LEVEL

INDEPENDENT STABILITY AND CONTROL ANALYSIS
OF NAVIGATION DEVELOPMENT SATELLITES
FOR THE GLOBAL POSITIONING SYSTEM

FINAL REPORT

VOLUME II: APPENDIXES

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This report documents the results of an independent stability and control analysis of the control systems and procedures of the navigation development satellite (NDS) for the global positioning system (GPS). The analysis consisted of detailed analytical investigation substantiated with sophisticated satellite system modeling and simulation of the following GPS mission events:

(continued on reverse)
20. ABSTRACT (concluded)

- Spin phase
- Despin
- Earth acquisition
- Panel deployment
- Sun acquisition
- On-orbit operations

The results of the analysis indicated satisfactory performance in all mission phases analyzed. A redesign of the jet control system was recommended to facilitate earth acquisition operations.
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APPENDIX A

DATA BASELINE DEFINITION, 15 JULY 1977
# APPENDIX A

## DATA BASELINE DEFINITION, 15 JULY 1977

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<thead>
<tr>
<th>DATA ITEM</th>
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<th>DATE</th>
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<tbody>
<tr>
<td>2. Jet Control System</td>
<td>Rockwell Block Diagram MC476-0146, Rev. F, Seq. 4</td>
<td>Received 29 March 1977</td>
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<tr>
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<td>Rockwell Block Diagram MC476-0146, Rev. F, Seq. 4</td>
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<tr>
<td>a. Thermal Variation</td>
<td>(See Solar Array Drive)</td>
<td>± 0.6 percent</td>
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<tr>
<td>6. Thruster Locations</td>
<td>Scaled from Rockwell Drawing V505-900003, Rev. C</td>
<td>27 January 1975</td>
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<tr>
<td>7. Thruster Characteristics</td>
<td>GPS 0.1 lbf REA, Qualification Test Report CDRL, Item A065, Contract F04701-74-C-0527</td>
<td>31 March 1976</td>
</tr>
<tr>
<td>a. 0.1 lb</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. 5.0 lb</td>
<td>GPS 5.0 lbf REA, Qualification Test Report CDRL, Item A065, Contract F040701-74-C-0527</td>
<td>31 March 1976</td>
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<td>7. c. RCS Tank Blowdown Profile, 5.0 lb and 0.1 lb Thruster Performance</td>
<td>GPS Propellant Budget</td>
<td>Received from SAMSO, June 1977</td>
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<tr>
<td>8. Wheel Dynamics</td>
<td>Motor Torque Speed, Curve Plot No. 5, Design Review Data, Item R-005 Bendix to Rockwell</td>
<td>14 January 1975</td>
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<tr>
<td>a. Pulse Width Normal 50 ± 5 milliseconds</td>
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<tr>
<td>b. Pulse Width Extended 85 ± 8.5 milliseconds</td>
<td></td>
<td></td>
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<tr>
<td>c. Slew Command 64 pulses</td>
<td></td>
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<tr>
<td>d. Expected Slew Normal 11.85° ± 6°</td>
<td></td>
<td></td>
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<tr>
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<tr>
<td>f. Max Pulse Rate 1 to 1.85</td>
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<tr>
<td>g. Deadband, 1 to 1.7°</td>
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<tr>
<td>h. Driving Torque 145 oz-in min</td>
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<td></td>
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<tr>
<td>i. Coulomb Friction 65 oz-in max</td>
<td></td>
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<tr>
<td>j. Solar Array Drive Torques</td>
<td>Ball Brothers Memo</td>
<td>5 February 1976</td>
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<td>k. Minimum Pulse Rate</td>
<td>Phil Talley Memo</td>
<td>22 October 1975</td>
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<td>10. Nutation Damper</td>
<td>Blue Print Damper Assembly</td>
<td>15 January 1976</td>
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<tr>
<td>a. Radius of Curvature</td>
<td>Nutation GPS Rockwell Drawing No. V505-520016</td>
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<tr>
<td>158.25&quot; + 10&quot; to 18&quot;</td>
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<td></td>
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<tr>
<td>b. Length of tube 15.75&quot;</td>
<td>Rockwell Viewgraph</td>
<td>CDR</td>
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<td>c. Mass of Ball .018 slugs</td>
<td>Honeywell Memo</td>
<td>29 September 1975</td>
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<tr>
<td>d. Dissipation Coefficient</td>
<td></td>
<td></td>
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<tr>
<td>$c_D = 0.017 \pm 30%$</td>
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<td>11. Pitch Sun Sensor</td>
<td>Ball Brothers Test Data</td>
<td>Received</td>
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<td>Ball Brothers Memo</td>
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<td>15 July 1977</td>
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<td>12. Alignment Specs</td>
<td>Global Positioning System Orbital Operations Handbook, Volume 1, Section 2.4: Control System, Description CDRL, Item A00G, Contract F04701-74-C-0527, P2.4.138</td>
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<tr>
<td>a. CES</td>
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<td>b. SSS</td>
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<td>c. YSS</td>
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<td>d. Reaction Wheel</td>
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<tr>
<td>c. Thrusters</td>
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<tr>
<td>13. Precession Procedure</td>
<td>See Item 12</td>
<td></td>
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<tr>
<td>a. Firing Pulse Width</td>
<td></td>
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<tr>
<td>93.6 msec</td>
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<td>b. Expected Precession Angle/Pulse, 0.12&quot;</td>
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<td>c. Thrusters Used</td>
<td></td>
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<tr>
<td>17 and 18</td>
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<td>d. Firing Delay, 0 to 5.24 sec (increments of 10.25 msec)</td>
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<td>a. Thrusters Used-- 11 and 13</td>
<td>Modified from Item 12</td>
<td>SAMSO Review 15 July 1977</td>
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<td>b. Sequence</td>
<td>95 to 85 rpm</td>
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<tr>
<td>Calibrate</td>
<td>85 to 75 rpm</td>
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<tr>
<td>Recalibrate</td>
<td>75 to 20 rpm</td>
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<td>20 to 10 rpm</td>
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<td>10 to 6 rpm</td>
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<td>a. Enable roll axis control</td>
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<td>b. Wait 5.0 minutes</td>
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<td>16. -y Panel Deployment Procedure with roll and pitch jet control, deploy -y panel, slew 90 deg</td>
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<td>a. Automatic dump of pitch and roll axis when momentum is greater than 0.25 ft-lb-sec</td>
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<td>c. Roll axis firing times 0.8, 2.4, 4.0, or 5.6 sec (2.4 nominal)</td>
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APPENDIX B

ONE-BODY MODEL EQUATIONS
APPENDIX B

ONE-BODY MODEL EQUATIONS

This appendix presents the derivation of equations of motion used in the one body simulation. For Definition of Symbols, see Figure B-1.

Vector torque equations are given by

\[ \dot{r} \times F = \int \frac{d\tau}{dt} \, (dr) \, dm \]

\[ \dot{\tau} = \frac{d\hat{H}}{dt} \]

where \( \hat{H} \) is defined as

\[ \hat{H} = \int \frac{\dot{\tau} \times (dr)}{dt} \, dm \]

\[ = \left[ \bar{J}_o + m_1 M(\bar{r}_o, \bar{r}_o) + \sum_i \bar{J}_i + \sum_i m_i M(\bar{r}_i, \bar{r}_i) \right] \bar{\omega}_o + \sum_i \bar{J}_i \bar{\omega}_i \]

where, in body 0 coordinates,

\[ M(a,b) = a^T b I - a b^T \]

\[ J \triangleq \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{xy} & I_y & -I_{xy} \\ -I_{xz} & -I_{xy} & I_z \end{bmatrix} \]

\[ J_i = T_{oi}^T I_i T_{oi} \] (\( T_{oi} \) is transformation from wheel i to body o)
Body CM's \( \text{CM}_0, \text{CM}_i \)  
Total CM \( \text{CM}_1 \)  
Masses \( m_0, m_i \)  
Inertia Tensors \( \bar{J}_0, \bar{J}_i \) (\( J_0, J_i \) are inertia tensors in reference body coordinates of inertias about \( \text{CM}_0 \) and \( \text{CM}_i \) respectively)

Figure B-1. Notation
\[ \omega_i = \frac{T_i \omega_i}{I_{oi}} \]

\[ I_x = \int (y^2 + z^2) \, dm = I_{yy} + I_{zz}, \text{ etc.} \]

\[ I_{xy} = \int xy \, dm, \text{ etc.} \]

The rotational equations of motion are then given by

\[ \ddot{\omega}_i = \frac{d}{dt} \left[ \omega_i \right], \text{ etc.} \]

\[ T = I_{o} \omega_o + \sum_{i} I_{i} \omega_i + \omega_0 x H \]

Wheel torques may be described by

\[ \int \bar{x}_i x dF = \int \bar{x}_i x \frac{d^2}{dt^2} \bar{x}_i \, dm \]

\[ = \frac{d}{dt} \int \bar{x}_i x \frac{dx_i}{dt} \, dm \]

\[ T_i = \frac{d\bar{H}_i}{dt} \]

Wheel momentum is given by

\[ H_i = \int \bar{x}_i x \frac{d}{dt} \bar{x}_i \, dm \]

\[ = \int \bar{x}_i x \left[ \frac{d}{dt} \bar{x}_i + \omega_0 \times \bar{x}_i \right] \, dm \]
Differentiating

\[ T_i = J_i \dot{\omega}_i + J_i \dot{\omega}_o + \omega \times J_i \omega_i + \omega \times J_i \omega_o \]  

(B-5)

Torques about the wheel axis are given by

\[ T_i = J_i \dot{\omega}_i + J_i \dot{\omega}_o \]  

(B-6)

\[ - J_i \dot{\omega}_i \]  

(B-7)

where \( T_i \) is the sum of motor, bearing and windage torques on wheel \( i \).

Equations B-7 and B-3 are combined to compute the body rate derivatives.

\[ T = I \dot{\omega} + \sum_{i} T_i + \omega \times H \]  

(B-8)

**Kinematics**

The inertial coordinate system and the Euler sequence are consistent with the Rockwell convention presented in the AVCS splinter meeting notes at the preliminary design review. Specifically, Rockwell specified a yaw pitch roll Euler sequence with the equations:

\[ \dot{\theta} = \omega \cos \psi + q \cos \phi - r \sin \phi \]

\[ \dot{\psi} = (\omega \sin \psi \sin \theta + q \sin \phi + r \cos \phi) \left(\frac{1}{\cos \theta}\right) \]

\[ \dot{\phi} = p + \dot{\psi} \sin \theta + \omega \sin \psi \cos \theta \]
where

\[ \omega = \text{orbit rate} \]
\[ p, q, r = \text{body rates} \]
\[ \phi, \theta, \psi = \text{Euler angles} \]
\[ \dot{\phi}, \dot{\theta}, \dot{\psi} = \text{Euler rates} \]

These equations are consistent with:

1. A local vertical system with \( z \) axis pointing toward the earth and \( x \) axis in the orbit plane and positive on the direction of motion.
2. Euler angles specifying body orientation relative to the local vertical frame.

The inertial coordinate system is the Geocentric Equatorial Frame (e.g., \( z \) axis positive North and \( x \) axis positive in the direction of the Vernal Equinox). The local vertical system is consistent with Rockwell's (e.g., \( z \) axis positive toward the earth and \( x \) axis in the orbit plane).

Two sets of Euler angles, both having a yaw pitch roll sequence, are used. The first set \( \Psi, \Theta, \Phi \) define the inertial to body transformation \( E \). The second set, \( \Psi_0, \Theta, \Phi \) specify the orientation of the body relative to the local vertical frame and should be consistent with Rockwell's Euler angles. The inertial Euler angles are integrated in the simulation via quaternions.

**Linear Motion**

The simulation uses two subroutines (POSITN and RADVEC) to compute the ephemerides for the satellite and the sun. The ephemerides for the satellite and the sun are calculated using the six Kepler elements: semi major axis \( a \),
eccentricity (e), inclination (i), right ascension of the ascending node (Ω), argument of perigee (ω), and mean anomaly (M). The update is a simple rate term on the latter three. Table B-1 presents the six elements, their units and values for the vernal equinox condition. The rate terms are derived in the program from the fixed parameters and are given by

\[
\dot{M} = k \frac{\sqrt{\mu/a^3}}{[a(1-e^2)]^2} \sqrt{1-e^2} (1.5 \sin^2 i-1)
\]

\[
\dot{\Omega} = k \frac{\sqrt{\mu/a^3}}{[a(1-e^2)]^2} \cos i
\]

\[
\dot{\omega} = k \frac{\sqrt{\mu/a^3}}{[a(1-e^2)]^2} (2.5 \sin^2 i-2)
\]

where

\[\mu = \mu_m = 62630.3949\]

\[k = R^2 E J_2 \text{ (first term of perturbing force = -19255.96124 due to Earth's oblateness)}\]

The rate terms for the sun are fixed and are included in Table B-1. The simulation updates M, Ω, ω by

\[
M(t) = M(t_0) + \dot{M} \cdot (t-t_0) \quad -\pi < M < \pi
\]

\[
\Omega(t) = \Omega(t_0) + \dot{\Omega} \cdot (t-t_0) \quad -\pi < \Omega \leq \pi
\]

\[
\omega(t) = \omega(t_0) + \dot{\omega} \cdot (t-t_0) \quad -\pi < \omega \leq \pi
\]

A seventh term, the Greenwich Hour Angle (GHA), which relates the earth-fixed coordinate system to the inertial system, given the six elements is updated via:

\[
\text{GHA}(t) = \text{GHA}(t_0) + \omega_e \cdot (t-t_0)
\]

\[\omega_e = \text{earth rotation rate}\]
### TABLE B-1. GPS AND SOLAR EPHEMERIDES

<table>
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<td></td>
<td></td>
<td>GPS</td>
<td>SUN</td>
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<tr>
<td>a</td>
<td>Semi Major Axis</td>
<td>Miles</td>
<td>14342.</td>
</tr>
<tr>
<td>e</td>
<td>Eccentricity</td>
<td>--</td>
<td>0.</td>
</tr>
<tr>
<td>i</td>
<td>Inclination</td>
<td>Degrees</td>
<td>63.</td>
</tr>
<tr>
<td>M</td>
<td>Mean Anomaly</td>
<td>Degrees</td>
<td>0.</td>
</tr>
<tr>
<td>Ω</td>
<td>Right ascension</td>
<td>Degrees</td>
<td>0.</td>
</tr>
<tr>
<td></td>
<td>of ascending node</td>
<td></td>
<td></td>
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<tr>
<td>ω</td>
<td>Argument of perigee</td>
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<td>˙M</td>
<td>Degrees/sec</td>
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<td>1.14074x10^{-5}</td>
</tr>
<tr>
<td>˙Ω</td>
<td>Degrees/sec</td>
<td>--</td>
<td>0.544773x10^{-9}</td>
</tr>
<tr>
<td>˙ω</td>
<td>Degrees/sec</td>
<td>--</td>
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<tr>
<td>GHA</td>
<td>Greenwich Hour Angle</td>
<td>Degrees</td>
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* Astronomical unit
APPENDIX C

THREE-BODY DYNAMIC MODEL
APPENDIX C

THREE-BODY DYNAMIC MODEL

This appendix contains the equations of motion for the GPS three-body model.

ROTATIONAL DYNAMICS

Let \( \vec{\omega} = \begin{pmatrix} p \\ q \\ r \end{pmatrix} \) angular velocity of reference body in reference body coordinates

Notation: \( \vec{x} = \) vector or tensor
\( \vec{x} \) components of \( \vec{x} \) in reference body coordinates
\( \vec{x}_i \) components of \( \vec{x} \) in body \( i \) coordinates
\( \frac{d\vec{x}}{dt} = \) derivative of \( \vec{x} \) with respect to inertial frame
\( \frac{\vec{x}}{dt} = \) derivative of \( \vec{x} \) with respect to reference body frame
\( \vec{x} = \) componentwise differentiation of \( x \). Note that \( (\frac{\vec{x}}{dt})^0 = \dot{x} \).

(See Figure C-1.)

MOMENT OF NEWTON'S LAW ABOUT CM

\[
\int \vec{r} \times d\vec{F} = \int \vec{r} \times \frac{d^2 \vec{r}}{dt^2} \, dm = \frac{d}{dt} \int \vec{r} \times \frac{d\vec{r}}{dt} \, dm
\]

\[
\vec{T} = \frac{d\vec{H}}{dt} \quad \text{(C-1)}
\]

EVALUATION OF \( \vec{H} \)

\[
\vec{H} = \int \vec{r} \times \left( \frac{d\vec{r}}{dt} \right) \, dm
\]

\[
= \int \left( \vec{r}_0 + \vec{x}_0 \right) \times \left( \frac{\delta \vec{r}_0}{\delta t} + \vec{\omega}_0 \times (\vec{r}_0 + \vec{x}_0) \right) \, dm
\]

\[
+ \sum_{i=1}^{n} \left( \vec{r}_0 + \vec{r}_i \right) \times \left( \frac{\delta \vec{r}_i}{\delta t} + \vec{\omega}_i \times (\vec{r}_0 + \vec{r}_i + \vec{x}_i) \right) + \vec{\omega}_i \times \vec{x}_i \right) \, dm
\]
Figure C-1. Notation
Noting that
\[ m \vec{r}_o + \sum_i m_i (\vec{r}_i + \vec{r}_o) + \sum_j m_j (\vec{r}_j + \vec{r}_o) = 0 \]  
(C-3)

Equation (C-2) becomes
\[ \vec{H} = \vec{I}_o \omega_o + \sum_i \vec{I}_i \omega_i + \sum_j \vec{J}_j \omega_j + \sum \vec{H}_n \]  
(C-4)

where
\[ \begin{align*}
I_o &= J_o + \sum_i J_i + \sum_j J_j - mM(r_o, r_o) + \sum_i M(r_i, r_i) \\
&\quad + \sum_j M(r_j + \vec{r}_j, r_j + \vec{r}_j) + \sum_j M(r_j + \vec{r}_j, r_j + \vec{r}_j) \\
I_j &= J_j + m_j M(r_j + \vec{r}_j, r_j + \vec{r}_j) \\
\vec{H}_n &= m \left( \vec{r}_o + \vec{r}_n + \vec{r}_n \right) \times \left( 5\vec{r}_n / \ell_n \right) + \int \left( \vec{r}_o + \vec{r}_n + \vec{r}_n + \vec{r}_n \right) \times \left( 5\vec{r}_n / \ell_n \right) \, dm \\
\end{align*} \]

and
\[ M(a, b) \equiv b^T aI - ba^T \]

**COMPUTATION OF H**

The vectors \( \vec{r}_i, \vec{r}_j, \vec{r}_n, \vec{r}_p \) are given. The vector \( \vec{r}_o \) can be computed from Equation (C-3).

**Wheel Momenta**
\[ I_{j_1 \omega_1} = T_{1j} J_{1j} \omega_1 \]

**Panel Momenta**
\[ I_{j_1 \omega_2} = T_{1j} J_{1j} \omega_2 + m_j M(r_j + r_j + \vec{r}_j, \vec{r}_j) \omega_2 \]
where
\[ \omega_j = T_{ij} \left\{ \begin{array}{c} 0 \\ \dot{\omega}_j \\ 0 \end{array} \right\} = T_{ij} \omega_j^i \]

Nutation Dampers

Assume: \( J_n = 0 \) (point mass)
\[ \gamma H_n = m_n (\ddot{\gamma}_{\theta} + \ddot{\gamma}_{\rho} + \ddot{\gamma}_{n}) \times \frac{\delta_{\theta}^n}{\delta t} \]

ROTATIONAL DIFFERENTIAL EQUATION IN SATELLITE COORDINATES

\[
\ddot{T} = \frac{d\ddot{H}}{dt} = \frac{5\ddot{H}}{5t} + \frac{\ddot{\omega}_o \times H}{\delta t} \\
T = I_{ij} \dot{\omega}_o + I_{ij} \dot{\omega}_o + \sum I_{ij} \dot{\omega}_i + \sum I_{ij} \dot{\omega}_j + \sum I_{ij} \dot{\omega}_n + \sum \ddot{H}_n \\
\]

EVALUATION OF THE DERIVATIVES

Inertia Tensors

\[
\ddot{\omega}_j = m_{ij} \left[ M(r_{ij} + \rho_{ij}, \dot{\rho}_{ij}) + M(r_{ij} + \rho_{ij}, \dot{\rho}_{ij}) \right] \\
+ \sum_{n} \left[ M(r_{ij} + \rho_{ij}, \dot{\rho}_{ij}) + M(r_{ij} + \rho_{ij}, \dot{\rho}_{ij}) \right] \\
\ddot{\omega}_j = \dot{J}_j + m_{ij} \left[ M(r_{ij} + \rho_{ij}, \dot{\rho}_{ij}) + m_{ij} M(r_{ij} + \rho_{ij}, \dot{\rho}_{ij}) \right] \\
\]

where
\[
\dot{\rho}_j = \omega_j \times \rho_j \\
\ddot{\omega}_j = \omega_j \times \omega_j \\
\ddot{\omega}_j = \frac{1}{m} \left[ \sum_{j} \dot{\omega}_j + \sum_{n} \dot{\rho}_{ij} \right] \\
\]

WHEEL TORQUES

\[
\int_i \dddot{\omega}_j = \frac{d}{dt} \dddot{\omega}_j = \frac{d^2}{dt^2} \dddot{\omega}_j \\
= T_{ij} = J_{i} \dot{\omega}_i + J_{i} \dot{\omega}_i + \omega_o x [J_{i} (\omega_o + \omega_i)] \\
\]

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The magnitude of the wheel torques about the axis is given by

\[ T_{ia} = h_i \cdot T_i \]

where \( h_i \) is a unit vector along the spin axis. For example,

\[ h_i = T_{oi} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \]

**PANEL TORQUES**

\[ \int (\tilde{\rho}_j + \tilde{x}_j) \times d\tilde{F} = \int (\tilde{\rho}_j + \tilde{x}_j) \times \frac{d^2}{dt^2} (\tilde{\rho}_j + \tilde{x}_j + \tilde{r}_j \tilde{p}_j) \, dm \]

\[ \bar{T}_{ij} = m_j \tilde{\rho}_j \times \frac{d^2 \rho_i}{dt^2} + d\bar{H}_j \]

where

\[ \bar{H}_j = [\bar{J}_j + m_j M(\tilde{\rho}_j, \tilde{\rho}_j)] (\tilde{\omega}_j + \tilde{\omega}_o) \]

\[ T_j = m_j \tilde{\rho}_j \times \frac{d^2 \rho_i}{dt^2} + [\bar{J}_j + m_j [M(\tilde{\rho}_j, \tilde{\rho}_j) + M^T(\tilde{\rho}_j, \tilde{\rho}_j)] (\tilde{\omega}_j + \tilde{\omega}_o) \]

\[ + [J_j + m_j M(\tilde{\rho}_j, \tilde{\rho}_j)] (\tilde{\omega}_j + \tilde{\omega}_o) + \omega_o \times H_j \]

\[ T_{ja} = h_j \cdot T_j \]

where

\[ h_j = \text{unit vector along panel hinge} \]

\[ = T_{oj} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \]

and

\[ \frac{d^2 \rho_i}{dt^2} = \rho_j \times a_{cm} + \rho_j \times \left[ \frac{\delta r_o}{\delta t^2} + \omega_o \times \frac{\delta r_o}{\delta t} \right] + [M(\tilde{\rho}_j, r_o) + M(\rho_j, \tilde{r}_j)] \omega_o \]

\[ + M(\rho_j, r_o + r_j) \omega_o + \omega_o \times M(\rho_j, r_o + r_j) \omega_o \]

\[ \frac{\delta^2 r_o}{\delta t^2} = -\frac{1}{m} \left[ \Sigma \frac{m}{j} p_j \right] + \Sigma \frac{m}{n} \left( \right] \]
\[ \dot{\hat{n}}_j = \omega_j \times \hat{\rho}_j + \omega_j \times \dot{\hat{\rho}}_j \]

**Nutation Damper Forces**

\[ \Sigma F = \frac{d^2}{dt^2}(r_{cm} + r_o + r_n + \rho_n) \]

\[ = a_{cm} + \frac{\delta r_o}{2} + \frac{\delta \rho_n}{2} + \omega_x (r_o + r_n + \rho_n) \]

\[ + \omega \times (\dot{r}_o + \dot{\rho}_n) \]

\[ + \omega \times \left[ \frac{\delta r_o}{2} + \frac{\delta \rho_n}{2} + \omega \times (r_o + r_n + \rho_n) \right] \]

\[ \Sigma F \cdot h_n = -k_{1n} \dot{\rho}_n - k_{2n} \ddot{\rho}_n \]

where

\[ \rho_n = \hat{\rho}_n \cdot h_n \]

and

\[ h_n = \text{unit vector along } \rho_n. \]
APPENDIX D

NUTATION DAMPER MODEL

This appendix summarizes the nutation damper model implemented for the digital simulation evaluation of GPS stability and control analysis.

The nutation damper is used for passive GPS spin axis stabilization during the initial mission phases by dissipating the spacecraft precession energy. The nutation damper adopted for GPS application is of the type of ball in gas filled tube. The tube is made of aluminum and the ball of tungsten carbide. The gas is a 9:1 ratio mixture of nitrogen and helium at 1 atm pressure. Physical parameters of this damper is shown in Figure D-1.

A similar damper of this type was used in the Telstar satellite, Reference D-1. Discussions on the design and analysis of ball-in-tube type nutation dampers can be found in Reference D-2.

Derivation of the torque equations presented here follows the same procedure of Reference D-2 which contains numerous typographical errors. The results, in the torque form, are compatible for integration into the three body model described in Appendix C.

**DAMPER MODEL DERIVATION**

To facilitate the model derivation, we shall define a vehicle fixed $B$-frame whose origin is at the vehicle's center of mass (excluding damper ball) with the $(X_B - Y_B - Z_B)$ axes aligned with its principal axes. Also in the following consideration of vehicle rotational motions, it is assumed that the translational motion of vehicle's center of mass resulted from reactional
**Type**  Ball in Gas Filled Tube  
**Tube**  Curved to provide Centrifugal Spring

---

**Approximate Half Scale Sketch**

---

**Tube**
- I.D. 1.256 Inch  
- O.D. 1.656 Inch  
- Length 16 Inches  
- \( r = \text{radius of curvature} \) 159 Inches = 12.23 ft.  
- Material: Aluminum

**Ball**
- Diameter 1.25 Inch  
- \( r_D = 0.625 \text{ inches} \)  
- Material: Tungsten Carbide

**Gas**
- 9:1 Mixture Nitrogen: Helium  
- 1 Atmosphere

**Weight**
- 1.75 lbs

---

*Figure D-1. Nutation Damper Characteristics*
forces of the damper ball can be neglected. By reference to the geometrical
description of damper location in B-frame, the position of center of ball,
$R_b$ can be written as, Figure D-2,

$$
\begin{pmatrix}
0 \\
-a + \gamma \cos \alpha \\
b + \gamma \sin \alpha
\end{pmatrix}
$$

\hspace{1cm} (D-1)

where

$\gamma = 159 \text{ in.} = 13.25 \text{ ft.}$

$(a,b) = \text{coordinates of tube center of curvature}$

Equation (1) implies that $Y_B - Z_B$ plane contains the damper tube.

