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THE EFFECT OF WEATHER ON A SHIP'S SPEED (U)

BY

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SUMMARY

The need for better dead-reckoning data from several trials on HMAS KIMBLA, which is not fitted with a speed log, led to this attempt to model the effect of wind and sea-state on ship's speed. Weather data comprised only normal ship's log observations, which imposed a constraint on the factors considered. In particular, no specific data on wave and swell periods were recorded. Equations were set up to allow for augmentation of resistance due to waves, swell and applied rudder. In addition, the effects of the direct push of wind and waves on the forward and transverse components of ship's velocity were considered. The parameters were fitted to observed data on the set of the ship in varying weather conditions. Some recommendations for future work are included.
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1. INTRODUCTION

The effect of wind, swell and waves on the speed of a ship through the water is quite complex and the simple study presented here was made primarily to reduce the navigational data from two trials. The data for this particular study were obtained from HMAS KIMBLA Trial 19/74 during the period 8-13 December 1974. HMAS KIMBLA is not fitted with either a distance or speed log. If she had been, it would not have been necessary to make the study and until she is so fitted, insufficient actual data are available to warrant further refinement in the theoretical model. At the time, however, the need for some current measurements in the neighbourhood of an eddy warranted a reasonable attempt to deduce ships' speed from engine revolutions and standard weather observations. The principal reference for an appreciation of the problem has been the work by Kent (1958). His work discusses the problem with reference to ships somewhat larger than HMAS KIMBLA, but nevertheless is most instructive. His Figure 75 is shown as our Figure 1, illustrative of what may happen to the speed of a low powered ship when the weather shifts its direction to her course and about which he makes the following comments:

(1) "In all winds below 50 knots the greatest loss in speed occurred with the weather about 30° on the bow, although the differences in ship speed were small between 'head on' and 40° on the bow.

(2) For wind velocities of 50 knots and upwards, the ship lost speed rapidly as the weather direction approached 'broadside on'. This was primarily due to the rapid increase in helm angle necessary to keep the vessel on her course, which makes the rudder resistance a major retarding force.

(3) In following wind and waves, the increase in ship speed over that in smooth water was greatest at the lower wind speed of 30 knots, with the weather between 'tail on' and 30° on the quarter. This was due to the greater 'push' on the stern by following waves when they are short by comparison with the ship's length, than the very much diminished push of longer waves.
(4) The ship speed in 30-knot winds would have been from 0.3 to 0.5 of a knot more in wave directions ahead or on the bow but for the drop in propulsive efficiency due to the greater augment in resistance in short waves. In higher winds the ship's speed would have been less by approximately the same amount but for the drop in the augment of resistance in waves longer than the vessel, which reduced the thrust needed to drive the ship and raised her propulsive efficiency.

(5) With the weather ahead or on the bow the average percentages of the total resistance due to wind, hull and rudder were as follows:

<table>
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<th>Percentages of the total ship resistance with the weather 'head on'.</th>
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<td>Windspeed (knots)</td>
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<tr>
<td>Hull resistance in smooth water</td>
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<td>Extra hull resistance in rough water</td>
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As the weather direction veered towards the beam, the percentages of the wind and the extra hull resistances dropped sharply and that due to the rudder rose rapidly.

He then discusses the loss of ship speed with change in wave length, something which our model could not pursue owing to lack of recorded data.

The approach used here was to write down a set of equations involving only the available weather data in order to compute the modifying effect of the weather on the smooth water speed of the ship. It was then hoped that a critical appraisal of all the data (bridge log, fixes, expected currents etc from KIMBLA Trial 19/74) would enable some actual values to be assigned to the constants in the equations. This was achieved by reasonable estimates and trial and error, but a more sophisticated multivariate correlation analysis could be applied if the need and the data (as well as the equations) warranted it.

The equations were set down with some cognizance of the physical forces involved - e.g. one might expect that the energy transmitted to the ship head on into waves would increase as the square of the wave height - but they are not meant in any way to be regarded as analytic solutions to the overall problem. The approach is largely empirical. It became apparent to the author when writing up this work that the analysis and model calculations could be improved in several ways. These have been noted appropriately in the text.
2. FORM OF EQUATIONS

The usual pertinent weather observations consist of wind speed and
direction, sea height and swell height and direction. Sea and swell wave-
length is not noted, so that possible resonance effects cannot be considered.

Use the following notation and units:

\[ v = \text{actual ship's velocity through the water (km)} \]

\[ v_0 = \text{ship's speed in absolutely calm weather (km)} \]

\[ h = \text{observed wave height (m)} \]

\[ h_s = \text{observed swell height (m)} \]

\[ w = \text{wind speed (kn)} \]

\[ \theta_w = \text{direction from which wind and sea are coming (degrees true)} \]

\[ \theta_s = \text{direction from which swell is coming (degrees true)} \]

\[ \phi = \text{course made good through the water (degrees true)} \]

\[ \theta = \text{course steered by the ship (degrees true)} \]

In addition, resolve \( v \) into two components, \( v_1 \) in the direction \( \phi' \) and a
normal component \( v_2 \).

