Computer Simulation of Light Propagation through a Scattering Medium

by

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June 1978

Thesis Advisor: William M. Tolles

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ABSTRACT

A Monte Carlo computer model was developed to simulate the propagation of light through a scattering/absorbing medium using various parameters and phase functions. The model permits characterization of the spatial and temporal spread of light traversing plane-parallel clouds. It was found that both the time and spatial spread of light in a scattering medium are independent of the details of the phase function for a cloud thickness of greater than 15 extinction lengths.
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TABLE OF CONTENTS

I. INTRODUCTION ......................................................... 14
   A. HISTORY ................................................................. 14
   B. PURPOSE ............................................................... 15

II. THEORETICAL PREMILINARIES ...................................... 17
   A. INTRODUCTION ........................................................ 17
   B. PRINCIPAL CHARACTERISTICS OF SCATTERING/ABSORBING MEDIA .................................................. 18
      1. Parameters ....................................................... 19
      2. Types of Scattering ............................................. 20
      3. Phase Function .................................................. 22
   C. MIE SCATTERING ..................................................... 27
   D. RADIATION TRANSFER ............................................... 28
   E. ANALYTICAL METHODS .............................................. 29
   F. NUMERICAL METHODS ............................................... 30
   G. MONTE CARLO METHODS ............................................. 31
   H. THEORETICAL RESULTS ............................................. 32
      1. Angular Spreading ............................................... 33
      2. Spatial Spreading ............................................... 33
      3. Multipath Time Spreading ..................................... 33
      4. Total Transmission ............................................. 34

III. STATEMENT OF THE PROBLEMS ..................................... 36
   A. INTRODUCTION ....................................................... 36
   B. THE PROBLEMS ........................................................ 36
      1. Dependence of Spatial Spread on Phase Function .......... 36
2. Dependence of Time Spread on Phase Function........................................36
3. Transition Region from Forward to Multiple Scattering........................37
4. Spatial Characterization of Light Traversing Finite Clouds....................37
5. Model to Study the Above Problems.....................................................37

IV. METHODS OF SIMULATION...............................................................38

A. MONTE CARLO METHOD.................................................................38
1. The Model.........................................................................................38
2. Phase Functions................................................................................42
   a. Henyey-Greenstein Phase Function..............................................42
   b. Numerical Data Input for Arbitrary Phase Functions......................42
   c. Rayleigh Phase Function...............................................................43
3. Automation of Contour Plotting.........................................................43

B. ANALYTICAL METHODS........................................................----------43
1. Effective Attenuation Coefficient Model............................................44
2. Closed Form Time Spread Expression.................................................45

C. COMPARISON OF METHODS EMPLOYED.............................................45
1. Monte Carlo vs. Analytical.............................................................46
   a. Spatial Characterization............................................................46
      (1) Description of Relative Flux Contours..................................46
      (2) Comparison..............................................................................47
      (3) Conclusions.............................................................................47
   b. Temporal Characterization.........................................................52
      (1) Comparison.............................................................................52
2. This Monte Carlo Model vs. Bucher's Monte Carlo Model [1]..........................58
   a. Spatial Characterization......................58
      (1) Description of Scaling Method..............58
      (2) Comparison..................................59
      (3) Conclusions.................................59
   b. Temporal Characterization....................59
      (1) Comparison..................................61
      (2) Conclusions.................................64

V. RESULTS..............................................65
   A. SPATIAL SPREAD DEPENDENCE ON PHASE FUNCTION........................................65
   B. TEMPORAL SPREAD DEPENDENCE ON PHASE FUNCTION.........................................70
   C. REGION OF TRANSITION FROM FORWARD TO MULTIPLE SCATTER....................................76
   D. EFFECT ON SPATIAL CHARACTER OF LIGHT WHEN PASSING THROUGH A CLOUD....................84

VI. DISCUSSION...........................................89

VII. CONCLUSIONS........................................90

APPENDIX A: Generation of a Random Scattering Angle Weighted by an Arbitrary Phase Function.................................................................91

APPENDIX B: Description and Documentation of Program to Adapt MIE Theory to Machine Computation.................................................................96

APPENDIX C: Simulation of a Cloud in Monte Carlo Routine LITE..................................106

APPENDIX D: Automation of Relative Flux Contour Plotting in DRLITE Routine..................109
APPENDIX E: Derivation, Documentation and Verification of Effective Attenuation Coefficient Method......................112
APPENDIX F: Checks on Possible Errors..........................122
APPENDIX G: Derivation of Closed Form Expression for Time Spread........................................125

COMPUTER PROGRAMS

MIE Scattering Program..............................................127
Monte Carlo Input-Output Routine..............................135
Monte Carlo Simulation of Light Propagation Routine..................................................140
Effective Attenuation Coefficient Routine....................153

LIST OF REFERENCES......................................................157
INITIAL DISTRIBUTION LIST...........................................160
LIST OF TABLES

I.  Scattering Coefficients and Albedo
    Used in Simulation..........................42

II. Tabulation of Parameters and Phase
    Function Used in Figures 19-36............66

III. Summary of Time Spreading by
     One Kilometer Thick Cloud..................83
## LIST OF FIGURES

1. Henyey-Greenstein Phase Function .................................. 24  
3. NOSC Fog Phase Function ........................................... 26  
4. Monte Carlo Model ................................................... 39  
5. Cloud Model .................................................................. 40  
6. EAC Model vs. Monte Carlo Model ................................. 48  
7. EAC Model vs. Monte Carlo Model ................................. 49  
8. EAC Model vs. Monte Carlo Model ................................. 50  
9. EAC Model vs. Monte Carlo Model ................................. 51  
10. Three Model Time Spread Comparison .............................. 53  
11. Three Model Time Spread Comparison .............................. 54  
12. Three Model Time Spread Comparison .............................. 55  
13. Three Model Time Spread Comparison .............................. 56  
14. Three Model Time Spread Comparison .............................. 57  
15. This Monte Carlo vs. Bucher's Monte Carlo [1] Spatial Spread Comparison ................................................. 60  
16. This Monte Carlo vs. Bucher's Monte Carlo [1] Spatial Spread Comparison ................................................. 60  
19. Spatial Spread Dependence on Phase Function ................. 67  
20. Spatial Spread Dependence on Phase Function ................. 68
21. Spatial Spread Dependence on Phase Function

22. Spatial Spread Dependence on Phase Function

23. Spatial Spread Dependence on Phase Function

24. Time Spread Dependence on Phase Function

25. Time Spread Dependence on Phase Function

26. Time Spread Dependence on Phase Function

27. Time Spread Dependence on Phase Function

28. Time Spread Dependence on Phase Function

29. Time Spread Dependence on Phase Function

30. Time Spread Dependence on Phase Function

31. Time Spread Dependence on Phase Function

32. Time Spread Dependence on Phase Function

33. Effect of Finite Clouds on Spatial Character of Light

34. Effect of Finite Clouds on Spatial Character of Light

35. Effect of Finite Clouds on Spatial Character of Light

36. Effect of Finite Clouds on Spatial Character of Light

37. Diagram of Random Generation of Theta

38. Geometry of EAC Method

39. Verification of EAC Method

40. Verification of EAC Method
LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>absorption coefficient</td>
</tr>
<tr>
<td>$a$</td>
<td>(Appendix B) constant of particle size distribution</td>
</tr>
<tr>
<td>$a_n$</td>
<td>Mie scattering coefficient</td>
</tr>
<tr>
<td>$A_i$</td>
<td>(Appendix A) intercept of line connecting $(\theta_i, p_i)$ and $(\theta_{i+1}, p_{i+1})$</td>
</tr>
<tr>
<td>$A_j$</td>
<td>(Appendix B) Mie scattering amplitude</td>
</tr>
<tr>
<td>$b$</td>
<td>constant of particle size distribution</td>
</tr>
<tr>
<td>$b_n$</td>
<td>Mie scattering coefficient</td>
</tr>
<tr>
<td>$B_i$</td>
<td>slope of line connecting $(\theta_i, p_i)$ and $(\theta_{i+1}, p_{i+1})$</td>
</tr>
<tr>
<td>$\langle \cos \theta \rangle$</td>
<td>average cosine of the scatter angle $\theta$ for phase function</td>
</tr>
<tr>
<td>$C_i$</td>
<td>factor used in generating $\theta_R$</td>
</tr>
<tr>
<td>$D$</td>
<td>mean free path between collisions</td>
</tr>
<tr>
<td>$F$</td>
<td>total flux through an aperture</td>
</tr>
<tr>
<td>$G$</td>
<td>Henyey-Greenstein parameter $= \langle \cos \theta \rangle$</td>
</tr>
<tr>
<td>$h_n^2$</td>
<td>spherical Bessel function of order $n$</td>
</tr>
<tr>
<td>$H_B$</td>
<td>beam spread function</td>
</tr>
<tr>
<td>$i_j(\theta)$</td>
<td>dimensionless intensity parameters</td>
</tr>
<tr>
<td>$I$</td>
<td>beam intensity</td>
</tr>
<tr>
<td>$j_n$</td>
<td>spherical Bessel function of order $n$</td>
</tr>
<tr>
<td>$k$</td>
<td>wave number</td>
</tr>
<tr>
<td>$K_{abs}$</td>
<td>absorption coefficient</td>
</tr>
<tr>
<td>$K_{ext}$</td>
<td>extinction coefficient</td>
</tr>
<tr>
<td>$K_{Mie}$</td>
<td>portion of $K_{sca}$ due to Mie scattering</td>
</tr>
<tr>
<td>$K_{Ray}$</td>
<td>portion of $K_{sca}$ due to Rayleigh scattering</td>
</tr>
</tbody>
</table>
\( K_{\text{sca}} = \) scattering coefficient
\( L = \) multipath time spread
\( m = \) complex index of refraction
\( n(x) = \) particle size distribution as a function of Mie size parameter
\( N = \) number of phase function data pairs
\( \text{NORM} = \) normalization factor used for individual panel
\( P(\theta) = \) normalized scattering phase function
\( P(\theta, \phi) = \) Normalized scattering phase function
\( P_i = \) value of scattering phase function at ith data pair
\( P_{ij}(\theta) = \) elements of normalized scattering matrix
\( Q_{\text{ext}} = \) extinction efficiency factor
\( Q_s = \) scattering efficiency factor
\( Q_{\text{sca}} = \) scattering efficiency factor
\( r = \) radius of scattering particle
\( \bar{r}, \bar{r}, \bar{r} = \) spatial coordinates, time adjusted for numerical method
\( R = \) range of distance
\( R = \) (Appendix A) uniform random number in interval \([0,1]\)
\( R_l = \) random number related to \( R \)
\( \text{RPT} = \) ratio of particulate to total scatter
\( s = \) scattering coefficient
\( S_j = \) \( kA_j \)
\( T = \) physical thickness of cloud
\( W_i = \) weight of panel \( i \), used in generating \( \theta_R \)
\( x = \) Mie size parameter
\( \alpha = (\text{Appendix E}) \) extinction coefficient
\( \alpha = (\text{Appendix B}) \) constant of particle size distribution
\( \alpha_e = \) effective extinction coefficient
\( \beta_{\text{ext}} = \) volume extinction cross section
\( \beta_{\text{sca}} = \) volume scattering cross section
\( \gamma = \) constant of particle size distribution
\( \gamma_0 = \) RMS scatter angle
\( \epsilon = \) absolute value of index refraction of particle
\( \theta = \) angle off direction of incidence
\( \theta_i = \) value of theta in ith data pair
\( \theta_R = \) weighted scatter angle
\( \lambda = \) wavelength of light
\( \mu = \) absolute value of index of refraction of medium surrounding particle
\( \pi_n = \) angular functions used in Mie Series
\( \sigma_{\text{sca}} = \) total scattering cross section
\( \tau = K_{\text{ext}} \)  
\( \tau_{\text{d}} = \) effective scattering thickness
\( \tau_n = \) angular functions used in Mie Series
\( \phi = \) longitudinal angle around incident direction
\( \omega_o = \) single scatter albedo
\( \Omega_x = \) coordinate transformation factor
\( \Omega_y = \) coordinate transformation factor
\( \Omega_z = \) coordinate transformation factor
I. INTRODUCTION

A. HISTORY

The existence of clouds and fog in many regions of the earth presents a formidable problem to the designer of an optical communication system whose transmission channel is the atmosphere. A scattering medium can inhibit system performance by inducing beam spread, dispersion in angle-of-arrival, degradation of spatial coherence and dispersion in time and frequency of the modulated optical beam. The development of an optical communication system for atmospheric applications requires an accurate knowledge of the effects of scattering on light propagation. Numerous studies have been made in an attempt to provide this knowledge. Different aspects of the problem have been assaulted using various mathematical models. Monte Carlo computer simulation has been used by Bucher [1], Plass and Kattawar [2] through [5], Junge [6] and Danielson, Moore and van de Hulst [7], to mention only the few used extensively in this work. Equally prominent attempts have been made using analytical methods such as those by Arnush [8], Gordon [9], Stotts [10], Hansen [11], Ishimaru [12], Kennedy [13] and Lutomirski and Yura [14]. Complicated numerical techniques have been used only by Dell-Imagine [15] and Zachor [16]. Each attempt has contributed to the stock-pile of knowledge required but additional problems remain to be investigated. Many books have been published on the subject
of light scattering. Those found most useful to this work were van de Hulst [18], Kerker [19], Chandrasekhar [20] and Deirmendjian [21]. Interested parties and reasons for their interest are briefly discussed in the following paragraphs.

B. PURPOSE

Current navy operational communications systems suffer from a number of problems. There is no operational system which is not subject to jamming, intercept, spoofing and direction finding. In addition, it appears that optical communications systems have great promise in solving these problems for many applications [17]. Any information that could be used in evaluating such systems is desired.

The Navy's effort to create a worldwide satellite-to-submarine communication network is another rapidly progressing area requiring information of the nature being investigated in this thesis. In this situation, light propagation through water further complicates the matter.

Among other parties that may require information on light propagation are the United States Coast Guard in their aids to navigation system, NASA in satellite communications and researchers trying to determine the composition of the atmosphere by analyzing scattered radiation.

The purpose of this work is to characterize both the spatial and temporal aspects of a light beam propagating through a scattering/absorbing medium using an analytically verified Monte Carlo model of the system. Models were created
and tested against each other until an adequate level of confidence in each was obtained. The models were then used to study more specific questions concerning time and spatial spread. More explicitly, the models were used to observe what effect different phase functions had on the temporal and spatial aspects of light scattering. It was believed that the pronounced forward peak of some phase functions would generate different time and spatial character than a moderately peaked phase function. An investigation attempting to verify this belief was an important part of this work.

Spatial characterization of light passing through a cloud of finite thickness, and transition from the forward scatter to diffusion region was also investigated.

Once confidence was established in the models used, the general areas of agreement or disagreement were studied in an attempt to specify when one model may be used more efficiently than another for generating information concerning a specific situation. The drawbacks and strong points of each model type have been determined and mentioned where appropriate.
II. THEORETICAL PRELIMINARIES

A. INTRODUCTION

The purpose of this section is to review the various theoretical formulations currently used to characterize optical propagation in a single and multiple scattering medium. A complete coverage of any one of the topics in the following sub-sections is most certainly not contained here but sufficient references are provided for the reader inclined to investigate any topic in more detail. Only those details related closely to this work are described in any detail and even then only the important conclusions are mentioned in many cases.

The order of presentation is as follows. The principal characteristics of the scattering/absorbing medium are mentioned briefly, thereby defining frequently used terms. A brief summary of Mie theory is included to present the valuable insight necessary in investigating the problems posed. Radiative transfer theory is mentioned in passing because the solution of the complicated radiative transfer equation is essentially the subject of many analytical attempts at solving the scattering problem. This section then probes into mathematically modeling the geometry of a real atmosphere. Three general categories of models are discussed.

Analytical models, which made various assumptions allowing closed form expressions for certain characteristics of the
medium, are discussed. The approximations in such models normally detract from the total worthiness of such an approach.

The discussion then progresses into the use of numerical methods in evaluating analytical solutions as well as in evaluating the radiative transfer equation directly. This often overlooked powerful technique has been used by theoreticians because of its ease in solving complicated integro-differential equations for which exact solutions have been proven impossible.

Monte Carlo methods, used predominantly in this work, are then discussed. This technique is used extensively due to its ease in adaptation to odd geometries of the scattering medium. In this work this benefit was used only to model finite clouds in an otherwise homogeneous atmosphere.

This section is completed with a general discussion of the results predicted by theoretical insight into the problem of light transfer in a random medium.

B. PRINCIPAL CHARACTERISTICS OF SCATTERING/ABSORBING MEDIA

This section divides the characteristics of a scattering medium into three sub-sections. The first defines the parameters of the atmosphere which during the course of this work were considered constants. The second considers the types of scattering encountered in a scattering medium and the third briefly describes the significance of the phase function in scattering theory.
1. **Parameters**

In general, a medium may exhibit both scattering and absorption. Some of the energy of a collimated incident beam on a particle will be scattered over all possible directions; the rest will be absorbed by the particle and lost from the radiation field. In the single scattering problem the intensity of radiation at any point along the direction of plane wave propagation is given by:

\[
I(R) = I(0)e^{-(K_{ext})R} = I(0)e^{-(K_{abs}+K_{sca})R},
\]

where the absorption loss and scattering loss together is called extinction. The ratio of scattering coefficient to the extinction coefficient is called the albedo of single scatter. Thus,

\[
\omega_0 = \frac{K_{sca}}{K_{abs}+K_{sca}}, \quad 0 < \omega_0 < 1 \tag{2}
\]

It is sometimes more convenient to discuss optical dimensions in terms of mean free paths of photons, i.e., the average distance between collisions. The mean free path is often called an extinction length and is given by the reciprocal of the extinction coefficient. Optical thickness of a given medium is a dimensionless number representing its thickness in extinction lengths or mean free paths.

The transmittance of the atmosphere is the fractional intensity remaining in a beam after traversing a path \( R \) units
long. Explicitly, with $K_{\text{ext}}$ in km$^{-1}$ and $R$ in kilometers,

$$\tau = \frac{I(R)}{I(0)} = e^{-K_{\text{ext}}R}.$$  \hfill (3)

The term visibility is defined as [22]

$$V = \frac{1}{K_{\text{ext}}} \ln \frac{1}{.02} = \frac{3.912}{K_{\text{ext}}},$$  \hfill (4)

thus the transmittance for a path length just equal to the visibility is two per cent. In many atmospheric light propagation problems the albedo is close to unity and $K_{\text{ext}}$ in the above equations can be expressed by $K_{\text{sca}}$ with little error.

In a real atmosphere there is a contribution to scattering by small particles as well as a contribution by large particles. $K_{\text{sca}}$ then is defined in even more detail. The following paragraphs present some of these details.

2. Types of Scattering

Atmospheric light scattering can be classified in two general categories. Small particle scattering, where the particle radius is much smaller than the wavelength of the incident light, is called Rayleigh scattering, and other scattering from larger particles is termed Mie scattering. The terminology of this work refers to $K_{\text{Ray}}$ as the portion of the scattering coefficient due to Rayleigh scattering and $K_{\text{Mie}}$ as the portion due to Mie scattering. Therefore,
\[ K_{\text{ext}} = K_{\text{abs}} + K_{\text{Mie}} + K_{\text{Ray}}, \quad (5) \]

Mie scattering is often called particulate scattering while Rayleigh is often referred to as molecular scattering. A ratio of particulate to total scattering is defined for use in this thesis as:

\[ \text{RPT} = \frac{K_{\text{Mie}}}{K_{\text{Mie}} + K_{\text{Ray}}}. \quad (6) \]

Scattering may be isotropic (scatters in all directions equally) or anisotropic (scatters as a function of angle off incident direction). The latter only is used here. The problem of defining scatter for single particles is well understood. The theory of scattering has been extended to a medium of many equal sized particles, a monodispersion, and to a medium of many different sized particles, a polydispersion [18, 21].

When a photon undergoes only one collision in traversing a scattering medium, single scattering has taken place. On the other hand, when a photon undergoes more than one collision, multiple scattering has occurred. This work involves independent single and multiple scattering in spherical polydispersions, where no scattering event affects other scattering events. For further details related to scattering parameters, types of scattering and other more specific topics
the reader is referred to an excellent text on the subject, McCartney [22]. The following paragraphs focus on the angular dependence of scattering for scattering of different types.

