NTS-2 INDEPENDENT STABILITY AND CONTROL ANALYSIS

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FINAL REPORT
VOLUME II APPENDICES

Prepared for
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This is Volume II of two volumes. Volume I contains the Introduction, Summary of Results and Recommendations, Model Development, Presentation of Results, and Conclusions (288 pages).

Volume II presents appendices in this order:
- NTS-2 Mass Properties
- One Body Mode Equations
- Three Body Model
- Nutation Damper Model
- Attitude Control Electronics System Model
- Tilted, Centered Dipole Magnetic Torque Model
Solar Torque Model
NTS-2 Gravity Gradient Rod Thermal Analysis

Details of Solar Array Flexibility Modeling
Modified NTS-2 Solar Array Torsion Model
Solar Array Torsion Modes, Solution for Response to Torque
Motor Angular Accelerations
Combined Earth Sensor Model
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APPENDIX A

NTS-2 MASS PROPERTIES

The mass properties for NTS-2, Tables A-1 and A-2, consist of launch and orbit conditions. DRY signifies NO hydrazine in that condition. Hydrazine usage can be obtained from Table A-3 and be added to any dry condition by

$$I_{\text{Total}} = I_c + WT_c x d_c^2 + I_h + WT_h x d_h^2$$

where

- $C = $ Condition
- $H = $ Hydrazine
- $d = $ distance CG is transferred

Individual component mass properties are shown in Tables A-1 and A-2, which along with hydrazine usage can be added to the last two conditions shown in Figure A-1.

EXAMPLE:

USING COND (DRY), AKM BURN, AKM SEP, WITHOUT SOLAR ARRAY, GRAVITY GRADIENT, NUTATION DAMPER, MAGNETOMETER, AND REACTION WHEELS, FIND MASS PROPERTIES WITH SOLAR ARRAYS STOWED AND 50 PERCENT HYDRAZINE USAGE.

New C.G.Z. = \( \frac{WT \times CGZ}{TWT} \)

\[
= \frac{(799.7) (25.69 + (60.6) (22.00) + (26)(31.68)}{886.3}
\]

= 25.61 in.

\[
I_{xx} = 70.02 + (799.7) (0)^2 + 11.82 + (60.6) (0)^2 + 0.1 + (26) (0)^2
\]

= 81.94 Slug-Ft.²

\[
I_{yy} = 56.32 + \frac{(799.7)(.08)^2}{(32.2)(144)} + 8.65 + \frac{(60.6)(3.61)^2}{(32.2)(144)} + 3.1 + \frac{(26)(6.07)^2}{(32.3)(144)}
\]

= 68.45 Slug-Ft.²

\[
I_{zz} = 102.40 + \frac{(799.7)(.08)^2}{(32.2)(144)} + 14.47 + \frac{(60.6)(3.61)^2}{(32.2)(144)} + 3.1 + \frac{(26)(6.07)^2}{(32.2)(144)}
\]

= 120.35 Slug-Ft.²
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<th>PRODUCT OF INERTIA (slug-ft²)</th>
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<td></td>
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<td>Y</td>
<td>Z</td>
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<td>DEPLOYED 60 FT</td>
<td>16.82</td>
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<td>9.06</td>
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<td>PRODUCT OF INERTIA (slug-ft^2)</td>
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<td>191.51</td>
<td>1995.23</td>
<td>202.09</td>
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<tr>
<td>COND (DRY)</td>
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<td>25.53</td>
<td>70.62</td>
<td>56.58</td>
<td>102.87</td>
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<td>NUTATION DAMP</td>
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</tr>
<tr>
<td>COND (DRY)</td>
<td>799.7</td>
<td>25.69</td>
<td>70.02</td>
<td>56.32</td>
<td>102.40</td>
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Figure A-1. Hydrazine Usage

Hydrazine Usage

<table>
<thead>
<tr>
<th>Hydrazine (Pounds)</th>
<th>34.13</th>
<th>32.37</th>
<th>31.68</th>
<th>30.85</th>
<th>30.35</th>
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<td>Inches (C.O.D.)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
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</tbody>
</table>

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Prep. Chk. App. Date: 6/2/76

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APPENDIX B

ONE BODY MODEL EQUATIONS

This appendix presents the derivation of equations of motion used in the one body simulation. For Definition of Symbols, see Figure B-1.

Vector torque equations are given by

\[ \tau = \int_{x} \left( \frac{d}{dt} \frac{d^2 r}{dt^2} \right) \, dm \]

\[ = \frac{d}{dt} \left( \int_{x} \frac{d}{dt} \frac{d r}{dt} \right) \, dm \]

\[ \tau = \frac{d\vec{H}}{dt} \]  

(B-1)

where \( \vec{H} \) is defined as

\[ \vec{H} = \int_{x} \left( \frac{d}{dt} \frac{d r}{dt} \right) \, dm \]

\[ = \left[ \vec{J}_o + m_o \vec{M}(\vec{r}_o, \vec{r}_o) + \sum i \vec{J}_i + \sum m_i \vec{M}(\vec{r}_i, \vec{r}_i) \right] \vec{\omega}_o + \sum i \vec{J}_i \vec{\omega}_i \]

\[ = \vec{I}_o \vec{\omega}_o + \sum i \vec{I}_i \vec{\omega}_i \]  

(B-2)

where, in body 0 coordinates,

\[ M(a,b) \triangleq a^T \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{xy} & I_y & -I_{xy} \\ -I_{xz} & -I_{xy} & I_z \end{bmatrix} b^T - a^T b \]

\[ J \triangleq \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{xy} & I_y & -I_{xy} \\ -I_{xz} & -I_{xy} & I_z \end{bmatrix} \]

\[ J_i = T_{oi}^T J_i T_{oi} \quad \text{(} T_{oi} \text{ is transformation from wheel } i \text{ to body } o \text{)} \]
Body CM's: \( \mathbf{C}_0, \mathbf{C}_1 \)
Total CM: \( \mathbf{C} \)
Masses: \( m_0, m_1 \)
Inertia Tensors: \( \mathbf{J}_0, \mathbf{J}_1 \) (\( \mathbf{J}_0, \mathbf{J}_1 \) are inertia tensors in reference body coordinates of inertias about \( \mathbf{C}_0 \) and \( \mathbf{C}_1 \) respectively)

Figure B-1. Notation
\[ \omega_i = T_{oi} \frac{\omega_i}{I} \]

\[ I_x = \int (y^2 + z^2) \, dm = I_{yy} + I_{zz}, \text{ etc.} \]

\[ I_{xy} = \int xy \, dm, \text{ etc.} \]

The rotational equations of motion are then given by

\[ \bar{T} = \frac{d\bar{H}}{dt} \]

\[ = \frac{d}{dt} \left[ I_0 \omega_o + \sum_i I_i \omega_i \right] \]

\[ = \frac{\delta}{\delta t} \left[ I_0 \omega_o + \sum_i I_i \omega_i \right] - \omega_o \times \left[ I_0 \omega_o + \sum_i I_i \omega_i \right] \]

\[ T = I_0 \omega_o + \sum_i I_i \omega_i + \omega_o \times H \]

Wheel Torques may be described by

\[ \int \bar{x}_i \times \bar{F} = \int \bar{x}_i \times \frac{d^2}{dt^2} \bar{x}_i \, dm \]

\[ = \frac{d}{dt} \int \bar{x}_i \times \frac{dx_i}{dt} \, dm \]

\[ \bar{T}_i = \frac{d\bar{H}_i}{dt} \]

Wheel momentum is given by

\[ \bar{R}_i = \int \bar{x}_i \times \frac{d}{dt} \bar{x}_i \, dm \]

\[ = \int \bar{x}_i \times \left[ \frac{\delta}{\delta t} \bar{x}_i + \bar{\omega}_o \times \bar{x}_i \right] \, dm \]

7
\begin{align*}
  &= \int \mathbf{x}_i x [\mathbf{\omega}_i \times \mathbf{x}_i + \mathbf{\omega}_o \times \mathbf{x}_i] \, dm \\
  &= \overline{J}_i \mathbf{\omega}_i + \overline{J}_i \mathbf{\omega}_o \quad (B-4)
\end{align*}

Differentiating
\begin{equation}
  T_i = \overline{J}_i \mathbf{\omega}_i + \overline{J}_i \mathbf{\omega}_o + \mathbf{\omega}_o \times J_i \mathbf{\omega}_i + \mathbf{\omega}_o \times J_i \mathbf{\omega}_o \quad (B-5)
\end{equation}

Torques about the wheel axis are given by
\begin{equation}
  T_i = \overline{J}_i \mathbf{\omega}_i + \overline{J}_i \mathbf{\omega}_o \quad (B-6)
\end{equation}

\begin{equation}
  \dot{J}_i \mathbf{\omega}_i \quad (B-7)
\end{equation}

where \( T_i \) is the sum of motor, bearing and windage torques on wheel \( i \).
Equations B-7 and B-3 are combined to compute the body rate derivatives.
\begin{equation}
  T = I_0 \mathbf{\omega}_o + \sum_{i} T_i + \mathbf{\omega}_o \times H \quad (B-8)
\end{equation}

**Kinematics**

The inertial coordinate system and the Euler sequence are consistent with the Rockwell convention presented in the AVCS splinter meeting notes at the preliminary design review. Specifically, Rockwell specified a yaw pitch roll Euler sequence with the equations:
\begin{align*}
  \dot{\phi} &= \omega \cos \psi + q \cos \phi - r \sin \phi \\
  \dot{\psi} &= (\omega \sin \psi \sin \theta + q \sin \phi + r \cos \phi) (1/\cos \theta) \\
  \dot{\theta} &= p + \dot{\psi} \sin \theta + \omega \sin \psi \cos \theta
\end{align*}
where

\[ \omega = \text{orbit rate} \]
\[ p, q, r = \text{body rates} \]
\[ \phi, \theta, \psi = \text{Euler angles} \]
\[ \dot{\phi}, \dot{\theta}, \dot{\psi} = \text{Euler rates} \]

These equations are consistent with:

1. A local vertical system with z axis pointing toward the earth and x axis in the orbit plane and positive on the direction of motion.
2. Euler angles specifying body orientation relative to the local vertical frame.

The inertial coordinate system is the Geocentric Equatorial Frame (e.g., z axis positive North and x axis positive in the direction of the Vernal Equinox). The local vertical system is consistent with Rockwell's (e.g., z axis positive toward the earth and x axis in the orbit plane).

Two sets of Euler angles, both having a yaw pitch roll sequence, are used. The first set PSI, THETA1, PHI1 define the inertial to body transformation E. The second set, PSI, THETA, PHI specify the orientation of the body relative to the local vertical frame and should be consistent with Rockwell's Euler angles. The inertial Euler angles are integrated in the simulation via quaternions.

Linear Motion

The simulation uses two subroutines (POSITN and RADVEC) to compute the ephemerides for the satellite and the sun. The ephemerides for the satellite and the sun are calculated using the six Kepler elements: semi major axis (a),
eccentricity (e), inclination (i), right ascension of the ascending node (Ω), argument of perigee (ω), and mean anomaly (M). The update is a simple rate term on the latter three. Table B-1 presents the six elements, their units and values for the vernal equinox condition. The rate terms are derived in the program from the fixed parameters and are given by

\[
\begin{align*}
\dot{M} &= \sqrt{\frac{\mu}{a^3}} - \frac{k}{a(1-e^2)^2} \sqrt{1-e^2} (1.5 \sin^2 i-1) \\
\dot{\Omega} &= \frac{k}{a(1-e^2)^2} \cos i \\
\dot{\omega} &= \frac{k}{a(1-e^2)^2} (2.5 \sin^2 i-2)
\end{align*}
\]

where

\[
\begin{align*}
\mu &= Gm = 62630.3949 \\
k &= \frac{R_E^2 J_2}{6} \text{(first term of perturbing force due to Earth's oblateness)} = -19255.96124
\end{align*}
\]

The rate terms for the sun are fixed and are included in Table B-1. The simulation updates M, Ω, ω by

\[
\begin{align*}
M(t) &= M(t_0) + \dot{M} \cdot (t-t_0) \\
\Omega(t) &= \Omega(t_0) + \dot{\Omega} \cdot (t-t_0) \\
\omega(t) &= \omega(t_0) + \dot{\omega} \cdot (t-t_0)
\end{align*}
\]

A seventh term, the Greenwich Hour Angle (GHA), which relates the earth-fixed coordinate system to the inertial system, given the six elements is updated via:

\[
GHA(t) = GHA(t_0) + \omega_e \cdot (t-t_0)
\]

\[\omega_e = \text{earth rotation rate}\]
TABLE B-1. GPS AND SOLAR EPHEMERIDES

<table>
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<th>SYMBOL</th>
<th>NAME</th>
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<th>STANDARD VALUE</th>
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<tr>
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* Astronomical unit
APPENDIX C
APPENDIX C

THREE BODY MODEL

STRUCTURE

The overall "three" body structure is as shown in Figure C-1. It differs from that given in Reference C-1 in the addition of the gravity gradient rods and tip mass components. The vector $\vec{r}_k$ extends from the center of the space vehicle, CM, to the support point of rod $k$. The vector $\vec{r}_k$ extends from the support point to the center of tip mass $k$. The vector $\vec{r}_k$ extends from the center of tip mass $k$ to a differential mass element. Since all torques will be referenced to the total system center of mass, CM, the vector $\vec{r}_o$ will contain the effects of center of mass variation due to rod/tip mass movement.

EQUATION DERIVATION

In modeling the dynamics associated with the gravity gradient rods and tip masses, the form, procedure, and notation developed in Reference C-1 was followed as closely as possible.

The system momentum, with respect to inertial space is given by:

$$\vec{H} = \vec{I}_o \omega_o + \sum_i I_i \omega_i + \sum_j I_j \omega_j + \sum_k \vec{H}_k + \sum_n \vec{H}_n$$  \hspace{1cm} (C-1)

where the inertia tensors

$$\vec{I}_o = J_o + \sum_i J_i + \sum_j J_j + \sum_k J_k + \sum_n J_n - m \vec{M}(r_o, r_o) + \sum_i m_i \vec{M}(r_i, r_i)$$

$$+ \sum_j m_j \vec{M}(r_j, r_j) + \sum_k m_k \vec{M}(r_k + r_k) + \sum_n m_n \vec{M}(r_n + r_n)$$

$$\vec{I}_1 = J_1$$  \hspace{1cm} (C-2)

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GRAVITY GRADIENT ROD SUPPORT POINT $P_k$
SOLAR PANEL PIVOT POINTS $P_j$
BODY CM'S: $CM_i$, $CM_j$, $CM_k$; CM = TOTAL CM
MASSES: $m_i$, $m_j$, $m_k$, $m_n$; $m$ = TOTAL MASS
INERTIA TENSORS (ABOUT CM'S) $J_0$, $J_i$, $J_j$, $J_k$, $J_n$

Figure C-1. Three Body Structure
I, = j + m_j M(r_j + r_j + p_j, J)

and

M(a,b) = b^T a T - b a^T

and where \( \dot{r}_o \) is computed from

\[
\dot{m}_o r_o + \varepsilon_i m_i (r_i + \dot{r}_i) + \varepsilon_j m_j (\dot{r}_j + p_j, \dot{p}_j) + \varepsilon_k m_k (\dot{r}_k + \dot{r}_k + \dot{p}_k)
\]

\[
+ \varepsilon_m n (\dot{r}_n + \dot{p}_n) = 0
\]

If we assume a massless rod and the sphere is a point mass, then

\[
J_k = 0
\]

The momentum \( \vec{H}_k \) may be represented by

\[
\vec{H}_k = m_k (r_o + \dot{r}_k + p_k) + \frac{\delta \rho_k}{\delta t} + \int_k (\dot{r}_o + \dot{r}_k + \dot{p}_k) \times \frac{\delta x_k}{\delta t} dm
\]

Since we have assumed the tip mass may be represented by a point mass, the second term in equation C-7 may be set to zero.

The torque equations which are the ultimate goal of the model are derived from

\[
\vec{T} = \frac{d\vec{H}}{dt} = \frac{\delta \vec{H}}{\delta t} + \vec{\omega} \times \vec{H}
\]

or, differentiating (1)

\[
\vec{T} = I \vec{\omega} + \int \vec{\omega} + \Sigma_i I_i \vec{\omega} + \Sigma_j I_j \vec{\omega} + \Sigma_k \vec{H}_k + \Sigma_n \vec{H}_n + \vec{\omega} \times \vec{H}
\]

where the inertia tensor derivatives

\[
i_o = \varepsilon_j \dot{j}_j - m(M(\dot{i}_o, r_o) + M^T(\dot{i}_o, r_o))
\]

\[
+ \varepsilon_j m_j (M(r_j + \dot{p}_j, \dot{p}_j) + M^T(r_j + \dot{p}_j, \dot{p}_j))
\]

\[
+ \varepsilon_k m_k (M(r_k + \dot{p}_k, \dot{p}_k) + M^T(r_k + \dot{p}_k, \dot{p}_k))
\]

\[
+ \varepsilon_m n (M(r_n + \dot{p}_n, \dot{p}_n) + M^T(r_n + \dot{p}_n, \dot{p}_n))
\]
\[ \dot{\mathbf{r}}_j = \mathbf{\dot{r}}_j + \sum m_j \mathbf{M}(\mathbf{r}_0^j + \mathbf{r}_j^i, \mathbf{v}_j^i) + \sum m_j \mathbf{M}(\mathbf{\dot{r}}_0 + \mathbf{\dot{r}}_j^i, \mathbf{\dot{v}}_j^i) \]  
(C-11)

where

\[ \dot{\rho}_j = \omega_j \times \rho_j \]  
(C-12)

\[ \ddot{\mathbf{r}}_0 = -\frac{1}{m} \left[ \sum m_j \mathbf{\dot{\rho}}_j + \sum \mathbf{m}_k \mathbf{\dot{\rho}}_k + \sum \mathbf{n}_m \mathbf{\dot{\rho}}_n \right] \]  
(C-13)

The derivatives of the rod/tip mass momentum shown in equation C-9 is given by

\[ \dot{\mathbf{H}}_k = m_k \left( \frac{\delta \mathbf{r}_O}{\delta t} + \frac{\delta \rho_k}{\delta t} \right) \times \frac{\delta \rho_k}{\delta t} + m_k (\mathbf{\dot{r}}_0 + \mathbf{\dot{r}}_k + \mathbf{\dot{\rho}}_k) \times \frac{\delta^2 \rho_k}{\delta t^2} \]  
(C-14)

\[ \frac{\delta^2 \rho_k}{\delta t^2} \] is the acceleration of tip mass \( k \) with respect to the space vehicle.

At this point we introduce rod flexure considerations. Consider the location of the tip mass to be defined

\[ \mathbf{\rho}_k = \mathbf{\rho}_{k_0} + \mathbf{\eta}_a_k \]  
(C-15)

where

\[ \mathbf{\rho}_{k_0} = \text{initial position of tip mass } k \]

\[ \mathbf{\eta}_a_k = \text{displacement of tip mass } k \text{ due to flexure} \]

Differentiating (15) with respect to body

\[ \frac{\delta \mathbf{\rho}_k}{\delta t} = \frac{\delta \mathbf{\eta}_a_k}{\delta t} \]  
(C-16)

and

\[ \frac{\delta^2 \mathbf{\rho}_k}{\delta t^2} = \frac{\delta^2 \mathbf{\eta}_a_k}{\delta t^2} \]  
(C-17)
The flexure model for the gravity gradient rods is described in Appendix I. It is a 2-node model which may be described by

\[ M_k \ddot{\bar{r}}_k + C_k \dot{\bar{r}}_k + K_k \bar{r}_k = \bar{F}_{\text{ext}} - T_k^\omega \]  

(C-18)

where \( \bar{r}_k \) is a displacement vector for rod/tip mass \( k \)

\[ \bar{r}_k = (\bar{r}_k^a, \bar{r}_k^b) \]

\[ \bar{r}_k^a = \text{displacement at node A (tip mass center)} \]

\[ \bar{r}_k^b = \text{displacement at node B (mid-point of rod)} \]

\( m_k \) is the mass matrix defined by

\[ m_k = \begin{bmatrix} m_k^a & 0 \\ 0 & m_k^b \end{bmatrix} \]

\[ m_k^a = m_k^\text{tip mass } k + k m_{\text{rod } k} \]

\[ m_k^b = k m_{\text{rod } k} \]

\( C_k \) is the damping matrix given by

\[ C_k = C_k \begin{bmatrix} C_{k,a} \\ C_{k,b} \end{bmatrix} \]

where \( C_{k,a} \) and \( C_{k,b} \) have been defined in Appendix I as

\[ C_{k,a} = \begin{bmatrix} 201.06 \times 10^{-6} & 0 & -0.75 \times 10^{-6} & 0 \\ 0 & 201.06 \times 10^{-6} & 0 & -0.75 \times 10^{-6} \\ -0.75 \times 10^{-6} & 0 & 12.17 \times 10^{-6} & 0 \\ 0 & -0.75 \times 10^{-6} & 0 & 12.17 \times 10^{-6} \end{bmatrix} \]

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and $C_k$ is a scalar used for a conversion factor and for changing damping ratios. NOTE: It has been assumed that there are no deflections in the $Z$ direction, thus there are only two degrees of freedom at each node.

$K_k$ is the stiffness matrix

$$K_k = \left[ \begin{array}{c} K_{k_a} \\ K_{k_b} \end{array} \right]^{-1}$$

where $K_{k_a}$ is a conversion factor and

$$K_{k_a} = \begin{bmatrix} \frac{4604\xi^3}{EI_y} + \frac{1728\xi}{JG} \left( y_{a k}^o - 2y_{b k}^o \right)^2 & 0 & \frac{1440\xi^3}{EI_y} \\ 0 & \frac{4604\xi^3}{EI_x} + \frac{1728\xi}{JG} \left( x_{a k}^o - 2x_{b k}^o \right)^2 & 0 & \frac{576\xi^3}{EI_x} \end{bmatrix}$$

$$K_{k_b} = \begin{bmatrix} \frac{1440\xi^3}{EI_y} & 0 & \frac{576\xi^3}{EI_x} \\ 0 & \frac{1440\xi^3}{EI_x} & 0 \\ 0 & \frac{576\xi^3}{EI_x} \end{bmatrix}$$

$\xi$ = rod half length = 30 ft (fully deployed)

$EI_y$ = rod bending stiffness about y axis 3000±10 percent lb-in$^2$

$EI_x$ = rod bending stiffness about x axis 3000±10 percent lb-in$^2$

$JF$ = rod torsional stiffness 25 lb-in$^2$

$x_{a k}^o$, $y_{a k}^o$ = initial x and y components of $\eta_k$ representing unstressed position from x, y axes respectively, due to thermal and manufacturing curvatures.

$\hat{\omega} = \begin{bmatrix} \dot{\rho} \\ \dot{q} \end{bmatrix}$ is the angular acceleration of the spacecraft which is translated to the node point of rod k by transformation $T_k$. 

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\[ T_k = \begin{bmatrix}
    r_k^z + 2\xi \text{sgn}(r_k^z) & (y_o - \eta y_k) \\
    -r_k^z + 2\xi \text{sgn}(r_k^z) & 0 \\
    0 & (r_k^x + \eta x_k - x_o^k) \\
    -r_k^z + \text{sgn}(r_k^z) & 0 \\
    0 & (r_k^x + \eta x_k - x_o^k)
\end{bmatrix} \]

where

\[ \tilde{r}_k = \tilde{r}_o + \tilde{r}_k \]

The external forces, \( F_{\text{ext}} \), are composed of gravity gradient forces and solar pressure forces. The solar pressure forces acting on the tip masses are defined in Appendix C.

The gravity gradient forces are discussed below.

![Figure C-2. Tip Mass Configuration](image-url)
In a central inverse square gravitational field the force acting on a particle of mass \( m \) is given by

\[
F = \frac{-mk\ddot{r}}{|R|^2} = \frac{-mk\ddot{\bar{R}}}{|R|^3}
\]  

(C-19)

where:

- \( m \) = mass of particle
- \( k \) = gravitational constant
- \( \bar{R} \) = vector from the particle to the mass center of the attracting body

If we assume two tip masses, the vector \( \bar{R} \) would break down into \( \bar{R}_1 \) and \( \bar{R}_2 \) as shown in Figure C-2.

\[
\bar{R}_1 = \bar{R}_0 + \bar{r}_1
\]  

(C-20)

\[
\bar{R}_2 = \bar{R}_0 + \bar{r}_2
\]  

(C-21)

where \( \bar{r}_1 \) and \( \bar{r}_2 \) are the vectors from the center of mass of the spacecraft/rod system to the center of mass of the tip masses and \( \bar{R}_0 \) is a vector from the center of the earth to the center of mass of the space vehicle. Referring to Figure C-1

\[
\bar{R}_1' = (\bar{r}_0 + \bar{r}_k_1 + \bar{r}_k_1)
\]  

(C-22)

\[
\bar{R}_2' = (\bar{r}_0 + \bar{r}_k_2 + \bar{r}_k_2)
\]  

(C-23)

Thus

\[
\bar{F}_1' = \frac{-m_1k\ddot{r}_1}{|R_1|^2} = \frac{-m_1k\ddot{\bar{R}}_1}{|R_1|^3}
\]  

(C-24)

\[
\bar{F}_2' = \frac{-m_2k\ddot{r}_2}{|R_2|^2} = \frac{-m_2k\ddot{\bar{R}}_2}{|R_2|^2}
\]  

(C-25)
Now

\[ |R_1|^2 = (R_o + \tilde{r}_o + \tilde{r}_{k_1} + \tilde{\rho}_{k_1}) \cdot (R_o + \tilde{r}_o + \tilde{r}_{k_1} + \tilde{\rho}_{k_1}) \]

\[ = |R_o|^2 + |\tilde{r}_o + \tilde{r}_{k_1} + \tilde{\rho}_{k_1}|^2 + 2R_o \cdot (\tilde{r}_o + \tilde{r}_{k_1} + \tilde{\rho}_{k_1}) \]

\[ = |R_o|^2 \left[ \frac{(1 + 2R_o \cdot (\tilde{r}_o + \tilde{r}_{k_1} + \tilde{\rho}_{k_1}) \cdot (\tilde{r}_o + \tilde{r}_{k_1} + \tilde{\rho}_{k_1})^2)}{|R_o|^2 + |R_o|^2} \right] \]

The last term may be neglected leaving

\[ |R_1|^2 \approx |R_o|^2 \]

Take the \(-3/2\) power of both sides

\[ |R_1|^{-3} \approx |R_o|^{-3} \]

Using the binomial theorem

\[ |R_1|^{-3} \approx \frac{1}{|R_o|^3} \]

Therefore

\[ F_{1G} \approx \frac{m_1 k}{|R_o|^3} \frac{(1 - 3R_o \cdot (\tilde{r}_o + \tilde{r}_{k_1} + \tilde{\rho}_{k_1}))}{|R_o|^2} \]

and

\[ F_{2G} \approx \frac{m_2 k}{|R_o|^3} \frac{(1 - 3R_o \cdot (\tilde{r}_o + \tilde{r}_{k_2} + \tilde{\rho}_{k_2}))}{|R_o|^2} \]
The forces $\mathbf{F}_{1G}$ and $\mathbf{F}_{2G}$ can be separated into a gravity component and a gravity gradient component or

$$\mathbf{F}_{1G} = \mathbf{F}_{1G}^{\text{gravity}} + \mathbf{F}_{1G}^{\text{gravity gradient}}$$

where

$$\mathbf{F}_{1G}^{\text{gravity}} = \frac{m_{1}k}{|\mathbf{R}|^{3}} \mathbf{R}$$

and

$$\mathbf{F}_{1G}^{\text{gravity gradient}} = -\frac{m_{1}k}{|\mathbf{R}|^{3}} (\mathbf{R} \cdot \mathbf{R} + \mathbf{R} \mathbf{R}_{1} + \mathbf{R}_{1} \mathbf{R})$$

Over the n body model the gravity forces will cancel out, leaving only the gravity gradient forces.