Let the vehicle moment of inertia be such that

$$
\begin{align*}
I_{XX} &= I_{YY} = A \\
I_{ZZ} &= C
\end{align*}
$$

\hspace{1cm} (D-2)

The rotational energy of the vehicle becomes,

$$
T_V = \frac{1}{2} A(W_X^2 + W_Y^2) + \frac{1}{2} CW_Z^2
$$

\hspace{1cm} (D-3)

The velocity of damper ball can be written through differentiation of (1) as,

$$
\begin{pmatrix}
\dot{V}_b^B \\
\dot{W}_b^B
\end{pmatrix} = T_{BI}(\dot{R}_b^B) = T_{BI}(\dot{R}_b^B)
$$

with $(\dot{R}_b^B) = T_{BI}^T(\dot{R}_b^B)$, we have

$$
\begin{pmatrix}
\dot{R}_b^I \\
\dot{R}_b^B
\end{pmatrix} = T_{BI}^T(\dot{R}_b^B) + T_{BI}^T(\dot{R}_b^B)
$$

$$
= -T_{BI}^T S \left[ (\dot{w}_b^B) \right] (\dot{R}_b^B) + T_{BI}^T (\dot{R}_b^B)
$$

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NOTE:  \( Z_B = \text{SPIN AXIS} \)
\( cMv = \text{VEHICLE CENTER OF MASS} \)
\( \text{DAMPER CONTAINED IN PLANE} \ (\gamma_B - Z_B) \)

Figure D-2.  Nutation Damper in Vehicle Body Fixed Frame
or
\[ \mathbf{\ddot{x}_b} = -\mathbf{S} \left( \mathbf{\dot{w}_b} \right) \left( \mathbf{\ddot{R}_b} \right) + \left( \mathbf{\ddot{R}_b} \right) \] (D-4)

In component forms, (4) becomes,
\[
\begin{bmatrix}
V_x \\
V_y \\
V_z
\end{bmatrix}
= \begin{bmatrix}
0 & -W_Z & W_Y \\
W_Z & 0 & -W_X \\
-W_Y & W_X & 0
\end{bmatrix}
\times
\begin{bmatrix}
0 \\
-a + \gamma \cos \alpha \\
b + \gamma \sin \alpha
\end{bmatrix}
+ \begin{bmatrix}
0 \\
-\gamma \sin \alpha \\
\gamma \cos \alpha
\end{bmatrix}
\] (D-5)

Hence the translational kinetic energy of the damper ball is
\[
T_b = \frac{1}{2} M \left( V_x^2 + V_y^2 + V_z^2 \right)
= \frac{1}{2} M \left[ \left( -a + \gamma \cos \alpha \right) W_z + (b + \gamma \sin \alpha) W_y \right]^2
+ \left[ (b + \gamma \sin \alpha) W_x - \gamma \sin \alpha \dot{\omega} \right]^2
+ \left[ (-a + \gamma \cos \alpha) W_x + \gamma \cos \alpha \dot{\omega} \right]^2
\] (D-6)

where \( M = \) mass of the ball
\[
= \frac{4}{3} \pi \gamma_b^3 \rho = 263 \text{ gm} = 0.018 \text{ slug}
\]
\( \rho = \) density of tungsten carbide (WC) = 15.7 gm/cm\(^3\)

Assuming the ball is rolling without slipping in the tube, the magnitude of the angular velocity of the ball is related to the translational velocity of the center of the ball as
\[
\dot{\omega}_b = \frac{1}{\gamma_b} \left[ (\mathbf{\ddot{R}_b})^T (\mathbf{\dot{R}_b}) \right]^{1/2}
= \frac{1}{\gamma_b} \left[ \gamma^2 \sin^2 \alpha \dot{\alpha}^2 + \gamma^2 \cos^2 \alpha \dot{\alpha} \right]^{1/2}
= \frac{\dot{\alpha}}{\gamma_b} \quad \text{where } \gamma_b = \text{radius of the ball} = 0.625 \text{ in.} \] (D-7)
Hence, the rotational energy of the ball is

\[ T = \frac{1}{2} I_b \omega^2_b = \frac{1}{2} \left( \frac{2}{5} M \gamma b^2 \right) \left( \frac{\gamma^2}{\gamma b} \right)^2 \]

\[ = \frac{1}{5} M \gamma^2 a^2 \]  \hspace{1cm} (D-8)

The drag on the ball is viscous, Reference D-3, since the tube is gas filled, the clearance between ball and tube is small and the ball moves much slower than the speed of sound. Hence, the viscous drag on the ball can be represented by Rayleigh's dissipation function as

\[ F = \frac{1}{2} C_D \left( \frac{\gamma^2}{\gamma b} \right) T \left( \frac{\gamma b}{\gamma^2} \right) \]

\[ = \frac{1}{2} C_D \gamma^2 a^2 \]  \hspace{1cm} (D-9)

where \( C_D \) is a constant to be described later.

The total energy of the system then becomes

\[ T = T_v + T_b + T_\gamma \]

\[ = W_x^2 \left\{ \gamma^2 a + \gamma M \left( \gamma^2 b^2 + a^2 + 2b \gamma \sin a - 2a \gamma \cos a \right) \right\} \]

\[ + W_y^2 \left\{ \gamma^2 a + \gamma M \left( b^2 + 2b \gamma \sin a + \gamma^2 \sin^2 a \right) \right\} \]

\[ + W_z^2 \left\{ \gamma^2 a + \gamma M \left( a^2 - 2a \gamma \cos a + \gamma^2 \cos^2 a \right) \right\} \]

\[ + W_x \dot{a} \left\{ M \left( \gamma^2 - a \gamma \cos a + b \gamma \sin a \right) \right\} \]

\[ + a^2 \left( \frac{7}{10} M \gamma^2 \right) \]  \hspace{1cm} (D-10)

With the total vehicle energy given in (D-10) and the damper dissipation function given in (D-9), the equation of motion of the vehicle and the damper system can be obtained as
The rotational equation of motion of the vehicle can be written in the form of Euler's rigid body equation as

\[
\begin{align*}
\frac{\text{d}}{\text{d}t} & \left( \frac{\partial T}{\partial W_X} - W_Z \frac{\partial T}{\partial W_Y} + W_Y \frac{\partial T}{\partial W_Z} \right) = 0 \\
\frac{\text{d}}{\text{d}t} & \left( \frac{\partial T}{\partial W_Y} - W_X \frac{\partial T}{\partial W_Z} + W_Z \frac{\partial T}{\partial W_X} \right) = 0 \\
\frac{\text{d}}{\text{d}t} & \left( \frac{\partial T}{\partial W_Z} - W_Y \frac{\partial T}{\partial W_X} + W_X \frac{\partial T}{\partial W_Y} \right) = 0 \\
\frac{\text{d}}{\text{d}t} & \left( \frac{\partial T}{\partial \alpha} - \frac{\partial T}{\partial \omega} + \frac{\partial T}{\partial \beta} \right) = 0
\end{align*}
\]

\[\text{(D-11)}\]

where the torques \( T_X \), \( T_Y \), and \( T_Z \) are,

\[
T_X = -M \left[ W_X \left( \gamma^2 + b^2 + a^2 + 2b \gamma \sin \alpha + 2a \gamma \cos \alpha \right) \right. \\
+ \left. \omega \left( 2W_X + \omega \right) \left( b \gamma \cos \alpha + a \gamma \sin \alpha \right) \right. \\
+ \left. \omega \left( \gamma^2 - a \gamma \cos \alpha + b \gamma \sin \alpha \right) \right. \\
+ \left. W_Y W_Z \left( a^2 - b^2 - 2a \gamma \cos \alpha - 2b \gamma \sin \alpha - \gamma^2 \sin^2 \alpha + \gamma^2 \cos^2 \alpha \right) \right. \\
+ \left. (W_Y^2 - W_Z^2) \left( ab + a \gamma \sin \alpha - b \gamma \cos \alpha - \gamma^2 \sin \alpha \cos \alpha \right) \right]
\]

\[\text{(D-13)}\]

\[
T_Y = -M \left[ W_Y \left( b^2 + 2b \gamma \sin \alpha + \gamma^2 \sin^2 \alpha \right) \right. \\
+ \left. (W_Z - W_X W_Y) \left( ab + a \gamma \sin \alpha - b \gamma \cos \alpha - \gamma^2 \sin \alpha \cos \alpha \right) \right]
\]
\[ + W_X W_Z \left( \gamma^2 + b^2 + 2b \gamma \sin \alpha - \gamma^2 \cos^2 \alpha \right) \]
\[ + 2W_Y \dot{a} (b \gamma \cos \alpha + \gamma \sin \alpha \cos \alpha) \]
\[ + 2W_Z \dot{a} (b \gamma \sin \alpha + \gamma^2 \sin^2 \alpha) \]
\[ (D-14) \]

\[ T_Z = -M \left[ \dot{W}_Z \left( a^2 - 2a \gamma \cos \alpha + \gamma^2 \cos^2 \alpha \right) \right. \]
\[ + (W_Y + W_X W_Y) \left( ab + a \gamma \sin \alpha - b \gamma \cos \alpha - \gamma^2 \sin \alpha \cos \alpha \right) \]
\[ + W_X W_Y \left( -a^2 + 2a \gamma \cos \alpha - \gamma^2 \cos^2 \alpha \right) \]
\[ + 2W_Z \dot{a} (a \gamma \sin \alpha - \gamma^2 \sin \alpha \cos \alpha) \]
\[ - 2W_Y \dot{a} (b \gamma \sin \alpha + \gamma^2 \sin^2 \alpha) \]  
\[ (D-15) \]

With ball's equation of motion in tube given as,
\[ \frac{7}{5} \gamma \ddot{a} + \dot{W}_X (\gamma - a \cos \alpha + b \sin \alpha) \]
\[ - W_X^2 (b \cos \alpha + a \sin \alpha) - W_Y^2 (b \sin \alpha + \gamma \sin \alpha \cos \alpha) \]
\[ - W_Z^2 (a \sin \alpha - \gamma \sin \alpha \cos \alpha) \]
\[ - W_Y W_Z (a \cos \alpha + b \sin \alpha - \gamma \cos^2 \alpha + \gamma \sin^2 \alpha) \]
\[ + W_X \dot{a} (a \sin \alpha + b \cos \alpha - a \sin \alpha - b \cos \alpha) \]
\[ + \left( \frac{C_D a}{N} \right) \dot{\alpha} = 0 \]  
\[ (D-16) \]

The drag force on the ball consists of two components: the pressure drag and the viscous drag. For the case where the gap between tube and ball is small, as in the GPS nutation damper, and the speed of the ball in the tube is small compared with the speed of sound, it has been determined in Ref. D-3 that the drag force is primarily due to pressure drag. The drag force \( D \),
can be written as

\[ \ddot{\alpha} = \frac{135}{4x64} \left( \frac{\pi^3 \mu \gamma_b}{g^{5/2}} \right) S \dot{\alpha} \hat{C}_D S \]

where
\[
\begin{align*}
\mu &= \text{viscosity of gas in tube} \\
g &= (\gamma_f - \gamma_b) \gamma_b = \text{gap parameter} \\
\gamma_f &= \text{inside radius of tube} \\
S &= \text{speed of ball relative to gas} \\
C_D &= \text{dissipation coefficient} \\
&= \frac{135}{4x64} \frac{\pi^3 \mu \gamma_b}{g^{5/2}}
\end{align*}
\]

For GPS nutation damper,
\[
\begin{align*}
\mu &= 178.1 \times 10^{-6} \times 2.089 \times 10^{-3} \text{ lb}_f \text{ sec/ft}^2 \\
g &= (0.633 - 0.625) / 0.625 \\
\gamma_f &= 0.633 \text{ in.}
\end{align*}
\]

Hence, value of dissipation coefficient,
\[ C_D = 0.0171 \text{ lb}_f / (\text{ft/sec}) \]

To handle the discontinuity at either end of the damper tube, let
\[ \dot{\alpha} = -\epsilon \dot{\alpha} \quad \text{at } \alpha = \pm \alpha_{\text{max}} \]

where
\[
\begin{align*}
\alpha_{\text{max}} &= (8/159) \text{ rad} = 2.9 \text{ deg.} \\
\epsilon &= 0 \text{ for inelastic collision} \\
&= 1 \text{ for elastic collision.}
\end{align*}
\]
The GPS requirement on half cone angle of nutation is to be less than 2 deg. The discontinuity situation is not expected to be encountered often if the 2 deg. requirement is met. The value of \( \varepsilon = 1 \) would give a conservative result in that less precession energy is dissipated at bottoming. Hence, use of \( \varepsilon = 1 \) is recommended for this reason.

**NUTATION DAMPING TIME CONSTANT VERIFICATION**

**Nominal Solution Undamped Rigid Body Motion**

Consider a rigid body with symmetric moment of inertia about the traverse axes, i.e., \( I_{XX} = I_{YY} \) in a torque free situation. The Euler's equation is reduced to:

\[
\begin{align*}
\dot{w}_x &= -A w_y w_z \\
\dot{w}_y &= A w_z w_x \\
\dot{w}_z &= 0
\end{align*}
\]

where

\[
A = \begin{pmatrix} I_{ZZ} - I_{XX} \\ I_{XX} \end{pmatrix} = \begin{pmatrix} I_{ZZ} - 1 \end{pmatrix} = (n - 1)
\]

The solution of the above equation can be written as:

\[
\begin{align*}
\dot{w}_x &= -B \sin \Omega t \\
\dot{w}_y &= B \cos \Omega t \\
\dot{w}_z &= w_z(0)
\end{align*}
\]

with \( \Omega = A w_z(0) \)
The angular momentum vector is written as:

\[
\mathbf{H} = I_{XX} \omega_X \mathbf{X}_B + I_{YY} \omega_Y \mathbf{Y}_B + I_{ZZ} \omega_Z \mathbf{Z}_B
\]  

(D-20)

Since there is no external torque, the angular momentum is constant. The nominal motion of the rigid body can be visualized as a constant precession about the angular momentum vector with a precession rate \( \dot{\omega} \) and a nutation angle \( \theta \) which is defined as:

\[
\tan \theta = \frac{H_I}{H_S} = \frac{\sqrt{I_{XX} \omega_X^2 + I_{YY} \omega_Y^2}}{I_{ZZ} \omega_Z} = \frac{I_{YY} B}{I_{ZZ} \omega_Z}
\]

(D-21)

Therefore:

\[
B = \left( \frac{I_{ZZ}}{I_{YY}} \times \tan \theta \right) \omega_Z
\]

(D-22)

For small value of \( \theta \), (6) becomes

\[
B \approx \left( \frac{I_{ZZ}}{I_{YY}} \times \theta \right) \omega_Z
\]

(D-23)

A relationship between the rigid body's angular momentum and rotational energy useful for later development of the energy sink approximation is:

\[
\begin{align*}
T &= \frac{1}{2} I_{XX} (\omega_X^2 + \omega_Y^2) + \frac{1}{2} I_{ZZ} \omega_Z^2 = \frac{H_I^2}{2 I_{XX}} + \frac{H_S^2}{2 I_{ZZ}} \\
H^2 &= H_t^2 + H_s^2
\end{align*}
\]

(D-24)

Hence

\[
H_t^2 = \left( \frac{I_{XX} I_{ZZ}}{I_{ZZ} - I_{XX}} \right) \left( 2T - \frac{H^2}{I_{ZZ}} \right)
\]

(D-25)

From (5)

\[
\sin^2 \theta = \frac{H_t^2}{H^2} = \left( \frac{I_{XX} I_{ZZ}}{I_{ZZ} - I_{XX}} \right) \left( 2T - \frac{I_{ZZ}}{H^2} \right)
\]

(D-26)
**Nutation Damping Time Constant**

The GPS nutation damper consists of a tube aligned with respect to the vehicle spin axis. A ball inside the tube is dissipating the nutation energy via viscous drag force retarding the ball's motion. An approximation procedure known as the energy sink concept assumes that the dissipation of nutation energy does not change the system's angular momentum (which is dominantly spin angular momentum). Hence equation (D-26) can be differentiated with terms \( \dot{H} \) neglected in this approximation, i.e.,

\[
2 \sin \theta \cos \theta \dot{\theta} = \frac{2I_{zz}}{(n-1)H^2}
\]

(D-27)

For small value of \( \theta \),

\[
2 \sin \theta \cos \theta \dot{\theta} = \frac{2I_{zz}}{(n-1)H^2}
\]

(D-28)

A further approximation involves

\[
H^2 = H_s^2 = I_{zz} \omega_z^2
\]

(D-29)

Hence

\[
\dot{\theta} = \left[ \frac{1}{(n-1)I_{zz} \omega_z^2} \right] \dot{\theta}
\]

(D-30)

The energy dissipation \( \dot{T} \) can be expressed as the product of the drag force the ball experienced and the velocity of the ball, i.e.,

\[
\dot{T} = -fv
\]

(D-31)
with drag force,
\[ f = C_D V \]
and the ball velocity,
\[ V = \dot{\alpha} \]

Hence the energy dissipation rate,
\[ \dot{T} = -C_D \gamma^2 \alpha^2 \]

where \( \alpha \) is the position of ball in tube defined as in Figure D-2,
\[ \gamma = \text{is radius of damper tube}, \]
\[ C_D = \text{is dissipation coefficient} \]

From analysis of nutation damper, the motion of the ball in tube was obtained as
\[
\frac{7}{5} \gamma \dot{\alpha} + \dot{W}_x (\gamma - a \cos \alpha + b \sin \alpha) - W_y^2 (b \cos \alpha + \gamma \sin \alpha \cos \alpha) - W_x^2 (b \cos \alpha + a \sin \alpha) - W_z^2 (a \sin \alpha - \gamma \sin \alpha \cos \alpha) - W_y W_z (a \cos \alpha + b \sin \alpha - \gamma \cos^2 \alpha + \gamma \sin^2 \alpha) + \frac{C_D^2}{M} \dot{\alpha} = 0
\]

With the geometry of mounting such that \( b = 0 \) and introducing the small angle approximations \( \sin \alpha = \alpha \), \( \cos \alpha = 1 \) and neglecting terms \( W_x^2 \) and \( W_y^2 \) as they are small for small nutation angle \( \theta \), equation (17) becomes
\[
\left( \frac{7}{5} \right) \ddot{\alpha} + \left( \frac{C_D^2}{M} \right) \ddot{\alpha} + W_z^2 (\gamma - a) \alpha = W_y W_z (a - \gamma) - W_x (\gamma - a)
\]

Using the expression of \( W_y \) as given in equations (2) and (7)
\[ W_y = \omega n W_z \cos \omega t \]
Similarly,

\[ \dot{W}_X = -\theta n W_Z \Omega \cos \Omega t \]

Equation (18) can be reduced to the following form:

\[
\begin{align*}
\dot{\alpha} + & \left\{ \frac{5C_D}{7M} \right\} \dot{\alpha} + \left\{ \frac{5(\gamma - a)W_Z^2}{7\gamma} \right\} \alpha \\
= & - \left\{ \frac{5(\gamma - a) (2n^2 - n^2)W_Z^2}{7\gamma} \right\} \cos \Omega t 
\end{align*}
\]

The solution of the standard second order linear differential equation of the form

\[ \ddot{\alpha} + 2b \dot{\alpha} + a^2 \alpha = c \theta \cos \Omega t \]

has been obtained in standard textbooks as:

\[ \alpha = \frac{c \theta}{\sqrt{(a^2 - b^2)^2 + 4b^2}} \cos (\Omega t + \phi) = \theta \cos (\Omega t + \phi) \]

Compare (20) and (21),

\[
\begin{align*}
2b & = 5 C_D / (7M) \\
a^2 & = 5(\gamma - a)W_Z^2 / (7\gamma) \\
c & = -5(\gamma - a) (2n^2 - n^2)W_Z^2 / (7\gamma)
\end{align*}
\]

From (22),

\[ \dot{\alpha} = -5 \alpha_0 \Omega \sin (\Omega t + \phi) \]

Substitute (24) into (16)

\[ \dot{\theta} = -b^2 C_D \gamma^2 \alpha_0^2 \Omega^2 \sin^2 (\Omega t + \phi) \]
The average value over a full period, i.e.,

$$\tilde{t}_p = 2\pi$$,

is

$$\tilde{t} = -\frac{1}{2} C_D \gamma^2 \alpha_o^2 \Omega^2 \theta^2$$

(D-42)

since

$$\frac{1}{t_p} \int_0^{t_p} \sin^2 (\Omega t + \phi) \, dt = \frac{1}{2}$$

Substitution of (26) into (14) gives

$$\dot{\theta} = \left[ \frac{1}{(n-1)I_{ZZ}W_Z^2} \right] \left[ -\frac{1}{2} C_D \gamma^2 \alpha_o^2 \Omega^2 \theta^2 \right]$$

or

$$\dot{\theta} = \left[ \frac{C_D \gamma^2 \alpha_o^2 \Omega^2}{2(n-1)I_{ZZ}W_Z^2} \right] \theta$$

(D-43)

The time constant of nutation damping can now be written as:

$$T_c = \frac{2(n-1)I_{ZZ}W_Z^2}{C_D \gamma^2 \alpha_o^2 \Omega^2}$$

(D-44)

where

$$\alpha_o = \sqrt{(\alpha^2 - \Omega^2)^2 + 4b^2 \Omega^2}$$

$$2b = 5 C_D / (7M)$$

$$\alpha^2 = 5(\gamma - a)W_Z^2 / (7\gamma)$$

$$c = -(2n - n^2) \alpha^2$$

$$\Omega = (n - 1)W_Z$$
The predicted nutation damping time constant for the two cases of $W_Z = 100$ RPM and $W_Z = 10$ RPM are computed as follows:

**Case 1:**

\[
\begin{align*}
  r &= 13.25 \text{ ft.} \\
  a &= 10.25 \text{ ft.} \\
  m &= 0.018 \text{ slug} \\
  C_D &= 0.0171 \text{ lb}_f/(\text{ft/sec}) \\
  I_{XX} &= I_{YY} = 77 \text{ slug-ft}^2 \\
  I_{ZZ} &= 94 \text{ slug-ft}^2 \\
  W_Z &= 100 \text{ RPM} = 10.47 \text{ rad/sec.} \\
  n &= 1.22 \\
  \Omega &= 2.31 \\
  2b &= 0.6789 \\
  a^2 &= 17.728 \\
  c^2 &= -16.84 \\
  \alpha_0 &= -1.348 \\
  T_c &= 155.755 \text{ sec.}
\end{align*}
\]

**Case 2:**

\[
\begin{align*}
  W_Z &= 10 \text{ RPM} = 1.047 \text{ rad/sec.} \\
  n &= 1.22 \\
  \Omega &= 0.231 \\
  2b &= 0.6786 \\
  a^2 &= 0.17728
\end{align*}
\]
\[ c = -0.1684 \]
\[ \alpha_o = -0.84275 \]
\[ T_c = 398.497 \text{ sec.} \]

The nutation half cone angle history corresponding to Case 1 and Case 2 considered above are plotted in Figures D-3 and D-4 respectively. These results are obtained using the nutation damping simulation program where the detailed dynamics of the damper ball and rigid body dynamics are considered.
100 RPM

\[
\begin{align*}
I_{xx} &= 76.3 \\
I_{yy} &= 78.3 \\
I_{zz} &= 94.1
\end{align*}
\]

"Time constant" = 150 sec.
(155.75 sec. was predicted)

Figure D-3. Simulation Data
10 RPM

\[
\begin{align*}
I_{xx} &= 76.3 \\
I_{yy} &= 78.3 \\
I_{zz} &= 94.1
\end{align*}
\]

"Time Constant" ≈ 365 sec.
(398.5 sec. was predicted)

Figure D-4. Simulation Data
REFERENCES


APPENDIX E

MODEL FOR GPS DIGITAL SPIN SUN SENSOR (SSS)
APPENDIX E

MODEL FOR GPS DIGITAL SPIN SUN SENSOR (SSS)

This appendix documents the GPS spin sun sensor (SSS) model. It has been programmed on a digital computer, and the simulation performs satisfactorily. Figure E-1 shows a flowchart of the math model which is based on the data of the references and the assumptions listed in this appendix. The main simulation program will provide the model with the sun vector in the spin reference frame and the spin rate at each sample instant. The model will provide a flag indicating whether or not a pipper command pulse will occur during the next sample interval, the time to the pipper pulse from the current sample instant, and the solar aspect angle at the pipper pulse occurrence.

DISCUSSION OF THE MODEL

A flowchart of the preliminary math model for the SSS is shown in Figure E-1. It accepts from the main program the spin rate and the sun vector in the spin reference frame at the sampling instant. To simulate the asynchronous pipper pulse within a synchronous simulation, both a flag and the predicted pipper pulse time from the current sampling instant is output when the pipper pulse is predicted to occur within the next sample period, $T_{\text{samp}}$. The measured solar aspect angle, nominally the angle between the Z axis and the sun vector and specifically the angle between the sun vector and the sensor Z axis, will be computed at the time of the pipper pulse, quantized, and output to the main program along with the flag and pipper pulse time. To perform these computations, the model must be initialized with the angular alignment of the sensor and the orientation of the spin vector with respect to the vehicle.

The SSS model described in Figure E-1 is based on the 1/8/75 spin sun sensor procurement specification and the assumptions listed in Table E-1. Table E-1 lists the initialization data required by the model. Table E-2 lists the input data to the model from the main program, and Table E-3 lists the outputs computed by the model. The intermediate variables internal to the model are described in Table E-4.

