LEVEL

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REMOTE INFRARED ATMOSPHERIC PROFILING SYSTEM (RIAPS)

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BY

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The subject contained in this annual report covers the task "Statistical Improvement of Ground-Based Infrared Profiles." During the past year, we have developed a simple, yet sufficiently accurate atmospheric transmission model to be used in the 15 μm CO₂ absorption region for the inversion of atmospheric temperature profile. The result of this study has been accepted for publication in June issue of Optics Letters. A preprint of this article is included in this report. Basically, we included temperature effect of the transmission function in the model such that the RIAPS could be operated with seasonal as well as geographical variations. If a set of transmission functions is known either analytically or empirically for a given temperature distribution, the first-order approximation method developed here may be used to calculate the transmission functions for a different temperature distribution. Since coefficients of the linear function are predetermined, this procedure can be implemented in an iteration process without significantly increasing the computation time.

The new transmission model is yet to be incorporated in the RIAPS software program. With current available measurement data (taken in the summer at San Diego and Pt. Mugu), it is difficult to verify effectiveness of the new transmission model. However, numerical simulation approach may be employed to compare the retrieved profiles using existing and modified transmission functions. With existing line-by-line calculation code, one can further look into combinations of various frequency channels with distinct transmission characteristics. The
discrepancy between calculated and measured radiances at certain frequency regions is a known fact, combining RIAPS measurements and analytic code, one might be able to shed some light on this problem.
Temperature effect on the atmospheric transmission function in the 15-\(\mu\)m region

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Abstract

The method of first-order Taylor expansion is used to study the temperature effect on the transmission function for several narrow spectral intervals in the 15-\(\mu\)m CO\(_2\) absorption region with direct application to the problem of thermal remote sounding. Numerical analysis indicates that the computed transmittances are in good agreement with those obtained by line-by-line calculation, especially for spectral intervals with relatively strong temperature dependence.
Information on the transmission characteristics of a real atmosphere in narrow spectral intervals is necessary to derive atmospheric temperature and humidity profiles from satellite or ground-based radiation measurements. The 15-μm CO₂ absorption band has been chosen for atmospheric thermal remote sounding, since carbon dioxide is uniformly distributed (assuming 330 ppm in this study) through the entire atmosphere. In a previous study, a set of ground-based irradiance measurements with spectral resolution around 7 cm⁻¹ were taken; the band centers are located at 669, 677.5, 688, 693, 700, 711.5, 724, and 730 cm⁻¹. These measurements were related to the atmospheric temperature distribution through the radiative transfer equation. With linearization of the Planck function, the temperature distribution was retrieved through a linear matrix inversion of the Fredholm integral equation of the first kind. An iteration process was included to correct the influence of the water vapor distribution obtained from irradiance measurements in the 18-μm region. Since this inversion problem involves an iteration process, atmospheric transmission functions need to be computationally efficient and highly accurate. Originally these functions were empirically determined, assuming temperature independence; obviously, this treatment of the transmission functions is not adequate under conditions of substantial seasonal or geographic temperature variations. One way to improve formulation of the transmission function is to include the temperature effect.

There are numerous methods to compute the atmospheric transmittance and the most accurate one is line-by-line calculation. In this procedure, the transmittances in narrow spectral intervals are computed as sums of contributions from each of the lines in the absorption band and are integrated over the atmospheric path. This approach is much too laborious and time consuming for use in real-time processing of remotely sensed data.

Alternative approaches include the Curtis-Godson method and the effective mass method, which replace the inhomogeneous path with an equivalent homogeneous path by defining effective values of pressure, temperature, and quantity of absorber. This approach is efficient but not sufficiently accurate.