Summation of forces

$$\frac{m_{1}}{\delta^{2}} \frac{\delta^{2}}{\mathbf{R}_{1}} = \mathbf{F}_{1G} + \mathbf{F}_{1S}$$

(gravity gradient)

Where $\mathbf{F}_{1S}$ = solar force

All vectors are expressed in body axes, so $\mathbf{R} = T_{LV}^{B}$

Where $T_{LV/B}$ is the transformation from local vertical axes to body.
REFERENCES

This appendix summarizes the nutation damper model implemented for the digital simulation evaluation of GPS stability and control analysis.

The nutation damper is used for passive GPS spin axis stabilization during the initial mission phases by dissipating the spacecraft precession energy. The nutation damper adopted for GPS application is of the type of ball in gas filled tube. The tube is made of aluminum and the ball of tungsten carbide. The gas is a 9:1 ratio mixture of nitrogen and helium at 1 atm pressure. Physical parameters of this damper are shown in Figure D-1.

A similar damper of this type was used in the Telstar satellite, Reference D-1. Discussions on the design and analysis of ball-in-tube type nutation dampers can be found in Reference D-2.

Derivation of the torque equations presented here follows the same procedure of Reference D-2 which contains numerous typographical errors. The results, in the torque form, are compatible for integration into the three body model described in Appendix C.

**DAMPER MODEL DERIVATION**

To facilitate the model derivation, we shall define a vehicle fixed B-frame whose origin is at the vehicle's center of mass (excluding damper ball) with the \((x_B, y_B, z_B)\) axes aligned with its principal axes. Also in the following consideration of vehicle rotational motions, it is assumed that the translational motion of vehicle's center of mass resulted from reactional
TYPE: BALL IN GAS FILLED TUBE
TUBE: CURVED TO PROVIDE CENTRIFUGAL SPRING

APPR. HALF SCALE SKETCH

<table>
<thead>
<tr>
<th>TUBE</th>
<th>BALL</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.D. 1.256 INCH</td>
<td>DIAMETER 1.25 INCH</td>
</tr>
<tr>
<td>O.D. 1.566 INCH</td>
<td>r_D = 0.625 inches</td>
</tr>
<tr>
<td>LENGTH 16 INCHES</td>
<td>MATERIAL TUNGSTEN CARBIDE</td>
</tr>
<tr>
<td>r = RADIUS OF CURVATURE 159 INCHES = 13.25 ft.</td>
<td></td>
</tr>
<tr>
<td>MATERIAL = ALUMINUM</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GAS</th>
<th>WEIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:1 MIXTURE NITROGEN: HELIUM</td>
<td>1.75 LBS</td>
</tr>
<tr>
<td>1 ATMOSPHERE</td>
<td></td>
</tr>
</tbody>
</table>

Figure D-1. Nutation Damper Characteristics
forces of the damper ball can be neglected. By reference to the geometrical description of damper location in B-frame, the position of center of ball, \( \mathbf{R}_b \) can be written as, Figure D-2,

\[
(\mathbf{R}_b^B) = \begin{bmatrix}
0 \\
-a + \gamma \cos \alpha \\
b + \gamma \sin \alpha 
\end{bmatrix}
\] (D-1)

where

\( \gamma = 159 \text{ in.} = 13.25 \text{ ft.} \)

\((a,b) = \text{coordinates of tube center of curvature}\)

Equation (1) implies that \( Y_B - Z_B \) plane contains the damper tube.

Let the vehicle moment of inertia be such that

\[
\begin{cases}
I_{XX} = I_{YY} = A \\
I_{ZZ} = C
\end{cases}
\] (D-2)

The rotational energy of the vehicle becomes,

\[
T_V = \frac{1}{2} A(W_X^2 + W_Y^2) + \frac{1}{2} C W_Z^2
\] (D-3)

The velocity of damper ball can be written through differentiation of (1) as,

\[
(\dot{\mathbf{R}}_b^B) = T_{BI} (\dot{\mathbf{R}}_b^B) = T_{BI} (\mathbf{R}_b^I)
\]

with \((\mathbf{R}_b^I) = T_{BI} (\mathbf{R}_b^B)\), we have

\[
(\dot{\mathbf{R}}_b^I) = T_{BI} T (\dot{\mathbf{R}}_b^B) + T_{BI} T (\dot{\mathbf{R}}_b^B)
\]

\[
= -T_{BI} T [ (\mathbf{R}_B^B) ] (\mathbf{R}_b^B) + T_{BI} T (\mathbf{R}_b^B)
\]
NOTE: \( Z_B = \) SPIN AXIS
\( cMv = \) VEHICLE CENTER OF MASS
DAMPER CONTAINED IN PLANE \( (Y_B - Z_B) \)

Figure D-2. Nutation Damper in Vehicle Body Fixed Frame
or
\[
\dot{R}_b^B = -s \left[ \left( \hat{\omega}_b^B \right) \right] (R_b^B) + (\hat{\nu}_b^B) \tag{D-4}
\]

In component forms, (4) becomes,
\[
\begin{bmatrix}
V_x \\
V_y \\
V_z
\end{bmatrix} =
\begin{bmatrix}
0 & -W_z & W_y \\
W_z & 0 & -W_x \\
-W_y & W_x & 0
\end{bmatrix}
\begin{bmatrix}
0 \\
-a + \gamma \cos \alpha \\
b + \gamma \sin \alpha
\end{bmatrix}
\]
\[
+ \begin{bmatrix}
0 \\
-\gamma \sin \alpha \\
\gamma \cos \alpha
\end{bmatrix}
\tag{D-5}
\]

Hence the translational kinetic energy of the damper ball is
\[
T_b = \frac{1}{2} M (v_x^2 + v_y^2 + v_z^2)
\]
\[
= \frac{1}{2} M \left[ \left( - (a + \gamma \cos \alpha) W_z + (b + \gamma \sin \alpha) W_y \right)^2 \\
+ \left( - (b + \gamma \sin \alpha) W_x + \gamma \sin \alpha \hat{v}_b \right)^2 \\
+ \left( - (a + \gamma \cos \alpha) W_x + \gamma \cos \alpha \hat{v}_b \right)^2 \right] \tag{D-6}
\]

where \( M \) = mass of the ball
\[
= \frac{4}{3} \pi \frac{3}{2} \rho \pi \frac{3}{2} = 263 \text{ gm} = 0.018 \text{ slug}
\]
\[
\rho = \text{density of tungsten carbide (WC) = 15.7 gm/cm}^3
\]

Assuming the ball is rolling without slipping in the tube, the magnitude of the angular velocity of the ball is related to the translational velocity of the center of the ball as
\[
\omega_b = \frac{1}{\gamma_b} \left[ (\hat{R}_b^B)^T (\hat{R}_b^B) \right]^{\frac{1}{2}}
\]
\[
= \frac{1}{\gamma_b} \left[ \gamma^2 \sin^2 \alpha \hat{\omega}^2 + \gamma^2 \cos^2 \alpha \hat{\omega}^2 \right]^{\frac{1}{2}}
\]
\[
= \frac{\omega}{\gamma_b} \text{ where } \gamma_b = \text{radius of the ball = 0.625 in.} \tag{D-7}
\]
Hence, the rotational energy of the ball is

\[ T = \frac{1}{2} \mathcal{I} \omega_b^2 = \frac{1}{2} \left( \frac{2}{5} M \gamma_b^2 \right) \left( \frac{\gamma b^2}{\gamma_b} \right)^2 \]

\[ = \frac{1}{5} M \gamma^2 \omega^2 \]  

(D-8)

The drag on the ball is viscous, Reference D-3, since the tube is gas filled, the clearance between ball and tube is small and the ball moves much slower than the speed of sound. Hence, the viscous drag on the ball can be represented by Rayleigh's dissipation function as

\[ F = \frac{1}{4} C_D \left( \frac{\gamma^2}{b^2} \right)^2 \left( \frac{\gamma b^2}{\gamma_b} \right) \]

\[ = \frac{1}{4} C_D \gamma^2 \omega^2 \]  

(D-9)

where \( C_D \) is a constant to be described later.

The total energy of the system then becomes

\[ T = T_v + T_b + T_y \]

\[ = \omega_x^2 \left\{ \frac{1}{2} A + \frac{1}{2} M \left( \gamma^2 + b^2 + a^2 + 2b \gamma \sin \alpha - 2a \gamma \cos \alpha \right) \right\} \]

\[ + \omega_y^2 \left\{ \frac{1}{2} A + \frac{1}{2} M \left( b^2 + 2b \gamma \sin \alpha + \gamma^2 \sin^2 \alpha \right) \right\} \]

\[ + \omega_z^2 \left\{ \frac{1}{2} C + \frac{1}{2} M \left( a^2 - 2a \gamma \cos \alpha + \gamma^2 \cos^2 \alpha \right) \right\} \]

\[ + \omega_x \omega_y \left\{ -M (-ab - a \gamma \sin \alpha + b \gamma \cos \alpha + \gamma^2 \sin \alpha \cos \alpha) \right\} \]

\[ + \omega_y \omega_z \left\{ -M (\gamma^2 - a \gamma \cos \alpha + b \gamma \sin \alpha) \right\} \]

\[ + \omega_x \omega_x \left\{ \frac{7}{10} M \gamma^2 \right\} \]  

(D-10)

With the total vehicle energy given in (D-10) and the damper dissipation function given in (D-9), the equation of motion of the vehicle and the damper system can be obtained as
The rotational equation of motion of the vehicle can be written in the form of Euler's rigid body equation as

\[
\begin{align*}
\frac{d}{dt} \frac{\partial T}{\partial W_Y} - W_Z \frac{\partial T}{\partial W_Z} + W_Y \frac{\partial T}{\partial W_Y} &= 0 \\
\frac{d}{dt} \frac{\partial T}{\partial W_X} - W_Y \frac{\partial T}{\partial W_Y} + W_Z \frac{\partial T}{\partial W_X} &= 0 \\
\frac{d}{dt} \frac{\partial T}{\partial W_Z} - W_Y \frac{\partial T}{\partial W_Y} + W_X \frac{\partial T}{\partial W_X} &= 0 \\
\frac{d}{dt} \frac{\partial T}{\partial \alpha} - \frac{\partial T}{\partial \beta} + \frac{\partial T}{\partial \gamma} &= 0
\end{align*}
\]  

(D-11)

where the torques \( T_X, T_Y, \) and \( T_Z \) are,

\[
T_X = -M \left[ \dot{W}_X (r'^2 + b^2 + a^2 + 2b \gamma \sin \alpha - 2a \gamma \cos \alpha) \\
+ \dot{\alpha} (2W_X + \dot{\alpha}) (b \gamma \cos \alpha + a \gamma \sin \alpha) \\
+ \dot{\alpha} (r'^2 - a \gamma \cos \alpha + b \gamma \sin \alpha) \\
+ W_Y W_Z (a^2 - b^2 - 2a \gamma \cos \alpha - 2b \gamma \sin \alpha - r'^2 \alpha + \gamma^2 \cos^2 \alpha) \\
+ (W_Y^2 - W_Z^2) (ab + a \gamma \sin \alpha - b \gamma \cos \alpha - \gamma^2 \sin \alpha \cos \alpha) \right] \quad (D-13)
\]

\[
T_Y = -M \left[ \dot{W}_Y (b^2 + 2b \gamma \sin \alpha + \gamma^2 \sin^2 \alpha) \\
+ (\dot{W}_Z - W_X W_Y) (ab + a \gamma \sin \alpha - b \gamma \cos \alpha - \gamma^2 \sin \alpha \cos \alpha) \right]
\]

\[29\]
\[
\begin{align*}
+ \dot{W}_X W_Z (\gamma^2 + b^2 + 2b \gamma \sin \alpha - \gamma^2 \cos^2 \alpha) \\
+ 2W_Y \dot{\alpha} (b \gamma \cos \alpha + \gamma^2 \sin \alpha \cos \alpha) \\
+ 2W_Z \dot{\alpha} (b \gamma \sin \alpha + \gamma^2 \sin^2 \alpha) 
\end{align*}
\]

(D-14)

\[
\begin{align*}
T_Z = -M [\dot{W}_Z (a^2 - 2a \gamma \cos \alpha + \gamma^2 \cos^2 \alpha) \\
+ (\dot{W}_Y + W_X W_Y) (ab + a \gamma \sin \alpha - b \gamma \cos \alpha - \gamma^2 \sin \alpha \cos \alpha) \\
+ W_X W_Y (-a^2 + 2a \gamma \cos \alpha - \gamma^2 \cos^2 \alpha) \\
+ 2W_Z \dot{\alpha} (a \gamma \sin \alpha - \gamma^2 \sin \alpha \cos \alpha) \\
- 2W_Y \dot{\alpha} (b \gamma \sin \alpha + \gamma^2 \sin^2 \alpha)]
\end{align*}
\]

(D-15)

With ball's equation of motion in tube given as,

\[
\frac{7}{5} \gamma \alpha + \dot{W}_X (\gamma - a \cos \alpha + b \sin \alpha) \\
- W_X^2 (b \cos \alpha + a \sin \alpha) - W_Y^2 (b \sin \alpha + \gamma \sin \alpha \cos \alpha) \\
- W_Z^2 (a \sin \alpha - \gamma \sin \alpha \cos \alpha) \\
- W_Y W_Z (a \cos \alpha + b \sin \alpha - \gamma \cos^2 \alpha + \gamma \sin^2 \alpha) \\
+ W_X \dot{\alpha} (a \sin \alpha + b \cos \alpha - a \sin \alpha - b \cos \alpha) \\
+ (\frac{c_D \gamma}{N}) \dot{\alpha} = 0
\]

(D-16)

The drag force on the ball consists of two components: the pressure drag and the viscous drag. For the case where the gap between tube and ball is small, as in the GPS nutation damper, and the speed of the ball in the tube is small compared with the speed of sound, it has been determined in Ref. D-3 that the drag force is primarily due to pressure drag. The drag force D,
can be written as
\[ D = \frac{135}{4 \times 64} \left( \frac{3 \mu \gamma_b}{g^{5/2}} \right) S \hat{\alpha} = C_D S \]

where
- \( \mu \) = viscosity of gas in tube
- \( g = (\gamma_i - \gamma_b) \gamma_b = \) gap parameter
- \( \gamma_i = \) inside radius of tube
- \( S = \) speed of ball relative to gas
- \( C_D = \) dissipation coefficient

\[ = \frac{135}{4 \times 64} \frac{3 \mu \gamma_b}{g^{5/2}} \]

For GPS nutation damper,
\[ \mu = 178.1 \times 10^{-6} \times 2.089 \times 10^{-3} \text{ lb}_f \text{ sec/ft}^2 \]
\[ g = (0.633 - 0.625) / 0.625 = 0.633 \text{ in.} \]

Hence, value of dissipation coefficient,
\[ C_D = 0.0171 \text{ lb}_f / (\text{ft/sec}) \]

To handle the discontinuity at either end of the damper tube, let
\[ \hat{\alpha} = -\varepsilon \hat{\alpha} \quad \text{at} \quad \alpha = \pm \alpha_{\text{max}} \]

where
- \( \alpha_{\text{max}} = (8/159) \text{ rad} = 2.9 \text{ deg.} \)
- \( \varepsilon = 0 \) for inelastic collision
- \( = 1 \) for elastic collision.
The GPS requirement on half cone angle of nutation is to be less than 2 deg. The discontinuity situation is not expected to be encountered often if the 2 deg. requirement is met. The value of $c = 1$ would give a conservative result in that less precession energy is dissipated at bottoming. Hence, use of $c = 1$ is recommended for this reason.

**NUTATION DAMPING TIME CONSTANT VERIFICATION**

**Nominal Solution Undamped Rigid Body Motion**

Consider a rigid body with symmetric moment of inertia about the traverse axes, i.e., $I_{XX} = I_{YY}$ in a torque free situation. The Euler's equation is reduced to:

\[
\begin{align*}
\dot{W}_X &= -A W_Y W_Z \\
\dot{W}_Y &= A W_Z W_X \\
\dot{W}_Z &= 0
\end{align*}
\]

where

\[
A = \left( I_{ZZ} - I_{XX} \right) = \left( I_{ZZ} - \frac{1}{I_{XX}} \right) \Delta = (n - 1)
\]

The solution of the above equation can be written as:

\[
\begin{align*}
W_X &= -B \sin \Omega t \\
W_Y &= B \cos \Omega t \\
W_Z &= W_Z(0)
\end{align*}
\]

with $\Omega = A W_Z(0)$
The angular momentum vector is written as:

\[ \mathbf{H} = I_{XX} \mathbf{w}_X \mathbf{X}_B + I_{YY} \mathbf{w}_Y \mathbf{Y}_B + I_{ZZ} \mathbf{w}_Z \mathbf{Z}_B \]  
(D-20)

Since there is no external torque, the angular momentum is constant. The nominal motion of the rigid body can be visualized as a constant precession about the angular momentum vector with a precession rate \( \Omega \) and a nutation angle \( \theta \) which is defined as:

\[
\tan \theta = \frac{H_I}{H_S} = \frac{\sqrt{I_{XX}^2 \omega_X^2 + I_{YY}^2 \omega_Y^2}}{I_{ZZ} \omega_Z} = \frac{I_{YY} B}{I_{ZZ} \omega_Z} \]
(D-21)

Therefore:

\[
\mathbf{B} = \left( \frac{I_{ZZ}}{I_{YY}} \times \tan \theta \right) \mathbf{w}_Z
\]
(D-22)

For small value of \( \theta \), (6) becomes

\[
\mathbf{B} \approx \left( \frac{I_{ZZ}}{I_{YY}} \times \theta \right) \mathbf{w}_Z
\]
(D-23)

A relationship between the rigid body's angular momentum and rotational energy useful for later development of the energy sink approximation is:

\[
\begin{align*}
T &= \frac{1}{2} I_{XX} (\omega_X^2 + \omega_Y^2) + \frac{1}{2} I_{ZZ} \omega_Z^2 = \frac{H_T^2}{2 I_{XX}} + \frac{H_S^2}{2 I_{ZZ}} \\
H^2 &= H_T^2 + H_S^2
\end{align*}
\]
(D-24)

Hence

\[
H_T^2 = \left( \frac{I_{XX} I_{ZZ}}{I_{ZZ} - I_{XX}} \right) \left( 2T - \frac{H_T^2}{I_{XX}} \right)
\]
(D-25)

From (5)

\[
\sin^2 \theta = \frac{H_T^2}{H^2} = \left( \frac{I_{XX} I_{ZZ}}{I_{ZZ} - \frac{1}{I_{XX}}} \right) \left( 2T - \frac{1}{I_{ZZ}} \right)
\]
(D-26)
Nutation Damping Time Constant

The GPS nutation damper consists of a tube aligned with respect to the vehicle spin axis. A ball inside the tube is dissipating the nutation energy via viscous drag force retarding the ball’s motion. An approximation procedure known as the energy sink concept assumes that the dissipation of nutation energy does not change the system’s angular momentum (which is dominantly spin angular momentum). Hence equation (D-26) can be differentiated with terms $\dot{H}$ neglected in this approximation, i.e.,

$$2 \sin \theta \cos \theta \dot{\theta} = \frac{2I_{ZZ} \dot{\xi}}{(n-1)H^2}$$  \hspace{1cm} (D-27)

For small value of $\theta$,

$$\theta \dot{\theta} = \frac{2I_{ZZ} \dot{\xi}}{(n-1)H^2}$$  \hspace{1cm} (D-28)

A further approximation involves

$$H^2 = H_s^2 = I_{ZZ} \frac{W^2}{W_z^2}$$  \hspace{1cm} (D-29)

Hence

$$\dot{\theta} = \left[ \frac{1}{(n-1)I_{zz}W_z^2} \right] \dot{T}$$  \hspace{1cm} (D-30)

The energy dissipation $\dot{T}$ can be expressed as the product of the drag force the ball experienced and the velocity of the ball, i.e.,

$$\dot{T} = -f\psi$$  \hspace{1cm} (D-31)
with drag force,
\[ f = C_D V \]
and the ball velocity,
\[ V = \gamma a \]

Hence the energy dissipation rate,
\[ \dot{I} = -C_D \gamma^2 \alpha^2 \]
where \( \alpha \) is the position of ball in tube defined as in Figure D-2,
\[ \gamma = \text{radius of damper tube}, \]
\[ C_D = \text{dissipation coefficient} \]

From analysis of nutation damper, the motion of the ball in tube was obtained as
\[
\frac{7}{5} \gamma \ddot{a} + \dot{W}_X(\gamma - a \cos \alpha + b \sin \alpha) - W_Y^2(b \cos \alpha + \gamma \sin \alpha \cos \alpha) \\
-\dot{W}_X^2(b \cos \alpha + a \sin \alpha) - W_Z^2(a \sin \alpha - \gamma \sin \alpha \cos \alpha) \\
-\dot{W}_X W_Z(a \cos \alpha + b \sin \alpha - \gamma \cos^2 \alpha + \gamma \sin^2 \alpha) \\
+ \frac{C_D \gamma}{M} \ddot{a} = 0 \tag{D-33}
\]

With the geometry of mounting such that \( b = 0 \) and introducing the small angle approximations \( \sin \alpha = \alpha, \cos \alpha = 1 \) and neglecting terms \( \dot{W}_X^2 \) and \( \dot{W}_Y^2 \) as they are small for small nutation angle \( \theta \), equation (17) becomes
\[
\left( \frac{7\gamma}{5} \right) \ddot{a} + \left( \frac{C_D}{M} \right) \ddot{a} + W_Z^2(\gamma - a)a = \dot{W}_Y W_Z(a - \gamma) - \dot{W}_X(\gamma - a) \tag{D-34}
\]

Using the expression of \( W_Y \) as given in equations (2) and (7)
\[ W_Y = \dot{W}_Z \cos \omega t \tag{D-35} \]
Similarly,

\[ \dot{W}_x = -b n W_Z \Omega \cos \Omega t \]

Equation (18) can be reduced to the following form:

\[
\ddot{\alpha} + \left[ \frac{5C_D}{7M} \right] \dot{\alpha} + \left[ \frac{5(\gamma - a)W_z^2}{7\gamma} \right] \alpha = -\theta \left[ \frac{5(\gamma - a)(2n - n^2)W_Z^2}{7\gamma} \right] \cos \Omega t \tag{D-36}
\]

The solution of the standard second order linear differential equation of the form

\[
\ddot{\alpha} + 2b \dot{\alpha} + \alpha = c \cos \Omega t \tag{D-37}
\]

has been obtained in standard textbooks as:

\[
\alpha = \frac{c}{\sqrt{\left( \frac{a^2}{\Omega^2} - \frac{c^2}{\omega^2} \right)^2 + 4b^2 \omega^2}} \cos (\Omega t + \phi) \tag{D-38}
\]

Compare (20) and (21),

\[
\begin{align*}
2b &= \frac{5C_D}{7M} \\
\alpha^2 &= \frac{5(\gamma - a)W_z^2}{7\gamma} \\
c &= -5(\gamma - a)(2n - n^2)W_z^2/(7\gamma) \tag{D-39}
\end{align*}
\]

From (22),

\[ \dot{\alpha} = -\theta \alpha_0 \Omega \sin (\Omega t + \phi) \tag{D-40} \]

Substitute (24) into (16)

\[ \dot{\theta} = -\theta^2 C_D \gamma^2 \alpha_0^2 \Omega^2 \sin^2(\Omega t + \phi) \tag{D-41} \]
The average value over a full period, i.e.,

\[ \Omega_{t_p} = 2\pi \], is

\[ t = -\frac{1}{2} C_D \gamma^2 \alpha_o^2 \Omega^2 = t_p \tag{D-42} \]

since \( \frac{1}{2} \int_0^t \sin^2 (\Omega t + \phi) dt = \frac{1}{2} \)

Substitution of (26) into (14) gives

\[ \dot{\theta} = \left[ \frac{1}{(n-1)^2 I_{zz} w_z^2} \right] \left[ -\frac{1}{2} C_D \gamma^2 \alpha_o^2 \Omega^2 \right] \theta \]

or

\[ \dot{\theta} = \left[ \frac{C_D \gamma^2 \alpha_o^2 \Omega^2}{2(n-1)^2 I_{zz} w_z^2} \right] \theta \tag{D-43} \]

The time constant of nutation damping can now be written as:

\[ T_c = \frac{2(n-1)^2 I_{zz} w_z^2}{C_D \gamma^2 \alpha_o^2 \Omega^2} \tag{D-44} \]

where

\[
\begin{align*}
\alpha_o &= \sqrt{\left(\alpha^2 - \Omega^2\right)^2 + 4b^2 \Omega^2} \\
2b &= 5 \frac{C_D}{(7\gamma)} \\
\alpha^2 &= 5(\gamma - a)W_z^2/(7\gamma) \\
c &= -(2n - n^2)\alpha^2 \\
\Omega &= (n - 1)W_z
\end{align*}
\]
The predicted nutation damping time constant for the two cases of $W_z = 100$ RPM and $W_z = 10$ RPM are computed as follows:

Case 1:

\[ r = 13.25 \text{ ft.} \]
\[ a = 10.25 \text{ ft.} \]
\[ m = 0.018 \text{ slug} \]
\[ C_D = 0.0171 \text{ lb}_f/(\text{ft/sec}) \]
\[ I_{xx} = I_{yy} = 77 \text{ slug-ft}^2 \]
\[ I_{zz} = 94 \text{ slug-ft}^2 \]
\[ W_z = 100 \text{ RPM} = 10.47 \text{ rad/sec.} \]
\[ n = 1.22 \]
\[ \Omega = 2.31 \]
\[ 2b = 0.6789 \]
\[ \Delta^2 = 17.728 \]
\[ \epsilon^2 = -16.84 \]
\[ \alpha_0 = -1.348 \]
\[ T_c = 155.755 \text{ sec.} \]

Case 2:

\[ W_z = 10 \text{ RPM} = 1.047 \text{ rad/sec.} \]
\[ n = 1.22 \]
\[ \Omega = 0.231 \]
\[ 2b = 0.6786 \]
\[ \Delta^2 = 0.17728 \]
\[ c = -0.1684 \]
\[ \alpha_0 = -0.84275 \]
\[ T_c = 398.497 \text{ sec.} \]

The nutation half cone angle history corresponding to Case 1 and Case 2 considered above are plotted in Figures D-3 and D-4 respectively. These results are obtained using the nutation damping simulation program where the detailed dynamics of the damper ball and rigid body dynamics are considered.
100 RPM

\[
\begin{align*}
I_{xx} &= 76.3 \\
I_{yy} &= 78.3 \\
I_{zz} &= 94.1
\end{align*}
\]

"Time constant" = 150 sec.
(155.75 sec. was predicted)

Figure D-3. Simulation Data
10 RPM

\[
\begin{align*}
I_{xx} &= 76.3 \\
I_{yy} &= 78.3 \\
I_{zz} &= 94.1
\end{align*}
\]

"Time Constant" = 365 sec.
(398.5 sec. was predicted)

Figure D-4. Simulation Data
REFERENCES


APPENDIX E
APPENDIX E

ATTITUDE CONTROL ELECTRONICS SYSTEM MODEL

The unmodified model of the Attitude Control Electronics system as implemented in the computer simulation differs only from that shown on the Spacetae blueprints in the absence of voltage units and wheel signal monitoring elements. It is given in block diagram form in Figures E-1, E-2, E-3, and E-4.
Figure E-1. ACE Systems Block Diagram
Figure E-2. Wheel Command Transformation Logic
\[ \text{a} = 1.57 \text{ rad/sec} \]
\[ \text{b} = 2.09 \text{ rad/sec} \]
\[ \text{slope} = 1.0 \]

(equivalent to \(0.14^\circ\) db in pitch and roll, 2.58\(^\circ\) db in yaw, for single axis inputs)

Figure E-3. Nonlinear Element
$G = \frac{125}{181.4}$

$K_G \rightarrow \text{Stiction} \rightarrow K_m \rightarrow \text{Motor} \rightarrow \text{Wheel}$

$\frac{1}{1 + 0.08}$

$T \rightarrow \frac{1}{\tau_w s + 1}$

$\frac{1}{1 + 1.4}$

$K_T \rightarrow \frac{1}{\tau_T s + 1}$

$K_G = 33.333$

$K_m = 125$

$K_T = 0.00776$

$\tau_w = 30 - 90 \text{ sec.}$

$\tau_T = 1.0$

$\text{Stiction Gain} = \begin{cases} 
0, & \text{if } s < 3.93 \\
\frac{1}{s - 3.93}, & \text{if } 3.93 \leq s \leq 3.14 \\
\frac{1}{s - 3.14}, & \text{if } 3.14 < s \leq 3.93 \\
1, & \text{if } s > 3.93 
\end{cases}$

Figure E-4. Wheel Dynamics with Tach Feedback
APPENDIX F

TILTED, CENTERED DIPOLE MAGNETIC TORQUE MODEL

The instantaneous magnetic disturbance torque is \( T_m = \mathbf{M} \times \mathbf{B} \) where
\( \mathbf{M} \) is the spacecraft effective dipole moment
\( \mathbf{B} \) is the local magnetic induction or flux density.