Write

\[ \theta_w' = \theta_w - \phi' \]

and

\[ \theta_s' = \theta_s - \phi' \]

as the relative directions (to the ship's bow) of wind and swell respectively,
such that

\[ |\theta'| \leq 180^0. \]

Assume that the effects of the weather can be treated separably in the form

\[ v_1 = v_0 \cdot f(h_w, \theta_w', h_s, \theta_s', w) + f(w, \theta_w', \phi) \] \hspace{1cm} (1)

\[ v_2 = f_5(w, \theta_w', v_1) \] \hspace{1cm} (2)

where the function \( f \) can be written

\[ f = f_1(h_w, \theta_w') \cdot f_2(w, \theta_w') \cdot f_3(h_s, \theta_s') \] \hspace{1cm} (3)
Thus, we have defined retardation (or enhancement) factors $f_1$, $f_2$ and $f_3$ for wave, wind and swell effects respectively, acting on the forward speed $v_0$ and an additional additive function $f_4$, due to wind and dependent on ship's velocity, also contributing to $v_1$. The normal component of ship's speed $v_2$ (the leeway) is defined by $f_5$ as a function of $v_1$ and wind velocity.

The question now arises as to the form of the functions $f_1$ to $f_5$. Factors $f_1$, $f_2$ and $f_3$ form a similar group, so let us consider them together. Assume that the power delivered from the ship's engine is dependent only on the product of engine revolutions and shaft torque and is independent of variation in ship's speed (HMAS KINBLA, for example, sets a constant steam pressure and throttle setting is only changed when different revolutions are called for.)

The effective power $P_e$ is used to drive the ship forward at speed $v_1$ and, in general,

$$P_e = F v_1$$

where $F$ is the forward thrust. Hence, for constant throttle setting

$$F = \frac{1}{v_1} \quad \text{for weather no worse than moderate.}$$

As the sea state increases, $v_1$ decreases and $F$ increases. This corresponds to an increasing propeller slip ratio and increasing angle of incidence of the water on the propeller blade; so $F$ increases. (See Todd, 1967).

Now consider the equilibrium between forward thrust $F$ and the overall drag or resistance $D$ of the ship to forward motion. Write $D$ in the form

$$D = C_d A v_1^2$$

so that $C_d$ takes the form of a drag coefficient, $A$ is some constant (dimension $l^2$) for the ship.

Equating $F$ and $D$ gives us

$$C_d v_1^3 = \text{constant}$$

$$\quad = \frac{1}{3}$$

or

$$v_1 = C_d \quad \ldots \ldots (4)$$
Call $C_d$ the resistance coefficient which will vary with the weather (wind, sea state etc.). Suppose in calm weather it has the value $C_{do}$.

Write 
\[ C_d = C_{do} (1 + \beta) \] \hspace{1cm} \ldots (5)

where $\beta$ represents an augmentation of our resistance coefficient due to weather. Thus in calm weather $\beta = 0$ and $v_1$ takes a value $v_0$ say.

Substituting in (4) we obtain 
\[ v_1 = v_0 (1 + \beta) \] \hspace{1cm} \ldots (6)

This is the form in which we shall write the retardation factors $f_1$, $f_2$ and $f_3$.

3. RETARDATION DUE TO WAVE RESISTANCE

The function $f_1(h_w, \theta_w')$ is assumed to have the form
\[ f_1 = \left[ 1 + g_w(\theta_w') \cdot \left( \frac{h_w}{H_w} \right)^2 \right]^{-y_3} \] \hspace{1cm} \ldots (7)

where $H_w$ is an empirically set normalization constant and $g_w(\theta_w')$ describes the variation with incident angle, such that $g(0) = 1.0$.

(The value of the index used in (7) in the model calculations was actually -0.75 instead of -0.33, but the difference is not significant.)

When $h_w = H_w$ and $\theta_w' = 0$, $f_1 = 0.79$. $H_w$ was finally set at 2.44 m (8 ft).

The function $g_w(\theta_w')$ controlling the variation of $f_1$ with relative angle of incidence is shown in Figure 2, along with the corresponding function for swell effect, $g_s(\theta_s')$, in the factor $f_3$. These functions have been set up entirely on the basis of "reasonableness", lacking much in the way of observed data. Note that the effect of swell is assumed to be zero once the incidence is abaft the beam. Some retardation from wave action is still assumed, however, for $|\theta_w'| > 90^\circ$. This will be counteracted to a varying degree by the positive push allowed for in the function $f_4$, augmentation of resistance and push being considered independently. The differences between swell and wave action are introduced because of the difference in wavelengths.
4. RETARDATION CAUSED BY APPLIED RUDDER

According to Kent, under strong beam winds, the applied rudder necessary to hold a given course can cause sufficient resistance to dominate the other resistance effects. It may not even be possible to hold a course in beam winds of above, say, 50 knots, as illustrated in Figure 1. Such retardation is shown as reaching a maximum somewhat abaft the beam, at about 110° from the bow. This would also accord with the more usual situation in which steering is more difficult with the seas coming from abaft rather than forward of the beam. Greater use of the rudder will increase the average resistance.

