A PRACTICAL GUIDE TO THE USE OF GROUP PROBLEM
SOLVING AS A DECISION MAKING OPTION IN
PROGRAM/PROJECT MANAGEMENT OFFICES

STUDY PROJECT REPORT
PMC 77-2

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MAJOR
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FORT BELVOIR, VIRGINIA 22060

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## Title

A PRACTICAL GUIDE TO THE USE OF GROUP PROBLEM SOLVING AS A DECISION MAKING OPTION IN PROGRAM/PROJECT MANAGEMENT OFFICES

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## Abstract

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STUDY TITLE: A PRACTICAL GUIDE TO THE USE OF GROUP PROBLEM SOLVING AS A DECISION MAKING OPTION IN PROGRAM/PROJECT MANAGEMENT OFFICES

STUDY PROJECT GOALS:

To describe the organizational conditions under which group problem solving is appropriate; to specify the variables which favor greater or lesser participatory decision making styles; to describe the effect of each phase in the systems acquisition process on participatory decision making styles; and to identify the appropriate techniques for amalgamating group opinion.

STUDY REPORT ABSTRACT:

The purpose of this project was to investigate the conditions which favor use of group problem solving and decision making to examine the effect of differing organizational climates during the various phases of the systems acquisition process on selection of appropriate participatory decision making styles, to evaluate and identify quantitative methods to assist in amalgamating group opinion, and to understand the appropriate steps in utilizing this decision making approach in the systems acquisition process. A literature survey was conducted of participative decision making and of quantitative methods for amalgamating group opinion.

This project shows that it is possible to list a set of questions, the answers to which assist the principal decision maker in selecting an appropriate participatory style. It shows it is possible to develop a system of techniques useful in amalgamating opinions. It also shows that it is possible to list the steps which a principal decision maker may follow in his decision process when utilizing group problem solving. A decision process flow chart and a decision—flow diagram are included in the report to aid in selecting an appropriate participatory style and in selecting an appropriate technique for amalgamating opinion.

SUBJECT DESCRIPTORS:

Group and Intergroup Relations (10.03.03)
Program/Project Management (10.02)
Decision/Statistical Analysis Methods (10.04)

NAME, RANK, SERVICE
William B. Whipple, Jr., Maj, USA

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A PRACTICAL GUIDE TO THE USE OF GROUP PROBLEM SOLVING AS A DECISION MAKING OPTION IN PROGRAM/PROJECT MANAGEMENT OFFICES

Individual Study Program
Study Project Report
Prepared as a Formal Report

Defense Systems Management College
Program Management Course
Class 77-2

by
William Benjamin Whipple, Jr.
Major USA

November 1977
Study Project Advisor
LTC Donald S. Fujii, Ph.D., USAF

This study project report represents the views, conclusions and recommendations of the author and does not necessarily reflect the official opinion of the Defense Systems Management College or the Department of Defense.
EXECUTIVE SUMMARY

The primary aim of this report is to acquaint the reader with the conditions favoring use of group problem solving and decision making and to provide some techniques with which principal decision makers or managers may amalgamate group opinion where it is crucial in reaching an effective decision. The techniques are tailored to the requirements of a principal decision maker located in a program/project management office (PMO). This report examines the changing organizational environment as the program moves through the phases of the systems acquisition process. The changing environment is related to group problem solving and decision making with the finding that the more participatory styles are most effective during the conceptual and demonstration—validation phases. The lesser participatory styles are more relevant during the full-scale engineering development phase. While the guidelines were tailored for the principal decision maker in the PMO, the findings have a wider, more general application. The quantitative aspects of the report may be omitted without sacrificing understanding of the other features. Conversely, the quantitative techniques described are valid for any situation in which opinions of multiple experts require amalgamation, and these techniques may be considered at the exclusion of the other features.
The very deepest appreciation is expressed to Lieutenant Colonel Don Fujii, the author's advisor, for his guidance and inspiration during the conduct of this project. His knowledge and support were invaluable in clarifying the author's thinking and refining the manner in which this report is presented. A special thank you is extended to Janice, my wife, and Jennifer, my daughter, for their understanding and patience as this project consumed a disproportionate amount of available time. Finally, to Wilma Collins, who expertly typed and retyped this report, goes my sincere gratitude.
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<th>Description</th>
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<tr>
<td>$A, B, \ldots, n$</td>
<td>Designations of objectives.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Risk level associated with Type - 1 error in hypothesis testing.</td>
</tr>
<tr>
<td>$a_{ij}, b_{ij}$</td>
<td>Coefficient of the $i$-th judge's ranking of the $j$-th objective in a matrix $A$ or a matrix $B$.</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>Chi-Square test statistic.</td>
</tr>
<tr>
<td>$\text{cov}_{ij}$</td>
<td>Covariance of the distributions of values assigned to the $i$-th and $j$-th objectives.</td>
</tr>
<tr>
<td>$d.f.$</td>
<td>Degrees of freedom.</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$ - test statistic.</td>
</tr>
<tr>
<td>$f(i\mid j)$</td>
<td>Frequency of preference of the $i$-th objective to the $j$-th objective.</td>
</tr>
<tr>
<td>$m$</td>
<td>Number of judges in the sample.</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of objectives in the attribute.</td>
</tr>
<tr>
<td>$P_i$</td>
<td>Probability of the $i$-th event.</td>
</tr>
<tr>
<td>$P(i\mid j)$</td>
<td>Probability of preference of the $i$-th objective to the $j$-th objective.</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Spearman's Rho or the rank correlation coefficient.</td>
</tr>
<tr>
<td>$R_j$</td>
<td>Sum of ranks given by the set of $m$ judges for the $j$-th objective.</td>
</tr>
<tr>
<td>$r_{ij}$</td>
<td>Correlation coefficient expressing the degree of association of the values of the $i$-th objective with those of the $j$-th objective.</td>
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1Throughout this Individual Study Program (ISP) report, various symbols, equations, diagrams, and tables will be presented using as a medium pen and ink. This technique of presentation was approved by DR. Andrew P. Mosier, ISP Director, Defense Systems Management College.
In Kendall's Concordance Test, the sum of squares of deviations of the $R_j$ from the mean of the $R_j$.

$s_{i^2}$ Sample variance of the values of the i-th objective.

$s_i$ Sample standard deviation.

t "Student's" - test statistic.

$\tau$ Kendall's Tau or the rank correlation coefficient.

$v_i$ Value of the i-th objective.

$W$ Kendall's Coefficient of Concordance.

$\bar{x}$ Sample mean.

$\neq$ Not equal to.

$>$ Greater than; relation of preference for one objective over another.

$\geq$ Greater than or equal to.

$<$ Less than.

$\leq$ Less than or equal to.

$\sum_{i=1}^{n}$ Summation from the first through the n-th entities.

