RELATIONSHIP BETWEEN THE REAL CONCENTRATION
AND THE MEASURED CONCENTRATION INSIDE
A SAMPLING CHAMBER

by
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SHIP MATERIALS ENGINEERING DEPARTMENT
RESEARCH AND DEVELOPMENT REPORT

June 1978  DTNSRDC/SME-78/19
A mathematical relationship between the real concentration and the measured concentration of a material in a stream flowing into a sampling chamber is derived. It shows how the flow rate and the size of the sampling chamber affect the determination of the measured concentration. Graphical and numerical representations of this relationship are presented. The relationship is expressed as a function of the flow rate and chamber volume.
(Block 20 continued)

relationship can be used to predict possible errors involved in sampling. It also can be used by designers of processes, equipment, and instruments to size sample or mixing chambers when the flow rate and the variation in concentrations are known.
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NOTATION

C Concentration
Ci Concentration of incoming fluid
Co Concentration initially inside a chamber
n Number of flushings
Q Flow rate
t Time
V Volume of sampling chamber
ABSTRACT

A mathematical relationship between the real concentration and the measured concentration of a material in a stream flowing into a sampling chamber is derived. It shows how the flow rate and the size of the sampling chamber affect the determination of the measured concentration. Graphical and numerical representations of this relationship are presented. The relationship can be used to predict possible errors involved in sampling. It also can be used by designers of processes, equipment, and instruments to size sample or mixing chambers when the flow rate and the variation in concentrations are known.

ADMINISTRATIVE INFORMATION

This work was carried out under Program Element 62765N, Task ZF-57-572-003, David W. Taylor Naval Ship Research and Development Center Work Unit 1-2860-101.

INTRODUCTION

One of the elements required to control a continuous process is the knowledge of the changes that occur in the properties and characteristics of the flowing stream, so that the necessary adjustments can be made before the controlled process variable deviates from the intended range. The measurable properties are often temperature, pressure, pH, viscosity, density, reflective index, flow rate, concentration, etc. It is recognized that, if the measurement of the real property of the flowing stream is required, a good sample and a good mixing device are essential. When the constituents in the stream are homogeneous, the dependence on mixing may be less important than in a poorly mixed stream where the distribution of the properties of the constituents in the flowing stream are not well defined. When the latter is the case, the mixing device, the size of the mixing chamber, and the rate the mixing chamber in the sampling system is being filled can all affect the
measured property "seen" by a sensor. This report presents the depending relationship of such a measured property on the flow rate and the size of the chamber containing the sensor.

THEORY

EFFECT OF THE CONCENTRATION OF AN INFLUENT STREAM ON THE MEASURED CONCENTRATION INSIDE A MEASURING CHAMBER

Consider a rigid measuring chamber in which a sensor is placed. It is assumed that the mixing of the influent fluid (whose constituents are of conservative nature)* with the fluid which is already inside the chamber is so effective that the properties of the fluid leaving the chamber are the same as the mixed fluid within the chamber. Other effects on the fluid inside the chamber are assumed to be negligible.

A relationship between the concentration of the fluid inside the measuring chamber and the concentration of the fluid entering the measuring chamber can be derived for a constant rate of flow, provided the volume of the measuring chamber is known. Figure 1 is a schematic diagram on which this derivation is based.

* A conservative substance is defined as one whose property does not change with time.
DERIVATION

When a sample measuring chamber of volume \( V \) filled with fluid of concentrational initially at \( C_0 \) is fed with fluid at a rate of \( Q \) but of a different concentration \( C_1 \), the sensing device inside the chamber would measure the concentration \( C \) inside the chamber as it changes from \( C_0 \) to \( C_1 \). If it is assumed that the mixing inside the chamber is effective and other effects are negligible, the fluid leaving the chamber can be assumed to have the same concentration \( C \) as the concentration of the fluid inside the chamber at that instant.

The change occurring inside the chamber over an infinitesimal amount of time can be expressed mathematically as:

\[
\frac{d(CV)}{dt} = QC_1 dt - QC dt
\]  

When the volume of the measuring chamber \( V \) and the flow rate into the chamber \( Q \) are fixed, Expression (1) can be rearranged as:

\[
(\frac{Q}{V}) \frac{dt}{dt} = \frac{dC}{C_1 - C}
\]  

(2)

When this expression is used to express the variation in concentration of the fluid inside the chamber from \( C_0 \) initially at time zero to \( C \) at time \( t \), Equation (2) can be integrated and becomes:

\[
\frac{Q}{V} t = \frac{C_1 - C_0}{C_1 - C}
\]  

(3)

The relation between the flow rate \( Q \) of the stream and the volume of the sampling chamber \( V \) is such that

\[
Qt = nV
\]  

(4)

That is, during the time \( t \), the chamber of volume \( V \) has been flushed \( n \) times by a fluid flowing at a rate \( Q \). Utilizing this relation, Equation (3) becomes:
\[ n = \ln \left( \frac{C_i - C_o}{C_i - C} \right) \]  

or

\[ e^n = \frac{C_i - C_o}{C_i - C} = \frac{1 - \frac{C_o}{C_i}}{1 - \frac{C}{C_i}} \]  

Rearranged, Equation (6) becomes:

\[ \frac{C}{C_i} = 1 - e^{-n} \left( 1 - \frac{C_o}{C_i} \right) \]  

Equation (7) shows that, under a steady flow condition, the variation in concentration of the fluid leaving a well-mixed chamber can be predicted if the concentration change of the fluid flowing into the chamber is known. It also shows that, under this condition, one can estimate how soon, or after how many flushes, the concentration inside a sample measuring chamber, or the concentration of the effluent leaving a mixing chamber, becomes the same as the influent.

RESULTS OF THE DERIVED EXPRESSION

Graphical representations of Equation (7) are shown in Figures 2 and 3. They are based on the calculated results in Table 1. Figure 2 shows that it requires a minimum of four flushings for the concentration inside a sample measuring chamber, or the effluent leaving a mixing chamber to approach the same concentration as the influent. Figure 3 shows the relation between \( C/C_i \times 100 \) (measured concentration in percent of influent concentration) and \( C_i/C_o \) (ratio of influent concentration to initial concentration inside a chamber) when a different number of flushings was used.
Figure 2 - Flushings Required to Equalize Incoming and Outgoing Concentration of a Sample Chamber

C/C₀: MEASURED CONCENTRATION IN % OF INFLUENT CONCENTRATION

NUMBER OF FLUSHINGS, n

n = 0
n = 1
n = 4
Figure 3 - Error Prediction Under Different Operating Conditions
### TABLE 1 - NUMERICAL SOLUTION TO EQUATION (7) ILLUSTRATING THE EFFECT OF FLUSHING OF THE VARIATION BETWEEN INFLUENT AND EFFLUENT

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### APPLICATION

This relationship can be used by designers of processes, equipment, or instruments. If the variation in the ratio of influent concentration to concentration of fluid originally inside a sample measuring chamber, or the concentration initially inside a mixing chamber, are known, the designer can predict the error, as well as other operating conditions, and can select appropriate chamber sizes for different flow rates. A process control manufacturer, therefore, can design his control system by selecting a sensing device which has an appropriate delay in its response time.

7
This derived relation is also useful to evaluate a process in which the concentration in the flowing streams is varying.
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