Hence, rather crudely, we define

\[
\theta_r' = \frac{9}{11} |\theta_w'| \quad \text{for} \quad |\theta_w'| < 110^\circ \quad \ldots \ldots (8a)
\]

\[
\theta_r' = |\theta_w'| - 20^\circ \quad \text{for} \quad |\theta_w'| \geq 110^\circ \quad \ldots \ldots (8b)
\]

so that \( \sin \theta_r' = 0 \) for \( \theta_w' = 0 \),

\( = 1.0 \) for \( |\theta_w'| = 110^\circ \) (maximum effect),

\( = 0.34 \) for \( |\theta_w'| = 180^\circ \) (wind astern)

and now define \( f_2(w, \theta_w') \) by

\[
f_2 = \left[ 1 + C_r(w \sin \theta_r')^3 \right]^{-0.5} \quad \ldots \ldots (9)
\]

where \( C_r \) is an empirically set constant.

The term \( w \sin \theta_r' \) was raised to the third power inside the bracket as a step towards satisfying the general form of the effect in Figure 1. Note the rapid increase in this effect shown by Kent as the wind increases from 40 to 50 knots. Also, the effect is strong with \( \theta_w' \) between 80° and 140°. The whole term was included with the index set at -0.5 instead of -0.33 (suggested by (6)) to further strengthen the function. The resultant form of \( f_2 \) as defined by (9) and used in the calculations is shown in Fig. 3 for winds varying from 0 to 51 knots. In retrospect, it might have been better to leave the index at -0.33 and to raise \( w \) and \( \sin \theta_r' \) separately to higher powers to better match Figure 1.
A consideration of the data along part of the track of Trial 19/74 (leg V-W', see Figure 6), in which beam winds of 25 knots were encountered, showed that the effect did not appear marked under those conditions. Hence $C_\phi$ was set at $1 \times 10^{-5}$.

5. RETARDATION DUE TO SWELL

This is similar to factor $f_2$. Define

$$f_3 = \left[1 + g_s(\theta_s') \cdot \left(\frac{h_s}{H_s}\right)^2 \right]^{\frac{1}{3}} \ldots \text{(10)}$$

$g_s(\theta_s')$ is shown in Figure 2.

There were insufficient data to really distinguish between the effects of waves and swell, but as the ship's log records these independently, with directions noted for each, the factors $f_1$ and $f_3$ have been separately included. Figure 4a shows a plot of swell against wave height observed during Trial 19/74. Swell height seems roughly correlated with wave height such that $h_s = 2h_w$. Hence the normalization height in (10), $H_s$, has been set at twice the value used for $H_w$ in (7). Thus the factors $f_1$ and $f_3$ tend to be of the same magnitude under average conditions. This is also consistent with the observation that waves (coming from bow on, say) have a greater retardation effect than swell of the same height because the former have shorter wavelengths and, being steeper, lose more energy to the ship. Also, the separation of the sea state into swell and waves is a matter of visual judgement by the ship's officers and is often most imprecise. Insofar as Trial 19/74 is concerned, the absolute accuracy of the estimations of sea and wind state is not important, as the various constants in the model equations were adjusted to suit; however, if this model were used with data from other trials the absolute accuracy of the weather observations would be important. Fortunately, as the weather observations are made by a number of watch-keeping officers, individual tendencies to under or over-estimate the parameters are probably evened out.

6. THE PUSH OF WIND AND WAVES

Apart from the retardation already discussed which the ship suffers due to travelling through a disturbed sea, the vessel also receives a direct push due to incident wind and waves. This can be conveniently resolved into two components, parallel and normal to the ship's heading. At the time of this
study and for the purpose of these formulae, there seemed little point in trying to distinguish between wind and waves in the open ocean, as they are usually quite well correlated and a common direction is recorded for both. In addition, it is not uncommon in practice for one to be estimated from the other. Usually the wind is estimated both in magnitude and direction from the observed sea. Thus the recorded wind should be an absolute measurement, independent of the ship's motion. Should the wind be recorded by instrumentation on the ship, however, the apparent velocity would have to be corrected for ship's motion. The relation between recorded wave height and wind speed for Trial 19/74 is shown in Figure 4b. A reasonable estimate of wave height can be obtained from

\[ h \text{ (metres)} = 0.06 \times w \text{ (knots)} \] ....(11)

Supposing that the swell reached twice this figure, the heights are still considerably less than those expected of a fully developed sea. This is consistent with the rapidly changing nature of the weather at the time.