3. Phase Function

The phase function expresses in a formal manner the angular dependence of scattering. The phase function, denoted here by \( P(\theta) \), is defined by van de Hulst [18] as the ratio of the energy scattered per unit solid angle in a given direction to the average energy scattered per unit solid angle in all directions. This definition requires that the integral of the phase function be normalized to unity, which is to say that,

\[
\frac{1}{4\pi} \int_{0}^{2\pi} \int_{0}^{\pi} P(\theta) \sin \theta \, d\theta \, d\phi = 1 \quad (7)
\]

When expressed in this way the phase function is a scalar function. In a more complex analysis of scattering theory, there are phase functions of different types for different polarizations of light. These functions form elements of the normalized scattering matrix which is used to transform Stoke's parameters. Detailed discussion of the parameters is given by Refs. 18 and 21. In Appendix B a program to evaluate the Stoke's parameters for the spherical polydispersions used in this work is presented and explained. The scalar phase function is obtained by averaging the two Stoke's parameters obtained using this computer adapted Mie theory.

In many cases the phase function is easily approximated as a closed form function of scatter angle. In these cases,
the Henyey-Greenstein function is often used as the approximating function because of its simple form. For the Henyey-Greenstein phase function (Figure 1),

\[ P(\theta) = \frac{(1-G^2)}{4\pi(1+G^2-2G\cos\theta)^{1.5}}, \quad -1 < G < 1 \quad (8) \]

where \( G \) is a parameter which equals \( \langle \cos\theta \rangle \) for the phase function. A \( G \) value between .80 and .87 approximates real atmospheric phase functions quite well in most cases. However, the following phase functions were not represented well using the Henyey-Greenstein phase function no matter what parameter was used.

Phase functions used in this work were of Water Cloud C.2 [21] at a wavelength of .53 microns (Figure 2) which was calculated using the program of Appendix B, and NOSC Fog (Figure 3) which was calculated in a similar manner using an experimentally determined particle distribution near San Diego, California. For further discussion of phase functions, see "Methods of Simulation" later in this thesis. References 18 and 21 study the phase function and its generation extensively.

In general the phase function is quite symmetric in the forward and backward direction for particles small compared to wavelength. For large particles the phase function has an increasingly complicated dependence on angle of scatter.

As one might imagine, the scalar phase function plays a significant role in single and multiple scattering problems.
Figure 1
Henyey-Greenstein Phase Function

Henyey-Greenstein

$\text{AVG } \cos \theta = 0.83$

$\text{RMS } \theta = 0.65$

ALBEDO $= 1.00$

$P(0.00) = 5.00 / \text{sr}$

WAVELENGTH $= .53 \text{ micrometers}$
WATER CLOUD C.2

AVG COS $\theta = .83$
RMS $\theta = .65$
ALBEDO = 1.00
EXT COEFF = 11.2 / KM
P [0,0] = 115.5 / SR
WAVELENGTH = .53 MICROMETERS

SCATTERING ANGLE [ DEGREES ]

Figure 2
Water Cloud C.2 Phase Function [21]
NOSC Fog

AVG COS $\theta = .83$
RMS $\theta = .69$
ALBEDO = 1.00
EXT COEFF = 6.4 / KM
WAVELENGTH = .53 MICROMETERS
P [0,0] = 244.8 / SR

Figure 3

NOSC Fog Phase Function
Much attention has been focussed on defining the conditions under which the entire Mie series can be approximated by analytic expressions such as the Henyey-Greenstein function mentioned previously [1, 7, 11, 18]. This thesis is part of the quest to define these conditions.

C. MIE SCATTERING

Mie theory considers the scattering and absorption which occurs when an electromagnetic wave is incident on a spherical particle. It generates desired parameters of the atmosphere, namely the extinction and scattering cross sections, as well as the phase function. Mie theory is extremely versatile in that it can be applied at any particle size to wavelength ratio and can be extended to many particles and more importantly polydispersions. Appendix B explains the adaptation of Mie theory to machine computation for collecting scattering data for polydispersions of a given particle distribution. As mentioned before, Mie theory was used to generate optical parameters of the Water Cloud C.2 [21] model at a wavelength of .53 microns. It is prudent at this time to define the Mie scattering parameter as

\[ x = \frac{2\pi r}{\lambda} \]  

(9)

where \( r \) is the radius of the particle and \( \lambda \) is the wavelength.

For the reader interested in complete derivations of Mie theory, see Chandrasekhar [20], Sekera [23], Born and Wolf [24] or van de Hulst [18].
D. RADIATION TRANSFER

Any discussion of light propagation through the atmosphere would not be complete without at least a mention of the radiative transfer equation. This equation models mathematically what is happening as light transverses a scattering medium. The radiative transfer equation in its simplest form is [10],

\[
(\mu \frac{d}{d\tau} + 1) I(\tau, \vec{r}, \mu, \theta, t) = \omega_o I_o(\tau, \vec{r}, \mu, \theta, t) + \omega_o \int_{-1}^{1} \int_{0}^{2\pi} P(\theta, \theta'; \theta'', \theta') I(\tau, \vec{r}, \mu', \theta', t) d\mu' d\theta'
\]

where \( I \) is the scattered radiance, \( I_o \) is the radiance due to the distributed source produced by the incident beam transversing the region, \( \mu = \cos \theta, \tau = k_ext z \) is the optical thickness, \( \vec{r} \) are the spatial coordinates, \( \theta, \theta' \) are angular coordinates, \( \omega_o \) is the albedo, \( P \) is the scalar phase function, and \( t \) is time.

Approximations can be made to solve this equation for \( I(\tau, \vec{r}, \mu, \theta, t) \) in closed form. The equation has basically two components. One represents the field caused by the incident spatially distributed field and the other represents the field scattered out of the direct beam but redirected back into the same direction by other scattering events. For a complete derivation and explanation of this equation, Chandrasekhar [20] is suggested. Under specific conditions, approximate solutions to this equation can be found. The next section mentions a few of the efforts made toward finding useful solutions.
E. ANALYTICAL METHODS

As mentioned previously, several authors have attempted to provide knowledge of the effects of particulate multiple scattering on light propagation by applying analytical methods [8-13, 20, 25, 26]. One analytical development by Arnush [8] utilized radiative transfer theory and the small angle approximation to characterize the light. Arnush made two assumptions that greatly weakened his model: (1) He assumed the incident signal did not experience pulse broadening while traversing an optically thick medium, and (2) he assumed the incident beam never directly created internal emission sources. Stotts [10] does consider the previous details but still makes the small angle approximation, claiming that the phase function is highly peaked in the forward direction. Other attempts at gathering knowledge include that by Ishimura and Hong [12] who reported an analytical study of coherence using first order approximations. The results are valid for weak fluctuations in the medium.

Zachor [16] uses a double-integral transform method which is evaluated recursively to obtain the aurole radiances contributed by successive scattering orders. He has assumed in his calculations a homogeneous unbounded atmosphere and his results are good for short ranges only.

In any case, because of the complexity of the integrals involved, exact analytical methods yield results only after repeated numerical integration. Even in the small angle
approximation, the closed form solution [27] requires seven successive integrations to obtain a single value of the irradiance caused by a unidirectional point source of light. All this numerical work, besides being tedious, tends to mask the functional dependence of the results on the underlying physical geometry and optical parameters.

There are many complications in solving the multiple scattering problem analytically. Nevertheless, analytical work does contribute significantly to overall knowledge of the problem because in many cases the approximations made do not substantially deviate from reality.

In this thesis, two of the more straightforward analytical methods are used; Gordon [9] and Stotts [26]. The former concerns spatial spread and the latter, time spread.

Dell-Imagine [15] derives a solution to the radiative transfer equation analytically in the usual fashion using no assumptions until he has to actually get numerical results. His numerical approach to evaluating the transfer equation seems quite attractive and is briefly mentioned in the following section.

F. NUMERICAL METHODS

The solution of the radiative transfer equation is sufficient to specify the properties of a received signal which has passed through a multiple scattering region. The solution, however, requires numerical computation on a digital computer. The solution has the general form [15],
\[ I(x,y,z,t,\theta,\phi) = I(\vec{r},\theta,\phi,0) \exp \left[ -Q_s c \int_0^t \vec{\kappa}(\vec{r}) d\lambda \right] \]
\[ + \frac{Q_s c}{4\pi} \int_0^t \int_0^{2\pi} n(\vec{r}) \exp \left[ -Q_s c \int_0^t n(\vec{r}) d\gamma \right] I(\vec{r}',\theta',\phi',\lambda) \]
\[ P(\theta,\phi; \theta',\phi') \sin \theta' d\theta' d\phi' d\lambda \quad (11) \]

where,

\[ \vec{r} = \{(x-\Omega x ct),(y-\Omega y ct),(z-\Omega z ct)\} \quad (12) \]
\[ \vec{r}' = \{(x-\Omega x c(t-\gamma)),(y-\Omega y c(t-\gamma)),(z-\Omega z c(t-\gamma))\} \]
\[ \vec{r}'' = \{(x-\Omega x c(t-\lambda)),(y-\Omega y c(t-\lambda)),(z-\Omega z c(t-\lambda))\} \]
\[ \Omega_x = \sin \theta \cos \phi \quad \Omega_y = \sin \theta \sin \phi \quad \Omega_z = \cos \phi \]
\[ Q_s = \text{scattering efficiency factor} \quad c = \text{speed of light} \]

which has not been restricted to any shape of cloud. By establishing boundary conditions, initial conditions and density of particles \(n\), a solution for \(I(x,y,z,t,\theta,\phi)\) can be obtained by approximating the equation by a group of simultaneous algebraic equations. Dell-Imagine describes this approximation and the numerical methods used in solving the equation and the reader is referred to his work for further details.

G. MONTE CARLO METHODS

The Monte Carlo method can be applied to any problem if one knows the probability for each step in the sequence of
events and desires the probability of the total of all possible events. Thus, the Monte Carlo method may be used to study problems in radiative transfer. As mentioned in the introduction, many authors have described their attempts to do so. The Monte Carlo calculations are relatively easier to model than other methods especially when the geometry becomes difficult. The Monte Carlo method is very consumptive of computer time and only approximate information is obtained. Some generally useful results have been presented by Bucher [1], Junge [6] and Hansen [11]. Monte Carlo results generally yield excellent agreement with experimental data; however, the results deteriorate statistically at large ranges or narrow observation angles.

Details of a program for generation and curve fitting of Monte Carlo data are contained in Junge [6]. This thesis uses the groundwork of that report to extend study into regions other than ultraviolet light in a homogeneous space.

H. THEORETICAL RESULTS

The purpose of this section is to summarize the effects scattering has on light propagation through clouds as predicted by the theories of the previous sections. In doing this the definitions of effects investigated are given. Actual quantitative relations used in this thesis are reserved for later sections and qualitative descriptions are found here.
Previous Monte Carlo, analytic, and numeric studies of the multiple scattering problem have found that light pulses are distorted in the following manner:

1. **Angular Spreading**

   Angular spreading constitutes an effective decollimation of the incident radiation. It contributes to beam spread, dispersion in angle-of-arrival and loss of spatial coherence.

2. **Spatial Spreading**

   Spatial spreading indicates the dimensional increase of the beam's finite cross section. It is related to the above parameter in that angular spread inside the medium produces spatial spread as the pulse propagates on. Spatial coherence and beam spread are related to this parameter.

3. **Multipath Time Spreading**

   Different distances along various possible propagation paths imply different transit times for a photon. Thus, a short optical pulse will incur pulse broadening after traversing a multiple scattering region. This pulse broadening is called multipath time spreading. The maximum pulse frequency that can be used in communicating is determined by this parameter.

   Multipath time spread is often defined as the difference between the average transit time incurred from multiple scattering and the normal transit time in the absence of scattering. This definition is normally used when the time dependence of the input is that of a delta function. The
previous description of multipath time spread concerns input pulses of finite width.

4. Total Transmission

Total transmission describes the amount of irradiance left in the pulse after traversing the medium. It is inversely related to the beam attenuation.

Bucher [1] found that the amount and distribution of multipath time spreading was essentially independent of the detailed shape of the scattering function for sufficiently thick clouds. He also observed that the propagation parameters for sufficiently thin clouds were dependent both on the cloud parameters and on the scattering function.

Gordon [9] found that within certain ranges and angles of practical interest, the flux and beam spread function can be adequately approximated by closed form expressions. His work was related to scattering under water but can be applied equally well to atmospheric scattering when the phase function is very forward peaked.

Time spreads on the order of microseconds to milliseconds have been reported by Bucher [1], Stotts [10,26] and Ishimura and Hong [12]. Danielson, Moore and van de Hulst [7] and Hansen [11] found that the Henyey-Greenstein function adequately approximated the true cloud or haze phase function in determining the reflection and transmission characteristics of clouds.

It is the purpose of this thesis to further verify results found previously and to quantify some of the more
important cloud characteristics with respect to time and spatial spread.
III. STATEMENT OF THE PROBLEMS

A. INTRODUCTION

There are many areas in the study of multiply scattered light that could easily be subject to further study. It was necessary to select a few problems which were assailable using available groundwork and techniques.

B. THE PROBLEMS

The Monte Carlo routine of Ref. 6 was employed to investigate the following problems:

1. **Dependence of Spatial Spread on Phase Function**

   Characterize the spatial spread of light traversing scattering media of various types. Select phase functions of high, moderate and low forward peakedness all with the same $\langle \cos \theta \rangle$ and the same root mean squared scatter angle. Determine the effect on spatial character of light of using the different phase functions. Determine the conditions concerning dependence on phase function.

2. **Dependence of Time Spread on Phase Function**

   Characterize the time spread of light traversing scattering media of various types. Use the same phase functions selected above to determine the effect on temporal character of light of using the different phase functions. Determine the conditions concerning dependence on phase function.
3. **Transition Region from Forward to Multiple Scattering**

Using the results of the previous two characterizations, determine information concerning the transition from the region of primarily forward scatter to the diffusion region where scatter is directed in all directions. Estimate the accuracy of the information and verify its agreement with other theories.

4. **Spatial Characterization of Light Transversing Finite Clouds**

Characterize the spatial spread of light traversing a homogeneous medium. Characterize the spatial spread of light traversing the same medium except that a dense cloud of scatterers of finite thickness has been added. Compare the two characterizations and discuss the results.

5. **Model the Above Problems**

Create a model to simulate the geometries of the above problems and verify at each step that the model is indeed generating results consistent with existing theory.
IV. METHODS OF SIMULATION

A. MONTE CARLO METHOD

The computer routine of Ref. 6 was available for use at the outset of this endeavor. The routine could calculate both spatial and temporal characteristics of light traversing a scattering medium. It required as input the scattering parameters and phase function parameters of the scattering particles of the medium. The routine was restricted to homogeneous atmospheres utilizing a Henyey-Greenstein phase function or a Modified Henyey-Greenstein phase function [16], and a given portion of Rayleigh scattering. It was necessary to modify the routine so that it adequately represented the geometry of a finite cloud and so that any arbitrary phase function could be adapted to it. These changes are summarized in the following sub-sections.

1. The Model

Figure 4 represents the model used in this simulation. The model is identical to that of Ref. 6 with respect to photon path generation and accountability. The figure depicts a cloud on whose left boundary the photons are incident at the center of the accountability shells, and at whose right boundary time and spatial information is tabulated. Figure 5 expands the cloud and lists the names of the parameters used in the simulation. KSCA1 and KSCA2 represent the scattering coefficients in inverse kilometers of the inside and outside
Figure 4
Monte Carlo Model
when phase function other than Henyey-Greenstein is used, it is noted on the respective graph

Figure 5
Cloud Model
media respectively. Likewise, KABS1 and KABS2 represent the absorption coefficients of the two media. RPT1 and RPT2 represent the ratio of particulate to total scatter and G1 and G2 represent the Henyey-Greenstein phase function parameters when used in this simulation. THICK is the physical thickness of the cloud in kilometers. Appendix C gives details on the actual modification of the routine.

At each crossing of a photon from inside of the cloud to beyond the cloud, the total distance traveled is determined and tabulated in time of arrival bins. Time spread and delay information is presented using these bins. A running summation and counter are used to calculate the mean and standard deviation of time required to reach the cloud boundary.

One other change from the original routine is conversion of dimensions from units of extinction lengths to units of kilometers. Only minor changes in program logic were required to make this change.

Typical values of atmospheric parameters were used in most simulations. A summary of the parameters used is given in Table I. All diagrams of spatial and temporal character include parameters used in the specific case.

The Monte Carlo results of Bucher [1] were very useful as guidelines for determining correct operation of the model created. Comparisons of the two models appear later in this section.
TABLE I

Scattering Coefficients used in the Simulation

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<th></th>
<th>KSCA (km⁻¹)</th>
<th>Albedo</th>
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<td>Light Haze</td>
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<td>NOSC Fog</td>
<td>6.37</td>
<td>1.0</td>
</tr>
<tr>
<td>Water Cloud C.2</td>
<td>11.18</td>
<td>1.0</td>
</tr>
</tbody>
</table>

2. Phase Functions

As mentioned in the statement of the problem, three phase functions of different peakedness yet equal in \( \langle \cos \theta \rangle \) were needed to investigate the problem. The following phase functions were employed:

a. Henyey-Greenstein Phase Function

Figure 1 depicts the functional dependence of scatter on angle off direction of incidence, \( \theta \). The closed form expression for the Henyey-Greenstein function has been stated earlier in Equation 8. A G value of .83 was selected so as to match other phase functions used in the simulation. The peak of the phase function is only five units. Reference 6 explains its use in the Monte Carlo routine.

b. Numerical Data Input for Arbitrary Phase Functions

The other two phase functions required development of a method for inputting phase functions consisting only of data pairs and no closed functional form. They are that representing Water Cloud C.2 [21] at .53 microns and that of
NOSC Fog. The first was generated by use of the program of Appendix B, the second by a similar calculation using an experimentally obtained water particle distribution. Appendix A describes the details of generating a random scatter angle weighted by these arbitrary phase functions defined by data pairs.

c. Rayleigh Phase Function

This routine can also simulate a fractional amount of the total scatter as Rayleigh scatter. In this work the use of this capacity was minimal because in most cases water clouds of predominately large particles were simulated.

3. Automation of Contour Plotting

Contours of equal relative photon flux, as described in Ref. 6, were used in this work for spatial characterization. A method of machine computation was adapted to the DRLITE routine. This allowed the computer to perform the tedious interpolation made before by pocket calculator. Although useful in some instances, the procedure proved quite computer time consumptive. Its usefulness was very pronounced in comparing the Monte Carlo routine with Gordon [9] as will be discussed later. Appendix D gives details of the adaption of this method to the routine.

B. ANALYTICAL METHODS

As explained in the previous section on theoretical preliminaries, analytical methods ordinarily lead to solutions which are applicable only under certain conditions. This is so
because in order to reach a usable solution, approximations have to be made. Nevertheless, in the regions of their applicability, analytical solutions are excellent sources of the information needed to verify results generated by other more universally applicable methods. In this work two analytical treatments were employed for just that reason. Gordon's [9] paper on practical approaches to underwater scattering was used to verify the spatial output of the Monte Carlo routine created and Stott's [26] closed form expression for time spread was used to verify the temporal output of the program. The two treatments are summarized in the following paragraphs.

1. Effective Attenuation Coefficient (EAC) Method

Gordon's paper [9] presents the concept of an effective attenuation coefficient which considerably simplifies underwater multiple scattering calculations. Concise expressions for the total flux through an aperture and the beam-spread function are defined in terms of the EAC. The derivation of these expressions is included for easy reference as Appendix E as is a documentation and description of the program adapting them to machine computation. It is interesting to note that Gordon's treatment is actually a truncated version of the exact analytical solution of the multiple scattering problem using the small angle approximation provided by him [28]. The truncation, in effect, considers the scattering phase function as a forward scattering delta function, thus does not consider higher order terms of the exact solution. It is not surprising then that the result is only applicable under certain conditions.
Fortunately these conditions are found often in the underwater optical world. The comparison of this method to the Monte Carlo method is found in the next section. It should be kept in mind that Gordon's treatment, even in the truncated manner, closely agrees with experimental data [29]. The reader is referred to Appendix E for further details on the EAC method.