The notation $[\theta]_j$ denotes a rotational transformation by the angle $\theta$ about the $j$ axis. Transformations used in the model and those suggested to be used in the main program to obtain $S_W$, the sun vector in the spin frame at the sampling instant $t$, are defined in Figures E-2 through E-7.
Figure E-1. GPS SSS Model Top Level Flowchart
### TABLE E-1. INITIALIZATION DATA

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_S$</td>
<td>Degrees</td>
<td>Figure E-6</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>Degrees</td>
<td>Figure E-6</td>
</tr>
<tr>
<td>$\gamma_S$</td>
<td>Degrees</td>
<td>Figure E-6</td>
</tr>
<tr>
<td>$\alpha_W$</td>
<td>Degrees</td>
<td>Figure E-7</td>
</tr>
<tr>
<td>$\beta_W$</td>
<td>Degrees</td>
<td>Figure E-7</td>
</tr>
</tbody>
</table>

### TABLE E-2. INPUT DATA FROM MAIN PROGRAM

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{SAMP}$</td>
<td>Seconds</td>
<td>SSS Sample Period</td>
</tr>
<tr>
<td>$W_S$</td>
<td>Radians/Second</td>
<td>Spin Rate</td>
</tr>
<tr>
<td>$S_W(t)$</td>
<td>Second</td>
<td>Sun Vector in Spin Reference Frame at Sample Instant</td>
</tr>
</tbody>
</table>

### TABLE E-3. SSS OUTPUT TO MAIN PROGRAM

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{pipper}$</td>
<td>Seconds</td>
<td>Time from beginning of current sample interval to time of pipper pulse.</td>
</tr>
<tr>
<td>$SUN_{ANG}$</td>
<td>Degrees</td>
<td>Angle between Z sensor axis and sun vector (solar aspect angle).</td>
</tr>
<tr>
<td>$i_{pipper}$</td>
<td></td>
<td>Flag set to 1 if proper pulse occurs during current interval. Set = 0 otherwise.</td>
</tr>
<tr>
<td>Variable</td>
<td>Units</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
<td>-------</td>
<td>-------------</td>
</tr>
<tr>
<td>$[T_{V/S}]$</td>
<td></td>
<td>Sensor to vehicle transformation. Transpose of $[T_{S/V}]$. (See Figure E-6.)</td>
</tr>
<tr>
<td>$[T_{W/V}]$</td>
<td></td>
<td>Vehicle to spin transformation. (See Figure E-7.)</td>
</tr>
<tr>
<td>$[T_{W/S}]$</td>
<td></td>
<td>Sensor to spin transformation $[T_{W/S}] = [T_{W/V}] [T_{S/V}]^T$</td>
</tr>
<tr>
<td>$\overline{B_w}$</td>
<td></td>
<td>Boresight vector in spin frame. (See Figure E-5.)</td>
</tr>
<tr>
<td>$\overline{N_s}$</td>
<td></td>
<td>Normal to sensor plane in sensor frame. (See Figure E-5.)</td>
</tr>
<tr>
<td>$\bar{\theta}$</td>
<td>Radians</td>
<td>Sensor measured value of complement of the sun angle.</td>
</tr>
<tr>
<td>$\overline{P_w}$</td>
<td></td>
<td>Sensor plane normal vector in spin frame.</td>
</tr>
<tr>
<td>$\overline{S_s}$</td>
<td></td>
<td>Sun vector in sensor frame.</td>
</tr>
<tr>
<td>ITER</td>
<td></td>
<td>Iteration counter.</td>
</tr>
<tr>
<td>$\overline{K_s}$</td>
<td></td>
<td>Unit vector along sensor $Z$ axis.</td>
</tr>
<tr>
<td>$T_p$</td>
<td>Seconds</td>
<td>Value of $T_{pipper}$ from previous iteration.</td>
</tr>
<tr>
<td>$\bar{\theta}'$</td>
<td>Radians</td>
<td>Value of $\theta$ from previous iteration.</td>
</tr>
</tbody>
</table>

The method of solution for the pipper command and pulse time and the analysis performed to develop the model are documented in the subsection entitled "Method of Solution." The analysis showing the effect of the quasi-stationary spin axis orientation is also documented. The model does not include sensor dynamics or anomalies in the optics.

**ASSUMPTIONS**

- The orientation of the spin axis to the vehicle axes is defined in Figure E-7 and is constant. Nominally, the spin axis and the $Z$ vehicle axis are aligned.
- The sensor reference frame and orientation to the vehicle axes are as defined in Figures E-5 and E-6. Nominally, the $Z$ sensor axis is parallel to the spin axis.
INERTIAL FRAME

$X_I$ - in celestial equatorial plane pointed toward vernal equinox

$Y_I$ - in celestial equatorial plane at 90 degrees right ascending

$Z_I$ - perpendicular to equatorial plane (nominally toward Polaris)

ORBIT FRAME

$X_0$ - ascending node

$Y_0$ - lies in orbit plane and 90 degrees from $X_0$

$Z_0$ - perpendicular to orbit plane (right-hand triad)

TRANSFORMATION FROM INERTIAL FRAME TO THE ORBIT ORIENTED INERTIAL FRAME IS

$$\begin{bmatrix} T_{0/I} \end{bmatrix} = \begin{bmatrix} i_1 \end{bmatrix} \times \begin{bmatrix} \alpha_I \end{bmatrix} _Z$$

where $\alpha_I = \text{right ascension of ascending node}$

$i_1 = \text{orbit inclination}$

Figure E-2. Definition of Transformation from the Stellar Inertial Frame to the Orbit Frame
Figure E-3. Transformation from the Orbit (O) Frame to the Momentum (H) Frame.

\[
\begin{bmatrix}
\mathbf{i}_{H/O} \\
\mathbf{j}_{H/O} \\
\mathbf{k}_{H/O}
\end{bmatrix} =
\begin{bmatrix}
\beta_0 \\
\gamma_0 \\
\phi
\end{bmatrix}
\begin{bmatrix}
\mathbf{i}_O \\
\mathbf{j}_O \\
\mathbf{k}_O
\end{bmatrix}
\]

This is a time varying transformation driven by the control system where:

- \( \phi = 0 \)
- \( \beta_0 \) = the angle between the spin vector and the orbit plane normal
- \( \gamma_0 \) = the angle from \( \mathbf{i}_O \) about \( \mathbf{k}_O \) to the projection of the spin vector on the orbit plane \( \mathbf{i}_O \mathbf{j}_O \).
- \( \mathbf{i}_H \) = along spin vector
- \( \mathbf{j}_H \) = intersection of orbit plane and plane normal to spin vector.
- \( \mathbf{k}_H \) = completes right-hand orthogonal triad
Figure E-4. Transformation from the Momentum (H) Frame to the Spin (ω) Frame

\[
\begin{bmatrix}
T_{\omega/H} \\
\omega_t
\end{bmatrix} = \begin{bmatrix}
\omega_t & \omega_s \\
\omega_s & \omega_s
\end{bmatrix}
\]

**SPIN (ω) FRAME**

- \( Z_\omega \) - along spin vector
- \( Y_\omega = \omega_s t \) about \( Z_H \) or \( Z_\omega \) from \( Y_H \)
- \( X_\omega = \omega_s t \) about \( Z_H \) from \( X_H \)

\[ X_H \quad Z_H \quad Z_W \]

\[ X_\omega \quad \omega_t \quad \omega_s \]

\[ Y_H \quad Y_\omega \quad \omega_s t \]

\[ \omega_s \quad \omega_s \quad \omega_s \]
Figure E-5. Definition of GPS SSS Sensor Reference Frame

SENSOR FRAME

\( y_s \) is along \( N \), the normal to the \( \theta = 0 \) plane (plane of the narrow field of view).

\( x_s \) is along \( B \), the boresight which is the intersection of the plane \( \theta = 0 \) and the plane \( \beta = 0 \).

\( z_s \) completes the right-hand orthogonal triad.
This is a constant transformation where

\[ \alpha = \text{angle between the } Z_w \text{ axis and the projection of the spin vector on the } X_w - Z_w \text{ plane} \]

\[ \omega = \text{angle between the spin vector and the } X_v - Z_v \text{ plane} \]

Figure E-7. Definition of GPS Spin Reference Frame and Transformation from Vehicle to Spin Reference Frame
The sample period $T_{\text{SAMP}}$ will be 0.05 sec or shorter to preclude the sun vector from entering the active region of the sensor from the opposite side of the vehicle during one sample period.

The rate of angular displacement of the spin vector in inertial space will be less than $0.01 \text{ deg}/T_{\text{SAMP}}$ so that angular displacement of the spin axis during $T_{\text{SAMP}}$ will be negligible. This will allow prediction of the pipper pulse from the spin rate and geometry without accounting for spin vector displacement.

At each sample instant, the simulation main program will provide the SSS model with
1. Spin rate $W_S$ about the spin axis in rad/sec, and
2. $\mathbf{S}(t)$, the sun vector in the spin frame.

The pipper command pulse occurs $0.25 \text{ deg}/\cos \beta$ ahead of the crossing of the sensor ($\beta = 0$) plane. The repeatability of the pulse with respect to $\beta = 0$ is 3 arc minutes for repeated crossings at constant $\beta$.

METHOD OF SOLUTION

The method of solution is to predict, from the sun vector, sensor plane unit normal vector and the spin rate, the time of the pipper command pulse, and the measured solar aspect angle at that instant. It is assumed that the spin axis is quasi-stationary as explained in the next subsection.

Define the pipper command plane with unit normal $\mathbf{P}_S$ to contain the sensor $Z$ axis and the sun vector at the time of the pipper command pulse. The unit normal $\mathbf{P}_S$ can be defined in terms of $\mathbf{N}_S$, the normal to the sensor ($\beta = 0$) plane shown in Figure E-5. $\mathbf{P}_S$ is the result of the rotation of $\mathbf{N}_S$, $0.25 \text{ deg}/\cos \beta$ about the $Z$ sensor axis, where $\beta$ is the complement of the measured solar aspect angle also shown in Figure E-5.

$$\mathbf{P}_S = (-0.25/57.3)(1/\cos \beta) Z \mathbf{N}_S$$ \hspace{1cm} (E-1)

This relationship between the pipper command plane, sensor plane, and aspect angle is defined in Reference E-1. At the time of the pipper command pulse, the sun vector lies in the pipper command plane; therefore, the sun vector $\mathbf{S}$ and the normal $\mathbf{P}$ are perpendicular at this instant. Thus,
The solution of Equation (E-2) will allow the prediction of the pipper pulse time. If sensor, vehicle, and spin reference frames all have Z axes aligned, the solution is of closed form. However, these frames, in general, will not be aligned, so an iterative solution is shown in Figure E-2. The solution of Equation (E-2) will be done in the spin reference frame so that only rotations about one axis need to be considered. The sun vector in the spin frame,

\[ \vec{S}_W = [T_{W/H}] [T_{H/O}] [T_{O/I}] \vec{S}_I \]  

(E-3)

where the transformations from the inertial frame to the spin frame are defined in Figures E-3, E-4, and E-5, will be input from the main program at the sampling instant. The pipper command plane unit normal in the spin frame

\[ \vec{P}_W = [T_{W/V}] (T_{S/V})^T (-0.25/57.3) 1/\cos \beta \vec{S}_S \]  

(E-4)

will be computed in the model. The transformations \([T_{W/V}]\) and \([T_{S/V}]\) are defined in Figures E-7 and E-8. The initial value of \(\beta\) is computed from the sun vector \(\vec{S}_W\) at the sample instant:

\[ \beta = \sin^{-1} (\hat{S}_S \cdot [T_{S/V}] [T_{W/V}]^T \vec{S}_W) \]  

(E-5)

At the instant of the pipper pulse, \(T_{\text{pipper}}\), the sun vector in the spin frame will be

\[ \vec{S}_W (T_{\text{pipper}}) = [W_S T_{\text{pipper}}]_Z \vec{S}_W \]  

(E-6)

because the spin frame rotates about the Z axis. \(W_S\) is the spin rate (about the Z spin axis) and \(T_{\text{pipper}}\) is measured from the sample instant. Incorporating (E-4) and (E-6) into (E-2),

\[ \begin{bmatrix} W_S T_{\text{pipper}} \end{bmatrix}_Z \vec{S}_W \cdot \vec{P}_W = 0 \]  

(E-7)

Evaluating Equation (E-7), one obtains

\[ (\sin W_S T_{\text{pipper}}) (S_Y P_x - S_X P_y) + (\cos W_S T_{\text{pipper}}) (S_X P_y + S_Y P_x) = -S_Y P_z \]  

(E-8)
which has the form
\[
\sin \theta B + \cos \theta A = -K
\]

Equation (E-8) can be reduced by trigonometry identities to
\[
\sin(W_s T_{\text{piper}} + \phi) = \frac{-S_z P_z}{\sqrt{(S_x^2 P_x^2 + S_y^2 P_y^2 + S_z^2 P_z^2)^2}}
\]
\[
= \frac{-S_z P_z}{\sqrt{(1-S_z^2)(1-P_z^2)}}
\]

where
\[
\phi = \sin^{-1} \frac{S_x P_x + S_y P_y}{\sqrt{(1-S_z^2)(1-P_z^2)}}
\]

Then,
\[
T_{\text{piper}} = \left[ \sin^{-1} \left( \frac{-S_z P_z}{\sqrt{(1-S_z^2)(1-P_z^2)}} \right) - \sin^{-1} \left( \frac{S_x P_x + S_y P_y}{\sqrt{(1-S_z^2)(1-P_z^2)}} \right) \right] \frac{1}{W_s} \quad (E-9)
\]

Examining the second term, we see that the opposite sign is needed so that, for positive spin rate and \( P_x, P_y, P_z \) positive, the time \( T_{\text{piper}} \) will be positive for the time required for the sensor to rotate so that the sun vector is perpendicular to the plane normal \( \overrightarrow{P} \) (Figure E-9). Examining the first term, we see also that the sign is wrong because the sensor must be rotated farther to cross the sun vector when \( S_z \) and \( P_z \) are of the same sign than when they are either zero or of opposite signs (Figure E-9). Thus,
Figure E-9. Sun Vector Sensor Geometry

\[ T_{\text{piper}} = \frac{1}{W_S} \left( \sin^{-1} \left( \frac{S_z P_z}{\sqrt{(1-S_z^2)(1-P_z^2)}} \right) + \sin^{-1} \left( \frac{S_x P_x + S_y P_y}{\sqrt{(1-S_x^2)(1-P_y^2)}} \right) \right) \] (E-10)

Because the sensor frame is not aligned to the spin frame, the measured \( \delta \) is recomputed for the sun vector in the sensor frame at \( T_{piper} \). Equation (E-10) is then iterated with \( \overline{P} \) based on the new value of \( \delta \). This iterative process can be repeated until the \( T_{piper} \) and/or \( \delta \) converge to acceptable tolerances or for a maximum number of iterations.

If \( T_{piper} > T_{\text{samp}} \), the piper command will occur following the next sampling. If \( T_{piper} < 0 \), the piper command has already occurred in a previous sample period.

There is also a random error which must be added to \( T_{piper} \). It is the 3 arc minute repeatability of the piper command at \( \delta = 0 \). This repeatability is assumed to be the same for repeated passes through any angle \( \delta \). The random time error will be modeled by a nominally distributed random number representing the uncertainty in the time of the piper pulse. The 1 sigma value of this time uncertainty is

\[ \sigma^t_{\text{rand}} = \frac{1/20}{57.3} \frac{1}{W_S} \text{ sec} \]

If the spin rate is variable, a random number generator which has a variable standard deviation will be needed.
SIMULATION ACCURACY WITH ASSUMPTION OF QUASI-STATIONARY SPIN AXIS

The GPS SSS is designed to operate in the spin rate range of 5 to 100 rpm. To model the sun crossing pipper pulse time but maintain a relatively long sample period $T_{\text{SAMPLE}}$, it is necessary to predict the time within the sample period during which the pipper pulse occurs. It is assumed that the rate of change of the spin vector orientation is sufficiently small (less than $0.01 \text{ deg}/T_{\text{SAMPLE}}$) so that the angular motion of the sensor with respect to the sun vector can be approximated by the angular motion about the spin axis location at the last sample instant. Then the prediction of pipper pulse time is a straightforward computation from the spin rate and the sun vector in the spin frame at the last sample instant. If the spin vector orientation will be changing faster than $0.01 \text{ deg}/T_{\text{SAMPLE}}$, then the model needs to be changed to use an iterative method or to extrapolate the sun vector to the spin frame at the instant of pipper crossing.

The maximum spin axis orientation rate according to the Reference E-3 precession control mode is $0.136 \text{ deg per jet firing}$. The duration of firing is $0.111 \text{ sec}$ and firing occurs once per revolution. Since the precession equations are derived from

$$\dot{T} = \frac{\vec{H}}{\kappa t} + \vec{W} \times \vec{T}$$

or

$$\dot{T} = \vec{W} \times h k$$

$$T_x = W_y h$$

$$T_y = -W_x h$$

the jet torque causes rate rather than acceleration. The precession rate will be $0.136 \text{ deg}/0.111 \text{ sec} = 1.22 \text{ deg/sec}$. This rate of change of the spin axis orientation does not occur as a continuous function, but the rate is present for about 60 deg out of each rotation. With a sampling period of $0.005 \text{ sec}$, the error in spin axis orientation knowledge at the predicted pipper pulse time would be about $0.008 \text{ deg}$ in the worst case. Worst case will occur when the pipper pulse occurs at the end of the sample interval. This will be an error of $0.008 \text{ deg}$ in the aspect angle measurement with negligible error in the pipper pulse time.
REFERENCES


APPENDIX F

REACTION JET MODEL FOR GPS INDEPENDENT ANALYSIS
APPENDIX F

REACTION JET MODEL FOR GPS INDEPENDENT ANALYSIS

This appendix presents the model for the GPS reaction jets. This model is based on the model used in the 3XATS simulation. Both ATS and GPS use Rocket Research Corporation monopropellant hydrazine thrusters. Any data that was not available from Rockwell was extracted from ATS documents.

MODEL DESCRIPTION

The model consists of an initialization section and a normal operation section. The initialization section performs one-time computations and clears outputs and counters. The normal operation section receives jet commands from the controller and outputs total jet force and total jet torque on the spacecraft. The following are system characteristics included in the model:

- Thrust increases with catalyst temperature which increases when the jet is firing and decreases when the jet is not firing. The time constant for heating is less than that for cooling.
- No thrust is developed if the jet command is less than the minimum on time.
- Thrust start and stop times lag the jet command start and stop times due to valve response time and hydrazine burn response.
- Eighteen thrusters are used.
- Thruster torque depends on variable spacecraft cg location.

The following are jet system characteristics not included in the model:

- Dynamics of the thrust rise at turn-on and thrust decay at turn-off must be considered because of the impact on computer run time. The static model will be adequate for all but difficult limit cycle cases. These limit cycle cases should be studied using a single-axis simulation and correlated with static model results. A dynamic model can be provided if necessary.
• Changes in thrust due to supply pressure reduction with fuel consumption is not modeled. The supply pressure changes will be very slow. Stability should be analyzed with both high and low supply pressure by changing the thrust level initialization inputs.

• Dynamic changes in thruster position and angular alignment are not considered. The spacecraft is very stiff. Cases of extreme position and angular alignment should be run.

The jet model flow diagram is shown in Figure F-1. The left arrow (→) within boxes indicates replacement, i.e., the expression on the right replaces the variable on the left. Mathematical subscripts are used. Symbols used in the flow diagram are defined in the sections below. Comments appear outside the boxes to describe the operations within the boxes.

INITIALIZE DATA

Table F-1 lists the initialization parameters, nominal values, tolerance, and source of data. This data is to be input at the start of each run.

INPUT AND OUTPUT DATA

Table F-2 lists the input data that the jet model expects from the calling program each time the jet subroutine is executed in the normal mode.

Table F-3 lists the output data from the jet model subroutine. Other subroutine variables may be outputs if the user desires.

INTERNAL VARIABLES

Internal variables used in the flow diagram are defined in Table F-4.
Figure F-1. Flow Diagram of Reaction Jet Model

```
ENTER SUBROUTINE JETS

MODE
1(NORMAL)
1(INITIALIZE)

JET THRUST ALIGNMENT DIRECTION COSINES
CACR = \cos A \cos \beta
CASH = \cos A \sin \beta
S = \sin A

COM, 1 = \sin \theta ON,
COFF, 1 = \sin \theta OFF,
JET THRUST RISE AND DECAY INCREMENTS

JETCTR, 0 = 0
TOTJT = 0
FSAV, = FMN,
PTDIF = 0
INITIALIZE JET PULSE COUNTER.
TOTAL JET ON TIME, INITIAL THRUST AND TIME UNTIL JET TURNS OFF.

M, INTEGER
ON DELAY,
CONRAIN T 0, M, 10

T, J = 0
INITIALIZE JET COMMAND HISTORY

T, J = 0
J = J + 1
LOOP FOR 20 PREVIOUS COMMANDS.

T20 =
T20 =
J = J + 1
LOOP FOR 20 JETS.

TKJET = 0
TVJET = 0
TJET = 0
FXTOT = 0
FVTOT = 0
FZTOT = 0
JETCT = 0
CLEAR TORQUE, FORCE AND JET COUNTER OUTPUTS.

TM = 0
INITIALIZE JET SUBROUTINE TIME COUNTER

ICMN = 1 = INTEGER
MINIMUM ON TIME

ICMN - 1 = INTEGER
MINIMUM ON TIME

RETURN
END OF INITIALIZATION
```
Figure F-1. Flow Diagram of Reaction Jet Model (continued)
PTOFF₁ ← TYM + OFF DELAY 
- INTEGER (ON DELAY / Δ₁) \* 1 

COMPUTE ACTUAL 
OFF TIME.

DON - DEL

YES

PTOFF₁ ≥ TYM + DEL

NO

DON - PTOFF₁ - TYM

DON

DON

≥ 0

(JET WILL 
STAY ON AT 
LEAST 
ONE MORE 
INTERVAL)

DON - PTOFF₁ - TYM

≤ 0

(JET WILL 
STAY ON FOR 
PART OF 
INTERVAL)

(JET IS 
OFF.)

FSAV₁ - FSAV₁ + (FMIN₁ - FSAV₁) \* COFF₁

DECREASE JET THRUST 
POTENTIAL DUE TO 
CATALYST COOLING.
SET JET THRUST TO 0.

FJET₁ - 0

G

H

Figure F-1. Flow Diagram of Reaction Jet Model (continued)
FROM SHEET 2

\[ j \rightarrow 20 \cdot M_i \]

\[ JET_{i,j} \]

\( \neq 1 \)

\( JET_{i,j} = 1 \)

\( JET_{i,j} \)

\( j \rightarrow j + 1 \)

\( i \)

\[ 19 \cdot M_i^{+} ICMin \geq \]

\[ 19 \cdot M_i^{+} ICMin < \]

\[ (OK \ TO \ TURN \ JET \ ON) \]

\[ JETCTR_i = JETCTR_{i-1} \]

\[ FJET_i = FSAV_i \]

\[ DON = DEL \]

\[ PTOFF_i \]

\[ TYM + DEL \]

\( \geq PTOFF_i \)

\[ DON = \text{DEL} \cdot \text{ON DELAY} \]

\[ + \text{INTEGER} \cdot \text{ON DELAY} \]

\[ \Delta t \]

\[ \Delta t \]

\[ (JET \ IS \ STILL \ ON \ FROM \ A \ PREVIOUS \ COMMAND.) \]

\[ SET \ INCREMENTAL \ ON \ TIME \ TO \ ACCOUNT \ FOR \ TRUE \ ON \ DELAY. \]

Figure F-1. Flow Diagram of Reaction Jet Model (continued)
SET JET THRUST, SET INCREMENTAL ON TIME TO T.

INCREASE JET THRUST DUE TO CATALYST HEATING. TAKE FRACTION OF THRUST EQUAL TO FRACTION OF ON TIME.

INCREMENT TOTAL JET ON TIME. SAVE THRUST LEVEL.

COMPUTE COMPONENTS OF INDIVIDUAL JET FORCES.

SUM TO GET TOTAL JET FORCE ON SPACECRAFT.

COMPUTE NOZZLE MOMENT ARM COMPONENTS.

SUM TO GET TOTAL JET TORQUE ON SPACECRAFT.

SUM TO GET TOTAL JET PULSES.

END OF JET SUBROUTINE.

Figure F-1. Flow Diagram of Reaction Jet Model (concluded)
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**Notes:**

2. Rockwell GPS PDR handout.
3. Coordination meeting notes, 3/20/75.
6. GPS 0.1 LBF REA, Qualification Test Report, CDRL Item A005, March 31, 1976.
7. GPS 5.0 LBF REA, Qualification Test Report, CDRL Item A005, March 31, 1976.
8. Pairs match to ±5%, 3σ. Pairs are 9 and 14, 10 and 13, 12 and 15, 11 and 16, 3 and 6, 4 and 5, 1 and 7, 2 and 8, 6 and 7, 2 and 4, 1 and 3, 5 and 8, 9 and 15, 10 and 16, 12 and 14, 11 and 13.
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<td>lbs</td>
<td>4.0</td>
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<td>18</td>
<td></td>
<td>4.0</td>
<td>Notes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thrust rise time constant, +ON</td>
<td>1</td>
<td>sec</td>
<td>0.4</td>
<td>Notes</td>
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<td>0.4</td>
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<td>Notes</td>
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<td>Notes</td>
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<td>None</td>
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<td>Notes</td>
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### TABLE F-1. INITIALIZATION DATA FOR JET MODEL (concluded)

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<th>Parameter</th>
<th>Subscript</th>
<th>Units</th>
<th>Nominal Value</th>
<th>Source</th>
<th>Tolerance</th>
<th>Source</th>
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<td></td>
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<tr>
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<td>18</td>
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<td>0.025</td>
<td></td>
<td></td>
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</tr>
<tr>
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<td>18</td>
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</tr>
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### TABLE F-2. INPUT DATA FOR JET MODEL

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<th>Definition</th>
<th>Flow Diagram Symbol</th>
<th>Dimension</th>
<th>Type</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode control flag</td>
<td>Mode</td>
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<td>Integer</td>
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</tr>
<tr>
<td>Jet command from controller (Subscript is jet number)</td>
<td>Jet 18</td>
<td>Integer</td>
<td>Feet</td>
<td></td>
</tr>
<tr>
<td>Spacecraft eg X coordinate</td>
<td>XCG</td>
<td>Real</td>
<td>Feet</td>
<td></td>
</tr>
<tr>
<td>Spacecraft eg Y coordinate</td>
<td>YCG</td>
<td>Real</td>
<td>Feet</td>
<td></td>
</tr>
<tr>
<td>Spacecraft eg Z coordinate</td>
<td>ZCG</td>
<td>Real</td>
<td>Feet</td>
<td></td>
</tr>
</tbody>
</table>

1 = Normal
1 = On
0 = Off
### TABLE F-3. OUTPUT DATA FOR JET MODEL

<table>
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<th>Definition</th>
<th>Flow Diagram Symbol</th>
<th>Dimension</th>
<th>Type</th>
<th>Units</th>
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<tr>
<td>Total number of jet operations</td>
<td>JETCTR</td>
<td></td>
<td>Integer</td>
<td></td>
</tr>
<tr>
<td>Number of operations for each jet</td>
<td>JETCTR</td>
<td>18</td>
<td>Integer</td>
<td></td>
</tr>
<tr>
<td>Accumulated on time for each jet</td>
<td>TOTJT</td>
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<td>Real</td>
<td>Seconds</td>
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<tr>
<td>X component of total jet force on spacecraft</td>
<td>FXTOT</td>
<td></td>
<td>Real</td>
<td>Pounds</td>
</tr>
<tr>
<td>Y component of total jet force on spacecraft</td>
<td>FYTOT</td>
<td></td>
<td>Real</td>
<td>Pounds</td>
</tr>
<tr>
<td>Z component of total jet force on spacecraft</td>
<td>FZTOT</td>
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<td>Real</td>
<td>Pounds</td>
</tr>
<tr>
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<td>Real</td>
<td>Foot pounds</td>
</tr>
<tr>
<td>Total jet torque about Y axis</td>
<td>TY.JET</td>
<td></td>
<td>Real</td>
<td>Foot pounds</td>
</tr>
<tr>
<td>Total jet torque about Z axis</td>
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<td>Real</td>
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</tr>
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<td>Definition</td>
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<td>Dimension</td>
<td>Type</td>
<td>Units</td>
</tr>
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<tr>
<td>Thrust direction cosine</td>
<td>CACB</td>
<td>18</td>
<td>Real</td>
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<tr>
<td>Thrust direction cosine</td>
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<td>Thrust rise increment</td>
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<td>Thrust decay increment</td>
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<td>Potential jet thrust from previous cycle</td>
<td>FSAV</td>
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<td>Pounds</td>
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<tr>
<td>Predicted time for jet thrust to go off</td>
<td>PTOFF</td>
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<td>Real</td>
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<tr>
<td>Quantized on delay</td>
<td>M</td>
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<tr>
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<td>Integer</td>
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<td>Time since start of run</td>
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<td>Quantized minimum on time</td>
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<td>Incremental jet on time</td>
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<td>Actual jet thrust</td>
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<td>Pounds</td>
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<td>Pounds</td>
</tr>
<tr>
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<td>Pounds</td>
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<td>Z component of jet torque</td>
<td>TZJ</td>
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<td>Foot pounds</td>
</tr>
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<tr>
<td>General index</td>
<td>j</td>
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APPENDIX G

GPS DEPLOYMENT MODEL
APPENDIX G

GPS DEPLOYMENT MODEL

The following equations will be used for simulation of the GPS rotational dynamics during deployment. These equations will be incorporated in the one-body simulation to investigate the effect of deployment on the pitch and roll jet channels.

