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THE GROWTH AND PREDICTION OF NOCTURNAL INVERSIONS. (U)

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THE GROWTH AND PREDICTION OF NOCTURNAL INVERSIONS

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By

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The growth of nocturnal inversions is investigated by utilizing concepts originally proposed by Taylor, with respect to experimental data extracted from the literature. Generally, it was found that the Taylor formulation adequately describes the growth of nocturnal inversion with time in the form $z_1 = 2 (k_t t)^{1/2}$, where $k_t$ is the eddy viscosity and $t$ is time in minutes, after inversion onset.

This investigative effort is applicable to the dispersion and transport of smoke screening and obscurant aerosols and particulates released in nocturnal conditions.
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The formation of the nocturnal inversion was probably first examined formally by Taylor [1]. His investigation was based upon the eddy transfer of heat with the eddy conductivity $K_H$ independent of height, which can be stated as

$$\frac{\partial T}{\partial t} = K_H \frac{\partial^2 T}{\partial z^2} \quad (1)$$

where $T$ is temperature, $t$ time, and $z$ height above the surface. For the case of a nocturnal inversion, where the rate of temperature change at the surface can be considered to be fairly uniform, Taylor suggested that the temperature diminishes $n$-degrees per unit time so that at time $t$ the temperature, $T$, is $T_O - nT$ where $T_O$ is the temperature at $t_0$.

At time $t$, the temperature at $z$ will be

$$T = T_O - \delta z - nt \left[ \left( 1 + \frac{z^2}{2K_H t} \right) \left( 1 - \frac{2}{\sqrt{\pi}} \int_0^{z/\sqrt{4K_H t}} e^{-\mu^2} \, d\mu \right) - \frac{2}{\sqrt{\pi}} \frac{z}{\sqrt{4K_H t}} e^{-z^2/4K_H t} \right] \quad (2)$$

where $\delta$ is a constant lapse rate, and $\mu$ the kinematic viscosity. Taylor found that the term multiplying $nt$ was unity at the surface and 0.1 at $z(4K_H t)^{1/2} = 0.8$ and suggested that no effect existed beyond $z(4K_H t)^{1/2} = 1$, so that surface changes of temperature over time $t$ extend to a height defined by

$$z_I^2 = 4K_H t \quad (3)$$

where $z_I$ is the top of the inversion.

Sutton [2], using a multilayer approach, but similar reasoning, verifies the approximation of Eq. (3) for the growth of nocturnal inversions with time where $K_H$ is a maximum at some height $h$ above the surface.
Estimates of the depth of nocturnal inversions are of some importance to transport and dispersion of smoke/obscuration materials released into the atmosphere during stable conditions. Crosswind integrated concentrations, source strengths, and downwind diffusion of materials released into the atmosphere are highly dependent upon stability and inversion depths. In turn, these parameters determine munition expenditures and obscuration.

BASIC CONSIDERATIONS FOR THE SURFACE BOUNDARY LAYER

The lower portion of the atmosphere may be considered to consist of two layers, the surface boundary layer \( z_0 < z < h \) and the planetary layer proper extending to the gradient wind level \( h < z < Z \), where \( z_0 \) is the roughness length, \( h \) the top of the surface layer, and \( z \) the gradient wind level. It may also be assumed that the exchange coefficients for heat and momentum reach a maximum at \( h \), decreasing again to a residual value at \( z \).

In a thermally stratified stable regime, vertical fluxes of heat and momentum can be considered to be functions of mechanical turbulence alone, which suggests that \( K_H = K_M \), where \( K_M \) is the eddy viscosity, with the dimensionless parameters of the Obukhov [3] similarity theory given by

\[
\frac{Z}{L} = Ri \Phi_M; \quad \Phi_H = \Phi_M; \text{ and } R_f = \frac{K_H}{K_M}
\]

where \( L \) is the Obukhov scaling length, \( Ri \) and \( R_f \) the gradient and flux Richardson numbers, respectively, with \( \Phi_M \) a dimensionless wind shear and \( \Phi_H \) a dimensionless lapse rate. Hansen [4] has demonstrated that for stable flow the wind profile may be written as

\[
V = \frac{u_*}{k} \left[ \ln \frac{Z}{Z_0} + (\Phi_M - 1) \right]
\]
where $\bar{V}$ is the mean horizontal windspeed, $u_*$ a friction velocity, and $k$ Karman's constant. Furthermore, Hansen [4] also found that

$$\frac{Z}{L} = R_1 + 15 R_1^2$$

(5)

$$\phi_M = 1 + \frac{Z}{L} = 1 + 15 R_1$$

(6)

where $\bar{\beta}$ is the average over a height interval of a variable given by

$$\bar{\beta} = \frac{Z/L - R_1}{R_1}$$

(7)

The mean reduces to

$$\bar{\beta} = \frac{15}{1 + 15 R_1}$$

(8)

If the wind profile in finite difference form is written for stable flow as

$$\frac{\Delta V}{h \Delta \ln h} = \frac{u_*}{k} R_1^{-1} L^{-1}$$

(9)

where $z = h = L R_1 \phi_M$, then

$$h = L \frac{k}{u_*} R_1 \frac{\Delta V}{\Delta \ln z}$$

(10)