2. Closed Form Time Spread Expression

Stott's treatment [26] of time spread in a multiple scattering region is short, concise and seems to agree well with existing time spread data. It was therefore chosen as an additional means for verification of time spread results given by the Monte Carlo method. The details in deriving Stott's closed form solution for time spread are located in Appendix G for easy reference. The result is the average additional multipath time required for a photon to traverse a scattering medium with physical thickness $z$, optical thickness $\tau$, albedo $\omega_o$ and RMS scatter angle $\gamma_o$. The time spread is

$$L = \frac{z}{c} \left( \frac{30}{\omega_o \tau \gamma_o^2} \left[ (1 + \frac{9}{4} \omega_o \tau \gamma_o^2)^{1.5} - 1 \right] - 1 \right)$$ (12)

This result is compared to Monte Carlo results in later sections.

C. COMPARISON OF METHODS EMPLOYED

A problem usually encountered in mathematically modeling any physical situation is to ensure at all times that the model
employed is turning out reasonable answers. To verify that this was in fact the case, frequent cross-checks were made between the models. The proof of the theory follows, of course, in agreement between the models and/or agreement with experimental observations. This section compares the results of the Monte Carlo model with the analytical models as well as with the results of Bucher's [1] independently created Monte Carlo model. Once confidence is established, conclusions can be drawn about the regions of effectiveness of each model. Other more routine checks for error are noted in Appendix F.

1. **Monte Carlo vs. Analytical**

   a. **Spatial Characterization**

   The EAC method derived and verified in Appendix E is compared to the Monte Carlo method in this section. First, the method of spatial characterization adapted from Ref. 6 is briefly explained.

   (1) **Description of Relative Flux Contours.** At each angular bin of each reference shell the relative flux is given by

   \[
   \text{Relative Flux} = \frac{\text{# Photons Passing Through Bin}}{\text{Total Number Ejected} \times \text{Area of Bin}}
   \] (13)

   The negative logarithm of this quantity is calculated for each bin. This array of values is interpolated as described in Appendix D, and contours of equal relative photon flux are drawn. Likewise, the negative logarithm of the beam spread
function evaluated by the EAC method is evaluated at a similar array of spatial positions and equal flux contours are drawn. The two sets of contour lines are compared in the following paragraphs.

(2) Comparison. Flux contours calculated using Henyey-Greenstein G parameters of .70 and .95 for each albedo of .80 and .90 were drawn at integer values of the negative logarithm. Figures 6 through 9 show these comparisons. The dotted flux contours are those of the EAC method from zero to 45 degrees and the solid contours are those of the Monte Carlo method. The term S on the graph is the scattering coefficient as well as the albedo since the extinction coefficient is unity in each case. Dimensions are in kilometers but in this case one kilometer is equivalent to one extinction length.

In general, agreement is good between the methods for highly peaked, $G = .95$, phase functions at distances less than 10 extinction lengths and angles varying from one to 20 degrees. This comparison agrees with results given in Gordon [9]. It should be noted that angular agreement is nearly independent of albedo whereas range agreement is better when large absorption is present.

(3) Conclusions. Inside of a few degrees at moderate to large extinction lengths the EAC method becomes undependable in most cases. The Monte Carlo and EAC methods agree well in regions where agreement should be expected. The Monte Carlo method has the advantage over the EAC method when
Figure 6

EAC Model vs. Monte Carlo Model

Position perpendicular to direction of position of EAC model (kilometers)

Position along direction of incidence (kilometers)

- 10.0 - 5.0 0.0 5.0 10.0 15.0 20.0 25.0

E 80 / KM E 30 / KM E 70 / KM E 100 / KM
Figure 9

EAC Model vs. Monte Carlo Model

G = .95
R = .10 / KM
S = .90 / KM
E = 1.00 / KM.

— MONTE CARLO

—— EAC MODEL
considering atmospheric scattering because in this case absorption is small and large optical thicknesses are encountered often. Confidence can be placed in the Monte Carlo routine for spatial characterization of light in a multiple scattering region.

b. Temporal Characterization

Stott's [26] closed form expression for time spread which is summarized in Appendix G is compared to time spread found using the Monte Carlo method in this section. The technique used in tabulating time spread information in the Monte Carlo model was explained earlier in the section titled "The Model".

(1) Comparison. Multipath time spread is defined as the difference between the average transit time incurred from multiple scattering and the normal transit time in the absence of scattering. The expression for time spread derived by Stotts is given in Equation 12. This equation is plotted in Figures 10 through 14 with an albedo of one, an RMS scatter angle of .65 and optical thickness, \( \tau \), ranging from zero to 100 for physical thickness values, \( z \), of .5, 1.0, 2.0, 3.0 and 4.0 kilometers. Monte Carlo data points are marked with X's using various \( \tau \) values with other parameters the same. Also plotted are Bucher's [1] data which is explained later. In general, agreement is within one microsecond for optical thicknesses less than 10 and within five microseconds for optical thicknesses between 10 and 60.
Three Model Time Spread Comparison

Figure 10

Optical Thickness [Extinction Lengths]

Additional Time Required to Pass Through Cloud [Microseconds]
Figure 11

Three Model Time Spread Comparison
Figure 12

Three Model Time Spread Comparison
Figure 13

Three Model Time Spread Comparison

Z = 3.00 KM

Optical Thickness (Extinction Lengths)

Additional Time Required to Pass Through Cloud (Micro-seconds)
Three Model Time Spread Comparison

Figure 14

Z = 4.00 KM

STOTTS

BUCHER

OPTICAL THICKNESS [ EXTINCTION LENGTHS ]

ADDITIONAL TIME REQUIRED TO PASS THROUGH [ MICRO-SECONDS ]
(2) Conclusions. The Monte Carlo and closed form methods agree well in regions where agreement should be expected. Because of this agreement, confidence can be placed in the Monte Carlo routine for temporal characterization. The Monte Carlo routine has the advantage over the closed form expression in that different phase functions can be simulated, thus allowing study of the effects of the details of the phase functions on time spread of light in a scattering medium. Because time spread is expressed quite well by the closed form expression, little dependence on these details is expected. The dependence is discussed in later sections.

2. This Monte Carlo Model vs. Bucher's Monte Carlo Model [1]
   a. Spatial Characterization

The Monte Carlo method is tested for agreement with Bucher's [1] Monte Carlo results. Because both methods are built with nearly the same geometry, nearly identical results are expected. First the graphic representation of data used by Bucher needs explanation.

(1) Description of Scaling Method. Bucher [1] defines for a scattering medium the diffusion distance

$$D_d = \frac{D}{1-\langle \cos \theta \rangle}$$  \hspace{1cm} (14)

where $D$ is the mean free path between collisions and $\langle \cos \theta \rangle$ is the average cosine of the scattering angle as before. Using this formula, the effective scattering thickness $\tau_d$ is given by the relation
\[ \tau_d = \frac{T}{D_d} = \tau(1 - \langle \cos \theta \rangle) \]  

(15)

where \( T \) is the physical thickness of the cloud and \( \tau \) is the usual optical thickness. The spatial results are then presented by graphically displaying the average displacement \( <r> \) in relation to the diffusion thickness, \( D_d \), as a function of effective scattering thickness \( \tau_d \). Also presented is the critical transverse displacement, \( r_c \), in relation to \( D_d \) inside which 50 per cent of the photons exit the cloud at effective scattering thickness, \( \tau_d \). The reader is referred to Bucher [1] for further details.

(2) Comparison. Monte Carlo calculations were made and interpreted in terms of the units described above. Figures 15 and 16 show the results at various effective scattering thicknesses using an albedo of unity and a Henyey-Greenstein function for which \( G = .83 \). The X's represent data collected here and the continuous line is a best fit curve to Bucher's data using the same parameters. As can be seen, agreement is excellent in both cases.

(3) Conclusion. Spatial characterization by this Monte Carlo routine is consistent with a previously developed routine. This creates confidence in the routine. It can now be used to study the effect phase function details have on the spatial character of light.

b. Temporal Characterization

The time spread data of the Monte Carlo routine is compared to Bucher [1] in this section. Two methods of
Figure 15

Figure 16

This Monte Carlo vs. Bucher's Monte Carlo [1] Spatial Spread Comparison
presentation are used. One uses time bins, the other uses Bucher's best fit equation for time spread.

(1) Comparison. Bucher's equation for multipath time spread as defined in an earlier section is,

\[
L(\tau_d) = \frac{67.2 T(\tau_d)}{c} \cdot T(\tau_d)^{0.9} = 3.91 \times 10^{-10} T(\tau_d)^{0.9} (16)
\]

where \(L(\tau_d)\) is the time spread in seconds, \(T\) is the physical thickness of the cloud in meters, \(\tau_d\) is the effective scattering thickness of the cloud and \(c\) is the speed of light in meters per second. This function is plotted alongside Monte Carlo data and Stott's closed form equation in Figures 10 through 14 for an albedo of unity, a Henyey-Greenstein phase function with \(G = 0.83\), and a physical thickness of 0.5, 1.0, 2.0, 3.0 and 4.0 kilometers.

Agreement is well within one microsecond for nearly the entire range which is well within statistical error. In these trials only 50 - 500 photon histories were tabulated which seems to be an indication that collection of time spread information does not require unwieldy amounts of computer time.

Another comparison of time spread information is graphically displayed in Figures 17 and 18. The number of photons arriving in each time bin is normalized to the maximum number in any bin and plotted as a function of time in microseconds. Bucher's data is superimposed by a dotted line on the time bin data presented. Figure 17 plots results for an optical thickness of 15 and Figure 18 for an optical thickness of 30. Other parameters are listed on the graphs. Agreement once again is very good between the methods.
Additional time required to pass through cloud [micro-seconds]

This Monte Carlo vs. Bucher's Monte Carlo [1] Time Spread Comparison
ADDITIONAL TIME REQUIRED TO PASS THROUGH CLOUD [MICRO-SECONDS]

Figure 18

This Monte Carlo vs. Bucher's Monte Carlo [1] Time Spread Comparison
(2) **Conclusion.** Once again, confidence is ensured in temporal characterization of time spread and pulse spread by the Monte Carlo routine. The routine can now be used in investigating time spread dependence on phase function.
V. RESULTS

A. SPATIAL SPREAD DEPENDENCE ON PHASE FUNCTION

In this section five graphs are used to display the dependence of the spatial spread of light on phase function when traversing a scattering medium. All five diagrams show the relative flux contours described earlier in this thesis and in Ref. 6. Each figure lists the parameters and the phase function used in the simulation. Each axis is in units of kilometers. Incidence of photons is at the origin and to the right in all cases. The left edge of the cloud, if it exists, is along the axis perpendicular to direction of incidence and the right edge of the cloud is indicated by a dotted line if it is within the boundaries of the graph. For easy reference, Table II tabulates the parameters and phase function used in Figures 19 through 36.

The phase function of Water Cloud C.2 is used in Figure 19 to show the peakedness of the relative flux contours out to at least 20 extinction lengths when the albedo is only .90. A "toe" appears on the contours at small angles due to the high degree of forward scattering of Water Cloud C.2.

Figures 20 and 21 compare the flux contours due to NOSC Fog and Henyey-Greenstein using G = .83, respectively. The flux contour for NOSC Fog is more forward peaked than that of the Henyey-Greenstein function for distances less than three kilometers or 19 extinction lengths but flux contours are very
## TABLE II

Tabulation of Parameters and Phase Functions used in Figures 19 - 36

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<tr>
<th>Figure</th>
<th>KSCA1 (km$^{-1}$)</th>
<th>KSCA2 (km$^{-1}$)</th>
<th>KABS1 (km$^{-1}$)</th>
<th>KABS2 (km$^{-1}$)</th>
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<th>RPT2</th>
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<td>0.50</td>
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(In Figures 20-23, 35 and 36 the dots represent the data points to which the contours were fit)
Figure 19
Spatial Spread Dependence on Phase Function
Figure 20
Spatial Spread Dependence on Phase Function
similar at distances greater than three kilometers. Outside of 20 extinction lengths the contours and therefore the spatial character of the light they represent become independent of the phase function used. The contours of these two graphs are much less forward peaked than that of Figure 19 because the albedo in the former is unity.

Figures 22 and 23 show similar results when comparing results when Water Cloud C.2 [21] and Henyey-Greenstein were used, respectively. In all figures the cloud was extended to infinity at the right of the origin, explaining the discontinuity in contour slope.

B. TEMPORAL SPREAD DEPENDENCE ON PHASE FUNCTION

In this section nine histograms and one table are used to display the dependence of time spread of light on phase function when traversing a scattering medium. All nine diagrams list the parameters in the same fashion as in the last section. The essential parameters can also be found in tabulated form in Table II. The information is plotted as the number of photons arriving in any bin normalized to the bin of most frequent arrival versus the time spread incurred. Note that the bins are logarithmic and therefore represent much less span in time to the left than to the right.

Figures 24, 25 and 26 compare time spread for NOSC Fog, Water Cloud C.2 and Henyey-Greenstein, respectively in a cloud one kilometer thick of optical thickness four. It can be seen by comparing the graphs that at this optical thickness the
Figure 24

Time Spread Dependence on Phase Function
Figure 25

Time Spread Dependence on Phase Function
Figure 26
Time Spread Dependence on Phase Function
higher peaked phase functions have more photons arriving at earlier times.

Figures 27, 28 and 29 similarly compare time data for a cloud with a physical thickness of one kilometer and an optical thickness of eight. The time spread diagrams begin to look very much alike but still vary slightly with peakedness. At this thickness the average transit time is nearly identical for the three cases but there are still photons of the higher peaked phase functions which traverse the cloud with very little time delay. This indicates some dependence on phase function for this thickness.

Figures 30, 31 and 32 conclude the graphical time spread presentation with a cloud one kilometer thick with an optical thickness of 15. There is no noticeable difference in the graphs that can not be attributed to the statistical deviation expected in a Monte Carlo routine. Table III summarizes the time spread data.

From these results it can be said that the time spread becomes independent of the details of the phase function at optical thicknesses greater than 15.

C. REGION OF TRANSITION FROM FORWARD TO MULTIPLE SCATTER

The region of transition from forward to multiple scatter can be defined as that region for which (1) distances of lesser magnitude show time and spatial spread depending markedly on the phase function of the scattering particles and (2) distances of greater magnitude show that time and spatial spread do not depend on the phase function of the scattering particles.
KSCA1: 8.00 / KM
KSCA2: 0.00 / KM
KABS1: 0.00 / KM
KABS2: 1.00 / KM
RPT1: 1.00
THICK: 1.00 KM
MEAN: 2.92E-06 SEC
SIGMA: 4.16E-06 SEC
NOTE
NOSC FOG PHASE FUNCTION

Figure 27
Time Spread Dependence on Phase Function
Figure 28

Time Spread Dependence on Phase Function

KSCA1: 0.00  /  KM
KSCA2: 0.00  /  KM
KABS1: 0.00  /  KM
KABS2: 1.00  /  KM
RPT1:  1.00

THICK:  1.00  KM
MEAN:  2.90E-06  SEC
SIGMA:  4.54E-06  SEC
NOTE
WATER CLOUD C.2
PHASE FUNCTION
WAVELENGTH:  .53  MICROMETERS

ADDITIONAL TIME REQUIRED TO PASS THROUGH
CLOUD [SECONDS]
Figure 29

Time Spread Dependence on Phase Function
Figure 30

Time Spread Dependence on Phase Function
Figure 31

Time Spread Dependence on Phase Function
Figure 32
Time Spread Dependence on Phase Function
TABLE III

Summary of Time Spreading by One Kilometer Thick Cloud

<table>
<thead>
<tr>
<th>KSCAl (km^{-1})</th>
<th>H.G. G=.83 (\mu-sec)</th>
<th>W.C. C.2 (\mu-sec)</th>
<th>NOSC Fog</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 Mean</td>
<td>1.7</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Sigma</td>
<td>3.1</td>
<td>2.4</td>
<td>2.8</td>
</tr>
<tr>
<td>8 Mean</td>
<td>2.9</td>
<td>2.9</td>
<td>2.9</td>
</tr>
<tr>
<td>Sigma</td>
<td>4.2</td>
<td>4.5</td>
<td>4.1</td>
</tr>
<tr>
<td>15 Mean</td>
<td>4.4</td>
<td>4.6</td>
<td>4.3</td>
</tr>
<tr>
<td>Sigma</td>
<td>4.4</td>
<td>5.3</td>
<td>4.7</td>
</tr>
<tr>
<td>30 Mean</td>
<td>7.5</td>
<td>7.1</td>
<td>7.9</td>
</tr>
<tr>
<td>Sigma</td>
<td>5.3</td>
<td>5.6</td>
<td>5.5</td>
</tr>
</tbody>
</table>
From the previous two sections the region of transition from forward to multiple scatter is between about 10 - 20 optical thicknesses. This agrees favorably with observations made by other authors [40].

D. EFFECT ON SPATIAL CHARACTER OF LIGHT WHEN PASSING THROUGH A CLOUD

In this section four graphs are used to display the effect of finite thickness clouds on the spatial spread of light. All four diagrams use the relative flux contours and the list of parameters defined in earlier sections of this report.

Figure 33 shows the relative flux contours of an infinite very clear atmosphere. Figure 34 shows the effect when a cloud of optical thickness about nine is placed in the light's path. Deformation of the contours is very obvious.

Likewise, Figures 35 and 36 show the effect of a .5 kilometer Water Cloud C.2 when placed in the path of light in a clear atmosphere. The sharp peaks on the flux contours are rounded and as expected, light is significantly redirected when passing through the cloud. Clouds of large optical thickness would eventually cause the Lambertian irradiance well known in the theory of light propagation.
Figure 33

Effect of Finite Clouds on Spatial Character of Light
Effect of Finite Clouds on Spatial Character of Light
Figure 35

Effect of Finite Clouds on Spatial Character of Light
VI. DISCUSSION

In this thesis Monte Carlo methods and simple analytical methods were used to characterize light transmission through a multiple scattering medium. Complicated numerical methods were not used nor were highly mathematical models. This type of approach remains for future investigations. It is anticipated that results drawn from present methods will be confirmed.

Much information on the multiple scattering problem remains to be gathered. The present Monte Carlo routine can be adapted to numerous useful geometries and atmospheres with gradients and time variation of physical parameters. Propagation over and around solid bodies such as mountains and the earth itself can be modeled. Layered media such as cloud, air and water interfaces can also be simulated by the routine.
VII. CONCLUSIONS

A computer routine based on a Monte Carlo model was developed to simulate light propagation through a scattering absorbing medium. The routine can simulate a plane parallel cloud of scatterers with the desired parameters located within a medium with another set of parameters. The phase functions of each medium can be arbitrarily defined using a set of data pairs or can be approximated by closed form expressions. The data generated by the routine has been checked for accuracy against other theories using analytical methods. Each comparison shows adequate agreement between theories where agreement can be expected.

The routine will automatically plot spatial information necessary in characterizing light transfer through the medium by use of contours of equal photon flux. It will also generate histograms depicting time spread information for light passing through finite clouds.

Using the Monte Carlo routine created and inputting phase functions of different peakedness, it has been found that both spatial and temporal spread are independent of the details of the phase function for thicknesses greater than 15 extinction lengths. The region of transition from forward scatter to multiple scatter is between 10 - 20 extinction lengths.

The routine has also been used to study the effect finite thickness clouds have on the spatial character of light.
APPENDIX A

I. GENERATION OF A RANDOM SCATTER/ANGLE WEIGHTED BY AN ARBITRARY PHASE FUNCTION

A. STATEMENT OF THE PROBLEM

Reference 6 outlines the theory and calculations necessary to generate random numbers weighted in accordance with functions that represent characteristics of a scattering medium. At each collision new scatter angles and a distance were generated by closed form expressions which weight a uniformly distributed random number. However, in the more general case of a polydispersion, the computed phase function could not be represented adequately in closed form. This made it impossible to invert the function enabling analytical generation of a weighted scatter distribution. It was necessary to create a method for generation of a random theta weighted by an arbitrary phase function.