The net addition to the ship's forward speed is assumed to be

\[ f_4(w, \theta_w) = p \cos (\theta_w + 180^\circ) \] ....(12)

where

\[
\begin{align*}
p &= C_p \frac{w}{w} & \text{for} \ w \leq 25 \text{kn} \\
or &= (C_p \frac{w}{w})^{0.5} & \text{for} \ w > 25 \text{kn}
\end{align*}
\] ....(13)

and \( C_p \) and \( C'_p \) are fitted constants.

The change in the form of \( p(w) \) at 25 kn is based on the concept that in strong winds the waves get longer as well as higher and thus the pushing effect of the waves drops below a linear increase. A half-power law for all wind speeds, however, over-estimates the effect of light winds.

In retrospect, the following analysis is more soundly based:

From Section 2, we have for the propulsive force on the ship

\[ F = C_d \cdot v_1^2 \]
and, under given weather conditions, assume the drag coefficient \( C_d \) to remain constant.

\[
F = C'v_1^2,
\]

where \( C' \) is some constant. Therefore

\[
\Delta F = 2C'v_1 \Delta v_1
\]

Suppose \( \Delta F \), due to wind and waves, is of the form

\[
\Delta F = p(h_w, w, \theta'_w, \gamma)
= q(h_w, w') \cdot r(\theta'_w)
\]

\[\ldots (14a)\]

where \( w' \) is the relative wind speed. Function \( q \) can reasonably be made dependent on \( w'^2 \), but the dependence on waveheight is more uncertain, owing to the variation of wavelength. Suppose for simplicity,

\[
q(h_w, w') = C_1 h_w + C_2 w'^2
\]

The variation of \( r(\theta'_w) \) is not a simple cosine as used in (12). As the angle of the apparent wind to the bow \( \theta'_w \), varies from zero the projected area of the ship increases rapidly and \( r(\theta'_w) \) increases to a maximum at about 30° off the bow. Such a function is given by Todd (1967) reporting the work of Hughes (1930). Using this function, we have

\[
p = q(h_w, w') \cdot r(\theta'_w)
\]

and

\[
f_4(h_w, w, \theta'_w, \gamma) = \Delta v_1
\]

\[= \frac{p}{2C'v_1}\]

\[\ldots (14b)\]

so long as the ship is under way. Now suppose the engine is stopped in moderate winds and \( \Delta F \) contributes the total propulsive force.

We have

\[
\Delta F = C'(\Delta v_1)^2
\]

hence

\[
\Delta v_1 = \left(\frac{p}{C'}\right)^{\frac{1}{2}}
\]

\[\ldots (14c)\]

(Even at low speeds the Reynolds number is large and the flow will be turbulent.)

This shows why the ship may move through the water at several knots with moderate winds and seas astern, but no engine power; whereas the additive effect under normal steaming conditions is much less and decreases with increasing speed. It also suggests a way of evaluating \( p/C' \). With engines off, \( \Delta v_1 \) could be measured under varying conditions in which (14c) applies. Then (14b) could be evaluated. For example, suppose with 25 kn winds astern \( \Delta v_1 = v_1 \) is measured at 3 kn. The addition under similar conditions while under way at 9 kn
would be, by (14b), only 0.5 kn. A full and proper analysis of these effects is beyond the capacity of this paper (and of the author). The calculations were made using (13), however, before Eqs. (14) were appreciated. The errors are not so severe as to warrant repeating the analysis on this account.

An effective push while under way of 1 kn in head (or tail) winds of 25 kn seemed reasonable at the time and did not appear inconsistent with the data, so, using (13), $C_p = C'_p = 0.04$. In view of (14), this value of $f_n$ (while under way) now appears a little large; however it should be noted that Kent (see Figure 1) appears to predict larger values.

The normal (sideways) component of this effect has an interesting aspect. Elementary hydrodynamical considerations suggest that, provided the ship is not stalled in the water, the sideways speed is also reduced linearly as the forward speed increases. Thus under normal conditions, the leeway angle should fall off as the square of the forward speed. Consider Figure 5. We suppose that the ship is travelling through the water with a velocity

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

The ship can be considered stationary with the water stream incident with velocity $-v$. The ship makes leeway angle $\alpha$, given by

$$\alpha = \tan^{-1}\frac{v_2}{v_1}$$

Suppose the sideways force due to wind and sea is $F_w$. Under equilibrium conditions this will be balanced by a hydrodynamic sideways 'lift' $L$, which for small $\alpha$ is given by

$$L = \alpha \left| \frac{v_2}{v_1} \right|^2$$

$(15)$

$F_w$ depends on the wind and sea state and the relative heading of the ship. For given conditions, $F_w$ (and hence $L$) is independent of $\left| \frac{v_2}{v_1} \right|$, so from (15) we have

$$\alpha = \frac{1}{\left| \frac{v_2}{v_1} \right|^2}$$

$(16)$

To a sufficient approximation, we can substitute

$$\alpha = \frac{1}{v_2^2}$$

$(17)$

hence

$$v_2 = \frac{1}{v_1}$$

$(18)$
Now, for the model, initially define the normal velocity component \( v_2 \) by

\[
v_2 = C_q p \sin(\theta' + 180^\circ)
\]

....(19)

where \( p \) is defined as in (13) (or preferably, for new work, by an equation similar to (14a) using absolute wind velocity) and \( C_q \) is a constant.