The following primary assumptions were made in developing these equations:

1. Panels are treated as point masses.
2. Mass of the boom is neglected.
3. A "linear" deployment linkage is assumed.

ROTATIONAL DYNAMICS

Referring to Figure G-1 and Appendix C for notation, the angular momentum during deployment is given by

\[ \bar{H} = \bar{I}_0 \omega + \sum_i m_i (\bar{r}_i + \bar{r}_o + \bar{r}_1) \times \bar{v}_i \]  \hspace{1cm} (G-1)

where

\[ I_0 = J_0 - m \sum_i M(r_i + r_o + r_1) + \sum_i m_i (r_i + r_o + r_1) \]  \hspace{1cm} (G-2)

The rotational differential equations of motion are given by

\[ T = I \omega + I \omega + \sum_i m_i (r_i + r_o + r_1) \times \dot{v}_i \]  \hspace{1cm} (G-3)

The deployment linkage, Figure G-2, is such that \( \theta = 2\alpha \). With this linear relationship, derivatives appearing in Equations (G-1) through (G-3) can be evaluated as follows:

\[ \dot{\alpha}_i \begin{cases} 0 \\ (-1)^j (a + b) \sin \gamma_1 \\ b - a \cos \gamma_1 \end{cases} \]

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Figure G-1. Notation for Deployment Model

Figure G-2. Deployment Mechanism Linkage Model

\[ \beta = 2 \alpha \]
\[
\begin{align*}
\dot{q}_1 &= \begin{cases} 
0 \\
(-1)^i (a + b) \cos \frac{\alpha_i}{2} \dot{\alpha}_i_1 \\
(a - b) \sin \frac{\alpha_i}{2} \dot{\alpha}_i_1
\end{cases} \\
\ddot{\varphi}_1 &= \begin{cases} 
0 \\
(-1)^i [(a + b) \cos \frac{\alpha_i}{2} \ddot{\alpha}_i_1 - (a + b) \sin \frac{\alpha_i}{2} \ddot{\alpha}_i_1] \\
(a - b) \sin \frac{\alpha_i}{2} \ddot{\alpha}_i_1 + (a - b) \cos \frac{\alpha_i}{2} \ddot{\alpha}_i_2
\end{cases}
\end{align*}
\]

\[
\begin{align*}
r_o &= -\sum_i m_i \frac{1}{i} (r_i + \varphi_i) \\
\dot{r}_o &= -\sum_i m_i \frac{1}{i} \ddot{\varphi}_i \\
r_o &= -\sum_i m_i \frac{1}{i} \dddot{\varphi}_i
\end{align*}
\]

\[
\begin{align*}
\dot{c}_o &= -m \left[ \sum_i \left( \frac{1}{i} \right) \left( M(\dot{r}_i, r_o) + M^T(\dot{r}_i, r_o) \right) + \sum_i \left( \frac{1}{i} \right) \left( M(\ddot{c}_i, r_i + \varphi_i) + M^T(\ddot{c}_i, r_i + \varphi_i) \right) \right]
\end{align*}
\]

The hinge torques are given by

\[
\begin{align*}
\bar{T}_{Di} &= \bar{c}_o \times m_i \frac{1}{i} \frac{d^2 R_i}{dt^2} = \bar{c}_i \times \frac{d^2 (\bar{r}_o + \bar{r}_i + \bar{\varphi}_i)}{dt^2} + \bar{\varphi}_i \times m_i \bar{\alpha}_i \bar{\alpha}_i \bar{\alpha}_i \\
T_{Di} &= \omega_i \times \left[ \bar{r}_o + \bar{r}_i + \omega_o \times (r_o + r_i + \varphi_i) + 2 \omega_o \times (\dot{r}_o + \varphi_i) \\
&+ \omega_o \times \omega_o \times (r_o + r_i + \varphi_i) \right] + \varphi_i \times \alpha_i \alpha_i \alpha_i
\end{align*}
\]

The hinge torques are produced by spring forces which will be modeled as

\[
\begin{align*}
T_{Di} \cdot \left\{ \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right\} &= d_{\varphi} + d_{\alpha} \alpha_1 + d_{\beta} \beta_1
\end{align*}
\]
APPENDIX H

SOLAR PRESSURE TORQUE AND FORCE MODEL
APPENDIX H

SOLAR PRESSURE TORQUE AND FORCE MODEL

This appendix summarizes the preliminary solar pressure force and torque model developed for the digital simulation evaluation of GPS stability and control analysis.

The solar force results from the photo momentum transfer to the vehicle surfaces exposed to solar illumination. The magnitude and direction of the solar force developed on a given vehicle surface are functions of the vehicle-sun attitude, the geometry of the surface, and the physical characteristics of the surface. The solar torque will be developed if the vehicle center of mass is not coincident with the center of total net force.

The model described in this appendix was based on the vehicle geometrical configurations deduced from the Rockwell drawing No. V505-900003 titled "Spacecraft General Arrangement Baseline Configuration - GPS," January 27, 1975.

VEHICLE GEOMETRY FOR SOLAR TORQUE EVALUATION

The GPS spacecraft configuration has been approximated by surfaces of simple geometry to facilitate the evaluation of solar force and torque. This simplified geometry consists of five flat surfaces, six cylindrical surfaces, one segment of a cone, and fourteen complete cylindrical surfaces for the representation of vehicle main body, solar panels, thruster cone, and antenna in on-orbit configuration. The stowed vehicle during the spinning phase is approximated by a complete cylindrical surface. The geometries of the simplified vehicle configurations are shown in Figure II-1.

Total solar pressure force and torque experienced by the vehicle can be evaluated as the vector sum of the forces and torques exerted on the approximating surfaces. Nominal geometries including size, location, and orientation of the surfaces for the GPS vehicle are summarized in a later section of this appendix. These data are based on the Rockwell drawing No. V505-900003 titled "Spacecraft General Arrangement Baseline Configuration - GPS," January 27, 1975. The computations involved in the evaluation of the force and torque contributed by these elementary surfaces are described in the sections that follow.
Figure H-1. Simplified GPS Geometry and B-Frame Definition
Solar Pressure on Flat Surface

A flat surface, defined in terms of the surface normal $\mathbf{N}$ and geometrical center $\mathbf{C}$, develops a solar pressure force $\mathbf{F}$ and torque $\tau$ about vehicle center of mass as given in the following equations:

$$\mathbf{F} = -F_n \mathbf{N} + F_t \frac{[\mathbf{(N \cdot S)N - S}]}{|(\mathbf{N \cdot S})\mathbf{N - S}|} \quad (H-1)$$

$$\tau = \mathbf{C} \times \mathbf{F} \quad (H-2)$$

where

- $\mathbf{S}$ = LOS vector to sun
- $F_n = [1 + 5 \rho (1 - s) \cos \phi] \mathbf{PA}$
- $F_t = [1 - s \rho (1 - s) \cos \phi] \mathbf{PA}$
- $\rho = 1 \times 10^{-7} \text{ lb/ft}^2$
- $s = \text{fraction of incident radiation reflected}$
- $\mathbf{S} = \text{fraction of reflected radiation that is specular}$
- $\mathbf{C} = \text{position vector of surface geometrical center from vehicle center of mass}$

$$\cos \phi = (\mathbf{N} \cdot \mathbf{S})$$

Note that, for the case $\mathbf{N} = \mathbf{S}$,

$$| (\mathbf{N} \cdot \mathbf{S}) \mathbf{N - S} | = 0$$

This portion of computation in Equation (H-1) should be bypassed. This is when there is no tangential force.
Solar Force on Cylindrical Surfaces

From the geometry of Figure H-2, the location, orientation and size of a cylindrical surface can be completely specified in terms of the following parameters:

\[ C = \text{position vector of the geometrical center of the surface} \]
\[ N = \text{normal vector at } C \text{ (orientation)} \]
\[ T = \text{tangent vector at } C \text{ (orientation)} \]
\[ r = \text{radius of the cylinder (size)} \]
\[ \delta_o = \text{angular width of the cylinder (size)} \]
\[ l = \text{length of the cylinder (size)} \]

The evaluation of solar force on a cylindrical surface can be carried out by the subdivision into \( n \) flat surface elements as shown in Figure H-3. The \( i \)th flat surface element is defined in terms of the location of its geometrical center \( C_i \) and the normal vector \( N_i \) which are related to the cylinder parameters as

![Figure H-2. Surface Numbering Notation](image-url)
Figure H-3. Definition of Antenna Rods
\[ C_i = C - rN_i + rN_i \]  
\[ N_i = \cos \theta_i N + \sin \theta_i T \]  
\[ \theta_i = \frac{\pi}{2} - i/2 + \pi i, i = 1, 2, \ldots, n \]  
\[ \Lambda = \frac{\pi}{n} \]

The area of \( i \) th surface element is
\[ A_i = r \cdot \Lambda \cdot z \]

The solar forces and torques developed on these flat surface elements can be computed using Equations (H-1) and (H-2). Let the forces and torques be denoted as \( F_i \) and \( \tau_i \).

The total force and torque developed by the cylindrical surface are \( F_c \) and \( \tau_c \) evaluated as
\[ F_c = \sum_{i=1}^{n} F_i \]  
\[ \tau_c = \sum_{i=1}^{n} \tau_i \]

For the case of a full cylindrical surface, such as the antenna, the cylinder is defined in terms of following parameters:

\( C \) = position vector of the center of the cylinder from vehicle center of mass
\( L \) = unit vector along the cylinder axis
\( r \) = radius of cylinder
\( \ell \) = length of cylinder

The solar force \( F \) and torque \( \tau \) about vehicle center of mass become
\[ F = \frac{n}{n} F N - F L \]  
\[ \tau = D \times F \]

where
\[ N = \frac{|S - (S \cdot L)L|}{|S - (S \cdot L)L|} \]  
\[ F_n = [(1 + s p/3) \sin^2 a + s (1 - s)(\pi/6) \sin 2a] PA \]
\[ F_t = [(1 - s_p) \sin \alpha \cos \theta] PA \]

\[ A = \text{projected cylinder area} = 2lr \]

\[ \cos \theta = (S \cdot L) \]

\[ D = C + rN \]

Again, the computations associated with \( |S - (S \cdot L) L| \) should be bypassed for \( S = L \). This is when there is no transverse force.

**Solar Force on Conic Surface**

From the geometry of Figure 11-4, the location, orientation, and size of a conic surface can be completely specified in terms of the following parameters:

- \( \frac{N}{c} \): unit vector along cone axis
- \( r_a \): radius of circle on top
- \( r_b \): radius of circle on bottom
- \( o_a \): center of circle on top
- \( h \): height of conical surface

The evaluation of the solar force on a conical surface can be carried out through approximations using flat surface elements. Each surface element is a trapezoid which can be represented by a surface normal \( N_i \) and the location of its geometrical center \( C_i \) as depicted in Figure 11-4. These parameters are related to the defining parameters of the conical surface as

\[ \frac{N_i}{c} = \cos \gamma \frac{X_i}{c} + \cos \gamma \frac{X_i}{c} \sin \gamma \frac{Y_i}{c} + \sin \gamma \frac{Z_i}{c} N_c \]  \quad (H-11)

\[ C_i = A_i + \left( \frac{2r_b + r_i}{(3r_b + 3r_i)} \right) [B_i - \Lambda_i] \]  \quad (H-12)

where \( \frac{X_i}{c} \) and \( \frac{Y_i}{c} \) are unit vectors that make \( \frac{X_i}{c} - \frac{Y_i}{c} - \frac{Z_i}{c} \) a right-handed orthogonal triad. For convenience, let

\[ \frac{Y_i}{c} = (\frac{N_i}{c} \times \frac{X_i}{c}) / |\frac{N_i}{c} \times \frac{X_i}{c}|, \quad \frac{X_i}{c} = -\frac{Y_i}{c} - \frac{Z_i}{c} \]

\[ o_i = i\delta - \delta/2, \quad i = 1, 2, \ldots, n \]

\[ \delta = \pi/n \]
GEOMETRICAL CENTER OF TRAPEZOID:

\[
c = \frac{2S_b + S_a}{3S_b + 3S_a}
\]

\[
= \frac{2r_b + r_a}{3r_b + 3r_a}
\]

Figure II-4. Geometry of Thruster Cone

\[
\Lambda_i = \frac{O_a + r_a \cos \alpha_i X_c + r_a \sin \alpha_i Y_c}{C}
\]

\[
B_i = \frac{(O-a - l_i N_c) + r_b \cos \alpha_i X_c + r_b \sin \alpha_i Y_c}{C}
\]

\[
\delta_c = \tan^{-1} \left( \frac{r_b - r_a}{b/a} \right)
\]

\[
r_1 = \frac{r_a + \delta (r_b - r_a) \sqrt{r_b - r_a}^2 + h^2}{C}
\]

\[
\ell_1 = \text{distance between } \Lambda_1 \text{ and } B_1
\]

\[
\ell_1' = \text{distance between } \Lambda_1' \text{ and } B_1'
\]

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The area of the trapezoid is
\[
A_i = \frac{1}{2} (r_1 + r_b) s_i
\]

The solar force and torque contributed by each of the trapezoidal surface elements can then be computed using Equations (H-1) and (H-2) with the total force and torque contributed by the thruster cone evaluated in the same manner as Equations (H-7) and (H-8).

Vehicle Nominal Attitude and Shading Considerations

The GPS spacecraft on-orbit attitude is sun and earth oriented. The pitch and roll control allows the vehicle yaw axis to be aligned with local vertical. The vehicle yaw motion is such that the solar array surface normal can be driven to be aligned with sun line for maximum solar exposure. Attitude accuracy requirements are 0.5 deg for roll and pitch and 3 deg for yaw.
In order for the vehicle main body to cast shadow on the solar array, the minimum yaw attitude error must be $\theta_s$.

$$\theta_s = \tan^{-1}\left(\frac{f}{b/2}\right) \approx \tan^{-1}\left(\frac{12}{23}\right) \approx 27.5 \text{ deg}$$

where

- $f =$ clearance between main body and solar array
- $b =$ length of main body

Similarly, the main body will not be shaded by the solar array panels. Also, the solar array attitude is driven to within 5 deg of maximum exposure. Hence it can be assumed that the solar array is completely free of shadow. Since the sun vector can be anywhere within the pitch plane, the shading of the thruster cone and the flat surface $P_1$ interfacing the thruster cone need to be considered. The shading of the antenna is ignored due to the relatively small areas involved.

**Shading of Thruster Cone by Surface $P_1$**—The shading of the thruster cone can be considered in terms of the $i^{th}$ flat surface element described earlier. Referring to the geometry of Figure II-5, the projection of point $B_i$ on the surface $P_1$ is $D_i$. Any point located on the surface $P_1$ satisfies the vector equation

$$(D_i - C_{P1}) \cdot N_{P1} = 0$$

(H-14)

Also, the vector $(D_i - B_i)$ must be pointed along the sun line, i.e.,

$$(D_i - B_i) = rS$$

(H-15)

where $r =$ distance between $B_i$ and $D_i$. The projection of vector $(B_i - A_i)$ on $P_1$ along $S$ is the vector $(E_i - A_i)$ which intersects the edge of $P_1$ at $E_i$. Hence $E_i$ satisfies the vector equations

$$(E_i - C_{P1}) \cdot (E_i - C_{P1}) = r_c^2$$

(H-16)

$$(E_i - A_i) = t(D_i - A_i)$$

(H-17)

where $t =$ ratio of distance between $E_i$ and $A_i$ to that between $D_i$ and $A_i$. 91
Solution of the vector equations for \( D_1 \) and \( E_1 \) is included in this appendix. Once the point \( E_1 \) is determined, the shade of \( E_1 \) on the surface element, denoted as the point \( A_1' \), can be computed, i.e.,

\[
A_1' = A_1 + \left( \frac{Z_{AE}}{Z_{AD}} \right) (I_1 - A_1)
\]

where

\[
Z_{AE} = \text{distance between } A_1 \text{ and } E_1
\]

\[
Z_{AD} = \text{distance between } A_1 \text{ and } D_1
\]

To evaluate the solar force contributed by the unshaded portion of the \( i \)th surface element, Equations (H-11), (H-12), and (H-13) can be used with \( A_1 \) replaced by \( A_1' \). It may happen that the entire \( i \)th surface element is in the shadow of \( P_1' \). The condition for this situation is that the distance between \( C_{P1} \) and \( D_i \) must be less than the radius of surface \( P_i \)'s circular edge \( r_c \), i.e.,

Figure II-5. Shading of Thruster Cone by Plane \( P_1 \) Illustrated in Terms of Elementary Surface \( A_i - B_i \).
\[(C_{\text{P1}} - D) \cdot (C_{\text{P1}} - D) < r_c^2 \quad (H-19)\]

If the inequality, Equation (H-19), is valid, the computations associated with the \(i^{th}\) surface element can be bypassed since it contributes nothing to the total force and torque. Also, it may happen that the entire \(i^{th}\) surface element is not shaded by \(P_1\). This is the case when the sun vector has positive projection along the normal to \(P_1\), i.e.,

\[
\cos \alpha_c > (S \cdot N_{P1}) > 0 \quad (H-20)
\]

If the inequality, Equation (H-20), is valid, the point \(A_i\) is set to be identical to \(A_i\).

Shading of Surface \(P_1\) by Thruster Cone—Since the circle defining the open end of the thruster cone is parallel to the surface \(P_1\), the projection of this circle on \(P_1\) is always a circle of identical size as shown by the parallelograms of Figure H-6. The geometries of the shadow on surface \(P_1\) for four different sun line configurations are considered for the following discussions. The shadow of thruster cone on \(P_1\) will be characterized as a flat surface of area \(A_s\) with normal \(N_s\) and geometrical center \(C_s\).

Equations (H-1) and (H-2) are then used to compute the contribution of solar force \(F_s\) and torque \(T_s\) as if \(A_s\) were not shaded. Hence, the effective solar force and torque contributed by the shaded surface \(P_1\) is

\[
\frac{F_{P1}}{F_s} = \frac{F_{P1}^{(0)}}{F_s} - F_s
\]

\[
\frac{T_{P1}}{T_s} = \frac{T_{P1}^{(0)}}{T_s} - T_s
\]

where

\[
F_{P1}^{(0)} \quad \text{and} \quad T_{P1}^{(0)}
\]

are force and torque computed for a completely unshaded surface \(P_1\).

Let the center of the circle where thruster cone joins \(P_1\) be \(C_a\), and let the center of the circular shadow of the end of thruster cone on \(P_1\) be \(C_b\). Position of \(C_b\) can be solved from the vector equation

\[
(C_b - C_a) \cdot N_{P1} = 0
\]

\[
C_b - (C_a + h N_{P1}) = \xi
\]

where \(\xi\) = length of \([C_b - (C_a + h N_{P1})]\).
Figure II-6. Shading of Plane $P_1$ by Thruster Cone

The normal of the shadow on $P_1$ is of course $N = N_{P1}$. To evaluate $F_s$ and $\tau_s$, the quantities $C_s$ and $A_s$ are defined for four cases:

Case 1: $l_{ab} < r_b - r_a$

$$C_s = C_b$$
$$A_s = \pi r_b^2$$

$$l_{ab} + r_b \leq r_c$$

$r_b \cdot r_a \leq r_c \cdot r_b$
Case 2:

\[ A_1 = \frac{1}{2} r_a^2 [2\beta - \sin 2\beta] \]

\[ C_1 = \frac{C_a - C_b}{I_{ab}} = \frac{2/3 r \sin^3 \beta}{\beta - \cos \beta \sin \beta} \]

\[ A_2 = \frac{1}{2} (2r_a \sin \beta + 2r_b \sin \beta) \frac{L}{r_b} \sin^2 \beta \]

\[ C_2 = \left[ \frac{C_a + L(r_a \cos \beta)}{r_a} \right] - \frac{L}{2} \left( \frac{r_b + r_a}{3} \right) \left( \frac{3r_b + 3r_a}{3} \right) \]

\[ A_3 = \frac{1}{2} r_b^2 [2\beta - \sin 2\beta] \]

\[ C_3 = \frac{C_b + L(\frac{2}{3} r_b \sin^3 \beta)}{2 \cos \beta \sin \beta} \]

Total area of shadow:

\[ A_s = A_1 + A_2 + \frac{1}{2} \pi r_b^2 - A_3 \]

Centroid position of \( A_s \):

\[ C_s = \frac{1}{A_s} \left[ A_1 C_1 + A_2 C_2 + \frac{1}{2} \pi r_b^2 C_b - A_3 C_3 \right] \]
Case 3: $r_b + r_c \geq L_{ab} > r_c - r_b$

\[ r_b + r_c \geq L_{ab} > r_c - r_b \]

\[ A_4 = \frac{1}{3} r_b^2 \left[ 2 \alpha_b - \sin 2 \alpha_b \right] \]

\[ C_4 = C_b - L \left( \frac{(2/3)r_b \sin^3 \alpha_b}{\alpha_b - \cos \alpha_b \sin \alpha_b} \right) \]

\[ A_5 = \frac{1}{2} r_c^2 \left[ 2 \alpha_c - \sin 2 \alpha_c \right] \]

\[ C_5 = C_a - L \left( \frac{(2/3)r_c \sin^3 \alpha_c}{\alpha_c - \cos \alpha_c \sin \alpha_c} \right) \]

Total area of shadow:

\[ A_s = A_1 + A_2 + \frac{1}{2} \pi r_b^2 - A_3 - A_4 + A_5 \]

Centroid of $A_s$:

\[ C_s = \frac{1}{A_s} \left[ A_1 C_1 + A_2 C_2 + \frac{1}{2} \pi r_b^2 C_b - A_3 C_3 - A_4 C_4 + A_5 C_5 \right] \]

where

\[ \cos 2\alpha_b = \left( \frac{D_1 - C_b}{D_2 - C_b} \right) \cdot \left( \frac{D_1 - C_b}{D_2 - C_b} \right) \]

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\[
\cos 2\theta_c = [(D_1 - C_a) \cdot (D_2 - C_a)] N ((D_1 - C_a) \cdot (D_1 - C_a))
\]

\[D_1\] and \[D_2\] satisfy the vector equation
\[
(D - C_a) \cdot (D - C_a) = r_c^2
\]
\[
(D - C_b) \cdot (D - C_b) = r_b^2
\]
\[
(D - C_a) \cdot N_{D_1} = 0
\]

\(A_1, A_2, A_3, C_1, C_2\) and \(C_3\) are as defined in Case 3.

Case 4: \(r_{ab} > r_b + r_c\)

\[
A_6 = \frac{1}{2} \left( 2r_c \sin \beta + 2r_a \sin \beta \right) (r_c \cos \beta_a + r_a \cos \beta)
\]
\[
C_6 = \left[ \frac{C_a + \frac{1}{2} (r_a \cos \beta)}{r_c \sin \beta_c + r_a \sin \beta} \right]
\]

Total area of shadow:
\[
A_s = A_1 + A_5 + A_6
\]
Centroid of $A_s$:

$$C_s = \frac{1}{A_s} [A_1 C_1 + A_5 C_5 + A_6 C_6]$$

where

$$\cos 2\alpha_c = [(E_1 - C_a) \cdot (E_2 - C_b)] \sqrt{[(E_1 - C_a) \cdot (E_1 - C_a)]}$$

$$E_i = F_i \sqrt{r_i^2 - r_a^2} (G_i - F_i) / |(G_i - F_i)| \quad i = 1, 2$$

$$F_i = C_a + r_a \cos \theta L + (-1)^{i+1} r_a \sin \theta M$$

$$G_i = C_b + r_b \cos \theta L + (-1)^{i+1} r_b \sin \theta M$$

$$M = N_{P1} \times L$$

SIMPLIFIED GPS VEHICLE GEOMETRY FOR SOLAR FORCE AND TORQUE EVALUATION

Planar Surfaces

The five planar surfaces include $P_1$ where thruster cone is located, $P_2$ where antennae are located, $P_3$ and $P_4$ adjacent to the solar panels in the directions of $+Y_B$ and $-Y_B$, respectively, and the plane introduced at the open end of the thruster cone to simplify the complex geometry involved. The vehicle body frame (B-frame), as defined in the Rockwell drawing for spacecraft general arrangement configuration, has its $Y_B$ axis parallel to the line of the solar array drive, the $Z_B$ axis along the AKM thruster centerline, and the $X_B$ axis to complete a right-handed orthogonal triad. The origin of the B-frame is located at the center of the open end of the thruster cone (not the AKM nozzle) as shown in Figure 11-7. Let each plane be defined in terms of its surface normal $N_{j}$, location of surface centroid $C_{j}$, and area of surface $A_{j}$ in B-frame coordinates.