The tilted, centered dipole model is used to represent the flux density. The flux density is a function of latitude and longitude. These parameters were not being calculated in the previous model. Hence a new coordinate system, the \( E \) frame, was defined. The \( E \) frame has its center at the center of mass of the vehicle with the reference plane perpendicular to the earth's radius vector.

The \( x \) axis points east, the \( y \) points south, and the \( z \) axis points towards the center of the earth.

The flux density, \( \mathbf{B} \) is defined in the \( E \) frame as

\[
B_{E_x} = -\left( \frac{Re}{R} \right)^3 \left( g_1 \sin \lambda - h_1 \sin \theta \right)
\]

\[
B_{E_y} = \frac{Re}{R}^3 \left( g_1 \cos \theta + g_1 \sin \theta \cos 2 + h_1 \sin \theta \sin \lambda \right)
\]

\[
B_{E_z} = -2 \left( \frac{Re}{R} \right)^3 \left( g_1 \cos \theta + g_1 \cos \lambda + h_1 \cos \theta \sin \lambda \right)
\]

where \( R_e \) = radius of earth = 2.09029 x \( 10^7 \) ft.
\( R \) = orbit radius 8.71627 x \( 10^7 \) ft.
\( \theta \) = latitude
\( \lambda \) = longitude
where if \( B \) is in webers/m²

\[
\begin{align*}
\gamma_1 &= -3.04012 \times 10^{-5} \\
\gamma_1' &= -2.1638 \times 10^{-6} \\
\gamma_1'' &= 5.7782 \times 10^{-6}
\end{align*}
\]

Latitude and longitude may be calculated from the following expressions.

\[
\begin{align*}
\dot{\theta} &= \omega_0 \cos \psi_H \\
\dot{\lambda} &= \omega_0 \frac{\sin \psi_H}{\cos \psi_H} - \omega_e
\end{align*}
\]

where \( \omega_0 \) = orbital rate = \( 1.454441 \times 10^{-4} \) rad/sec = \( \sqrt{\frac{R}{R^3}} \)

\( \omega_e \) = earth spin rate = \( 7.272205 \times 10^{-5} \) rad/sec

\( \psi_H \) = heading angle

The rate of change of the heading angle is given by

\[
\dot{\psi}_H = \sin \psi_H \tan \theta_0 \omega_0
\]

Initial conditions may be defined as

\[
\begin{align*}
\theta_0 &= 0 \\
\lambda_0 &= 0 \\
\psi_{H_0} &= (90^\circ - i), \ i = \text{inclination angle} - 63^\circ
\end{align*}
\]

The flux density must be transformed into body coordinates or

\[
\mathbf{B}_B = T_L V_B T_E / L_V B_E
\]
where

\[ \text{TE/LV} = \text{transformation from E frame to local vertical frame} \]

\[
\begin{bmatrix}
\sin \psi_H - \cos \psi_H & 0 \\
\cos \psi_H - \sin \psi_H & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[ \text{TLV/B} = \text{transformation from local vertical frame to body frame} \]

(already computed in program)

The resulting torque expression then is

\[
\text{T}^*_B = M_B \times B_B
\]

Prior to launch the spacecraft dipole moment will be calculated and reduced to less than .5 ampere-meter\(^2\) (500 pole-cm) per axis. Thus if \(M_B\) is given ampere-meter\(^2\) and \(B_B\) is in Weber/m\(^2\), \(T_B\) is in newton meter. The conversion factor between newton meters and ft. lbs. is .737757 ft-lb/nt-m, which will be applied to \(T_B\), or

\[
\text{T}_B \text{(ft. lbs.)} = 0.737757 \text{T}_B \text{(nt m)}
\]

Operating Range

Although it is planned to measure the spacecraft dipole moment prior to launch and compensate to obtain values less than 500 pole-cm per axis.

Table F-1, which was taken from NASA SP-8018 indicates there may be a large disparity between prelaunch measurements and on-orbit values. In magnitude alone, the values given in Table F-1 range from a factor of .25 to 12 for spin axis.
### Table F-1. Change in Spacecraft Dipole Moment

<table>
<thead>
<tr>
<th>Spacecraft</th>
<th>Spin-axis dipole moment measured prior to transportation to launch site</th>
<th>Spin-axis dipole moment in orbit (computed from motion of spin axis)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A-m²</td>
<td>(pole-cm)</td>
</tr>
<tr>
<td>Tiros II</td>
<td>+0.1</td>
<td>(+100)</td>
</tr>
<tr>
<td>Tiros III</td>
<td>+0.34</td>
<td>(+340)</td>
</tr>
<tr>
<td>Tiros V</td>
<td>-0.40</td>
<td>(-400)</td>
</tr>
<tr>
<td>Tiros IX</td>
<td>+0.08</td>
<td>(+80)</td>
</tr>
<tr>
<td>ESSA II</td>
<td>+0.01</td>
<td>(+10)</td>
</tr>
<tr>
<td>ESSA III</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>ESSA IV</td>
<td>+0.03</td>
<td>(+30)</td>
</tr>
<tr>
<td>ESSA V</td>
<td>(a)</td>
<td></td>
</tr>
<tr>
<td>ESSA VI</td>
<td>+0.17</td>
<td>(+170)</td>
</tr>
</tbody>
</table>

**NOTE:** Specified spin axis dipole moment for all spacecraft listed is 0.1 A-m² (100 pole-cm).

(a) Information is not available.

### Table F-2. Factors for Estimating Spacecraft Dipole Moment (M)

<table>
<thead>
<tr>
<th>Category of magnetic properties control</th>
<th>Estimate of dipole moment per unit mass for non-spinning spacecraft</th>
<th>Estimate of dipole moment per unit mass for spinning spacecraft</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A-m²/kg</td>
<td>(pole-cm/lb)</td>
</tr>
<tr>
<td>Class I</td>
<td>1.0x10⁻³</td>
<td>(0.45)</td>
</tr>
<tr>
<td>Class II</td>
<td>3.0x10⁻³</td>
<td>(1.6)</td>
</tr>
<tr>
<td>Class III</td>
<td>10x10⁻³</td>
<td>(4.5)</td>
</tr>
<tr>
<td>(and higher)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table F-2, also taken from NASA SP-8018 suggests using a value of 1.6 pole cm. per lb. of spacecraft which would suggest a value of 1600 pole-cm. for NTS-2. The Class I category is for cases where magnetic torques are comparable to other torques.

Although Table F-1 does indicate factor of 10 variations, it appears that these occurred for systems who were tuned to finer than 500 pole cm. An operating range of ± 5000 pole cm. is clearly unreasonable. However, based on Table F-2, an operating range of ± 2000 pole-cm. would be realistic if slightly conservative value.
The relative position of the satellite's surfaces are given in Figure G-1 below.

The solar force on a flat surface is given by:

\[ \vec{F} = -\vec{F}_N + \vec{F}_T \]

where:

\[ \vec{F}_N = \left[ (1+\sigma) \cos \psi + \frac{2}{3} (1-\sigma) \right] PA \cos \psi \]

\[ \vec{F}_T = (1-\sigma) \cos \psi \sin \theta \cdot PA \]

\[ P = \text{Solar Pressure Constant} = 1. \times 10^{-7} \text{ lb/ft}^2 \]
A = Area of the flat surface
\( \rho \) = Total reflected of the incident light
\( \sigma \) = That part of \( \rho \) that is specular reflection (there is also the diffused reflection \( \delta \) and \( \rho = (\sigma + \delta) \))
\( \mathbf{N} \) = Normal unit vector to the flat surface
\( \mathbf{T} \) = Shear (tangent) unit vector on the flat surface given by:
\[
\mathbf{T} = \frac{(\mathbf{N} \cdot \mathbf{S}) \mathbf{N} - \mathbf{S}}{(\mathbf{N} \cdot \mathbf{S}) \mathbf{N} - \mathbf{S}}
\]
(See Figure G-2)
\( \mathbf{S} \) = Unit vector on the sun LOS.
\( \psi \) = \( \cos^{-1} (\mathbf{N} \cdot \mathbf{S}) \)

Figure G-2. Unit Vector on Flat Surface

The torque due to (1) is:
\[
\mathbf{T} = \mathbf{c} \times \mathbf{\hat{p}} \quad \text{where:}
\]
(6-2)

\( \mathbf{c} \) = Vector of the center of pressure of the surface (usually the geometrical center).
The force and the torque on a full cylindrical surface are given by:

\[
\vec{F} = -F_N \vec{N} + F_T \vec{T}
\]  \hspace{1cm} (G-3)

\(\vec{T}\) = Unit vector along cylinder axis

\(\vec{N}\) = Normal unit vector in middle of the cylindrical surface given by:

\[
\vec{N} = \frac{\vec{S} - (\vec{S} \cdot \vec{T}) \vec{T}}{\vec{S} - (\vec{S} \cdot \vec{T})}
\]

\(\vec{S}\) = Unit vector on LOS to the sun

\[F_N = \left[ (1 + \sigma p/3) \cos \psi + \rho (1-\sigma) \frac{P}{6} \right] PA \cos \psi\]

\[F_T = (1-\sigma) \cos \psi \sin \psi PA\]

\(A\) = Projected area of the cylinder = \(2RH\)

\(R\) = Cylinder radius

\(H\) = Cylinder height

\(\psi = \cos^{-1}(\vec{S} \cdot \vec{N})\)

\(P\) = Solar constant \((1. \times 10^{-7} \text{ lb/ft}^2)\)

The torque due to G-3 is:

\[\vec{T} = \vec{D} \times \vec{F}\]  \hspace{1cm} (G-4)

\(\vec{D} = \vec{C} + R \vec{N}\)

\(\vec{C}\) = Vector to the geometrical center of the cylinder (see Figure G-3)
Figure G-3. Vector to Geometrical Center of Cylinder
TIP MASSES

The total solar force on a sphere goes through its geometrical center. Due to the symmetry each quarter (of the half surface of the sphere hit by the sunlight) will produce one quarter of this force along LOS.*

Let us divide this quarter into k equal slices by means of k meridians at \[ \Delta \phi = \frac{\pi}{2k} \] angle intervals (Figure G-4).

Then let us divide each slice into n spherical trapezia by means of n parallel circles at \[ \Delta \theta = \frac{\pi}{2n} \] angle intervals, from the equator to the north pole.

As \( k \to \infty \) and \( n \to \infty \) the spherical trapezia can be approximated by flat trapezia.

The dimensions of the ith trapezium on a slice are: The two equal sides:
\[
\frac{k_\theta}{r} = r \Delta \phi, \quad r = \text{radius of sphere}
\] (G-5)

the unequal sides of the trapezium (its bases):
\[
\frac{k_\phi}{r_i} = r_i \Delta \phi, \quad r_i = \text{radius of ith parallel circle}
\]

or

*Other forces (tangential) cancel out due to the symmetry of the sphere and the assumption that sun rays are parallel at \( \infty \).
Figure G-4. Angle Intervals
\[
\ell_i = r \cos (i \Delta \theta) \Delta \phi, \quad i = 1, 2, \ldots, n \quad (G-6)
\]

A. From (G-5) and (G-6) we find the area of the \textit{i}th trapezium:

\[
A_i = \left( \frac{r \phi_{r_{i-1}} + r \phi_{r_1}}{2} \right) \sqrt{\frac{r^2 \phi_{r_{i-1}} - (r \phi_{r_{i-1}} - r \phi_{r_1})^2}{4}} / 2 \quad (G-7)
\]

\[i = 1, 2, \ldots, n; \text{ where } r_o = r.\]

B. CG Location

We have: \( \hat{\mathbf{w}} = \hat{\mathbf{R}} + \hat{\mathbf{U}} \) (See Figure G-5)

\[
\hat{\mathbf{R}} = (r \cos \phi, r \sin \phi, 0) \quad (G-8)
\]

\[
\mathbf{U} = (-x \cos \phi, -x \sin \phi, \sin ((i-1)\Delta \theta)) \quad (G-9)
\]

Figure G-5. CG Location
From Figure G-6: \( X = r - r \cos ((i-1)\Delta \theta) \) or \( X = r(1-\cos((i-1)\Delta \theta)) \) (G-10)

![Figure G-6. Value of X](image)

\[ \vec{\mathbf{w}} = \vec{\mathbf{r}} + \vec{\mathbf{u}} = ((r-X)\cos \phi, (r-X)\sin \phi, r\sin ((i-1)\Delta \theta)) \] (G-11)

Now the CG of the \( i \)th trapezium is:

\[ \vec{\mathbf{C}}_{\mathbf{G}} = \vec{\mathbf{w}} + \vec{\mathbf{v}} \] where:

\[ \vec{\mathbf{v}} = (c_x \cos \phi, c_x \sin \phi, c_z) \] where:

\[ c_z = \frac{c}{h} z_i, \quad c_x = \frac{c}{h} \sqrt{h^2 - z_i^2}, \quad c = \frac{h(2a + \beta)}{3(a+\theta)} \] (G-14)

See Figure G-7.

where:
\( h = \sqrt{\left( \frac{r^2 - \frac{1}{\theta \phi \theta} r_{i-1}^2}{4} \right)} \)  

See also G-6 and G-7  

\[(G-15)\]

and:

\[
z_i = r \sin(i\Delta \theta) - r \sin((i-1)\Delta \theta)
= r \left[ \sin(i\Delta \theta) - \sin((i-1)\Delta \theta) \right]
\]  

\[(G-16)\]

Substituting G-11 and G-13 into G-12 we obtain the \( CG \) vector of the \( i \)th trapezium

\[
CG = (xcos \phi, xsin \phi, c_z + r \sin((i-1)\Delta \theta))
\]  

\[(G-17)\]

where: \( x = r - X - c_x \)
1) \( c_x \) and \( c_z \) are given by G-14, G-15, and G-16

2) \( X \) is given by (5a) and

3) \[ \phi = (j-1)\Delta \phi + \frac{\Delta \phi}{2}, \quad j = 1, 2, \ldots, (k-1) \]
   \[ = (j-0.5)\Delta \phi \]

\( j = \) jth slice \((j = 1, 2, \ldots, k)\)

\( i = \) ith trapezium on this slice from equator to north pole.
\((i = 1, 2, \ldots, n)\)

C. The normal unit vector to the ith trapezium is:

\[ \hat{U}_i = \frac{\vec{C}^*}{\vec{C}_G} \]

where \( \vec{C}^* \) is given by G-17 and the norm:
\[ \vec{C}_G = r \] or

\[ \vec{C}_G = \sqrt{[xcos \phi]^2 + [xsin \phi]^2 + [c_z + rsin((i-1)\Delta \theta)]^2} \]
\[ \quad = \sqrt{x^2 + [c_z + rsin((i-1)\Delta \theta)]^2} \]

GRAVITY GRADIENT RODS

Let \( S \) be the length of the rod. The rod is a full cylindrical surface and when it is curved due to thermal bending we can approximate it by \( N \) equal small cylinders, each having height \( S/N \) and which we assume to be right cylinders. See Figure G-9.
Let:

X Y Z - Body Frame

x y z - Rod Frame (same as XYZ at nominal position)

R = Radius of curvature of rod.

We have: \( \alpha = \frac{S}{R} \), \( \Delta \psi = \frac{\alpha}{N} \) and

\[
\phi = i \Delta \psi + \frac{\Delta}{2}, \quad i = 0, 1, 2, \ldots, (N-1)
\]  \hspace{1cm} \text{(G-19)}

\( 0 < \phi < \alpha \)

From Figure G-9 we determine the normal unit vector \( \hat{N}_1 \) to the ith subdivision in Rod Frame.
\[ \hat{N}_1 = (-R \cos \phi, 0, -R \sin \phi)/R = (-\cos \phi, 0, -\sin \phi) \]

where \( \phi \) is given by G-19.

Let \( T_{RB} \) be the transformation from Rod Frame to Body Frame.

Then \( \hat{N} = T_{RB} \hat{N}_1 \) is the normal unit vector in Body Frame.

We must determine \( T_{RB} \), i.e., the three Euler angles.

Let \( \hat{S} = (S_1, S_2, S_3) \) be the sun unit vector, and \( \hat{S}_{XY} \) its projection vector on the XY plane. Then (see Figure G-10) we have

\[ \tan \theta = \frac{-S_2}{-S_1} \]

\[ \theta = \tan^{-1} \left( \frac{S_2}{S_1} \right) \] (G-20)

\[ \psi = \text{(see Figure 8)} \]

and \( \varphi = 0 \)

and we have the Euler angles for \( T_{RB} \).
Let us determine the unit normal vector $\mathbf{\hat{N}}$ of the $i$th rod element in Body Frame from $\mathbf{\hat{N}}_1$ (corresponding one in Rod Frame) and the known Euler angles $(\theta, \psi, \varphi)$ given by G-20.

We had: $\mathbf{\hat{N}}_1 = (-\cos \psi, 0, \pm \sin \psi), \quad \psi = (i-0.5)\Delta \varphi$ \hspace{1cm} (G-23)

or $\mathbf{\hat{N}}_1 = (x_i, 0, z_i) \quad i = 1, 2, \ldots, N$

$\mathbf{\hat{N}}_1 = (x_0, 0, z_0)$ in xyz rod frame

i) Find $\mathbf{\hat{N}}_1$ in x'y'z' frame (call it $\mathbf{\hat{N}}_2$). See also Figure C-11. For this project $x_0$ on x'-y'-z' axes actually x'-z'-axes (since the y' projection is always zero because $\mathbf{\hat{N}}_1$ is always on the x'z' plane).

This gives: $(x_0 \cos \alpha, 0, \mp x_0 \sin \alpha)$

Project $z_0$ on the x'z'-plane: $(\mp z_0 \sin \alpha, 0, z_0 \cos \alpha)$

$\therefore \mathbf{\hat{N}}_2 = (x_0 \cos \alpha \mp z_0 \sin \alpha, 0, \mp x_0 \sin \alpha \pm z_0 \cos \alpha)$ \hspace{1cm} (G-24)

ii) Find $\mathbf{\hat{N}}_2$ in XYZ Body Frame (call it $\mathbf{\hat{N}}$)

For this we need only project $(x_0 \cos \alpha + z_0 \sin \alpha)$ on the X, Y axes since Z and Z' are identical.

$\therefore \mathbf{\hat{N}} = ((x_0 \cos \alpha \mp z_0 \sin \alpha) \cos \theta, (x_0 \cos \alpha \mp z_0 \sin \alpha) \sin \theta, \pm x_0 \sin \alpha \pm z_0 \cos \alpha)$ \hspace{1cm} (G-25)

where:

$x_0 = -\cos \phi; \quad \phi = (i-0.5)\Delta \varphi, \quad i = 1, 2, \ldots, N$

$z_0 = \mp \sin \phi; \quad \Delta \varphi = \frac{\Delta}{N}$
where: the top sign applies to the rod on -Z axis and the bottom sign applies to the rod on +Z axis.

Finally we must determine the CENTER vector in body frame at the i\textsuperscript{th} element of the rod as well as its AREA.

From Figure G-12 we have

\[ \text{CENTER} = R_{\text{curv}} (\hat{N} + \hat{T}) + R \hat{N} \]

Figure G-12

where \[ \hat{T} = ( -\frac{S_1}{\|\hat{S}\|}, -\frac{S_2}{\|\hat{S}\|}, 0) \]
and \[ \hat{S} = (S_1, S_2, 0_3) \] unit vector on LOS to sun.

R = Rod Radius

\( \hat{N} \) is given by G-23 and \( R_{\text{curv}} \) is the input radius of curvature of the rod due to the thermal bending.
\[ \text{AREA}_1 = \Delta \varphi \times \text{R}_{\text{curv}} \times \text{WR}\text{ØD} \]
\[ \text{WR}\text{ØD} = \text{Rod Diameter} \]

When \( \text{R}_{\text{curv}} = \) i.e., straight rod the formulae above must be modified as follows:

\[ \theta = \tan^{-1} \left( \frac{S_2}{S_1} \right) \]
\[ \alpha = 0 \]
\[ \Delta \varphi = 0 \]
\[ \phi = 0 \]
\[ \vec{n}_1 = (-1, 0, 0) \]
\[ \vec{n} = (-\cos \theta, -\sin \theta, 0) \]
\[ \text{CENTER} = (0, 0, \frac{\text{SR}\text{ØD}}{2} + \text{RBIAS}) \]
\[ \text{SR}\text{ØD} = \text{S} \ (\text{Rod Height}) \]
\[ \text{AREA} = \text{SR}\text{ØD} \times \text{WR}\text{ØD} \]

\[ \text{SOLAR PANELS} \]

**Definition 1:** \( \theta_p \) = angle between the panel normal and LOS to the sun and is positive if rotation is \( \text{CCW} \) from LOS to the sun, when standing at the origin of the body frame and looking at \(+Y\) axis.

**Definition 2:** Panels are at nominal position when their normal unit vector is \( \vec{n} = (1, 0, 0) \)
From Figure G-13 we have: 
\[ \psi = \cos^{-1} \left( \frac{S_1}{\|S\|} \right) \]

\[ \hat{S} = (S_1, S_2, S_3) \text{ unit vector along LOS.} \]

Then \( \psi = \phi + \theta \) and

\[ \hat{N} = (\cos \psi, 0, \sin \psi) \text{ is the normal unit vector to the panel} \]
From Figure G-14 we see that:
\[ \text{CENTER} = \vec{CP} + \vec{D_2} + \vec{\Delta_0} \]

where:
- \( \vec{CP} \) is input
- \( \vec{\Delta_0} \) = Solar pressure shift vector from the panel geometrical center (given in Body Frame when panel is at nominal position) and is input.
- \( \vec{D_2} = (0, d/2, 0) \)
- \( d \) = length of panel
- \( h \) = width of panel
- \( \text{AREA} = d \times h \)
APPENDIX H

NTS-2 GRAVITY GRADIENT ROD THERMAL ANALYSIS

The BI-STEM boom configuration and 2-D thermal model are shown in Figure H-1. The NRL specification quotes a maximum solar absorptivity of 0.15; no emissivity data was available—a nominal value of 0.05 was assumed. Preliminary calculations showed that compared with peripheral conduction, internal radiation could be neglected. Radiation was included between the overlapping sections of the boom elements. A hot case solar flux of 455 Btu/hr. ft.\(^2\) was assumed. Earth-albedo and IR fluxes were excluded, providing a major analysis simplification. Inclusion of these terms would result in a slight reduction of the predicted temperature gradients.

Note that the model is two dimensional, i.e., axial temperature gradients in the boom are not considered.

Temperature Predictions

Figures H-2 to H-7 show steady state temperature for two solar orientations and for various values of \(\alpha\), \(\varepsilon\). Figure H-2 is for nominal \(\alpha\), \(\varepsilon\) and indicates a gradient of only 2\(^\circ\)F from sunside to backside. Figure H-3 shows the effect of severe degradation in both \(\alpha\) and \(\varepsilon\): the gradient increases to 4.1\(^\circ\)F. (Note that the mean boom temperature is unchanged because the \(\alpha/\varepsilon\) ratio is the same.) Figure H-4 shows the effect of increasing the emissivity only; although the mean boom temperature drops significantly, the gradient is still only 2\(^\circ\)F.

Figures H-5, H-6 and H-7 show temperatures for the same property combinations but for a different solar orientation. Hot side to cold side gradients are unchanged.
Figure H-1. Bi-Stem Boom Configuration and Thermal Model
Figure H-2. Nominal $\alpha$ and $\epsilon$
Figure H-3. Degraded $\alpha$ and $\epsilon$
$\alpha = 0.15$
$\epsilon = 0.1$

Figure H-4. Degraded $\epsilon$
\( \alpha = 0.15 \)  
\( \epsilon = 0.05 \)

\[ 
\text{SUN} 
\]

Figure H-5. Nominal \( \alpha \) and \( \epsilon \)
\[ \alpha = 0.3 \]
\[ \varepsilon = 0.7 \]

Figure H-6. Degraded \( \alpha \) and \( \varepsilon \)
$\alpha = 0.15$
$\epsilon = 0.1$

Figure H-7. Degraded $\epsilon$
Figure H-8 shows the transient response for entering and leaving a 55 minute period in the earth's shadow. Initial temperatures were assumed to be those shown in Figure H-5; at time zero the solar flux was removed and the temperature histories computed for 55 minutes. At this time the sun was switched on again and the response computed for a further 200 seconds. The results in Figure H-8 show that the 2°F gradient decays within 10 seconds of eclipse and re-establishes within 10 seconds of re-entering sunlight. In actuality, the change would be slower, since the transition from sunlight to darkness (and vice versa) would take a minute or so rather than being a step function as assumed in the analysis.