$\sim$ Relation of indifference between two objectives.
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SECTION I

INTRODUCTION

Purpose

The intent of this paper is to provide the members of the systems acquisition community with an understanding of the conditions which favor the use of group problem solving and decision making. Another specific objective is to provide techniques by which program members can amalgamate group opinion where such amalgamation is crucial in reaching an effective decision. In a study of the decision making process of weapon systems acquisition, Holland (31:3) quoted then Deputy Secretary of Defense Packard as stating, "Improvement in the execution of these programs will be made to the extent the Services are willing and able to improve their management practices." Although the quote was made during the Spring of 1970, it is still true today.

Specific Goals

The specific goals of this project are (1) to describe the organizational conditions under which group problem solving is appropriate, (2) to specify the variables which favor greater or lesser participatory

2This notation will be used throughout the report for sources of quotations and major references. The first number is the source listed in the cited reference portion of the bibliography. The second number is the page in the reference.
decision making styles, (3) to describe the effect of each phase in the system acquisition process within which the problem situation is occurring, and (4) to identify the appropriate techniques for amalgamating group opinion.

**Definitions**

The reader is encouraged to review the List of Symbols (page vi) and the terms and their meanings found in Appendix A, Glossary, before proceeding further.

**Application**

There are no "cookbook" solutions to management and to decision making. However, it is believed that a systematic, objective approach to diagnosing the situation and applying appropriate participatory style and quantifying techniques will aid the principal decision makers within the Program/Project Management Office (PMO) in making effective decisions relative to the systems acquisition process. Rose Glubin, a Department of Defense manager writing in the Defense Management Journal, has asserted that "the science of statistics is not an alternative to experienced judgement; rather it aids and supplements by creating a more informed judgement based on objective data, principles, and techniques" (24:49).

Because the PMO environment is becoming increasingly complex, the individual principal decision maker is often unable to make effective decisions based only on his own information and experiences. Just as he
alone does not have all the technical information and expertise to solve today's problems, neither do any one of his subordinates. Fischer (7:12) has shown that amalgamated opinion, obtained by consolidating and averaging the assessments of individuals, typically outperforms almost all individual assessments. Although the audience for whom this paper is tailored are the principal decision makers within a PMO, the basic concepts expressed have a wider, more general application as well.

**Scope of the Project**

The project will investigate group problem solving and decision making. It will also evaluate amalgamating techniques. This investigation will be a literature search utilizing material principally from the Defense Systems Management College Library, The Defense Documentation Center, and The Defense Logistics Studies Information Exchange.

**Limitations**

The results of this project will be based upon information found within the literature referenced in the Bibliography. This is an initial investigation to tie together separate, yet related work by many authors. Validation of the concepts proposed within this paper will not be undertaken but is considered a fruitful follow-on effort. The reader does not have to be qualified in operations research methodology to follow the discussion and examples in this paper; however, he should, as a prerequisite, understand the basic premises of statistics. Mathematical proofs will not be included since this paper is application oriented.
The reader desiring proof of formulae is encouraged to avail himself of the references found in the Bibliography. The reader wishing to skip the quantitative aspects of this report may do so and still gain an appreciation of group problem solving and decision making in a PMO. Conversely, a reader seeking techniques for amalgamating opinion may exclude the other features.

Organization of the Report

The description of related research in Section II provides both the cornerstone and the foundation upon which this report is built. Appendices B through E complement Sections II and III for the reader interested in amalgamating techniques. Section II also provides the background discussion which unfolds into use of a decision process flowchart to aid the principal decision maker in selecting appropriate participatory styles. Section III builds upon the discussion of Section II, provides a decision flow diagram for use in selecting an amalgamating technique, and suggests a sequence of steps which the principal decision maker may follow in the decision process. Finally, the report closes by making some concluding remarks and recommending some areas for additional research.
SECTION II

REVIEW OF CURRENT LITERATURE AND RELATED RESEARCH

General

This section addresses the research findings and methodologies that are pertinent to the objectives of this paper. The libraries and data banks are rich with associated research; however, no research was found tying together the separate, yet related areas of participative management or group problem solving and the quantitative techniques to assist in amalgamating opinion. Furthermore, these areas have not adequately been related to the organizational environment of the PMO.

Conditions Which Favor Group Problem Solving and Decision Making

Group problem solving and decision making, usually called participative decision making (PDM), received extensive favorable attention in management literature in the late forties and throughout the fifties and to a lessor degree during the sixties and seventies. Military applications of this style of management or decision making process were also widely studied as is evidenced by a proliferation of papers written by students attending various service schools up through and including the senior service colleges during the sixties and seventies. The early writings described PDM as being highly desirable because the involvement of organization
members in the decision process led to increased morale, motivation, and productivity. Later work, while still indicating PDM can be effective, does not categorize it as a panacea. The later findings suggest the application of PDM only after careful scrutiny and determination that the organizational environment possesses variables conducive to its successful employment (32:4).

The organizational environment which favors use of PDM is one in which some or all of the following variables exist:

1. The decision is concerned with complex system(s) (13:190);
2. The interpersonal and intergroup relationships are not conflict-free (13:319 & 27:7);
3. The decisions are associated with diverse tasks involving rapidly changing, and/or unpredictable technology (32:5);
4. The decision to be reached is not overly constrained in time (26:19 & 27:3-4);
5. Others have data or knowledge not possessed by the principal decision maker (27:7); and
6. Commitment (not merely compliance) is required and cannot, with a high probability, be expected to be achieved if the decision is made solely by the principal decision maker (27:7).

Another consideration in examining conditions favoring PDM is the question of the quality requirement (see Glossary). In other words, does it make a difference to the principal decision maker which alternative is adopted? This one variable may override the use of the more participatory extremes of PDM if the principal decision maker
perceives possible goal incongruence between the goals of the group he might look to for PDM and the organization's goals as he sees them. He might be reluctant to relinquish his prerogatives when there is a strong possibility that the group's selected alternative would not be compatible with his view of the organization's objectives for decisions he considers extremely important (27:5).

During the consideration of conditions which favor PDM, interaction factors must also be considered since they may affect the effectiveness of the group's performance. Assuming that the variables which were previously listed favor PDM, Hackman (10:27-28) has postulated that there are three interim criteria for which there may be process losses or process gains. He concludes that interventions may be made which will improve group internal processes by focusing on that criteria of special saliency to the kind of decision upon which the group is working. Conversely, intervention by the principal decision maker or a non-salient aspect of the group process could direct the attention and energy of group members away from the issues of importance and toward phenomena of little impact on eventual group effectiveness. He lists some possible process gains and losses related to the interim criteria effort, knowledge and skill, and performance strategy as follows:

1. When effort is salient:
   Process loss: Members fail to coordinate their efforts in applying them to the (decision) task, resulting in a "coordination decrement."
   Process gain: Members develop strong commitment to each other and to the group that increases the amount of effort they are willing to expend in task work.
2. When knowledge and skill are salient:
   Process loss: The group imperfectly assesses and weighs the inputs of members who have differential task-relevant talent.
   Process gain: Members share uniquely-held knowledge and skill, and cooperate to gain new learning — thereby increasing the total pool of talent available to the group.