B. METHOD USED IN WEIGHTING THETA

Reference 21 and Appendix B explain in great detail a method for calculating the representative phase function of a polydispersion given the particle size distribution and composition. Using this computer adaptation of Mie theory, values of the averaged normalized phase function were generated at selected angles of scatter. Given this data and a random number, R, weighted uniformly over the closed interval [0, 1], the problem is to solve the following equation for $\theta_R$.
where $P(\theta)$ is the averaged normalized phase function and $\theta$ is the variable of integration. Notice that,

$$\theta = 0 \text{ when } R = 0,$$

$$\theta = \pi \text{ when } R = 1,$$

as you would expect on the closed interval. $P(\theta)$ is not a continuously defined function over the interval $[0, \pi]$ in this case so the integrals are represented numerically by the trapezoidal rule. Figure 37 diagrams the basic procedure used. $N$ values of $P(\theta)$ are selected so as to closely approximate the phase function. Of course, more values are selected near the small scatter angles to adequately describe the sharp peak.

The $N$ values are multiplied by the sine of the respective scatter angle ($P(\theta)$ includes $\sin \theta$ implicitly in Figure 37). $N-1$ trapezoids are established using these $N$ values. The area of each trapezoid is divided by the total area of all trapezoids thus establishing a weight for the respective interval.

$$W_i = \frac{(P_i \sin \theta_i + P_{i+1} \sin \theta_{i+1})(\theta_{i+1} - \theta_i)}{\sum_{i=1}^{N-1} (P_i \sin \theta_i + P_{i+1} \sin \theta_{i+1})(\theta_{i+1} - \theta_i)}$$

This of course requires that

$$\sum_{n=1}^{N-1} W_n = 1.$$

92
GENERATION OF A RANDOM THETA
WEIGHTED FOR AN ARBITRARY
PHASE FUNCTION

Points generated by
Computerized Mie Theory
N-1 trapezoids are constructed
from N data pairs

\[ B_i = \frac{P_{i+1} - P_i}{\theta_{i+1} - \theta_i} \]

Area of Trapezoid number i

R = Area of portion
between these lines
under curve

\[ R_i = \text{Area between lines} \]

Figure 37
Diagram of Random Generation of Theta
With the weight of each panel known, the uniform random number \( R \) is used to solve for the first value of \( n \) such that,

\[
\sum_{i=1}^{n} W_i > R \tag{5}
\]
is satisfied. The random \( \theta_R \) required is somewhere between \( \theta_n \) and \( \theta_{n+1} \). Within the panel, \( \theta \) is weighted linearly in a fashion as explained in Ref. 6. For a given panel, \( \theta_i, \theta_{i+1}, p_i \) and \( p_{i+1} \) are known. From these the slope and intercept are established.

\[
B_i = \frac{p_{i+1} - p_i}{\theta_{i+1} - \theta_i} \quad A_i = p_i - B_i \theta_i \tag{6}
\]
The panel is normalized as follows:

\[
\text{NORM} \int_{\theta_i}^{\theta_{i+1}} (A_i + B_i \theta) d\theta = 1 \tag{7}
\]
so that,

\[
\text{NORM} = \frac{1}{A_i (\theta_{i+1} - \theta_i) + B_i \left( \frac{1}{2} \theta_{i+1}^2 - \theta_i^2 \right)} \tag{8}
\]
All that remains is to solve the following for \( \theta_R \),

\[
R_1 = \frac{(R - \sum_{i=1}^{n} W_i)}{(\sum_{i=1}^{n+1} W_i - \sum_{i=1}^{n} W_i)} = \text{NORM} \int_{\theta_i}^{\theta_R} (A_i + B_i \theta) d\theta \tag{9}
\]
After a few algebraic steps, the solution is

\[ \theta_R = \frac{-A_i \pm \left( \frac{A_i}{B_i} \right)^2 + \left( \frac{2C_i}{B_i} \right)^{1/2}}{B_i} \begin{cases} + B_i > 0 \\ - B_i < 0 \end{cases} \]

where,

\[ C_i = R_1 \left( A_i \theta^{i+1} + \frac{B_i \theta_i^2}{2} \right) + [1-R_1] \left( \frac{B_i \theta_i^2}{2} + A_i \theta_i \right) \]

This \( \theta_R \) has the desired properties. The FORTRAN coding of this method is found in the Subroutine RANTH.
UNCLASSIFIED

COMPUTER SIMULATION OF LIGHT PROPAGATION THROUGH A SCATTERING MEDIA

M A MILLBACH

JUN 78
APPENDIX B

I. DESCRIPTION AND DOCUMENTATION OF PROGRAM
TO ADAPT MIE THEORY TO MACHINE COMPUTATION

A. INTRODUCTION

Mie theory has been adapted to machine computation on many occasions. Deirmendjian [21] provides an excellent guideline for using Mie theory on spherical polydispersions. Using Deirmendjian's guideline, a computer routine was created to generate the volume scattering and extinction cross sections and the corresponding elements of the normalized scattering matrix for a polydispersion where the number of particles per unit volume, per unit radius is given by,

\[ n(r) = a r^a e^{-br^\gamma} \quad 0 < r < \infty \]  

(1)

where \( r \) is the particle radius. The four constants \( a, a, b \) and \( \gamma \) are positive and real and \( a \) is an integer. They are not independent of each other, and are related to quantities in the particle frequency distribution. The radius which is most frequently found in the particle distribution is \( r_c \) and \( N \) is the total number of particles per unit volume. Both \( N \) and \( r_c \) can be found by experimental measurement. In terms of \( N \) and \( r_c \) the constants of the distribution are found using:

\[ N = a \int_0^\infty r^a e^{-br^\gamma} dr = \frac{a}{(a+1) \gamma b} \Gamma(a+1) \]  

(2)

\[ \gamma \]

96
and by choice of $\alpha$ and $\gamma$ to best fit the experimental distribution function. $\Gamma$ is the usual gamma function.

**B. EQUATIONS TO BE CALCULATED**

The equations used to generate scattering data for a polydispersive system are extensions of those used in a monodispersion. The distribution has been created in a manner requiring that

$$N = \int_{r_1}^{r_2} n(r)dr$$  \hspace{1cm} (4)

where $n(r)$ is a continuous and integrable function within the range and represents the partial concentration per unit volume per unit increment of radius $r$. The values of interest are volume scattering cross sections and the corresponding elements of the normalized scattering matrix [18, 21]. These values can be computed using

$$\beta_{sca}[\lambda, n(x)] = \pi k^{-3} \int_0^\infty x^2 n(x)Q_{sca}(x)dx$$  \hspace{1cm} (5)

and

$$\beta_{ext}[\lambda, n(x)] = \pi k^{-3} \int_0^\infty x^2 n(x)Q_{ext}(x)dx$$  \hspace{1cm} (6)

$$P_j(\theta) = \frac{4\pi}{k^2 \beta_{sca}} \int_0^\infty n(x) i_j(\theta)dx$$  \hspace{1cm} j=1,2  \hspace{1cm} (7)
where

\[ Q_{\text{sca}}(x) = \text{scattering efficiency factor} \]
\[ Q_{\text{ext}}(x) = \text{extinction efficiency factor} \]
\[ i_j(x) = \text{dimensionless intensity parameters} \]

Each of the above terms will be examined in detail in following sections.

C. CALCULATION OF SCATTERING EFFICIENCY FACTORS AND DIMENSIONLESS INTENSITY PARAMETERS

To perform numerical integration, the integrand must be evaluated at many different values within the range of integration. In this case the scattering and extinction efficiency factor must be evaluated for numerous Mie size parameters \( x \).

Scattering efficiency is a name given to the ratio of total scattering cross section to geometrical cross section. A similar definition is used to define extinction efficiency. For any particle size and composition, Mie theory gives the total scattering cross section as [18, 21],

\[ \delta_{\text{sca}}(m,x) = \frac{1}{2} \int_\Omega (A_1 A_1^* + A_2 A_2^*) d\omega \]  \hspace{1cm} (8)

where

\[ kA_1 = S_1(m,x,\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (a_n \tau_n + b_n \pi_n) \]  \hspace{1cm} (9)

\[ kA_2 = S_2(m,x,\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (b_n \pi_n + a_n \tau_n) \]  \hspace{1cm} (10)
using the complex index of refraction of the particle \( m \), the Mie coefficients \( a_n, b_n \) and angular coefficients \( \pi_n, \tau_n \), each of which is described in detail in later sub-sections. Using these equations, the scattering efficiency factors are [18, 21],

\[
q_{\text{ sca}}(m, x) = 2 \sum_{n=1}^{\infty} (2n+1)\left( |a_n|^2 + |b_n|^2 \right), \tag{11}
\]

\[
q_{\text{ ext}}(m, x) = 2 \sum_{n=1}^{\infty} (2n+1)\Re\{a_n + b_n\}. \tag{12}
\]

The dimensionless intensity parameters are given by the expressions,

\[
i_1(x, m, \theta) = k^2 A_1^* A_1 = S_1^* S_1^* \tag{13}
\]

\[
i_2(x, m, \theta) = k^2 A_2^* A_2 = S_2^* S_2^*
\]

where \( A_1, A_2, S_1 \) and \( S_2 \) are the same as above. The procedure used in calculating \( a_n, b_n, \tau_n \) and \( \pi_n \) is described next.

1. Computation of \( \tau_n \) and \( \pi_n \)

The two angular functions \( \tau_n \) and \( \pi_n \) are required before the dimensionless intensity parameters can be calculated. \( \tau_n \) and \( \pi_n \) are functions of \( u = \cos\theta \) only and as the subscript implies, there are actually many different angular functions. Both functions are defined in terms of Legendre polynomials [18] and their derivatives. The following recursion relations are used in calculating \( \tau_n \) and \( \pi_n \),
\[ \pi_n(\theta) = \cos^2 \frac{2n-1}{n-1} \pi_{n-1}(\theta) - \frac{1}{n-1} \pi_{n-2}(\theta) \]  
(14)

\[ \tau_n(\theta) = \left[ \cos \theta [\pi_n(\theta) - \pi_{n-2}(\theta)] \right] - [(2n-1)\sin^2 \theta \pi_{n-1}(\theta)] 
+ \tau_{n-2}(\theta) \]  
(15)

with

\[ \pi_0(\theta) = 0 \quad \tau_0(\theta) = 0 \]
\[ \pi_1(\theta) = 1 \quad \tau_1(\theta) = \cos \theta \]  
(16)
\[ \pi_2(\theta) = 3 \cos \theta \quad \tau_2(\theta) = 3 \cos 2\theta \]

2. Computation of \( a_n \) and \( b_n \)

The Mie coefficients \( a_n \) and \( b_n \) have a complex dependence on the index of refraction of the particle, and the surrounding medium and also depend on the size parameter, \( x \). There exists many different expressions for the Mie coefficients in terms of known mathematical functions. The form used in this program is described in terms of spherical Bessel functions each of which is described in later sections. The relations for \( a_n \) and \( b_n \) are

\[ a_n = \frac{u \left[ y_{n-1}(y) - nj_n(y) \right] x_n(x) - \varepsilon y_j_n(y) x_{n-1}(x) - nj_n(x)}{u \left[ y_{n-1}(y) - nj_n(y) \right] x_n(x) - \varepsilon y_j_n(y) x_{n-1}(x) - nj_n(x)} \]  
(17)

\[ b_n = \frac{\varepsilon \left[ y_{n-1}(y) - nj_n(y) \right] x_n(x) - \mu y_j_n(y) x_{n-1}(x) - nj_n(x)}{\varepsilon \left[ y_{n-1}(y) - nj_n(y) \right] x_n(x) - \mu y_j_n(y) x_{n-1}(x) - nj_n(x)} \]  
(18)
where $\mu$ is the absolute value of the complex index of refraction outside the sphere, $\varepsilon$ is the absolute value of the complex index of refraction inside the sphere, $x = 2\pi r/\lambda$ and $y = mx$.

The functions $j_n$, $y_n$, $h_n^2 = j_n - iy_n$ are the spherical Bessel and Neuman functions. A procedure for calculating their values is outlined in the following sections.

3. Computation of $j_n$, $y_n$, $h_n^2$

The argument of these Bessel related functions is complex in some cases, so much care is required in their calculation. Each function is calculated using recursion relations and the lowest order functions as follows. For $j_n$,

\[ j_{n+1}(z) = \frac{(2n+1)j_n(z)}{z} - j_{n-1}(z) \quad (19) \]

\[ j_0(z) = \frac{\sin z}{z} \]

\[ j_1(z) = \frac{\sin z}{z} - \frac{\cos z}{z} \quad (20) \]

\[ j_2(z) = (\frac{3}{z^2} - \frac{1}{z})\sin z - \frac{3}{z^2} \cos z \]

likewise for $y_n$,

\[ y_{n+1}(z) = \frac{(2n+1)y_n(z)}{z} - y_{n-1}(z) \quad (21) \]

\[ y_0(z) = -\frac{\cos z}{z} \]

\[ y_1(z) = -\frac{\cos z}{z} - \frac{\sin z}{z} \quad (22) \]

\[ y_2(z) = (\frac{-3}{z^3} + \frac{1}{z})\cos z - \frac{3}{z^2} \sin z \]
and these two yeild $h_n^2$,

$$h_n^2(z) = j_n(z) - iy_n(z)$$  \hspace{1cm} (23)

Because the argument of the sine and cosine in evaluating the spherical Bessel function is complex, the following relations are needed,

$$z = a + ib$$

$$\cos z = \cos(a)cosh(b) - i\sin(a)\sinh(b)$$ \hspace{1cm} (24)

$$\sin z = \sin(a)cosh(b) + i\cos(a)\sinh(b)$$

where $\cosh$ and $\sinh$ are the usual hyperbolic trigonometric functions.

D. DESCRIPTION AND DOCUMENTATION OF PROGRAM

The program adapting Mie theory to machine computation is composed of the MAIN routine and numerous subroutines described in the following sub-sections.

1. **MAIN Routine**

   The MAIN routine handles the input and output functions necessary in program operation. The input parameters include indices of refraction inside and outside of the spheres, constants of the particle distribution and the most frequently occurring value of Mie size parameter in the distribution, $x_c$. Through $x_c$, the wavelength of the incident light is entered because $r_c$ is known for any desired particle distribution.
Other necessary inputs are the number of scatter angles desired in the output scattering matrix elements $P_j(\theta)$, the smallest value of $x$, the increment in $x$ to be used for numerical integration and the odd number of $x$ values at which the integrand is to be evaluated. The MAIN program accepts the input values and uses the distribution parameters to assign each designated $x$ value a weight. Because the particle distribution is normalized, the sum of all weights assigned is unity.

Simpson's 1/3 rule is used to evaluate integrals 5, 6 and 7. At each value of $x$ the scattering and extinction efficiency factors of equations 11 and 12 are calculated through use of a subroutine MIEM. Similarly, at each desired scattering angle the dimensionless intensity parameters of 13 are calculated through MIEM. Printout can be called at each value of $x$ if so desired. The MAIN program cumulatively sums the areas of an even number of panels to get the desired results which are then printed.

2. MIEM Subroutine

MIEM is a subroutine composed to calculate the scattering and extinction efficiency factors in the integrand of equations 5 and 6 and the dimensionless intensity parameters of equation 13. Required inputs of MIEM, transferred to it by MAIN, are the index of refraction of the particle (henceforth the index of refraction of the outside medium will be unity), the size parameter $x$, and the number of scattering angles at which calculation of the intensity parameters is desired. MIEM
uses equations 11 and 12 to calculate the efficiency factors and equation 13 to calculate the intensity parameters. Because equations 9, 10, 11 and 12 require summation of series, each element of the series must be evaluated in turn. The terms involved in finding the efficiency factors are \( a_n \) and \( b_n \) which are calculated for each \( n = 1, 2, 3, 4, \ldots \) until the next term of the series is adequately small or the total number of terms exceeds 120. After each calculation of \( a_n \) and \( b_n \) its contribution to \( Q_{\text{sca}}(x) \) and \( Q_{\text{ext}}(x) \) is added to the previous total. MIEM uses function subroutines JN, FN and HN to calculate each of the \( a_n \)'s and \( b_n \)'s. Arrays JX, FX and HX are used to store values of the corresponding spherical Bessel functions during each recursion step.

After all the parameters \( a_n \) and \( b_n \) are calculated, MIEM turns them over to ANGLE to complete the remaining dimensionless intensity parameter calculations.

3. **ANGLE Subroutine**

ANGLE is a subroutine called by MIEM to calculate the dimensionless intensity parameters at each desired angle of scatter. ANGLE requires as inputs the Mie scattering coefficients generated in MIEM and the number of scatter angles desired. When called, ANGLE uses equations 9 and 10 to find \( S_1 \) and \( S_2 \) at each scatter angle. To do so ANGLE evaluates \( \tau_n \) and \( \pi_n \) using recursion relations 14 and 15 with initial order functions, 16. At each \( n \), the corresponding Mie coefficients \( a_n \) and \( b_n \) are used with \( \tau_n \) and \( \pi_n \) and their contributions to \( S_1 \) and \( S_2 \) are added.
The process is repeated for each scatter angle and the resulting arrays are used to evaluate equation 13 at each scatter angle. The resulting dimensionless intensity parameters are then returned to MIEM and in turn to MAIN for integration using equation 7.

4. \( JN, FN, HN, CCOS, CSIN \) Complex Functions

These functions are called by MIEM in evaluation of the Mie coefficients \( a_n \) and \( b_n \). \( JN, FN \) and \( HN \) contain logic to perform the recursion operation of equations 19 through 23. \( CCOS \) and \( CSIN \) are complex trigonometric functions drawn upon as needed by \( JN, FN \) and \( HN \).

E. CPU TIME CONSIDERATIONS

CPU time depends largely on the wavelength to particle size ratio. This, of course, varies with the particle size distribution used. Thus, if the distribution includes many particles of large size compared to the wavelength the CPU time is great and vice versa. Typical CPU time requirements were on the order of 15 to 30 minutes for wavelengths of .53 to .28 microns, with about 300 \( x \) values in a distribution of Water Cloud C.2 of Deirmendjian [21]. The dependence on particle size to wavelength ratio is due to the fact that many terms are required for convergence of series for large values of \( x \). As expected, CPU time goes up quite linearly with the number of \( x \) values used as integration points. There is little dependence on the number of scatter angles required.
APPENDIX C

I. SIMULATION OF A CLOUD IN MONTE CARLO ROUTINE LITE

A. INTRODUCTION

Reference 6 describes Monte Carlo simulation of light propagation through an infinite, homogeneous atmosphere. Many problems can be sufficiently investigated using this model, but one of the advantages of a Monte Carlo model is its relative ease in adaptation to irregular geometries and inhomogeneous atmospheres. This Appendix gives one method for simulation of a cloud with plane parallel homogeneous medium. All macroscopic properties are the same everywhere inside of the cloud and another set of macroscopic parameters are the same everywhere outside of the cloud. This model circumvents the entire problem of describing and locating boundaries and inhomogeneities in real clouds. Figure 5 depicts the general structure of the cloud model giving the names of various parameters of the computer simulation. A point source of light is incident normal to the left edge of the cloud and its path is randomly generated until it exits the outermost sphere of the reference volume. Reference 6 describes the random path generation and accountability also used in this model so the terminology of that reference is used here for continuity whenever possible.
B. MODELING CLOUD BOUNDARIES AND PROPERTIES

The sets of parameters needed in defining two different scattering media are implemented by use of storage arrays. A binary logic code is used that switches whenever a photon crosses a boundary. Each array of characteristic parameters has two columns, one for inside the cloud and one for outside the cloud. The logic switch determines at each collision from which column the parameter is to be drawn.

Upon crossing a boundary, the photon is stopped and a new distance is randomly generated in accordance with the parameters of the newly entered medium. The method used for determination of whether or not a boundary was crossed is described in the following section.