From (1) and (3) we have

\[
v_1 = v_o \cdot f_1 \cdot f_2 \cdot f_3 + f_4
\]

If \(|v_2'| > v_1\) (ie \( \alpha > 45^\circ \)) the ship is almost certainly hydrodynamically stalled and there is no justification for reducing the initial value \( v_2 \) according to (18). Somewhat arbitrarily, write

\[
v_2 = v_2' \text{ for } |v_2'| > v_1 \text{ (stalled)}
\]

....(20a)

or,

\[
v_2 = \frac{v_2}{v_1} \text{ for } |v_2'| < v_1 \text{ (underway)}
\]

....(20b)

A different transition (stall) angle could be chosen, or a non-linear relationship between \( L \) and \( \alpha \) (15) could be incorporated, but in the absence of measured data on leeway angles, such refinements are not presently justified.

After some consideration, we set \( C_q = 2.5 \). This gives results that do not conflict with impressions gained over several trials. According to this value a 25 kn wind broadside \( \alpha \) to a stopped ship would cause it to drift downwind at 2.5 kn. This may be a little high, but the stalled condition was not of prime importance. Underway at 10 kn however, with the same wind on the beam, \( v_2 \) reduces to 0.25 kn and the leeway to 1.4°.

7. SHIP'S SPEED IN SMOOTH WATER

In the absence of any data to the contrary, it was assumed that the apparent propeller slip ratio was constant over the normal operating range of engine revolutions. This is defined by

\[
S_a = 1 - \frac{v_o}{Pn}
\]

....(21)

where \( P \) is the propeller pitch and \( n \) the propeller revolutions (in appropriate units). A constant value of \( S_a \) implies a simple linear relationship between \( v_o \) and engine revolutions, ie

\[
v_o = \frac{n}{K}
\]

....(22)
K can be regarded as constant over the course of any one trial, but its value will vary with time depending on the state of the ship's hull, loading, state of the propeller etc. The degree of hull fouling is usually the major factor. HMAS KIMBLA, with a clean hull, usually works with a value K = 15.0 r.p.m./knot. Sufficient data existed from Trial 19/74 to make an independent determination of this figure. This is given in Section 9.

8. PROGRESS THROUGH THE WATER

The progress through the water is given by

$$|v| = \sqrt{v_1^2 + v_2^2}$$

...(23)

in the direction

$$\phi = \phi' + \alpha$$

where \(\alpha\) is the leeway angle.

9. FITTING THE MODEL TO THE DATA

Figure 6 shows a plot of the ship's track for Trial 19/74. A total of 66 fixes were obtained from a satellite navigation system (accuracy = 0.3 n.miles) during the course of the trial (114 hr). The fixes of significance to this work are lettered. Associated with each labelled fix is a figure giving the number of hours into the trial starting from 0001 hrs (time zone K) on 08 December 1974. Figure 7 shows the dynamic topography of the area, derived from Expendable Bathythermograph (XBT) data taken during the trial. The expected surface currents are inversely proportional to the contour spacing.

Now the determination of the model parameters rested on two propositions. Firstly, that the surface currents over the legs CDEF were small enough for the net effect to be ignored. Secondly, that the average surface current over the leg RS equalled that over the leg ST. The reasonableness of these propositions can be judged by referring to Figure 7, bearing in mind that a topographic gradient of 10 dyn. cm. (0/900 m) per 18.5 km (10.0 n.miles) is estimated to produce a surface current of 0.75 m sec\(^{-1}\) (1.46 kn). This figure includes a factor of 1.15 to allow for the fact that the dynamic ocean structure
extends below 900 m, but does not include any enhancement due to the curvature of the topographic contours.