3. When performance strategy is salient:
   Process loss: Members imperfectly assess task requirements, and implement task-inappropriate strategies.
   Process gain: Members invent new creative ways of proceeding with work on the task. (10:27)

Thus, it is important, once a decision has been made to employ the fully participatory style of PDM, to not indiscriminantly intervene in the group's decision process. Otherwise, the interaction of the group may be adversely affected.

While the preceding discussion indicates that the group's effectiveness may be affected by the principal decision maker's intervention, it also indicates that concerted action on the part of the group can influence actions taken by the principal decision maker. Richards and Greenlaw have developed a systems view of work group behavior in terms of inputs and outputs which they assert are dynamically and reciprocally interdependent in an open systems sense (18:207-209). Management decisions in their model would relate to decisions of the principal decision maker as defined in this paper. A schematic model of their systems view is shown in Figure 1.
These variables, factors, and the pertinence of intervening actions are considerations which must be examined in exploring whether or not conditions favor use of PDM. The group, if used, must be encouraged and helped to develop and mature into an effective unit. According to Schein (20:96) if the principal decision maker ignores these variables, factors, and the pertinence of intervening actions, he may "land in the trap" of expecting people called together to be able to automatically perform as an effective psychological group. Further, he asserts, if they fail to perform, he may then erroneously conclude that groups are ineffective. He would then be failing to realize he had only an aggregate of people -- not a group.
Finally, in examining conditions favoring PDM, consideration is given to the spectrum of possible participation the principal decision maker may desire to obtain from his subordinates (referred to in this paper as principal decision maker subordinates). This can be cast in a matrix, adapted from the work of Weiss, and is shown in Figure 2. The columns have arbitrarily been numbered one to five, with the higher numbers equating to a lessening of authority on the part of the principal decision maker and an increase in the freedom subordinates have in the decision making process. The five style categories can be described as follows (27:3 & 26:18-19):

1 - This style is characterized by completely independent action on the part of the principal decision maker, using whatever information is available to him at that time.

2 - The principal decision maker resolves a situation alone, but he obtains any necessary information of a specific nature from his subordinates before making the decision himself. He may or may not divulge the purpose of the questions he asks in gathering data he needs. In any case, the people from whom he seeks data play no role in analyzing the problem or making the decision.

3 - Under this style, the situation is explained to each person from whom data is solicited. Although they provide data, the principal decision maker makes the decision. The decision may or may not reflect the influences of those who have been consulted.

4 - This style, like style 3, is a variant of the consultative process. The principal decision maker's subordinates are assembled in a group meeting
Figure 2. A Spectrum of Participatory Styles
at which the situation is explained, data provided, and alternatives proposed. Again, the final decision is that of the principal decision maker and may or may not reflect the group's influence.

5 - This style corresponds with Norman Maier's concept of group decision making (15:217-75) in which the principal decision maker's role is that of a chairperson of a group meeting aimed at reaching consensus. The principal decision maker facilitates the flow of data while the group members try to reach a consensus as a solution to the situation. The principal decision maker may or may not try to influence group consensus. In any case, he accepts and implements whatever conclusions the group reaches.

Phases of The Systems Acquisition Process and Their Relation to Group Problem Solving and Decision Making

The results from a recent study of the organizational climate within a number of Air Force System Program Offices (SPOs) has a bearing on the use of PDM. While one style may work for some people because of their personality it may not for others, and the individual situation, such as situations with great time pressures, may dictate the participatory style (27:3).

In essence, the amount of participation the principal decision maker subordinates may be called upon to provide will increase or decrease in accordance with the guide displayed in Figure 3.
Figure 3. A Guide for Changing Participation Levels

<table>
<thead>
<tr>
<th>IF</th>
<th>Participation Should</th>
<th>But Only IF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. There is a high quality requirement.</td>
<td>Increase</td>
<td>Others have data needed for a high quality decision.</td>
</tr>
<tr>
<td>2. The situation is unstructured.</td>
<td>Increase</td>
<td>Others have data needed for a high quality decision.</td>
</tr>
<tr>
<td>3. There is low goal congruence.</td>
<td>Decrease</td>
<td>There is strong quality requirement.</td>
</tr>
<tr>
<td>4. There is a high level of commitment (not merely compliance) required.</td>
<td>Increase</td>
<td>Others would not be committed to a decision made by the leader alone.</td>
</tr>
<tr>
<td>5. There is a high probability of commitment to a decision made by the leader alone.</td>
<td>Decrease</td>
<td>NO CONDITIONS</td>
</tr>
<tr>
<td>6. There is a high level of conflict.</td>
<td>Increase</td>
<td>Others would not be committed to a decision made by the leader alone.</td>
</tr>
<tr>
<td>7. There is a strong need for training or team building.</td>
<td>Increase</td>
<td>NO CONDITIONS</td>
</tr>
</tbody>
</table>

Vroom indicates that the validity thus far documented has been 65% successful decisions when the style was from the feasible set.

The payoff of the tree is a feasible set of styles which may be used.

The variables described in the previous subsection can be addressed in a decision process flowchart adapted from Vroom (26:19). See Figure 4.
(35% unsuccessful when a choice of style was made from the feasible set) and 29% successful decisions when the style was outside the feasible set (71% unsuccessful when a choice of style was other than those included in the feasible set). The important point to recognize is that, for most situations, there is a feasible set or range of styles which are appropriate and the principal decision maker does not have to search for the "perfect style" of participatory decision making (27:8).

There are five phases of the systems acquisition process. Since it is presumed the reader is familiar with these phases, they will be described in summary fashion. The first phase is the conceptual phase. The primary objective of this phase is to identify those system concepts which warrant further development to satisfy a valid mission element need. The primary efforts within the PMO during this phase are as follows: (1) Identify and systematically explore and develop alternative system concepts, or modifications to an existing capability; (2) Develop the acquisition strategy, including development of the Program Master Plan; (3) Perform cost/schedule estimates which are to be used for comparison of alternatives and for providing initial cost and schedule information for the proposed program; (4) Translate operational/mission element needs into system technical performance requirements; (5) Develop logistic and maintenance concepts for alternative systems; (6) Perform trade-off analysis between cost, schedule, and technical performance alternatives; and (7) Analyze alternate procurement and management approaches, including procurement and business planning (33:9).
(adapted from 26:19)

To question P is yes, is within the feasible set only when the answer

Figure 4: Decision Process Flowchart (Feasible Set of Participatory Styles)
The second phase is the demonstration/validation phase. During this phase, the primary objective is to conduct demonstrations of viable alternative systems or critical subsystems and to provide a basis for selection of a system for full scale engineering development. The activities of the PMO are centered around the following: (1) Develop alternative system(s) design(s); (2) Establish relationships between need, urgency, risk and worth to permit practical trade-offs among cost, schedule, and technical performance; (3) Reduce or solve the risk of the proposed system through the conduct of competitive demonstrations of full-scale prototypes of systems or critical subsystems in realistic environments, when feasible and practical, and alternatively through detailed studies and analyses; and (4) Expand and refine the acquisition strategy (33:9-10).