**Boom Deflections**

To get an estimate of tip deflection, consider the relative expansion of "strips" on the sunside and shadeside:

\[ \theta = \frac{\ell}{R} = \frac{\ell + \delta \ell}{R + d} \]

i.e.

\[ \frac{R + d}{R} = \frac{\ell + \delta \ell}{\ell} \]

\[ R = \frac{d \times \ell}{\delta \ell} \]
But

\[ \delta \ell = B \times \Delta T \times \ell \]

where \( B \) is the coefficient of expansion

\[ \bullet \quad R = \frac{d}{B \times \Delta T} \]

For Beryllium Copper Alloy No. 25,

\[ B = 9.3 \times 10^{-6} \text{ in/in}^\circ F \]

Thus for a \( \Delta T \) of \( 2^\circ F \), and \( d = 0.5" \)

\[ R = \frac{0.5}{9.3 \times 10^{-6} \times 2} = 26,881.7 \text{ inches} = 2240 \text{ ft.} \]

The tip deflection,

\[ \delta = R \left( 1 - \cos \theta \right) \]

and \( \theta = \frac{61}{2240} = 1.56^\circ \)

\[ \bullet \quad \delta = 0.83 \text{ ft. (For } 2^\circ F \text{ gradient)} \]

[For comparison, the formula on P.13 of NASA CR-2516 gives a static tip deflection of 1.05 ft. for this geometry.]

It had been planned to generate a STARDYNE model of a finite length of the boom and impose the computer cross section temperature profile and determine displacements. In view of the small gradients and tip deflection, this is not presently considered necessary.
Figure H-8. Transient Response
Implementation

The thermal model was set up to run a series of cases for solar incidence angles varying from 0° to 80°. The configuration was simulated with "nominal" thermal properties. Temperature gradients are summarized in the table below and plotted in Figure H-9. Full thermal maps are given in Figures H-10 through H-18.

<table>
<thead>
<tr>
<th>SOLAR INCIDENCE</th>
<th>0</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>50°</th>
<th>60°</th>
<th>70°</th>
<th>80°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunside Temp. °F</td>
<td>251.59</td>
<td>248.42</td>
<td>240.12</td>
<td>225.89</td>
<td>205.06</td>
<td>176.40</td>
<td>137.52</td>
<td>83.26</td>
<td>-1.51</td>
</tr>
<tr>
<td>Shadeside Temp. °F</td>
<td>249.53</td>
<td>246.39</td>
<td>238.18</td>
<td>224.11</td>
<td>203.49</td>
<td>175.07</td>
<td>136.49</td>
<td>82.56</td>
<td>-1.86</td>
</tr>
<tr>
<td>T °F</td>
<td>2.06</td>
<td>2.03</td>
<td>1.94</td>
<td>1.78</td>
<td>1.57</td>
<td>1.33</td>
<td>1.03</td>
<td>0.70</td>
<td>0.35</td>
</tr>
<tr>
<td>Static Tip Deflection (Feet)</td>
<td>1.0</td>
<td>0.985</td>
<td>0.942</td>
<td>0.864</td>
<td>0.762</td>
<td>0.646</td>
<td>0.50</td>
<td>0.34</td>
<td>0.17</td>
</tr>
</tbody>
</table>

The boom static tip deflections shown assume that a 2°F gradient gives approximately a 1 ft. deflection.
NOMINAL $\alpha$, $\varepsilon$

$\alpha = 0.15$

$\varepsilon = 0.05$

$0^\circ$ INCIDENCE ANGLE

Figure H-10
Figure H-11
Figure H-12
Figure H-13
Figure H-14
Figure H-15
Figure H-16
Figure H-17
Figure H-18
Summary and Conclusions

For nominal external thermal properties ($\alpha = 0.15$, $\varepsilon = 0.05$) the steady state temperature gradient across the boom is predicted to be only $2^\circ F$. This would produce a deflection of about 1 foot at the boom tips. With degraded thermal properties ($\alpha = 0.3$, $\varepsilon = 0.1$) this gradient would increase to $4^\circ F$ and the tip deflection to 2 feet.

Transient thermal response was evaluated for step function sun on/sun off conditions. The results show that the $2^\circ F$ sun side to back side gradient is established/or decays very rapidly (within 10 seconds).

This analysis suggests that deflection of the boom as a result of thermal gradients should not be a problem. The effect on the spacecraft control system could be assessed by imposing a 1 - 2 ft. step displacement on the boom tips. If problems occur the displacement could be ramped over time periods of 10 seconds to 1 minute instead of assuming a step function.
APPENDIX I

GRAVITY GRADIENT ROD FLEXIBILITY MODEL

I-1 NTS-2 GRAVITY GRADIENT ROD DYNAMICS

This appendix contains details of Stardyne simulations of a rigid satellite with two flexible gravity gradient rods. These simulations showed that modeling each rod with one flexible beam and one tip mass gives questionable results. Using two-beam elements or four-beam elements gives almost identical results. Therefore, a two-beam model was further explored for addition to the orbit simulation model (GPS). (See Section I-2).

Contents

- Summary Table - Key Results (Table I-1)
- Deformed Geometry Sketches, for modes responsive to solar array drive torques (Figures I-1, I-2, and I-3)
- Math Models:
  - 2 Nodes/Rod (1 beam/rod), original (Figure I-4)
  - 2 Nodes/Rod (1 beam/rod), updated (Figure I-5)
  - 3 Nodes/Rod (2 beam/rod) (Figure I-6)
  - 5 Nodes/Rod (4 beam/rod) (Figure I-7)
- Modal Data Summaries, Tables I-6 and I-7
- Mass Property Calc. for Dynamic Rod Models

95
### TABLE I-1. SUMMARY TABLE--RESULTS OF FLEXIBLE GRAVITY GRADIENT ROD STUDIES

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Original)</td>
<td>0.00546</td>
<td>0.02878</td>
<td>446.18</td>
<td>0.03677</td>
<td>1.858</td>
</tr>
<tr>
<td>1 (Updated)</td>
<td>0.00554</td>
<td>0.02442</td>
<td>303.42</td>
<td>0.02533</td>
<td>1.799</td>
</tr>
<tr>
<td>2</td>
<td>0.00553</td>
<td>0.02808</td>
<td>16.54</td>
<td>-0.00686</td>
<td>0.1424</td>
</tr>
<tr>
<td>4</td>
<td>0.00554</td>
<td>0.02804</td>
<td>16.8</td>
<td>-0.006899</td>
<td>0.1479</td>
</tr>
</tbody>
</table>

Good agreement in these parameters for both 2 and 4 beam/rod models shown that 2 beams are adequate. Also see transient response simulations.
Figure 1-1. Beam/Rod Model
NODE 11

$W_{11} = 7.5$

1st ELASTIC MODE SHAPE

$\theta_1 = \frac{\pi}{4}$ radian

$X_1 = 0.02834$ Hz

$\tau_1 = 7.02$ sec

4th ELASTIC MODE SHAPE

$\theta_4 = \frac{\pi}{4}$ radian

$f_4 = 0.02834$ Hz

$\tau_4 = 35.28$ sec

10th ELASTIC MODE SHAPE

$\theta_{10} = \frac{\pi}{4}$ radian

$f_{10} = 0.005527$ Hz

$\tau_{10} = 35.28$ sec

Figure I-2. 2 Beam/Rod Model
Figure I-3. Gravity Gradient Rod Bending Mode Shapes
2 NODES/ROD

BEAM 19 (RIGID)

NODE 11

3.8125

BEAM 3

720"

ORIGIN

19.957

C.G.

NODE 31

.37147

19.75"

BEAM 7

720"

2.0"

BEAM 20 RIGID

NODE 21

MODE 22 (MASSLESS)

IS AT SYSTEM C.G.

X = 0.0

Y = 0.0

Z = 19.86 in.

Figure I-4
FOR DYNAMIC MODEL WITH 2 NODES/ROD

Node 6

.45 lbs. @ center of upper mass

Rod Weight = .90 lbs.

Node 2

.45 lbs.

ADDITIONAL INERTIA FOR NODE 31:

\[
\Delta I_{xx} = (0.45) \frac{2 \left(19.86^2 + 11.14^2\right)}{518.5} \\
= 466.67 \text{ lbs-in}^2
\]

\[
\Delta I_{yy} = (0.45) (2) (28.34^2 + 22.95^2) \\
= 1196.87 \text{ lbs-in}^2
\]

\[
\Delta I_{zz} = (0.45) (2) [19.75]^2 \\
= 351.06 \text{ lbs-in}^2
\]

\[
\Delta W = .90 \text{ lbs. total added to Node 31}
\]
For cantilever beam of mass $m_b$ and tip mass $m$:

Equivalent tip mass weight = $m + \frac{1}{3}m_b$


Above is based on Rayleigh-Ritz energy method.

Therefore, transform $\frac{1}{3}m_b$ beam weight to end of beam but do not transform $\frac{1}{3}m_b$ inertia, since this is accounted for in the $\frac{1}{3}m_b$ approximation.

Node 11

$$\Delta W = 0.225 \text{ lbs.}$$

$$\Delta I_{xx} = I_{yy} = 0.225(3.8125)^2 = 3.27 \text{ lbs-in}^2$$

Node 21

$$\Delta W = 0.225 \text{ lbs.}$$

$$\Delta I_{xx} = \Delta I_{yy} = 0.225(2)^2 = 0.9 \text{ lbs-in}^2$$
Node 31 (Spacecraft c.g.)

Assume 3/4 of rods attached and full inertia transformed to base.

\[ \Delta W = 2 \times .675 = 1.35 \text{ lbs.} \]

From upper rod

\[ \Delta I_{YY} = \frac{1}{3} W \ell^2 + W (19.96^2 + 19.75^2) \]

\[ = \frac{1}{3}(.675)540^2 + .675(19.96^2 + 19.75^2) \]

\[ = 65,610 + 532.2 \]

\[ = 66,142.2 \text{ lbs-in}^2 \] from upper rod only

From lower rod

\[ \Delta I_{YY} = \frac{1}{3} W \ell^2 + W (11.14^2 + 19.75^2) \]

\[ = 65,610 + 65,957 \]

Total \( I_{YY} \) added to Node 31

\[ = 66,142.2 + 65,957 \]

\[ \Delta I_{YY \text{ TOTAL}} = 132,099 \text{ lbs-in}^2 \]

For \( \Delta I_{XX} \):

From upper rod

\[ \Delta I_{YY} = \frac{1}{3} W \ell^2 + W 19.96^2 \]

\[ = 65,610 + .675 (19.96)^2 \]

\[ = 65,878.9 \]
From lower rod

\[ \Delta I_{yy} = \frac{1}{3} W \cdot l^2 + W(11.14)^2 \]
\[ = 65,610 + \frac{.675(11.14)^2}{83.76} \]
\[ = 65,693.76 \]

\[ \Delta I_{xx} \text{ TOTAL} = 131,572.7 \text{ lbs-in}^2 \]

For \( \Delta I_{zz} \):

\[ \Delta I_{zz} = 2 \cdot W \cdot (19.75)^2 \]
\[ = 2 \cdot .675 \cdot 19.75^2 \]
\[ \Delta I_{zz} = 526.584 \text{ lbs-in}^2 \]
3 NODES/ROD MODEL
RIGID SOLAR ARRAYS
FREE TO ROTATE ABOUT Y

Figure I-6
Program will automatically lump rod masses to Nodes
2, 4, 6 (upper rod)
12, 14, 16 (lower rod)

Will manually transform weights at dependent nodes 2, 6, 12 and 16 to adjacent independent nodes of rigid systems.

Node 11:
\[ \Delta W_{11} = \Delta I = \Delta I_x = \Delta I_y = 0.225 \times (3.8125)^2 = 3.27 \text{ lbs-in}^2 \]

Node 21:
\[ \Delta W_{21} = \Delta I = \Delta I_x = \Delta I_y = 0.225 \times (2.0)^2 = 0.90 \text{ lbs-in}^2 \]

Node 31:
\[ \Delta W_{31} = 2 \times 0.225 = 0.450 \text{ lbs.} \]
\[ \Delta I_x = \Delta I_y = \frac{0.225 \times (19.96)^2 + 0.225 \times (11.14)^2}{117.56 \text{ lbs-in}^2} \]
\[ \Delta I_y = \frac{0.225 \times (19.96)^2 + 0.225 \times (19.75)^2}{293.09 \text{ lbs-in}^2} \]
\[ \Delta I_z = 0.225 \times 2 \times (19.75)^2 = 175.53 \text{ lbs-in}^2 \]
MOVE Z MASSES TO SPACECRAFT

Program generates rod weights at nodes 2, 4, 6
12, 14, 16

Must manually transform weight to independent nodes of any rigid systems.

Node 11: (from Node 6)
\[ \Delta W_{11} = 0.225 \text{ lbs.} \quad W_x, W_y \]

Node 21: (from Node 16)
\[ \Delta W_{21} = 0.225 \text{ lbs.} \quad W_x, W_z \]

Total rod + tip mass weight in Z direction = 7.5 + .90 = 8.4 lbs.
Transform to Nodes 2 and 12, which are part of rigid systems. Hang Z weight on new Nodes at ends of stiff elastic beams tied to Node 31. Node 23 at same location as Node 2, Node 24 at same location as Node 12.

Thus total c.g. is correctly modeled and inertia of Z weight contribution is retained.

For beam from Node 23 - 31

\[ K = \frac{3EI}{L^3} - \text{cantilever beam} \]

Want \( f_n = 50 \) Hz

\[ W_n^2 = (2\pi f_n)^2 \frac{K 386.4}{16.8} \]

\[ K = \frac{16.8 \times (100 - 2)^2}{386.4} = 4291. \]

\[ E = 18.5 \times 10^6 \quad b = 12kI^{1/2} \]

\[ I = \frac{Kb^3}{3E} = \frac{4291 (19.75)^3}{3 \times 18.5 \times 10^6} = 0.03016 \quad \text{for } b = d \quad \text{for } b = .775 \]

\[ I = 1/12 b^{4} \]
Figure I-7

108
FOR DYNAMIC MODEL WITH 5 NODES/ROD

Nodes 11
\[ \Delta I_{xx} = \Delta I_{yy} = 0.1125 \times 3.8125^2 \times 50\% = 0.8176 \text{ lb-in}^2 \]

Node 21
\[ \Delta I_{xx} = \Delta I_{yy} = 0.1125 \times (2.0)^2 \times 50\% = 0.225 \text{ lb-in}^2 \]

ADDITIONAL INERTIA FOR NODE 31:
\[ \Delta I_{xx} = 0.1125 \times (19.86^2 + 11.14^2) = 116.67 \text{ lb-in}^2 \]
\[ \Delta I_{yy} = 0.1125 \times (28.94^2 + 22.95^2) = 299.22 \text{ lb-in}^2 \]
\[ \Delta I_{zz} = 0.1125 \times (19.75^2) = 87.764 \text{ lb-in}^2 \]
\[ \Delta W = 0.1125 \times 2 = 0.225 \text{ lbs to Node 31} \]
### TABLE 1-2. STARDYNE MODAL DATA 1 BEAM PER GRAY. GRAD. ROD. ROD EI = 3000. LBS-IN². SOLAR ARRAYS ASSUMED RIGID BUT FREE TO ROTATE ABOUT X2 AXIS.

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**NOTES:**

* at spacecraft c.g.
** lumped 1/2 of each rod weight at tip mass
  - lumped 1/2 of each rod weight at attach point on spacecraft
  - Ignored transfer of distributed rod inertias

(HQR run TSXST65, not saved on tape)
TABLE I-3. STARDYNE MODAL DATA 1 BEAM PER GRAY, GRAD, ROD. ROD EI = 3000, LBS-IN^2.
SOLAR ARRAYS ASSUMED RIGID BUT FREE TO ROTATE ABOUT X2 AXIS (updated)**

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NOTES:  
* at spacecraft c.g.  
** Lumped 1/4 of rod weight to tip mass  
Wt. and inertia of 3/4 of rod adjacent to spacecraft lumped with spacecraft c.g. (Node 31)  

(HBR run TSNST U, modes on tape X765)
### TABLE I-4: STARDYNE MODAL DATA 2 BEAMS PER GRAV. GRAD. ROD. ROD EI = 3000. LBS-IN².

SOLAR ARRAYS ASSUMED RIGID BUT FREE TO ROTATE ABOUT X2 AXIS.

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**NOTES:** spacecraft c.g.

BGR run TEDS/LS, modes on tape X417a
TABLE I-5. STARDYNE MODAL DATA, 4 BEAMS PER GRAV. GRAD. ROD. ROD EI = 3000. LBS-IN².
SOLAR ARRAYS ASSUMED RIGID BUT FREE TO ROTATE ABOUT X2 AXIS.

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<td>25</td>
<td>19</td>
<td>3.527</td>
<td>3.34</td>
<td>.0000000003</td>
<td>X2</td>
<td>Upper Mass, X1</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>26</td>
<td>20</td>
<td>3.527</td>
<td>3.34</td>
<td>-.0000000003</td>
<td>X1</td>
<td>Upper Mass, X2</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>21</td>
<td>4.645</td>
<td>6.71</td>
<td>.0000000003</td>
<td>X2</td>
<td>Lower Mass, X1</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>22</td>
<td>4.645</td>
<td>6.71</td>
<td>.0000000003</td>
<td>X1</td>
<td>Lower Mass, X2</td>
<td>16</td>
</tr>
</tbody>
</table>

NOTES:  *Spacecraft c.g.
(Run No. TSNSTA5, Moes on Tape X3381)
TABLE I. SPACECRAFT PARAMETER VALUES

<table>
<thead>
<tr>
<th>Mass Properties (orbital configuration)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total weight (lbs) = 910</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orbit correction - 75% hydrazine usage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>spacecraft moments and products of inertia, solar arrays deployed, gravity gradient rods not deployed (slug - ft²)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_{XX} = 208.89$</td>
<td>$I_{YY} = 167.46$</td>
<td>$I_{ZZ} = 78.98$</td>
</tr>
<tr>
<td>$I_{XX} = 0$</td>
<td>$I_{YY} = 0$</td>
<td>$I_{ZZ} = -3.04 = 14082.73$ lbs-in²</td>
</tr>
</tbody>
</table>

Spacecraft moments and products of inertia with arrays and rods deployed (slug - ft²)

| $I_{XX} = 209 = 968188.3$ lbs-in² | $I_{YY} = 1998 = 9.2557 \times 10^6$ | $I_{ZZ} = 1909 = 8.8434 \times 10^6$ |
| $I_{XY} = 0$ | $I_{XZ} = 0$ | $I_{YZ} = -3.04$ |

Solar array moment of inertia about array axis of rotation = 3.28 slug - ft² (total for two arrays) = 15194.5 lbs-in².

Spacecraft weights, center of gravity, moments and products of inertia (slug-ft²), and hydrazine usage for orbit connection in stowed condition.

<table>
<thead>
<tr>
<th>Total WT (lbs)</th>
<th>C G X (in)</th>
<th>I_{XX}</th>
<th>I_{YY}</th>
<th>I_{ZZ}</th>
<th>I_{YZ}</th>
<th>Hydrazine Usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>949</td>
<td>25.75</td>
<td>124.12</td>
<td>73.91</td>
<td>98.63</td>
<td>-3.31</td>
<td>0</td>
</tr>
<tr>
<td>936</td>
<td>25.58</td>
<td>122.55</td>
<td>71.92</td>
<td>98.34</td>
<td>-3.31</td>
<td>25%</td>
</tr>
<tr>
<td>923</td>
<td>25.44</td>
<td>120.94</td>
<td>70.08</td>
<td>97.99</td>
<td>-3.31</td>
<td>50%</td>
</tr>
<tr>
<td>910</td>
<td>25.33</td>
<td>119.34</td>
<td>68.33</td>
<td>97.54</td>
<td>-3.31</td>
<td>75%</td>
</tr>
<tr>
<td>897</td>
<td>25.26</td>
<td>117.75</td>
<td>66.69</td>
<td>97.10</td>
<td>-3.31</td>
<td>100%</td>
</tr>
</tbody>
</table>
TABLE I-7. INPUT TO RUN G2 MASS INERTIA PROPERTIES COMPUTER PROGRAM

NTS-2 with rigid deployed solar arrays less elastic grav. grad. rods
Total properties from G.E. report, NRL Table 4-1, c.g. from top of vehicle.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Weight</td>
<td>2</td>
<td>910</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total c.g.</td>
<td></td>
<td>19.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Mx</td>
<td></td>
<td>9255695.04</td>
<td>8843404.32</td>
<td>968188.32</td>
</tr>
<tr>
<td>Total Pred.</td>
<td></td>
<td>-14082.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uprmass Weight</td>
<td>2</td>
<td>-7.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uprmass c.g.</td>
<td></td>
<td>19.75</td>
<td>-723.8125</td>
<td></td>
</tr>
<tr>
<td>Uprmass Mx</td>
<td></td>
<td>-48</td>
<td>-48</td>
<td>-48</td>
</tr>
<tr>
<td>Upr rod Weight</td>
<td>2</td>
<td>-0.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upr rod c.g.</td>
<td></td>
<td>19.75</td>
<td>-360</td>
<td></td>
</tr>
<tr>
<td>Upr rod Mx</td>
<td></td>
<td>-38880.03</td>
<td>-38880.03</td>
<td>-.0558</td>
</tr>
<tr>
<td>Lwrmass Weight</td>
<td>2</td>
<td>-7.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lwrmass c.g.</td>
<td></td>
<td>19.75</td>
<td>753.0</td>
<td></td>
</tr>
<tr>
<td>Lwrmass Mx</td>
<td></td>
<td>-27</td>
<td>-27</td>
<td>-27</td>
</tr>
<tr>
<td>Lwr rod Weight</td>
<td>2</td>
<td>-0.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lwr rod c.g.</td>
<td></td>
<td>19.75</td>
<td>391.0</td>
<td></td>
</tr>
<tr>
<td>Lwr rod Mx</td>
<td></td>
<td>-38880.03</td>
<td>-38880.03</td>
<td>-.0558</td>
</tr>
</tbody>
</table>

Endgroup
Endcase
Stop
$    ENDJOB
$**DIS
$*DIS

RESULTS for Node 31, Flex Rod Models

Wt. = 893.2

c.g. =
x = -3.774 in.
y = 0.0
z = 19.957 in.

Inertias about c.g.

\[
I_{xx} = 744940 \text{ lbs-in}^2 \\
I_{yy} = 325980 - 15,195 = 310,785 \text{ lbs-in}^2 \text{ for solar arrays free to turn about drive axis} \\
I_{zz} = 961440 \text{ lbs-in}^2
\]
TABLE I-8. INPROP RUN G2 MASS

<table>
<thead>
<tr>
<th>RESULTS FOR THE TOTAL SYSTEM</th>
<th>Pounds</th>
<th>Inches</th>
<th>Inches</th>
<th>Inches</th>
<th>Pound - Inches</th>
<th>Pound - Inches</th>
<th>Pound - Inches</th>
<th>Pound - Inches</th>
<th>Pound - Inches</th>
<th>Pound - Inches square</th>
<th>Pound - Inches square</th>
<th>Pound - Inches square</th>
<th>Pound - Inches square</th>
<th>Pound - Inches square</th>
<th>Pound - Inches square</th>
<th>Pound - Inches square</th>
<th>Pound - Inches square</th>
<th>Pound - Inches square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wt. ~</td>
<td>0.89320E 03 ± 0.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c.g. Loc.</td>
<td>-0.37147E 00 ± 0.</td>
<td>0. ± 0.</td>
<td></td>
<td>0.19957E 02 ± 0.</td>
<td>-0.33180E 03 ± 0.</td>
<td>0. ± 0.</td>
<td>0.17826E 05 ± 0.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>0.11007E 07 ± 0.</td>
<td></td>
<td></td>
<td>0.68185E 06 ± 0.</td>
<td>-0.14083E 05 ± 0.</td>
<td>0. ± 0.</td>
<td></td>
<td>-0.48744E 04 ± 0.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Mass mom. of inertia about c.g.</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Square = I_{XX}</td>
<td>0.74494E 06 ± 0.</td>
<td></td>
<td></td>
<td>0.32598E 06 ± 0.</td>
<td>-0.14083E 05 ± 0.</td>
<td>0. ± 0.</td>
<td></td>
<td>0.17474E 04 ± 0.</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
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<tr>
<td>Square = I_{YY}</td>
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<td></td>
</tr>
<tr>
<td>Square = I_{ZZ}</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Square = I_{XY}</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Square = I_{YZ}</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Square = I_{XZ}</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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</tr>
</tbody>
</table>
I-2 DERIVATION OF INFLUENCE COEFFICIENT MATRIX, INCLUDING CROSS-COUPLING EFFECTS DUE TO ROD CURVATURE

Gravity Rod Simulation with Two Flexible Beam Elements

(Linear Theory)

Based on Stardyne model vibration analysis of NTS-2 with rigid solar arrays and flexible gravity gradient rods, the following flexible rod model with two beam elements of equal length was found to give very good results.

The influence coefficient matrix for the above case can be calculated using standard beam deflection equations, shown on the next page. Thus

\[
\begin{pmatrix}
y_a \\
y_b
\end{pmatrix} = \begin{pmatrix}
e_{1,1} & e_{1,2} \\
e_{2,1} & e_{2,2}
\end{pmatrix} \begin{pmatrix}
F_A \\
F_B
\end{pmatrix}
\]

where the \( e_{i,j} \) are influence coefficients.
Under dynamic loading cases, the net applied force at each panel point will be

\[ F_i = F_{\text{external}} - m_i \ddot{y}_i \]

where

\( F_{\text{external}} \) could be due to solar pressures or the component of gravity gradient torque (forces) normal to the rod.

From Roark, "Formulas for Stress and Strain," 4th Ed., the following beam equations are presented.

**TABLE I-9. SHEAR, MOMENT, AND DEFLECTION FORMULAS FOR BEAMS; REACTION FORMULAS FOR RIGID FRAMES**

Notation: \( W = \) load (lb.); \( w = \) unit load (lb. per linear in.). \( M \) is positive when clockwise; \( V \) is positive when upward; \( y \) is positive when upward. Constraining moments, applied couples, loads, and reactions are positive when acting as shown. All forces are in pounds, all moments in inch-pounds; all deflections and dimensions in inches. \( \theta \) is in radians and \( \tan \theta = \theta \)

**STATICALLY DETERMINATE CASES**
Note that no external beam torque loads are included.

**Definitions:**
- \( e_{1,1} \) = deflection of point A due to unit load at point A
- \( e_{1,2} \) = deflection of point A due to unit load at point B
- \( e_{2,2} \) = deflection of point B due to unit load at point B
- \( e_{2,1} = e_{1,2} \) by reciprocity theorem.
- \( e_{2,1} \) = deflection of point B due to unit load at point A

By Case 1, for \( F_A \) applied positive in y direction

\[
y_a = \frac{1}{3} \frac{PL^3}{EI}
\]

but

\[
L = 2\ell
\]

\[
y_a = \frac{1}{3} \frac{(1.0)(2\ell)^3}{EI} = \frac{8\ell^3}{3 EI}
\]

\[
e_{1,1} = \frac{8\ell^3}{3 EI}
\]
By Case 1

\[ y_b = \frac{1}{3} \frac{P \ell^3}{EI} = \frac{1}{3} (1.0) \frac{\ell^3}{EI} \]

Rotation at point b, (by Case 1)

\[ \theta = \frac{1}{2} \frac{P \ell^3}{EI} \]

Deflection at point A due to load at point B is then

\[ y_A = y_b + \theta \cdot \ell \]

\[ = (1.0) \frac{\ell^3}{3EI} + \frac{\ell^2}{2EI} \ell = \]

\[ = (\frac{1}{3} + \frac{1}{2}) \frac{\ell^3}{EI} \]

\[ e_{1,2} = 5 \frac{\ell^3}{6 EI} \]

From first calculation for \( e_{1,2} \)

\[ e_{2,2} = \frac{1}{3} \frac{\ell^3}{EI} \]
For load at A, deflection at point B is found by Case 2.

\[ y_b = \frac{1}{6} \frac{P}{EI} (-a^3 + 3a^2L - 3a^2x) \]

but \( a = \ell \)

\[ L = 2\ell \]

\[ x = 0 \]

\[ y_b = \frac{1}{6} \frac{P}{EI} (-\ell^3 + 3\ell^2 \cdot 2\ell - 3\ell^2 \cdot 0) \]

\[ = \frac{1}{6} \frac{P}{EI} (-\ell^3 + 6\ell^3) \]

\[ = \frac{5}{6} \frac{P}{EI} \ell^3 \]

\[ e_{2,1} = \frac{5}{6} \frac{\ell^3}{EI} \text{ for unit load P.} \]

\[ = e_{1,2} \]

\[
\begin{bmatrix}
e_{1,1} & e_{1,2} \\
e_{2,1} & e_{2,2}
\end{bmatrix}
\]

\[ E = \frac{\ell^3}{EI} \begin{bmatrix}
8 & 5 \\
5 & 6 \\
\end{bmatrix} \]

for NTS-2 \( \ell = 720/2 = 360 \)"

\[ EI = 3000 \text{ lbs-in}^2 \]

\[ \frac{\ell^3}{EI} = \frac{(360)^3}{3000} = 15,552.0 \]
Numerically, for straight rods, NTS-2

\[
[E] = \begin{bmatrix}
41,472 & 12,960 \\
12,960 & 5,184
\end{bmatrix} \text{ inches/lb.}
\]

Note:
Define total rod panel point deflection as follows

\[y_{\text{TOT}} = y_{\text{elastic}} + y_{\text{rigid body}} + y_{\text{thermal}} + y_{\text{non-linear}}\]

Define inertia loads in terms of \(y_{\text{total}}\)

\[y_{\text{total}} = \ddot{y}_{\text{elastic}} + \ddot{y}_{\text{rigid body}} + \ddot{y}_{\text{thermal}} + \ddot{y}_{\text{non-linear}}\]

\(y_{\text{elastic}}\) is determined by means of influence coefficient matrix above.