C. DETERMINATION OF BOUNDARY CROSSING

The following equation is used to determine at each collision the angle between the original direction of incidence and the present position vector, \( \vec{R} \),

\[
\theta = \cos^{-1} \left( \frac{\vec{R} \cdot (\vec{R} - \vec{R}_K)}{r} \right)
\]

where \( r \) is the distance between the collision point and the point of incidence on the cloud. \( \vec{R}_K \) is an orientation vector from the point of collision to the \((0, 0, 1)\) point of the fixed coordinate system. The \( r \) and \( \theta \) values are computed at each collision which allows, due to cylindrical symmetry, calculation of the projected distance along the direction of incidence.
This distance is

\[ \text{Projected distance} = r \cos \theta = \text{DIST}_{\text{new}} \]

which is compared to the thickness of the cloud. The medium in which the photon exists is determined by,

\[
\begin{align*}
\text{DIST}_{\text{new}} < 0 & \quad \text{Behind} \\
0 < \text{DIST}_{\text{new}} < \text{THICKNESS} & \quad \text{Inside} \\
\text{DIST}_{\text{new}} > \text{THICKNESS} & \quad \text{Beyond}
\end{align*}
\]

The location of the collision relative to the cloud is compared to the location of the previous collision and a boundary crossing is found if it has occurred between collisions. Logic was created to then stop the photon at the boundary and project it along the same path using new scattering parameters. There are nine different combinations of old and new positions, three possibilities for the old position and three possibilities for the new position. Each specific situation is investigated at each collision and upon boundary crossing the correct stopping formula is applied.
APPENDIX D

I. AUTOMATION OF RELATIVE FLUX CONTOUR PLOTTING IN DRLITE ROUTINE

A. INTRODUCTION

One very important output of the Monte Carlo routine is the relative flux per unit area at numerous grid points within the reference spheres with respect to the original energy projected by the point source. Knowing the relative flux at numerous grid points allows generation of equal photon flux contours. Calculation of the contours was a tedious job requiring many hours on a programmable pocket calculator and at the drawing board. Accuracy of the contours suffered (not to mention the one doing the work) therefore a method was created to produce many of the contour plots found within this report.

B. AUTOMATION

A subroutine, CONISD, was found in the computer program library that was designed to produce a contour map of irregularly spread data points. Each data point is a triad of x, y and z values where

\[ z = f(x,y) \]

This procedure is nicely suited for plotting the flux contours required since the flux per unit area has been calculated at
each increment in range and scattering angle. In transforming equally spaced range and angle intervals from polar to cartesian coordinates, irregularly spaced intervals are formed. By inspection the flux varies quite linearly in the range direction.

The region to be contoured is subdivided into triangular areas, using acute triangles as much as possible. The method of triangularization is based on that of Ref. 30. The list of triangles is then analyzed for adjacencies. For a given contour level, each outer boundary of the area to be contoured is checked for a possible entry value. Upon finding a value, the contour line is traced from triangle to triangle until it exits the area. As the line is traced from segment to segment, the four nearest values are linearly interpolated and the resulting value stored. After all lines have been found and intersections stored, another subroutine is called to fit smooth curves to the stored values. The resulting curves are the desired contours of equal photon flux.

C. ADDITIONS TO DRLITE

A section was created within DRLITE to drive the subroutine CONISD after computation of relative flux at desired grid points. Inputs required by CONISD are the number of grid points, each corresponding triad of values, the value of each contour desired, scaling factors for plotting the output and designation of which data points are to be boundary points.

Because data used to generate the contours was generated by a Monte Carlo routine, a measure of statistical significance had
to be included before actual plotting. The measure used was based purely on the number of photon crossings at each particular angular bin at every range. If the number of crossings was less than three, the corresponding data triad was removed from the list to be used in generation of contours.

D. DISCUSSION

The automation method explained above, although useful in reducing manual labor, is quite consumptive of CPU time. Computer time goes up almost exponentially with the number of grip points used in the routine. This can be attributed to the number of searches necessary in passing from segment to segment. Although the general idea behind automation of the contour plotting is good, the actual implementation of such methods must be done with caution.

As an aside, the subroutine CONISD also has the capability to have internal cut out boundaries placed into its logic. Future application of this feature may be to draw equal photon flux contours for light propagation around defined objects.
APPENDIX E

I. DERIVATION, DOCUMENTATION AND VERIFICATION OF EFFECTIVE ATTENUATION COEFFICIENT METHOD

A. INTRODUCTION

Gordon [9] presents the concept of an effective attenuation coefficient (EAC) which considerably simplifies multiple scattering calculations. Concise expressions for the total flux through an aperture and the beam-spread function are derived in terms of the EAC. A program was written to evaluate these expressions so that comparison with other light scattering models could be accomplished. This Appendix derives the expressions given by Gordon, documents the routine written to evaluate the expressions and shows verification of program accuracy.

B. DERIVATION

Gordon first suspected that simple formulas might describe multiple scattering after examining Duntley's [29] measurements of the fraction of power emitted from a collimated source which reaches a circular aperture subtending a cone of half-angle $\theta$ when viewed from the source and located a distance $r$ from it. Gordon noticed that on semi-log paper the data formed straight lines to about 15 extinction lengths when plotted as a function of range. The expression for flux reaching the aperture where $F_0$ is the source strength can be expressed as [9]:
\[ F(\theta, R) = F_o e^{-\alpha_e(\theta)R}, \]  

(1)

where \( \alpha_e(\theta) \) is the EAC as a function of \( \theta \).

Consider the geometry of Figure 38. A unidirectional point source is centrally located on the plane \( x = 0 \). On the plane \( x = R \) is located a circular aperture which subtends a cone of half-angle \( \theta \) with the point source. The total flux through the aperture is given by [9]:

\[
F = F_o \exp \left[ -\left( 1 - \frac{s}{\sigma} \int_0^{2\pi} \int_0^{\theta'} P(\theta) d\omega \right) \alpha R \right]
\]  

(2)

where,

\[
d\omega = \sin \theta \, d\theta \, d\phi = \text{differential solid angle}
\]
\[
s = \text{scattering coefficient in km}^{-1}
\]
\[
\sigma = \text{extinction coefficient in km}^{-1}
\]
\[
t = \frac{x}{R} = \text{normalized range}
\]
\[
\theta' = \tan^{-1} \left( \frac{\tan \theta}{1 - x/R} \right)
\]

and \( P(\theta) \) is the normalized phase function, thus it satisfies the relation,

\[
\int_0^{2\pi} \int_0^{\pi} P(\theta) \, d\omega = 1
\]  

(3)
Figure 38
Geometry of EAC Method
A simple physical description of equation 2 is that it accounts for flux absorbed and scattered from the beam in the first term of the exponent and also accounts for flux that is scattered back through the aperture in the integral term of the exponent.

Comparing equation 1 with equation 2 the effective attenuation coefficient is found to be

\[ \alpha_e(\theta) = \alpha \left(1 - \frac{G}{\sigma} \int_0^{2\pi} \int_0^\infty P(\theta) d\omega \right) dt \] (4)

In adapting this equation to machine computation, a normalized phase function is necessary. A reasonable choice is the Henyey-Greenstein phase function [7],

\[ P(\theta) = \frac{(1-G^2)}{4\pi(1+G^2-2G\cos\theta)^{1.5}} \quad -1 < G < 1 \] (5)

Because the Monte Carlo model makes use of this phase function also, comparison of the two models is possible.

Now that the flux through an aperture is known, it can be used to derive the irradiance caused by a unit strength unidirectional point source at each point of a selected target plane. This irradiance distribution is called the beam spread function, \( H_B(\theta, R) \). Let the initial point source have unit strength. By considering the flux through a small annulus of width \( R \, d\theta \) located at \((R, \theta)\), \( H_B(\theta, R) \) can be written as [9],
\[ H_B(\theta,R) = \frac{1}{2\pi R \sin \theta} \frac{dF}{d\theta} \]  

where \(\frac{dF}{d\theta}\) is the derivative of equation 2 with \(F_0 = 1\). To evaluate this derivative, Leibnitz rule is used [33]. It states that if

\[ F(t) = \int_{a(t)}^{b(t)} \phi(x,t) \, dt \]  

where \(a(t)\) and \(b(t)\) are differentiable functions of \(t\) and where \(\phi(x,t)\) and \(\frac{\partial \phi}{\partial t}\) are continuous in \(x\) and \(t\), then

\[ \frac{dF}{dt} = \int_{a(t)}^{b(t)} \phi \left( x, t \right) \frac{\partial}{\partial t} \left( \frac{\partial}{\partial t} \left( \phi \left( x, t \right) \right) \right) \, dx - \phi \left[ b(t), t \right] \frac{db(t)}{dt} - \phi \left[ a(t), t \right] \frac{da(t)}{dt} \]  

Take the derivative of equation 2,

\[ \frac{dF}{d\theta} = F \frac{d}{d\theta} \left[ \left( \frac{1}{\theta} \right) \int_{0}^{\theta} \int_{0}^{\theta} P(\theta) \, d\theta \right] \right) \frac{\partial}{\partial t} \left( \frac{\partial}{\partial t} \left( P(\theta) \sin \theta \right) \right) \frac{\partial}{\partial R} \left( P(\theta) \sin \theta \right) \right) \frac{\partial}{\partial R} \]  

and apply Leibnitz rule where

\[ \phi(t,\theta) = \int_{0}^{\theta} P(\theta) \sin \theta \, d\theta \quad a(t) = 0 \quad b(t) = 1 \]  

it is now apparent that the derivative becomes,
\[
\frac{dF}{d\theta} = 2\pi Fr \left( \int_0^\theta \left( \frac{\partial}{\partial \theta} \int_0^{\theta'} P(\theta') \sin \theta' \, d\theta' \right) \, d\theta \right) \, dt
\]  
(12)

The derivative that remains is of an integral whose range is over the same independent variable, \( \theta \), therefore the result is the product of the integrand evaluated at the upper limit and the derivative of the upper limit, i.e.,

\[
\frac{\partial}{\partial \theta} \int_0^{\theta'} P(\theta') \sin \theta' \, d\theta' = P(\theta') \sin(\theta') \frac{\partial}{\partial \theta} (\theta') \]  
(13)

but

\[
\frac{\partial}{\partial \theta} \left[ \tan^{-1} \left( \frac{\tan \theta}{1-t} \right) \right] = \frac{1}{1 + \left( \frac{\tan \theta}{1-t} \right)^2} \frac{1}{(1-t)\cos^2 \theta} \]  
(14)

so it is clear that

\[
\frac{dF}{d\theta} = 2\pi Fr \int_0^1 P(\theta') \sin(\theta') \frac{1}{1 + \left( \frac{\tan \theta}{1-t} \right)^2} \frac{1}{(1-t)\cos^2 \theta} \, dt \]  
(15)

Using the small angle approximation, \( \tan \theta = \theta = \sin \theta \) and letting

\[
u = \tan^{-1} \left( \frac{\theta}{1-t} \right) \]  
(16)

\[
d\nu = \frac{dt}{\left[ 1 + \left( \frac{\theta}{1-t} \right)^2 \right] (1-t)} \]  
(17)

equation 15 takes the form
\[
\frac{dF}{d\theta} = 2\pi F_s R \int_0^{\pi/2} P(u) \cos(u) \, du \tag{18}
\]

Using equation 18 in equation 6 the beam spread function is

\[
H_B(\theta, R) = \frac{s_F}{R^9} \int_0^{\pi/2} P(u) \cos u \, du \tag{19}
\]

A useful calculation in comparing this beam spread function with that found using the Monte Carlo method is taking the relative logarithm of the beam spread function.

C. ADAPTATION OF EFFECTIVE ATTENUATION COEFFICIENT METHOD TO MACHINE COMPUTATION

The Effective Attenuation Coefficient method lends itself nicely to machine computation. The inputs required by the program are the scattering coefficient, the extinction coefficient and the Henyey-Greenstein G parameter. Also entered are the initial range and theta, the increment in range and theta and the number of grid points desired in each dimension.

The routine written is called EAC. EAC calls on a library subroutine DQG32 which integrates functions FCC and FCT inside of which the integrands of equation 4 and 19 are defined, respectively. DQG32 uses 32 point Gaussian quadrature which integrates polynomials up to degree 63 exactly.

The output of EAC can be received by numerical matrix tabulation or by graphical contour plotting as described in Appendix D. Examples of these plots are found in Figures 6 through 9.
D. VERIFICATION OF EAC

The results from EAC, the NPS routine, were compared to figures obtained by Gordon [9] using the same method. There is a small dependence on phase function for small optical thicknesses for which this method is applicable. Thus in comparing results with Gordon, a slight deviation is noticed due to the fact that the phase function used may not have been exactly the same. A Henyey-Greenstein G parameter of .94 was used to generate the figures presented here. Figure 39 compares the results of EAC with Gordon for irradiance as a function of $\theta$ at a range of 2.12 extinction lengths. Agreement is seen to be quite good. Figure 40 compares the results of EAC with Gordon for irradiance as a function of range with a scatter angle of 10 degrees. Agreement is good here also. These are a few of many checks on routine accuracy which lead to much confidence in the routine.
Figure 39
Verification of EAC Method
Figure 40

Verification of EAC Method
APPENDIX F

I. CHECKS ON POSSIBLE ERRORS

A. GENERAL

Each computational procedure employed in this simulation was verified when possible and every effort was made to ensure the entire simulation functioned properly. Results of various theories were compared using identical parameters to build confidence at intermediate steps along the path toward final results. Procedures used to document the correct behavior of the computer routines used in this simulation are described in this Appendix.

B. SPECIFIC CHECKS

One of the secondary objectives of this report was to compare results of different theories in predicting light transfer through a scattering absorbing medium. The ability to compare results of separate theories stands by itself as a verification of the accuracy of each theory.

Reference 6 states numerous initial methods used to verify the mechanics of the computer simulation in its first stages. In converting the routine from one that simulates an infinite homogeneous medium to one that contains a cloud, additional checks were required. Many photons were traced manually by hand calculations to ensure that all nine possible combinations of transfer across boundaries were correctly handled by the
routine. Errors were found and corrected on many an occasion.
In the situation where the transmission parameters and the
phase function of the cloud were the same as those of the
surrounding medium, one would expect the results to be identi-
cal to those found when simulating a homogeneous medium. This
in fact was the case. Results were identical to well within
statistical scatter.

Changeover from measurements in terms of extinction lengths
to measurements in terms of real dimensions also required brief
checking. One would expect the results to be identical when
comparing a medium with an extinction coefficient of one in-
verse kilometer to a medium described in terms of optical
thickenss. This is so because one unit of distance is the
same in both cases, one kilometer. This is the case. In com-
paring trials where the first has a distance between accounta-
bility shells of twice the second trial but an extinction
coefficient of half the second trial with the albedo the same,
the number of photons crossing each bin should be the same.
Once again this is the case.

In some cases the results were checked by intuition only
as no previous work with similar presentation was found.
Specifically, the equal photon flux contours when passing
through clouds were checked only by what one would expect them
to look like.

Many test runs were made using the model generating a
scatter angle weighted by an arbitrary phase function. The
angles generated are in fact weighted by areas of panels and linearly within, panels.

Confidence in the statistical nature of a Monte Carlo routine is related to the number of photon histories used in each trial. The smallest number of photons possible were used to create outputs of the desired accuracy in order to minimize computer time consumption. In most cases of spatial spread thousands of photon histories were generated. In time spread cases as few as 100 were required at times.

Other checks made are found in Ref. 6 as well as within the body of this work.
APPENDIX G

I. DERIVATION OF CLOSED FORM EXPRESSION FOR TIME SPREAD

Stotts [26] derives a closed form expression for time spread as follows. From vector analysis, the average pathlength of a photon per unit time is

$$\frac{dR}{dt} = \left[ (\frac{dz}{dt})^2 + (\frac{dv}{dt})^2 + (\frac{dz}{dt})^2 \right]^{\frac{1}{2}}$$

which reduces easily, where \( \overrightarrow{r} = x\hat{i} + y\hat{j} \), to

$$\frac{dR}{dt} = \left[ 1 + \left| \frac{d\overrightarrow{r}}{dz} \right|^2 \right]^{\frac{1}{2}} dz$$

From basic scattering theory [31] the projected angle of scattering is approximated by

$$[\omega_o \gamma_o^2]^{\frac{1}{2}}$$

where \( \omega_o \) is the albedo of single scattering, \( \tau \) is the optical thickness of the scattering region and \( \gamma_o \) is the RMS scatter angle. Thus the projected mean-squared transverse displacement is

$$|d\overrightarrow{r}| = z(\omega_o \gamma_o^2)^{\frac{1}{2}}$$
thus

\[ \frac{\overline{d\tau}}{dz} = \left( \frac{9}{4} \omega_0 \tau \gamma_0^2 \right)^{1/2} \]

and

\[ dR = \left[ 1 + \frac{9}{4} \omega_0 \tau \gamma_0^2 \right]^{1/2} dz \]

Integrating results in

\[ R = \frac{8z}{27\omega_0 \tau \gamma_0^2} \left[ \left( 1 + \frac{9}{4} \omega_0 \tau \gamma_0^2 \right)^{1.5} - 1 \right] \]

The average multipath time spread is defined as the difference between the average transient time incurred from multiple scattering and the normal transient time in the absence of scattering. Thus, Stotts has for time spread,

\[ L = \frac{2z}{\omega_0 \tau \gamma_0^2} \left[ \left( 1 + \frac{9}{4} \omega_0 \tau \gamma_0^2 \right)^{1.5} - 1 \right] - 1 \]
C**** MAIN ROUTINE OF MIE SCATTERING PROGRAM

C THIS ROUTINE CALCULATES THE MIE SCATTERING FUNCTIONS AVERAGED
C OVER A PARTICLE DISTRIBUTION FUNCTION OF THE FORM
C
C \( \alpha^* \exp(-B^* X) \) [SEE DEIRMENDUJAN, ELSEVIER, 1969]
C INPUT: NX,NTH,ICODE,RXO,DELX,M,ALPHA,GAM,XC [13,12,11,F4.2,E65.3]
C NX = NUMBER OF VALUES OF PARTICLE SIZES TO BE CALCULATED OVER
C NTH = NUMBER OF VALUES OF THETA AT WHICH TO CALCULATE SCATTERING
C INTENSITY EVENLY INCREMENTED FROM 0 TO 180 DEGREES
C ICODE = 0 FOR MINIMUM OUTPUT
C GT. 0 TO GET SCATTERING AND EXTINCTION PRINTED AT EACH X
C GT. 5 TO GET SCATTERING FUNCTION PRINTED AT EACH X
C RXO = INITIAL VALUE OF X, REDUCED PARTICLE SIZE; 2*PI*R/LAMBDA
C DELX = INCREMENT OF PARTICLE SIZE
C M = INDEX OF REFRACTION (2F5.3), REAL AND IMAGINARY
C GAM = M
C XC = MOST PROBABLE VALUE OF X IN DISTRIBUTION FUNCTION
C NOTE, B = ALPHA/(GAM*X*X*GAM)
C
C IF NX = 0, PROGRAM STOPS EXECUTION
C
C REAL *4AS1(90),AS2(90),WEIGHT(501),S1(90),S2(90)
C DATA PI/3.1415926536/

C**** READ IN DATA
C 20 READ (5,320) NX,NTH,ICODE,RXO,DELX,M,ALPHA,GAM,XC
C IF (NX.EQ.0) STOP
C WRITE (6,340) NX,NTH,RXO,DELX,M,ALPHA,GAM,XC
C IF ((NX/2).EQ.0.NE.NX) GO TO 40
C WRITE (6,360) GC TO 20

C**** INITIALIZE
C 4C SAV = (ALPHA+1.0)/GAM
C B = ALPHA/(GAM*X*X*GAM)
C ANORM = (GAM*B**SAV)/(GAMMA(SAV))
C RX = RXC-DELX

C**** CALCULATE DISTRIBUTION FUNCTION (WEIGHT(I))
C 360 RX = RX+DELX
C 370 WEIGHT(I) = ANORM*RX**ALPHA*EXP(-E*RX**GAM)
C 380 IF (WEIGHT(I).LT.1.00E-08) GO TO 6C
C 390 IF (RX.LT.XC) GO TO 60
C 400 N* = I