During the next phase, the full scale engineering development phase, the primary objective is to complete, prior to commitment to full scale production, the design, the assembly, and the testing of the system and its support requirements to ascertain whether the required operational capability can be achieved subject to the imposed cost constraints. Normally, the physical size of the PMO, in terms of number of people, expands during this phase to be able to perform the planning of future requirements (engineering and production design, test and evaluation, and required training and support planning), procuring long lead time items, administering the numerous contracts, managing the on-going program, and complying with the reporting procedures of higher authorities (33:13-14).
The fourth of the five phases is the production phase. The primary objective of this phase is to produce and deliver operational systems (including all necessary items of logistic support) for deployment by the operational forces. The tasks in which the PMO personnel are involved are as follows: (1) Production management of the weapon system, training equipment, spares and support equipment, including monitoring of contractor progress and performance, resolving encountered difficulties, and controlling schedules and costs; (2) Implementation of logistic support activities, such as provisioning; and (3) Commencing preparation for transitioning and turning over management responsibilities for the system, if applicable (33:14-16).

The fifth phase is deployment. Its primary objective is to provide systems and support to operational units. This period commences with user acceptance of the first operational unit and ends with the systems phase-out of the inventory. As far as the PMO is concerned, transition terminates most of the program management responsibilities and the size of the office shrinks to a size sufficient to accomplish residual tasks which remain. Again, the transition process and transfer of responsibility to a logistic or readiness command is service dependent; however, in most cases, the PMO is eventually disbanded (33:16). While the foregoing description of the phases of the systems acquisition process has been brief, assuming reader familiarity, it should be sufficient for even those less conversant in the life cycle of system acquisitions to understand the following comments.
In a study of the PMO organizational climate during the various phases of the systems acquisition process, Larson and Ruppert examined 13 Air Force SPOs, which will be assumed for the purpose of this paper to be representative of PMOs of all services. They used an instrument developed by Rensis Likert of the Institute for Social Research at the University of Michigan, the "Likert Profile of Organizational Characteristics," to determine the nature of the organizational climate. It uses questions to measure individual member perceptions of the organizational climate. The climate is measured along a continuum from negative (System 1), in which all decisions are made by top management, communications flow only downward, and personnel within the organization do not contribute to, or reject, the organization's goals; to positive (System 4), in which the decision making process is shared, communications flow not only downward, but upward and horizontally, and the individuals accept the goals of the organization, adopt these as their own, and contribute toward achieving these goals (33:19-20). They concluded that the System Program Offices have a tendency to more nearly practice a participative style (System 4) during the conceptual and demonstration/validation phases than in any other phase. As the program progressed into full scale engineering development, they found that individual perceptions of the organizational climate changed. They attributed the change to organizational expansion which results in the organization's moving towards a System 1 style of management. When System 4 management is no longer, or can no longer, be practiced, they deduced that individuals have a lesser tendency to identify with the organization goals and to replace them with their individual goals. They found that, as the program
moves into the final two phases, it was interesting to note that, instead of the organization becoming more structured, the climate reverses its trend and progresses back toward System 4. However, they observed that it never reaches the participative management level attained during the first two phases (33:57-58).

Although the Larson and Ruppert study used a range of styles from one through four while this paper uses a continuum containing five levels, the point to be made is that generally the climate is more favorable for the more participatory styles of decision making during the conceptual and demonstration/validation phases, to a lesser degree during the production and deployment phases, and least favorable during the full scale engineering development phase.

Methodologies for Amalgamating Opinions

The principal decision maker may require tools to assist in combining the opinions of those he calls upon to assist him in reaching a decision. The process of combining these opinions into a single joint order is the function of amalgamation. Those to whom he looks to for information have been called, in the earlier sub-sections, principal decision maker subordinates. Throughout this sub-section, they will be referred to as judges. This terminology is consistent with the majority of the literature on the subject of amalgamating opinion, and is used here as a somewhat wider label to encompass those individuals the principal decision maker may look to for information but who are not his true subordinates. Because the principal decision maker is not going to be receptive to learning a large number of different techniques to assist in reaching his decision, the following constraints were imposed for the purpose of evaluating techniques
to be recommended in this report:

(1) There must be only a small number of techniques for the principal decision maker to learn, and

(2) Considered collectively, the techniques must be sufficiently general to encompass the wide spectrum of decision making situations in the systems acquisition process.

It is conceivable that the techniques which will be described could be used for participatory styles two through five; however, it would probably have its greatest application for styles two or three, and to a lesser degree four.

With the previously listed constraints in mind, a number of techniques were reviewed, examined and tested. Some of the more representative of these are now discussed. Algorithms and examples illustrating how these techniques may be employed are included for three of the techniques meeting the constraints and being recommended to the principal decision maker. The same hypothetical example is used for Arrow's Method of Majority, for the Football Poll, and for Thurstone's Law of Comparative Judgement.

Thurstone's Law of Comparative Judgement

Thurstone (23:19-49) originally published his "Law of Comparative Judgement" in 1927, initially intending it for use in the field of

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3For the algorithm and an example of its usage, see Appendix D.
psychometrics. It has also found wide use elsewhere, in such diverse fields as econometrics, sociometrics, and utility theory. Within this law, there are five cases. The method is permitted to differ in each of these cases as it pertains to assumptions, approximations, and degree of simplification. The provision of differing cases makes this technique an extremely versatile tool. The five cases may be summarized as follows:

Case 1. - Used when a single judge makes repeated judgements.
Case 2. - Used when rankings are by different judges.
Case 3. - If the discriminant deviations of the same judgment are uncorrelated, the equation may be simplified.
Case 4. - If, in addition to the assumption of Case 3, the variances of the discriminant deviations are not subject to gross differences, the equation may be further simplified.
Case 5. - Used in the simplest version when equispacing, or equal discriminant dispersions, may be assumed; the equation may be reduced to its simplest form.

Another strong attraction of this technique is that it is simple to employ, and it is possible to cardinaly rank the joint order. However, it does require transitive orders and requires normality of the joint order of the rankings. Although Guilford (9:154-96) presents some scaling methods which would permit normalizing the rankings, this additional transformation is not recommended. There are other techniques which do not require the joint order developed from the ranks assigned to have a particular underlying statistical distribution. The next two techniques to be discussed are amalgamating methods which do not require the rankings to fit a specified distribution.
Arrow's Method of Majority and Football Poll Method of Ranking.\footnote{For the algorithm and an example of its usage, see Appendix B.}

Arrow proposed a simple majority rule that makes no assumptions concerning the rankings performed by the judges. The Football Poll technique derives its name because it is used by the various press associations for rating football teams. These two techniques are best described by contrasting the features of one with those of the other (25: xviii–xx). Each technique has both advantages and disadvantages. Which is to say, that one is not necessarily "good" and the other "bad." Rather, each technique may be either good or bad dependent solely upon whether the conditions under which it is employed fit the assumptions which must be made concerning the data with which it is used.

