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ABSTRACT

A relativistic computer simulation demonstrates that the application of a DC electric field enhances the excitation of plasma waves through runaway electrons. The presence of an elongated tail in the distribution function gives rise to acoustic-type modes which interact strongly with the runaway population. The waves produce a strong drag which reduces the runaway momentum increase from 2/3 to 1/2 of the free fall value. A threshold DC electric field is found above which there is no collective slowdown.
The evolution of runaway electrons taking into consideration the self-consistent excitation of large amplitude collective oscillations is of considerable interest\textsuperscript{1,2,3} at the present time. This interest has been stimulated by recent experimental observations\textsuperscript{4,5,6} in tokamak devices of runaway electrons accompanied by the spontaneous emission of intense radiation in various\textsuperscript{4,5} frequency intervals. The study of runaway electrons provides a suitable arena to test and develop various theories of collective interactions in plasmas whose applicability can extend beyond just the runaway problem. Recent theoretical studies\textsuperscript{1} rely on assumptions concerning a flat momentum distribution parallel to the applied electric field. However, many details associated with the wave-particle interactions that generate and alter such distributions still remain to be illustrated. One approach previously used to examine some of the relevant physics has been the Fokker-Planck type of simulation\textsuperscript{7,8} in which the wave-particle interactions are assumed to be described by quasilinear theory. The latter assumption is also characteristic of most of the theoretical treatments of the problem. In yet another work, non-relativistic particle simulations without DC field and starting from a flat distribution have been performed\textsuperscript{8} to verify that collective effects could lead to a positive slope.

This study examines some of the wave-particle interactions and the nature of the collective modes present in the evolution of runaway electrons, employing a full particle computer simulation. The major effects found are: (1) the runaway electrons exhibit large excursions in momentum space not amenable to a quasilinear description, (2) the momentum distribution function develops an elongated tail which gives rise to modes with frequencies lower than the plasma frequency,\textsuperscript{9} (3) the excitation of large amplitude waves reduces the rate of increase of the average runaway momentum from 1/2 to 2/3 of the free fall value.
A one dimensional relativistic electrostatic particle code is used. The system is initialized with a low density cold electron beam and a Maxwellian background distribution. The external DC electric field $E_{DC}$ is applied selectively to fast particles with momentum above a set value, chosen smaller than the initial beam momentum and larger than the initial average momentum associated with the thermal velocity of the background particles $v_{T0}$. This technique models the effect of collisions which would prevent the slower particles from running away in a real system. In the runs investigated we have considered two different values of the beam to background density ratio, $n_b/n_o = 1/9$ and $1/180$. The system length is $256 \lambda_{DO}$, where $\lambda_{DO}$ is the Debye length at $t = 0$; 256 beam electrons and 2304 bulk electrons are followed. Appropriate measures are taken to handle the discreteness of the beam electrons. Typical parameter values are as follows. The beam momentum at $t = 0$ is $p_b = 7.63 mv_{T0}$, where $m$ is the electron mass. The limiting value of the momentum above which $E_{DC}$ is applied is $6.0 mv_{T0}$. The speed of light $c$ is taken as $9.0 v_{T0}$, and $E_{DC} = \frac{eE_{DC}}{m_w v_{T0}} = 0.1$, where $\omega_p$ is the electron plasma frequency and $e$ the electron charge. Each electron is represented by a Gaussian-shaped finite size particle of width $1.0 \lambda_{DO}$. A typical run extends to $t = 10^3 \omega_p^{-1}$.

The early evolution of the system is dominated by the two-stream instability triggered by the low density runaway beam as shown in Fig. 1(a). During the early phase one observes the exponential growth characteristic of such an instability. Unlike the $E_{DC} = 0$ case, the instability does not saturate by particle trapping$^{10}$, but rather the wave energy grows secularly eventually reaching a value roughly 10 times larger than the non-runaway beam saturation level, as illustrated in Fig.1(a). This figure exhibits the time evolution of the total wave energy for $n_b/n_o = 1/180$; the continuous...
curve corresponds to $E_{\text{DC}} = 0.1$ and the smaller dotted curve to $E_{\text{DC}} = 0$. The latter curve is the well-known trapped particle saturation pattern. The reason for the enhanced excitation of plasma waves is that the free energy of the beam continues to be replenished through the work done by the DC field, hence the beam nonlinearities act as a converter of DC to AC energy. Together with the secular increase in the total wave energy one observes that the wave spectrum cascades toward smaller wavenumbers $k$ eventually reaching $k < \omega_p/c$.