C****
GO TO 80  
C 60 CONTINUE  
80 CONTINUE  
IF (NX.EQ.1) WEIGHT(I)=1.00  
WRITE (6,380) WEIGHT(I),I=1,NX  
RX = RXC-DELX  
ICOEF = 2  
C**** START LOOP FOR EACH VALUE OF X  
C  DO 240 IC=1,NX  
   RX = RX+DELX  
   CALL MLEM (ICODE,NTH,RX,M,AS1,AS2,GSCA,QEXT)  
C**** PRINT OUT FOR EACH VALUE OF X IF DESIRED (ICODE.GE.5)  
   IF (ICODE.LT.5) GO TO 100  
   WRITE (6,400) (AS1(J),J=1,NTH)  
   WRITE (6,400) (AS2(J),J=1,NTH)  
100 IF (ICODE.LT.1) GO TO 120  
   WRITE (6,420) RX,QSCA,QEXT  
C**** BEGIN INTEGRATION USING SIMPSON'S RULE, FIRST VALUE STARTS HERE  
120 IF (IC.NE.1) GO TO 160  
   SWT = WEIGHT(I)  
   BETASC = PI*RX**2*GSCA*WEIGHT(IC)  
   BETAX = PI*RX**2*QEXT*WEIGHT(IC)  
C  DC 140 I=1,NTH  
   S1(I) = AS1(I)*WEIGHT(IC)  
   S2(I) = AS2(I)*WEIGHT(IC)  
C  DC 240  
C**** COMPLETE INTEGRATION USING SIMPSON'S RULE; LAST VALUE ENTERED HERE  
160 IF (IC.NE.NX) GO TO 200  
   SWT = (SWT+WEIGHT(IC))*DELX/3.0  
   BETAX = (BETAX+WEIGHT(IC)*PI*RX**2*QEXT)*DELX/(3.0)  
   BETASC = (BETASC+WEIGHT(IC)*PI*RX**2*GSCA)*DELX/(3.0)  
C  DC 180 I=1,NTH  
   S1(I) = (S1(I)+WEIGHT(IC)*AS1(I))*DELX/(3.0)  
   S2(I) = (S2(I)+WEIGHT(IC)*AS2(I))*DELX/(3.0)  
C  DC 240  
C**** INTERMEDIATE STEPS IN INTEGRATION USING SIMPSON'S RULE  
200 ICOEF = 8/ICOF  
   SWT = SWT+ICOEF*WEIGHT(IC)  
   BETASC = BETASC+ICOEF*PI*RX**2*GSCA*WEIGHT(IC)  
   BETAX = BETAX+ICOEF*PI*RX**2*QEXT*WEIGHT(IC)  
C  DC 220 I=1,NTH
S1(I) = S1(I) + ICCEF*WEIGHT(I)*AS1(I)

220 S2(I) = S2(I) + ICCEF*WEIGHT(I)*AS2(I)

C**** PRINT OUT S1 EVERY 1/0 TH CALCULATION
IF (IC/10)*10, NE, IC) GO TO 240
C WRITE (6,998) RX, BETASC
C WRITE (6,997) (S1(I), I = 1, NTH)
C 240 CONTINUE
C
C**** COMPLETE LOOP
C**** PRINT OUT RESULTS
WRITE (6,440) SWT, BETASC, BETAEX
WRITE (6,460) WRITE (6,460) S1(I), I = 1, NTH
WRITE (6,480) S2(I), I = 1, NTH
C DO 260 I = 1, NTH
S1(I) = S1(I)/BETASC
S2(I) = S2(I)/BETASC
C 260 CONTINUE
C WRITE (6,500) WRITE (6,500) S1(I), I = 1, NTH
WRITE (6,480) S2(I), I = 1, NTH
C DO 280 I = 1, NTH
S1(I) = 0.5*(S1(I) + S2(I))
C WRITE (6,520) WRITE (6,520) S1(I), I = 1, NTH
WRITE (6,480) S2(I), I = 1, NTH
C
C**** THIS PROVIDES PUNCHC CARDS USABLE IN MONTE CARLO ROUTINE
C DO 300 J = 1,2
C 300 WRITE (7,340) S1(I), I = 1, NTH
C GC TO 20
C
320 FORMAT (I3,12.11,F4.2,6F5.3)
340 FORMAT (1I11,13*13,*2,13,*RXO = ',F10.4,*
1 'DELX = ',F10.4,*M = ',2F10.4,/1'ALPHA = ',F12.4,*GAM = ',',2F12.4,*XC = ',F12.4)
360 FORMAT ('NC. OF POINTS FOR NX MUST BE ODD TO APPLY SIMPSON'S RUL
11E')
380 FORMAT ('WEIGHTING FUNCTION: ',/(1X,10F12.6))
400 FORMAT (1X,10F12.6)
420 FORMAT (1X = ',F10.4,*QSCA = ',F12.6,*GEXT = ',F12.6)
440 FORMAT (1X = ',F12.6,*BETASC = ',F12.6,*BETAEX = ',F12.6)
460 FORMAT (1X = ',THE AVERAGED SCATTERING FUNCTIONS ARE: ',/)}
C**** SUBROUTINE ANGLE CF MIE SCATTERING PROGRAM ****

C
SUBROUTINE ANGLE (A,B,NS,NTH,AS1,AS2)
REAL *4, AS1(90), AS2(90)
COMPLEX *8A(I20), B(I20), S1(90), S2(90)
DATA PI/3.1415926536/

C
IF (NS.LT.12) GO TO 20
WRITE (6,980)
RETURN

C
DO 60 J=1,NTH
THETA = THETA+DTHETA
C
CP
WRITE(6,997) THETA

C
CP997 FORMAT(1X,997), Theta = PI/(NTH-1)
C
C
DO 59 J=1,NTH
THETA = THETA+DTHETA
C
CP
WRITE(6,999) (A(I), B(I), I = 1,NS)

CP999 FORMAT(1X,999), A(I), B(I)

C
FORMAT (1X,10F12.5)

C
FORMAT (/,* VALUES OF P/4PI TO COMPARE WITH DEIRMENDJIAN ARE *)

C
FORMAT (/,* THE AVERAGED NORMALIZE SCATTERING PHASE FUNCTION IS *)

C
END
P = (2.0*L-1.0)*CT/PM1/L-(L-1.0)*PM2/L
PP = ((2.0*L-1.0)*CT/PM1P-(L-1.0)*PM2P+(2.0*L-1.0)*PM1P)/L
PPP = ((2.0*L-1.0)*CT/PM1PP-(L-1.0)*PM2PP+(4.0*L-2.0)*PM1PP)/L
C = (2.0*C/L+1.0)/(L*(L+1.0))
SC = CT/PP-ST**2*PPP
S1(J) = S1(J)+(A(N)*PP+B(N)*SC)*C
S2(J) = S2(J)+(B(N)*PP+A(N)*SC)*C
4C CONTINUE
CP WRITE(6,998) S1(J), S2(J)
CP998 FORMAT(5X,2(2F8.4,5X))
AS1(J) = CABS(S1(J))**2
AS2(J) = CABS(S2(J))**2
6C CONTINUE
C CP WRITE(6,110) (AS1(J), J = 1,NTH)
CP WRITE(6,110) (AS2(J), J = 1,NTH)
CP110 FORMAT(1X,1 CF12.6)
RETURN
C 80 FORMAT(* *** ERROR WILL OCCUR IN ANGLE DUE TO IMPROPER INDEXING*)
END

C***** SUBROUTINE MIEM OF MIE SCATTERING PROGRAM *****
C
C THE FOLLOWING FUNCTIONS MUST BE INCLUDED
C JN(N,X) CALCULATES PSI, RELATED TO BESSEL FUNCTION
C JNP(N,X) CALCULATED DERIVATIVE OF PSI
C FN(N,X) CALCULATES CHI, RELATED TO NEUMAN FUNCTION
C FNP(N,X) CALCULATED DERIVATIVE OF CHI
C HN(N,X) CALCULATES XI, INVOLVING HAENKEL FUNCTION
C HNP(N,X) CALCULATED DERIVATIVE OF XI
C CSIN(X) CALCULATES THE SIN OF X, X MAY BE COMPLEX
C CCOS(X) CALCULATES COSINE OF X, X MAY BE COMPLEX
C
C INPUT CONSISTS OF
C N,NTH,RX,EP (II,I3,IX,3F5.2)
C N = 0 TO START A NEW PAGE. = 1 IF NO NEW PAGE TO START
C NTH = NUMBER OF VALUES OF THETA BETWEEN 0 AND PI
C RX = X (REDUCED PARTICLE SIZE IN NOTATION OF VAN DE HULST)
C EP = DIELECTRIC CONSTANT = SQUARE CF INDEX CF REFRACTION
C
C M IS THE COMPLEX REFRACTIVE INDEX
C I READ(5,100) N,NTH,RX,M
C 100 FORMAT(II,13,IX,6F5.2)
SUBROUTINE PIEM (ICODE, NTH, RX, M, AS1, AS2, QSCA, QEXT)
COMPLEX X, JN, FN, HN, JAF, FNP, HNP, M, Y
COMPLEX JX(3), JY(3), HX(3), JPX, JPY, HPX, AN, BN
COMPLEX MU, EP, RMU, REP
COMPLEX *8 A(120), B(120)
REAL *4 AS1(50), AS2(90)

N = 1
X = RX
DEL = 0.0
MU = (1.0, 0.0, 0.0)
EP = M**2
RMU = CSORT(MU)
REP = CSORT(EP)
IF (NTH.LE.0) STOP
N = 1-N
Y = M**X
EPSIL = 1.0E-6*X**2/(2*(2*N+1))
QEXT = 0.0
QSCA = 0.0

C SET UP FIRST THREE FUNCTIONS OF EACH KIND REQUIRED

JX(1) = JN(0, X)
JX(2) = JN(1, X)
JX(3) = JN(2, X)
JY(1) = JN(0, Y)
JY(2) = JN(1, Y)
JY(3) = JN(2, Y)
HX(1) = HN(0, X)
HX(2) = HN(1, X)
HX(3) = HN(2, X)

C DE 20 N=1:120
JFX = (N+1)*JX(2)/X-JX(3)
JPY = (N+1)*JY(2)/Y-JY(3)
HPX = (N+1)*HX(2)/X-HX(3)

C CALCULATE THE COEFFICIENTS OF THE SCATTERED WAVE FUNCTION

AA = (RMU*JY*JX(2)-REP*JY(2)*JPY)/30*RP*JY(2)*HX(2)-REP*JY(2)*HPX
BN = (REP*JY*JX(2)-RMU*JY(2)*JPY)/30*REP*JY(2)*HX(2)-RMU*JY(2)*HPX
A(N) = AN
B(N) = BN
DELQ = (2*N+1)*(AN+BN)
DELQS = (2*N+1)*(CABS(AN)**2+CABS(BN)**2)
QEXT = QEXT + DELQ
QSCA = QSCA + DELQS
CONVERGENCE CRITERION TESTED HERE

IF (ABS(DELQ) .LE. EPSIL) GO TO 40

CALCULATE THE NEXT HIGHER ORDER FUNCTION FOR NEXT TERM IN SERIES

JX(1) = JX(2)
JX(2) = JX(3)
JY(1) = JY(2)
JY(2) = JY(3)
HX(1) = HX(2)
HX(2) = HX(3)
JX(3) = (2*N+3)*JX(2)/X-JX(1)
JY(3) = (2*N+3)*JY(2)/Y-JY(1)
HX(3) = (2*N+3)*HX(2)/X-HX(1)

20 CONTINUE

WRITE (6,60) N, DELQ, X

CONTINUE

CALL ANGLE (A, B, NS, NTH, AS1, AS2)
QEXT = QEXT*2.0/X**2
QSCA = QSCA*2.0/X**2

60 FORMAT (/15,14, I4, " TERMS WERE USED TO OBTAIN CONVERGENCE. LAST TERM W
1AT X = ", 2G12.3)

END

C**** COMPLEX FUNCTIONS JNP, HN, FN, JN, CSIN, CCCS CF MIE SCAT PROGRAM

COMPLEX FUNCTION JNP(N,X)
COMPLEX X,N
N1 = N+1
JNP = (N+1.0)*JN(N,X)/X-JN(N1,X)
RETURN
END

FLOWCHART FOR FNP
COMPLEX FUNCTION FNP(N,X)
COMPLEX X, F
N1 = N+1
FNP = (N+1.0)*FN(N,X)/X-FN(N1,X)
RETURN
END
COMPLEX FUNCTION HN(N,X)
COMPLEX X, JN, FN
HN = JN(N,X) + (0.0,1.0)*FN(N,X)
RETURN
END

COMPLEX FUNCTION HNP(N,X)
COMPLEX X, JNP, FNP
HNP = JNP(N,X) + (0.0,1.0)*FNP(N,X)
RETURN
END

COMPLEX FUNCTION FNP(N,X)
COMPLEX X, CCOS, CSIN, F0, F1
IF (CABS(X).GE.1.0E-9) GO TO 20
FN = 0.0
RETURN
20 F0 = -CCOS(X)/X
IF (N.NE.0) GO TO 40
FN = -F0*X
RETURN
40 F1 = (F0-CSIN(X))/X
IF (N.NE.1) GO TO 60
FN = -F1*X
RETURN
C 60 DO 80 I=2,N
   FN = (2.0*I-1.0)*F1/X-F0
   F0 = F1
   F1 = FN
80 CONTINUE
C
FN = -FN*X
RETURN
END

COMPLEX FUNCTION JN(N,X)
COMPLEX X, CCOS, CSIN, JO, J1
IF (CABS(X).GE.1.0E-9) GO TO 40
JC = (1.0,0.0)
IF (N.NE.0) GO TO 20
JN = (1.0,0.0)
RETURN
20 JN = (0.0,0.0)
RETURN
40 JO = CSIN(X)/X
IF (N.NE.0) GO TO 60
JN = JO*X
RETURN
60 J1 = (JO-CCOS(X))/X
IF (N.NE.1) GO TO 80
C

JN = J1*X
RETURN

C

80 DC 100 I=2-N
JA = (2.0*I-1.0)*J1/X-J0
J0 = J1
J1 = JN
100 CONTINUE
C

JN = JN*X
RETURN
END

C

COMPLEX FUNCTION CSIN(X)
COMPLEX X
A = X
B = AIMAG(X)
CSIN = SIN(A)*COS(B)+(0.0,1.0)*COS(A)*SINH(B)
RETURN
END

C

COMPLEX FUNCTION CCS(X)
COMPLEX X
A = X
B = AIMAG(X)
CCCS = COS(A)*COS(B)+(0.0,1.0)*SIN(A)*SINH(B)
RETURN
END

C**** MCNTE CARLO DRIVER ROUTINE DRLITE ****
C

THIS ROUTINE IS INTENDED TO DRIVE THE MONTE CARLO ROUTINE
CALLED LITE, WHICH CALCULATES THE DISTRIBUTION OF PHOTONS
FROM A UNIDIRECTIONAL LIGHT SOURCE. IT IS USED TO READ
INFORMATION INTO THE PROGRAM AND TO CONTROL THE OUTPUT.
THE DISTRIBUTION OF PHOTONS AND THE TOTAL NUMBER OF PHOTONS
IN EACH SHELL ARE STANDARD OUTPUT INFORMATION.
FUNCTIONAL OUTPUT IS CONTROLLED BY METHOD AND IPRT STATE-
MENTS. METHOD CONTROLS THE SHAPE OF THE THETA BINS,
WHILE IPRT CONTROLS THE ACTUAL OUTPUT. PLOT INDICATES IF
PLOT CF SPATIAL INFORMATION IS DESIRED.
REAL *8GIN
DIMENSION G(2), SCA(2), ABB(2), M(2), RPT(2)
DIMENSION TIM(80), XVAL(80)
REAL *8 THA(20), X(200), Y(200), Z(2CC), C(8), RFLUX(10,20,1)
REAL*8 TL(12) /*PLCT CF */, "NEG LOG", "OF RELAT", "IVE FLUX",
M A MDRL00080
*I*, 'LLBACH ', 6 */
REAL *8XAX/ KMETER !/
REAL *8YAX/ KMETER !/
INTEGER *I*18(4), NB(1)
INTEGER NAI(10) /1, 2, 3, 4, 0, 1, 0, 1, 1, 0, 1/
INTEGER*# BINS(10, 20, 1), BINS(10, 20, 20)
COMMON WEIGHT(2, 70), PHASE(2, 70), SLOPE(2, 70), THETA(2, 70)
EQUIVALENCE (BINS, RFLUX)
DATA PI/3.1415926536 /
CALL ERRSET (209, 100, -1, 1, 0, 200)
1IX
READ (5, 560) NPHOT, NTHETA, NFLOVW, NSHLS, N(1), N(2), PLOT, IPRT, METHOD, DRL00090
1PI(1), RPT(2), BACK, GIN
IF (METHOD.EQ.0) METHOD=1
IF (NPHOT.EQ.0) STCP
IF (N(1).EQ.0) GO TO 100
C
DC 80 J=1,2
READ (5, 600) (THETA(J, I), I=1, 70)
READ (5, 600) (PHASE(J, I), I=1, 70)
T_NORM = 0.0
TWT = 0.0
PHASE(J, 1) = 0.0
M = N(1)-1
C
DC 40 I=1, M
PHASE(J, I+1) = PHASE(J, I+1)*SIN(THETA(J, I+1)*PI/180.0)
P_AVG = (PHASE(J, I)+PHASE(J, I+1))/2.0
T_DIF = (THETA(J, I+1)-THETA(J, I))*PI/180.0
40 T_NORM = T_NORM*P_AVG*T_DIF
C
DC 60 I=1, M
P_AVG = (PHASE(J, I)+PHASE(J, I+1))/2.0
T_DIF = (THETA(J, I+1)-THETA(J, I))*PI/180.0
WEIGHT(J, I) = P_AVG*T_DIF/T_NORM
SLOPE(J, I) = (PHASE(J, I+1)-PHASE(J, I))/(THETA(J, I+1)-THETA(J, I))
1*180.0/PI
60 TWT = TWT+WEIGHT(J, I)
C
80 CONTINUE
C
100 WRITE (6, 626)
CALL LIFP (NPHOT, NTHETA, NFLOVW, NSHLS, DI_STSH, THKNES, SCA, ABB, G, RPT, DRL00370
BACK, IX, BINS, METHOD, IPRT, N, PTIM, SIGTIM, AVGTIM)
WRITE (6, 640)
C
DC 240 I=1, NSHLS
  ISUM = 0
C
DC 120 J=1, NTHETA
C
DC 120 K=1, NFLDVW
  ISUM = ISUM+BINS(I, J, K)
  120 CONTINUE
C
WRITE (6,660)
WRITE (6,680) I, ISUM
IF (NFLDVW.EQ.1) CC TC 180
C
DC 140 J=1, NTHETA
  ISUM = 0
C
DC 140 K=1, NFLDVW
  ISUM = ISUM+BINS(I, J, K)
  140 CONTINUE
C
WRITE (6,700) J, ISUM
WRITE (6,840) (BINS(I, J, K), K=1, NFLDVW)
  160 CONTINUE
C
GO TO 200
180 WRITE (6,840) ((BINS(I, J, K), K=1, NFLDVW), J=1, NTHETA)
  220 CONTINUE
C
220 IF (IPRT.LE.3) GO TC 240
  WRITE (6,720)
C
DC 220 J=1, NTHETA
  220 CONTINUE
C
WRITE (6,740) (BINDST(I, J, K), K=1, 20)
C
240 CONTINUE
C
IF (IPRT.NE.3) GO TO 280
C
WRITE (6,780)
WRITE (6,890)
WRITE (6,930) PTIM
WRITE (6,960) TKNES, AVGTIM, SIRTIM
WRITE (6,920)
WRITE (6,780)
DO 260 I=1, 80
260 XVAL(I) = -9.0750 + I*.0750
C
MAX=0.0
C
DC 270 J=2, 80
270 IF (PTIM(J) .GT. MAX) MAX = PTIM(J)
C
275 DC 275 I = 1, 80
C
PTIM(I) = PTIM(I) / MAX
C
CALL PLOT(I, XVAL, PTIM, 80, 8)
C
CONTINUE
C
IF (IPRT .GE. 6) GO TO 380
IF (IPRT .LT. 1) GO TO 20
WRITE (6, 800)
C
DO 320 I = 1, NSHLS
R = DISTSH*I
C
DO 300 J = 1, NTHETA
TH1 = (PI*(J-1))/NTHETA
IF (METHOD .EQ. 3) TH1 = (TH1*(J-1))/NTHETA
TH2 = (PI*J)/NTHETA
IF (METHOD .EQ. 3) TH2 = (TH2*J)/NTHETA
AREA = 2.0*PI*R**2*(COS(TH1)-COS(TH2))
THA(J) = (TH2+TH1)/2.0
C
DO 300 K = 1, NFLDVW
IF (BINS(I,J,K) .LT. 3) RFLUX(I,J,K) = 0.0
IF (BINS(I,J,K) .LT. 3) GO TO 300
RFLUX(I,J,K) = -ALCG10(BINS(I,J,K)/(AREA*NP*CT))
C
WRITE (6, 820) ((RFLUX(I,J,K), K = 1, NFLDVW), J = 1, NTHETA)
300 CONTINUE
320 CONTINUE
380 CONTINUE
420 IF (PLOT .EQ. 0.0) GO TO 20
C
***** THIS SECTION IS USED TO DRIVE THE PLOTTING SUBROUTINE CONISO
C
CC 540 K = 1, NFLDVW
INC = 4
C
DO 460 I = 1, NTHETA
C
CC 440 J = 1, NSHLS
IF (RFLUX(I,J,K) .EQ. 0.0) GO TO 440
INC = INC + 1
K(INC) = J*DISTSH*COS(THA(I))
Y(INC) = J*DISTSH*SIN(THA(I))
Z(INC) = RFLUX(I,J,K)
440 CONTINUE
460 CONTINUE
C
C  X(1) = -DISTSH+NSHLS-1.0
X(2) = -DISTSH+NSHLS-1.0
X(3) = +DISTSH+NSHLS+1.0
X(4) = +DISTSH+NSHLS+1.0
Y(1) = 0.0
Y(2) = +DISTSH+NSHLS+1.0
Y(3) = +DISTSH+NSHLS+1.0
Y(4) = 0.0
Z(1) = 500.0
Z(2) = 500.0
Z(3) = 500.0
Z(4) = 500.0
NPTS=1 NC
C
DC 500 ICORN=1,4
IE(ICORN) = ICORN
C
500 CONTINUE
C
NF = 1
NB(1) = 4
NC = 8
C
DC 520 I=1,NC
C(I) = -3.0+I*1.0
C
520 CONTINUE
C
CF = .6
XS = 9.0
YS = 4.5
CALL CONISOI (X,Y,Z,NPTS,IB,NR,NB,C,NC,CF,XS,YS,TL,XAX,YAX,NA,IER)
WRITE (6,880) IER
C
540 CONTINUE
C
GC TO 20
C
560 FORMAT (16,5I2,31I1,110)
580 FFORMAT (11F5.3,D5.3)
600 FFORMAT (7F1G.6)
620 FFORMAT (1HI)
640 FFORMAT ("' DISTRIBUTION OF PHOTONS IS GIVEN BY'")
660 FFORMAT (1X)
680 FFORMAT (1X,'TOTAL NUMBER IN SHELL 'I2,' IS 'I5'')
700 FFORMAT (1X,'NO. IN BIN 'I2,' IS 'I5')
720 FFORMAT (1X,' TIME OF ARRIVAL BINS GIVEN BY',/)
C**** MCNE CARLO ROUTINE L** ****

THIS SUBROUTINE SIMULATES A PHOTON WHICH RANDOMLY COLLIDES WITH PARTICLES, SCATTERED AT ANGLES WEIGHTED BY VARIOUS FUNCTIONS, AND DETERMINES LOCATION OF INTERSECTION WITH VARIOUS SPHERES.