Plane $P_1$:

$$B_{(C_{P1})} = \begin{bmatrix} 0 \\ 0 \\ 23.88 \end{bmatrix} \quad B_{(N_{P1})} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \quad \Lambda_{P1} = 43.8 \times 41.8 \quad + (32)^2 \begin{bmatrix} 86.4 \\ 57.3 \end{bmatrix} \sin 86.4$$
Figure II-7. Simplified GPS Geometry and B-Frame Definition
Plane $P_2$:
\[
\begin{bmatrix}
B \\
\mathbf{C}_{P2}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
61.38
\end{bmatrix} \quad \begin{bmatrix}
B \\
\mathbf{N}_{P2}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} \quad A_{P2} = A_{P1}
\]

Plane $P_3$:
\[
\begin{bmatrix}
B \\
\mathbf{C}_{P3}
\end{bmatrix} = \begin{bmatrix}
0 \\
21.8 \\
42.63
\end{bmatrix} \quad \begin{bmatrix}
B \\
\mathbf{N}_{P3}
\end{bmatrix} = \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix} \quad A_{P3} = 37.5 \times 41.8 = 1567.5
\]

Plane $P_4$:
\[
\begin{bmatrix}
B \\
\mathbf{C}_{P4}
\end{bmatrix} = \begin{bmatrix}
0 \\
-21.8 \\
42.63
\end{bmatrix} \quad \begin{bmatrix}
B \\
\mathbf{N}_{P4}
\end{bmatrix} = \begin{bmatrix}
0 \\
-1 \\
0
\end{bmatrix} \quad A_{P4} = 37.5 \times 41.8 = 1567.5
\]

Plane $P_5$:
\[
\begin{bmatrix}
B \\
\mathbf{C}_{P5}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} \quad \begin{bmatrix}
B \\
\mathbf{N}_{P5}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
-1
\end{bmatrix} \quad A_{P5} = \pi (14.6)^2 = 669.7
\]

Cylindrical Surfaces

The six segments of cylindrical surfaces include two main body surfaces, $C_1$ and $C_2$, in the directions of $X_B$ and $-X_B$, respectively, and two solar panel segments for each of the two solar arrays. Each segment of cylindrical surface is defined in terms of the location of the geometrical center $C$, the normal and tangent vectors at $C$, $N$, and $T$, respectively, the radius, length, and the angular width of the cylinder segment, $r$, $\ell$, and $\delta$, respectively. The cylindrical surfaces are defined as follows:

Cylindrical Surface $C_1$:
\[
\begin{bmatrix}
B \\
\mathbf{C}_{C_1}
\end{bmatrix} = \begin{bmatrix}
30.2 \\
0 \\
42.63
\end{bmatrix} \quad \begin{bmatrix}
B \\
\mathbf{N}_{C_1}
\end{bmatrix} = \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} \quad \begin{bmatrix}
B \\
\mathbf{T}_{C_1}
\end{bmatrix} = \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}
\]

\[
r_{C_1} = 32.0, \quad \ell_{C_1} = 37.5, \quad \delta_{C_1} = 2 \sin^{-1}(21.9/32.0) = 86.4 \text{ deg}
\]
Cylindrical Surface $C_2$:

\[
\begin{align*}
B \begin{bmatrix} C_2^B \end{bmatrix} &= \begin{bmatrix} -30.2 \\ 42.63 \end{bmatrix}, \\
B \begin{bmatrix} N_2^B \end{bmatrix} &= \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \\
B \begin{bmatrix} T_2^B \end{bmatrix} &= \begin{bmatrix} 0 \\ -1 \end{bmatrix}
\end{align*}
\]

$r_{C_2} = 32.0$, $\theta_{C_2} = 37.5$, $\delta_{C_2} = 86.4$ deg

Cylindrical surfaces $C_3$ through $C_6$ are solar panel surfaces. In the following, the normal and tangent vectors, $N$ and $T$, will be defined in terms of the panel fixed frames. Their B-frame definition will be obtained through a transformation as

\[
\begin{align*}
B \begin{bmatrix} N_3^B \end{bmatrix} &= T_{BP}^B \begin{bmatrix} N_3^P \end{bmatrix} \\
B \begin{bmatrix} T_3^B \end{bmatrix} &= T_{BP}^B \begin{bmatrix} T_3^P \end{bmatrix}
\end{align*}
\]

The positions of centroid of the cylindrical surfaces will be defined in P-frame with respect to origin of P-frame as

\[
\begin{bmatrix} P^P \end{bmatrix} = \begin{bmatrix} \overline{O}^P_C \end{bmatrix} + \begin{bmatrix} \overline{C}^P_C \end{bmatrix}
\]

Their P-frame definitions are then obtained as

\[
\begin{align*}
B \begin{bmatrix} \overline{C}^B_C \end{bmatrix} &= T_{BP}^B \begin{bmatrix} \overline{C}^P_C \end{bmatrix}
\end{align*}
\]

Each cylindrical segment is individually defined so that misalignments between segments of arrays and between arrays can be simulated.

Cylindrical Surface $C_3$:

\[
\begin{align*}
P^3 \begin{bmatrix} N_3^P \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \\
P^3 \begin{bmatrix} T_3^P \end{bmatrix} &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
P^3 \begin{bmatrix} \overline{C}^P_3 \end{bmatrix} - \begin{bmatrix} \overline{O}^P_3 \end{bmatrix} &= \begin{bmatrix} g_3^3 \\ \frac{l_{C3}}{2} \end{bmatrix} = \overline{df}
\end{align*}
\]
Nominally,
\[ Y_{P3} = Y_B \]
\[ \alpha_3 \text{ defines } X_{P3} \text{ with respect to } X_B \text{ or } Z_P \text{ with respect to } Z_B \]
\[ g_3 = r_{C3}^2 - r_{C3} \cos (\alpha_{C3}/2) \]
\[ d_3 = 2r_{C3} \sin (\alpha_{C3}/2) \]
\[ r_{C3} = 32.7 \text{ in} \]
\[ \alpha_{C3} = 58.6 \text{ deg} \]
\[ l_{C3} = 67 \text{ in} \]
\[ (\overline{O}_3) = (0, 21.9, 42.38)^T \]
\[ T_{BP3} = \begin{bmatrix} \cos \alpha_3 & 0 & \sin \alpha_3 \\ 0 & 1 & 0 \\ -\sin \alpha_3 & 0 & \cos \alpha_3 \end{bmatrix} \]

Cylindrical Surface C4:
\[ \frac{P4}{(X_{C4})} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \frac{P4}{(T_{C4})} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \]
\[ \frac{P4}{(C_{C4})} - (\overline{O}_4) = \begin{bmatrix} g_{4} \\ l_{C4}/2 \\ d_{4}/2 \end{bmatrix} \]

Nominally,
\[ Y_{P4} = Y_B \]
\[ \alpha_4 \text{ defines } X_{P4} \text{ with respect to } X_B \]
\[ g_4 = r_{C4}^2 - r_{C4} \cos (\alpha_{C4}/2) \]
\[ d_4 = 2r_{C4} \sin (\alpha_{C4}/2) \]
\[ r_{C4} = 32.7 \text{ in} \]
\[ \alpha_{C4} = 58.6 \text{ deg} \]
$\varepsilon_{C4} = 67$ in

$\begin{bmatrix} B \\ \overline{O}_4 \end{bmatrix} = (0, \ 21.8, \ 38)^T$

$T_{BP4} = \begin{bmatrix} \cos \sigma_4 & 0 & \sin \sigma_4 \\ 0 & 1 & 0 \\ -\sin \sigma_4 & 0 & \cos \sigma_4 \end{bmatrix}$

**Cylindrical Surface** $C_5$: $P_5$: Panel fixed frame

$\begin{bmatrix} P_5 \\ \overline{N}_{C5} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} P_5 \\ \overline{T}_{C3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} P_5 \\ \overline{C}_{C5} \end{bmatrix} - \overline{O}_5 = \begin{bmatrix} g_5 \\ -\varepsilon_{C5}/2 \\ -d_5/2 \end{bmatrix}$

Nominally,

$\Sigma_{P5} = \Sigma_B$

$\sigma_5$ defines $\Sigma_{P5}$ with respect to $\Sigma_B$

$g_5 = r_{C5} \cos \left( \varepsilon_{C5}/2 \right)$

$d_5 = 2r_{C5} \sin \left( \varepsilon_{C5}/2 \right)$

$r_{C5} = 32.7$ in

$\varepsilon_{C5} = 53.6$ deg

$\varepsilon_{C5} = 67$ in

$\begin{bmatrix} B \\ \overline{O}_5 \end{bmatrix} = (0, -21.8, 38)^T$

$T_{BP5} = \begin{bmatrix} \cos \sigma_5 & 0 & \sin \sigma_5 \\ 0 & 1 & 0 \\ -\sin \sigma_5 & 0 & \cos \sigma_5 \end{bmatrix}$

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Cylindrical Surface $C_6$: 

\[
\begin{bmatrix}
\begin{array}{c}
\mathbf{P}_6^C \\
\mathbf{N}_6^C \\
\mathbf{T}_6^C
\end{array}
\end{bmatrix} = \begin{bmatrix}
\begin{array}{c}
1 \\
0 \\
0
\end{array}
\end{bmatrix} \begin{bmatrix}
\begin{array}{c}
\mathbf{P}_6^C \\
\mathbf{N}_6^C \\
\mathbf{T}_6^C
\end{array}
\end{bmatrix} = \begin{bmatrix}
\begin{array}{c}
0 \\
0 \\
1
\end{array}
\end{bmatrix}
\]

\[
\mathbf{P}_6^C - \mathbf{N}_6^C = \begin{bmatrix}
\mathbf{g}_6 \\
-\mathbf{d}_6^C/2 \\
\mathbf{d}_6^C/2
\end{bmatrix}
\]

Nominally,

\[
\frac{\mathbf{Y}_6^P}{\mathbf{Y}^B} = \mathbf{Y}_6^B
\]

$\alpha_6$ defines $\mathbf{Y}_6^P$ with respect to $\mathbf{Y}^B$.

\[
\mathbf{g}_6 = r_6^C - r_6^C \cos(\theta_6^C/2)
\]

\[
d_6 = 2r_6^C \sin(\theta_6^C/2)
\]

$r_6^C = 32.7$ in

$\theta_6^C = 58.6$ deg

$\mathbf{B}_6^B = (0, -21.3, 38)^T$

\[
\mathbf{T}_{BP6} = \begin{bmatrix}
\cos \alpha_6 & 0 & \sin \alpha_6 \\
0 & 1 & 0 \\
-\sin \alpha_6 & 0 & \cos \alpha_6
\end{bmatrix}
\]

Thruster Cone $T_C$

The orientation, location, and physical dimensions of the thruster cone surface are defined in terms of the parameters $(N_C', \gamma_a, \gamma_b, O_a, h)$ as defined in Section V. In terms of B-frame coordinates, we have

\[
\mathbf{B}^B_{N_C} = \begin{bmatrix}
0 \\
0 \\
-1
\end{bmatrix} \quad \mathbf{B}^B_{O_a} = \begin{bmatrix}
0 \\
0 \\
h
\end{bmatrix}
\]

104
\[ \nu_a = 2.6/2 = 15.0 \]
\[ \nu_b = 3.2/2 = 18.5 \]
\[ h = 23.88 \]

**Antenna Rods**

Antenna rods are simplified as cylindrical surfaces defined in terms of the parameters \((C, L, \nu, \xi)\) as described in Section II. Antenna rods \(A_1\) through \(A_{12}\) are arranged symmetrically about an axis \(L_C\) through a point \(C_C\) as shown in Figure H-3. Hence,

\[
\begin{align*}
B & \quad (L_C) = (0, 0, 1)^T \\
B & \quad (C_C) = (3.27, 0, 61.38)^T
\end{align*}
\]

For \(A_1\) through \(A_{12}\), i.e., \(i = 1, 12\),

\[
\begin{align*}
\nu_i &= 1.25 \text{ in.} \\
L_i &= 13.6 \text{ in.} \\
B & \quad (L_i) = (0, 0, 1)^T \\
B & \quad (C_i) = (X_i, Y_i, 61.38)^T + (C_C)
\end{align*}
\]

with

\[
\begin{align*}
X_i &= -r_o \cos \theta_i \\
Y_i &= -r_o \sin \theta_i \\
r_o &= 17.25 \text{ for } i = 1, \ldots, 8 \\
&= 6.0 \text{ for } i = 7, \ldots, 12 \\
\theta_i &= (i-1) \times 45^\circ + 22.5^\circ \text{ for } i = 1, \ldots, 8 \\
&= (i-9) \times 90^\circ + 45^\circ \text{ for } i = 9, \ldots, 12
\end{align*}
\]

For \(A_{13}\),

\[
\begin{align*}
r_{13} &= 1.25 \text{ in.} \\
L_{13} &= 44.3
\end{align*}
\]
\[
B(\hat{L}_{13}) = (0, 0, 1)^T
\]

\[
B(\hat{C}_{13}) = (8.27, 17.25, 61.38 + \ell_{13}/2)^T
\]

For \(A_{14}\),

\[
\gamma_{14} = 0.8 \text{ in.}
\]
\[
\ell_{14} = 19.1 \text{ in.}
\]

\[
B(\hat{L}_{14}) = (0, 0, -1)^T
\]

\[
B(\hat{C}_{14}) = (29.2, 0, 23.88 - \ell_{14}/2)^T
\]

**Stowed Vehicle as Cylindrical Surface**

During the initial spinning phase, the GPS configuration has its solar arrays stowed. A cylindrical surface approximation can be used for evaluating solar force. The cylinder is defined in terms of the following parameters:

\[
B(\hat{L}) = (0, 0, 1)^T
\]

\[
B(\hat{C}) = (0, 0, \ell/2)^T
\]

\[
\gamma = 35.7
\]

\[
\ell = 67
\]

**cg Locations**

To evaluate the torque due to solar pressure, the moment arm is the vector difference of location of center of force and the location of vehicle center of mass. In B-frame coordinates, the locations of center of mass for GPS are summarized in Table H-1.

**Other Simplifications**

The approximating surfaces are assumed to have uniform physical characteristics. For GPS spacecraft exterior surfaces, the coating material is of the Al-Teflon type which, for solar arrays, consists of a cover of AR coat and a layer of UV filter coat.
# TABLE H-1. SEQUENCE MASS PROPERTIES

<table>
<thead>
<tr>
<th>Item</th>
<th>Weight (lb)</th>
<th>Center of Gravity (in.)</th>
<th>Moment of Inertia (slug feet²)</th>
<th>Product of Inertia (slug feet⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X</td>
<td>Y</td>
<td>Z</td>
</tr>
<tr>
<td>Launch</td>
<td>1436</td>
<td>0.00</td>
<td>0.01</td>
<td>35.81</td>
</tr>
<tr>
<td>Less Transfer Orbit Propellant</td>
<td>-6</td>
<td>0.00</td>
<td>0.00</td>
<td>39.40</td>
</tr>
<tr>
<td>AKM Ignition</td>
<td>1430</td>
<td>0.00</td>
<td>0.01</td>
<td>35.79</td>
</tr>
<tr>
<td>Less AKM Propellant</td>
<td>-567</td>
<td>0.00</td>
<td>0.00</td>
<td>31.80</td>
</tr>
<tr>
<td>On-Orbit</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Array Stowed</td>
<td>863</td>
<td>-0.01</td>
<td>0.02</td>
<td>38.42</td>
</tr>
<tr>
<td>Array Deployed</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In X-X Plane</td>
<td>863</td>
<td>-0.01</td>
<td>0.07</td>
<td>38.11</td>
</tr>
<tr>
<td>In Z-Z Plane</td>
<td>863</td>
<td>-0.52</td>
<td>0.02</td>
<td>38.48</td>
</tr>
<tr>
<td>Less RCS Propellant</td>
<td>-51</td>
<td>0.00</td>
<td>0.00</td>
<td>39.40</td>
</tr>
<tr>
<td>End Mission</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Array Deployed</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In X-X Plane</td>
<td>811</td>
<td>0.00</td>
<td>0.07</td>
<td>38.03</td>
</tr>
<tr>
<td>In Z-Z Plane</td>
<td>811</td>
<td>-0.56</td>
<td>0.02</td>
<td>38.43</td>
</tr>
</tbody>
</table>

*Principal axis angular offset
The values of parameters $s$ and $p$ for solar pressure evaluation will be furnished by Rockwell.
APPENDIX I

SOLAR ARRAY FLEXIBILITY MODEL
APPENDIX I

SOLAR ARRAY FLEXIBILITY MODEL

A STARDYNE (Control Data version of NASTRAN) finite element program was used to generate a mathematical model of the +Y solar array model. Only the +Y array was modeled, with the pivot point considered to be fixed. The +Y array is structurally identical to the -Y array. However, since each array has independent pitch drive actuators, the model considers each array to be responding separately and independently. The model will be discussed first, with respect to the original coarse model that was used for determining the model structure (i.e., how many structural modes should be included), followed by the final fine mesh model.

COARSE MESH SOLAR ARRAY FLEXIBILITY MODEL

The coarse NDS solar array structural model is shown in Figure I-1. Nodes 5 through 8 are on the boom centerline, at the approximate location of the solar panel to boom hinges. The solar panel sun sensors are located close to Node 8.

![Diagram of solar array model]

Figure I-1. Math Model—Coarse Mesh
Figures I-2 and I-3 present a layout schematic drawing of the +Y array and the Rockwell vibration test panel orientation, respectively. Based on this test, the equivalent viscous damping ratio for one mode was estimated to be 3.5 percent (Zeta = 0.035). This damping estimate was obtained informally from Rockwell.

The solution for solar array dynamic response can be obtained by direct integration of the equations of motion of the structural math model shown in Figure I-1. However, such a model would have 11 independent nodes with six degrees of freedom per node. Instead, a solution can be obtained using a reduced set of generalized coordinates. The generalized coordinates are the natural frequencies (rad/sec) of the structure. The transformation from the generalized coordinates to the physical math model are the normal modes (mode shapes) of vibration.

The deployed NDS solar array flexibility model is represented by a selected set of n eigenvectors and eigenvalues. The solution for the eigenvector/eigenvalue problem is based on a finite element structural model of the NDS deployed solar array (+Y array) based on available structural drawings obtained from Rockwell through the SAMSO contract monitor office.

The numerical values tabulated in this memo are for a coarse mesh math model based on out-date drawings. The associated natural frequencies are known to be higher than the actual solar array structure, based on informal results of a ground vibration test performed at Rockwell. However this model is sufficiently close in dynamic response characteristics to allow checkout runs to be made.

EQUATIONS OF MOTION

For the NDS solar array model, the base of the STARDYNE elastic structure is at the following station (+Y array):

\[ X = 0.0 \]
\[ Y = 28.77 \text{ in.} \]
\[ Z = 38.00 \text{ in.} \]

This point is the location of the inner pivot point of the deployment mechanism and is the centerline of the circular, integrally machined drive tube as shown in Rockwell drawing No. V505-932004.
Figure 1-2. Solar Array Layout

TOTAL WEIGHT = 57.96 lbs

\[
\begin{align*}
X_{cg} &= .27 \\
Y_{cg} &= 64.27 \\
Z_{cg} &= 34.24
\end{align*}
\]
For convenience, the attach point of the elastic structure will be considered to be on the vehicle centerline at Z station 38.0. Thus modal participation factors and the array response torques, moments, and forces will be transformed to X = 0.0, Y = 0.0, and Z = 38.00.

This will simplify the equations required to couple the elastic array model to the rigid body controls simulation model.

Figure I-3. Rockwell Test Results

Reference: Rockwell report N18231-26, 2/17/77.
The equations of motion in matrix notation for the $+Y$ array are

$$[m] \{ \ddot{u} \} + [c] \{ \dot{u} \} + [k] \{ u \} = [m][\Gamma] \{ u \} \quad (I-1)$$

where

$$[u] = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ \delta X_i \\ \delta Y_i \\ \delta Z_i \end{bmatrix}$$

are motions of panel points on the array relative to the base point. The accelerations of the base point are $\{ \ddot{u} \}$. $\{ \ddot{u} \}$ has six possible components: $\ddot{X}$, $\ddot{Y}$, $\ddot{Z}$, $\ddot{\delta X}$, $\ddot{\delta Y}$, $\ddot{\delta Z}$.

Let the panel points on the array be described by the normal mode coordinate transformation:

$$\{ u \} = \{ \phi \} \{ \eta \} \quad (I-2)$$

$$\{ \dot{u} \} = \{ \phi \} \{ \dot{\eta} \} \quad (I-3)$$

$$\{ \ddot{u} \} = \{ \phi \} \{ \ddot{\eta} \} \quad (I-4)$$

where $\{ \phi \}$ represents the normal modes and the $\{ \eta \}$ are a set of generalized coordinates which are uncoupled, second-order, ordinary differential equations.

The response of the $n^{th}$ "mode" or generalized coordinate is expressed as

$$\ddot{\eta}_n + 2\zeta_n \omega_n \dot{\eta}_n + \omega_n^2 \eta_n = -\{ \gamma_n \}^T \{ \ddot{u} \} \quad (I-5)$$

where

- $\zeta_n$ = equivalent viscous damping factor assuming 3.5 percent ($\zeta = 0.035$)
- $\omega_n$ = $n^{th}$ natural frequency (rad/see)
- $\{ \gamma_n \}_{6 \times n}$ = model participation factor for base acceleration in each of the six input axes: $\ddot{X}$, $\ddot{Y}$, $\ddot{Z}$, $\ddot{\delta X}$, $\ddot{\delta Y}$, $\ddot{\delta Z}$.
Equation (1-5) must be integrated with the satellite main body equations of motion. If each array is represented by the first four elastic modes, then four sets of equations such as Equation (1-5) must be integrated. If solar arrays are pulsed separately, then a total of eight such equations will be required. The +Y array is assumed identical to the -Y array. However, the participation factors will be different (of opposite sign) for rotational base accelerations in $\xi X$ and $\xi Z$.

The rotational position of the sun sensor at the outer end of the array can be obtained by using Equation (1-2).

ARRAY REACTION FORCES AND MOMENTS

Array dynamic responses are coupled to the spacecraft base point by the elastic forces produced at the attach point by array relative responses: $[u] = [\sigma][\eta]$

For each generalized coordinate $\eta$, a set of three forces and three moments are produced at Node 1 in the STARDYNE math model. These have been transformed to the spacecraft centerline at Z station 38.0. Thus, for the +Y array,

\[
\begin{bmatrix}
F_x \\
F_y \\
F_z \\
M_x \\
M_y \\
M_z_{base, n^{th} mode}
\end{bmatrix} = \begin{bmatrix}
-VA2 \\
-P \\
VA3 \\
-MA2 \\
-Torque \\
MA3
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
+28.77^\circ VA3 \\
0 \\
+28.77^\circ VA2
\end{bmatrix} \cdot [\eta^n_{n}]
\]

Modal reactions at base point per unit model deflection of +Y elastic array.
For the -Y array, the modal (or generalized) reaction forces at the base point (X = 0.0, Y = 0.0, Z = 38.0) are as follows:

\[
\begin{bmatrix}
  F_x \\
  F_y \\
  F_z \\
  M_x \\
  M_y \\
  M_z
\end{bmatrix}_{\text{base}} = \begin{bmatrix}
  -VA2 \\
  +P \\
  VA3 \\
  +MA2 \\
  -Torque \\
  -MA3
\end{bmatrix} + \begin{bmatrix}
  0 \\
  0 \\
  0 \\
  -28.77\times VA3 \\
  0 \\
  +28.77\times VA2
\end{bmatrix} \cdot [\eta_n]
\]

Modal reactions at base point per unit modal deflection of -Y elastic array.

SIGN CONVENTION

The sign convention for the loads acting on spacecraft and the loads acting at +Y array drive point is shown in Figure 1-4.

Figure 1-4. Structural Analysis Sign Convention
For the +Y array (STARDYNE, Beam 1 loads, end A),

\[
\begin{align*}
F_x &= -VA_2, & +X_1 \text{ tip deflection gives } +F_x \\
F_y &= -P, & +X_2 \text{ deflection gives } +F_y \\
F_z &= VA_3, & +X_3 \text{ tip deflection gives } +F_z \\
M_x &= -MA_2, & +X_3 \text{ tip deflection gives } +M_x \\
M_y &= -\text{Torque,} & +Y \text{ deflection gives } +M_y \\
M_z &= MA_3, & +X_3 \text{ tip deflection gives } -M_z \\
\end{align*}
\]

For the -Y array,

\[
\begin{align*}
F_x &= VA_2, & +X_1 \text{ tip deflection gives } +F_x \\
F_y &= P, & +X_2 \text{ deflection gives } +F_y \\
F_z &= VA_3, & +X_3 \text{ tip deflection gives } +F_z \\
M_x &= MA_2, & +X_3 \text{ tip deflection gives } +M_x \\
M_y &= -\text{Torque} & \text{ } \\
M_z &= -MA_3 \\
\end{align*}
\]

EQUATIONS OF MOTION, FLEXIBLE APPENDAGES COUPLED TO A RIGID SPACECRAFT

The previous derivation may now be extended to a general math model as follows.

The total displacement set for the spacecraft is

\[
[U_a] = \begin{bmatrix} U_a \vspace{-3pt} \\ U_a \end{bmatrix}
\]

where the spacecraft rotations \( \{ \theta_r \} \) are a subset of \( \{ U_p \} \). For no external loads acting on structure, the undamped equations of motion for displacement set \( \{ U_a \} \) become

\[
[M_{aa}] \ddot{U}_a + [K_{aa}] U_a = 0 \tag{I-6}
\]

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Equation I-6 may be partitioned as follows:

\[
\begin{bmatrix}
M_{LL} & M_{LR} \\
M_{LR}^T & M_{RR}
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_L \\
\ddot{u}_R
\end{bmatrix}
+ \begin{bmatrix}
K_{LL} & K_{LR} \\
K_{LR}^T & K_{RR}
\end{bmatrix}
\begin{bmatrix}
u_L \\
\nu_R
\end{bmatrix} = 0
\] (I-7)

where the subset \( \{U_r\} \) of the displacement vector is sufficient to define rigid body motion without additional constraints.

Now introduce the following "hybrid" transformation between modal coordinates \( \{\eta\} \) and physical coordinates \( \{u\} \):

\[
\begin{bmatrix}
u_L \\
\nu_R
\end{bmatrix} = \begin{bmatrix}
\phi_{Lr} & \phi_{Dr} \\
0 & I
\end{bmatrix}
\begin{bmatrix}
\eta_L \\
\eta_R
\end{bmatrix}
\] (I-8a)

Note that

\[
\{U_r\text{ relative to base}\} = [\phi_{Lr}]
\begin{bmatrix} \eta_L \\ \eta_R \end{bmatrix}
\] (I-8b)

where

\( [\phi_{Lr}] = \) matrix of eigenvectors of structure with \( \{U_r\} = 0 \)

\( [D_{Dr}] = \) rigid body transformation matrix

\( = [T] \) in STARDYNE write-up

\( [I] = \) identity matrix

\( \{\eta_L\} = \{\eta_R\} = \) vector of flexible body modal (or generalized) coordinates.
where

\[ M(x)(t), M_y(t), \text{ and } M_z(t) \text{ are jet torques or other external torque applied to spacecraft.} \]

**NUMERICAL VALUES OF FLEXIBILITY COEFFICIENTS**

Table I-1 presents the equations of motion in terms of generalized coordinates which must be integrated for each solar array. The coefficients for the first four modes are presented. For checkout runs, only the first three modes are necessary.