In using the Football Poll technique, points are awarded for position. For example, one point might be awarded for first place, two points for second place, etc; or, as with the press rankings, the reverse might be used, awarding points in diminishing order from first place to the lowest place considered. The points awarded a particular place is the R associated with that objective (the objective being a team, a proposal, or whatever is being ranked). The objectives are then ordered on the basis of the R\textsubscript{j}'s. Those with the lower sum (or higher sum, as the case may be) being ordered as the most preferred. The problem with this technique is
that the majority of the judges may feel a particular objective is better than all of the alternatives and yet it may ultimately be ranked second, third or lower. The reason for this happening is that artificial information, not necessarily truly representative of the judge's scalings of the objectives, is that which is added. The Football Poll procedures assume that there is equal spacing between preferences, which is to say, that the amount one objective is preferred over another is always equal. The rank ordering provided by the judges does not necessarily imply this. It also assumes that the $R_j$ are additive. Therefore, because it is easy to apply and its results easily displayed, the Football Poll Method of Ranking is widely accepted and misused by those not understanding its assumptions.

In contrast to the Football Poll, the Arrow's Method of Majority provides a similar, easily displayed result, but requires no assumptions to be made concerning the rankings. Each objective is considered as an entity and the number of times it is preferred to each other alternative is counted. The results are cast in an array which is then converted to a new array portraying a 0,1, or $1/2$, depending upon which entity was preferred less times, more times, or an equal number of times to its complement. For example, if "a" is preferred to "b" two times and "b" is preferred to "a" once; then, "a" is preferred to "b" by majority vote. In this event, "a" would receive a 1 and "b" would receive a 0.

Later work by Shannon and Wyatt (34:5-10) has added to the features of Arrow's Method of Majority Rule the ability to reflect, or take into
account, the differing expertise or knowledge of the judges for the various entities being ranked.\(^5\)

The greatest limitation of this technique is that it fails to consider whether one judge has a strong preference for one objective as opposed to one or more alternatives, while a second judge has a weak counter preference concerning these same objectives.

Some Rejected Techniques

Certainly all techniques for quantification of opinion cannot be discussed here. However, to ignore those which were considered but to be less appropriate would be tantamount to glossing over the very existence of techniques. These other techniques are described in Appendix E. As was previously set forth in establishing constraints on selecting techniques to be recommended to the principal decision maker, there must be only a limited number of techniques for the decision maker to learn and they must collectively cover the spectrum of his applications. Consequently, a number of techniques were rejected, not because of invalidity, but simply because they were less desirable than those selected in the accomplishment of similar purposes.

\(^5\)For the modifying algorithm and an example of its usage, see Appendix C.
SECTION III

GROUP PROBLEM SOLVING TECHNIQUES TO ACHIEVE EFFECTIVE DECISIONS

A Process for Amalgamating Opinions

The three techniques selected in the previous section collectively encompass the wide spectrum of decision making situations in the systems acquisition process. A few simple questions will be helpful in allowing the principal decision maker to select the most appropriate amalgamating technique for a given application. The following questions, used in conjunction with the accompanying decision-flow diagram (see Figure 5), are designed to assist in this determination:

1. Do the judges selected possess varying degrees of expertise in relation to the sub-categories reflected by the totality of objectives to be ranked?
2. Can it be assumed that the rankings of the individual judges may not be equispaced?
3. Can it be assumed that the $R_j$'s (which are the sum of ranks given by the set of $m$ judges for the $j$-th objective) may not be additive?
4. Is intransitivity a possibility in the rankings, and conversely is there no requirement for a cardinal ranking of the joint order?

Prior to making the final selection of an amalgamation technique to be employed, it is first necessary to perform some statistical tests on the data. These statistical tests must be performed to insure the validity of the joint order. Which is to say, the agreement of the judges must be
Figure 5. Decision-Flow Diagram (Amalgamation Method Selection)

Legend:
- Process = □
- Decision = □

The text to the questions listed in numerical symbols are keyed.

Select
Go to App C
Go to App D
Select
Go to App B
Prepare Case 1-4
Select
Insert
Case 5
Select
Yes
No
Yes
No
Yes
No
Yes
No
No
ascertained to be within acceptable limits. Also the data's distribution may have to be checked.

The statistical tests to be performed must be identical for most of the techniques which have been recommended in this paper. If they are not, the decision maker could not select a technique until after acquiring the rankings and testing of the joint order of this data. However, that would be at variance with the proper order of events in which selection of the amalgamation technique should come very early, before determining sample size and stratification, before developing interrogation methods, and before obtaining rankings. Therefore, it is imperative that these statistical tests be applicable to all of the recommended techniques. An exception to the requirement for statistical tests exists for Case 1 of Thurstone's Law of Comparative Judgements, since only a single judge is involved.

The first test that must be performed is a measurement of the degree of correspondence between two or more rankings. More specifically, a measurement of the intensity of rank concordance must be made. For correlation of two judges, or of a single judge and a standard, there are several tests which might be applied. The most widely accepted of these are Kendall's Rank Correlation Coefficient $\tau$ (Greek letter tau), and Spearman's Coefficient of Rank Correlation $\rho$ (Greek letter rho). For both of these, the numerical values of $\tau$ and $\rho$ range from +1.00 (perfect positive correlation, meaning every objective has the same rank in both), to 0.00 (no correlation, meaning independence), and down to -1.00 (perfect negative correlation, meaning one ranking is the inverse of the other).
An excellent discussion of these tests and a comparison of their uses has been made by Kendall (14:3-33). Both of these are too limited in application to be of use to us in the amalgamation of group opinion.

To be sufficiently general for our use, the test selected must handle not only two sets of rankings, but it also must handle multiple rankings. Averaging all the possible values \( t \) or \( \rho \) between pairs of observers would be a solution, but it would prove very tedious for large numbers of judges. Recognizing this, Kendall developed a Coefficient of Concordance \( W \) (14:94-116). \( W \) measures the commonality of judgements of multiple observers. Unlike the tests of two rankings, the range of numerical values of \( W \) varies from 0.00 to +1.00, and not from -1.00 to +1.00. Symmetrical opposites can not exist in the sense we are considering for rankings of more than two judges. If there are three judges, A may disagree with B on a comparison and also disagree with C, but then B and C must agree. This reasoning can be carried on for four or more judges also. Therefore, if all judges agree, \( W \) will approach +1.00 as a limit. The algorithm for Kendall's Coefficient of Concordance is found in Appendix B.

The non-parametric test described above is the only test required when using the Football Poll Technique or Arrow's Method of Majority. Recalling that Thurstone's Law of Comparative Judgement requires normality of the joint order of the rankings, when this technique is being utilized, a test for normality is thus required. The classical test for normality of a distribution, the Chi-Square Test for Goodness of Fit, is the test recommended for use. It will be employed in addition to Kendall's
Coefficient of Concordance when Thurstone's Law of Comparative Judgement, Cases 2 through 5 are being used. The procedure for computing the Chi-Square test statistic is included in Appendix D.