Table 1: Details of weather during Trial 19/74

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>$g_w$ (° true)</th>
<th>$w$ (kn)</th>
<th>$h_w$ (ft)</th>
<th>$g_s$ (° true)</th>
<th>$h_s$ (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.0</td>
<td>200</td>
<td>8</td>
<td>1</td>
<td>180</td>
<td>4</td>
</tr>
<tr>
<td>23.0</td>
<td>180</td>
<td>14</td>
<td>1</td>
<td>180</td>
<td>6</td>
</tr>
<tr>
<td>27.0</td>
<td>-</td>
<td>0</td>
<td>1</td>
<td>180</td>
<td>6</td>
</tr>
<tr>
<td>31.0</td>
<td>040</td>
<td>5</td>
<td>2</td>
<td>180</td>
<td>-</td>
</tr>
<tr>
<td>35.0</td>
<td>000</td>
<td>19</td>
<td>2</td>
<td>020</td>
<td>5</td>
</tr>
<tr>
<td>39.0</td>
<td>040</td>
<td>19</td>
<td>3</td>
<td>020</td>
<td>5</td>
</tr>
<tr>
<td>43.0</td>
<td>000</td>
<td>30</td>
<td>4</td>
<td>030</td>
<td>7</td>
</tr>
<tr>
<td>47.0</td>
<td>010</td>
<td>30</td>
<td>6</td>
<td>010</td>
<td>12</td>
</tr>
<tr>
<td>51.0</td>
<td>010</td>
<td>19</td>
<td>3</td>
<td>010</td>
<td>8</td>
</tr>
<tr>
<td>55.0</td>
<td>180</td>
<td>14</td>
<td>4</td>
<td>020</td>
<td>6</td>
</tr>
<tr>
<td>95.0</td>
<td>020</td>
<td>5</td>
<td>0</td>
<td>040</td>
<td>3</td>
</tr>
<tr>
<td>99.0</td>
<td>010</td>
<td>2</td>
<td>1</td>
<td>020</td>
<td>2</td>
</tr>
<tr>
<td>103.0</td>
<td>240</td>
<td>8</td>
<td>3</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>107.0</td>
<td>230</td>
<td>14</td>
<td>2</td>
<td>250</td>
<td>4</td>
</tr>
<tr>
<td>111.0</td>
<td>190</td>
<td>19</td>
<td>2</td>
<td>190</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 1 shows the recorded weather during those sections of the trial pertinent to this work. Original units are given. The weather at the start of the trial was relatively light (winds < 16 kn) from varying directions until point E was reached, after which the wind freshened to 30 kn from 010° around point E', with sea and swell reaching 1.8 m (6 ft) and 3.7 m (12 ft) respectively from the same direction. At F' the wind quietened and came in from the south. During legs RST the winds were again less than 16 kn from varying directions. Thus, variations in the parameters of the model only had a small effect on the calculated progress over legs RST, but a considerable effect over the leg EF, particularly as the wind and sea were almost head on at that time.

In view of the above, an iterative procedure was adopted to fit the various parameters to the data. First estimates were made and the set of the ship calculated along the various legs. If the data and model were perfect, the calculated set between successive fixes would be due to surface current plus any net error in the fixes. This calculated set has been termed the
'drift' in this discussion. In practice, this will also contain errors in the calculated dead reckoning (DR) data. The value of \( K \) (Eq.(22)) was adjusted to equalize the magnitude of the drift over the legs RS and ST. Now, with respect to the legs CDEF, errors in calculated ship's speed will show up directly in the comparison between distance travelled through the water, \( D_w \), and distance travelled over the ground, \( D_g \), assuming no net effect due to surface currents. The criterion for the model was to minimise \( \sum (D_w - D_g) \) over the three legs CD, DE and EF, separately and totally. Having adjusted the model appropriately, the drift over RST was recalculated, \( K \) readjusted and so on. This process converged after a few cycles, leading to \( K = 15.0 \pm 0.2 \) r.p.m./kn - a value identical to that traditionally used by HMAS KIMBLA's officers. The drifts during legs RS and ST are 0.75 m.sec\(^{-1}\) at 287° and 286° true respectively. The dynamic topography suggests a mean current of about 0.8 - 0.9 m.sec\(^{-1}\) at 285° true, so the calculated directional data (for which there were no direct constraints) are most encouraging. Interestingly, the exponents of the factors \( f_1 \) and \( f_4 \) ((7) and (10)) were initially each set at -0.5, due to a lack of appreciation at the time of (6). The data showed -0.5 was too severe and the magnitude of the exponents was reduced to -0.35. This gave a better fit and now can be seen to accord with the value of \( 1/3 \) suggested by (6).

The values finally accepted for \( \sum (D_w - D_g) \) for legs CDEF are given in Table 2, along with the time averages of \( v_0 \), \( f \), \( f_4 \) and \( f_5 \).

