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I. INTRODUCTION

In a previous report [1], the backscatter power density spectrum of a large rotating conducting cylinder of arbitrary cross-section was investigated and numerical data presented for orthogonal polygonal cylinders. Because orthogonal polygonal cylinders possess some symmetry properties, their backscatter power density spectra are symmetric about the incident frequency. These backscatter power density spectra consist of spectral lines located at uniformly spaced discrete frequencies, \( \omega = \omega_0 + m \omega_a \), where \( m = 0, \pm 1, \pm 2, \ldots \), \( \omega_0 \) is the incident angular frequency, \( N \) is the number of sides of the polygon and \( \omega_a \) is the angular velocity of the cylinder. The values of the spectral lines decrease sharply when \( |m| > \omega_0/\omega_a \), where \( \omega_D = 2a \omega_0/\omega_a \) and \( a \) is the largest radius of the cylinder.

Although orthogonal polygonal cylinders are not accurate models of the tracks of tracked vehicles, the backscatter power density spectra of the former do exhibit some degree of similarity to those measured of the latter. To more truthfully simulate the tracks, cylinders with cross-sections like the one shown in Figure 1 are probably more suitable than orthogonal polygonal cylinders. In this report, we shall study the backscatter power density spectra of such cylinders and cylinders of more complicated shapes.

Infinite cylinders are good models of tracks when incidence is either from the front or from the rear of the tracked vehicles. However, for other directions of incidence the tracks can not be considered as infinite cylinders. The open ends of the tracks will contribute to the backscatter power density spectra too. These two contributions to the backscatter power density spectra of tracked vehicles are very different. In this report, we shall also discuss this problem.

II. ROTATING CYLINDER

The cylindrical structures whose backscatter power density spectra will be studied in this section are shown in Figures 1 through 3. The cylinder in Figure 1 is a model for simulating the track of a tracked vehicle. The motion of the track can be simulated in the following way. Points \( A_i \) move along a circular arc of radius \( a \) at an angular velocity \( \omega_a \). Points \( P \) and \( Q \) move along the straight lines \( SS' \) and \( UU' \), respectively. The effect of a cover over the track and the effect of the ground on the backscatter power density spectrum are studied using the structures in Figures 2 and 3, respectively.

If the total number of points moving along the circular arc of Figure 1 is \( I \), then a complete circle will contain \( N \) points where

\[
N = \frac{360^\circ}{180^\circ - 36^\circ} I = 2.5I. \tag{1}
\]
Figure 1. Track model.
Figure 2. Track model with cover.
Figure 3. Orthogonal polygon-cylinder above a ground plane.
Thus we can compare the backscatter power density spectra of the cylindrical structures shown in Figures 1 through 3 with those of a corresponding orthogonal polygonal cylinder. These comparisons are shown in Figures 4 through 11. From these figures, we can make the following observations:

1) **For the orthogonal polygonal cylinder:**

   The spectrum is symmetric about \( m=0 \). For the TM case, the spectrum decreases monotonically, while for the TE case, the spectrum has a local maximum around \(|m| = \omega_D/Na\), where \( \omega_D \) is the Doppler angular frequency, defined as \( \omega_D = 2\alpha_0\omega_0/c \). For a more detailed discussion, see Reference [1].

2) **For the track:**

   The spectrum is essentially the same as that of a corresponding orthogonal polygonal cylinder. However, a large difference between the two sets of spectra occurs around \( m \approx \omega_D/Na \), much lower for the spectrum of the track. This is due to the change of the lower half geometry of the orthogonal polygonal cylinder which was the main contributor to the spectrum around \( m \approx \omega_D/Na \).

3) **For the track with a cover:**

   The addition of the cover blocks the contribution from the top of the track to the backscatter power density spectrum, thus reducing the level of the spectrum around \( m \approx \omega_D/Na \). The local maximum around \( |m| = \omega_D/Na \) found in the spectrum of an orthogonal polygonal cylinder for the TE case disappears in the spectrum of a covered track.

4) **For the orthogonal polygonal cylinder above a ground plane:**

   The ground plane generally will make the spectrum more irregular. But with the ground plane added, the general trend of the spectrum still follows the spectrum obtained for the case without the ground plane. Note that this set of spectra are calculated without taking into account the coupling between the cylinder and its image.
Figure 4. Power density spectrum.
ORTHOGONAL POLYGON-CYLINDER

ORTHOGONAL POLYGON-CYLINDER WITH GROUND PLANE

TRACK

TRACK WITH COVER

Figure 5. Power density spectrum.
Figure 6. Power density spectrum.
Figure 7. Power density spectrum.
Figure 8. Power density spectrum.
Figure 10. Power density spectrum.
Figure 11. Power density spectrum.
III. UNIFORMLY MOVING LINEAR ARRAY

Infinite cylinders are good models of tracks when incidence is either from the front or from the rear of the vehicles. For other directions of incidence, the tracks can not be considered as infinite cylinders. The open ends of the tracks will also contribute to the backscatter power density spectra too. The major part of this contribution is from a uniformly moving linear array. Unfortunately there are no simple models for the linear array. Hence the analysis of this contribution will be qualitative only.

Consider a linear array moving at a uniform velocity \( \mathbf{a} \) in the direction of its axis. The inter-element distance of the array is \( 2\mathbf{a}/N \). Let \( \phi \) be the angle of incidence measured from the direction of the array velocity. The backscattered field can be written as

\[
E_s = \bar{e}(t)e^{j\omega_0 t},
\]

where \( \omega_0 \) is the incident angular frequency. The function \( \bar{e}(t) \) is a periodic function of \( t \) with period \( T = 2\pi/\mathbf{\omega} \) and can be written as

\[
\bar{e}(t) \cong \bar{A} e^{j \frac{2\mathbf{a}}{c} \omega_0 t \cos \phi} = \bar{A} e^{j \omega_D t \cos \phi}, \quad 0 < t < T,
\]

where \( \bar{A} \) does not depend on \( t \), and \( \omega_D = 2\mathbf{a}\omega_0/c \) is the Doppler angular frequency. Thus the backscatter power density spectrum consists of spectral lines located at \( \omega = \omega_0 + m\mathbf{\omega} N \), \( m = 0, \pm 1, \pm 2, \ldots \). A single spectral line is proportional to the square of

\[
E_m = \left| \int_0^T e^{j(\omega_D t \cos \phi - m\mathbf{\omega} N t)} dt \right|^2 = 2 \left| \sin \left( \frac{\left| \omega_D t \cos \phi \right|}{\mathbf{\omega} \cos \phi - m\mathbf{\omega} N} \right) \right|^2,
\]

which peaks around \( m\mathbf{\omega} \cos \phi / N \) and decreases monotonically for \( m \) away from \( \mathbf{\omega} \cos \phi / N \). The contribution of the linear array to the backscatter power density spectrum accounts for the peak at \( \mathbf{\omega} \cos \phi \) and the spectrum beyond \( \mathbf{\omega} \cos \phi \) measured of a tracked vehicle with oblique incidence. The contribution of this linear array to the spreading of the spectrum vanishes for \( \phi = 90^\circ \).
REFERENCES

The backscatter power density spectrum from the track of a tracked vehicle is investigated in this report. The major contributors to the spectrum are modeled as a rotating cylinder and a uniformly moving linear array. Both spectra consist of discrete spectral lines. The former peaks around the incident angular frequency \( \omega_0 \), while the latter peaks around \( (\omega_0 + \nu' \cos \phi) \), where \( \omega_0 \) is the Doppler angular frequency and \( \phi \) is the angle of incidence measured from the direction of the array velocity.