor
Arlington, VA 22209
   ATTN: Herbert J. Mitchell

Rand Corporation, The
1700 Main Street
Santa Monica, CA 90406
   ATTN: Cullen Crain

Science Applications, Inc.
P. O. Box 2351
La Jolla, CA 92038
   ATTN: Daniel A. Hamlin
   ATTN: D. Sachs
   ATTN: E. A. Straker

Space Data Corporation
1331 South 26th Street
Phoenix, AZ 85034
   ATTN: Edward F. Allen