INPUT PARAMETERS:

NPHOT = NUMBER OF PHOTONS TO TRACE THROUGH

NTHETA = THE NUMBER OF THETA "BINS" TO KEEP TRACK OF

NFLOD = THE NUMBER OF "FIELD OF VIEW BINS" FOR EACH THETA BIN

NSHLS = THE NUMBER OF SHELLS TO BE INTERSECTED BY EACH PHOTON

DISTSH = THE DISTANCE BETWEEN EACH SHELL (AND FROM ORIGIN FOR FIRST)

THKNES = PHYSICAL THICKNESS OF CLOUD IN KILOMETERS

G1(1),G1(2) = G VALUES FOR HENGY-GREENSTEIN FUNCTION INSIDE AND OUTSIDE OF CLOUD, RESPECTIVELY

RPT(1), RPT(2) = RATIOS OF PARTICULATE TO TOTAL SCATTER INSIDE AND OUTSIDE, RESPECTIVELY

KSCA(1), KSCA(2) = SCATTERING COEFFICIENTS INSIDE AND OUTSIDE OF CLOUD, RESPECTIVELY ( INVERSE KILOMETERS )

KABS(1), KABS(2) = ABSORPTION COEFFICIENTS INSIDE AND OUTSIDE OF CLOUD, RESPECTIVELY ( INVERSE KILOMETERS )

N = NUMBER OF INPUT DATA PAIRS USED TO DEFINE AN ARBITRARY NORMALIZED PHASE FUNCTION

FITM(1) = TIME BINS FOR TABULATING PHOTON ARRIVAL AT EDGE OF CLOUD

SIGTIN = VARIABLE USED TO CALCULATE DEVIATION OF PHOTON ARRIVAL TIMES ABOUT THE MEAN FOR PLANE PARALLEL CLOUDS

AVGTIN = VARIABLE USED TO CALCULATE MEAN OF PHOTON ARRIVAL TIMES FOR PLANE PARALLEL CLOUDS

PLOT = INPUT VALUE TO INDICATE IF FLOT OF CONTOURS OF EQUAL
PHOTON FLUX IS DESIRED. PLOT UNEQUAL 0 CALLS FOR PLOT
INS, IBANG, IFAR, IFARN = VARIABLES USED IN BOUNDARY CROSSING
LOGIC
DCTI, DOTO = PROJECTIONS OF PHOTON POSITION ALONG DIRECTION
OF INCIDENCE AT EACH COLLISION. NEW AND OLD RES-
PECUTIVELY, USED IN BOUNDARY CROSSING LOGIC.
FBACK = RATIO OF BACKSCATTER TO TOTAL PARTICULATE SCATTERING.
IX IS A RANOM NUMBER TO START THINGS

X(I) = X COORDINATE; I=1 FOR ORIGIN, I=2 FOR X=1, I=3 FOR Z=1
(X=1 MEANS (1,0,0) POINT FOR ORGINAL COORD SYST, ETC.)
Y(I) = Y COORDINATE; DITO
Z(I) = Z """".
XT(I) = THE NEW COORDINATE AFTER ROTATION BY THETA, PHI, AND
TRANSLATION BY DISTANCE DR
VT(I), AND ZT(I), LIKewise
Z(R) = TEMPORARY COORDINATES HELD FOR BOUNDARY CROSSING ADJUSTMENT
XX(I) IS A TEMPORARY COORDINATE POINT, AT INTERSECTION WITH SPHERE
YX(I), ZX(I), LIKewise
TH = VALUE OF THETA BY WHICH COORDINATE SYSTEM IS ROTATED AFTER
SIMULATED COLLISION WITH PARTICLE
PH = VALUE OF PHI, LIKewise
THP = VALUE OF THETA PRIME, Theta RELATIVE TO PHOTON-FIXED COORD
PHP = VALUE OF PHI RELATIVE TO PHOTON FIXED COORD SYST
BINS(I,J,K) = NO. OF OCCURRENCES AT JTH SPHERE WITH JTH
BIN OF THETA AND KTH BIN OF THETA PRIME
RHSH(I) = DISTANCE OF ITH SHELL FROM ORIGIN
DHSH = NEW VALUE OF THETA RELATIVE TO OLD DIRECTION
DPI = NEW VALUE OF PHI RELATIVE TO OLD DIRECTION
CR = DISTANCE OF PHOTON TRAVEL AFTER A COLLISION
IX, IY = FIXED-POINT RANOM NUMBERS
RTDI = TOTAL DISTANCE A PHOTON TRAVELS
DIST = DISTANCE FROM ORIGIN FOR PREVIOUS CALCULATION
DPC1 = DISTANCE ACTUALLY TRAVELED FROM ORIGIN TO SHELF
ISAV = INTEGER CORRESPONDING TO NUMBER OF SHELL BEYOND WHICH THE
PHOTON IS LOCATED
XDFR = RATIO OF DISTANCE FROM POINT OF COLLISION TO A SHELL
RELATIVE TO TOTAL DISTANCE A PHOTON TRAVELS
 AFTER A COLLISION
DIMENSION PTIM(80)
DATA PI/3.1415926536/, TPI/6.2831853072, R007PI/1.772453851/
DATA EPSIL/1.00E-03/
WRITE (6,960)
WRITE (6,1000) NPHCT, NTHETA, NFLDVW, NSHLS, N(1), N(2), IPRT, METHOD, IX
WRITE (6,980) DISTSH, THKNES, SCA, ABB, GR, PRT, FEACK, GIN
C**** INITIALIZE ARRAYS
C  DO 60 I=1, NSHLS
C  DC 60 J=1, NTHETA
C  DC 20 K=1,20
  20 BINEST(I,J,K) = 0
C  DC 40 K=1, NFLDVW
  40 BINS(I,J,K) = 0
C  6C CONTINUE
C  DO 80 I=1,8C
  80 PTIM(I) = 0.0
C  TIMT = 0.0
T IMD = 0.0
C T = 0.0
D = -DISTSH
NUP = NSHLS+1
C  DC 100 I=1, NUP
D = D+DISTSH
  100 RSHL(I) = D
C C**** START LOOP FOR EACH PHOTON
C  DC 940 NPH=1, NPHOT
IXS = IX
NSCA = 0
RTOT = 0.0
ISAV = 1
C  DO 120 I=1,4
  X(I) = 0.0
  Y(I) = 0.0
120 Z(I) = 0.0
C
X(2) = 1.0
Y(4) = 1.0
Z(3) = 1.0
C***** CHOOSE DISTANCE OF INITIAL PHOTCN
INS = 1
IBANG = 0
140 TAU = 1.0/(ABB(INS)+SCA(INS))
CR = RANEXP(IY,TAU)
IF ((CR.GT.THKNES).AND.(INS.EQ.1)) GO TO 200
C
DG 160 I=1,4
160 Z(I) = Z(I)+CR
C
180 IF (Z(I).LT.RSHL(ISAV+1)) GO TO 240
ISAV = ISAV+1
IF (ISAV.GT.NSHLS) GO TO 940
GO TO 180
200 CR = THKNES
C
DC 220 I=1,4
220 Z(I) = Z(I)+CR
C
INS = 2
GO TO 140
240 CONTINUE
RTOT = Z(I)
SDIST = RTCT
T = THKNES
C***** DETERMINE IF PHOTCN IS ABSORBED OR SCATTERED
260 RATIO = SCA(INS)/(ABB(INS)+SCA(INS))
CALL RANDU (IX, IY, RN)
IX = IV
IF (RN.GT.RATIO) GO TO 940
C***** IF PHOTCN IS SCATTERED, DETERMINE NEW ANGLES AND DISTANCES
280 NSCA = NSCA+1
DR = RANEXP(IX, IY, TAU)
DTHETA = RANth(IX, IY, G, RPT, FBACK, N, INS)
DFHI = RANPH(IX, IY)
STH = SIN(DTHETA)
CTh = COS(DTHETA)
SPH = SIN(DFHI)
CPH = COS(DFHI)
C***** ROTATE TO NEW PHOTCN POSITION AND TRANSLATE
DC 280 I=1,4
X(I) = X(I)*CTh*CPH-Y(I)*CTh*SPH-Z(I)*STH
Y(I) = X(I)*SPH+Y(I)*CPH
C***** HOLE PRESENT Z COORDINATES UNTIL BOUNDARY CROSSING DETERMINATION
C IS MADE
Z(H(I)) = X(I)*STH*CPH-Y(I)*STH*SP+Z(I)*CTH

280 CONTINUE
C
C******** DETERMINE IF A BOUNDARY HAS BEEN CROSSED
DISTO = DKRT((X(1)**2+Y(1)**2+Z(1)**2)
VALO = (X(1)**2+X(1)**2+Y(1)**2+Y(1)**2+Z(1)**2)/CISTO
IF (VALO.GT.1.0) VALO=1.0
IF (VALO.GT.1.0) VALO=-1.0
THEO = PI-ARCCOS(VALO)
IF (THEO.LT.0.0) THEO=0.0
300 DIST1 = VRIT(X(1)**2+Y(1)**2+Z(1)**2)
VALN = (X(I)**2+X(1)-X(3)**2+Y(I)**2+Y(1)-Y(3)**2+Z(I)**2+Z(1)-Z(3)**2)/DIST1
IF (VALN.GT.1.0) VALN=1.0
IF (VALN.LT.-1.0) VALN=-1.0
THEN = PI-ARCCOS(VALN)
IF (THEN.LT.0.0) THEN=0.0
C******** CHECK TO SEE IF PHOTON HAS CROSSED A BORDER

C DCTO = DISTO*COS(THEO)
C
C DOTT = DISTO*COS(THEN)
IF (DOT0.GT.0.0) IFAR=3
IF (DOT0.LE.0.0) AND. (.THEN.LT.PI/2.0) IFAR=2
IF (THEO.GE.PI/2.0) IFAR=1
IF (DOT1.GT.0.0) IFARN=3
IF (DOT1.LE.0.0) AND. (.THEN.LT.PI/2.0) IFARN=2
IF (.THEN.GE.PI/2.0) IFARN=1
IF (IANG.EQ.2) GO TO 500
IF (IFARN.EQ.2) GO TO 560
C******** NEW PHOTON POSITION IS INSIDE SCATTERING CLOUD
IF (IFAR.EQ.2) GO TO 500
C******** PHOTON PASSED FROM OUTSIDE TO INSIDE OF SCATTERING CLOUD
INS = 1
IF (IFAR.EQ.1) GO TO 440
C******** PHOTON PASSED FROM INSIDE THICK TO INSIDE THICK
32C FRACI = (I-DOTT)/(DOTO-DOTT)
C
DC 340 J=1,4
340 ZT(J) = ZT(J)-FRACI*DR
C TAU = 1.0/(ABB(INS)+SCA(INS))
DR = RANEXP(IX,IY,TAU)
C
DO 360 J=1,4
360 ZT(I,J) = ZT(I,J)+DR
C
36C IBANG = 1
GO TO 300
36C IBANG = 0
C
36F IF (1FARM-EQ.2) GO TO 700
C
COUTSIDE AGAIN SO IT NEEDS TO BE CHECKED AGAIN
INS = 2
FRAC = -DOT1/(T-DOT1)
C
400 ZT(I) = ZT(I)-FRACT*DR
C
TAU = 1.0/(ABB(INS)+SCA(INS))
CR = RANEXP(IY,TAU)
C
420 ZT(I) = ZT(I)+DR
C
GC TO 700
C
COUTSIDE AGAIN AND A SECOND CHECK IS NEEDED
INS = 2
FRAC = (DOT1-T)/DOT1
C
520 ZT(I) = ZT(I)-FRACT*DR
C
CT = CT+1.0
TIM = ((RTOT+(1-FRACT)*DR)-T)/3.0E+05
IF (TIM.LT.1.0E-09) TIM=1.0E-09
IF (TIM.GT.1.0E-03) TIM=1.0E-03
NBIN = (ALOGI(TIM)+9.0)*1.3333+1
IF (NBIN.LT.1) NBIN=1
PTIM(NBIN) = PTIM(NBIN)+1.0
TIMT = TIMT+TIM
TIMD = TIMD+TIM**2
TAU = 1.0/(ABB(INS)+SCA(INS))
DR = RANEXP(Ix, Iy, TAU)

C
DG 540 I=1.4
540 ZT(I) = ZT(I)+DR
C
GO TO 700
C*****NEW PHOTON POSITION IS OUTSIDE OF SCATTERING CLOUD
560 IF ((IFARN.EQ.1).OR.(IFARN.EQ.3)) GO TO 680
C*****PHOTON PASSED FROM INSIDE CLOUD TO OUTSIDE CLOUD
INS = 2
IF (IFARN.EQ.1) GO TO 620
C*****PHOTON PASSED FROM WITHIN CLOUD TO A POSITION GREATER THAN THKNESS
FRACT = (DOT1-T1)/(DOT1-DOT0)

C
DG 580 I=1.4
580 ZT(I) = ZT(I)-FRACT*DR
C
CT = CT+1.0
TIM = (RDOT+1.-FRACT)*DR-T1/3.0E+05
IF (TIM.LT.1.0E-09) TIM=1.0E-09
IF (TIM.GT.1.0E-03) TIM=1.9E-03
NBIN = (ALOGI(TIM)+9.0)*1.3333+1
IF (NBIN.LT.1) NBIN=1
PTIM(NBIN) = PTIM(NBIN)+1.0
TIMT = TIMT+TIM
TIMD = TIMD+TIM**2
TAU = 1.0/(ABB(INS)+SCA(INS))
DR = RANEXP(Ix, Iy, TAU)

C
DG 600 I=1.4
600 ZT(I) = ZT(I)+DR
C
GO TO 700
C*****PHOTON PASSED FROM WITHIN CLOUD TO A POSITION BEHIND CLOUD
620 FRACT = -DOT1/(DOTC-DCT1)
C
DG 640 I=1.4
640 ZT(I) = ZT(I)-FRACT*DR
C
TAL = 1.0/(ABB(INS)+SCA(INS))

L02500
L02510
L02520
L02530
L02540
L02550
L02560
L02570
L02580
L02590
L02600
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L02870
L02880
L02890
L02900
L02910
L02920
L02930
L02940
L02950
L02960
L02970
DR = RAMEXP(IX, IY, TAU)

C

DO 660 I = 1, 4

660 ZT(I) = ZT(I) + DR

C

GO TO 700

C** PHOTON PASSED FROM OUTSIDE CLOUD TO OUTSIDE CLOUD

680 IF (IFAR.EQ.1) GO TO 700

C** PHOTON PASSED COMPLETELY THROUGH CLOUD

IF (IFAR.EQ.3) GO TO 440

C** PHOTON PASSED FROM FRONT OF CLOUD TO BEHIND CLOUD

GC TO 320

C** END OF CLOUD CROSSED OR CHECKING HAS BEEN ACCOMPLISHED

7CC DR = SQRT((ZT(1)-ZT(1))**2)

CIST = SQRT(XT(I)**2+YT(I)**2+ZT(I)**2)

FF = 1.0-ZT(1)/DR

C** CHECK TO SEE IF PHOTON HAS PENETRATED A SHELL

C** CONSIDER IF PHOTON HAS PENETRATED IN AND OUT OF SHELL

C FF IS RATIONAL DISTANCE BETWEEN TWO POINTS GIVING

C THE DISTANCE OF CLOSEST APPROACH

IF ((FF.GT.0.0).AND.(FF.GT.1.0)) GO TO 720

DC = SQRT((XT(1)**2+YT(1)**2+ZT(1)**2)

IF (DC.GT.RSHL(ISA)) GO TO 720

SISAV = ISA

IF (DC.GT.RSHL(ISA)) GO TO 800

GC TO 720

C** CONSIDER IF PHOTON HAS PENETRATED OUTER SHELLS

720 IF (((ISA).GT.(NSHLS))) GO TO 940

740 IF (((DIST.GT.RSHL(ISA)) .GT. 8CC)) GC TO 8CC

C** TIME TO UPDATE COORDINATES - GET READY FOR ANOTHER COLLISION

760 RTOT = RTOT+DR

SCLST = DIST

C

DO 780 I = 1, 4

780 CONTINUE

C

GC TO 20 FOR ANOTHER COLLISION

GC TO 260

C** CALCULATE COORDINATES OF POINT AT WHICH PHOTON PENETRATES SHELL

800 ISA = ISA+1

XFDR = (DR-ZT(1)+SQRT((ZT(1)-DR)**2-SDIST**2+RSHL(ISA)**2))/DR
C
DC 820 I=1,4
XS(I) = XT(I)
YS(I) = YT(I)
ZS(I) = ZT(I) - (1.0 - XPCR)*DR
820 CONTINUE