Recently several approximate methods have been proposed for fast calculation of the atmospheric transmittance for satellite radiation measurements. An alternative method is presented here using transmission functions for the ground-based measurements in the numerical analysis. The current approach, originally suggested by Kondratyev and Timofeyev, expands the
transmittance at a certain pressure level in Taylor series using temperature as the variable. This method is especially useful when the transmission functions for a particular temperature distribution are accurately known, and when the approximate method is used to compute a correction term of the second-order of smallness for a different temperature distribution. Transmittance between \( x = 0 \) and \( x_1 \) of a nonhomogeneous atmosphere over a spectral interval \( \Delta \nu \) may be written as

\[
\tau_{\Delta \nu} (x_1) = \int_{\Delta \nu} \phi(\nu) \exp \left[ -\int_0^{x_1} u(x) \sum_i K_i (\nu, T, x) \, dx \right] d\nu / \int_{\Delta \nu} \phi(\nu) \, d\nu
\]  

(1)

where \( \phi(\nu) \) is the instrument response function, assumed to be triangular in this analysis; \( u \) is the optical thickness in the unit of atm-cm; and \( K_i \) is the absorption coefficient of the \( i \)th line. In the lower atmosphere, the Lorentz line profile for the absorption coefficient is adequate:

\[
K_i (\nu, T, x) = \frac{\alpha_i S_i (T)}{\pi \left( (\nu - \nu_i)^2 + \alpha_i^2 \right)}
\]  

(2)

The line intensity \( S_i (T) \) for a linear molecule may be written

\[
S_i (T) = S_i (T_o) \left( \frac{T_o}{T} \right)^2 \exp \left[ -1.439 E_i \left( \frac{1}{T} - \frac{1}{T_o} \right) \right]
\]  

(3)

where \( T_o \) is a reference temperature, \( E_i \) is the lower energy level of the absorption line, and the intensity is presented in the unit of atm\(^{-1}\) cm\(^{-2}\) at temperature \( T \). The induced emission and the vibrational partition function have been ignored in Eq. (3). The line half-width \( \alpha_i \) is approximately related to a reference value \( \alpha_{i0} \) at pressure \( P_o \) and temperature \( T_o \) as:

\[
\alpha_i = \frac{P}{\alpha_{i0} P_o}
\]  

(4)

The temperature dependence of the half-width is neglected in Eq. (4), since its influence on the transmittance is much smaller than that due to the line intensity for the temperature range being considered. Substituting Eqs. (2)-(4) into Eq. (1), the transmission function is expressed explicitly in terms of the tem-
temperature distribution \( T(x) \). For the method of first-order Taylor expansion, the transmittance may be written approximately as

\[
\tau_{\Delta \nu \nu} \left[ T(x) \right]_{x=x_1} = \tau_{\Delta \nu \nu} \left[ T_0(x) \right]_{x=x_1} + \int_0^{x_1} \frac{\delta \tau_{\Delta \nu \nu} [T(x')]}{\delta T} \left| T_0(x') \right| \delta T(x') \, dx'
\]

(5)

where

\[
\delta T(x') = T(x') - T_0(x').
\]

(6)

The term \( \delta \tau_{\Delta \nu \nu} / \delta T \) is the variational derivative which may be expressed

\[
\frac{\delta \tau_{\Delta \nu \nu}}{\delta T(x')} \bigg|_{T_0(x')} = \int_{\Delta \nu} \phi(v) \exp \left[ - \int_0^{x_1} u(x'') \sum_i K_i (\nu, T_0, x'') \, dx'' \right]
\]

\[
\times \sum_i u(x') K_i (\nu, T_0, x') \left[ - \frac{2}{T_0} + \frac{1.439 E_i}{T_0^2} \right] \, d\nu / \int_{\Delta \nu} \phi(v) \, d\nu .
\]

(7)

From Eqs. (5) through (7), the transmission function for a temperature distribution \( T(x) \) may be estimated when the transmittances for the reference temperature distribution \( T_0(x) \) are known. For a homogeneous atmosphere, the formulation in Eq. (5) is reduced to the familiar form

\[
\tau_{\Delta \nu \nu} (T) = \tau_{\Delta \nu \nu} (T_0) + \frac{\partial \tau_{\Delta \nu \nu} (T)}{\partial T} \bigg|_{T_0} (T - T_0).
\]

(8)