### Table 2

<table>
<thead>
<tr>
<th>Start time</th>
<th>leg</th>
<th>( \frac{D}{D_g} )</th>
<th>( \frac{D-D_g}{D_w} )</th>
<th>( \Delta t )</th>
<th>speed error</th>
<th>( &lt; v_0 &gt; )</th>
<th>( &lt; f &gt; )</th>
<th>( &lt; f_4 &gt; )</th>
<th>( &lt; f &gt; )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.50 C-D</td>
<td>79.2</td>
<td>+0.3</td>
<td>8.23</td>
<td>0.04</td>
<td>9.82</td>
<td>0.969</td>
<td>0.15</td>
<td>-0.01</td>
<td></td>
</tr>
<tr>
<td>33.73 D-E</td>
<td>32.9</td>
<td>-0.9</td>
<td>3.37</td>
<td>-0.27</td>
<td>9.91</td>
<td>0.970</td>
<td>-0.12</td>
<td>-0.17</td>
<td></td>
</tr>
<tr>
<td>37.10 E-F</td>
<td>88.8</td>
<td>+0.6</td>
<td>14.80</td>
<td>0.04</td>
<td>8.04</td>
<td>0.854</td>
<td>-0.84</td>
<td>+0.14</td>
<td></td>
</tr>
<tr>
<td>C-F</td>
<td>200.9</td>
<td>0.0</td>
<td>26.40</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Unfortunately, these figures convey an impression which is too optimistic. We should be able to obtain similar agreement for any portion of the track along, say BDEF'. There is the limitation, however, that the weather observations were only made every 4 hr and the weather changed quite rapidly over this period. Hence, one might not expect agreement over any leg much shorter than \( = 8 \) hr. For this reason not much notice was taken of the short...
leg DE compared with the two longer ones in adjusting the model and the quite good agreement shown in Table 2 for leg DE is of little moment. As a test, the section BDEF' has been subdivided in a different manner to that used in Table 2. Using the same model, the values for $\sum(D_w - D_g)$ and the time averages of the main functions were recalculated for the new divisions and are given in Table 3. Each division occupies about 8 hr.

Table 3

<table>
<thead>
<tr>
<th>Start time</th>
<th>leg</th>
<th>$\sum D_g$</th>
<th>$\sum(D_w - D_g)$</th>
<th>$\Delta t$</th>
<th>Spec error</th>
<th>$&lt;v_o&gt;$</th>
<th>$&lt;f&gt;$</th>
<th>$&lt;f_4&gt;$</th>
<th>$&lt;f_5&gt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.80</td>
<td>B-C'</td>
<td>70.5</td>
<td>0.8</td>
<td>7.53</td>
<td>0.11</td>
<td>9.89</td>
<td>.974</td>
<td>-1.17</td>
<td>0.04</td>
</tr>
<tr>
<td>29.33</td>
<td>C'-E</td>
<td>75.2</td>
<td>-1.0</td>
<td>7.77</td>
<td>-0.13</td>
<td>9.80</td>
<td>.965</td>
<td>0.09</td>
<td>-0.09</td>
</tr>
<tr>
<td>37.10</td>
<td>E-E'</td>
<td>45.9</td>
<td>8.9</td>
<td>8.70</td>
<td>1.02</td>
<td>8.02</td>
<td>.884</td>
<td>-0.81</td>
<td>0.13</td>
</tr>
<tr>
<td>45.80</td>
<td>E'-F'</td>
<td>61.6</td>
<td>-9.0</td>
<td>8.50</td>
<td>-1.06</td>
<td>8.13</td>
<td>.839</td>
<td>-0.65</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>B-F'</td>
<td>253.2</td>
<td>-0.3</td>
<td>22.50</td>
<td>-0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is at once apparent that although the net value of $\sum(D_w - D_g)$ over the section EE'F' is satisfactorily small (-0.1 n.miles), the errors in each leg separately are quite gross (8.9 and -9.0 n.miles respectively). These correspond to errors in ship's speed of about 1.0 kn. To take a closer look at these errors, suppose for a start that the values of $f_4$ (the component of the push received from the wind and waves along the ship's course) are correct. To satisfy the observed values of distance travelled $D_g$, the values of $f$ for legs EE' and E'F' would have to be 0.76 and 0.97 respectively. The figures for the observed weather however, indicate that the winds were about the same and the seas (from nearly head-on) were more severe during the latter leg, resulting in a lower value of $f$. (Each weather observation is assumed to be a mid-interval observation, ie to apply for 2 hr before and after the noted time.) Adoption of (14) to calculate $f_4$ might improve matters a little, but no amount of sensible adjustment of the parameters in the model will fully satisfy these data. A major error in the fixed position of E' would provide an easy explanation for this discrepancy, but an examination of that and neighbouring fixes gives no evidence of such an error. With the limitation of both the model and the data we should rest with the overall minimization of $\sum(D_w - D_g)$. 
10. **SUMMARY: THE MODEL FOR SPEED AS A FUNCTION OF WEATHER**

The values for the various parameters contained in the model equations were obtained from the procedure discussed in Section 9. These values have been given with the appropriate formulae in Section 3-7. Figure 8 shows the calculated variation of speed $v_1$ for HMAS KIMBLA (in the direction of ship's head) as a function of wind velocity and relative heading. It is of interest to compare this with Figure 1, which shows the variation for a large vessel given by Kent (1958). The calculations for Fig. 8 were made assuming that wind, waves and swell all come from the same direction and, furthermore, that wave height is proportional to wind speed (given by Eq. (11)) and swell height is twice the wave height.

