Interaction of a surface wave with a dielectric slab discontinuity

Cover: Ice accumulation on helicopter blade after winter flight. (Photograph courtesy of U.K. Aeroplane and Armament Experimental Establishment.)
Interaction of a surface wave with a dielectric slab discontinuity

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The interaction of a 5.1-GHz transverse electric surface wave with a dielectric slab discontinuity is experimentally investigated. The wave is initially supported by a dielectric substrate resting upon a metallic groundplane. A slab, made of the same dielectric material as the substrate and variable in height, is then placed upon the waveguide. The results for a small slab sitting on the substrate showed that the discontinuity was a very inefficient launcher of reflected surface waves. Investigations of these reflections with a trough waveguide showed that, for values of slab height comparable to the exponential decay height of the surface wave, the reflections remain very small. However, as the slab height is increased beyond the decay height, the reflected amplitude approaches the theoretical value for a plane wave reflected.
20. Abstract (cont'd)

from the interface between air and the same dielectric. The results are applicable to surface wave methods of microwave deicing of wings and helicopter rotors.
PREFACE

This report was prepared by Dr. Steven A. Arcone, Geophysicist, and Allan J. Delaney, Physical Sciences Technician, of the Physical Sciences Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory. The research described in this report was funded by DA Project 4A161102AT24, Research in Snow, Ice and Frozen Ground, Task A3, Research in Terrain and Climatic Constants, Scientific Effort E1, Cold Environment Factors, Work Unit 002, Adhesion and Physics of Ice.

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SUMMARY

A method of current interest in wing and helicopter rotor deicing is the application of microwave energy propagating along the leading edge of the airfoil in a surface wave mode. This mode can be established using a thin dielectric substrate or coating upon a metallic surface. The problem investigated experimentally in this report deals with the interaction of such a surface wave with a slab (or step) discontinuity resting upon a dielectric waveguide.

A piece of plexiglass of known dielectric constant was placed in a section of a rectangular waveguide which had one wall removed. The structure then supported a surface wave propagating at 5.1 GHz in the lowest order transverse electric mode. A short slab of the same dielectric constant was placed on the waveguide and the resulting disturbed electric field strength was recorded above the waveguide along its entire length. The field strength profiles revealed strong perturbations taking place at the forward and rear edges of the slab but a comparatively weak reflected wave from the slab. A further experiment was then conducted with a trough waveguide designed to guide most of the reflected energy in a surface mode. The results of this experiment showed that, for values of slab height comparable to the exponential decay height of the surface wave, the reflections are weak. As the slab height was increased, however, the reflected wave amplitude approached the theoretical value for plane wave reflection from a simple air dielectric interface. These studies suggest that surface waves propagating at microwave frequencies along rotors or wings may strongly couple into abrupt accumulations of ice.
INTERACTION OF A SURFACE WAVE WITH A DIELECTRIC SLAB DISCONTINUITY

Steven A. Arcone and Allan J. Delaney

INTRODUCTION

Background

The application of microwaves to the deicing of wings and rotors where ice rapidly accumulates on the leading edge of the airfoil (see cover) is of current interest. Pure ice is very absorptive (i.e. it has a high loss factor) in the electromagnetic spectrum below 100 kHz, but only a marginal loss factor exists at microwave wavelengths. However, if enough heat can be generated to cause a local melting, then deicing may accelerate due to the higher loss factor of liquid water in the microwave spectrum (1-100 GHz). Deicing may also result when the ice reaches approximately ~4 °C, at which point the ice-substrate adhesion bond may weaken sufficiently to cause ice shedding.

The basic engineering problem of the design of a microwave airfoil deicing system is to generate and confine sufficient microwave energy in a manner that will not interfere with the mechanical requirements and aerodynamic design of the wing or rotor in question. Currently, consideration is being given to the use of the rotor or wing as a dielectric surface waveguide. A thin dielectric coating upon a metallic surface (e.g. rotor) is capable of confining propagating electromagnetic energy in a surface wave mode (so-called because the field strength decays with height above the guide) if the frequency, thickness and dielectric constant are properly chosen. Confined microwave radiation may then couple into an ice layer accumulating on the airfoil and cause the ice to shed as a result of internal dielectric heating.