C
PDIST = RTOT*XPDR*DR
C
C*** CALCULATE VALUE OF THETA
VAL = (((XS(1)**(XS(1)-XS(3))+YS(1)**(YS(1)-YS(3)))+ZS(1)**(ZS(1)-ZS(3)))
1)/RSHEL(ISAV))
IF (ABS (VAL).GT.1.1) CALL PERR(1,IXS,VAL,IXS,YS,ZS)
IF (ABS (VAL).GT.1.1) WRITE (6,1020) VAL,NPH,IXS
IF (VAL.GT.1.0) VAL=1.0
IF (VAL.LE.-1.0) VAL=-1.0
TH = PI-ARCOS(VAL)
IF (TH.LT.0.0) TH = 0.0
C
C*** CALCULATE VALUE OF PHI
DENOMS = (RSHEL(ISAV)**2-(XS(1)**(XS(1)-XS(3))+YS(1)**(YS(3)-YS(1)))+ZL)**2
15(I)**(ZS(3)-ZS(1))**2)
IF (DENOMS.LT.-EPSIL) CALL PERR(5,IXS,DENOMS,YS,ZS)
IF (DENOMS.LT.-EPSIL) IF (DENOMS.EQ.0.0) GO TO 840
DENOM = SQRT(DENOMS)
IF (DENOM.EQ.0.0) PH=0.0
IF (DENOM.EQ.0.0) GO TO 840
VAL = (((XS(1)**(XS(1)-XS(2))+YS(1)**(YS(1)-YS(2)))+ZS(1)**(ZS(1)-ZS(2)))
1)/DENOM)
IF (ABS (VAL).GT.1.1) CALL PERR(2,IXS,VAL,IXS,YS,ZS)
IF (ABS (VAL).GT.1.1) WRITE (6,1020) VAL,NPH,IXS
IF (VAL.GT.1.0) VAL=1.0
IF (VAL.LE.-1.0) VAL=-1.0
PH = ARCOS (VAL)
IF ((XS(1)**(XS(1)-XS(4))+YS(1)**(YS(1)-YS(4)))+ZS(1)**(ZS(1)-ZS(4)))
1)*PH=PH
840 CONTINUE
C
C*** CALCULATE VALUE OF THETA PRIME (RECEIVER)
VAL = (ZS(1)/RSHEL(ISAV))
IF (ABS (VAL).GT.1.1) CALL PERR(3,IXS,VAL,IXS,YS,ZS)
IF (ABS (VAL).GT.1.1) WRITE (6,1020) VAL,NPH,IXS
IF (VAL.GT.1.0) VAL=1.0
IF (VAL.LE.-1.0) VAL=-1.0
THP = ARCOS (VAL)
C
C*** CALCULATE VALUE OF PHI PRIME (RECEIVER)
PARAM = ((XS(1)**(XS(3)+YS(1)**(YS(3)+ZS(1)**(ZS(3)))
13)**2+YS(3)**2+ZS(3)**2))
1)*PARAM.LT.0.999) GO TO 860
VAL = XS(2)-XS(1)
GC TO 1040
860 \( EF = -RSHL(ISAV)*2/(XS(3)\times XS(1)+YS(3)\times YS(1)+ZS(3)\times ZS(1)) \)

\[
\begin{align*}
VX(1) &= XS(1) + BP*XS(3) \\
VY(1) &= YS(1) + BP*YS(3) \\
VZ(1) &= ZS(1) + BP*ZS(3) \\
\text{ANGRM} &= \text{SORT}(VX(1)**2+VY(1)**2+VZ(1)**2) \\
VX(1) &= VX(1)/\text{ANORM} \\
VY(1) &= VY(1)/\text{ANORM} \\
VZ(1) &= VZ(1)/\text{ANORM} \\
VX(3) &= XS(1)/RSHL(ISAV) \\
VY(3) &= YS(1)/RSHL(ISAV) \\
VZ(3) &= ZS(1)/RSHL(ISAV) \\
VX(2) &= VY(3)*VZ(1)-VZ(3)*VY(1) \\
VY(2) &= VZ(3)*VX(1)-VX(3)*VZ(1) \\
VZ(2) &= VX(3)*VY(1)-VY(3)*VX(1) \\
\text{IF}(VZ(3)\times LT.0.9955)\ \text{GO\ TO\ 880} \\
\text{VAL} &= 1.0 \\
\text{GC\ TO\ 900} \\
\end{align*}
\]

880 \text{CONTINUE}

\[
\begin{align*}
\text{VAL} &= VZ(1)/\text{SQRT}(1.0-VZ(3)**2) \\
\text{IF}(ABS(\text{VAL})>1.0)\ \text{CALL \ PERR}(4,3,\text{RSHL(ISAV)},VX,VY,VZ) \\
\text{IF}(ABS(\text{VAL})<1.0)\ \text{CALL \ PERR}(4,\text{XS,VAL,YS,YS,ZS}) \\
\text{IF}(\text{VAL}>1.0)\ \text{VAL}=1.0 \\
\text{IF}(\text{VAL}<1.0)\ \text{VAL}=-1.0 \\
\text{PHF=ARCOS(VAL)} \\
\text{C**** TALLY LOCATION OF ANGULAR RESULTS} \\
\text{IF}(METHOD.GE.3) \text{NTH}=(\text{SQRT(th)}*\text{NTHETA})/\text{ROOTPI}+1 \\
\text{IF}(METHOD.LE.2) \text{NTHP}=2.0*\text{THP}\times \text{NFLDVW}/\text{PI}\times 1 \\
\text{IF}(\text{METHOD.GE.3}) \text{NTHP}=(\text{SQRT(2.0*THP)})/\text{NETHETA} \\
\text{IF}(\text{IPRT.NE.9})\ \text{GO\ TO\ 920} \\
\end{align*}
\]

920 \text{CONTINUE}

\[
\begin{align*}
\text{BINS(ISAV-1,NTH,NTHP)} &= \text{BINS(ISAV-1,NTH,NTHP)} \\
\text{CX XXXX} &= ((YS(3)-YS(1))*YS(1)-(ZS(3)-ZS(1))*ZS(1))*VX(1) \\
\text{CX YYYY} &= ((ZS(3)-ZS(1))*YS(1)-(XS(3)-XS(1))*XS(1))*VY(1) \\
\text{CX ZZXX} &= ((YS(3)-YS(1))*ZS(1)-(XZ(3)-XZ(1))*XZ(1))*VZ(1) \\
\text{C**** TALLY LOCATION OF INTERSECTION IN DISTANCE OF ARRIVAL BINS(}\text{)
\text{IF}(\text{PODIST.LE.1.00001*RSHL(ISAV))})\ \text{PODIST}=1.00001*RSHL(ISAV) \\
\text{XDIST} &= \text{ALOC10I}(\text{PODIST}/\text{RSHL(ISAV)})-1.0*4.0 \\
\text{IF}(\text{XDIST.LE.0.0})\ \text{XDIST}=3.0 \\
\text{IF}(\text{XDIST.GE.4.95})\ \text{XDIST}=4.99 \\
\text{ADIST} &= (\text{XDIST}*201/541) \\
\text{BINDS(\text{ISAV-1,NTH,PODIST})} &= \text{BINDS(\text{ISAV-1,NTH,ADIST}) +1} \\
\text{GC\ TO\ 720} \\
\end{align*}
\]

940 \text{CONTINUE}

\[
\begin{align*}
\text{AVGTIM} &= \text{TIMT}/\text{CT} \\
\end{align*}
\]
C

SIGTLM = SQRT(TIMC/CT-AVGTIM**2)
RETURN

C

96C FORMAT (' ENTERED LITE')
960 FORMAT (' DISTSH= ',F6.3,5X,'THICKNESS= ',F6.3,3X,'SCA(1)= ',F6.3,'SCA(2)= ',F6.3,3X,'ABBL(1)= ',F6.3,5X,'ABBL(2)= ',F6.3,3X,'6X='
2 '*G(1)= ',F6.3,8X,'G(2)= ',F6.3,3X,'RPT(1)= ',F6.3,5X,'RPT(2)= ',F6.3,3X,'FBCK= ',F6.3,6X,'GIN= ',D6.3,'/
1003 FORMAT (' NHOT= ',I5,9X,'NTETHA= ',I5,3X,'NFLDWR= ',I5,8X,'N5HS= ',LIT04450)
110 I5,= ' N(1)= ',I5,10X,'N(2)= ',I5,10X,'RPT(1)= ',I5,10X,'RPT(2)= ',I5,'/LIT04510
1102 FORMAT (' ARCOS GT. 1.1; VAL= ',F10.5,' NPPCT= ',I5,' IX= ',I10)
1110 ENDC

C

SUBROUTINE PERR (ERRNO,IXS,PARAM,XS,YS,ZZ)
INTEGER *4ERRNO
DIMENSION XS(4), YS(4), ZZ(4)
NUP = 4
IF (IXS,L.4) NUP = IXS
WRITE (6,20) ERRNO,PARAM,IXS
WRITE (6,40) (XS(I),YS(I),ZZ(I),I=1,NUP)
RETURN

C

20 FORMAT (' ERROR DETECTED, LOCATION NO. ',I5,' PARAM = ',E14.8,' IXS = ',I10)
40 FORMAT (' COORD. FROM PERR: ',12F9.5)
END

C**** FUNCTION SUBROUTINE RANEXP ****
C

FUNCTION RANEXP (IX,IY,TAU)
C

C**** THIS FUNCTION GENERATES A RANDOM NUMBER WEIGHTED EXPONENTIALY
CALL RANU (IX,IY,RN)
IX = IY
RANEXP = -TAU*ALOG(1.0-RN)
RETURN
END
C**** FUNCTION SUBROUTINE RANTH

C**** THIS FUNCTION GENERATES A RANDOM VALUE OF THETA,

C SUITABLY WEIGHTED

FUNCTION RANTH (IX, IY, G, RPT, FBACK, H, INS)
DIMENSION N(2), G(2), THETA(2,70), PHASE(2,70), SLCPE(2,70)
1T(2,70), RPT(2)
COMMON HEIGT, PHASE, SLOPE, THETA
DATA PI2/1.5707963268/, PI/3.1415926536/
CALL RANDU (IX, IY, RN)
IX = IY
IF (RN.GT.RPT(INS)) GO TO 80
IF (FBACK.EQ.0.0) GO TO 60
CALL RANDU (IX, IY, RN)
IY = IX
CRANM = 0.0

C DO 20 I=1,8
CRANTH = HENH1/(RN*HENC*HEND*FBACK*CRANM1*(1.0-CRANM1**2))**2
IF (ABS(CRANTH-CRANM1).LT.0.001) GO TO 40
CRANM2 = CRANM1
CRANM1 = CRANM1
20 CONTINUE

C CRANTH = 0.25*(CRANM2*CRANTH)0.5*CRANM1
40 CONTINUE
IF (ABS(CRANTH).LT.1.0) CRANTH=CRANTH/(ABS(CRANTH)*.00001)
RANTH = ARCS(CRANTH)
RETURN

C**** CALCULATE THETA USING FORWARD SCATTER FUNCTION HERE
60 IF (NL1).NE.01 GO TO 100
CALL RANDU (IX, IY, RN)
IX = IY
HENA = (1.0+G(INS)**2)/(2.0*G(INS))
HENB = (1.0-G(INS)**2)**2/(8.0*G(INS)**3)
HENC = (G(INS)+1.0)/(2.0*G(INS))
HENO = (1.0-G(INS)**2)/(4.0*(1.0+G(INS)**2)**1.5)
RANTH = ARCS(HENA-HENB)/(HENC-RN)**2
RETURN

C**** CALCULATE THETA ASSUMING RAYLEIGH SCATTERING HERE
80 CALL RANDU (IX, IY, RN)
IX = IY
RLB = 4.0*RN-2.0
RLBS = SQRT(RLB**2+1.0)
CAPA = -RLB*RLBS**(1.0/3.0)
CAPB = -(RLB*RLBS)**(1.0/3.0)
RANTH = ARCS(CAPA+CAPB)
RETURN

C**** CALCULATE THETA BY USE OF INPUTED DATA PAIRS HERE
100 CALL RANDU (IX, IY, RN)
     IX = IY
     W1 = 0.0
     W2 = WEIGHT(INS, 1)
     P = N(INS)-1
C
     DC 120 I=1,M
     NIT = 1
     IF ((RN.GE.W1).AND.(RN.LT.W2)) GO TO 140
     W1 = W2
     W2 = W2+WEIGHT(INS, I+1)
120 CONTINUE
C
     140 RN = (RN-W1)/(W2-W1)
     F = PHASE(INS, NIT)
     E = SLOPE(INS, NIT)
     T = THETA(INS, NIT)*PI/180.0
     T1 = THETA(INS, NIT+1)*PI/180.0
     A = P-E*T
     C = RN*(A*T1+B*T1**2/2.0)+(1.0-RN)*(B*T1**2/2.0+A*T)
     IF (B.GE.0.0) GO TO 160
     RANTH = -A/B+SQRT((A/B)**2+2.0*C/B)
     RETURN
C
     160 RANTH = -A/B+SQRT((A/B)**2+2.0*C/B)
     RETURN
     END

C***** FUNCTION SUBROUTINE RANPH *****
C
C***** THIS FUNCTION GENERATES A RANDOM VALUE OF PI
FUNCTION RANPH (IX, IY)
DATA TPI/6.283183072/
CALL RANDU (IX, IY, RN)
     IX = IY
     RANPH = TPI*RN
     RETURN
     END
C**** EFFECTIVE ATTENUATION COEFFICIENT PROGRAM

THIS PROGRAM CALCULATES THE EFFECTIVE ATTENUATION COEFFICIENT, THE
RELATIVE FLUX THROUGH AN APERTURE OF GIVEN HALF ANGLE AND RANGE.
AND THE BEAM SPREAD FUNCTION USING MULTIPLE GAUSSIAN QUADRATURE.

INPUTS REQUIRED ARE AS FOLLOWS:
G = HENNEY-GREENSTEIN PARAMETER OF PHASE FUNCTION USED
TH = INITIAL VALUE OF HALF ANGLE OF APERTURE
THI = INCREMENT IN TH
NTHN = NUMBER OF TH VALUES TO BE USED, INTEGER
R = INITIAL RANGE IN UNITS OF EXTINCTION LENGTHS
RI = INCREMENT IN R
RN = NUMBER OF R VALUES TO BE USED, INTEGER
NACC = ACCURACY OF NUMERICAL INTEGRATION-Doubles PRECISION
  1 = 32 PT GAUSSIAN QUADRATURE
  2 = 64 PT

** **

10 = 320 etc.

SCA = SCATTERING COEFFICIENT IN INVERSE KILOMETERS

EXT = EXTINCTION COEFFICIENT IN INVERSE KILOMETERS

DIMENSION HBSF(20,20), ALPHA(20), THETA(20), RANM(20,20), RHBSF(20
1,20), C(20), FLUX(20,20), X(500), Y(500), Z(500)
DOUBLE PRECISION HBSF, ALPHA, THETA, RANM, R, HBSF, RANGE, THA, G, SCA, EXT,
1ACC, FL, XU, XL, YR, PL
COMMON THA(10)
REAL *8 T1L(12), *PLOT0F, *NEGBSF, *MAM1, *LLBAC+, 8**/
INTEGER NA(10)/1, 0, 0, 0, 1, 1, 0, 2, 0, 2/
REAL*8 XAX, *KILOMTRS/
REAL*8 YAX, *KILOMTRS/
INTEGER *4 NAI(14), NB(1)
EXTERNAL FCC, FCT
DATA PI/3.141592653600/
2C WRITE (6,300)
READ (5,320) SCA, EXT, G
WRITE (6,320) SCA, EXT, G
IF (SCA.GE.0.0) GO TO 280
WRITE (6,340)
READ (5,360) TH, THI, NTHN, R, RI, RN, NACC, PLCT
WRITE (6,360) TH, THI, NTHN, R, RI, RN, NACC, PLCT
ACC = NACC

DO 100 I=1,NTHN
FL = 0.0
THA = (TH+(I-1)*THI)*PI/180.0

DC 40 J=1, NACC
XU = J/ACC
XL = (J-1)/ACC

100 CONTINUE

END
CALL DG32 (XL,XU,FCT,YR)
FL = FL+YR
40 CONTINUE
C
ALPHA(I) = EXT-SCA*FL
HINT = 0.0
C
DC 60 I=1,NAC
XL = DATAN(THTA)+((L-1)/ACC)*(PI/2.0-DATAN(THTA))
XL = DATAN(THTA)+((L/ACC)*(PI/2.0-DATAN(THTA))
CALL DG32 (XL,XL,FCC,YR)
HINT = HINT+YR
60 CONTINUE
C
DC 80 K=1,NRN
RAN = R+(K-1)*RI
FLX(I,K) = DEXP(-ALPHA(I)*RAN/EXT)
HB SF(I,K) = SCA*FLX(I,K)*HINT/(RAN*DATN(THTA)**EXT)
RH BSF(I,K) = DLOG10(HBSF(I,K))
RANM(I,K) = RAN/DCOS(THTA)
80 CONTINUE
C
100 CONTINUE
C
WRITE (6,380)
C
CC 120 I=1,NTHN
TH ETA(I) = ((I-1)*TH)+TH
WRITE (6,400) TH ETA(I),ALPHA(I)
120 CONTINUE
C
WRITE (6,420) (FLX(I,J),I=1,NTHN,2),J=1,NRN,2)
WRITE (6,440) (HB SF(I,J),I=1,NTHN,2),J=1,NRN,2)
WRITE (6,460) (RH BSF(I,J),I=1,NTHN,2),J=1,NRN,2)
WRITE (6,480) (RANM(I,J),I=1,NTHN,2),J=1,NRN,2)
C
NEXT LINES USED IF PLOT WAS REQUESTED BY SETTING PLOT > 0
IF (PLOT.NE.0) GO TO 140
STOP
C
140 CC 160 I=1,NTHN
C
CC 160 K=1,NRN
Y(I-1)*NRN+K) = RANM(I,K)*DSIN(TH ETA(I)*PI/180.0)
X(I-1)*NRN+K) = RANM(I,K)*DCOS(TH ETA(I)*PI/180.0)
Z(I-1)*NRN+K) = R*BSF(I,K)
160 CONTINUE
C
N = NR*NTHA
C
DC 180 I=1,NTHA
IB(I) = (I-1)*NR+1
180 CONTINUE
C
M = NRN-1
C
DC 200 I=1,M
IB(NTHN+I) = (NTHN-1)*NRN+1+1
200 CONTINUE
C
M1 = NTHN-1
C
DC 220 I=1,M1
IB(NTHN+NRN-1+1) = (NTHN-1)*NRN
220 CONTINUE
C
M2 = NRN-2
C
DC 240 I=1,M2
IB(2*NTHN+NRN-2+1) = NRN-1
240 CONTINUE
C
NE = 1
NE(1) = 2*NTHN+2*NRN-4
NC = 20
C
DC 260 I=1,NC
C(I) = -1.0+I*.5
260 CONTINUE
C
CF = .5
XS = -5.0
YS = -5.0
CALL CONISD (X,Y,Z,N,IB,NR,NB,C,NC,CF,XS,YS,TL,XAX,YAX,NA,IER)
WRITE (6,520) IER
GC TO 20
280 CONTINUE
STCP
C
300 FORMAT (/,' INPUT SCA,EXT,E, USING 3F6.3,/,)
320 FORMAT (3F6.3)
340 FORMAT (/,' INPUT TH,Th,THI,NTHN,F,RI,NRN,NACC,PLOT USING 2(3F6.3,13
11,13,13)/)
360 FORMAT (2(2F6.3,13),13,13)
380 FORMAT (/,'* EFFECTIVE ATTENUATION COEFFICIENTS ARE ',/)
400 FORMAT (2F12.3)
420 FORMAT (/,'* R FLUX IS ',/)
440 FORMAT (10E12.3)
460 FORMAT (/,'* HBSF IS ',/)
480 FORMAT (/,'* NEG LCG OF HBSF IS ',/)
500 FORMAT (10F12.3)
520 FORMAT (/,'* ERROR IER RETURNED=',I5,/)
      END
      ECUBLE PRECISION FUNCTION FCT(X)
      IMPLICIT REAL*8(A-H,C-Z)
      COMMON THTA,G
      D1 = 1.0+CTAN(THTA)**2/(1.0-X)**2
      D2 = DSQRT(1.0+X**2-2.0*G/1EN1)
      D1 = (1.0-G**2)/2.0*G
      D2 = (1.0/1.0-G)-1.0/DEN2
      FCT = CM1*DP2
      RETURN
      END
      DOUBLE PRECISION FUNCTION FCC(X)
      IMPLICIT REAL*8(A-H,C-Z)
      COMMON THTA,G
      DATA PI/3.1415926536D0/
      FCC = ((1.0-G**2)/(4.0*PI*(1.0+G**2-2.0*G*DCOS(X))**1.5))*DCOS(X)
      RETURN
      END
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