**TABLE I-1. NUMERICAL VALUES FOR GENERALIZED COORDINATE SOLUTION, NDS COARSE MESH SOLAR ARRAYS**

a. +Y Array

\[
[1] \dddot{x} + [2\omega_n^2] \dot{\dot{y}} + [w_n^2] \ddot{y} = -[ ]^T \dddot{\ddot{u}}
\]

\[ [1] = \text{unity matrix}; \quad \zeta = 0.035 \text{ or less} \]

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>( F_n ) (Hz)</th>
<th>( W_n ) (rad/sec)</th>
<th>( \dddot{X} )</th>
<th>( \dddot{Y} )</th>
<th>( \dddot{Z} )</th>
<th>( \dddot{X}^{+} )</th>
<th>( \dddot{Y}^{+} )</th>
<th>( \dddot{Z}^{+} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.074</td>
<td>19.317</td>
<td>0</td>
<td>0.1676</td>
<td>1.501</td>
<td>10.983</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3.491</td>
<td>21.933</td>
<td>1.245</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.7963</td>
<td>-0.59</td>
</tr>
<tr>
<td>3</td>
<td>4.534</td>
<td>28.491</td>
<td>0.0337</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.4983</td>
<td>-0.344</td>
</tr>
<tr>
<td>4</td>
<td>10.05</td>
<td>63.171</td>
<td>0.0483</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.2149</td>
<td>-0.155</td>
</tr>
</tbody>
</table>

b. -Y Array

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>( F_n ) (Hz)</th>
<th>( W_n ) (rad/sec)</th>
<th>( \dddot{X} )</th>
<th>( \dddot{Y} )</th>
<th>( \dddot{Z} )</th>
<th>( \dddot{X}^{+} )</th>
<th>( \dddot{Y}^{+} )</th>
<th>( \dddot{Z}^{+} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>-10.983</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>0</td>
<td>-0.7963</td>
<td>+0.59</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>0</td>
<td>0.4983</td>
<td>0.344</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>0</td>
<td>-0.2149</td>
<td>0.155</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Reference: Equation (I-10).

"STARDYNE DYNRE2 values divided by 12.0 to convert inches to feet."
Table 1-2 presents the model displacements at node point 8 in the STARDYNE model so that the sun sensor displacements and angles relative to the base point can be computed. If only sun sensor rotations are needed, use only the values in the columns titled $\delta X$, $\delta Y$, and $\delta Z$.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>$F_n$</th>
<th>$X$ (ft/ft)</th>
<th>$Y$ (ft/ft)</th>
<th>$Z$ (ft/ft)</th>
<th>$\delta X$ (rad/ft)</th>
<th>$\delta Y$ (rad/ft)</th>
<th>$\delta Z$ (rad/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.074</td>
<td>0</td>
<td>0.0212</td>
<td>1.00</td>
<td>0.0209</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3.491</td>
<td>0.0825</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.0088</td>
<td>-0.0169</td>
</tr>
<tr>
<td>3</td>
<td>4.534</td>
<td>1.000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.2087</td>
<td>-0.0198</td>
</tr>
<tr>
<td>4</td>
<td>10.05</td>
<td>-0.1109</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.1284</td>
<td>0.0125</td>
</tr>
</tbody>
</table>

Reference: Equation (1-7).

Example: $\delta Y_{+Y\text{ array}} = [0, -0.0088, 0.2087, 0.1284]$

where

$\eta_i = \text{generalized displacement, }$

$\eta_i$th mode

$\begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{bmatrix}$

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Tables I-3 and I-4 present the forces and moments applied to a point at the intersection of the solar array pitch drive axis and the spacecraft centerline due to unit generalized deflections of each solar array. These forces and moments should be resolved into the torque summation about the spacecraft cg. For use with elastic arrays, the rigid-body mass of the spacecraft and its cg should not include the solar arrays.

### TABLE I-3a. COARSE MESH MODEL INTERNAL FORCES AND MOMENTS, BEAM 1, NODE 1 FOR FIRST FOUR MODES (STARDYNE)

<table>
<thead>
<tr>
<th>Beam Load Component</th>
<th>Internal Beam Loads</th>
<th>Conversion to Loads Acting on Base Point</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode 1</td>
<td>Mode 2</td>
</tr>
<tr>
<td>VA2, lb</td>
<td>0</td>
<td>-26.619</td>
</tr>
<tr>
<td>P, lb</td>
<td>-1.96</td>
<td>0</td>
</tr>
<tr>
<td>VA3, lb</td>
<td>17.5628</td>
<td>0</td>
</tr>
<tr>
<td>MA2, in-lb</td>
<td>-1036.8</td>
<td>0</td>
</tr>
<tr>
<td>Torque, in-lb</td>
<td>0</td>
<td>+204.335</td>
</tr>
<tr>
<td>MA3, in-lb</td>
<td>0</td>
<td>-1695.2</td>
</tr>
</tbody>
</table>

### TABLE I-3b. FORCES AND MOMENTS AT (X = 0, Y = 0, Z = 38.0 inches) DUE TO UNIT GENERALIZED DISPLACEMENTS

<table>
<thead>
<tr>
<th>Force Component</th>
<th>Units</th>
<th>Internal Beam Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode 1</td>
<td>Mode 2</td>
</tr>
<tr>
<td>FX</td>
<td>lb</td>
<td>0</td>
</tr>
<tr>
<td>FY</td>
<td>lb</td>
<td>1.96</td>
</tr>
<tr>
<td>FZ</td>
<td>lb</td>
<td>17.5628</td>
</tr>
<tr>
<td>MX</td>
<td>ft-lb</td>
<td>128.507</td>
</tr>
<tr>
<td>MY</td>
<td>ft-lb</td>
<td>0</td>
</tr>
<tr>
<td>MZ</td>
<td>ft-lb</td>
<td>0</td>
</tr>
</tbody>
</table>

Reference: Run SSNSOHC

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### TABLE I-4. FORCES AND MOMENTS AT X = 0, Y = 0, Z = 38.0 INCHES DUE TO UNIT GENERALIZED DISPLACEMENTS OF -y ARRAY

<table>
<thead>
<tr>
<th>Force Component</th>
<th>Units</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX</td>
<td>lb</td>
<td>0</td>
<td>26.619</td>
<td>19.044</td>
<td>155.82</td>
</tr>
<tr>
<td>FY</td>
<td>lb</td>
<td>-1.96</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>FZ</td>
<td>lb</td>
<td>17.5628</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>MX</td>
<td>ft-lb</td>
<td>-128.507</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>MY</td>
<td>ft-lb</td>
<td>0</td>
<td>-17.028</td>
<td>281.69</td>
<td>-693.29</td>
</tr>
<tr>
<td>MZ</td>
<td>ft-lb</td>
<td>0</td>
<td>205.09</td>
<td>194.64</td>
<td>501.08</td>
</tr>
</tbody>
</table>

If the solar array Z axis is shifted more than 5 or 10 deg from the spacecraft Z axis due to pitch rotation of the arrays, then the appropriate coordinate transformations are required. In Table I-1, the array participation factors for \( \dot{\bar{X}}, \dot{\bar{Z}}, \dot{\bar{Z}}, \) and \( \dot{\bar{Y}} \) must be rotated. Also the FX, FZ, MX, and MZ forces and moments in Tables I-3b and I-4 must be rotated.

**CHECKOUT RUNS**

Figures I-5 through I-7 represent transient responses obtained at node 8 due to transient angular acceleration pulses.

For Figure I-5, the array pitch torque is assumed to be 1.0 ft-lb about \( \bar{Y} \) for 50 msec. Using the Rockwell L inertia value of 2.2335 slug/ft\(^2\), this causes an angular acceleration of 0.4477 rad/sec\(^2\). A reverse input of -0.153846 rad/sec\(^2\) was applied for 325 msec to kill off the rigid body velocity. This forcing function approximates a solar array drive pulse input.

Two STARDYNE simulations were made, one using the first three modes (up to 4.5 Hz) and a second run using the first four modes (up to 10.0 Hz).
Figure I-5. Relative Angular Displacement in Torsion of NDS Coarse Model to a Single Drive Torque Pulse, STARDYNE DYNREI Solution

Figure I-6. Relative Boom Tip X Displacement, NDS Coarse Model, due to Thruster Jet Pulse, STARDYNE DYNREI Solution

Reference: Run SSNSOQ9
Figure 1-6 presents the simulation for the array excited by the thruster jet firing about the yaw axis. Using an inertia estimate of 100 slug ft and a torque of 0.53 ft-lb, an acceleration of 0.0053 rad/sec about the array Z axis was input into the array base. The X response of node 8 at the tip of the boom in plotted at 0.42 sec is the maximum displacement observed.

Figure 1-7, also for the same yaw thruster pulse, shows the X array torsional response which is coupled with the X deflections due to the deployed boom offset from the pivot axis. The DYNE1 solution was carried out to 2.5 sec and the peak X response plotted at 0.42 sec is the maximum displacement observed.

Reference: Hunseng SSNSQ9124

Figure 1-7. Boom Tip and Sun Sensor Y Rotation Due to Yaw Thruster Pulse. STARDY NE DYNE1 Solution.
Figure 1-8. Relative Boom Tip Displacement, Z, Due to Single Roll Thruster Pulse

Note that these checkout runs were made with the array Z axis oriented along the spacecraft Z axis.

FINE MESH SOLAR ARRAY FLEXIBILITY MODE

After the solar array flexibility structure had been determined and the latest boom and array stiff data obtained, a fine mesh solar array flexibility model was constructed. Table 1-5 presents numerical values for the generalized or modal coordinates of the +Y and the -Y array. (Separate array models are used since they have individual pitch drive control systems and are not physically connected for torsion.)

Table 1-6 presents the appropriate mode shape displacements and rotations at the outer ends of each boom (Y = 104.97 inches) which is close to the sun sensor.

Table 1-7 presents the coupling terms which are used in torque contributions due to array generalized accelerations.
TABLE I-5. NUMERICAL VALUES, GENERALIZED COORDINATE SOLUTION, NDS FINE MESH SOLAR ARRAYS

a. +Y Array

\[
[I]\ddot{\eta} + [2\zeta w_n]\dot{\eta} + [w_n^2]\eta = -[T]\ddot{\upsilon}
\]

\[
[I] = \text{unity matrix; } \zeta = 0.035 \text{ or less}
\]

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>(F_n) (Hz)</th>
<th>(W_n) (rad/sec)</th>
<th>Participation Factors, ([\eta]^T), for Each Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(X)</td>
</tr>
<tr>
<td>1</td>
<td>3.9133</td>
<td>24.588</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3.9138</td>
<td>24.591</td>
<td>1.3305</td>
</tr>
<tr>
<td>3</td>
<td>5.086</td>
<td>31.958</td>
<td>0.3143</td>
</tr>
<tr>
<td>4</td>
<td>9.75</td>
<td>61.289</td>
<td>0</td>
</tr>
</tbody>
</table>

b. -Y Array

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>(F_n) (Hz)</th>
<th>(W_n) (rad/sec)</th>
<th>Participation Factors, ([\eta]^T), for Each Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(X)</td>
</tr>
<tr>
<td>1</td>
<td>3.9133</td>
<td>24.588</td>
<td>-12.159</td>
</tr>
<tr>
<td>2</td>
<td>3.9138</td>
<td>24.591</td>
<td>-0.00039</td>
</tr>
<tr>
<td>3</td>
<td>5.086</td>
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<td>0</td>
</tr>
<tr>
<td>4</td>
<td>9.75</td>
<td>61.289</td>
<td>-0.1847</td>
</tr>
</tbody>
</table>

Reference: Equation (I-5).

STARDYNE DYNRE2 values divided by 12.0 to convert inches to feet.
TABLE I-6. FINE MESH SOLAR ARRAY MODEL, MODE SHAPE AT SUN SENSORS
(Displacement and Rotations per Unit Generalized Displacement)

a. +Y Array

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>$F_n$</th>
<th>Modal Values at Node 8, Fine Mesh Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$X$ (ft/ft)</td>
</tr>
<tr>
<td>1</td>
<td>3.9133</td>
<td>-0.00031</td>
</tr>
<tr>
<td>2</td>
<td>3.9138</td>
<td>0.9613</td>
</tr>
<tr>
<td>3</td>
<td>5.086</td>
<td>0.2432</td>
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<tr>
<td>4</td>
<td>9.75</td>
<td>0</td>
</tr>
</tbody>
</table>

b. -Y Array

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>$F_n$</th>
<th>Modal Values at Node 8, Fine Mesh Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$X$ (ft/ft)</td>
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<tr>
<td>1</td>
<td></td>
<td>-0.01224</td>
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<td></td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>0.00709</td>
</tr>
</tbody>
</table>

*Reference: Equation (I-2).
†Reference: Run SSNSOQY
### TABLE I-7a. NDS FINE MESH SOLAR ARRAY MODEL COUPLING TRANSFORMATIONS FOR SPACECRAFT TORQUE SUMMATIONS DUE TO GENERALIZED ACCELERATIONS, FIRST FOUR MODES

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>General Mass (lb-sec^2/in)</th>
<th>( \frac{M_n}{\gamma_{nr}} ) (Participation Factors)</th>
<th>( \ddot{X} )</th>
<th>( \ddot{Y} )</th>
<th>( \ddot{Z} )</th>
<th>( \ddot{X} )</th>
<th>( \ddot{Y} )</th>
<th>( \ddot{Z} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.028288</td>
<td>0.00608</td>
<td>0.0512</td>
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<td>0.000016</td>
<td>0.00016</td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td>0.044165</td>
<td>0.05876</td>
<td>0.0</td>
<td>0.000208</td>
<td>-0.5072</td>
<td>-5.0756</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.022699</td>
<td>0.0071</td>
<td>0.0</td>
<td>0.0</td>
<td>0.7085</td>
<td>-0.7786</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.011422</td>
<td>0.0032</td>
<td>0.0052</td>
<td>0.0253</td>
<td>0</td>
<td>0</td>
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<td></td>
</tr>
</tbody>
</table>

Divide by 12 if used.

### TABLE I-7b. \( \frac{M_n}{\gamma_{nr}}^T \) FOR TORQUE SUMS DUE TO GENERALIZED ACCELERATIONS

<table>
<thead>
<tr>
<th>Axis</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
<th>Array</th>
</tr>
</thead>
<tbody>
<tr>
<td>TXX</td>
<td>4.1274</td>
<td>0.000208</td>
<td>0</td>
<td>0.0253</td>
<td>+Y</td>
</tr>
<tr>
<td>TYY</td>
<td>0.000016</td>
<td>-0.5072</td>
<td>0.7085</td>
<td>0</td>
<td>-Y</td>
</tr>
<tr>
<td>TZZ</td>
<td>0.00016</td>
<td>-5.0756</td>
<td>-0.7786</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>TXX</td>
<td>-4.1274</td>
<td>-0.000208</td>
<td>0</td>
<td>-0.0253</td>
<td></td>
</tr>
<tr>
<td>TYY</td>
<td>0.000016</td>
<td>-0.5072</td>
<td>0.7085</td>
<td>0</td>
<td></td>
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<tr>
<td>TZZ</td>
<td>-0.00016</td>
<td>5.0756</td>
<td>0.7786</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

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Figures 1-9 through I-11 present geometry plots of the fine mesh math model. The deployment arm and fittings were not changed from the previous model (see Figure I-12). The honeycomb aluminum solar panels were modeled as shown in Figure I-13. Triangular plate elements were used because the STARDYNE quadrilateral plate element cannot model a honeycomb panel.

Each panel has a set of four hinges, similar to door hinges, to allow folding for launch. At each end of the panel a linkage is used to hold the panels out in the deployed position for orbit. These links were modeled by scaling off the drawings, and some error in the torsion mode could be caused by inaccuracies in the linkage model.
Figure I-10. NDS +Y Array Model, X1 Rotated View
Figure I-11. NDS +Y Array Model End View, Looking Down Y Axis
HINGE BEAMS AND LINKAGES

VIEWS OF LINKAGES

OUTER END

INNER END

Figure I-12. NDS Solar Array, Fine Mesh Model
Figure i-13. Math Model--Solar Panels
SUPPORTING DATA

Supporting data used in development of the fine mesh solar array flexibility model is presented below.

Solar Panel

(Reference: Drawing No. V505-93200)

\[ R_o = 32.50 \text{ in.} \]
\[ R_i = 32.00 \text{ in.} \]
\[ R_{\text{mean}} = 32.25 \text{ in.} \]
\[ \text{chord} = 30.59 \]
\[ \phi = \sin^{-1}(30.59/2)/32.25 = 28.3 \text{ deg} \]
\[ t_{\text{core}} = 0.50 \text{ in.} \]
\[ t_{\text{face sheets}} = 0.010 \text{ in.} \]
Face sheets = aluminum \hspace{1cm} E = 10 \times 10^6
core = 1/4 cell Al honeycomb, 2.3 lb/ft³ density
\[ G_c \ (\text{core shear modulus}) = 27,000 \text{ psi average in } "L" \text{ direction} \]
\[ \text{average } G_c = \frac{27,000 + 14,000}{2} = 20,500 \text{ psi} \]
Assume isotropic core properties.
Origin for Array Cylindrical Coordination System

ICYL = 2

X2 = H
A = 0 deg at +X3 axis
X3\text{cg} = Z station = 33.0

approximately 1.25 in. above inside mean radius

R_{mean} = 32.25
X3 = (32.25 - 1.25) + 33.0
= 64.0 in. (Adjust origin after geometry check)

For -X Array

Point A: X1 = -17.7
X2 = 0
X3 = 64.

From angle = 180 deg + 28.32 = 208.32
To angle = 180 deg - 28.32 = 151.68

Point B: X1 = 0.0
X2 = 0.0
X3 = 64.

Point C: X1 = -10.
X2 = +10.
X3 = 64.

For +X Array

Point A: X1 = +17.7
X2 = 0.
X3 = 64.

Point B: X1 = 50.
X2 = 0.
X3 = 64.

Point C: X1 = 30.
X2 = 10.
X3 = 64.
<table>
<thead>
<tr>
<th>VEH</th>
<th>FC</th>
<th>DESCRIPTION</th>
<th>MC</th>
<th>NG</th>
<th>WEIGHT X</th>
<th>Y</th>
<th>Z</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>RXY</th>
<th>RY</th>
<th>RY</th>
<th>PXY</th>
<th>PXY</th>
<th>PXY</th>
<th>PXY</th>
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<td>GPS FPA ARRAY</td>
<td>1 E</td>
<td>6.42</td>
<td>15.5</td>
<td>71.3</td>
<td>32.0</td>
<td>16.1</td>
<td>0.0</td>
<td>9.0</td>
<td>21.1</td>
<td>C.</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>GPS FPA ARRAY</td>
<td>1 E</td>
<td>6.42</td>
<td>15.5</td>
<td>71.3</td>
<td>32.0</td>
<td>16.1</td>
<td>0.0</td>
<td>9.0</td>
<td>21.1</td>
<td>C.</td>
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<td>GPS FPA 30002 SUBSTATE</td>
<td>1 C</td>
<td>6.22</td>
<td>15.5</td>
<td>71.3</td>
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<td>16.1</td>
<td>0.0</td>
<td>9.0</td>
<td>21.1</td>
<td>C.</td>
<td>0.0</td>
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<tr>
<td>GPS FPA 45002 SUBSTATE</td>
<td>1 C</td>
<td>8.22</td>
<td>15.5</td>
<td>71.3</td>
<td>32.0</td>
<td>16.1</td>
<td>0.0</td>
<td>9.0</td>
<td>21.1</td>
<td>C.</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GPS FPA SLR PNL SUBST 1</td>
<td>1 E</td>
<td>0.20</td>
<td>15.5</td>
<td>71.3</td>
<td>32.0</td>
<td>16.1</td>
<td>0.0</td>
<td>9.0</td>
<td>21.1</td>
<td>C.</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>GPS FPA SLR PNL SUBST 1</td>
<td>1 E</td>
<td>0.20</td>
<td>15.5</td>
<td>71.3</td>
<td>32.0</td>
<td>16.1</td>
<td>0.0</td>
<td>9.0</td>
<td>21.1</td>
<td>C.</td>
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<tr>
<td>GPS FPA 2994 SHUNT</td>
<td>1 E</td>
<td>2.00</td>
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<td>66.8</td>
<td>35.0</td>
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<td>0.28</td>
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<td>0.4</td>
<td>C.</td>
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<td>0.1</td>
<td>0.4</td>
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<td></td>
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<td></td>
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<tr>
<td>GPS ABC 340018 STRAP ART</td>
<td>S</td>
<td>1 C</td>
<td>0.20</td>
<td>0.0</td>
<td>71.5</td>
<td>37.5</td>
<td>14.0</td>
<td>9.6</td>
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<td>0.20</td>
<td>0.0</td>
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<td>0.20</td>
<td>0.0</td>
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<td>1 C</td>
<td>0.20</td>
<td>0.0</td>
<td>105.0</td>
<td>37.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>C.</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE I-8. ROCKWELL WEIGHTS DATA**
### Table I-8. Rockwell Weights Data (concluded)

<table>
<thead>
<tr>
<th>Item</th>
<th>HCA 340105 ETO *Y</th>
<th>HCA 340105 COND ASSY</th>
<th>HCA A CR PILOTAIL</th>
<th>HCA L 451201 SCW XCSS</th>
<th>HCA F 451201 SOL THERM</th>
<th>HCA F 451201 SOL J-BOX</th>
<th>HCA L 451201 SOL PROW</th>
<th>HCA I 451201 SOL DRLMT</th>
<th>HCA I ANSOPES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass, Lbs</td>
<td>54.66</td>
<td>67.0</td>
<td>0.0</td>
<td>71.5</td>
<td>71.5</td>
<td>71.5</td>
<td>71.5</td>
<td>71.5</td>
<td>71.5</td>
</tr>
<tr>
<td>Height, in.</td>
<td>28.0</td>
<td>28.9</td>
<td>28.9</td>
<td>28.9</td>
<td>28.9</td>
<td>28.9</td>
<td>28.9</td>
<td>28.9</td>
<td>28.9</td>
</tr>
</tbody>
</table>

**TOTAL HEIGHT = 57.96**

<table>
<thead>
<tr>
<th>X Moment (in. Lbs in. Lbs)</th>
<th>-15.27</th>
<th>XCG = 0.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y Moment (in. Lbs in. Lbs)</td>
<td>3725</td>
<td>YCG = 0.0</td>
</tr>
<tr>
<td>Z Moment (in. Lbs in. Lbs)</td>
<td>0.0</td>
<td>ZCG = 0.0</td>
</tr>
</tbody>
</table>

**STATES:****POUNDS PERCENT OF TOTAL WEIGHT**

<table>
<thead>
<tr>
<th>Item</th>
<th>Estimated</th>
<th>Actual</th>
<th>Calculated</th>
<th>High Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass, Lbs</td>
<td>54.66</td>
<td>67.0</td>
<td>71.5</td>
<td>71.5</td>
</tr>
<tr>
<td>Weight, Lbs</td>
<td>57.96</td>
<td>57.96</td>
<td>57.96</td>
<td>57.96</td>
</tr>
</tbody>
</table>

** Moments of Inertia:**

<table>
<thead>
<tr>
<th>Component</th>
<th>Moment of Inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td>IX =</td>
<td>43934.1172</td>
</tr>
<tr>
<td>IY =</td>
<td>10347.7539</td>
</tr>
<tr>
<td>IZ =</td>
<td>53453.6016</td>
</tr>
</tbody>
</table>

** Products of Inertia:**

<table>
<thead>
<tr>
<th>Component</th>
<th>Product of Inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td>IXY =</td>
<td>353.2314</td>
</tr>
<tr>
<td>IYZ =</td>
<td>-50.2000</td>
</tr>
<tr>
<td>IZX =</td>
<td>-50.2000</td>
</tr>
<tr>
<td>IYX =</td>
<td>0.0108</td>
</tr>
<tr>
<td>IYZ =</td>
<td>-0.0108</td>
</tr>
<tr>
<td>IZX =</td>
<td>-0.0108</td>
</tr>
</tbody>
</table>
One Solar Panel (-X Array)

Weight = 15.34 lb excluding boom

\[
\begin{align*}
X_{cg} &= -15.5 = X_1 \\
Y_{cg} &= 71.3 = X_2 \\
Z_{cg} &= 33.0 = X_3
\end{align*}
\]

STARDYNE Notation

Total area, one panel = 2133.576 in\(^2\)

Modify material density (MATLG 2 card) to make face sheet density match total density.

Total volume, face sheets = AREA \times 0.020 in.

= 2133.576 \times 0.020

= 42.67152 in\(^3\)

Composite gross density \(= \frac{15.34 \text{ lb}}{42.67152 \text{ in}^3} = 0.35949 \text{ lb/in}^3\)

Solar Panel Distributed Weights

Honeycomb core material = 2.3 lb/ft\(^3\)

\[
\rho = \frac{2.3}{1728} = 0.001331 \text{ lb/in}^3
\]

\[
\begin{align*}
t_c &= 0.50 \text{ in. (core)} \\
t_{\text{face sheet}} &= 0.010 \text{ each side}
\end{align*}
\]

\(2t_f = 0.020 \text{ in at } 0.10 \text{ lb/in}^3\)

Composite density based on face sheet thickness is, for 1 square inch of panel,

\[
\begin{align*}
\rho_{\text{face}} &= 0.10 \text{ lb/in}^3 \times 0.020 \text{ in} = 0.002 \text{ lb} \\
\rho_{\text{core}} &= 0.001331 \times 0.50 = \frac{0.0006655}{1000} \text{ lb}
\end{align*}
\]
Increase density of face sheets:

\[
\zeta = \frac{0.0028655}{0.002} \times 0.10 = 0.13328 \text{ lb/in}^3
\]

to account for core material and face sheets.