The performance of a microwave deicing system is affected by the undesirable backscattering of a wave when inhomogeneities in the electrical properties of a material are encountered. The intensity of the scattering is determined by the size, shape and electrical parameters of the obstacle encountered and also the wavelength and nature (e.g. plane or spherical wave) of the incident radiation. In particular, the nature of a surface wave is such that the amplitude of the field strength varies exponentially along a plane of uniform phase situated perpendicularly to the direction of propagation. This nonuniformity presents a unique complication in the scattering process. In the ice adhesion application it is then necessary to investigate how such a wave will interact with an obstacle placed upon the waveguide. If the height of the obstacle is comparable or higher than the characteristic decay height, then one might expect severe reflections.

Objective and procedure

The objective of this research was to investigate the ability of a microwave surface wave to couple into a dielectric slab resting upon a uniform dielectric substrate. A plexiglass dielectric surface waveguide was designed and constructed to allow only the first order transverse electric mode surface wave to be propagated at 5.1 GHz. This wave mode was launched from within the dielectric waveguide. Plexiglass slabs of various heights were then placed on the waveguide. The surface wave electric field strength was measured along the entire length of the waveguide and slab. The results were used to estimate the amount of power lost by scattering from the slab discontinuity.

THEORY OF PLANE SURFACE WAVES

Surface waves have been extensively treated in the literature ever since Sommerfeld (1909) first established their theoretical basis. Subsequently, other investigators such as Norton (1936, 1937) refined the theory. These investigators intended their research to explain ground wave propagation at frequencies below about 10^7 Hz. With the advent of microwave technology in the 1940's, however, their results were also applied to laboratory surface waveguides. Much of this application has
involved designing efficient surface wave launchers (Cullen 1954, Rich 1955, Cohn et al. 1960) because simple antennas placed above a conducting or dielectric surface will radiate most of their power into space.

Investigations concerning the effect of inhomogeneities in the path of propagation have mainly been confined to flat surfaces containing discontinuities in electrical properties (Millington 1949, Wait 1956 and 1957, Feinberg 1959, King et al. 1973). Theoretical treatments of topographic irregularities have been given by Ott and Berry (1960) and Ott (1971), but experimental investigations of these irregularities have not been treated and are essentially the subject of this report.

The ensuing discussion briefly reviews the general features of plane surface wave transmission as described in Cartesian coordinates. Other coordinate geometries such as spheres and cylinders are also capable of supporting surface waves. Extensive treatises covering the theory of many surface waveguide possibilities may be found in the texts by Stratton (1941), Collin (1960), and Wait (1962).

Surface waves occur when an antenna is placed above or on the surface of some material, usually referred to as a half-space, of known dielectric permittivity, magnetic permeability and electrical conductivity. This situation is demonstrated in Figure 1. Three different means of radiation on and above the half-space then occur: 1) a direct space wave generated throughout the free space region, 2) an indirect reflected wave similarly generated throughout the free space region, and 3) a surface wave confined to the ground plane. At sufficient distance along the surface from the antenna, the space and reflected waves will cancel each other. This is due to the phase shift of 180° caused by the ground reflection when the indirect ray is incident upon the surface at a grazing angle of less than a few degrees. The only means of surface transmission is then the surface wave.*

Depending on the antenna type, transverse magnetic (TM) and/or transverse electric (TE) waves will be generated. This nomenclature refers to the field quantity that is exclusively orthogonal to the direction of wave propagation. These two types of wave modes are illustrated in Figure 2, where $E$ refers to the electric field components and $H$ to the magnetic field components. Simple electric and magnetic dipoles generate only the TM or TE modes, respectively, as shown in the figure, whereas more complex antennas such as arrays and horns may generate both mode types. It is also possible to generate both mode types by placing either an electrical (e.g. dielectric or conductive) or geometric discontinuity in the path of propagation.

Within the TM and TE mode classifications an infinite number of higher order modes can exist. When a dielectric coating is placed upon a metallic ground plane, however, the higher order surface wave modes may be suppressed upon selection of the proper signal frequency, permittivity, and thickness of the coating.

Since this investigation will be concerned with TE modes, only the mathematical formulation of these modes is presented. Equations for TM modes follow a similar development for which the reader is referred to the text by Collin (1960).