Similar expression will also govern elastic beam bending in the other beam bending axis. That is, for uniform \(EI_{xx} = EI_{yy}\)

\[
\begin{bmatrix}
x_a \\
x_b
\end{bmatrix} = \frac{3}{EI} \begin{bmatrix}
8/3 & 5/6 & 0 & 0 \\
5/6 & 1/3 & 0 & 0
\end{bmatrix} \begin{bmatrix}
F_{x_A} \\
F_{x_B} \\
F_{y_A} \\
F_{y_B}
\end{bmatrix}
\]

Note that x's and y's are uncoupled.
Now examine effect of slight bending of rod due to out-of-straightness, thermal deflections, or other sources, causing an equilibrium position of the rod with a slight curvature.

**LINEAR ELASTIC CROSS-AXIS COUPLING**

\[ \theta = \frac{2\alpha}{R} \]

at Node A

\[ x_{OA} = R - R \cos \theta = R(1 - \cos \theta) \]

at Node B

\[ x_{OB} = R(1 - \cos \left(\frac{\theta}{2}\right)) \]

Choose positive \( x \) in direction toward sun, resulting thermal bending is away from sun.

Now, a force perpendicular to \( x-z \) plane will result in a linear \( y \) direction deflection, as before plus additional deflections due to twisting about the \( z \) axis.
In 3-D View

For torsion
\[ \theta = \frac{TL}{JG} \]

\( T \) = torque
\( L \) = beam length
\( J \) = torsion section stiffness
\( G \) = shear modulus of material

In Side View, in X-Z Plane

\[ \phi_A = \frac{x_{0A} - x_{0B}}{\ell} \]
\[ \phi_B = \frac{x_{0B}}{\ell} \]

For loading into the plane of paper, in the y direction, no torsion exists in the beams if \( \phi_A = \phi_B \).

In upper beam segment, no torsion exists due to \( F_{yA} \).

Pure bending about point B is about an axis to upper beam, line \( A' \) - \( A' \).
This moment at B about \( A'A' \) must be resolved into a moment about axis \( B' - B \) and a torque.

\[
T = (\text{Moment at B referred to } A'-A') \cdot \sin (\phi_A - \phi_B)
= F_{yA} \cdot \ell \sin (\phi_A - \phi_B)
= \ell \sin \left( \frac{x_{0A} - x_{0B}}{\ell} \right) - \frac{x_{0B}}{\ell} \]
\[ F_{yA} = (x_{0A} - 2x_{0B})F_{yA} \]

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At Node B

\[ T = (x_0 - 2x_{OB}) \cdot F_{yA} \]

Twist of lower beam is \( \theta_B = \frac{T\ell}{JG} \)

\[ \theta_B = \frac{(x_{0A} - 2x_{OB})\ell}{JG} F_{yA} \]

Additional deflection in y direction at Node B = 0

However, twist \( \theta_B \) about lower beam will cause some additional y deflection at upper node.

Arm = \( \ell \sin (\phi_A - \phi_B) \)

\[ = \ell (\phi_A - \phi_B) \]

\[ = \ell \left( \frac{x_{0A} - x_{OB}}{\ell} - \frac{x_{OB}}{\ell} \right) = (x_{0A} - 2x_{OB}) \]

Deflection at end of arm (Node A) is \( \theta_B \cdot \text{arm} \)

\[ \Delta y_A = \frac{(x_{0A} - 2x_{OB})^2 \ell}{JG} F_{yA} \]

for \( x_{0A} = 36 \)

\( x_{OB} = 9 \)

\( \ell = 360 \cdot \)

\( JG = 25 \)

above term is \( \frac{(36 - 18)^2 \cdot 360}{25} = 4665.6 \text{ in/lb.} \)

Checks very well with 3 node Stardyne static model. Low for 5 Node static.
Coupling with Acceleration along Z Axis (Rod Axis)

As shown, a positive force $F_z$ will tend to straighten the rod. A negative oriented Z force will increase the rod deflection.

Buckling check:


$$\frac{W}{P_{cr}} = \frac{4L^2mg}{EI} \quad \text{for } F_z = mg \text{ down}$$

$p_{cr} = \text{column buckling load}$

$m = \frac{(7.5 + .45)}{386.4} = 186.4 \text{ lbs-sec}^2/\text{in.}$

$g = \text{dynamic load factor acting downward due to reactions from solar array drive torques, inches/sec}^2$.

$mg = \frac{2EI}{4L^2}$

$$g_{allow} = \frac{\frac{2EI}{4L^2}}{\frac{2EI}{4L^2}} = \frac{3900}{720} = 5.51 \text{ in/sec}^2$$
\[ g_{\text{allow}} = 5.51 \text{ in/sec}^2 \]

\[ r = \text{dist. from spacecraft c.g. to rod attack point,} \]
\[ 19.75 \text{ in.} \]

\[ \theta_{\text{allow}} = \frac{g_{\text{allow}}}{r} \]
\[ = \frac{5.51 \text{ in/sec}^2}{19.75 \text{ in}} \]
\[ = .279 \text{ rad/sec}^2 \]

\[ I_{\text{YY}} = 311,160 \text{ lb-in}^2 \text{ input to Stardyne run} \]

If peak applied torque = 1 ft-lb for max pulse duration of 45 millisec

\[ \theta = \frac{\text{Torque}}{I} \quad T = I \theta \]
\[ = \frac{12.0 \text{ lb-in} \times 386.4 \text{ in}}{311,160 \text{ lb-in}^2 \text{ sec}^2} \]
\[ = .0149 \text{ rad/sec}^2 \]

Fraction of buckling load = \[ \frac{.0149}{.279} = .0534 \]
Coupled X-Z Beam Deflection

Linear theory will be used. Exact buckling solution involving theory of elastic stability is not suitable for application at hand, especially since column loadings are very low, by prev. calculations.

\[
\sin \theta = \frac{2L}{R}
\]

\[x_{OA} = R - R \cos \theta = R(1 - \cos \theta)\]

\[\cos \theta = 1 - \frac{x_{OA}}{R}\]

\[\tan \theta = \frac{\left(\frac{2L}{R}\right)}{1 - \frac{x_{OA}}{R}} = \frac{L}{R - x_0}\]

\[\theta = \tan^{-1}\left(\frac{L}{R - x_0}\right)\]

\[x_{OB} = R(1 - \cos \frac{\theta}{2})\]

No closed form solution available to solve for \(R\) in terms of \(L, x_{OA}\). Must iterate.
Assume $\gamma_{OA}, \gamma_{OB}$ are known, estimated, or previously calculated.

$$\phi_A = \frac{x_{OA} - x_{OB}}{\ell}$$

for $\phi$ small

Moment at $B = F_{ZA} \cdot \phi_A \cdot \ell = M_B$ for $\sin \phi_A \approx \phi_B$

Resolving vertical force $F_{ZA}$ transferred to point $B$, lateral component is

$$F_{XB} = F_{ZA} \cdot \sin \phi_B \approx F_{ZA} \cdot \phi_B$$
Z-X Coupled Deflections, continued

Deflections of point B (elastic)

\[ x_B = \frac{1}{3} \frac{F_{XB}}{EI} \ell^3 + \frac{1}{2} \frac{M_{YB}}{EI} \ell^2 \]

Rotation of point B (elastic) = \( \alpha_B \) (intermediate variable)

\[ \alpha_B = \frac{M_{YB}}{EI} + \frac{F_{XB}}{2 EI} \ell^2 \]

Deflection of point A due to vertical \( F_{zA} \) force

\[ x_A = x_B + \ell \cdot \alpha_B \cdot \cos \phi_A \]

\[ = \frac{\ell^3}{EI} \left( \frac{1}{3} \phi_B + \frac{1}{2} \phi_A \right) F_{zA} + \frac{\ell^3}{EI} \left( \phi_A + \phi_B F_{zA} \right) \cos \phi_A \cdot \frac{5}{6} \]

For \( \cos \phi_A = 1.0 \)

\[ x_A = \frac{\ell^3}{EI} \left( \frac{3}{2} \phi_A + \frac{5}{6} \phi_B \right) F_{zA} \]
SUMMARY EQUATIONS

For un-bent rod, $E I$ same in both axes

\[
\begin{bmatrix}
  x_A \\
  x_B \\
  y_A \\
  y_B
\end{bmatrix} = \frac{k}{E I} \begin{bmatrix}
  \frac{8}{3} & \frac{5}{6} & 0 & 0 \\
  5 & \frac{1}{3} & 0 & 0 \\
  0 & 0 & \frac{8}{3} & \frac{5}{6} \\
  0 & 0 & \frac{5}{6} & \frac{1}{3}
\end{bmatrix} \begin{bmatrix}
  F_{x_A} \\
  F_{x_B} \\
  F_{y_A} \\
  F_{y_B}
\end{bmatrix}
\]

For bent rod, with point A initially at unstressed position having $x$ and $y$ components of $x_{OA}$, $y_{OA}$ and point B having corresponding initial unstressed deflection components of $x_{OB}$, $y_{OB}$ write out deflection equations in algebraic form.

\[
x_A = \frac{k}{E I y} \left( \frac{3}{2} F_{x_A} + \frac{5}{6} F_{x_B} \right) \quad \text{in-plane bending}
\]

\[
+ \frac{k}{J G} (y_{OA} - 2 y_{OB})^2 F_{x_A} \quad \text{torsion}
\]

\[
+ \frac{k}{E I y} \left( \frac{3}{2} \left( \frac{x_{OA} - x_{OB}}{\ell} \right) + \frac{5}{6} \frac{x_{OB}}{\ell} \right) F_{z_A} \quad \text{Z-X coupling}
\]

Similarly:

\[
y_A = \frac{k}{E I y} \left( \frac{3}{2} F_{y_A} + \frac{5}{6} F_{y_B} \right) + \frac{k}{J G} (x_{OA} - 2 x_{OB})^2 F_{y_A}
\]

\[
+ \frac{k}{E I y} \left( \frac{3}{2} \left( \frac{y_{OA} - y_{OB}}{\ell} \right) + \frac{5}{6} \frac{y_{OB}}{\ell} \right) F_{z_A}
\]
For Point B (elastic deflections)

\[ x_B = \frac{L^3}{EI_y} \left( \frac{5}{6} F_{x_A} + \frac{1}{3} F_{x_B} \right) \]

+ \[ + \frac{L^3}{EI_y} \left( \frac{1}{2} \left( \frac{x_{0A} - x_{0B}}{L} \right) + \frac{1}{3} \left( \frac{x_{OB}}{L} \right) \right) F_{z_A} \]

in-plane bending

(no torsional contribution at B) (1st order approx.)

\[ y_B = \frac{L^3}{EI_x} \left( \frac{5}{6} F_{y_A} + \frac{1}{3} F_{y_B} \right) \]

+ \[ + \frac{L^3}{EI_x} \left( \frac{1}{2} \left( \frac{y_{0A} - y_{0B}}{L} \right) + \frac{1}{3} \left( \frac{y_{0B}}{L} \right) \right) F_{z_A} \]

z-x coupling

Note: Rod bending stiffnesses \( EI_x \) and \( EI_y \) are designated as follows:

\( EI_x \) is stiffness for bending rotations about x axis, resulting in y deflections

\( EI_y \) is stiffness for bending rotations about y axis, resulting in x deflections.
## Final 2-Beam Gravity Rod Influence Coefficient Matrix

\[
\begin{bmatrix}
  x_A \\
  x_B \\
  y_A \\
  y_B
\end{bmatrix} = 
\begin{bmatrix}
  \frac{8}{3} \frac{3}{E I_y} + \frac{5}{6} \frac{3}{E I_y} \\
  \frac{5}{6} \frac{3}{E I_y} \\
  0 \\
  0
\end{bmatrix} \begin{bmatrix}
  \frac{1}{2} \frac{2}{E I_y} \\
  \frac{9}{10} x_{0A} - 4 x_{0B} \\
  \frac{1}{2} \frac{2}{E I_y} \\
  \frac{3}{10} x_{0A} - x_{0B}
\end{bmatrix}
\begin{bmatrix}
  y_A \\
  y_B \\
  z_A
\end{bmatrix}
\]

\[
\begin{bmatrix}
  \frac{8}{3} \frac{3}{E I_x} + \frac{5}{6} \frac{3}{E I_x} \\
  \frac{5}{6} \frac{3}{E I_x} \\
  0 \\
  0
\end{bmatrix} \begin{bmatrix}
  \frac{1}{2} \frac{2}{E I_x} \\
  \frac{9}{10} y_{0A} - 4 y_{0B} \\
  \frac{1}{2} \frac{2}{E I_x} \\
  \frac{3}{10} y_{0A} - y_{0B}
\end{bmatrix}
\begin{bmatrix}
  x_{OA} \\
  x_{OB} \\
  y_{OA} \\
  y_{OB}
\end{bmatrix}
\]
Numerical Check

For manufacturing tolerance, min radius of curvature for BI-STEM > 1800. foot per NTS-2 spec.

For \( L = 2 \) and \( \lambda = 720'' \)

\[
\text{tie deflection} = 12.0 \text{ in.} \quad \theta = 1.91021317^\circ
\]

Add 24 in. thermal deflection (per R.G. Langton memo)

Total \( x_{0A} = 36 \text{ in.} \quad \theta = 5.7248^\circ \)

\( R = 601.5 \text{ ft.} = 7218. \text{ in.} \) (radius of curvature)

\( x_{0B} = 9.005 \text{ inches} \quad y_{0A} = y_{0B} = 0.0 \)

\( E I_x = 3000. = E I_y \)

\( JG = 25. \)

\( \ell = 360. \)

A 2-beam Stardyne finite element model was run using the above parameters, including the radius of curvature. Two straight beam element connect the base, Node B, and Node A (top). Run TSNS T6Z.

An additional run, using twice as many beams and nodes, was also run (Run TSNS T9D). Both results are presented in Tables I-1, I-2, and I-3.
STARYDNE CHECK CASE, GRAV. ROD INFLUENCE MATRIX. [E]

TABLE I-10. COMPUTED [E] ~ 1bs/inch

\[
\begin{align*}
\begin{bmatrix}
    x_A \\
    x_B \\
    y_A \\
    y_B \\
\end{bmatrix} &=
\begin{bmatrix}
    41,472. & 12,960. & 0 & 0 & 2073. \\
    12,960. & 5,184. & 0 & 0 & 713. \\
    0 & 0 & 46,138. & 12,960. & 0 \\
    0 & 0 & 12,960. & 5,184. & 0 \\
\end{bmatrix}
\begin{bmatrix}
    F_{x_A} \\
    F_{x_B} \\
    F_{y_A} \\
    F_{y_B} \\
\end{bmatrix}
\end{align*}
\]

TABLE I-11. STARDYNE 2 BEAM SOLUTION
(INCLUDES Z DEFLECTIONS)

\[
\begin{align*}
\begin{bmatrix}
    x_A \\
    x_B \\
    y_A \\
    y_B \\
    z_A \\
    z_B \\
\end{bmatrix} &=
\begin{bmatrix}
    41,498. & 12,993. & 0 & 0 & 2464. & 6x1 \\
    12,993. & 5,205. & 0 & 0 & 715. & 5x1 \\
    0 & 0 & 46,294. & 13,011. & 0 & 5x1 \\
    0 & 0 & 13,011. & 5,208. & 0 & 5x1 \\
    +2,465. & 715. & 0 & 0 & 149. & 5x1 \\
    +325. & 130. & 0 & 0 & 18. & 5x1 \\
\end{bmatrix}
\begin{bmatrix}
    F_{x_A} \\
    F_{x_B} \\
    F_{y_A} \\
    F_{y_B} \\
    F_{z_A} \\
\end{bmatrix}
\end{align*}
\]
TABLE I-12. STARDYNE 4 BEAM STATIC CHECK RESULTS
FOR FLEXIBLE GRAVITY GRAD. ROD

(COMPARE TO TABLE 2 & 1)
(INCLUDES Z DEFLECTIONS)

\[
\begin{bmatrix}
  x_A \\
x_B \\
y_A \\
y_B \\
z_A \\
z_B
\end{bmatrix} =
\begin{bmatrix}
  41,494. & 12,992. & 0 & 0 & 2,562. \\
  12,992. & 5,204. & 0 & 0 & 739. \\
  0 & 0 & 48,336. & 13,886. & 0 \\
  0 & 0 & 13,886. & 5,354. & 0 \\
  2,562. & 739. & 0 & 0 & 162. \\
  398. & 154. & 0 & 0 & 23.
\end{bmatrix}
\]

Run TSIdST9D
In terms of previously defined parameters the dynamic loads imposed on spacecraft by rod motions are as follows:

For loads in Y-Z plane:
  (shown in positive direction)

**Loads on spacecraft**
are equal and opposite:

\[ V_y = F_{yA} + F_{yB} \]
\[ V_z = F_z \]
\[ M_x = -(\ell \cos \phi_A + \ell \cos \phi_B) \cdot F_{yA} \]
\[ - \ell \cos \phi_B \cdot F_{yA} \]
\[ - (y_{OA}) F_{zB} \]
\[ + F_y \text{ causes negative } M_x \text{ applied to spacecraft.} \]

For 1st order non-linear analysis, update moment to include deflection of point A. Assume \( \cos \phi_A \approx 1 \)

\[ M_x = -2 \cdot \ell \cdot F_{yA} - \ell \cdot F_{yA} - (y_{OA} - y_A) \cdot F_{zA} \]
Loads in X-Z Plane

 Loads shown at base of rod to maintain equilibrium.

 Loads on spacecraft are opposite and equal.

\[ V_x = F_x A + F_x B \]
\[ V_z \text{ (previously calculated) = } F_z \]
\[ M_y = + (\ell \cos \phi_A + \ell \cos \phi_B) F_x A + \ell \cos \phi_B F_x B + F_z (x_{0A}) \]

For 1st order non-linear analysis, update last term to include current deflection of point A (tip mass) which could be changing fastest. Also \( \cos \phi_A = \cos \phi_B \approx 1 \)
\[ M_y = 2 \ell F_x A + \ell F_x B + F_z (x_{0A} - x_A) \]

Moments about Z Axis

From top, looking down
\[ M_z = y_{0A} \cdot F_x A + y_{0B} \cdot F_x B - x_{0A} \cdot F_y A - x_{0B} F_y B \]

For non-linear effects, update total distance from Z axis.
Non-linear

\[ M_z = (y_{OA} - y_A) F_{x_A} + (y_{OB} - y_B) F_{x_B} \]
\[ - (x_{OA} - x_A) F_{y_A} - (x_{OB} - x_B) F_{y_B} \]

In matrix notation, loads on spacecraft at rod attach point are:

\[
\begin{bmatrix}
V_x \\
V_y \\
V_z \\
M_x \\
M_y \\
M_z
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & -2\xi & -\xi & (y_{OA} - y_A) (y_{OB} - y_B) \\
2\xi & \xi & 0 & 0 & (x_{OA} - x_A) (x_{OB} - x_B) \\
(y_{OA} - y_A) (y_{OB} - y_B) & -(x_{OA} - x_A) & -(x_{OB} - x_B) & 0 & 0
\end{bmatrix}
\begin{bmatrix}
F_{x_A} \\
F_{x_B} \\
F_{y_A} \\
F_{y_B} \\
F_{z_A} \\
F_{z_B}
\end{bmatrix}
\]

where

\[ x_A', x_B', y_A', y_B \] are updated rod elastic deflection.

For non-linear Lagrangian analysis or do not include for linear (small deflection) analysis.
NUMERICAL CHECK, REACTIONS AND MOMENTS

Linear analysis

CALCULATED VALUES, STARDYNE CHECK CASE

\[
\begin{align*}
\begin{bmatrix}
V_x \\
V_y \\
V_z \\
M_x \\
M_y \\
M_z \\
\end{bmatrix} &=
\begin{bmatrix}
1.0 & 1.0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.0 & 1.0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & -720 & -360 & 0 & 0 \\
720 & 360 & 0 & 0 & 36 & 9 \\
0 & 0 & -36 & -9 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
F_x \\
F_y \\
F_z \\
F_A \\
F_B \\
F_A \\
\end{bmatrix}
\end{align*}
\]

COMPUTED REACTIONS FROM STARDYNE
2 BEAM MODEL (RUN USNS T6Z)

\[
\begin{align*}
\begin{bmatrix}
V_x \\
V_y \\
V_z \\
M_x \\
M_y \\
M_z \\
\end{bmatrix} &=
\begin{bmatrix}
1.00 & 1.00 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 1.00 & 1.00 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 1.0 \\
0.0 & 0.0 & -720.00 & -360.45 & 0.0 & 0.0 \\
720.0 & 360.45 & 0.0 & 0.0 & 36.0 & 9.006 \\
0.0 & 0.0 & -36.00 & -9.006 & 0.0 & 0.0 \\
\end{bmatrix}
\begin{bmatrix}
F_x \\
F_y \\
F_z \\
F_A \\
F_B \\
F_A \\
\end{bmatrix}
\end{align*}
\]
Convert Modal Damping to Discrete Damping

Matrix equations of motion for free vibration:

\[ [M] \ddot{x} + [C] \dot{x} + [K] x = 0 \]  \hspace{1cm} (I-1)

Let \( \{x\} = [\phi] \{q\} \)  \hspace{1cm} (I-2)

where

\( [\phi] = \) set of mode shapes

\( \{q\} = \) modal displacements

Also \( \{\dot{x}\} = [\phi] \{\dot{q}\} \)  \hspace{1cm} (I-3)

\( \{\ddot{x}\} = [\phi] \{\ddot{q}\} \)  \hspace{1cm} (I-4)

Substituting Eqs. I-2, I-3, and I-4 into Eq. I-1 and premultiplying by \( [\phi]^T \), the result is

\[ [M_{EQ}] \ddot{q} + 2 [M_{EQ}] \beta \omega_n \dot{q} + [M_{EQ}] \omega_n^2 q = 0 \]  \hspace{1cm} (I-5)

where

\( [M_{EQ}] = [\phi]^T [M] [\phi] \)  \hspace{1cm} (Generalized Mass Matrix) or equivalent

(Ref. STARDYNE Theoretical Manual, DYNREL Analytics)

For solution by the normal mode method, Eq. I-5, the damping must be assumed to be proportional to the mass or to the stiffness. For the first,

\( [C] = 2 [\beta] [M] \)  \hspace{1cm} (I-6)

If so, then

\[ [\phi]^T [C][\phi] = 2 [\beta] [M_{EQ}] \]  \hspace{1cm} (I-7)

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Note:
(Corollary for one d.o.f., oscillator)


\[
C_c = 2 m \omega_n \quad \text{ (critical damping coeff.)}
\]

\[
\rho = \frac{C'}{C_c} \quad \text{ (damping factor),}
\]

\[
C' = 2 \rho m \omega_n
\]  

(8)

From Equation I-7, assume a value of .005 for \( \beta_1, \beta_2, \ldots, \beta_n \).

Premultiply both sides of I-7 by \([\phi]^T^{-1}\). (Inverse at transpose of \([\phi]\)).

\[
[I] [C] [\phi] = ([\phi]^T)^{-1} [2 \beta] [M_{EQ}]
\]  

(9)

Post-multiply both sides of Eq. I-9 by \([\phi]^{-1}\).

\[
[C] = ([\phi]^T)^{-1} [2 \beta] [M_{EQ}] [\phi]^{-1}
\]  

(10)

Evaluate numerically.