These tests, when required, must be performed to assure that the technique initially selected is compatible with the joint order of the rankings. If, for instance, it was planned to use Thurstone's Law of Comparative Judgements, Cases 2-5, and the Chi-Square Test fails, it is still possible to use the data as submitted by the judges in an alternate technique such as Arrow's Method of Majority.

**Dimensions of Effective Decisions**

For a decision to be effective, two essential dimensions must be considered. Maier (15:275-77) describes decision effectiveness as being a function of **quality** and **acceptance**. He states that the quality dimension is the degree to which the decision represents objective facts, and the acceptance dimension is a measure of how the group that must execute the decision feels about the decision which was reached. For a decision to be effective, both high quality and high acceptance are required. Therefore, the group problem solving process must consider both facts and feelings. Since the method(s) of dealing with the two are not necessarily the same, the principal decision maker must be skillful in making diagnostic judgements and then dealing with each. The effectiveness of the decision can be described by the equation
Effective decision = Quality \times Acceptance

It will be noted that Maier used a multiplication sign in the relationship. He assumed that when both dimensions were positive that the effectiveness would be highly positive. Therefore, he established his equation in this vein. He postulated that if either the quality or the acceptance dimension is zero, the effectiveness is zero. He further felt that if either dimension is negative, the effectiveness will be negative. Finally, he asserts that if both dimensions are negative, the relationship will still provide a valid answer mathematically and conceptually since it will display negative quality (violation of objective facts) times negative acceptance (rejection by the group) and consequently positive effectiveness (a correct decision of non-implementation).

The Problem Solving Process

Recognizing the dimensions of an effective decision, we can now describe the steps necessary to reach such a decision. The problem solving process is described in many management texts, operations research texts, and military manuals. What will be described below is a synthesis of the ideas from many sources which together provide a logical and orderly means for arriving at a solution to a problem confronting a principal decision maker. (For the reader desiring further information not provided in
this paper, the following sources are suggested: 2:23-57; 12:12-14; 1:67-76; 5:11-13; and 21:46-58. It is suggested that the principal decision makers utilize the following steps:

1. Define and state the problem.
2. Define expectations.
3. Consider the organizational environment of the particular problem situation.
4. Evaluate and select an appropriate participatory style.
5. Dependent upon the selected style, review information sources which are feasible and suitable.
6. Collect data.
7. If appropriate, amalgamate opinions.
8. Develop conclusions.
10. Make decision.
11. Implement decision and evaluate outcome(s). (This may dictate reentering the process at step one).

Section II and the earlier portions of this section (Section III) will have afforded the reader a background for use of the eleven step process. The reader will recognize that Section II contained both a narrative description and a decision process flow chart (Figure 4), relating to the participatory styles appropriate for various situations. Those guidelines will aid in accomplishing Step 4. A decision-flow diagram, which was provided earlier in Section III (Figure 5) will assist
in selecting an appropriate amalgamating technique. This will facilitate accomplishing Step 7.

As a part of developing conclusions, Step 8, the decision maker(s) should forecast the consequences of the solutions proposed, testing and evaluating the possible solution before finalizing. Throughout the process, the effectiveness of the decision must be considered in terms of quality (facts) and acceptance (feelings). Neither of these dimensions can be permitted to become zero or negative for a highly effective decision to result.
SECTION IV

SUMMARY

Conclusions

The examination and discussion contained within this report considered the conditions which favor use of group problem solving and decision making, the phases of the systems acquisition process and their relation to group problem solving and decision making, techniques available for amalgamating group opinion, and the steps in the group problem solving process to achieve an effective decision.

This project shows that it is possible to list a set of questions the answers to which assist the principal decision maker in selecting an appropriate participatory style. It also shows that it is possible to develop a system of techniques, small in number but extensive in their applicability, which will aid the principal decision maker in quantifying, by ranking ordinally or cardinally, the opinions of those he may look to for assistance in his decision making role. The report synthesizes, from a variety of sources, the steps which a principal decision maker may follow in his decision process when utilizing group problem solving. It is concluded that the more participatory styles of decision making have their greatest application during the conceptual and demonstration/validation phases and the less participatory styles are found in greater favor in the full scale engineering development phase.
Areas of Additional Research

It is recommended that a follow-up project be undertaken to determine if the findings of the study of Air Force SPOs (33:56-58) are borne out by studies of PMOs in the other Services. Research should also be undertaken to validate or refute productivity improvements and/or decision effectiveness measures associated with use of styles of decision making appropriate to the situational environment. This should be accomplished across the widest possible variety of PMOs and during all phases of the systems acquisition process.
APPENDIX A

GLOSSARY

Algorithm - a simple to follow step-by-step direction for solving a problem.

Amalgamate - to unite; mix; combine.

Attribute - a characteristic; composed of more than one objective.

Cardinal - one of the highest of the measurement scales; numbers are assigned to objects in a manner which provides a representation of distance between them.

Certainty - the decision maker knows precisely the consequence attached to every course of action available to him.

Concordance - the mutual interrelationship between more than two variables.

Conflict - the behavior of an individual, a group, or an organization which impedes or restricts (at least temporarily) another party from attaining its desired goals (the consequences may be beneficial if they produce new information enhancing the decision making process).

Correlation - the mutual interrelationship between two variables.

Decision - a judgement or a choice between alternatives.

Discriminal deviation - difference that exists between objective rankings.

Goal congruence - a measure of the agreement of the interests of subordinates -- especially implementors -- with the same interests and/or goals of the organization in resolving the problem situation.

Group - any manifold of persons, identifiable over a period of time, and sufficiently integrated so that its actions and objectives are identifiable.
**Group role** - marked by decline in self-oriented behavior of members who are interacting with one another, are psychologically aware of one another, and perceive themselves to be a group.

**Hypothesis** - an unproven theory, tentatively accepted to explain certain facts; in contrast, **null hypothesis**, is that which is normally tested in statistics, the objective being to disprove it and thereby prove the hypothesis.

**Individual role** - the premise that, at least initially, when two or more people come together as a group, there will be a period of essentially self-oriented behavior such as identity selection, power or influence distribution, goal congruence examination, and acceptance and intimacy resolution.

**Interquartile range** - the middle 50 percent of whatever is being measured, for example, responses; so named because it defines the spread between the measures standing between the first two quarters and the last two quarters of a distribution, called quartiles.

**Intransitivity** - (See transitivity).

**Joint order** - the joint set acquired by combining the individual orders of the various judges.

**Judge** - the evaluator; expert; respondent; observer; or other description of the person with expertise in an area that performs the individual rankings.

**Mean** - a measure of central tendency; the measure computed by adding all measurements and dividing by the total number of cases.

**Median** - a measure of central tendency; the measure that divides the group into halves of the same number; the middle value in order of size if N
is odd, or the mean of the two middle items if N is even.

Normalizing - the process of placing a group of values on a common scale.

Objective - the variable; the alternative; the option; the subject of the judge's ranking process.