HMAS KIMBLA has the following measurements: 1122 tonnes (1002 tons) displacement, 45.7 m (150 ft) length overall, 9.8 m (32.2 ft) beam, 2.7 m (8.7 ft) draft forward and 4.6 m (15.2 ft) draft aft. Power is 350 I.H.P. at 175 r.p.m., a speed not usually attained except perhaps within 24 hr of reaching home port. Figure 8 indicates a larger initial susceptibility to sea state than that shown in Figure 1. A 30 kn wind/sea head on apparently reduces HMAS KIMBLA's speed from 10 kn (at 150 r.p.m.) to about 6.2 kn, a drop of 38%, whereas the reduction for the larger vessel is from 14.4 kn to 12.1 kn, i.e. 16%. This is not surprising, considering that the larger ship has about six times as much power. The most notable difference between the two is that of the apparent behaviour in strong beam winds. As discussed in Section 4, the factor for retardation due to applied rudder was not made to be as severe as that indicated by Figure 1. The author believes that HMAS KIMBLA could hold a beam course in winds above 50 kn, but much depends on the build-up of sea state. HMAS KIMBLA's rudder may also be proportionately larger than that of the ship whose characteristics are illustrated in Figure 1. Until actual data are available, there is little point in pursuing this in detail. From the viewpoint of our subsequent use of the model (and Figure 8), that is to improve the dead reckoning in normal weather, our main interest lies in the behaviour in winds less than 30 kn. In this area little comparison with Figure 1 can be made. Our model indicates, however, that HMAS KIMBLA should be less affected by light to moderate winds and seas astern than the larger vessel.

A model that describes the variations in HMAS KIMBLA's speed through the water as a function of weather has been described. The only variables required as input to the calculations are those routinely recorded by the ship's officers at sea.
11. **RECOMMENDATIONS**

It should be re-emphasised that this work was done in the manner described in order to make the best use of the available data from several past trials. Improved dead reckoning was needed in order to deduce surface currents. The obvious way to improve our understanding of how HMAS KIMBLA or any other ship performs as a function of sea-state is to obtain some direct measurements of speed through the water, along with a continuous record of the pertinent weather data. There is little point in attempting to refine this particular model without such data. Thus it is essential that HMAS KIMBLA be fitted with a speed log, as is the case for most ships. If a dual axis log is fitted actual data on the leeway could also be obtained. The weather data could be improved simply by recording the observations at the time of any significant change, in addition to the regular 4 hr entry. A continuous record of relative wind velocity would be most helpful. The main lack in sea-state observations concerns data on wave-period or wave-length. A reliable shipborne wave and swell recorder would be of value. However, a simpler approach may be quite useful for these purposes. The major factor pertaining to loss of speed due to sea-state is probably the pitching motion of the ship. A three-axis accelerometer installed in the bows of the ship could record pitch, roll/yaw and longitudinal deceleration and would have the advantage of automatically recording the resonance effects associated with waves and swell of certain relative frequencies. These observations could be correlated with measured speed and a new model derived, perhaps through the application of multivariate analysis. Such a model would be useful to reduce dead reckoning data on occasions when the speed log was inoperative, as sometimes occurs. Associated analysis would also be useful in evaluating and correcting for errors in the recorded log speed under certain conditions.

**Acknowledgments**

The author acknowledges the support and cooperation of the officers and men of HMAS KIMBLA. The loan of the SATNAV equipment by the University of NSW Geophysics Department and the enthusiastic assistance of Mr B. Plummer is gratefully acknowledged. Mr J. Andrews of Weapons Research Establishment (WRE) calculated the dynamic height anomalies from the XBT data and the author is grateful for his assistance and advice. Mr V. Hardy of WRE took part in Trial 19/74 and much of this work was done while the author was seconded to WRE. Mr A. Latham (RANRL) provided technical support. This work was carried out under Navy Project ANZUS Eddy.
References.


Fig. 1. Fall in ship speed with changes in the relative directions of the wind and waves (From Kent, 1958)
FIG. 2. THE FUNCTIONS $g_w(\theta)$, $g_s(\theta)$ FOR RETARDATION DUE TO WAVES AND SWELL.
WIND DIRECTION RELATIVE TO SHIP'S BOW

FIG. 3. RETARDATION FACTOR $f_2$ DUE TO APPLIED RUDDER.
Fig. 4a. CORRELATION BETWEEN SWELL AND WAVE HEIGHT, TRIAL 19/74.
Fig. 4b. Correlation between wave height and wind speed, Trial 19/74.
FIG. 5. LEEWAY FORCES ON A SHIP UNDERWAY.
Fig. 8. Model of HMAS Kimbla's speed (at 150 RPM) as a function of wind velocity (knots).