EQUIV. DAMPING CALCULATION

\[
W_A = 7.5 + .225 = 7.725 \text{ lbs.}
\]

\[
W_B = .45 \text{ lbs.}
\]
<table>
<thead>
<tr>
<th>Mode No., (n)</th>
<th>(M_{EQ_n}) lbs.</th>
<th>(x_{1A})</th>
<th>(x_{1B})</th>
<th>(f_n), Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.7969</td>
<td>1.000</td>
<td>.31352</td>
<td>.00550005</td>
</tr>
<tr>
<td>2</td>
<td>.47307</td>
<td>-.01526</td>
<td>1.000</td>
<td>.13555</td>
</tr>
</tbody>
</table>

\[
[\phi] = \begin{bmatrix} 1.0000 & -.01526 \\ .31352 & 1.0000 \end{bmatrix}
\]
Columns are for a mode

\[
[\phi]^T = \begin{bmatrix} 1.0000 & .31352 \\ -.01526 & 1.0000 \end{bmatrix}
\]

\[
[2 \beta] = \begin{bmatrix} .01 & 0 \\ 0 & .01 \end{bmatrix}
\]

\[
[M_{EQ}] = \begin{bmatrix} 7.7969 & 0 \\ 0 & .47307 \end{bmatrix}
\]

\[
[2 \beta] [M_{EQ}] = \begin{bmatrix} .01 & 0 \\ 0 & .01 \end{bmatrix} \begin{bmatrix} 7.7969 & 0 \\ 0 & .47307 \end{bmatrix} = \begin{bmatrix} .077969 & 0 \\ 0 & .0047307 \end{bmatrix}
\]

\[
[[\phi]^T]^{-1} = \begin{bmatrix} 1.000 & .31352 \\ -.01526 & 1.000 \end{bmatrix}^{-1} = \frac{1}{A} \begin{bmatrix} 1.000 & -.31352 \\ .01526 & 1.000 \end{bmatrix}
\]
where

\[ A = \text{determinant} \]
\[ = a_{11}a_{22} - a_{12}a_{21} \]
\[ = (1.0)^2 + (0.31352)(0.01526) \]
\[ = 1.0047843 \]

\[
\begin{bmatrix}
[\phi]^T \end{bmatrix}^{-1} = \begin{bmatrix}
0.995238 & -0.312027 \\
0.0151873 & 0.995238 \\
\end{bmatrix} \tag{14}
\]

and

\[
[\phi]^{-1} = \begin{bmatrix}
1.000 & -0.01526 \\
0.31352 & 1.000 \\
\end{bmatrix}^{-1} = \frac{1}{|A|} \begin{bmatrix}
1.000 & 0.01526 \\
-0.31352 & 1.000 \\
\end{bmatrix}
\]

\[ |A| = (1.0)^2 + (0.01526)(0.31352) \]
\[ = 1.0047843 \]

\[
[\phi]^{-1} = \begin{bmatrix}
0.995238 & 0.0151873 \\
-0.312027 & 0.995238 \\
\end{bmatrix} \tag{15}
\]

Performing numerical multiplication, from Eqs. I-13 and I-14,

\[
\begin{bmatrix}
[\phi]^T \\
\end{bmatrix}^{-2} \begin{bmatrix}
[2 \beta] \\
M_{EQ} \\
\end{bmatrix} = \begin{bmatrix}
0.995238 & -0.312027 \\
0.0151873 & 0.995238 \\
0.0775977 & -0.001476 \\
0.0011841 & 0.00470817 \\
\end{bmatrix} \begin{bmatrix}
0.0775977 & -0.001476 \\
0.0011841 & 0.00470817 \\
\end{bmatrix} = \begin{bmatrix}
0.0775977 & -0.001476 \\
0.0011841 & 0.00470817 \\
\end{bmatrix} = [c_M]
\]

Then post-multiplying by Eq. I-15,
\[
\begin{bmatrix}
.07768884 & -.00029057 \\
-.00029057 & .00470374
\end{bmatrix} \times \frac{1}{386.4} \text{ lb-sec/inch}
\]

(Divide by 386.4)

For 1 d.o.f. system, \( f_n = .0055 \) Hz
\[\omega_n = 2\pi f_n = .0346 \text{ rad/sec.}\]

Using Eq. 1-8
\[C' = 2 \beta m_n \omega_n = 2 (.005) (7.725/386.4) (.0346) = 6.917 \times 10^{-6} \text{ lb-sec}^2/\text{in. sec} \]

If divided between Node A and Node B
\[C_A = C_B = 3.4586 \times 10^{-6} \]

Dividing coupled damping matrix by 386.4 (since Stardyne \( M_{EQ} \) is expressed in lbs.)
\[
\begin{bmatrix}
2.01058 \times 10^{-4} & -.75199 \times 10^{-6} \\
-.75199 \times 10^{-6} & 1.21732 \times 10^{-5}
\end{bmatrix} \text{ lb-sec/in.}
\]

Note: if single d.o.f. damping were based on second mode only
\[C' = 2 \beta m_2 \omega_2 \]
\[\omega_2 = 2\pi (.13555) \]
\[C' = (.01) \frac{.45}{386.4} 2\pi (.13555) = 9.9187 \times 10^{-6} \text{ (see below)} \]
For single d.o.f. system

Damping factor = $C$ lb-sec/in
Damping force = $C \dot{x}$

For sine vibration

$\dot{x} = \omega x$ in/sec
Damping force = $C \omega x$ lb-sec/in ⋅ rad/sec in. = lbs.

$$C_{1,1}' = 2 \beta m_a = 2 \times (0.005) \frac{7.725}{386.4} = 1.9992 \times 10^{-4}$$

For large mass.

$$C_{2,2}' = 2 \beta m_b = 2 \times 0.005 \times \frac{45}{386.4} = 1.1646 \times 10^{-5}$$

CHECK DAMPING MATRIX

Assume constrained vibration mode, at resonance,

tip amplitude = $1.0 \sin \omega t$

$f = 0.0055$ Hz
$\omega = 2\pi f = 0.03455$

$\dot{x}_A = 1.0$
$\ddot{x}_A = \omega x_A = 0.3455$

$\dot{x}_B = 0.31352$
$\ddot{x}_B = \omega x_B = 0.010834$

$$\dddot{x}_A = -\omega^2 x_A = -0.0011942 \text{ in/sec}^2$$
$$\dddot{x}_B = -\omega^2 x_B = -0.00037441 \text{ in/sec}^2$$

Inertia forces:

$$\begin{bmatrix} M_A & 0 \\ 0 & M_B \end{bmatrix} \begin{bmatrix} \dddot{x}_A \\ \dddot{x}_B \end{bmatrix} = \begin{bmatrix} \frac{7.725}{386.4} \times 0.0011942 \\ \frac{45}{386.4} \times 0.00037441 \end{bmatrix} = \begin{bmatrix} -2.3875 \times 10^{-5} \\ -4.3604 \times 10^{-7} \end{bmatrix} \text{ lbs.}$$
Damping Forces:

\[
[C] \begin{bmatrix} \dot{x}_A \\ \dot{x}_B \end{bmatrix} = \begin{bmatrix} 2.01058 \times 10^{-4} & -0.75199 \times \mathbf{10}^{-6} \\ -0.75199 \times \mathbf{10}^{-6} & 1.21732 \times \mathbf{10}^{-5} \end{bmatrix} \begin{bmatrix} 0.03455 \\ 0.010834 \end{bmatrix} \begin{bmatrix} 6.9384 \times \mathbf{10}^{-6} \\ 1.059 \times \mathbf{10}^{-7} \end{bmatrix} \text{lbs.}
\]

For stiffness forces, must invert 2 x 2 influence matrix

\[
[E] = \frac{\ell^3}{EI} \begin{bmatrix} 8/3 & 5/6 \\ 5/6 & 1/3 \end{bmatrix} \quad [K] = [E]^{-1} = \frac{EI}{\ell^3} \begin{bmatrix} 13 & -5 \\ -5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 8/3 \cdot 1/3 - 5/6 \cdot 5/6 \end{bmatrix}
\]

\[
[K] = \frac{EI}{\ell^3} \begin{bmatrix} 12 & -30 \\ -30 & 96 \end{bmatrix}
\]

Stiffness forces:

Evaluate numerically:

\[
[K] = \frac{EI}{\ell^3} \begin{bmatrix} 12 & -30 \\ -30 & 96 \end{bmatrix}
\]

\[\ell = 360 \quad \frac{EI}{\ell^3} = \frac{3000}{7 \times (360)^3} = 9.18577 \times \mathbf{10}^{-6} \quad \text{lbs/inch} \]

\[
[K] = \begin{bmatrix} 1.10225 \times \mathbf{10}^{-4} & -2.7557 \times \mathbf{10}^{-4} \\ -2.7557 \times \mathbf{10}^{-4} & 8.8183 \times \mathbf{10}^{-4} \end{bmatrix}
\]

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\[
[K] \begin{bmatrix}
 x_A \\
 x_B
\end{bmatrix} = [K] \begin{bmatrix}
 1.6 \\
 .31352
\end{bmatrix} = \begin{bmatrix}
 2.38315 \times 10^{-5} \\
 8.9947 \times 10^{-7}
\end{bmatrix}
\]

Summary of check at 1st Natural

<table>
<thead>
<tr>
<th>NODE</th>
<th>INERTIA, lbs.</th>
<th>STIFFNESS, lb.</th>
<th>DAMPING</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(-2.3875 \times 10^{-5})</td>
<td>(2.38315 \times 10^{-5})</td>
<td>(6.9384 \times 10^{-6})</td>
</tr>
<tr>
<td>B</td>
<td>(-4.3604 \times 10^{-7})</td>
<td>(8.9947 \times 10^{-7})</td>
<td>(1.059 \times 10^{-7})</td>
</tr>
</tbody>
</table>

At resonance, inertia + stiffness forces are equal and opposite.
APPENDIX J

DETAILS OF SOLAR ARRAY FLEXIBILITY MODELING

SOLAR ARRAY TORSION

The solar array torsional equations of motion are described by a solution of the following system of matrix equations

\[
[M] \{\ddot{\theta}\} + [C] \{\dot{\theta}\} + [K] \{\theta\} = \{T(t)\} \tag{J-1}
\]

where the vector \{\theta\} is the rotation of each solar panel and the drive motor.

Applied torques \{T(t)\} may be zero at all locations except the drive motor.

The above equations in discrete coordinates are coupled because the stiffness matrix \([K]\) is coupled. The damping matrix \([C]\) may also be coupled. For six solar panels plus the drive motor, the equation has at least seven degrees of freedom. Now make the following transformation:

\[
\{\theta\} = [\psi] \{q\} \tag{J-2}
\]

where the columns of \([\psi]\) are the eigen vectors which result from the STARDYNE solution of the relationship

\[
[K] \{\psi\} = \omega^2 [M] \{\psi\}
\]

Substituting equation J-2 and its derivatives into Eqn. J-1 and premultiplying by the transpose of \([\psi]\) results in the following set of uncoupled equations in terms of the generalized coordinates \{q\}:
\[ [M_{\text{eq}}] \{q\} + [M_{\text{eq}} \omega_n^2] \{\ddot{q}\} + [M_{\text{eq}} \omega_n^2] \{\dot{q}\} = [\Phi]^T [T(t)] \]  

Note that by orthogonality relationships, the off-diagonal terms of the generalized mass, damping and stiffness matrices are zero.

The STARDYNE solution for the NTS-2 model with flexible solar arrays and flexible gravity gradient rods has six rigid body modes and one mechanism type rigid body mode because the solar arrays are considered free-wheeling in \( \theta_y \). The "free-wheeling" solar array mode in the STARDYNE modes is included as a linear combination of all seven STARDYNE rigid-body modes.

For simulation purposes, the free-wheeling solar array mode will be replaced by a single equivalent rigid body \( \theta_y \) mode whose amplitude is normalized to 1.0 and whose generalized mass is thus equal to its total inertia about the solar array \( \theta_y \) axis.

The elastic modes are taken from the STARDYNE run TSNSTRL. The only significant solar array elastic modes are the symmetric torsional modes at 1.095 Hz and 1.898 Hz.

Thus the total number of generalized coordinates to be solved is three, defined as follows:

\[
\{q\} = \begin{cases} 
q_1 \\
q_2 \\
q_3 
\end{cases}
\]

\( q_1 \) = rigid body torsion mode  
\( q_2 \) = first symm. torsion mode  
\( q_3 \) = second symm. torsion mode

Generalized Mass

\[ [M_{\text{eq}}] = [\Phi]^T [M] [\Phi] \]  

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The diagonals of the generalized weight matrix are printed by STARDYNE and are calculated for modes normalized to a maximum displacement or rotation of unity. For the above three solution modes:

<table>
<thead>
<tr>
<th>MODE NO.</th>
<th>EQUIV. WEIGHT $M_{eq}$, Lbs.</th>
<th>EQUIV. MASS $M_e$, lb·sec$^2$/ft</th>
<th>FREQUENCY $f$, Hz</th>
<th>$\omega_n$, rad/sec</th>
<th>$\omega_n^2$, (rad/sec)$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13956.7</td>
<td>3.01</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>9284.0</td>
<td>2.002</td>
<td>1.095</td>
<td>6.880</td>
<td>47.339</td>
</tr>
<tr>
<td>3</td>
<td>6971.0</td>
<td>1.5034</td>
<td>1.898</td>
<td>11.925</td>
<td>142.2</td>
</tr>
</tbody>
</table>

Premultiplying Eqn. J-3 by $[M_{eq}]^{-1}$, the response equation becomes

$$[1] \{\ddot{q}\} + [2\rho \omega_n] \{\dot{q}\} + [\omega_n^2] \{q\} = \left[ \begin{array}{c} \dot{T} \\ T(t) \end{array} \right]$$


 Extracting only the $\theta_y$ rotations at the drive motor (node 20), the $-Y$ outer panel center (node 2), and the $+Y$ outer panel center (node 31), these rotations are related to the generalized responses by

$$\begin{align*}
\theta_y \text{ drive motor} & \rightarrow \left[ \begin{array}{ccc} 1.0 & -1.0 & -.5 \\
1.0 & 1.0 & .5 \\
1.0 & 1.0 & .5 \\
\end{array} \right] \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \\
\theta_y \text{ -Y tip mass} & \rightarrow \left[ \begin{array}{ccc} 1.0 & -1.0 & -.5 \\
1.0 & 1.0 & .5 \\
1.0 & 1.0 & .5 \\
\end{array} \right] \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \\
\theta_y \text{ +Y tip mass} & \rightarrow \left[ \begin{array}{ccc} 1.0 & -1.0 & -.5 \\
1.0 & 1.0 & .5 \\
1.0 & 1.0 & .5 \\
\end{array} \right] \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}
\end{align*}$$

Equation J-4 in numerical form reduces to ($\rho = .005$):

$$\begin{bmatrix} 1.0 & \frac{0.0688}{1.1925} & \frac{0.0}{1.1925} \\
\frac{0.0688}{1.1925} & \frac{47.334}{142.2} & \frac{-0.995}{142.2} \\
\frac{0.0}{1.1925} & \frac{-0.995}{142.2} & \frac{-0.3326}{142.2} \\
\end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \frac{0.33222}{1.1925} \\
\frac{-0.0}{1.1925} \\
\frac{-0.3326}{1.1925} \\
\end{bmatrix} (T_{\text{drive}}(t))$$
Equation J-6 can be solved as three uncoupled second order linear differential equations.

**TABLE J-1. **$-Y$ SOLAR ARRAY, (BENDING ONLY)

<table>
<thead>
<tr>
<th>STIFFNESS MATRIX</th>
<th>For motion relative to drive motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\text{Units } = \text{lbs./ft.})$</td>
<td></td>
</tr>
<tr>
<td>$[K] {x} = \begin{bmatrix} .0085617 &amp; -.004860 \ -.004860 &amp; .630218 \end{bmatrix} \begin{bmatrix} x_1 \ z_1 \end{bmatrix}$</td>
<td></td>
</tr>
</tbody>
</table>

where

$x_1$ = motion of center of outer panel, normal to plane of panel

$z_1$ = motion of center of outer panel, in plane of the panel

<table>
<thead>
<tr>
<th>MASS MATRIX</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\text{Units } = \text{slugs (lb-sec}^2/\text{ft})$</td>
<td></td>
</tr>
<tr>
<td>$[M] {x} = \begin{bmatrix} .42176 &amp; 0 \ 0 &amp; .52608 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \ \ddot{z}_1 \end{bmatrix}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DAMPING MATRIX</th>
<th>$\rho_{R} = .005$ (1/2 of 1 percent critical damping)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[C] {\dot{x}} = \begin{bmatrix} .0006009 &amp; 0 \ 0 &amp; .005758 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \ \dot{z}_1 \end{bmatrix}$</td>
<td>$\text{(Units of } [C] \text{ are } \text{lb-sec/ft.})$</td>
</tr>
</tbody>
</table>

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TABLE J-2.  +Y SOLAR ARRAY, (BENDING ONLY)

**STIFFNESS MATRIX**
(Units = lbs./ft.)

\[
[K] \{\ddot{x}\} = \begin{bmatrix}
.008670 & -.010705 \\
-.010705 & 1.37791
\end{bmatrix} \begin{bmatrix}
\ddot{x}_2 \\
\ddot{z}_2
\end{bmatrix}
\]

For motion relative to drive motor

**MASS MATRIX**
(Units = slugs (lb-sec^2/ft))

\[
[M] (\ddot{x}) = \begin{bmatrix}
.42176 & 0 \\
0 & .52608
\end{bmatrix} \begin{bmatrix}
\ddot{x}_2 \\
\ddot{z}_2
\end{bmatrix}
\]

**DAMPING MATRIX**
(Units = (lb-sec/ft))

\[
[C] (\ddot{x}) = \begin{bmatrix}
.0006047 & 0 \\
0 & .008514
\end{bmatrix} \begin{bmatrix}
\ddot{x}_2 \\
\ddot{z}_2
\end{bmatrix}
\]
**TABLE J-3. NTS-2 SOLAR ARRAYS**

**COMPARISON OF MAXIMUM OUTER PANEL DEFLECTIONS**

**FOR STEP INPUT YAW TORQUE OF .04 FT-LBS FOR**

**3 SECONDS, RESPONSES ARE NORM. TO PANEL, AT CENTER**

<table>
<thead>
<tr>
<th>DYNREI RUN NO.</th>
<th>MAX. REL. RESPONSE, NODE 2 INCHES</th>
<th>TIME OF MAX. RESPONSE, SECONDS</th>
<th>MODE NOS. INCLUDED IN SOLUTION</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSNSTCO</td>
<td>-.009843</td>
<td>1.40</td>
<td>10, 14, 17, 24 (Tape X3350)</td>
<td>Includes 1st and 2nd anti- symmetric array bending modes</td>
</tr>
<tr>
<td>TSNSTI1</td>
<td>-.009855</td>
<td>1.40</td>
<td>10, 14, 17 (Tape X3350)</td>
<td>2nd anti-symmetric array mode deleted</td>
</tr>
<tr>
<td>TSNSTJO</td>
<td>-.009916</td>
<td>1.40</td>
<td>10, 14, 17</td>
<td>Simplified bending model</td>
</tr>
</tbody>
</table>
Figure J-1. Sturdyne Math Model for Flexible Solar Arrays
Figure J-2. Gravity Gradient Rod and Spacecraft Definition
Figure J-3. Solar Array Bending Flexure Model
Figure J-4. Dynamic Response of Outer Panel (Node 2) to Step Yaw Torque Input of \(0.04\) ft-lbs for 3 seconds.
MOTOR LIES ON +X AXIS
△ TO C.G. OF SPACECRAFT
△ FROM BBR DWG NO. 49006 DATED 6/17/75
△ NRL DWG NO. 144
△ NRL DWG NO. 136

Figure J-5. Solar Array Drive Motor
Figure J-6. Stardyne Model Mode 2, θY Response vs. Time
Figure J-7. Solar Array Torsion Model, 4-Pulse, Input Forcing Function (Normalized)
Figure J-8. Solar Array Torsion Model, Node 2 = Outer Panel Disp., Vel., & Accel. Response in Radian Units to Four-Pulse Excitation
Figure J-9. Solar Array Torsion Model Response at Drive Motor to 4 Pulses Applied to Drive Motor
Figure J-10. Solar Array Torsion Model Response of Rigid Spacecraft to 4 Pulses
Figure J-11. Response of -Z Gravity Rod Tip Mass (Node 48)
X Direction Response (Inch Units) Relative to Spacecraft Attach Point, 4 Pulses on Solar Array Drive Motor
Figure J-12. Mode Plots—Torsion (First Order Modes)
Figure J-13. Mode Plots--Torsion (Second Order Modes)
TABLE J-4. STARDYNE MODAL DATA: FLEXIBLE RODS—2 BEAMS/ROD  
FLEXIBLE ARRAYS—5 DOF/PANEL (EACH PANEL)

<table>
<thead>
<tr>
<th>Ref. TNSSTRL</th>
<th>Modes on Tape X3350</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIGIF. MODES</td>
<td>MODE NO.</td>
</tr>
<tr>
<td>FOR ARRAY/ROD</td>
<td>TOTAL ELAS.</td>
</tr>
<tr>
<td>RESPONSES</td>
<td></td>
</tr>
<tr>
<td>Major Rod</td>
<td>7 0</td>
</tr>
<tr>
<td>Modes</td>
<td>8 1</td>
</tr>
<tr>
<td></td>
<td>9 2</td>
</tr>
<tr>
<td>Minor Rod</td>
<td>10 3</td>
</tr>
<tr>
<td>Modes</td>
<td>11 4</td>
</tr>
<tr>
<td></td>
<td>12 5</td>
</tr>
<tr>
<td></td>
<td>13 6</td>
</tr>
<tr>
<td>Minor Rod Modes</td>
<td>14 7</td>
</tr>
<tr>
<td></td>
<td>15 8</td>
</tr>
<tr>
<td></td>
<td>16 9</td>
</tr>
<tr>
<td>Row Coupling</td>
<td>17 16</td>
</tr>
<tr>
<td>(Pitch)</td>
<td>18 11</td>
</tr>
<tr>
<td>Major Array</td>
<td>19 12</td>
</tr>
<tr>
<td>Torsion</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20 13</td>
</tr>
<tr>
<td></td>
<td>21 14</td>
</tr>
<tr>
<td></td>
<td>22 15</td>
</tr>
<tr>
<td>Minor Array</td>
<td>23 16</td>
</tr>
<tr>
<td>Bending</td>
<td>24 17</td>
</tr>
<tr>
<td></td>
<td>25 18</td>
</tr>
<tr>
<td>Major Array</td>
<td>26 19</td>
</tr>
</tbody>
</table>
# Table J-5. StarDyne Modal Data: Flexible Rods—2 Beams/Rod Flexible Arrays—4 DOF/Array (Each Array)

<table>
<thead>
<tr>
<th>Run</th>
<th>TSNST17 on Type X3719</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SIGIF. MODES FOR ARRAY/ROD RESPONSES</strong></td>
<td><strong>MODE NO.</strong></td>
</tr>
<tr>
<td></td>
<td><strong>TOTAL ELAS.</strong></td>
</tr>
<tr>
<td>Major Rod</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>9</td>
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<tr>
<td></td>
<td>10</td>
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<tr>
<td></td>
<td>11</td>
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<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>13</td>
</tr>
<tr>
<td>Minor Rod</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>16</td>
</tr>
<tr>
<td>Array Bending</td>
<td>17</td>
</tr>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Array Bending</td>
</tr>
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<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Major Array</td>
<td>19</td>
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<td></td>
<td></td>
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</tbody>
</table>
SOLAR ARRAY DRIVE TORQUE PULSE

Drive torque vs. time is as follows:

Torque pulses repeat at 2/second.

\[ T \]

Torque, ft-lb

\[ a = \begin{cases} \text{1.0 ft-lb} & \text{50 m sec} \\ 0 & \text{otherwise} \end{cases} \]

\[ \text{t, TIME} \]

\[ t, \text{milliseconds} \]

Figure J-15. Drive versus Torque

\( a \) is defined as level of stopping torque required to kill any rate built up during positive part of torque cycle.

\[ \dot{\theta} = \frac{T}{I} \quad \text{for unit I} \]

\[ \ddot{\theta} = \frac{\dot{\theta}}{t} \]

after .050 sec.

\[ \dot{\theta} = \frac{12.0 \text{ in-lb}}{1.0} \times 0.050 = 0.60 \text{ rad/sec} \]
To kill of \(0.60\) rad/sec in 325 msec.

\[
\ddot{\theta} = \frac{T_B}{I}
\]

\[
\dot{\theta} = \dot{\theta}_0 - \ddot{\theta} \cdot t
\]

\[
0 = 0.60 - \frac{T_B}{I} (0.325)
\]

\[
T_B = \frac{0.60}{0.325} = \frac{1.846154}{\text{in-lb.}}
\]

\[
\frac{T_B}{I} = \frac{-1.846154}{12} = -0.1538462
\]
NTS-2 SOLAR ARRAY STRUCTURAL ANALYSIS

- A linear Open Loop model of total NTS-2 spacecraft was run on STARDYNE.
  - All sub-structure stiffnesses combined.
  - Combined with previous defined beam models (2 beams/rod) of gravity gradient rods.
  - Above model was reduced to varying degrees of complexity by selecting fewer lumped mass points on solar arrays.

RESULTS

- Solar array flexible torsion model should have 4 D.O.F. + drive motor $\theta_Y$.
- Solar array flex. bending model can have 2 D.O.F per array with good results, 4 D.O.F per array with better results.
- Open-Loop transient response runs to be made to guide decision on bending model.
NTS-2 SOLAR ARRAY STRUCTURAL ANALYSIS

FOR EACH ARRAY, CONSIDERED

- 3 Solar Panels (identical)
- 1 Interface Panel
- 4 Hinges (Assumed identical)
- Drive Tube \((L = 35.53\ \text{in.}, -Y\ \text{Array})\)
  \((L = 14.87\ \text{in.}, +Y\ \text{Array})\)

Each identical structure was modeled in detail by finite element analysis (STARDYNE), and reduced to a sub-structure stiffness matrix element.