Using the coordinate system and waveguide parameters of Figure 3, the equations for the $E$ and $H$ fields for a transverse magnetic (TM) wave are:

\[ E_x = E_0 \frac{r_0^2 - r^2}{r_0^2} \cos \phi \]

\[ E_y = E_0 \frac{r_0^2 - r^2}{r_0^2} \sin \phi \]

\[ E_z = 0 \]

\[ H_x = 0 \]

\[ H_y = 0 \]

\[ H_z = E_0 \frac{r_0^2 - r^2}{r_0^2} \sin \phi \]

* Sometimes referred to as the "ground wave" in radio technology.
Figure 3. Coordinate reference system and waveguide parameters for development of the surface wave equations. \( d \) is the thickness of the substrate of dielectric permittivity \( \varepsilon_r \).

\[
H_{x1} = A_1 \frac{\alpha_e \csc (\alpha_e d)}{\omega \mu_0} \cos (\alpha_e x) P \\
H_{z1} = A_1 \frac{\beta_e \csc (\alpha_e d)}{\omega \mu_0} \sin (\alpha_e x) P
\]

where \( P = e^{-j(\omega x - \omega t)} \) for the lower waveguide medium. \( A_1 \) is an arbitrary constant, \( \omega \) the radian frequency (rad/s), \( t \) time in seconds, \( I = \sqrt{\varepsilon_r} \) and \( \mu_0 = 1.256 \times 10^{-6} \) H/m.

\( \alpha_e \) and \( \beta_e \) are the wave propagation vectors in the \( z \) (standing wave) and \( x \) (traveling wave) directions, respectively, for the lower medium. \( \gamma_e \) and \( \beta_e \) are the traveling wave propagation vectors in the \( z \) and \( x \) directions, respectively, for the upper medium. They are interrelated by the equations

\[
\omega^2 \mu_0 \varepsilon_1 = \alpha_e^2 + \beta_e^2
\]

and

\[
\omega^2 \mu_0 \varepsilon_0 = -\gamma_e^2 + \beta_e^2
\]

where \( \varepsilon_1 = \varepsilon_0 \kappa \). \( \kappa \) is the relative permittivity, \( \varepsilon_0 = 8.85 \times 10^{-12} \) F/m, and \( \alpha_e \) is determined from the transcendental equation

\[
\alpha_e \cot (\alpha_e d) = \sqrt{\omega^2 \mu_0 \varepsilon_0 (\kappa - 1) - \alpha_e^2}.
\]

The exponential decay factor \( \gamma_e \) is determined from \( \alpha_e \) by the relation

\[
\gamma_e = -\alpha_e \cot (\alpha_e d).
\]

Equations 3-6 may now be used to obtain the following set of equations:

\[
\gamma_e d = -\alpha_e d \cot (\alpha_e d)
\]

\[
(\gamma_e d)^2 + (\alpha_e d)^2 = (\kappa - 1)(\kappa_0 d)^2
\]

for the upper air medium, and

\[
E_{y0} = A_1 e^{-\gamma_e (x-d)} P
\]

\[
H_{x0} = A_1 \frac{\beta_e}{\omega \mu_0} e^{-\gamma_e (x-d)} P
\]

\[
H_{z0} = A_1 \frac{\beta_e}{\omega \mu_0} e^{-\gamma_e (x-d)} P
\]

\[
E_{y1} = A_1 \csc (\alpha_e d) \sin (\alpha_e x) P
\]

\[
H_{x1} = A_1 \frac{\alpha_e \csc (\alpha_e d)}{\omega \mu_0} \cos (\alpha_e x) P
\]

\[
H_{z1} = A_1 \frac{\beta_e \csc (\alpha_e d)}{\omega \mu_0} \sin (\alpha_e x) P
\]

Figure 4. Graphical solution of the TE mode equations 7 and 8. Intersections of the curves within the shaded regions are permissible solutions allowing exponential decay away from the waveguide surface. Example: at a value \( r = 11 \pi/4 \), three modes may propagate.