For numerical analysis, the transmittance \( \tau_{\Delta \nu \nu} [T(x)] \) and \( \tau_{\Delta \nu \nu} [T_0(x)] \), and the variational derivative \( \delta \tau_{\Delta \nu \nu} / \delta T \) are computed using the line-by-line calculation method. The approximate transmittance \( \tau_{\Delta \nu \nu} [T(x)] \) is then evaluated from Eq. (5) and compared with the value obtained from the line-by-line method. The line parameters were generated using molecular constants contained in the report of McClatchey, et al. The spectral intervals employed in Ref. 1 were chosen to investigate the accuracy of the current method. The interval at 669 cm\(^{-1}\) is practically opaque for surface temperature measurement, while the transmisson function at 724 cm\(^{-1}\) is very close to that at 730 cm\(^{-1}\). These two channels are therefore not considered in the current analysis.
Since the temperature dependence of the transmission function is determined chiefly by the line intensities expressed in Eq. (3), lines with different temperature dependence in a spectral interval could lead to difficulties in determining the dependence of the transmission function on temperature. The nature of this problem may be seen from Eq. (7), where the two terms \( \frac{2}{T} \) and \( \frac{1.439}{T^2} \) are opposite in sign. Thus, for a spectral interval where most of the lower energy levels of strong absorption lines are greater than a certain value \((-1.39T)\), the transmittance increases as the temperature increases within a certain temperature range. For the range 220K to 340K, the transmittance decreases as the temperature increases for the 677.5 cm\(^{-1}\) spectral interval, while other channels show the opposite trend. The temperature dependence at 688 cm\(^{-1}\) is very weak; the first-order approximation method failed to improve the transmittance, since a much more accurate numerical integration scheme is required. Figure 1 is a plot of the transmittances as a function of the optical thickness for a homogeneous atmosphere at 300K and 1 atm. These results were obtained by line-by-line calculations and first-order approximation using two reference temperatures, 260K and 340K. Usually, the difference between the reference temperature and the actual temperature is less than 40K (chosen in this example). Apparently use of a higher reference temperature gives better agreement between the approximate transmittance and the ideal transmittance. Furthermore, agreement between actual and approximate transmittances is always better in an optically thin atmosphere. The percentage of error of the approximate method is plotted in Figure 2. For those portions of the curves with large errors, the actual transmittance is less than 0.001; thus, the comparison is physically less significant.

This approximate method is also evaluated for a nonhomogeneous atmosphere. First, the transmission functions for the 1962 standard atmosphere and the tropical model atmosphere\(^7\) are computed using line-by-line calculations, and then the approximate transmission functions for the 1962 standard atmosphere are estimated using the tropical atmosphere as the reference temperature distribution. These results are listed in Table I where transmittance values less than 0.001 are entered as zero. In most cases, the accuracy of the approximate method is within one percent of the actual value, and the worst discrepancy is within 10 percent, which occurs at 677.5 cm\(^{-1}\). The approximate transmittances at 688 cm\(^{-1}\) were not computed, since a more accurate numerical integration scheme is required for this channel.
In summary, the temperature dependence of the transmission function for narrow spectral intervals in the 15-μm region has been investigated using the method of Taylor’s first-order approximation. Except for 688 cm⁻¹, the computed transmittance using the approximate method is in good agreement with that using the line-by-line calculation method when the reference temperature distribution is properly chosen. This method is highly accurate and computationally efficient for application to thermal remote sensing problems. It is especially suitable when transmission functions are accurately known for a specific model atmosphere and corrections are computed using the current method. Although the numerical analysis presented here is for a ground-based thermal sounding, it should apply equally to the problem of downward-looking satellite radiation measurements.

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References


Figure 1. Comparison of computed CO₂ transmittances using line-by-line calculation and first-order approximations — spectral resolution is 7 cm⁻¹.

Figure 2. Percentage of error of the computed transmittances using the approximate method for the same spectral intervals as in Figure 1.

Table I. CO₂ transmittances between the ground and selected altitudes for several spectral intervals in the 15-μm region.
Table I. CO₂ transmittances between the ground and selected altitudes for several spectral intervals in the 15-μm region.

<table>
<thead>
<tr>
<th>Frequency (cm⁻¹)</th>
<th>Approach</th>
<th>Temperature Profile</th>
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a L – line-by-line calculation method; F – first-order approximation
b S – 1962 standard atmosphere; T – tropical model atmosphere
Figure 1. Comparison of computed CO$_2$ transmittances using line-by-line calculation and first-order approximations — spectral resolution is 7 cm$^{-1}$.
Figure 2. Percentage of error of the computed transmittances using the approximate method for the same spectral intervals as in Figure 1.