An examination of phase space (not shown) indicates that individual runaway electrons experience large momentum oscillations due to the interaction with the large amplitude waves. In this manner the spatially averaged momentum distribution $f(p)$ develops an elongated tail which proceeds to expand in time toward large momenta. An example of such a distribution is shown in Fig. 1(b) for the case $n_b/n_o = 1/9$ and at $\omega_p t = 550$. As seen in Fig. 1(b), the elongated tail is not smooth, but rather it exhibits the transient formation and subsequent destruction of small bumps (beams). These bumps, which act as converters of DC energy into wave energy, produce the rapid growth and damping of the wave energy seen in Fig. 1(a) over the interval $\omega_p t > 500$. The measured relaxation oscillation period is $56 \omega_p^{-1}$ for $n_b/n_o = 1/180$. This relaxation period may be evaluated as follows. The Landau growth rate is given by

$$\gamma_L = \pi/2 \sum \frac{\omega_k^3}{k^2} \left| \frac{\partial f(p)}{\partial p} \right| v=\omega_k/k.$$

(1)

Here $\partial f/\partial p$ may be estimated as $n_R/(p_b p_M n_o)$, where $p_M$, the maximum momentum cut off, $p_b$, the effective gradient length of the bump, and $n_R/n_o$ are $90 \text{mv}_T 0$, $3 \text{mv}_T 0$ and 0.016 respectively from the simulations. Here $n_R$ is the number of runaway electrons.
The summation is taken over bumps in resonance with the dominant components of the wave spectrum (modes 4 and 5), which yields $\gamma_L - 1 \times 10^{-2} \omega_p$. A much smaller value would be obtained from the overall (negative) slope of the runaway tail. Since the wave energy varies by 40% over the trough value, this leads to a relaxation oscillation period $\tau_L - 2 \frac{2n \ln 1.4}{\gamma_L} = 54 \omega_p^{-1}$. In the case $n_b/n_o = 1/9$, the measured relaxation oscillation period is $24 \omega_p^{-1}$, while an argument similar to the above yields $\tau_L - 21 \omega_p^{-1}$. The relaxation oscillation period then increases as the beam to plasma density ratio is lowered, as expected from the growth rate expression, Eq.(1). The above estimates also imply that an argument based on a steady state picture might be inadequate.

An assumption which permeates the quasilinear description of runaway behavior is that the real part of the dispersion relation is determined by the background (slow) particles. However, as seen in Fig. 1(b), the self-consistent distribution consists of an elongated tail. The dispersion relation for such a distribution may be written in the highly relativistic limit as

$$1 - \frac{\omega_p^2 (1 - \eta)}{\omega^2 - 3k^2v_T^2} - \frac{mc}{p_M} \frac{n_0 \omega_p^2}{\omega(\omega - kc)} = 0,$$

(2)

where $\eta = n_R/(n_0 + n_R)$ and $p_M$ is the maximum (cut-off) momentum, $p_M >> mc$. The modes thus introduced may be called an electroacoustic branch due to the hot electron population (elongated tail). The appearance of a dispersionless low frequency branch is indicative of the Van Kampen effect, since the cut-offs in runaway electron momenta (at 0 and $p_M$) introduce the non-cancellation of the phases of the Van Kampen modes. In Fig. 2 we compare the prediction of Eq.(2) with $p_M/mc = 13.3$ and $\eta = 0.016$ (solid curve) with the simulation results (solid dots) obtained by standard correlation methods. It is clearly seen that the...
elongated tail strongly modifies the wave dispersion, as expected. Furthermore, the strong nonlinearity produced by the large amplitude waves gives rise to multiple-harmonic branches (crosses). A salient feature in Fig. 2 is that the system supports modes whose phase velocity is slower than the edge velocity of the expanding distribution for $k < \omega_p/c$. Consequently, the runaway particles are always in resonance with a collective mode. The excitation of such modes retards the free fall expansion of the runaway tail.

Although the interaction of an individual runaway electron with the large amplitude waves is quite complicated, the average behavior of the runaway population is remarkably simple, as shown in Fig. 3. The average runaway momentum $\bar{p}$ (i.e., average above the cut-off at 6.0 $mV/t_o$) is found to increase linearly with time, $\bar{p} = p_0 + \alpha eE_{DC} t$, where the coefficient $\alpha = 2/3$ for $n_b/n_o = 1/9$ and $\alpha = 1/2$ for $n_b/n_o = 1/180$. This behavior can be understood through a simple scaling argument by realizing that for $n_b/n_0 = 1/9$ the number of runaway electrons is increasing only slightly and that the time averaged distribution function approximately has a self-similar triangular form given by

$$f(p) = \frac{2}{3\bar{p}} (1 - \frac{p}{3\bar{p}})^3 \quad (0 \leq p \leq 3\bar{p}) \quad (3)$$