Components of a TE wave propagating along the waveguide in a source free region are

\[
E_{y0} = A_1 e^{-\gamma_e (x-d)} P
\]

\[
E_{y1} = A_1 \csc (\alpha_e d) \sin (\alpha_e x) P
\]

where \( \kappa_0 = \omega \sqrt{\mu_0 \varepsilon_0} \) or \( 2\pi/\lambda \) where \( \lambda \) is the free space wavelength. These equations are graphed in Figure 4. The shaded regions represent permissible solution areas and each solution is defined by the intersection of the two curves. At a value \( r = \pi/2 \), \( \gamma_e d \) becomes positive (thereby ensuring negative exponential growth above the waveguide) and the first order TE\(_1\) mode is allowed. This mode is then permitted for all values of \( \lambda \) such that

\[
\lambda < 4d \sqrt{\kappa - 1}.
\]

When \( r = 3\pi/2 \), the TE\(_2\) mode will also be excited; at \( r = 5\pi/2 \), the TE\(_3\) mode, etc.
Figure 5. Experimental apparatus for investigating TE surface waves. The plexiglass fills the waveguide. The mode exciter is placed before the plexiglass and the polyiron load is inserted in it.

It must be remembered that the above theory applies to the ideal case of dielectric slabs that extend infinitely in the $xy$ plane. However, when considering TE modes, perfectly conducting infinite waveguide walls may be inserted parallel to the $xz$ plane without disturbing the fields. $E_y$ is then terminated by an equivalent sheet of charge on the walls while $H$ is terminated by an equivalent sheet of current. The use of such walls will then prohibit TE surface modes from losing field intensity due to spatial expansion of the wavefront when the modes are excited by a finite source.

**WAVEGUIDE DESIGN AND CHARACTERISTICS**

**Physical apparatus**

Figure 5 diagrams the equipment used for producing a TE$_1$ surface wave. A 114-cm plexiglass strip with a cross section of 2.29 x 1.02 cm was placed in a standard rectangular piece of waveguide with one narrow wall removed. The dielectric constant of the plexiglass was measured at 2.85 using standard waveguide techniques. A small exciter was inserted at one end of the waveguide before the plexiglass to excite the radiation and a polyiron wedge load was inserted in the plexiglass at the other end to absorb the radiation. A double-stub tuner was used to match the signal generator to the exciter. No attempt was made to maximize the efficiency of energy transfer into the surface wave mode, as only its propagation characteristics were to be investigated. Since radiation was in a TE mode, the mode characteristics would ideally be unaffected by infinite conducting sidewalls orthogonal to the electric field vector. However, since these sidewalls were finite, they were expected to have some influence on the surface wave fields.

A small electric field probe was mounted on a motorized traverse to carry it the length of the guide. The probe was an extended center conductor of a coaxial cable that was oriented to measure $E_y$ as in Figure 3. The height of the probe above the guide was adjusted by moving the guide towards or away from the probe.

**Frequency characteristics**

According to Figure 4 propagation is possible at all frequencies above a cutoff frequency $f_c$ determined by the relation

$$f_c = \frac{c}{4d\sqrt{\kappa - 1}}$$

where $c = 3 \times 10^{10}$ cm/s. Thus, for $d = 2.29$ cm and $\kappa = 2.85, f_c = 2.41$ GHz. Figure 6 shows the measured surface wave amplitude (in volts of recorder gain) as a function of frequency along with the voltage standing wave ratio (VSWR) as measured with a slotted line behind the double stub tuner. The tuner was used to achieve the best possible match at each frequency measured. The VSWR then determined the percentage of power reflected from the tuner. The strength of the surface wave was determined with the probe placed just before the load. Figure 6 shows that only minimal energy propagated below 3.2 GHz, while energy propagated at all frequencies above 3.2 GHz except at 6.8 GHz. At this latter frequency a severe mismatch occurred at the tuner which could not be alleviated.
Figure 6. Frequency characteristics of the tuner-waveguide combination. No power could be measured below the theoretical cutoff frequency of 2.41 GHz. The minimal transmitted power at 6.8 GHz was due to a severe mismatch at the tuner.

Figure 7. Profiles of $E_y$ of the $TE_1$ surface wave at 5.1 GHz at various heights above the waveguide.