**Distributed Weight, One Panel**

**AREA** = 72 tri-plates x 29.633 in² = 2133.576 in²

From weights statement, -X panel

GPS FAA ARRAY = 6.92 lb

FAA 450002 SUBSTRATE = 8.22 lb

FDB SLR PNL SUBSTRATE = \( \frac{0.20}{15.34} \) lb

**Weight Shifts on Solar Panel** (STARDYNE Run SSNS05A)

(-X panel geometry check)

WT = 15.34 lb

\[
\begin{align*}
X_{cg} &= -17.7 \text{ in.} \\
Y_{cg} &= 71.47 \text{ in.} \\
Z_{cg} &= 33.1173 \text{ in.}
\end{align*}
\]

Desired cg:

\[
\begin{align*}
X_{cg} &= -15.5 \text{ in.} \\
Y_{cg} &= 71.3 \text{ in.} \\
Z_{cg} &= 33.0 \text{ in.}
\end{align*}
\]

The model can be simulated, from the distributed weights above, as follows:
### Node No. Weight (lb) X, in. W \cdot X

<table>
<thead>
<tr>
<th>Node No.</th>
<th>Weight (lb)</th>
<th>X, in.</th>
<th>W \cdot X</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>0.10653</td>
<td>-33.0</td>
<td>-3.51549</td>
</tr>
<tr>
<td>41</td>
<td>0.21306</td>
<td>-7.0</td>
<td>7.03098</td>
</tr>
<tr>
<td>61</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>81</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>101</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>121</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>141</td>
<td>0.10653</td>
<td>-33.0</td>
<td>-3.51549</td>
</tr>
</tbody>
</table>

Totals: 1.27836 lb \cdot -42.18588 lb

\[ \bar{W} \cdot \bar{X} = 15.34 \times -17.7 = -271.518 \]  - current

\[ \text{desired } \bar{W} \cdot \bar{X} = 15.34 \times -15.5 = -237.77 \]

\[ \bar{W} \cdot \bar{X} \text{ is } 33.748 \]

\[ W_{\text{edge}} \cdot (X_{\text{new}} - X_1) = 33.748 \text{ in-lb} \]

\[ (X_{\text{new}} - (-33.0)) = \frac{33.748}{1.27836} = 26.3994 \text{ in.} \]

\[ X_{\text{new}} = 26.39945 - 33.0 = -6.6005 \text{ in.} \]

### Weight Shift, -X Panel (X-Direction Shift)

\[ W_{\text{edge}} = 1.27836 \text{ lb} \quad X_{\text{edge}} = -33.0 \text{ in.} \]

\[ W_{\text{panel}} \cdot \bar{X}_c = 15.34 \times -17.7 = -271.518 \text{ (current)} \]

\[ W_{\text{panel}} \cdot \bar{X}_{\text{desired}} = 15.34 \times -15.5 = -237.77 \text{ (desired)} \]

\[ \Delta W \cdot X = 33.748 \text{ (to match Rockwell weight report)} \]

\[ W_1 = a \cdot W_{\text{edge}} = \text{weight left at outer edge of panel} \]

\[ W_2 = (1 - a)W_{\text{edge}} = \text{weight shifted to inner edge of panel} \]

\[ W_2 (X_{\text{new}} - X_{\text{old}}) = \Delta W \cdot X \]

\[ W_2 (-2.400744 + 32.99926) = 33.748 \]

\[ W_2 (30.59852) = 33.748 \text{ in-lb} \]

\[ W_2 = 1.102929 \text{ lb} \]
Subtract 1.102929 lb from outer edge.
Add 1.102929 lb to inner edge.

<table>
<thead>
<tr>
<th>Node No.</th>
<th>Node No.</th>
<th>ΔW</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>37</td>
<td>-0.09191075</td>
</tr>
<tr>
<td>41</td>
<td>57</td>
<td>-0.1838215</td>
</tr>
<tr>
<td>61</td>
<td>77</td>
<td>-0.09191075</td>
</tr>
<tr>
<td>81</td>
<td>97</td>
<td>-0.09191075</td>
</tr>
<tr>
<td>101</td>
<td>117</td>
<td>-0.09191075</td>
</tr>
<tr>
<td>121</td>
<td>137</td>
<td>-0.09191075</td>
</tr>
<tr>
<td>141</td>
<td>157</td>
<td>-0.09191075</td>
</tr>
</tbody>
</table>

Panel

Subtract 1.102929 lb from outer edge.
Add 1.102929 lb to inner edge.

<table>
<thead>
<tr>
<th>Node No.</th>
<th>Node No.</th>
<th>ΔW</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>31</td>
<td>+0.09191075</td>
</tr>
<tr>
<td>47</td>
<td>51</td>
<td>+0.1838215</td>
</tr>
<tr>
<td>67</td>
<td>71</td>
<td>+0.09191075</td>
</tr>
<tr>
<td>87</td>
<td>91</td>
<td>+0.09191075</td>
</tr>
<tr>
<td>107</td>
<td>111</td>
<td>+0.09191075</td>
</tr>
<tr>
<td>127</td>
<td>131</td>
<td>+0.09191075</td>
</tr>
<tr>
<td>147</td>
<td>151</td>
<td>+0.102929</td>
</tr>
</tbody>
</table>

Panel

Aluminum Tube, Strut Assembly (Deployment Arm)

Major portion is machined down to

\[ I_{x-x} = \pi R^3 t \]
\[ = \pi (2.54/2)^3 \times 0.040 \]
\[ = 0.2574 \text{ in}^4 \]

\[ I_{z-z} = I_{x-x} = 0.2574 \]

\[ A = \pi d t = 0.314 \text{ in}^2 \]

\[ I_{yy} = ZI_{xx} = 0.52 \text{ in}^4 \]

BPROPI No. 1

ID = 2.500 in, diameter
\[ t_{\text{wall}} = 0.040 \text{ in.} \]

mean diameter = ID + \( t_{\text{wall}} \)
\[ = 2.540 \text{ in.} \]

BEAM 3 SECTION PROPS

BPROP 1. No. 2

\[ I_y = (Ad^2)(2) = (1.5)(0.25)(2.12)^2 = 3.37 \]

\[ I_z = BH^3/12 = (0.50)(1.50)^3/12 = 0.140 \]

\[ I_x = \left\{ ab^3 \frac{16}{3} - 3.36 \frac{b}{a} \left[1 - \frac{b^4}{12a^4}\right]\right\} \frac{Z}{2} \]

\[ a = \frac{1.50}{2} = 0.75 \]

\[ b = \frac{0.25}{2} = 0.125 \]

\[ I_x = 0.014 \quad \text{ignores } 2 \times K \cdot d^2 \text{ stiffness} \]

\[ A = (1.50)(0.25)(2) = 0.75 \text{ in}^2 \]

\[ 3 = \frac{TL}{JG} = T \times K_d \]

\[ K_d = 9.952 \times 10^6 \text{ in}-\text{lb}/\text{radian} \]

Use matrix addition for \( \xi \)X3, nodes 3 and 4.

\[ K_1 = \frac{3EI}{Z} = \frac{3 \times 10.0 \times 10^6 \times 0.140}{1.5^3} = 1.244 \times 10^6 \]

\[ K_2 = 2 \times K \cdot d^2 = 2 \times 1.244 \times 10^6 \times 2^2 \]
Aluminum Channel Beam Along Long Sides of Panel

Along hinge side (between hinges):

**ALONG HINGE SIDE (BETWEEN HINGES)**

\[
\begin{align*}
X2 & \quad \text{BEAM} \\
\cdot040 \\
3 \text{ PLCs.} \\
\end{align*}
\]

\[
\begin{align*}
1 & \quad d_1 \\
2 & \\
3 & \quad X3 \quad \text{BEAM} \\
\end{align*}
\]

\[
d_1 = 0.42 \text{ in}
\]

**Segment** | **Area** | **A \times X2** | **X2 - \overline{X2}** | **A(X2 - \overline{X2})^2** | **I_{0X3} - X3** | **X2**
--- | --- | --- | --- | --- | --- | ---
\(1 + 2\) | 0.63 \times 0.080  \\
\(= 0.0504\) | 0.015875 | 0.07375 | 0.00027413 | \(\frac{1}{12} (0.08)(0.63)^3\)  \\
\(= 0.0016698\) | 0.315 | 3 \times 0.63 \times 0.040  \\
\(= 0.0168\) | 0.000336 | -0.22125 | 0.00082239 | \(\frac{1}{12} \times 0.42(0.040)^3\)  \\
\(=\) | 0.020 | Totals | 0.0672 | 0.016212 | 0.0010865 | 0.0016692 | 0.00292 in^4

Area = 0.0672 in^2

\[
\overline{X2} = \frac{0.016212}{0.0672} = 0.24125
\]

\[
I_{X2} = \frac{1}{12} \times 0.63 \times 0.50^3 = \frac{1}{12} \times 0.59(0.42)^3
\]

\[
= 0.0065625 - 0.00364266
\]

\[
I_{X2} = 0.00292 \text{ in}^4
\]

\[
J = \frac{1}{3} (0.63 + 0.63 + 0.42)(0.040)^3
\]

\[
= 0.000358 \text{ in}^4
\]

*BPROP1 No. 11, Reference: Rockwell Drawing No. V505-450003.
Math Model--Solar Panels

0 = Guyan Node Points
For reducing mass matrix
For modes

104.97

X1

X2

BOOM
EFFECT OF DRIVE LINK ADDITION

Because of the discrepancy in model versus test torsional frequency, the effect of adding a flexible torsion lock or drive link was investigated using the coarse mesh model. Figure I-14 shows the modified model with the flexible drive link introduced with nodes 28 and 29.

Table I-9 presents the results of such modification on natural frequencies. The torsional frequency was reduced from 6.84 Hz to from 5.13 to 6.16 Hz, depending on the model of the drive link used. Although test and model frequencies were still in slight disagreement, these results indicate that the torsional frequency would be higher in flight than the test indicates.

<table>
<thead>
<tr>
<th>BEAM #</th>
<th>PROPERTY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>8-11</td>
<td>5</td>
</tr>
<tr>
<td>53</td>
<td>5</td>
</tr>
<tr>
<td>54</td>
<td>8</td>
</tr>
</tbody>
</table>

Figure I-14. Present Coarse Model with Flexible Drive Link
TABLE 1-9. STARDYNE FREQUENCIES OF MODIFIED MODEL

<table>
<thead>
<tr>
<th>Mode Shape Description</th>
<th>Test Frequency (Hz)</th>
<th>STARDYNE Model Frequencies (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending, normal to panel Z-direction</td>
<td>4.1</td>
<td>3.07 3.75 3.75 3.75</td>
</tr>
<tr>
<td>Bending, in plane to panels X-direction</td>
<td>4.3</td>
<td>3.49 3.75 3.70 3.74</td>
</tr>
<tr>
<td>Torsion about Y-axis</td>
<td>3.1</td>
<td>4.53 6.84 5.13 6.16</td>
</tr>
</tbody>
</table>

Condition:

A  From reference (old boom stiffnesses).
B  With new boom stiffnesses.
C  With drive-link addition.
D  With expanded drive-link addition.
APPENDIX J

COMBINED EARTH SENSOR MODEL
APPENDIX J

COMBINED EARTH SENSOR MODEL

INTRODUCTION

This appendix describes the model developed for the GPS combined earth sensor. The model can be used to evaluate acquisition as well as normal operation.

This model is based on information obtained from Barnes Engineering Company at a meeting in St. Petersburg, Florida on March 13, 1975, and a subsequent phone call, and the Rockwell procurement specification for the sensor.

SENSOR DESCRIPTION

The Combined Earth Sensor is a two-axis, static sensor which incorporates two horizon crossing indicators for spinning mode attitude sensing. The adjective static means the sensor uses no moving parts in the measurement of attitude.

The two-axis sensor consists of an optical system, 24 thermopile detectors, and processing electronics. The 24 detectors are arranged on the focal surface of the optical system. The earth image is tangent to the inner edges of the A detectors at null at the design altitude. Two roll and two pitch measurements are computed sequentially by switching sums and differences of various detector outputs to the input of an integrator. Each measurement is sampled and held for comparison check and possible output. A new roll and new pitch measurement is computed each 1/4 second. Roll and pitch computations are separated by 1/8 second. The S detectors are for space reference and are used with the A detectors for accurate
measurements over $+1^\circ$ in the track mode. The A and B detectors are used in the acquisition mode for $+4^\circ$ range. The thermopile detectors have a time constant of 1.4 seconds and are sensitive to the 14 to 16 micron wavelengths ($\text{CO}_2$ band). The sensor continuously tests all signals and rejects spurious or incorrect signals due to sun, moon, or other anomalies.

The two horizon crossing indicators consist of 3 degree by 3 degree square fields of view. One diagonal of each square is parallel to the scan path. One field is $10^\circ$ from the $-X$ axis toward $+Z$ and the other is $10^\circ$ from $-X$ toward $-Z$ (Spin rotation is about the Z axis). The detectors are pyroelectric with insignificant time constant.

MODEL DESCRIPTION

The model consists of an initialization and two normal operation sections. The initialization section performs one-time computations and clears outputs and counters. One normal operation section is for the horizon crossing indicators used in the spin phase. The other normal operation section is for two-axis earth sensing.

Characteristics of the horizon crossing indicator model are as follows:

- Inputs to the model are spacecraft position vector from center of earth, and spacecraft attitude direction cosine matrix. Both inputs must be relative to the same reference frame.
- When a sensor line of sight crosses the edge of the earth, the time from the crossing to the sample time is output. If no crossing occurs in the sample interval, the output is set to $-\Delta t$.
- If a sensor line of sight crosses the earth very close to a tangent, there may not be an output. The sample, $\Delta t$, should be small.
enough so that this loss of output does not happen when attitude
determination is critical. Select $\Delta t$ as follows:

$$\Delta t < \frac{1}{W_s} \sqrt{1 - \frac{\ell}{r_e}}^2 \tan^{-1}\left[ \frac{1}{2 \left(1 - \frac{h}{r_e}\right)} \right]$$

where

- $W_s$ = spin rate in rad/sec
- $W_{RPM} = \frac{2\pi}{60} W_{RPM}$
- $W_{RPM}$ = spin rate in rev/min
- $h$ = satellite altitude in n. mi. (100 < $h$ < 10897)
- $r_e$ = earth radius in n. mi. (3440)
- $\ell$ = distance from earth center to chord of detector
  path in n. mi.

E.g.

let

- $W_{RPM} = 100$
- $h = 10897$
- $r_e = 3440$
- $\ell = 3405.6 = 0.99 r_e$

then $\Delta t < 0.00326$ seconds.
The equations used in the flow diagram are derived below:

LOS is a unit vector

\[ \frac{\mathbf{R} \times \mathbf{LOS}}{\mathbf{R} \cdot \mathbf{LOS}} = \sin(\text{ANG2}) \]

\[ \frac{\mathbf{R} \cdot \mathbf{LOS}}{\mathbf{R} \cdot \mathbf{LOS}} = -\cos(\text{ANG2}) \]

\[ \text{ANG2} = \tan^{-1} \left( \frac{\sin(\text{ANG2})}{\cos(\text{ANG2})} \right) \]

\[ \text{ANG2} = \tan^{-1} \left( \frac{\mathbf{R} \times \mathbf{LOS}}{-\mathbf{R} \cdot \mathbf{LOS}} \right) \]
\[ \text{ANG1} = \sin^{-1} \left( \frac{r_e}{|\mathbf{R}|} \right) \]
\[ = \tan^{-1} \left( \frac{r_e/|\mathbf{R}|}{\sqrt{1 - (r_e/|\mathbf{R}|)^2}} \right) \]
\[ = \tan^{-1} \left( \frac{r_e}{\sqrt{|\mathbf{R}|^2 - r_e^2}} \right) \]

\[ \text{ANG3} = \text{ANG2} - \text{ANG1} \]

Note that \[3440 + 100 \leq |\mathbf{R}| \leq 10897 + 3440\] n. mi.
\[13.8^\circ \leq \text{ANG1} \leq 76.4^\circ.\]

Characteristics of the two-axis earth sensor model are as follows:

- The sensor output is updated every 1/8 second.
- The sensor model has four states which are:
  
  \begin{align*}
  \text{State 1} & \quad \text{Compute Roll 1, Output Pitch 2 if valid} \\
  \text{State 2} & \quad \text{Compute Pitch 1, Output Roll 1 if valid} \\
  \text{State 3} & \quad \text{Compute Roll 2, Output Pitch 1 if valid} \\
  \text{State 4} & \quad \text{Compute Pitch 2, Output Roll 2 if valid}
  \end{align*}

  The state is advanced every 1/8 second.
- Each detector area is divided into \(1^\circ \times 1^\circ\) squares. The area coverage of each \(1^\circ \times 1^\circ\) square by the earth is computed from the distance between center of the earth circle on the focal surface and the center of the square. The orientation of the square is not considered.
- The position of the earth image on the focal surface (B&C) is computed from true attitude. The diameter of the earth image on the focal surface (RE) is computed from satellite altitude. Measurements on the focal surface are in equivalent radians.
Each detector output is the sum of the coverages of $1^0 \times 1^0$ squares within the detector.

The sensor output is computed from the detector outputs according to the functions in the electronics.

Functions of the system programmer to reject spurious or incorrect signals is included.

Subroutine must be initialized by calling with $MODE = 0$ prior to operating in two-axis mode.

The time increment, $\Delta t$, should be selected from one of the following to synchronize with the sensor update frequency:

<table>
<thead>
<tr>
<th>Recommended $\Delta t$</th>
<th>Increments per Update</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.125</td>
<td>1</td>
</tr>
<tr>
<td>0.0625</td>
<td>2</td>
</tr>
<tr>
<td>0.025</td>
<td>5</td>
</tr>
<tr>
<td>0.0125</td>
<td>10</td>
</tr>
<tr>
<td>0.01</td>
<td>12.5</td>
</tr>
<tr>
<td>$\leq 0.01$</td>
<td>$\geq 12.5$</td>
</tr>
</tbody>
</table>

Synchronization error $0.005$ sec. is insignificant

EARTH RADIANCE VARIATION

The variation of radiance over earth surface for the 14-16 $\mu\text{m}$ CO$_2$ band has been calculated in Figure J-1. Plots of the radiance as function of latitude and time of year for various altitudes is shown in Figure J-2. The model for GPS simulation is deduced from these results. For maximum radiance variation a January situation is considered. The radiance is assumed to be a function of latitude but not longitude. The radiance variation for southern hemisphere is assumed to be that for northern hemisphere six months later. The radiance variation for earth disk is approximated by piecewise linear segments using the values corresponding to 20 km altitude shown in Figure J-2. The radiance values and the normalized variations are summarized
Figure J-1. Radiance CO₂ 14-16μ
\[ v = y_1 + L = y_1 + \frac{\beta-x_1}{x_2-x_1} (y_2-y_1) \]
\[ \frac{L}{y_2-y_1} = \frac{\beta-x_1}{x_2-x_1} (y_2-y_1) \]

Figure J-2. Normalized Earth Radiance Variation with Latitude at January
in Table J-1. The normalized variation is used to be compatible with the existing earth sensor simulation that does not include the details of sensor optics and filter response. The radiance for the 15° latitude band is taken as that corresponding to a 220°K earth which, in conjunction with the sensor optics and filter response, provides 3.15 Jw when the "A" detector FOV is fully illuminated.

**TABLE J-1. EARTH RADIANCE VARIATION WITH LATITUDE AND TIME OF YEAR**

<table>
<thead>
<tr>
<th>LAT.</th>
<th>MON.</th>
<th>RADIANCE $W/M^2$</th>
<th>NORMALIZED VARIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>-75°</td>
<td>JAN</td>
<td>4.13</td>
<td>0.72</td>
</tr>
<tr>
<td>-60°</td>
<td>JAN</td>
<td>4.63</td>
<td>0.80</td>
</tr>
<tr>
<td>-45°</td>
<td>JAN</td>
<td>4.88</td>
<td>0.85</td>
</tr>
<tr>
<td>-30°</td>
<td>JAN</td>
<td>5.5</td>
<td>0.96</td>
</tr>
<tr>
<td>415°</td>
<td>JAN/JUL</td>
<td>5.75</td>
<td>1.00</td>
</tr>
<tr>
<td>30°</td>
<td>JUL</td>
<td>5.94</td>
<td>1.03</td>
</tr>
<tr>
<td>45°</td>
<td>JUL</td>
<td>6.19</td>
<td>1.08</td>
</tr>
<tr>
<td>60°</td>
<td>JUL</td>
<td>6.44</td>
<td>1.12</td>
</tr>
<tr>
<td>75°</td>
<td>JUL</td>
<td>6.94</td>
<td>1.21</td>
</tr>
</tbody>
</table>
Implementation of the earth radiance variation into the sensor model requires the following computations which are based on the geometry given in Figure J-3.

(a) LOS-vector for detector cell element centered at \((X_a, Y_a)\):

\[
U = \sqrt{X_a^2 + Y_a^2}
\]

\[
X_s = \frac{X_a \sin U}{U}
\]

\[
Y_s = \frac{-Y_a \sin U}{U}
\]

\[
Z_s = \cos U
\]

Denote

\[
(L_a^s) = \begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix}
\]

(b) Look point on earth surface along \(L_a\):

\[
(L_a^s) = T_{IB}^T S_B^I (L_a^s)
\]

\[
(L_a^I) = (r_a^I) + S (L_a^s)
\]

where

\[
(L_a^s) = \text{look point on earth surface along } L_a.
\]

\[
(r_a^I) = \text{vehicle position from center of earth}
\]

\[
S = \text{slant range from } R_V \text{ to } R_a
\]

\[
S = -r_v^I (L_a^s) - \gamma_e \left( -r_v^I (L_a^s) + (r_v^I (L_a^s))^2 \right)^{1/2}
\]

\[
\gamma_e = 3590.95 \text{ n. mi.}
\]
Figure J-3. Sensor Model Geometry
(c) Look point latitude:

\[
(R_a^E - E) = T_{EI} (R_a^I)
\]

Denote

\[
(R_a^E - E) = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}
\]

\[
\beta = \sin^{-1} \left( \frac{R_3}{\gamma_e} \right)
\]

= look point latitude

(d) Scaling factor for radiance variation:

\[ V = F(\beta) \]

where \( F(\beta) \) is the piecewise linear function defining the radiance variation with latitude; \( F(\beta) \) is plotted in Figure J-1.

(e) The value of AREA is modified from the original value by the scaling factor, i.e.:

\[ \text{AREA} \rightarrow \text{AREA} \times V \]

Figure J-4 presents a comparison of the GPS simulation radiance model and a model described in a Barnes Engineering document, "Technical Description Model 13-16901 and Model 13-16903 Synchronous Altitude Static-Sensors", 22 August, 1975. Models agree reasonably well.
Figure J-4. Comparison of Models
SENSOR ERROR MODELING

The sensor error modeling considered here includes detector and pre-amp noises, electronics parameter shift error, error due to temperature gradient across detector and sensor alignment error. Horizon uncertainty errors, though not a sensor originated error, are also included in the consideration. These are errors that exist in all operational situations.

Errors due to sun or moon in sensor detector field of view and errors resulted from variation of earth disk size due to vehicle orbit eccentricity exist only for a specific operational condition and are not included here. The detector and preamplifier noise has been calculated to be 0.379 μV (3σ) for sensor in track mode (A detectors only) and 0.464 μV (3σ) for sensor in acquisition mode (A+B detectors) with a detector scale factor of 3.15 μW/deg. and responsivity of 20 V/W. These noises are equivalent to 0.0060 deg. (3σ) and 0.0074 deg. (3σ) respectively. Use the output scale factors of 5 V/deg and 1.25 V/deg for track and acquisition modes respectively. The noise at output then becomes 0.03 V(3σ) and 0.0093 V(3σ). The noise error will be simulated as white noise.

Electronics parameter shift has been analyzed by Barnes, Reference J-2. The values at initial setting and at end of life have been calculated to be 0.0034 deg. (3σ) and 0.0219 deg. (3σ) respectively. These correspond to output errors of 0.017 V (3σ) and 0.1095 V (3σ) for track mode and 0.0043 V (3σ) and 0.0274 V (3σ) for acquisition mode. The error due to electronics parameter shift will be simulated as bias errors.

The temperature gradient considered by Barnes amounts to a temperature difference of 0.00048°F between A and S reference junctions. The corresponding attitude error evaluated for temperature of 70°C is 0.0103° or 0.0515 V
for track mode. For acquisition mode, the temperature change of 0.0028 deg. between detectors is calculated to cause attitude error of 0.0029 deg. which amounts to output error of 0.0035 V. All values quoted above are considered to be for worst case and will be treated as $3\sigma$ values. The error due to temperature gradient will be simulated as bias errors.

The achievable static sensor alignment accuracy has been estimated by Barnes to be in the order of 0.015 deg. ($3\sigma$). The equivalent output error is 0.075 V ($3\sigma$) for track mode and 0.019 V ($3\sigma$) for acquisition mode. The effect of alignment error will be simulated as bias errors. The horizon uncertainty error has been treated as an exponentially correlated noise with standard deviation 0.88 Km, correlation time 10 days and correlation distance 2500 N. Mi in Reference J-1, i.e.,

$$E[h(t,s)h(t+T, S+d)] = c_h 2 e^{-T/10} e^{-d/2500}$$

where

- $c_h = 0.88$ Km
- $T =$ time in days
- $d =$ distance in n. mi.

The term $e^{-T/10}$ can be approximated by 1 if $T$ is much less than 10 days. Factoring in the GPS orbit altitude of 10897 N. Mi the horizon uncertainty error amounts to 0.0020 deg. of earth disk angle error. In the evaluation of $P_1$ and $R_1$ detectors at opposite sides of the earth disk are involved. The separation between the look points of these detectors is:

$$d = \frac{(180^\circ - 14^\circ) \times 180^\circ}{180^\circ} \times 3461.6 \text{ N. Mi.}$$

$$= 10029 \text{ N. Mi.}$$
The separation is approximately four times the correlation distance. Hence the effects of horizon uncertainty error at opposing detectors are essentially uncorrelated. The equivalent error in evaluation of $P_1$ and $R_1$ is then $0.0020^\circ \times \sqrt{2} = 0.0028^\circ$ (lσ) or $0.0085^\circ$ (3σ). The output error becomes $0.0424$ V for track mode and $0.0106$ V for acquisition mode. The same values can be used for $P_2$ and $R_2$. (The distance between look points of every other detector cell is approximately 5383.1 N.Mi. The correlation of horizon uncertainty at points separated by this distance is only 0.12. It is neglected for simplicity of model.) For vehicle points at nadir and spins at a yaw rate of $W$ RPM, the sensor detector scans the earth surface at a rate of $358.87$ W N.Mi/sec. The 2500 N. Mi correlation distance would then be equivalent to a correlation time of $6.97/W$ sec.

The sensor output error $e(t)$ due to the horizon uncertainty will be modeled as an exponentially correlated noise as follows:

$$E[e(t) e(t+T)] = \sigma_0^2 e^{-T/T_c}$$

where

$\sigma_0 = 0.0424$ V for track mode
$\sigma_0 = 0.0106$ V for acquisition mode

$T_c = 6.97/W$ sec
$T_c = 3.485$ sec for $W = 2$ RPM

The above model will be used to account for the effect of horizon uncertainty even for cases where vehicle is not quite pointing at the nadir. (The change of correlation time due to the pointing offset from nadir by ± $4^\circ$ is expected to be small. For large offset from nadir the effect of detector not seeing the earth disk will be much more significant than the effect of horizon uncertainty.)

In summary, the sensor error will be simulated as an additive output error consists of a white noise component, a bias component and an exponentially
correlated random noise. The white noise is used to simulate the effect
of detector and preamplifier noises. The equivalent attitude error of the
white noise is $0.0060^\circ (3\sigma)$. The exponentially correlated noise is used to
simulate the effect of horizon uncertainty error. The equivalent attitude
error is $0.0085^\circ (3\sigma)$. The correlation time of this error for the vehicle
spin rate of 2 RPM at initiation of earth acquisition is 3.485 sec.