Order - to place in a relationship indicating preference, utilizing some scale of measurement.

Ordinal - the lowest scale of measurement permitting the property of rank order; if two objectives are unequal, the direction of their inequality, that is, \( a > b \) or \( b > a \) is indicated.

Participative decision making (PDM) - a decision making process in which those who will carry out the decision have a substantial input in the decision process or actually make the decision themselves. Within this report, the terms participative management and participation will be considered synonymously with PDM.

Preference - as contrasted with indifference, the former indicates a discernment of difference in two objectives, shown as \( a > b \) or \( b > a \), while indifference is displayed \( a \sim b \) or \( a=b \). Strong preference permits ordering only using inequality signs, while weak preference allows the use of indifference or ties in ordering to appear.

Principal decision maker - the manager charged with responsibility for a decision and having the authority to make the decision.

Principal decision maker subordinates - individuals subordinate to the manager categorized as the principal decision maker; for the purpose
of this report, these are individuals within a program/project management office (PMO) subordinate to a program manager or to a lower level manager. The subordinate may or may not, in turn, be the manager or supervisor of others.

Quality requirement - the question of whether or not it makes a difference which course of action is adopted.

Rank ordering - listing of opinions of judges.

Result of research - a term sometimes used indicating the conclusion drawn from the joint ordering; the making of the decision or recommendation.

Risk - the decision maker does not know the definite consequence attached to a course of action, but a probability may be attached to every possible consequence.

Transitivity - as contrasted with intransitivity, states that if a judge has any preference among three alternatives they must be consistent in the following sense:

If \( a \sim b \)

\( b \sim c \), then \( a \sim c \);

if \( a > b \)

\( b > c \), then \( a > c \);

and if \( a > b \)

\( b \sim c \), then \( a > c \).

Any other condition is intransitive. That is \( a > b, b > c \), cannot permit or imply \( c > a \) without being intransitive.
**Uncertainty** - decision situations in which either the probabilities associated with the outcomes are a matter of individual judgement, or no judgement at all can be made about them.

**Utility** - the perceived worth of an alternative to a judge; procedure for identifying preferences explicitly.
APPENDIX B

THE ARROW'S METHOD OF MAJORITY AND
THE FOOTBALL POLL METHOD OF RANKING

Football Pool Algorithm.

1. Let \( n \) = number of objectives to be ranked, and let \( m \) = number of judges assigning ranks. List each judge's preferences, then cast them in a matrix of rankings. If several objectives are tied, give them the value of the mid-point of the ranks for which they are tied (that is, if two objectives are tied for ranks 3 and 4, give them each 3.5; or if three objectives are tied for ranks 7, 8, and 9, give them each 8).

2. For each objective, determine \( R_j \), the sum of the ranks assigned to the \( j \)-th objective by the \( m \) judges.

3. Determine the mean of the \( R_j \) values by use of the relation

\[
\bar{R}_j = \frac{\sum R_j}{n}
\]

4. Express each \( R_j \) as a deviation from that mean, \( \bar{R}_j \), and square these deviations. Sum these squares to obtain \( A \):

\[
A = (R_1 - \bar{R}_j)^2 + (R_2 - \bar{R}_j)^2 + \ldots + (R_n - \bar{R}_j)^2.
\]
5. a. If there are few or no ties in the \( m \) sets of rankings, calculate the coefficient of concordance by the relation

\[
W = \frac{A}{\left(\frac{1}{12}\right) m^2 (n^3 - n)},
\]

where \( 0 \leq W \leq 1 \), and 0 means perfect disagreement, 1 means perfect agreement.

b. If there are a considerable number of ties relative to the number of objectives \( n \) in the \( m \) sets, use the equation

\[
W = \frac{A}{\left(\frac{1}{12}\right) m^2 (n^3 - n) - m \sum_{i=1}^{m} T_i},
\]

where

\[
T_i = \frac{\sum (t^3 - t)}{12}
\]

for each of the \( m \) sets or rows having ties;

\( t \) = number of observations in a group tied for a given rank; \( \sum \) means to sum over all the tied groups within any one row; and \( \sum_{i=1}^{m} T_i \) means sum the \( T \)'s over all \( m \) rankings.

(For example, if judge 1 was indifferent, in ranking two objectives, or had one tie; if judge 2 was indifferent in ranking two objectives ...)
twice, or had two ties; and if judge three was indifferent both in ranking three objectives and indifferent in ranking two objectives; then, the correction factor for ties would be as follows:

\[ T_1 = \frac{(2^3 - 2)}{12} = 0.5 \]

\[ T_2 = \frac{(2^3 - 2) + (2^3 - 2)}{12} = 1.0 \]

\[ T_3 = \frac{(2^3 - 2) + (3^3 - 3)}{12} = 2.5 \]

\[ \sum_{i=1}^{3} T_i = 0.5 + 1.0 + 2.5 = 4.0 \]

6. The test used to determine whether the calculated coefficient of concordance is significantly different from zero, (that is, whether there is agreement between the judges at a given level of significance), depends on the number of objectives which were ranked, the size of \( n \).

a. If \( n \leq 7 \), the \( \chi^2 \) calculated in step 4 is used instead of \( W \). Use table 1 or more complete tables found in (22:286).

b. If \( n > 7 \), a Chi-Square value is calculated using \( W \) in the relation

\[ \chi^2_{calc} = m(n - 1) W. \]
<table>
<thead>
<tr>
<th>n</th>
<th>m</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<tbody>
<tr>
<td>3</td>
<td></td>
<td>64.4</td>
<td>103.9</td>
<td>157.3</td>
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<td></td>
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<tr>
<td>4</td>
<td></td>
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<td>88.4</td>
<td>143.3</td>
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<td>136.1</td>
<td>221.4</td>
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<td></td>
<td>48.1</td>
<td>101.7</td>
<td>183.7</td>
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<td>231.2</td>
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<td>192.9</td>
<td>349.8</td>
<td>570.5</td>
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<td></td>
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<td>258.0</td>
<td>468.5</td>
<td>764.4</td>
<td>1158.7</td>
</tr>
</tbody>
</table>

**TABLE 1. CRITICAL VALUES OF $A$ AT THE .05 SIGNIFICANCE LEVEL; ADAPTED FROM (22:286).**

**LEVEL OF SIGNIFICANCE**

<table>
<thead>
<tr>
<th>d.f.</th>
<th>.10</th>
<th>.05</th>
<th>.01</th>
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<tbody>
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<td>7</td>
<td>12.02</td>
<td>14.07</td>
<td>18.47</td>
</tr>
<tr>
<td>8</td>
<td>13.36</td>
<td>15.51</td>
<td>20.09</td>
</tr>
<tr>
<td>9</td>
<td>14.68</td>
<td>16.92</td>
<td>21.67</td>
</tr>
<tr>
<td>10</td>
<td>15.99</td>
<td>18.31</td>
<td>23.21</td>
</tr>
<tr>
<td>12</td>
<td>18.55</td>
<td>21.03</td>
<td>26.22</td>
</tr>
<tr>
<td>14</td>
<td>21.06</td>
<td>23.68</td>
<td>29.14</td>
</tr>
<tr>
<td>16</td>
<td>23.54</td>
<td>26.30</td>
<td>32.00</td>
</tr>
<tr>
<td>18</td>
<td>25.99</td>
<td>28.87</td>
<td>34.80</td>
</tr>
<tr>
<td>20</td>
<td>28.41</td>
<td>31.41</td>
<td>37.57</td>
</tr>
</tbody>
</table>

**TABLE 2. CRITICAL VALUES OF $X^2$; ADAPTED FROM (25:xix).**
This value is tested against a tabulated value of \( \chi^2 \) for degrees of freedom \( n-1 \). If \( \chi^2_{\text{calc}} > \chi^2_{\text{table}} \), it is concluded that there is significant agreement. Use table 2 or complete tables from any statistics book.