Figure 8. Normalized distribution of the ratio of the field strengths at $z = 3.29$ cm and $z = 2.54$ cm in Figure 8 for 100 positions along the waveguide.
A frequency of 5.1 GHz was chosen for study because of the low VSWR. At this frequency, no modes higher than the TE₁ were possible for the waveguide parameters used.

Spatial distribution of $E_y$ above the guide

Initially there is a sharp spike near the launcher where the dielectric begins. The field then maintains itself between 2 and 4 mV (amplifier gain level) at the 0.25-cm height until the load is reached, whereupon it rapidly decays. The larger fluctuations appearing about every 5 to 10 cm are due to mechanical imperfections such as poor contact between the metallic waveguide and the plexiglass dielectric. The more rapid oscillations are due to a very small amplitude wave reflected from the load and producing a marginal standing wave pattern. The general loss in amplitude from right to left is due to resistive losses in the walls and scattering from the mechanical irregularities. As probe height is increased, many of these perturbations persist and the field still retains this general 50% amplitude loss from right to left.

At 5.1 GHz, the first order solution of eq 7 and 8 predicts an exponential decay rate $\gamma_e$ of 1.03 cm⁻¹. Therefore, at a height of 0.97 cm above the guide, the field should be at 0.37 of its value at the surface of the guide. Figure 8 gives the normalized distribution for the ratio of the field strengths at $z = 2.54$ cm and $z = 3.29$ cm heights as measured at 100 positions along the guide (variation in intensity from $z = 2.54$ cm to the guide surface was not measured). Fifty-seven percent of the values fall between 0.20 and 0.23, revealing a much more rapid decay than the theoretical value of $e^{-\gamma_e/0.25} = 0.46$.

Guide wavelength

At 5.1 GHz, the TE₁ solution to eq 7 and 8 predicts that $\beta_e = 1.49$ cm⁻¹ (eq 3). The corresponding guide wavelength in the x direction is 4.22 cm. A short circuit was placed before the load, leaving a propagation distance of 114 cm. The resulting standing surface wave field strength $E_y$ is shown in Figure 9. Not including the initial spike, 52 maxima (or minima) can be counted along the guide over a distance of 100 cm. This corresponds to a wavelength of 3.92 cm, which suggests that the dielectric constant of the waveguide was slightly larger than the value measured for the sample.

SURFACE WAVE INTERACTION WITH A SLAB DISCONTINUITY

Figure 10 shows the profiles of $E_y$ taken above the waveguide containing the slab discontinuity. The slab is made of the same plexiglass as the waveguide. The height of the probe above the waveguide (without the
slab) was fixed at 1 cm for all profiles and the length of the slab was fixed at 15 cm. In each successive profile going from top to bottom, the height \( h \) of the slab is increased by approximately 0.13 cm. Therefore, during this sequence the top of the slab moves closer to the fixed probe. The smallest vertical division represents 0.5 mV of recorder gain for the electric field strength.

Relative to the field trace at \( h = 0 \), the following observations may be made for the other traces:

1. The field over section A-A' in all cases is nearly unchanged except for a slight modulation. Since 20 modulated cycles can be counted over a 40-cm section within A-A', the modulation is a slight standing wave because \( \lambda = 4.00 \) cm.

2. Over section B-B' all the field traces are nearly identical.

3. Over section A-B, the average field strength progressively increases with increasing \( h \).

4. At point A there is a sharp decrease in field strength followed by a sharp increase to the left.

5. At point B there is a sharp increase followed by a sharp decrease to the left.

Points 4 and 5 can be elaborated upon by considering the simpler case of plane wave interaction for comparison. When a homogeneous plane wave of unit amplitude traveling in a semi-infinite medium of dielectric constant \( \kappa_1 \) is normally incident upon a semi-infinite medium of dielectric constant \( \kappa_2 \), the resulting electric field amplitude \( E \) at the interface is

\[
E = \frac{2\sqrt{\kappa_1}}{\sqrt{\kappa_2} + \sqrt{\kappa_1}}
\]

When \( \kappa_2 > \kappa_1 \), \( E \) is then less than 1 and the field strength has decreased. When \( \kappa_2 < \kappa_1 \), \( E \) is then greater than 1 and the field strength has increased.
Both these qualitative predictions hold true for the inhomogeneous plane wave field changes indicated above point A where $\kappa_2 = 2.85$ and $\kappa_1 = 1$, and above point B where $\kappa_2 = 1$ and $\kappa_1 = 2.85$. However, no further maxima or minima of these amplitudes persist to the right of these edges, demonstrating their inefficiencies as secondary surface wave launchers.