The bias error is introduced to account for the total effects of electronics
error, temperature gradient error and sensor alignment error. The equivalent
attitude error is $0.0285^\circ (3\sigma)$ evaluated as the RSS value of each individual
effects.

**CONCERNS**

- The detector geometry and output computations may not be correct
  since Barnes has two different designs. The one used here (Figure
  J-5) is one of their designs. The other has detector pattern rotated
  $22^\circ$ about the line of sight from the one shown here.

- The output update frequency for GPS is not certain. The one
  used here is the one proposed by Barnes for DSCS III.

- Earth image distortion and focus are not modeled. Excluding radiance,
  the earth image is assumed to be a perfect circle on the focal surface.

- The detector output approximation using $1^\circ \times 1^\circ$ squares causes
  very little error near null (normal operation) and up to $0.15^\circ$
  maximum detector error at extreme attitudes (during acquisition).
  The extent of the errors near null and the sensor output errors
during acquisition has not been analyzed. The approximation
errors off null are on the order the errors due to earth radiance
variation and earth image quality. Smaller squares may be used to
improve accuracy but computer cost will go up.
Figure J-5. Detector Geometry--2-Axis Earth Sensor
NUMBERING OF X-Y COORDINATES FOR POSITION OF 1° x 1° SQUARES.

X-Y COORDINATES OF ALL 160 1° x 1° SQUARES ARE OBTAINED FROM THESE 20 STORED VALUES

Figure J-5. Detector Geometry--2-Axis Earth Sensor (continued)
NUMBERING OF $A$ ELEMENTS

Figure J-5. Detector Geometry—2-Axis Earth Sensor (continued)
Figure J-5. Detector Geometry—2-Axis Earth Sensor (continued)
Figure J-5. Detector Geometry—2-Axis Earth Sensor (concluded)
Sun and moon interference are not included in this model description.

**FLOW DIAGRAM**

The combined earth sensor model flow diagram is shown in Figure J-6. The left arrow (→) within boxes indicates replacement, i.e., the expression on the right replaces the variable on the left. Mathematical subscripts are used. Symbols used in the flow diagram are defined in the sections below. Comments appear outside the boxes to describe the operations within the boxes.

**INITIALIZATION DATA**

Table J-2 lists the initialization parameters, nominal values, tolerance, and source of data. This data is to be input at the start of each run.

**INPUT DATA**

Table J-3 lists the input data the earth sensor model expects from the calling program each time the subroutine is executed in the normal mode.

**OUTPUT DATA**

Table J-4 lists the output data from the earth sensor model subroutine. Other subroutine variables may be outputs if the user desires.

**INTERNAL VARIABLES**

Internal variables used in the flow diagram are defined in Table J-5.
Initialize
time counters

Clear detector
outputs and saved detector
area coverages

Input values for $X_1$ thru $X_{20}$
$Y_1$ thru $Y_{20}$
$LOS_B^B$, $LOS_B^B$, $T_2$
$T_S^B$, $\Delta t$, $SIG_1$ thru $SIG_{10}$
$SF_1$ and $SF_2$

Detector dynamic
response increments

Clear outputs of 2-axis sensor

Set initial state
of 2-axis sensor

Figure J-6. Earth Sensor Flow Chart
Figure J-6. Earth Sensor Flow Chart (continued)
Save angle between earth tangent and LOS.

Loop for two horizon crossing indicators

\[ \text{for } i = 1 \text{ to } 2 \]

- If \( i \leq 2 \), continue
- If \( i > 2 \), set horizon crossing indicators to no crossing

Set horizon crossing indicators to no crossing

- If MODE \( \neq 0 \) (normal operation), continue
- If MODE = 0, return (initialize)

End of spin mode iteration

\[ \text{TPIP}_1 - \Delta t \]
\[ \text{TPIP}_2 - \Delta t \]

RETURN End of initialization

Figure J-6. Earth Sensor Flow Chart (continued)
Compute position and diameter of earth image on detector surface. Superscripts indicate reference frame:

- I: Inertial
- B: Body
- S: Sensor

\[
\text{RM SQRT} \left( R_x^2 + R_y^2 + R_z^2 \right) \\
\text{RS S TSB TIB TR} \\
\text{B RS}_x/RM \\
\text{B ATAN} \left( B/\text{SQRT} \left( 1 - B^2 \right) \right) \\
\text{C RS}_y/RM \\
\text{C ATAN} \left( C/\text{SQRT} \left( 1 - C^2 \right) \right) \\
\text{RE 3440/RM} \\
\text{RE ATAN} \left( \text{RE}/\text{SQRT} \left( 1 - \text{RE}^2 \right) \right)
\]

\[\text{SQRT is square root function} \]
\[\text{ATAN is single argument arc tangent function} \]

Check for state of 2-axis operation:

\[i = 1 \text{ thru } 10\]
\[A_1 \text{ AREA} \left( -Y_1, X_1 \right)\]
\[A_{i+10} \text{ AREA} \left( Y_1, -X_1 \right)\]
\[A_{i+20} \text{ AREA} \left( Y_1, X_1 \right)\]
\[A_{i+30} \text{ AREA} \left( -Y_1, -X_1 \right)\]

\[D_1 \ A_1 + A_2 + A_{21} + A_{22}\]
\[D_3 \ A_9 + A_{10} + A_{29} + A_{30}\]
\[D_4 \ A_{11} + A_{12} + A_{31} + A_{32}\]
\[D_6 \ A_{19} + A_{20} + A_{39} + A_{40}\]

Figure J-6. Earth Sensor Flow Chart (continued)
For $i = 3$ thru $8$

1. Compute area of earth coverage on B detectors

For $i = 1$ thru $6$

2. Compute detector output voltage and save detector coverage

3. Compute roll 1 attitudes

$\text{RIA SF}_2 \cdot (V_4 + V_5 - V_1 - V_2 + BS_2 + SIG_2 \cdot \text{RAND}(2) + REXP_2)$

$C2A = 0.25 + 0.2 \left( |R2A| + |P2A| \right)$

$|P1A - P2A| \leq C2A \text{ or } |R1A - R2A| \leq C2A$

PA = P2A

PA = P1A

Figure J-6. Earth Sensor Flow Chart (continued)
For $i = 1$ thru 10

$A_1$ AREA $(X_1, Y_1)$
$A_{i+10}$ AREA $(-X_i, -Y_i)$
$A_{i+20}$ AREA $(X_1, -Y_i)$
$A_{i+30}$ AREA $(-X_i, Y_i)$

Compute area of earth coverage on each $10 \times 10$ square on #1 and #5 detectors.

$D_1 = A_1 + A_2 + A_{21} + A_{22}$
$D_3 = A_9 + A_{10} + A_{29} + A_{30}$
$D_4 = A_{11} + A_{12} + A_{31} + A_{32}$
$D_6 = A_{19} + A_{20} + A_{39} + A_{40}$

Compute area of earth coverage on A and S detectors

$D_2 = 0$
$D_5 = 0$

For $i = 3$ thru 8

$D_2 = D_2 + A_i + A_{i+20}$
$D_5 = D_5 + A_i + 10*A_{i+30}$

Compute area of earth coverage on B detectors

For $i = 1$ thru 6

$j = i + 6$

$V_j = V_j + (D_1 - DSAV_j)_{CON2}$

$DSAV_j = D_i$

Compute detector output voltage and save detector coverage.

$PIA \cdot SF_2 \cdot (V_{10} + V_{11} - V_7 - V_8 + B_S \cdot SIG_4 \cdot \text{RAND}) (4) + FEXP_4$ Compute pitch 1 attitudes.

RAND is random number generator, normal distribution, unit variance.

Figure J-6. Earth Sensor Flow Chart (continued)
$C_2A = 0.25 + 0.2 \left( \frac{R_2A + P_2A}{2} \right)$

$|R_1A - R_2A| \leq C_2A$ or $|P_1A - P_2A| > C_2A.$

**TRUE** $R_1A = R_1A$

**FALSE** $R_1A = R_2A$

---

**Figure J-6. Earth Sensor Flow Chart (continued)**
Computer area of earth coverage on A and S detectors

For i = 3 thru 8
D_2 = D_2 + A_i + A_{i+40}
D_5 = D_5 + A_i+10 + A_{i+50}
D_8 = D_8 + A_i+20 + A_{i+60}
D_{11} = D_{11} + A_i+30 + A_{i+70}

For i = 1 thru 12
j = i + 12
V_j = V_j + (D_i - DSAV_j) * CON3
DSAV_j = D_i

Figure J-6. Earth Sensor Flow Chart (continued)
\[ R2A \cdot \frac{SF_2}{2} = V_{22} + V_{23} - V_{16} - V_{17} + V_{19} + V_{20} - V_{13} - V_{14} + SF_2 + BS_6 + SIG_6 \cdot \text{RAND}(6) + \text{REXP6} \]

\[ C1A = 0.25 + 2 \cdot \left( |P1A - |R1A| \right) \]

\[ |P1A - P2A| \leq C1A \quad \text{or} \quad |R1A - R2A| > C1A \]

\[ PA = P1A \]

\[ PA = P2A \]

Figure J-6. Earth Sensor Flow Chart (continued)
\[ SF_2 \]
\[ P_{2A} = \frac{V_9 + V_{20} - V_{19} - V_{13} - V_{14} + V_{16} - V_{17} - V_{22} - V_{23} + SF_2}{2} \times BS_8 + SIG_8 \times \text{RAND}(8) \times \text{REXP}8 \]

\[ C_{1A} = 0.25 + 0.2 \times (|P_{1A}| + |R_{1A}|) \]

\[ |R_{1A} - R_{2A}| \leq C_{1A} \text{ or } |P_{1A} - P_{2A}| \leq C_{1A} \]

**Figure J-6. Earth Sensor Flow Chart (continued)**
Figure J-6. Earth Sensor Flow Chart (concluded)
Where

\[ \text{SIG}_I = 0.0020 \text{ (deg., 1c) } I = 1, \ldots, 8 \]

\[ \text{BS}_I = \text{Bias error with worst case value 0.0285 (deg., 3c) } I = 1, \ldots, 8. \]

\[ \text{REXP}_I = \text{Exponentially correlated noise with standard deviation 0.00283° and correlation time 3.485 sec.} \]

The function \text{REXP}_I to be used to generate the exponentially correlated noise is given in the following:

\begin{verbatim}
FUNCTION REXP1
REXPIO = REXP1
BETA = 1./TMCNST
DREXP = -BETA*REXPIO
 + SQRT (2.*BETA)*RSIG1*RA
REXP1 = REXPIO + DREXP
RETURN
END
\end{verbatim}

Where

\[ \text{TMCNST} = \text{Correlation Time} \]

\[ \text{RSIG1} = \text{Standard Deviation} \]

\[ \text{RA} = \text{Random number generator with unit variance} \]

\[ \text{REXP1} \text{ to be assigned an initial value.} \]
### TABLE J-2. INITIALIZATION DATA FOR EARTH SENSOR MODEL

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>SUBSCRIPT</th>
<th>UNIT</th>
<th>NOMINAL VALUE</th>
<th>SOURCE</th>
<th>TOLERANCE</th>
<th>SOURCE</th>
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</thead>
<tbody>
<tr>
<td>Time constant of two axis earth sensor detectors, ( T_2 )</td>
<td></td>
<td>Seconds</td>
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<td>Note 1</td>
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<td>None</td>
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<tr>
<td>Line of sight of spin earth sensor field</td>
<td>( \text{LOS}_z )</td>
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<td>( \text{LOS}_x )</td>
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<td>-0.9848</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>( \text{LOS}_y )</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Transformation from body to sensor frame ( (T S R) )</td>
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<td></td>
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<td>Assumed (nominal)</td>
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<td>3</td>
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<td>9</td>
<td></td>
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<tr>
<td>Calling time increment, ( t )</td>
<td></td>
<td>Seconds</td>
<td>(&lt;0.00326) (spin mode) \</td>
<td></td>
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<tr>
<td></td>
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<td></td>
<td>(&lt;0.123) (2 axis) \</td>
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<tr>
<td>Center position of incremental detector area</td>
<td>( \text{X}_{\text{eq}} )</td>
<td></td>
<td>0.2530272415</td>
<td>Note 3</td>
<td>N/A</td>
<td>None</td>
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TABLE J-2. INITIALIZATION DATA FOR EARTH SENSOR MODEL (continued)

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<th>PARAMETER</th>
<th>SUBSCRIPT</th>
<th>UNIT</th>
<th>NOMINAL VALUE</th>
<th>SOURCE</th>
<th>TOLERANCE</th>
<th>SOURCE</th>
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<tbody>
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<td>( Y_1 )</td>
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<td>S/A</td>
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<td>7</td>
<td></td>
<td>0.008726666</td>
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<tr>
<td></td>
<td>8</td>
<td></td>
<td>0.0261799387</td>
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<td>15</td>
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<td>0.2734854881</td>
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TABLE J-2. INITIALIZATION DATA FOR EARTH SENSOR MODEL (concluded)

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>SUBSCRIPT</th>
<th>UNIT</th>
<th>NOMINAL VALUE</th>
<th>SOURCE</th>
<th>TOLERANCE</th>
<th>SOURCE</th>
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</thead>
<tbody>
<tr>
<td>Noise Standard Deviation</td>
<td>SIG om:</td>
<td>RIT 1</td>
<td>Volts</td>
<td>0.167</td>
<td>Note 4</td>
<td>N/A</td>
</tr>
<tr>
<td>SIG om:</td>
<td>RIA 2</td>
<td>Volts</td>
<td>0.0417</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SIG om:</td>
<td>PIT 3</td>
<td>Volts</td>
<td>0.167</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SIG om:</td>
<td>PIA 4</td>
<td>Volts</td>
<td>0.0417</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SIG om:</td>
<td>R2T 5</td>
<td>Volts</td>
<td>0.167</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SIG om:</td>
<td>R2A 6</td>
<td>Volts</td>
<td>0.0417</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SIG om:</td>
<td>P2T 7</td>
<td>Volts</td>
<td>0.167</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>SIG om:</td>
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<td>Volts</td>
<td>0.0417</td>
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</tr>
<tr>
<td>SIG om:</td>
<td>TPJP 1 9</td>
<td>Sec.</td>
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<td>Not Specified</td>
<td>Not Specified</td>
<td></td>
</tr>
<tr>
<td>SIG om:</td>
<td>TPJP 2 10</td>
<td>Sec.</td>
<td>0</td>
<td>Not Specified</td>
<td>Not Specified</td>
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</tr>
<tr>
<td>Two axis output scale factor</td>
<td>Track, SF 1</td>
<td>Volt/sq. deg.</td>
<td>1.25</td>
<td>Note 3</td>
<td>0.0625</td>
<td>Note 3</td>
</tr>
<tr>
<td></td>
<td>Acq., SF 2</td>
<td>Volt/sq. deg.</td>
<td>0.3125</td>
<td></td>
<td>0.025</td>
<td></td>
</tr>
</tbody>
</table>

NOTES TO TABLE J-2:

1. Barnes presentation to Honeywell in St. Petersburg on 3-13-75.
2. Rockwell Spec MC 432-0214, Rev. 8, 12-11-74.
3. Computed from data in Rockwell Spec MC 432-0214, Rev. 8, 12-11-74.

Track scale factor

\[ 1 \text{ degree attitude} = 4 \text{ sq. deg. coverage} = 5 \pm 0.25 \text{V}. \]

\[ \frac{500.25\text{V}}{4 \text{ sq. deg.}} = 1.25 \pm 0.0625 \text{ volt/sq. deg.} \]

Acquisition scale factor

\[ 4 \text{ degree attitude} = 16 \text{ sq. deg. coverage} \]

\[ \frac{1.25 \times 0.1 \text{V}}{4 \text{ sq. deg.}} = 0.3125 \pm 0.025 \text{ volt/sq. deg.} \]

4. Computed from data in Rockwell Spec MC 432-0214, Rev. 8, 12-11-74

Track mode

\[ \pm 0.1^\circ, 3 \text{ sigma} \]

\[ 0.1 \times 5 \text{ volt/deg} \times 1/3 = 0.167 \text{ volt, 1 sigma} \]

Acquisition mode

\[ \pm 0.1^\circ, 3 \text{ sigma} \]

\[ 0.1 \times 1.25 \text{ volt/deg} \times 1/3 = 0.0417 \text{ volt, 1 sigma} \]
TABLE J-3. INPUT DATA FOR EARTH SENSOR MODEL

<table>
<thead>
<tr>
<th>DEFINITION</th>
<th>FLOW DIAGRAM SYMBOL</th>
<th>DIMENSION</th>
<th>TYPE</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode control flag</td>
<td>MODE</td>
<td></td>
<td>Integer</td>
<td>0 = initialize</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1 = spin operation</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2 = 2 axis operation</td>
</tr>
<tr>
<td>Satellite position vector</td>
<td>R_x, R_y, R_z; R</td>
<td>3</td>
<td>Real</td>
<td>N Mi</td>
</tr>
<tr>
<td>Transformation from body to</td>
<td>TIB</td>
<td>9</td>
<td>Real</td>
<td>Inertial frame</td>
</tr>
<tr>
<td>Inertial frame</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# TABLE J-4. OUTPUT DATA FOR EARTH SENSOR MODEL

<table>
<thead>
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<th>DEFINITION</th>
<th>FLOW DIAGRAM SYMBOL</th>
<th>DIMENSION</th>
<th>TYPE</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin earth sensor output</td>
<td>TPIP</td>
<td>2</td>
<td>Real</td>
<td>Seconds</td>
</tr>
<tr>
<td>Pitch attitude in track mode</td>
<td>PT</td>
<td></td>
<td>Real</td>
<td>Volts</td>
</tr>
<tr>
<td>Pitch attitude in acquisition mode</td>
<td>PA</td>
<td></td>
<td>Real</td>
<td>Volts</td>
</tr>
<tr>
<td>Roll attitude in track mode</td>
<td>RT</td>
<td></td>
<td>Real</td>
<td>Volts</td>
</tr>
<tr>
<td>Roll attitude in acquisition mode</td>
<td>RA</td>
<td></td>
<td>Real</td>
<td>Volts</td>
</tr>
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</table>
### TABLE J-5. INTERNAL VARIABLES FOR EARTH SENSOR MODEL

<table>
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<tr>
<th>DEFINITION</th>
<th>SYMBOL</th>
<th>DIMENSION</th>
<th>TYPE</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two axis rise/decay increment for 0.25 sec.</td>
<td>CON2</td>
<td></td>
<td>Real</td>
<td>Volts per sq. deg.</td>
</tr>
<tr>
<td>Two axis rise/decay increment for 0.125 sec.</td>
<td>CON3</td>
<td></td>
<td>Real</td>
<td>Volts per sq. deg.</td>
</tr>
<tr>
<td>Area of incremental detector covered by earth</td>
<td>A</td>
<td>80</td>
<td>Real</td>
<td>Square degree</td>
</tr>
<tr>
<td>Time since start of run</td>
<td>TM</td>
<td></td>
<td>Dual prec</td>
<td>Seconds</td>
</tr>
<tr>
<td>Time of next 2-axis update</td>
<td>TYMUP</td>
<td></td>
<td>Dual prec</td>
<td>Seconds</td>
</tr>
<tr>
<td>Satellite distance to earth center</td>
<td>RM</td>
<td></td>
<td>Real</td>
<td>N. Mi</td>
</tr>
<tr>
<td>Satellite position vector in sensor frame</td>
<td>R</td>
<td>3</td>
<td>Real</td>
<td>N. Mi</td>
</tr>
<tr>
<td>Radius of earth image on focal surface</td>
<td>RE</td>
<td></td>
<td>Real</td>
<td>Equiv. radian</td>
</tr>
<tr>
<td>Position of earth image on focal surface, X component</td>
<td>B</td>
<td></td>
<td>Real</td>
<td>Equiv. radian</td>
</tr>
<tr>
<td>Position of earth image on focal surface, Y component</td>
<td>C</td>
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<td>Real</td>
<td>Equiv. radian</td>
</tr>
<tr>
<td>State of sequential processing within sensor</td>
<td>STATE</td>
<td></td>
<td>Integer</td>
<td></td>
</tr>
<tr>
<td>Area of detector covered by earth</td>
<td>D</td>
<td>12</td>
<td>Real</td>
<td>Square degree</td>
</tr>
<tr>
<td>Area of detector coverage from previous cycle</td>
<td>USAV</td>
<td>24</td>
<td>Real</td>
<td>Square degree</td>
</tr>
<tr>
<td>Dynamic Detector output</td>
<td>V</td>
<td>24</td>
<td>Real</td>
<td>Square Degree</td>
</tr>
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</table>
TABLE J-5. INTERNAL VARIABLES FOR EARTH SENSOR MODEL (concluded)

<table>
<thead>
<tr>
<th>DEFINITION</th>
<th>PLAN DIAGRAM SYMBOL</th>
<th>DIMENSION</th>
<th>TYPE</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch 1 track attitude</td>
<td>P1T</td>
<td>Real</td>
<td>Volts</td>
<td></td>
</tr>
<tr>
<td>Pitch 2 track attitude</td>
<td>P2T</td>
<td>Real</td>
<td>Volts</td>
<td></td>
</tr>
<tr>
<td>Roll 1 track attitude</td>
<td>R1T</td>
<td>Real</td>
<td>Volts</td>
<td></td>
</tr>
<tr>
<td>Roll 2 track attitude</td>
<td>R2T</td>
<td>Real</td>
<td>Volts</td>
<td></td>
</tr>
<tr>
<td>Pitch 1 acquisition attitude</td>
<td>P1A</td>
<td>Real</td>
<td>Volts</td>
<td></td>
</tr>
<tr>
<td>Pitch 2 acquisition attitude</td>
<td>P2A</td>
<td>Real</td>
<td>Volts</td>
<td></td>
</tr>
<tr>
<td>Roll 1 acquisition attitude</td>
<td>R1A</td>
<td>Real</td>
<td>Volts</td>
<td></td>
</tr>
<tr>
<td>Roll 2 acquisition attitude</td>
<td>R2A</td>
<td>Real</td>
<td>Volts</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Index</td>
<td>j</td>
<td>Integer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line of sight of spin sensor field in inertial frame</td>
<td>LOS^2</td>
<td>3</td>
<td>Real</td>
<td></td>
</tr>
<tr>
<td>Normalized cross product magnitude of R and LOS</td>
<td>SANG</td>
<td>Real</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normalized dot product of R and LOS</td>
<td>CANG</td>
<td>Real</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angle between R and tangent to earth</td>
<td>ANG1</td>
<td>Real</td>
<td>Radian</td>
<td></td>
</tr>
<tr>
<td>Angle between R and LOS</td>
<td>ANG2</td>
<td>Real</td>
<td>Radian</td>
<td></td>
</tr>
<tr>
<td>Angle between LOS and earth tangent</td>
<td>ANG3</td>
<td>Real</td>
<td>Radian</td>
<td></td>
</tr>
<tr>
<td>Saved angle between earth tangent and LOS</td>
<td>ANGSAV</td>
<td>2</td>
<td>Radian</td>
<td></td>
</tr>
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<td>Half dimension of 1° x 1° square</td>
<td>RD</td>
<td>Real</td>
<td>Radian</td>
<td></td>
</tr>
<tr>
<td>Area of earth covering 1° x 1° square</td>
<td>AREA</td>
<td>Real</td>
<td>Square Degree</td>
<td></td>
</tr>
</tbody>
</table>

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REFERENCES


APPENDIX K

WHEEL-MOMENTUM COUPLING
To examine the effects of wheel-momentum coupling, we begin with the rotational equations of motion, i.e.,

\[ T = I_\varepsilon + \omega \times (I_W + \dot{H}_W) + \dot{H}_W = T_J + T_d \quad \text{(K-1)} \]

where

- \( I \) = inertia tensor
- \( H_W \) = wheel momentum
- \( T_J \) = jet control torque
- \( T_d \) = disturbance torque

Linearizing about the nominal orbit rate \( \omega = \omega_o + \delta \omega \) gives

\[ I_{o \dot{x}} = -\omega_o \times (I_W + \dot{H}_W) - \omega_o \times (I_W + \dot{H}_W) - \omega_o \times I_{\delta \omega} \quad \text{(K-2)} \]

where

- \( T_W = -\dot{H}_W \) = wheel reaction torque

Solving for \( \delta \omega \) and substituting for

\[ T_o = (0 \ - \omega_e \ 0) \]

yields

\[
\delta \omega = \begin{bmatrix}
0 & -H_{WZ}/I_x & [H_{Wy} + (I_z - I_y) \omega_e]/I_x \\
-H_{WZ}/I_y & 0 & -H_{Wx}/I_y \\
[H_{Wy} + (I_z - I_y) \omega_e]/I_z & H_{Wx}/I_z & 0
\end{bmatrix} \delta \omega
\]
where

\( \omega_e = \text{earth rate} \)

We have two effects to examine—one static and the other dynamic. The static effect is that wheel momentum induces wheel momentum coupling torques in the roll and yaw axes. If necessary, this error source can be compensated for by using tachometer feedback to generate wheel commands in the cross axis, i.e.,

\[
T_{\text{Wc}} = -I_W \begin{bmatrix}
\hat{\omega}_{WZ} \\
0 \\
-\hat{\omega}_{WX}
\end{bmatrix} \omega_e
\]

where

\( I_W = \text{wheel inertia} \)

\( \hat{\omega}_W = \text{wheel-speed estimate from tachometer} \)

Assuming that wheel speed is known to within one percent of its true value, the effective disturbance due to wheel-momentum coupling is reduced, i.e.,

\[
T_{\text{dWMC}} = \begin{bmatrix}
H_{WZ} \\
0 \\
-H_{WX}
\end{bmatrix} \omega_e - I_W \begin{bmatrix}
\hat{\omega}_{WZ} \\
0 \\
-\hat{\omega}_{WX}
\end{bmatrix} \omega_e
\]

\[
= I_W \begin{bmatrix}
\hat{\omega}_{WZ} \\
0 \\
-\hat{\omega}_{WX}
\end{bmatrix} \omega_e
\]

\[
\leq H_{W\text{max}} \left( \frac{\tilde{\omega}_W}{\tilde{\omega}_{W\text{max}}} \right)
\]
where
\[ \tilde{\Omega}_W = \Omega_W - \hat{\Omega}_W = \text{true minus estimated wheel speed} \]

The other effect of wheel-momentum coupling is a dynamic one. It introduces nutation coupling between all axes. The maximum frequency of this nutation coupling is given by

\[ \Omega_{xz} = \frac{[H_{W\text{max}} + (I_x - I_y)m_c^2]}{I_z} \]
\[ \Omega_{xy} = \frac{H_{W\text{max}}}{\sqrt{I_x I_y}} \]
\[ \Omega_{yz} = \frac{H_{W\text{max}}}{\sqrt{I_z I_y}} \]

(K-7)

The effect of this nutation coupling on control loops which are designed to be independent should cause no problem, provided that these nutation frequencies are well below the crossover frequency of the wheel loop.