If $\Delta V \Delta \ln^{-1} z$, the critical gradient at the geometric mean height $h$, is assumed to occur over the layer $\Delta \ln z = \ln e = 1$, then $R_i \Delta V = \bar{\beta}^{-1}$, and

$$h = mL \bar{\beta}^{-1} = mL \frac{\phi_M}{15}$$

(11)

where $m$ is the profile slope $k/u_*$. If $h/L = R_i \phi_M$, then from Eq. (11)

$$R_i(h) = \frac{m}{15}$$

(12)
and from Eq. (5)

\[ h = L \left( R_i(h) + 15 R_i(h)^2 \right). \]  \hspace{1cm} (13)

The height \( h \) is taken to be the depth of the surface boundary layer in a stable flow regime and is the height where \( K_H \) or \( K_M \) maximum is evaluated for inversion depth estimates using Eq. (3). The eddy viscosity at \( h \) is given by

\[ K_M(h) = \frac{ku_z z}{\Phi_M(h)}. \]  \hspace{1cm} (14)

Lumley and Panofsky [5] suggest that the depth of the surface layer can be estimated from

\[ h = 20 \tau_0 \]  \hspace{1cm} (15)

where \( \tau_0 \) is the surface stress and assumed to be the equivalent of the Reynolds shearing stress, \( \tau = \rho u_*^2 \), where \( \rho \) is density. Calculations based upon stable regime data extracted from Lettau and Davidson [6] and Barad [7], as summarized in Figure 1, indicate that Eq. (15) provides estimates of \( h \) comparable with Eq. (13).

ESTIMATES OF INVERSION HEIGHTS

The scheme for estimating inversion heights was evaluated by using experimental data extracted from Lettau and Davidson [6]; Barad [7]; Bowne, Entrekin, and Smith [8]; and Stenmark and Drury [9]. Micrometeorological profile data from Lettau and Davidson, Stenmark and Drury, and Barad were utilized to calculate \( h \), \( K_M \) and \( z_I \) for the thermally stratified stable regime. Rawinsonde, aircraft soundings and wiresonde data summaries from all four data samples were used to obtain indicated inversion heights.
According to Milly [10], inversion conditions can be assumed to exist from approximately 60 minutes before sundown to about 60 minutes after sunrise. For those data samples where sunrise and sunset were not listed, these times were estimated from information in The American Ephemeris and Nautical Almanac, issued by the Nautical Almanac Office, US Navy Observatory, Washington, DC.

The calculated and observed values for \( z_1 \) from the 217 micrometeorological profiles and the 305 soundings used in this study were averaged by using two methods: (1) hourly, and (2) by means of a geometric progression where the overlapping time intervals serve to filter and smooth the data. The hourly averages of \( z_1 \) (observed) and \( z_1 \) (calculated) are shown in Figure 2 and the overlapping time averaged analysis in Figure 3. Note that the calculated values of \( z_1 \) overpredict inversion heights soon after sunset and underpredict in the early morning hours. No significant difference is apparent between the two averaging methods.

To simplify the prediction of inversion growth and height with time, a semiempirical formula based upon Eq. (3) where the value of \( K_M(h) \) is estimated from \( \Theta_M \) and \( V \) was developed and takes the form

\[
z_I = 7.75 \Theta_M^{-1} (V z t)^{1/3}
\]

(16)

where \( z \) is 2 m, \( V \) is windspeed in m sec\(^{-1}\) and \( t = \) minutes. Figure 4 shows values of \( z_I \) calculated by using Eq. (16) with respect to the observed average inversion heights with respect to time.
CONCLUSIONS

The results shown in Figures 2 through 4 are in good agreement with a study reported on by the US Weather Bureau [11] as summarized by Wanta [12]. It should be pointed out that the conditions indicated in Figures 2 through 4 are averages based upon four distinctly separate data samples observed in widely separated geographical locations over dissimilar terrain. The variations in observed or calculated inversion heights, as indicated by the one standard deviation bars about the meaned values of the figures, represent the variations that could occur in inversion heights, even in a single nocturnal period. This can be verified by considering the remotely sensed data of Hall [13] and the summary of the status of remote sensing prepared by Little [5]. The fluctuations in the inversion height with time can be easily recognized in Little's Figures 10 and 11.

One difficulty encountered while operating upon the profile data using Eqs. (3) and (16) was the low windspeed cases. Speeds less than 2 to 3 m sec\(^{-1}\) at 2 meters height tended to underestimate inversion heights badly, especially 6 to 8 hours into the nocturnal regime. Thus, the use of the prediction estimators is not recommended during periods of low windspeeds. Generally though, Eqs. (3) and (16) do a reasonably accurate job of estimating inversion heights as long as the atmosphere is stable and the 2-meter windspeed is greater than 3 m sec\(^{-1}\).

The results presented are applicable to US Army problems associated with the dispersion and transport of smoke/aerosols utilized for screening and obscuration.
Figure 3. Overlapping time averaged analysis of observed and calculated inversion depths.
FIGURE 4. INVERSION DEPTHS BASED UPON SEMIEMPIRICAL PREDICTION FORMULA.
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