One can elaborate on point 3 by comparing the increases of field strength observed over the slabs (as they increased in height and moved closer to the probe) to the increases predicted by the TE mode solutions given previously. Table I gives the observed $E_x$ values for varying slab heights and Figure 11 is a detail of the measuring arrangement. The values in the last column show that, as $h$ increased and the slab moved nearer the probe, $E_x$ did not increase as rapidly as theoretically expected. Since the slab was raised above the conducting lateral walls of the waveguide, it is probable that additional radiational losses occurred along the slab sides and thus decreased the observed field strength.

![Figure 11. Probe position and slab dimensions for the data in Table I.](image)

Table I. Comparison of theoretical and observed electric field strength ratios above the slab at $x = 68$ cm (see Fig. 10 and 11).

<table>
<thead>
<tr>
<th>$h$ (cm)</th>
<th>$d$ (cm)</th>
<th>$x_s$ (cm)</th>
<th>$E_x(z-s)/E_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Theoretical</td>
</tr>
<tr>
<td>0.0</td>
<td>2.29</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>0.13</td>
<td>2.42</td>
<td>0.87</td>
<td>1.12</td>
</tr>
<tr>
<td>0.25</td>
<td>2.54</td>
<td>0.75</td>
<td>1.24</td>
</tr>
<tr>
<td>0.38</td>
<td>2.67</td>
<td>0.62</td>
<td>1.41</td>
</tr>
<tr>
<td>0.51</td>
<td>2.80</td>
<td>0.49</td>
<td>1.59</td>
</tr>
<tr>
<td>0.64</td>
<td>2.93</td>
<td>0.36</td>
<td>1.71</td>
</tr>
<tr>
<td>0.76</td>
<td>3.05</td>
<td>0.24</td>
<td>1.88</td>
</tr>
<tr>
<td>0.89</td>
<td>3.18</td>
<td>0.11</td>
<td>2.06</td>
</tr>
</tbody>
</table>


**DISCUSSION AND CONCLUSIONS**

It is helpful for analyzing the data of Table II to compute the VSWR that results when a homogeneous plane wave in free space reflects normally off a dielectric of relative permittivity $\kappa$. The VSWR is computed from the formula

$$\text{VSWR} = \frac{1 + |R|}{1 - |R|}$$

(10)

where $R$, the reflection coefficient, is related to $\kappa$ by

$$R = \frac{\sqrt{\kappa - 1}}{\sqrt{\kappa + 1}}$$

(11)

For a value of $\kappa = 2.85$, as was investigated above, the VSWR = 1.69. The value is approached by the maximum observed VSWR for the slab of Figure 12b. In the most severe case, $h = 0.95$ cm while $\gamma_{sc}$ for the incident
wave is computed at 3.01 cm for $d = 1.27$ cm. Therefore, the height of this discontinuity needed to be only 32% of the vertical exponential decay height of the field strength in order to produce reflections comparable to the more ideal (and assumed) case of homogeneous waves and infinite boundaries.

Since the dielectric constant of ice is 3.2, it is therefore reasonable to expect that the maximum VSWR possible would be 1.79 from any severe ice buildup. This would correspond to a maximum power loss for the incident surface wave of only 0.35 db. As demonstrated in the results, this power would then be

Figure 12. Profiles of electric field strength at 5.16 GHz above a dielectric trough waveguide containing two cases (a and b) of steps of height $h$ but different lengths.
very inefficiently reflected back along the surface waveguide.

For the case of real rotors or airfoils, ice distributions are of course more gradual than the idealized model chosen here, and the waveguide will have to curve around the rotor edge. If sufficiently high frequencies are used (such as 22 GHz) where free water absorption is maximal, then the curvature of the waveguide should have a negligible effect on the propagation. Surface microwave energy might then reach almost 100% coupling efficiency into the more gradual ice buildup.

**LITERATURE CITED**


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