Substructures for \(-Y\) array checked out for comparison with NRL vibration test

- When gravity field stiffness included, natural frequency in \(X\) and \(Y\) agreed well.
- Model is somewhat stiffer in bending in \(Z\) direction (plane of panel) due to lack of defn. of brkt. at end of drive tube.
Figure J-18. NTS-2 Solar Panel Model
Figure J-19. Stardyne Model NTS-2 Hinge
Figure J-20. NTS-2 Solar Array Interface Panel Model
TABLE J-6. SUMMARY OF SOLAR ARRAY CANTILEVER VIBRATION TEST RESULTS @ NRL VS. STARDYNE MODEL (GRAVITY VECTOR ALONG Y AXIS)

<table>
<thead>
<tr>
<th></th>
<th>1ST TORSION FREQ. ( \frac{\theta}{J} ) Hz</th>
<th>1ST X BENDING FREQ., NORMAL TO PANELS ( H_z ) Hz</th>
<th>1ST Z BENDING FREQ. ( H_z ) Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>INFORMAL NRL TEST DATA, (REF. LARRY TURNER)</td>
<td>.40</td>
<td>.44</td>
<td>.68</td>
</tr>
<tr>
<td>STARDYNE, ORIG. HINGE STIFF. I G FIELD</td>
<td>.408</td>
<td>.48</td>
<td>2.07</td>
</tr>
<tr>
<td>STARDYNE, MODIFIED HINGE STIFF. I G FIELD</td>
<td>.384</td>
<td>.47</td>
<td>2.05</td>
</tr>
<tr>
<td>STARDYNE, ORIG. HINGE STIFF &quot;ZERO G&quot;</td>
<td>.408</td>
<td>.28</td>
<td>2.04</td>
</tr>
<tr>
<td>STARDYNE, MODIFIED HINGE &quot;ZERO G&quot;</td>
<td>.384</td>
<td>.26</td>
<td>2.03</td>
</tr>
<tr>
<td>MODEL DESCRIPTION (ARRAYS)</td>
<td>MAJOR ROD MODE, PITCH</td>
<td>1ST ARRAY BENDING MODE, YAW</td>
<td>MAJOR ARRAY TORSION MODE</td>
</tr>
<tr>
<td>---------------------------</td>
<td>------------------------</td>
<td>-------------------------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td></td>
<td>FREQ. Hz</td>
<td>GENR'LZD WT., LBS</td>
<td>FREQ. Hz</td>
</tr>
<tr>
<td>6 DOF/PANEL</td>
<td>.0263</td>
<td>19.56</td>
<td>.366</td>
</tr>
<tr>
<td>5 DOF/PANEL</td>
<td>.0264</td>
<td>19.97</td>
<td>.366</td>
</tr>
<tr>
<td>6 DOF/ARRAY (3 each @ Inner Outer)</td>
<td>.0264</td>
<td>19.96</td>
<td>.3499</td>
</tr>
<tr>
<td>4 DOF/ARRAY (2 Torsion) (2 Bending)</td>
<td>.0264</td>
<td>19.97</td>
<td>.3528</td>
</tr>
<tr>
<td>DOF/PANEL</td>
<td>SECOND ROD PITCH MODE</td>
<td>SECOND ARRAY BENDING MODE, YAW</td>
<td>SECOND/3RD ARRAY TORSION MODE</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------------</td>
<td>--------------------------------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td></td>
<td>FREQ. Hz</td>
<td>GENR'LZD WT., LBS</td>
<td>FREQ. Hz</td>
</tr>
<tr>
<td>6 DOF/PANEL</td>
<td>.1485</td>
<td>.872</td>
<td>1.979</td>
</tr>
<tr>
<td>5 DOF/PANEL</td>
<td>.1425</td>
<td>.92</td>
<td>1.978</td>
</tr>
<tr>
<td>6 DOF/ARRAY</td>
<td>.1425</td>
<td>.92</td>
<td>2.224</td>
</tr>
<tr>
<td>3 each @ Inner Outer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 DOF/ARRAY</td>
<td>.1425</td>
<td>.92</td>
<td>(——)</td>
</tr>
<tr>
<td>2 Torsion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Bending</td>
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</tbody>
</table>
### TABLE J-9. STARDYNE MODAL DATA: FLEXIBLE RODS—4 BEAMS/ROD
FLEXIBLE ARRAYS—6 DOF/PANEL

<table>
<thead>
<tr>
<th>Run</th>
<th>TSNSTQ</th>
<th>SIGCF, MODES FOR SOLAR ARRAY RESPONSES</th>
<th>MODE NO.</th>
<th>NAT. FREQ. Hz</th>
<th>CENTRIZED WEIGHT LBS</th>
<th>MODAL ROTATIONS OF SPACECRAFT C.G. (NODE 50 RADIANS)</th>
<th>MODE SHAPE DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>TOTAL &quot;TAS.&quot;</td>
<td></td>
<td></td>
<td></td>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>.00213</td>
<td>13.83</td>
<td>-.00103</td>
<td>.00011</td>
<td>-.00042</td>
<td>X2</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>.00516</td>
<td>10.20</td>
<td>.00037</td>
<td>.00002</td>
<td>-.00013</td>
<td>X2</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>.00671</td>
<td>16.21</td>
<td>.00089</td>
<td>.1x10^-6</td>
<td>.3x10^-6</td>
<td>X1</td>
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<tr>
<td>10</td>
<td>3</td>
<td>.01866</td>
<td>52.13</td>
<td>.00762</td>
<td>5.x10^-6</td>
<td>-1.4.x10^-6</td>
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<tr>
<td>11</td>
<td>4</td>
<td>.02635</td>
<td>19.36</td>
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<td>-0.00626</td>
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<td>X1</td>
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<tr>
<td>12</td>
<td>5</td>
<td>.1449</td>
<td>.828</td>
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<td>.8x10^-8</td>
<td>-.000011</td>
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<td>-.5x10^-6</td>
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<td>-.00005</td>
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<td>.0024</td>
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<td>11</td>
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<td>2.x10^-7</td>
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<td>19</td>
<td>12</td>
<td>.4592</td>
<td>.83</td>
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<td>4.8x10^-6</td>
<td>.87x10^-7</td>
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<td>13</td>
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<td>.71</td>
<td>7.1x10^-6</td>
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<td>.99x10^-7</td>
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<td>.71</td>
<td>1x000049</td>
<td>.9x10^-7</td>
<td>2.8x10^-6</td>
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<td>.6605</td>
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(Continued on Next Page)
TABLE J-9. STARDYNE MODAL DATA (concluded)

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<th>SIGIF. MODES FOR SOLAR ARRAY RESPONSES</th>
<th>MODE NO.</th>
<th>NAT. FREQU. TOTAL ELAS.</th>
<th>GEN'RLZD WEIGHT (LBS.)</th>
<th>MODAL ROTATIONS OF SPACECRAFT C.G. (NODE 50) RADIANS</th>
<th>MODE SHAPE DESCRIPTION</th>
<th>UNSYM.</th>
<th>SYMM.</th>
<th>COMMENTS</th>
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</thead>
<tbody>
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<td>High Array Response</td>
<td>27</td>
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<td>1.0954</td>
<td>928.4</td>
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<td></td>
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<td>High Array Response</td>
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<td>21</td>
<td>1.235</td>
<td>8126.8</td>
<td>-0.000024 0.3x10^-8 -0.17x10^-6</td>
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<td>&quot; &quot; &quot;</td>
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<tr>
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<td>22</td>
<td>1.898</td>
<td>6971.1</td>
<td>0.13x10^-7 0.3x10^-6 0.000055 0.1x10^-8</td>
<td>X1</td>
<td>&quot; &quot; &quot;</td>
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<td>23</td>
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<td>7510.1</td>
<td>-0.1x10^-6 0.53x10^-6 0.000002 0.000027</td>
<td>X1</td>
<td>2nd Array Torsion 1st Symm.</td>
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</tr>
<tr>
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<td>31</td>
<td>24</td>
<td>1.979</td>
<td>11.95</td>
<td>0.11x10^-6 -0.33x10^-6 0.000027</td>
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<td>2nd Array Torsion 2nd Unsymm.</td>
<td></td>
</tr>
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<td>25</td>
<td>2.007</td>
<td>11.95</td>
<td>0.47x10^-6 0.000089 0.000315</td>
<td>X1</td>
<td>2nd Array Bending</td>
<td></td>
</tr>
<tr>
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<td>26</td>
<td>2.422</td>
<td>19.54</td>
<td>0.00233 0.000337 0.0001</td>
<td>X3</td>
<td>1st Array Z Bending, In Plane of Panels</td>
<td></td>
</tr>
<tr>
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<td>27</td>
<td>4.286</td>
<td>22.64</td>
<td>0.00614 0.000015 0.000028</td>
<td>X3</td>
<td>1st Array Z Unsymm.</td>
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### TABLE J-10. STARDYNE MODAL DATA: FLEXIBLE RODS—2 BEAMS/ROD  
FLEXIBLE ARRAYS—3 DOF AT INNER, OUTER PANELS

<table>
<thead>
<tr>
<th>SICIF. MODES FOR ARRAY/ROD RESPONSES</th>
<th>MODE NO. TOTAL ELAS.</th>
<th>NAT. FREQ. ( h_2 )</th>
<th>GEN’RLZD WEIGHT LB.</th>
<th>MODAL ROTATIONS OF SPACECRAFT C.G. (NODE 50) RADIANS</th>
<th>MODE SHAPE DESCRIPTION</th>
<th>UNSYM.</th>
<th>SYMM.</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td></td>
<td></td>
<td></td>
<td>( x_{x10^6} )</td>
<td>( y_{x10^6} )</td>
<td>( z_{x10^6} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minor Rod</td>
<td>7 0</td>
<td>.00025</td>
<td>91.0</td>
<td>16.5</td>
<td>48.4</td>
<td>-1624.</td>
<td></td>
<td></td>
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<tr>
<td>Major Rod</td>
<td>8 1</td>
<td>.00556</td>
<td>15.61</td>
<td>0.7</td>
<td>-15</td>
<td>-51.4</td>
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<td></td>
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<td>Modes</td>
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<td>.00558</td>
<td>15.72</td>
<td>0.131</td>
<td>0.07</td>
<td>-22.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minor Rod</td>
<td>10 3</td>
<td>.01889</td>
<td>52.19</td>
<td>22.0</td>
<td>.19</td>
<td>.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modes</td>
<td>11 4</td>
<td>.02637</td>
<td>19.96</td>
<td>.17</td>
<td>-702.0</td>
<td>.05</td>
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<td></td>
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<td>Minor Rod</td>
<td>12 5</td>
<td>.1388</td>
<td>.87</td>
<td>.002</td>
<td>- .07</td>
<td>- .01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modes</td>
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<td>.1389</td>
<td>.87</td>
<td>.09</td>
<td>- .001</td>
<td>-10.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minor Rod</td>
<td>14 7</td>
<td>.1406</td>
<td>.89</td>
<td>- 169.</td>
<td>- .001</td>
<td>- .43</td>
<td></td>
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</tr>
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<td>Modes</td>
<td>15 8</td>
<td>.1425</td>
<td>.92</td>
<td>-.005</td>
<td>36.1</td>
<td>- .01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Array Couples w/Yaw (Bending)</td>
<td>16 9</td>
<td>.2670</td>
<td>21.26</td>
<td>.31</td>
<td>-25.6</td>
<td>-32.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Major Array Torsion</td>
<td>17 10</td>
<td>.3499</td>
<td>30.59</td>
<td>- 49.2</td>
<td>-1.9</td>
<td>5095</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Array Couples w/Yaw (Bending)</td>
<td>18 11</td>
<td>.6777</td>
<td>64.30</td>
<td>-.38</td>
<td>-.003</td>
<td>20.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Major Array Torsion</td>
<td>19 12</td>
<td>1.095</td>
<td>928.4</td>
<td>-.021</td>
<td>.32</td>
<td>.009</td>
<td></td>
<td></td>
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<tr>
<td>Minor Array Bending</td>
<td>20 13</td>
<td>1.2525</td>
<td>643.2</td>
<td>.038</td>
<td>.001</td>
<td>1.71</td>
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<td></td>
</tr>
<tr>
<td>Array Torsion Outer ends 180° out of Phage w/inner panels</td>
<td>21 14</td>
<td>2.2244</td>
<td>11.92</td>
<td>7.3</td>
<td>-105.8</td>
<td>-464.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minor Array Bending</td>
<td>22 15</td>
<td>2.2773</td>
<td>11.69</td>
<td>-7.6</td>
<td>-49.6</td>
<td>1021.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Major Array In-Plane Bending</td>
<td>23 16</td>
<td>2.4272</td>
<td>19.01</td>
<td>227.2</td>
<td>-36.8</td>
<td>9.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Major Array In-Plane Bending</td>
<td>24 17</td>
<td>4.2345</td>
<td>21.57</td>
<td>-5701</td>
<td>-16.4</td>
<td>-29.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

X1 1st Rod Bending  
X2 2nd Rod Bending  
X3 1st Array Bending  
X4 2nd Array X Bending  
X5 1st Array X3 Bending (In-Plane of Array)
\[ C x \{ K \theta \} = C x K \text{ in FT-LB/RADIAN} \]

where \( C = \frac{1}{12} \)

\[
\begin{bmatrix}
211.065 \\
-211.131 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
352.756 \\
0 \\
-141.625 \\
-141.788 \\
0
\end{bmatrix}
\begin{bmatrix}
\theta Y_1 \\
\theta Y_2 \\
\theta Y_3 \\
\theta Y_4
\end{bmatrix}
\]

(Final model is a modal solution using the more detailed structural model for torsion.)

Figure J-21. Solar Array Torsion Flexure Model
APPENDIX K

MODIFIED NTS-2 SOLAR ARRAY TORSION MODEL

Hinge stiffness increased from 326. to 92,000. in-lb/rad. Drive tube O.D. increased from 1.25 to 1.75 inches. Drive tube wall thickness increased from .049 to .063 inches. (Above information from Larry Turner, NRL by telephone calls 1/18/77 and 1/26/77.)

Previous model of solar arrays solved for torsion only (degree of freedom 5 = 6Y).

For existing S&R simulation model, update as follows:

<table>
<thead>
<tr>
<th>Mode No. (Symmetric Only)</th>
<th>Meq Equiv. Weight M_eq lb-in^2</th>
<th>Meq Equiv. Mass Slug-ft^2</th>
<th>Freq. Hz</th>
<th>( \omega_n ) rad/sec</th>
<th>( \omega_n^2 ) rad^2/sec^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13944.</td>
<td>3.007</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 (3)</td>
<td>7152.5</td>
<td>1.5426</td>
<td>7.9635</td>
<td>50.036</td>
<td>2503.6</td>
</tr>
<tr>
<td>3 (5)</td>
<td>6289.1</td>
<td>1.3563</td>
<td>15.5315</td>
<td>97.587</td>
<td>9523.3</td>
</tr>
</tbody>
</table>

The differential equation of motion of the solar array drive system (open loop) is given in terms of the generalized coordinates, q, as:

\[
I \ddot{q} + 2 \rho \omega_n \dot{q} + \omega_n^2 q = \frac{T}{M_{eq}}[T(t)]
\]

For \( \rho = 0.005 \) (½ percent critical damping), the generalized equation reduces to
This replaces Eq. 6 in previous memo. The discrete responses of several locations of interest are taken from the mode shapes and given below:

<table>
<thead>
<tr>
<th>LOCATION</th>
<th>STARDYNE MODE NO.</th>
<th>( \theta_y ) Rotation (Radians) per Unit ( q ) (Rigid Body)</th>
<th>( \theta_y ) Mode 1</th>
<th>( \theta_y ) Mode 2</th>
<th>( \theta_y ) Mode 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive Motor</td>
<td>20</td>
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<td>-1.0</td>
<td>-0.8856</td>
</tr>
<tr>
<td>Tip of (-Y) outer panel</td>
<td>1</td>
<td></td>
<td>1.0</td>
<td>1.0</td>
<td>-0.8856</td>
</tr>
<tr>
<td>Pitch sensor</td>
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<td></td>
<td>1.0</td>
<td>0.5683</td>
<td>0.32105</td>
</tr>
<tr>
<td>Tip of (+Y) outer panel</td>
<td>32</td>
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<td>1.0</td>
<td>1.0</td>
<td>-0.8856</td>
</tr>
</tbody>
</table>
Figure K-1. Mode Shapes for Solar Array Stiff Hinges

First Symmetric Torsion Mode
f = 7.96 Hz

Second Symmetric Torsion Mode
f = 15.53 Hz
### TABLE K-1. RIGID BODY TORSION MODE

<table>
<thead>
<tr>
<th>MODE NUMBER</th>
<th>FREQUENCY</th>
<th>GENERALIZED WEIGHT</th>
<th>MAXIMUM ROTATION IS AT NODE 1</th>
<th>MODAL PARTICIPATION FACTOR (X1) = 0.</th>
<th>GEN.WT. TIMES MODAL PART. FACT. (X1) = 0.</th>
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<td>X2</td>
<td>X3</td>
<td>X4</td>
<td>X5</td>
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<td>0.00000000</td>
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<tr>
<td>2</td>
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<td>0.00000000</td>
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<td>0.00000000</td>
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<td>0.00000000</td>
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### TABLE K-5. 2ND SYMMETRIC TORSION MODE

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<tr>
<th>MODE NUMBER</th>
<th>FREQUENCY</th>
<th>MODE SHAPE (EIGENVECTOR)</th>
<th>GENERALIZED WEIGHT</th>
<th>MAXIMUM ROTATION % AT NOUE</th>
<th>MAXIMUM TRANSLATION % AT NOUE</th>
<th>K/M</th>
<th>K/N</th>
<th>K/L</th>
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</table>

**MODAL PARTICIPATION FACTOR (x1) = 0.**

**MODAL PARTICIPATION FACTOR (x2) = 0.**

**MODAL PARTICIPATION FACTOR (x3) = 0.**
### TABLE K-6. NODAL COORDINATE TABLE

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<td>-1.203090E+04</td>
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</table>

*Warning* nodes 13 and 15 have identical coordinates.

--- MAXIMUM ALLOWABLE NODE NUMBERS BY ANALYSIS TYPE ---

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<tr>
<td>Substructure</td>
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Largest node number coded in this model = 33

**Next Table Header** (WSTG)

---
APPENDIX L
APPENDIX L

SOLAR ARRAY TORSION MODES,
SOLUTION FOR RESPONSE TO TORQUE
MOTOR ANGULAR ACCELERATIONS

SUMMARY INPUT DATA

Based on updated solar array hinge torsional stiffness, the arrays are
assumed to have symmetric responses for a rigid torque motor connection to
the spacecraft.

The modes of interest are obtained from STARDYNE finite element structural
model, using only the torsional masses and spring stiffness data, including
updated hinge stiffness. For a locked-up or grounded torque motor, the
following data is obtained for the two modes of interest:

<table>
<thead>
<tr>
<th>Mode No. (STARDYNE)</th>
<th>Mode No. (S &amp; R)</th>
<th>Equiv. Weight ( M_{eq} )</th>
<th>( M_{eq}^{\text{eq}} )</th>
<th>Freq. ( \omega_n )</th>
<th>( \omega_n^2 )</th>
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<td>2</td>
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<td>2.17225</td>
<td>2.5908</td>
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<td>7719.405</td>
<td>1.6648</td>
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</table>

The differential equations of motion of the motor array drive system in
terms of the generalized coordinates \( q \) is given by:

\[
\begin{bmatrix} I \end{bmatrix} \dddot{q} + \begin{bmatrix} 2 \rho \omega_n \end{bmatrix} \dot{q} + \begin{bmatrix} \omega_n^2 \end{bmatrix} q = \begin{bmatrix} -\frac{T}{M_{eq}} \end{bmatrix} \beta
\]  

(L-1)

where

\( \dddot{\beta} \) = angular acceleration of drive motor (absolute)

\( \dddot{\beta} = \sum_{\text{arrays}} \text{Torques applied to drive motor, not} \)  

\( \sum_{\text{arrays, total}} \text{pitch, total} \)  

\( \dddot{\beta} \text{spacecraft/pitch acceleration} \)  

(L-2)
Note that if torque motor is moving and torque is applied to cause $+ \vec{\beta}_{\text{arrays}}$, then a negative torque is applied to the spacecraft.

If
\[
I_P \cdot \beta_{\text{sc}} = \text{pitch moment of inertia of spacecraft, without arrays}
\]
\[
\beta_{\text{sc}} = -\left( \sum \text{Torques applied to drive motor} + \text{all other pitch torques} \right) / \left( I_{P_{\text{sc}}} \right)
\]

Also note that the net torque applied to drive motor = $T_{\text{applied}} - T_{\text{brake torque}}$

\[
\beta = \frac{\text{Drive motor Torque}}{I_{\text{array}}} + \theta_A
\]

\[
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2
\end{bmatrix}
+ \begin{bmatrix}
.16278 & 0 \\
0 & .56774
\end{bmatrix}
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2
\end{bmatrix}
+ \begin{bmatrix}
264.987 & 0 \\
0 & 3223.28
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_2
\end{bmatrix}
= \begin{bmatrix}
-.8076 \\
-.2076
\end{bmatrix}

\]

\[
\theta_{\text{pitch sensor}} = .95183 q_1 - .46754 q_2
\]

\[
\theta_A = \frac{T_m}{I} + \dot{\beta}_{\text{sc}}
\]

For $\rho = 0.005$ (½ percent critical damping) the generalized torsional equations of motion become

\[
\begin{bmatrix}
1. & 0. \\
0 & 1.
\end{bmatrix}
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2
\end{bmatrix}
+ \begin{bmatrix}
.16278 & 0 \\
0 & .56774
\end{bmatrix}
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2
\end{bmatrix}
+ \begin{bmatrix}
264.987 & 0 \\
0 & 3223.28
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_2
\end{bmatrix}
= \begin{bmatrix}
-.8076 \\
-.2076
\end{bmatrix}
\]

\[
\beta (L-4)
\]

197
The discrete responses of several locations of interest on the solar panels are taken from the mode shapes and the generalized coordinate responses

\[
\{\theta\} = [\phi] \{q\} + \beta
\]  

where \( \beta \) = absolute angular displ. of drive motor

<table>
<thead>
<tr>
<th>STARDYNE MODE NO.</th>
<th>LOCATION</th>
<th>( \phi ) MATRIX TERMS</th>
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</thead>
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</tr>
<tr>
<td>3</td>
<td>Pitch sensor</td>
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</tr>
<tr>
<td>32</td>
<td>Tip of (+Y) outer panel</td>
<td>1.000 \hspace{1em} -.98418</td>
</tr>
</tbody>
</table>
### Table 1-1: NTS-2 Solar Array Topology—Based on Anti-Symmetric Modes of Free-Free Model with Stiff Hinges

<table>
<thead>
<tr>
<th>NODE No. n</th>
<th>( M_n ) in lb-in²</th>
<th>( \phi_n,2 )</th>
<th>( \phi_n,4 )</th>
<th>( M )</th>
<th>( M_{eq} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>581.</td>
<td>1.000</td>
<td>-.98418</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1162.</td>
<td>.98788</td>
<td>-.83913</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>581.</td>
<td>.95183</td>
<td>-.46754</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>581.</td>
<td>.93482</td>
<td>-.28296</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1162.</td>
<td>.87591</td>
<td>.21696</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>581.</td>
<td>.79577</td>
<td>.65293</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>581.</td>
<td>.76368</td>
<td>.77438</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1162.</td>
<td>.66465</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>581.</td>
<td>.54951</td>
<td>.93087</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-</td>
<td>-.00457</td>
<td>-.00774</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \Sigma = 581 \times 4.99561 + 1162 \times 2.52844 \]

\[ \Sigma = 581 \times .6235 + 1162 \times .37783 \]

\[ \left[ \begin{array}{c} - \phi h \end{array} \right] M = \frac{406.7}{10072.3} = \frac{-40378 \times 2}{7719.4} = 0.8076 \]

Note: Generalized forcing function for base excitation \( \left[ \frac{T}{M} \right] \) is independent of units used for mass, as long as \( [M] \) (mass matrix), and \( [M_{eq}] \), generalized mass have same units.

For 8 side only = 4067. = 801.3

For both sides
APPENDIX M

COMBINED EARTH SENSOR MODEL

INTRODUCTION

This appendix describes the model developed for the GPS combined earth sensor. The model can be used to evaluate acquisition as well as normal operation.

This model is based on information obtained from Barnes Engineering Company at a meeting in St. Petersburg, Florida on March 13, 1975, and a subsequent phone call, and the Rockwell procurement specification for the sensor.

SENSOR DESCRIPTION

The Combined Earth Sensor is a two-axis, static sensor which incorporates two horizon crossing indicators for spinning mode attitude sensing. The adjective static means the sensor uses no moving parts in the measurement of attitude.

The two-axis sensor consists of an optical system, 24 thermopile detectors, and processing electronics. The 24 detectors are arranged on the focal surface of the optical system. The earth image is tangent to the inner edges of the A detectors at null at the design altitude. Two roll and two pitch measurements are computed sequentially by switching sums and differences of various detector outputs to the input of an integrator. Each measurement is sampled and held for comparison check and possible output. A new roll and new pitch measurement is computed each 1/4 second. Roll and pitch computations are separated by 1/5 second. The S detectors are for space reference and are used with the A detectors for accurate
measurements over $+1^\circ$ in the track mode. The $A$ and $B$ detectors are used in the acquisition mode for $+4^\circ$ range. The thermopile detectors have a time constant of 1.4 seconds and are sensitive to the 14 to 16 micron wavelengths (CO$_2$ band). The sensor continuously tests all signals and rejects spurious or incorrect signals due to sun, moon, or other anomalies.

The two horizon crossing indicators consist of 3 degree by 3 degree square fields of view. One diagonal of each square is parallel to the scan path. One field is $10^\circ$ from the $-X$ axis toward $+Z$ and the other is $10^\circ$ from $-X$ toward $-Z$ (Spin rotation is about the $Z$ axis). The detectors are pyroelectric with insignificant time constant.

MODEL DESCRIPTION

The model consists of an initialization and two normal operation sections. The initialization section performs one-time computations and clears outputs and counters. One normal operation section is for the horizon crossing indicators used in the spin phase. The other normal operation section is for two-axis earth sensing.

Characteristics of the horizon crossing indicator model are as follows:

- Inputs to the model are spacecraft position vector from center of earth, and spacecraft attitude direction cosine matrix. Both inputs must be relative to the same reference frame.
- When a sensor line of sight crosses the edge of the earth, the time from the crossing to the sample time is output. If no crossing occurs in the sample interval, the output is set to $-\Delta t$.
- If a sensor line of sight crosses the earth very close to a tangent, there may not be an output. The sample, $\Delta t$, should be small.
enough so that this loss of output does not happen when attitude
determination is critical. Select $\Delta t$ as follows:

$$\Delta t \leq \frac{1}{W_s} \sqrt{1 - \frac{r_e}{r}} \tan^{-1}\left[\frac{1}{2} \frac{h}{r} \frac{1}{r_e} \left(1 - \frac{h}{r_e}\right) - 1\right]$$

where

$W_s$ = spin rate in rad/sec

$= \frac{2\pi}{60} W_{RPM}$

$W_{RPM}$ = spin rate in rev/min

$h$ = satellite altitude in n. mi. ($100 \leq h \leq 10897$)

$r_e$ = earth radius in n. mi. ($3440$)

$r$ = distance from earth center to chord of detector

path in n. mi.

e.g.

let

$W_{RPM} = 100$

$h = 10897$

$r_e = 3440$

$r = 3405.6 = 0.99 r_e$

then $\Delta t \leq 0.00326$ seconds.
The equations used in the flow diagram are derived below.

\[ r_e = 3440 \text{ n.m.i.} \]

\[ \text{LOS} \] is a unit vector

\[ \frac{\mathbf{R} \times \text{LOS}}{\mathbf{R}} = \sin(\text{ANG2}) \]

\[ \frac{\mathbf{R} \cdot \text{LOS}}{\mathbf{R}} = -\cos(\text{ANG2}) \]

\[ \text{ANG2} = \tan^{-1} \frac{\sin(\text{ANG2})}{\cos(\text{ANG2})} \]

\[ \text{ANG2} = \tan^{-1} \frac{\mathbf{R} \times \text{LOS}}{\mathbf{R} \cdot \text{LOS}} \]
\[\text{ANG1} = \sin^{-1} \frac{r_e}{|\mathbf{r}|} = \tan^{-1} \frac{r_e/|\mathbf{r}|}{\left[1 - (r_e/|\mathbf{r}|)^2\right]^{1/2}} = \tan^{-1} \frac{r_e}{\left[|\mathbf{r}|^2 - r_e^2\right]^{1/2}}\]

\[\text{ANG3} = \text{ANG2} - \text{ANG1}\]

Note that \(3440 + 100 < |\mathbf{r}| < 10897 + 3440\) n. mi. \(13.8^\circ < \text{ANG1} < 76.4^\circ\).

Characteristics of the two-axis earth sensor model are as follows:

- The sensor output is updated every 1/8 second.
- The sensor model has four states which are:
  - State 1: Compute Roll 1, Output Pitch 2 if valid
  - State 2: Compute Pitch 1, Output Roll 1 if valid
  - State 3: Compute Roll 2, Output Pitch 1 if valid
  - State 4: Compute Pitch 2, Output Roll 2 if valid

The state is advanced every 1/8 second.

- Each detector area is divided into \(1^\circ \times 1^\circ\) squares. The area coverage of each \(1^\circ \times 1^\circ\) square by the earth is computed from the distance between center of the earth circle on the focal surface and the center of the square. The orientation of the square is not considered.

- The position of the earth image on the focal surface (B&C) is computed from true attitude. The diameter of the earth image on the focal surface (RE) is computed from satellite altitude. Measurements on the focal surface are in equivalent radians.
• Each detector output is the sum of the coverages of 1° x 1° squares within the detector.

• The sensor output is computed from the detector outputs according to the functions in the electronics.

• Functions of the system programmer to reject spurious or incorrect signals is included.

• Subroutine must be initialized by calling with MODE = 0 prior to operating in two-axis mode.

