Steady State Transport of High Current Beams in a Focused Channel

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By following the orbits of several thousand particles in their self-consistent space charge fields, the evolution of an unstable Kapchinskij-Vladimirskij distribution is examined until saturation is reached. A distribution is formed which is stable on a scale of hundreds of quadruple magnets.
STEADY STATE TRANSPORT OF HIGH CURRENT BEAMS
IN A FOCUSED CHANNEL

The possibility of using a heavy ion beam to ignite an inertially confined pellet to thermonuclear temperatures has increased the importance of transporting high current beams of charged particles. Though the required beam ignition power and energy can be achieved using relatively modest extensions of present technology, any instabilities in the beam being transported that occur as extrapolation is made to higher current, represent a major barrier to be overcome.

Extensive analysis of the transverse behavior of a transported beam in the presence of space charge effects has been conducted by numerically integrating the Kapchinskij-Vladimirskij (K-V) envelope equations. The solutions thus obtained have been found to unstable to several classes of perturbations. Evidence is presented here, through the use of computer simulations, that these instabilities saturate and a steady state evolves in which the beam remains stable over a scale of hundreds of transport lenses.

Figure 1 shows four views of the four-dimensional $x-p_x-y-p_y$ phase space of a K-V distribution. It is a property of this distribution that each of these projections is an ellipse of uniform density. The beam is assumed long compared with its transverse dimensions so that the evolution is assumed paraxial and variations along the beam are neglected.

The numerical model moves in a reference frame along with the beam so that the lens forces are electrostatic. The full Vlasov equation is simulated by following the orbits of several

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thousand simulation particles in their self consistent electric fields. The numerical algorithm is of the PPOWER type optimized for a Texas Instruments ASC computer. Numerical tests have shown the results to be essentially independent of variations in numerical parameters such as time step, system resolution, and number of particles when they are varied factors of two from the conditions used.

A thin lens quadrupole transport system is assumed with a phase advance of 90° per doublet. The phase space plot of figure 1 shows a sample 2K particles out of 16K in the run, with initial conditions corresponding to a matched system having enough space charge to bring the tune down to 30°. This system rapidly goes unstable and a complicated phase space structure evolves as shown in figure 2, which shows the system phase space after 20 magnet pairs. After about 40 magnet pairs the system ceases to evolve significantly. Fig. 3 shows the system phase space after 100 magnet pairs.

Figure 4 shows the evolution of the $x$ and $y$ emittances defined by the product of the root mean square values of $x$ and $p_x$, and $y$ and $p_y$, respectively. This evolution displays the rapid growth of the instability and its subsequent saturation. The difference in the $x$ and $y$ behavior appears to be due to a combination of statistical effects and the systematic favoring of the $x$-direction in the initial match because the initial conditions are chosen to minimize variation of the beam envelope between the first two $x$-focusing lenses. Once this anisotropy has developed, because of the lack of any dissipative mechanism for isotropization, it will persist.

A detailed characterization of the final distribution is beyond the scope of this communication. Though a plot such as $f(p_x)$ at $x_0$, integrated over $y$ and $p_y$, is somewhat Gaussian shaped, as are several other cross sections, the total picture may be somewhat more complex. For example, the ratio of rms to average absolute value of density in the $x$ direction is 1.23. This is the same as the rms to average ratio in the $y$ direction but different from both the $p_x$
and $p$, rms to average ratios which are 1.33. The contour plot in Fig. 5 of the $y-p$, phase space at an $x$-focusing lens shows that the growth in emittance is mostly due to a spreading in the velocity of the distribution function and also shows a marked squaring from the elliptical contours of a K-V distribution.

Figure 6 shows the evolution of a system which has a tune of 130° depressed by space charge to 50°. In this case the K-V envelope equations are themselves unstable and a more violent type of instability is observed. Here too, however, the system stabilizes rapidly but the emittance growth is considerably greater than in the first case.

The simulations discussed were performed using normalized variables. To relate them to a physical transport system, the variables must be reconverted to physical units. The transported power in a quadrupole channel is then,

$$ P = C \left( \frac{A}{z} \right)^{4/3} B_q^{2/3} \epsilon^{2/3} (\beta \gamma)^{7/3} (\gamma - 1) $$

where

- $P$ = power $(W)$
- $A, z$ = mass number, charge state
- $B_q$ = quadrupole pole tip field $(T)$
- $\epsilon = 4 <x^2>^{1/2} <x'^2>^{1/2}$, rms emittance $(m \cdot rad)$
- $x' = p_x/p_z$

The coefficient $C$ is dependent on the distribution function in the beam, the aperture size and the lens thickness. Since the thin lenses used in the simulation require an infinite pole tip strength and are not physical, some assumption must be made as to lens thickness. Laslett has shown that for a magnet system with half the space filled with magnets, with a 90° phase
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advance depressed to 30° by space charge, and with a K-V distribution filling the aperture, the
coefficient corresponds to

\[ C = 3.0 \times 10^{15}. \]

From this, an estimate of the coefficient corresponding to the distribution of Figure 3 can
be obtained. An aperture necessary to contain all the particles shown in Figure 3 must be
about twice as large as for the original K-V distribution so the pole tip field must be
correspondingly larger to maintain the same magnetic field gradient. This introduces a factor of
\(2^{-2/3}\) leading to a coefficient of \(C = 1.9 \times 10^{15}\). This is however, in terms of the initial emi-
ttance. In terms of the final rms emittance, another \(2^{-2/3}\) is introduced giving a coefficient of
\(C = 1.2 \times 10^{15}\).

The coefficient represents the distribution shown in Figure 5. A systematic search in
parameter space may in fact show that much greater currents can be stably transported and that
a proper choice of initial distribution will not lead to instability. The current result is, however,
important because the power levels observed are consistent with those required for thermonu-
clear pellet ignition. Since the simulations include most of the important physics, these results
seem to be a significant indication that transverse instabilities need not be a barrier to the use
of heavy ion beams as a source of pellet ignition energy.

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REFERENCES

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Fig. 1 — Four views of the initial $x p_x, y p_y$ phase space of a Kapchinskij-Valdimirshij distribution with the 90° phase advance depressed by space charge to 30°. Two thousand sample particles are shown.
Fig. 2 – Phase space of the distribution shown in Fig. 1 after passing through 20 doublets. The well developed structure of the instability is apparent.
Fig. 3 – Phase space after 100 doublets. Any evidence of the instability is no longer present.
Fig. 4 — Evolution of the $x_\text{rms}$ and $y_\text{rms}$ rms emittances showing the establishment of a steady state.
Fig. 5 — Contour plot of the $\gamma p_x$ phase space shown in Fig. 3. Emittance growth is seen to be primarily due to spreading in velocity space and the contours are noticeably squared from the initial K-V ellipses. Adjacent contours differ in density by a factor of four.
Fig. 6 – Evolution of the rms emittances for a system with a phase advance of 130° depressed to 50°. In this case the instability is of a more violent form and the emittance growth is considerably greater than in the first example.