Conservation of momentum, $(d/dt)(p^2/2) = eEp$, with

$$(d/dt)(p^2/2) = (d/dt) \left[ (1/2) \int_0^{3\bar{p}} f(p)p^2 dp \right] = (3/2) \bar{p} \frac{dp}{dt} \quad (4)$$

and

$$eEp = e \int_0^{3\bar{p}} f(p) p \cdot E_{DC} dp = eE_{DC} \bar{p} \quad (5)$$

yields

$$\frac{dp}{dt} = (2/3) eE_{DC} \quad (6)$$

in excellent agreement with the simulations (see Fig. 3).
In addition to the classical runaway threshold (i.e. the Dreicer field\(^{15}\)), we find another threshold DC field beyond which the electrons keep running away toward infinite momenta without any strong collective phenomena taking place. This critical DC field, \(E_{\text{cr}}\), is determined from the peak wave amplitude\(^{16}\) excited by a beam when \(E_{\text{DC}} = 0\), i.e.

\[
E_{\text{cr}} = [4\pi n_b \gamma_o mc^2 s(1 + s)^{-5/2}]^{1/2}
\]

(7)

where \(\gamma_o\) is the relativistic factor and \(s = \beta^2 \gamma_o (n_b/2n_o)^{1/3}\), \(\gamma_o = (1 - \beta^2)^{-1/2}\).

When \(E_{\text{DC}} > E_{\text{cr}}\), the waves fail to sustain oscillations and thus slow down the particles. In the case \(n_b/n_o = 1/9\), a strong two-stream instability is still experienced in the simulation for \(E_{\text{DC}} = 1.0\), while no two-stream instability nor any strong wave-particle interactions occur for \(E_{\text{DC}} = 2.0\).

In fact, it is observed that as \(E_{\text{DC}}\) approaches \(E_{\text{cr}}\), the level of AC fluctuations dramatically increases and then for \(E_{\text{DC}} > E_{\text{cr}}\) it remains at the thermal level. Simulation values of \(\gamma_o = 1.31\), \(c = 9.0 v_{\text{T}0}\), \(n_b/n_o = 1/9\) and \(s = 0.21\) (at \(t = 0\)) in Eq.(7) yield \(E_{\text{cr}} = 1.25\), confirming the simulation results.

For \(n_b/n_o = 1/180\), this threshold is found to lie between \(E_{\text{DC}} = 0.1\) and \(E_{\text{DC}} = 0.5\) in the simulation, while Eq.(7) predicts \(E_{\text{cr}} = 0.3\).

The physical picture which emerges out of this investigation is the following. The presence of a DC electric field \(E_{\text{DC}} < E_{\text{cr}}\) prevents the usual saturation of the beam-plasma instability. Consequently, large amplitude plasma waves are excited and cause large momentum fluctuations in the runaway population. The strong nonlocal mixing in momentum space produces an elongated momentum distribution which supports Van Kampen (electroacoustic) type of modes. The self-consistent excitation of these modes gives rise to the transient growth and decay of bumps in the distribution function accompanied by relaxation oscillations in the wave energy. The drag produced by these waves reduces the runaway momentum increase below the free fall value.
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13. Because the fluctuating cut-off velocities mix up the phases of the different wavenumbers, the electroacoustic branch has not been clearly observed in nonrelativistic simulations (e.g. Ref. 9). Here, however, cut-off velocities are already close to c.
14. The relative intensities of the harmonics are a steeply decreasing function of wavenumber. Unless the waves are strongly excited, as is the case here, a clear mapping of the multiple harmonic branches might be difficult.


FIG. 1  (a) Time evolution of the wave energy in the presence of runaway electrons. With $E_{DC} = 0.1$ (solid curve), the AC wave energy is sustained to a much higher level than with $E_{DC} = 0$ (dotted curve). $n_b/n_o = 1/180$. (b) The time averaged momentum distribution function at $t = 550 \omega_p^{-1}$ shows the elongated tail. $n_b/n_o = 1/9$.

FIG. 2  Electroacoustic dispersion relation measured (solid dots) and predicted by theory (solid curve) for $n_b/n_o = 1/180$. Note that there appear second, third and fourth harmonic branches (crosses) in the dispersion relation. The spectral intensity of the harmonic modes is highest at the crossing of the harmonic and electroacoustic branches.

FIG. 3  Evolution of the average runaway momentum. The solid dots represent the simulation results, the dashed line $\bar{p} = p_o + 2/3 eE_{DC} t$. The average runaway momentum first follows the free fall line and eventually asymptotes to $2/3$ of the free fall value.
FIGURE 2


PPG-274 "San Francisco Abstracts - Papers to be Presented at San Francisco Meeting of the American Physical Society Division of Plasma Physics, November 15-19 (1976)."


PPG-291 "Particle Orbits and Loss Regions in UCLA Tokamaks", C.P. Lee, January (1977)


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