• The time increment, \( \Delta t \), should be selected from one of the following to synchronize with the sensor update frequency:

<table>
<thead>
<tr>
<th>Recommended ( \Delta t )</th>
<th>Increments per Update</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.125</td>
<td>1</td>
</tr>
<tr>
<td>0.0625</td>
<td>2</td>
</tr>
<tr>
<td>0.025</td>
<td>5</td>
</tr>
<tr>
<td>0.0125</td>
<td>10</td>
</tr>
<tr>
<td>0.01</td>
<td>12.5</td>
</tr>
<tr>
<td>(&lt;0.01)</td>
<td>(\geq 12.5)</td>
</tr>
</tbody>
</table>

Synchronization error 0.005 sec. is insignificant

**EARTH RADIANCE VARIATION**

The variation of radiance over earth surface for the 14-16 \( \mu \text{CO}_2 \) band has been calculated in Figure M-1. Plots of the radiance as function of latitude and time of year for various altitudes is shown in Figure M-2. The model for GPS simulation is deduced from these results. For maximum radiance variation a January situation is considered. The radiance is assumed to be a function of latitude but not longitude. The radiance variation for southern hemisphere is assumed to be that for northern hemisphere six months later. The radiance variation for earth disk is approximated by piecewise linear segments using the values corresponding to 20 Km altitude shown in Figure M-2. The radiance values and the normalized variations are summarized.
Figure M-1. Radiance CO$_2$ 14-16µ
\[ v = y_1 + L = y_1 + \frac{\beta - x_1}{x_2 - x_1}(y_2 - y_1) \]

\[ \frac{L}{\frac{1}{2} y_1} = \frac{\beta - x_1}{x_2 - x_1}(y_2 - y_1) \]

**Figure M-2.** Normalized Earth Radiance Variation with Latitude at January
in Table M-1. The normalized variation is used to be compatible with the existing earth sensor simulation that does not include the details of sensor optics and filter response. The radiance for the 15° latitude band is taken as that corresponding to a 220°K earth which, in conjunction with the sensor optics and filter response, provides 3.15 W when the "A" detector FOV is fully illuminated.

<table>
<thead>
<tr>
<th>LAT.</th>
<th>MON.</th>
<th>RADIANCE W/M²</th>
<th>NORMALIZED VARIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>75°</td>
<td>JUL</td>
<td>6.94</td>
<td>1.21</td>
</tr>
<tr>
<td>60°</td>
<td>JUL</td>
<td>6.44</td>
<td>1.12</td>
</tr>
<tr>
<td>45°</td>
<td>JUL</td>
<td>6.19</td>
<td>1.08</td>
</tr>
<tr>
<td>30°</td>
<td>JUL</td>
<td>5.94</td>
<td>1.03</td>
</tr>
<tr>
<td>15°</td>
<td>JAN/JUL</td>
<td>5.75</td>
<td>1.00</td>
</tr>
<tr>
<td>+30°</td>
<td>JUL</td>
<td>5.5</td>
<td>0.96</td>
</tr>
<tr>
<td>+45°</td>
<td>JAN</td>
<td>4.88</td>
<td>0.85</td>
</tr>
<tr>
<td>+60°</td>
<td>JAN</td>
<td>4.63</td>
<td>0.80</td>
</tr>
<tr>
<td>+75°</td>
<td>JAN</td>
<td>4.13</td>
<td>0.72</td>
</tr>
</tbody>
</table>
Implementation of the earth radiance variation into the sensor model requires the following computations which are based on the geometry given in Figure M-3.

(a) LOS-vector for detector cell element centered at \((X_a, Y_a)\):

\[
U = \sqrt{X_a^2 + Y_a^2}
\]

\[
X_s = \frac{X_a \sin U}{U}
\]

\[
Y_s = \frac{-Y_a \sin U}{U}
\]

\[
Z_s = \cos U
\]

Denote

\[
\begin{pmatrix}
X_s \\
Y_s \\
Z_s
\end{pmatrix}
\]

(b) Look point on earth surface along \(L_a\):

\[
(L_a^s) = T_{IB} T_{SB} (L_a^i)
\]

\[
(R_a^i) = (R_V^i) + S (L_a^i)
\]

where

\[
(R_a^i) = \text{look point on earth surface along } L_a.
\]

\[
(R_V^i) = \text{vehicle position from center of earth}
\]

\[
S = \text{slant range from } R_V \text{ to } R_a
\]

\[
= -(R_V^i)^T (L_a^i) - (R_V^i)(R_V^i)^T (L_a^i) + [(R_V^i)^T (L_a^i)]^2)^{1/2}
\]

\[
\gamma_e = 3590.95 \text{ n. mi.}
\]
Figure M-3. Sensor Model Geometry
(c) Look point latitude:

\[
(\mathbf{R}^{-E}_a) = T_{EI} (\mathbf{R}^{-I}_a)
\]

Denote

\[
(\mathbf{R}^{-E}_a) = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}
\]

\[
\beta = \sin^{-1}(\frac{R_3}{y_e})
\]

= look point latitude

(d) Scaling factor for radiance variation:

\[
V = F(\beta)
\]

where \( F(\beta) \) is the piecewise linear function defining the radiance variation with latitude; \( F(\beta) \) is plotted in Figure M-1.

(e) The value of AREA is modified from the original value by the scaling factor, i.e.:

\[
\text{AREA} \rightarrow \text{AREA} \times V
\]

Figure M-4 presents a comparison of the GPS simulation radiance model and a model described in a Barnes Engineering document, "Technical Description Model 13-16901 and Model 13-16903 Synchronous Altitude Static-Sensors", 22 August, 1975. Models agree reasonably well.
Figure M-4. Comparison of Models
SENSOR ERROR MODELING

The sensor error modeling considered here includes detector and pre-amp noises, electronics parameter shift error, error due to temperature gradient across detector and sensor alignment error. Horizon uncertainty errors, though not a sensor originated error, are also included in the consideration. These are errors that exist in all operational situations.

Errors due to sun or moon in sensor detector field of view and errors resulted from variation of earth disk size due to vehicle orbit eccentricity exist only for a specific operational condition and are not included here. The detector and preamplifier noise has been calculated to be 0.379 μV (3σ) for sensor in track mode (A detectors only) and 0.464 μV (3σ) for sensor in acquisition mode (A+B detectors) with a detector scale factor of 3.15 μW/deg. and responsivity of 20 V/W. These noises are equivalent to 0.0060 deg. (3σ) and 0.0074 deg. (3σ) respectively. Use the output scale factors of 5 V/deg and 1.25 V/deg for track and acquisition modes respectively. The noise at output then becomes 0.03 V(3σ) and 0.0093 V(3σ). The noise error will be simulated as white noise.

Electronics parameter shift has been analyzed by Barnes, Reference N-2. The values at initial setting and at end of life have been calculated to be 0.0034 deg. (3σ) and 0.0219 deg. (3σ) respectively. These correspond to output errors of 0.017 V (3σ) and 0.1095 V (3σ) for track mode and 0.0043 V (3σ) and 0.0274 V (3σ) for acquisition mode. The error due to electronics parameter shift will be simulated as bias errors.

The temperature gradient considered by Barnes amounts to a temperature difference of 0.00048°F between A and S reference junctions. The corresponding attitude error evaluated for temperature of 70°F is 0.0103° or 0.0515 V
for track mode. For acquisition mode, the temperature change of 0.0028 deg. between detectors is calculated to cause attitude error of 0.0029 deg. which amounts to output error of 0.0035 V. All values quoted above are considered to be for worst case and will be treated as 3σ values. The error due to temperature gradient will be simulated as bias errors.

The achievable static sensor alignment accuracy has been estimated by Barnes to be in the order of 0.015 deg. (3σ). The equivalent output error is 0.075 V (3σ) for track mode and 0.019 V (3σ) for acquisition mode. The effect of alignment error will be simulated as bias errors. The horizon uncertainty error has been treated as an exponentially correlated noise with standard deviation 0.88 Km, correlation time 10 days and correlation distance 2500 N. Mi in Reference M-1, i.e.,

\[ E[h(t,s)h(t+T, S+d)] = \sigma_h^2 e^{-T/10} e^{-d/2500} \]

where

\[ \sigma_h = 0.88 \text{ Km} \]
\[ T = \text{time in days} \]
\[ d = \text{distance in n. mi.} \]

The term \( e^{-T/10} \) can be approximated by 1 if \( T \) is much less than 10 days.

Factoring in the GPS orbit altitude of 10897 N. Mi the horizon uncertainty error amounts to 0.0020 deg. of earth disk angle error. In the evaluation of \( P_1 \) and \( R_1 \) detectors at opposite sides of the earth disk are involved. The separation between the look points of these detectors is:

\[ d = \frac{(180° - 14°) \times 3461.6 \text{ N. Mi.}}{180°} \]

\[ = 10029 \text{ N. Mi.} \]
The separation is approximately four times the correlation distance. Hence the effects of horizon uncertainty error at opposing detectors are essentially uncorrelated. The equivalent error in evaluation of \( P_1 \) and \( R_1 \) is then \( 0.0020^\circ \times \sqrt{2} = 0.0028^\circ \) (1\( \sigma \)) or \( 0.0085^\circ \) (3\( \sigma \)). The output error becomes 0.0424 V for track mode and 0.0106 V for acquisition mode. The same values can be used for \( P_2 \) and \( R_2 \). (The distance between look points of every other detector cell is approximately 5383.1 N.Mi. The correlation of horizon uncertainty at points separated by this distance is only 0.12. It is neglected for simplicity of model.) For vehicle points at nadir and spins at a yaw rate of \( W \) RPM, the sensor detector scans the earth surface at a rate of 358.87 W N.Mi/sec. The 250C N. M correlation distance would then be equivalent to a correlation time of 6.97/W sec.

The sensor output error \( e(t) \) due to the horizon uncertainty will be modeled as an exponentially correlated noise as follows:

\[
E[e(t) e(t+T)] = \sigma_0^2 e^{-T/T_c}
\]

where

\[
\begin{align*}
\sigma_0 &= 0.0424 \text{ V for track mode} \\
&= 0.0106 \text{ V for acquisition mode} \\
T_c &= 6.97/W \text{ sec} \\
&= 3.485 \text{ sec for } W = 2 \text{ RPM}
\end{align*}
\]

The above model will be used to account for the effect of horizon uncertainty even for cases where vehicle is not quite pointing at the nadir. (The change of correlation time due to the pointing offset from nadir by \( \pm 4^\circ \) is expected to be small. For large offset from nadir the effect of detector not seeing the earth disk will be much more significant than the effect of horizon uncertainty.)

In summary, the sensor error will be simulated as an additive output error consists of a white noise component, a bias component and an exponentially
correlated random noise. The white noise is used to simulate the effect of detector and preamplifier noises. The equivalent attitude error of the white noise is 0.0060° (3σ). The exponentially correlated noise is used to simulate the effect of horizon uncertainty error. The equivalent attitude error is 0.0085° (3σ). The correlation time of this error for the vehicle spin rate of 2 RPM at initiation of earth acquisition is 3.485 sec.

The bias error is introduced to account for the total effects of electronics error, temperature gradient error and sensor alignment error. The equivalent attitude error is 0.0285° (3σ) evaluated as the RSS value of each individual effects.

CONCERNS

- The detector geometry and output computations may not be correct since Barnes has two different designs. The one used here (Figure M-5) is one of their designs. The other has detector pattern rotated 22½° about the line of sight from the one shown here.

- The output update frequency for GPS is not certain. The one used here is the one proposed by Barnes for DSCS III.

- Earth image distortion and focus are not modeled. Excluding radiance, the earth image is assumed to be a perfect circle on the focal surface.

- The detector output approximation using 1° x 1° squares causes very little error near null (normal operation) and up to 0.15° maximum detector error at extreme attitudes (during acquisition). The extent of the errors near null and the sensor output errors during acquisition has not been analyzed. The approximation errors off null are on the order the errors due to earth radiance variation and earth image quality. Smaller squares may be used to improve accuracy but computer cost will go up.
Figure 11-3. Detector Geometry—Single Axis Earth Sensor
NUMBERING OF X-Y COORDINATES FOR POSITION OF $1^0 \times 1^0$ SQUARES.
X-Y COORDINATES OF ALL 160 $1^0 \times 1^0$ SQUARES ARE OBTAINED FROM THESE 20 STORED VALUES

Figure M-5. Detector geometry—2-Axis Ruth Sensor (continued)
Figure M-5. Detector Geometry—2-Axis Earth Sensor (continued)
Figure M-5. Detector Geometry—2-Axis Earth Sensor (continued)
Figure M-5. Detector geometry--2-Axis Earth Sensor (concluded)
Sun and moon interference are not included in this model description.

**FLOW DIAGRAM**

The combined earth sensor model flow diagram is shown in Figure M-6. The left arrow (→) within boxes indicates replacement, i.e., the expression on the right replaces the variable on the left. Mathematical subscripts are used. Symbols used in the flow diagram are defined in the sections below. Comments appear outside the boxes to describe the operations within the boxes.

**INITIALIZATION DATA**

Table M-2 lists the initialization parameters, nominal values, tolerance, and source of data. This data is to be input at the start of each run.

**INPUT DATA**

Table M-3 lists the input data the earth sensor model expects from the calling program each time the subroutine is executed in the normal mode.

**OUTPUT DATA**

Table M-4 lists the output data from the earth sensor model subroutine. Other subroutine variables may be outputs if the user desires.

**INTERNAL VARIABLES**

Internal variables used in the flow diagram are defined in Table M-5.
Figure M-6. Earth Sensor Flow Chart
Figure M-6. Earth Sensor Flow Chart (continued)
Save angle between earth tangent and LOS.

Loop for two horizon crossing indicators

Set horizon crossing indicators to no crossing

Figure M-6. Earth Sensor Flow Chart (continued)
Compute position and diameter of earth image on detector surface. Superscripts indicate reference frame:

- \( B \) Inertial
- \( C \) Body
- \( S \) Sensor

\( \text{SQRT} \) is square root function

\( \text{ATAN} \) is single argument arc tangent function

For state of 2-axis operation

\[ A_i \text{ AREA } (-Y_i, X_i) \]
\[ A_{i+10} \text{ AREA } (Y_i, -X_i) \]
\[ A_{i+20} \text{ AREA } (Y_i, X_i) \]
\[ A_{i+30} \text{ AREA } (-Y_i, -X_i) \]

\[ D_1 = A_1 + A_2 + A_{21} + A_{22} \]
\[ D_2 = A_9 + A_{10} + A_{29} + A_{30} \]
\[ D_3 = A_{11} + A_{12} + A_{31} + A_{32} \]
\[ D_4 = A_{19} + A_{20} + A_{39} + A_{40} \]

See p. 234

Figure M-6. Earth Sensor Flow Chart (continued)
From p.227

Compute area of earth coverage on B detectors

For $i = 3$ thru 8

$$D_2 = D_2 + A_i + A_i + 20$$

$$D_5 = D_5 + A_i + 10 + A_i + 30$$

Compute detector output voltage and save detector coverage.

For $i = 1$ thru 6

$$V_i = V_i + (D_i - DSAV_i) \times CON2$$

$$DSAV_i = D_i$$

Compute roll 1 attitude. RAND is random number generator, normal distribution, unit variance

$$RIT_{SF_1} = (V_4 - V_1 + V_3 + BS_1 + SIG_1 \times RAND(1) + REXP_1)$$

$$RIA_{SF_2} = (V_4 + V_5 - V_1 + V_2 + BS_2 + SIG_2 \times RAND(2) + REXP_2)$$

Output pitch 2 result

PT P2T

PA P2A

If pitch 2 track is invalid, use pitch 1 track

If pitch 2 acq. is invalid, use pitch 1 acq.

PT P1T

PA P1A

Figure M-6. Earth Sensor Flow Chart (continued)
From p.22

For \( i = 1 \) thru \( 10 \)
\[ A_i \quad \text{AREA} (X_i, Y_i) \]
\[ A_i+10 \quad \text{AREA} (-X_i, -Y_i) \]
\[ A_i+20 \quad \text{AREA} (X_i, -Y_i) \]
\[ A_i+30 \quad \text{AREA} (-X_i, Y_i) \]

\[ D_1 = A_1 + A_2 + A_{21} + A_{22} \]
\[ D_2 = A_2 + A_{10} + A_{29} + A_{30} \]
\[ D_4 = A_{11} + A_{12} + A_{31} + A_{32} \]
\[ D_6 = A_{19} + A_{20} + A_{39} + A_{40} \]

For \( i = 3 \) thru \( 8 \)
\[ D_2 = D_2 + A_i + A_{i+20} \]
\[ D_5 = D_5 + A_i + 10 + A_{i+30} \]

For \( i = 1 \) thru \( 6 \)
\[ j = i + 6 \]
\[ V_j = V_j + (D_i - D_{SAV_j}) \cdot \text{CON2} \]
\[ D_{SAV_j} = D_i \]

PIT SF_1 \ast \left(V_{10} - V_{12} - V_7 + V_9 + BS_3 + SIG_3 \ast \text{RAND}(3) + \text{REXP}_3\right) \]
PIA SF_2 \ast \left(V_{10} + V_{11} - V_7 - V_8 + BS_4 + SIG_4 \ast \text{RAND}(4) + \text{REXP}_4\right)

RT RIT RA RIA

Output roll
1 results

Figure M-6. Earth Sensor Flow Chart (continued)
From p.229

\[ P1T - P2T < 0.5 \]
\[ \text{and} \]
\[ R1T - R2T > 0.5 \]

If roll 1 track is invalid, use roll 2 track

FALSE

\[ P1A - P2A < 0.125 \]
\[ \text{and} \]
\[ R1A - R2A > 0.125 \]

If roll 1 acq. is invalid, use roll 2 acq.

FALSE

p.234

From p.227

For \( i = 1 \) thru 10
\[ j = i + 10 \]
\[ A_{i} = \text{AREA}(X_{j}, Y_{j}) \]
\[ A_{i+10} = \text{AREA}(-X_{j}, Y_{j}) \]
\[ A_{i+20} = \text{AREA}(X_{j}, -Y_{j}) \]
\[ A_{i+30} = \text{AREA}(Y_{j}, -X_{j}) \]
\[ A_{i+40} = \text{AREA}(Y_{j}, X_{j}) \]
\[ A_{i+50} = \text{AREA}(-X_{j}, Y_{j}) \]
\[ A_{i+60} = \text{AREA}(-Y_{j}, X_{j}) \]
\[ A_{i+70} = \text{AREA}(X_{j}, -Y_{j}) \]

Compute area of earth coverage on each 10 x 10 square on #2, #4, #6, and #8 detectors
See p.234

p.231

Figure M-6. Earth Sensor Flow chart (continued)
Compute area of earth coverage on A and S detectors

For i = 3 thru 8

\[ D_2 = D_2 + A_i + A_{i+40} \]
\[ D_5 = D_5 + A_{i+10} + A_{i+50} \]
\[ D_8 = D_8 + A_{i+20} + A_{i+60} \]
\[ D_{11} = D_{11} + A_{i+30} + A_{i+70} \]

Compute area of earth coverage on B detectors

For i = 1 thru 12

\[ j = i + 12 \]
\[ V_j = V_j + (D_i - DSAV_j) \times CON3 \]
\[ DSAV_j = D_i \]

Compute detector output voltage and save detector coverage.

Figure M-6. Earth Sensor Flow Chart (continued)
Figure M-6. Earth Sensor Flow Chart (continued)
P2T $\frac{SF_1}{2} V_{19} - V_{21} + V_{13} + V_{15} + V_{16} - V_{18} - V_{22} + V_{24} + SF_1 BS_7 + SIG_7 * RND(7) + REXP7$

$P2A \frac{SF_2}{2} V_{19} + V_{20} - V_{13} - V_{14} + V_{16} + V_{17} - V_{22} - V_{23} + SF_2 BS_8 + SIG_8 * RND(8) * REXP8$

RT  R2T  Output roll 2 results

RA  R2A

If roll 2 track is invalid, use roll 1 track.

P1T-P2T > 0.5 and R1T-R2T > 0.5

FALSE

P1A-P2A > 0.125 and R1A-R2A > 0.125

FALSE

STATE 1  Return to state 1

P  p.234

Figure M-6. Earth Sensor Flow Chart (continued)
From pp. 228, 230, and 232

STATE  STATE + 1

Advance to the next state.

From p. 233

TYMUP  TYMUP + 0.125

Increment time of next update

From p. 227

TYM  TYM + Δt

Increment time

RETURN

End of 2-axis iteration

FUNCTION

AREA  (XA, YA)

Called on pp. 227, 229, and 230. B, C, RE are common to calling program. RD = 0.0087266 equiv. radian.

AREA  (RE+RD-SQRT (B-XA)^2 + (YA-C)^2) * 180

Compute approx. area of earth covering 10 x 10 square.

AREA  > 1

AREA  1

Limit AREA to be < 1 and > 0

AREA  0

Else

AREA  = AREA*V

RETURN

RETURN

Figure M-6. Earth Sensor Flow Chart (concluded)
Where

\[ \text{SIG}_I = 0.0020 \text{ (deg., 1:)} \quad I = 1, \ldots, 8 \]

\[ \text{BS}_I = \text{Bias error with worst case value 0.0285 (deg., 3:)} \quad I = 1, \ldots, 8. \]

\[ \text{REXP}_I = \text{Exponentially correlated noise with standard deviation 0.00283}^\circ \text{ and correlation time 3.485 sec.} \]

The function \( \text{REXP}_I \) to be used to generate the exponentially correlated noise is given in the following:

\[
\begin{align*}
\text{FUNCTION } \text{REXP}_I \\
\text{REXPIO} &= \text{REXP}_I \\
\text{BETA} &= 1./\text{TMCNST} \\
\text{DREXP} &= -\text{BETA}*\text{REXPIO} \\
&\quad + \text{SQRT} (2.*\text{BETA})*\text{RSIGI}*\text{RANDI} \\
\text{REXP}_I &= \text{REXPIO} + \text{DREXP} \\
\text{RETURN} \\
\text{END}
\end{align*}
\]

Where

\[ \text{TMCNST} = \text{Correlation Time} \]

\[ \text{RSIGI} = \text{Standard Deviation} \]

\[ \text{RANDI} = \text{Random number generator with unit variance} \]

\( \text{REXP}_I \) to be assigned an initial value.
### Table M-2. Initialization Data for Earth Sensor Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Subscript</th>
<th>Unit</th>
<th>Nominal Value</th>
<th>Source</th>
<th>Tolerance</th>
<th>Source</th>
</tr>
</thead>
<tbody>
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<td>Line of sight of spin earth sensor field</td>
<td>LOB, LOB_x</td>
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<td>-0.15 (2 axis)</td>
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236
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### TABLE M-2. INITIALION DATA FOR EARTH INSOR MODEL (concluded)

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<th>PARAMETER</th>
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<th>SOURCE</th>
<th>TOLERANCE</th>
<th>SOURCE</th>
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<tr>
<td>SIG om: R1    T</td>
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<td>Volts</td>
<td>0.167</td>
<td>Note 4</td>
<td>N/A</td>
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<tr>
<td>R1A</td>
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<td>Volts</td>
<td>0.0417</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R1T</td>
<td>3</td>
<td>Volts</td>
<td>0.167</td>
<td></td>
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<tr>
<td>R1A</td>
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<td>Volts</td>
<td>0.0417</td>
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<tr>
<td>R2T</td>
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<td>Volts</td>
<td>0.167</td>
<td></td>
<td></td>
<td></td>
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<td>R2T</td>
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<td>Volts</td>
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<td></td>
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<td>R2A</td>
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<td>Volts</td>
<td>0.167</td>
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<td>P21</td>
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<td>P21</td>
<td>9</td>
<td>Sec.</td>
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<td>Not</td>
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<td>P21</td>
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<td>Sec.</td>
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<td>Not</td>
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<td>Two axis output scale factor</td>
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<td></td>
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<tr>
<td>Track, SF</td>
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<td>Volt/sq. deg.</td>
<td>1.25</td>
<td>Note 3</td>
<td>0.0625</td>
<td>Note 3</td>
</tr>
<tr>
<td>Acq., SF</td>
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<td>Volt/sq. deg.</td>
<td>0.3125</td>
<td></td>
<td>0.025</td>
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</tbody>
</table>

**NOTES TO TABLE M-2**

1. Barnes presentation to Honeywell in St. Petersburg on 3-13-75.

Track scale factor

- 1 degree attitude = 4 sq. deg. coverage = $5 \times 0.25V$.

\[
\frac{5 \text{volts}}{4 \text{sq. deg.}} = 1.25 \pm 0.0625 \text{ volt/sq. deg.}
\]

Acquisition scale factor

- 4 degree attitude = 16 sq. deg. coverage

\[
\frac{1.25 \pm 0.1 \text{ V}}{4 \text{ sq. deg.}} = 0.3125 \pm 0.025 \text{ volt/sq. deg.}
\]

4. Computed from data in Rockwell Spec MC 432-0214, Rev. B, 12-11-74

Track mode

- $\pm 0.1^\circ$, 3 sigma

\[0.1 \times 5 \text{ volt/deg} \times \frac{1}{3} = 0.167 \text{ volt, 1 sigma}\]

Acquisition mode

- $\pm 0.1^\circ$, 1 sigma

\[0.1 \times 1.25 \text{ volt/deg} \times \frac{1}{3} = 0.0417 \text{ volt, 1 sigma}\]
<table>
<thead>
<tr>
<th>DEFINITION</th>
<th>FLOW DIAGRAM SYMBOL</th>
<th>DIMENSION</th>
<th>TYPE</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode control flag</td>
<td>MODE</td>
<td></td>
<td>Integer</td>
<td>0 = initialize</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td>1 = spin operation</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2 = 2 axis operation</td>
</tr>
<tr>
<td>Satellite position vector</td>
<td>$R_x, R_y, R_z; R$</td>
<td>3</td>
<td>Real</td>
<td>N Mi</td>
</tr>
<tr>
<td>Transformation from body to</td>
<td>TIB</td>
<td>9</td>
<td>Real</td>
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<tr>
<td>inertial frame</td>
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# TABLE M-4. OUTPUT DATA FOR EARTH SENSOR MODEL

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<th>TYPE</th>
<th>UNITS</th>
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<td>Real</td>
<td>Seconds</td>
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<td>Pitch attitude in track mode</td>
<td>PT</td>
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<td>Real</td>
<td>Volts</td>
</tr>
<tr>
<td>Pitch attitude in acquisition mode</td>
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<td>Real</td>
<td>Volts</td>
</tr>
<tr>
<td>Roll attitude in track mode</td>
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<td>Real</td>
<td>Volts</td>
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<tr>
<td>Roll attitude in acquisition mode</td>
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### TABLE M-5. INTERNAL VARIABLES FOR EARTI SENSOR MODEL

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<th>Type</th>
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<td>Two axis rise/decay increment for 0.125 sec.</td>
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<td>Time of next 2-axis update</td>
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<td>Satellite distance to earth center</td>
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<td>N. Mi</td>
</tr>
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<td>Satellite position vector in sensor frame</td>
<td>E&lt;sub&gt;x&lt;/sub&gt;, E&lt;sub&gt;y&lt;/sub&gt;, E&lt;sub&gt;z&lt;/sub&gt;</td>
<td>3</td>
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<td>N. Mi</td>
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<tr>
<td>Radius of earth image on focal surface</td>
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<td>Real</td>
<td>Equiv. radian</td>
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<tr>
<td>Position of earth image on focal surface, X component</td>
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<tr>
<td>Position of earth image on focal surface, Y component</td>
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<td>State of sequential processing within sensor</td>
<td>STATE</td>
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<td>Area of detector covered by earth</td>
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<tr>
<td>Area of detector coverage from previous cycle</td>
<td>DSAV</td>
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<td>Dynamic Detector output</td>
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TABLE M-5. INTERNAL VARIABLES FOR EARTH SENSOR MODEL (concluded)

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<th>SYMBOL</th>
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<td>Volts</td>
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<td>Roll 2 track attitude</td>
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<td>Pitch 2 acquisition attitude</td>
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<td>Function</td>
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REFERENCES