7. Determine rank order by ordering the objectives on the basis of the \( R_j \)'s. Those with the lower sum (or higher sum, if they were ranked initially with higher numerical values being assigned to the most preferred) are considered to be the most preferred.

**Arrow's Method of Majority Algorithm.**

Steps 1 through 6 are identical to those in the Football Poll Algorithm.

7. Generate a matrix, \( n \times n \), in which each of the objectives is considered one at a time and the number of times it was preferred to each of the other objectives is counted. The objective considered is listed on the left of the table and those with which it is considered are listed across the top. The array indicates how many times the objectives on the left of the table are preferred to the objectives with which they are compared. In the case of ties, the single vote is split and one-half vote is awarded to each objective for every pair of objectives tied. If three or more objectives are tied, they still receive one-half votes in pair-wise comparisons. The sum of the complement cells in the array will equal the number of judges.

8. Generate another \( n \times n \) matrix in which the number in the dominating cell of the previous matrix is replaced by a 1 and the number in the complement by a 0. Sum across the rows for each objective, obtaining the...
number of objectives assigned lower numerical values (higher preference) by majority vote. Those with the higher sum being highest preferred.

To illustrate these algorithms, an example opinion poll is now provided and the joint order is ranked using the Football Poll and Arrow's Method of Majority.

Example problem.

Rankings of six judges of four objectives

(1) B > A > C > D
(2) A > B > D > C
(3) A > D > B > C
(4) B > A > D > C
(5) A ∼ B > D > C
(6) B > A > C > D

Casting these judgments in a matrix of rankings:

<table>
<thead>
<tr>
<th>m</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0</td>
<td>1.0</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>2.0</td>
<td>4.0</td>
<td>3.0</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>3.0</td>
<td>4.0</td>
<td>2.0</td>
</tr>
<tr>
<td>4</td>
<td>2.0</td>
<td>1.0</td>
<td>4.0</td>
<td>3.0</td>
</tr>
<tr>
<td>5</td>
<td>1.5</td>
<td>1.5</td>
<td>4.0</td>
<td>3.0</td>
</tr>
<tr>
<td>6</td>
<td>2.0</td>
<td>1.0</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>R</td>
<td>9.5</td>
<td>9.5</td>
<td>23.0</td>
<td>18.0</td>
</tr>
</tbody>
</table>

45
Apply Kendall's Coefficient of Concordance:

Sum the $R_j$'s:

$$\sum R_j's = 9.5 + 9.5 + 23.0 + 18.0 = 60.0$$

Mean of the $R_j$'s:

$$\bar{R}_j = \frac{60}{4} = 15$$

Obtain $A$:

$$A = (9.5 - 15)^2 + (9.5 - 15)^2 + (23 - 15)^2 + (18 - 15)^2 = 133.5$$

Obtain $\sum_{i=1}^{m} T_i$:

$$T_1 = \frac{(2^3 - 2)}{12} = 0.5$$

$$T_5 = \frac{(2^3 - 2)}{12} = 0.5$$

$$T_6 = \frac{(2^3 - 2)}{12} = 0.5$$

$$\sum_{i=1}^{6} T_i = 0.5 + 0.5 + 0.5 = 1.5$$

Calculate $W$:

$$W = \frac{A}{\left(\frac{1}{12}\right) m^2 (n^3 - n) - m \sum_{i=1}^{m} T_i}$$

$$= \frac{133.5}{\left(\frac{1}{12}\right)(6)^2 (4^3 - 4) - (6)(1.5)}$$

$$= 0.7807$$
Where, \( 0 \leq W \leq 1 \), and 0 indicates perfect disagreement
1 indicates perfect agreement.

Testing for significance:
Since \( n \leq 7 \) we actually test \( \theta \) against the critical value from
Kendall's Coefficient of Concordance.

\[
\theta_{\text{calc}} = 133.5
\]

\[
\theta_{\text{Table 1}}(m = 6, n = 4) = 75.7
\]

Therefore, since \( \theta_{\text{calc}} > \theta_{\text{Table}} \), conclude there is a significant
agreement of the judgments.

Having passed Kendall's test of agreement, we are prepared to proceed
directly into either of two techniques of ordering and to proceed
with a test of normality for a third.

At this point we can make a Football Poll ranking by merely ordering
the \( R_j \)'s:

\[ A \sim B > D > C. \]

The ranking above concluded the sequence of operations for the
Football Poll. Proceeding with Arrow's Majority Rule
Method:
Generate a matrix of the times one option is preferred to another
option:
Generate a matrix for the greater number of preferences of one objective in comparison with another objective:

\[
\begin{array}{cccc}
A & B & C & D \\
\hline
A & - & 2.5 & 6 & 6 & 2 \\
B & 3.5 & - & 6 & 5 & 3 \\
C & 0 & 0 & - & 1 & 0 \\
D & 0 & 1 & 5 & - & 1 \\
\end{array}
\]

At this point we can make an Arrow's Majority Rule ranking of the objectives:

\[B > A > D > C\]

Note that the two techniques gave approximately the same joint order. The Football Poll saw indifference between A and B, while Arrow's Majority Rule saw a preference for B. The reason for this difference is the joint order formed from the Football Poll is influenced by whether the judges like an objective a great deal more than another objective, while Arrow's Majority Rule looks instead at how many judges liked an option.
APPENDIX C

WEIGHTED METHOD OF MAJORITY PROCEDURE

Modification of Arrow's Method of Majority Algorithm.

1. Two assumptions are necessary in order to modify rankings based upon weighting the opinion of the judges polled in relation to their degree of expertise concerning the various objectives being ranked.
   
   a. Assumption 1 - The judge's ability to accurately establish his preference for one objective over another is directly related to the least amount of knowledge he possesses concerning either objective. He will prefer (or not prefer) a particular objective of which he has considerable knowledge as opposed to an unknown objective based primarily upon his lack of appreciation of the unknown objective.
   
   b. Assumption 2 - The principal decision maker can assign a subjective weight to the level of knowledge or expertise of the judges. In other words, he can assign a numerical value, possibly based upon information provided by the judges themselves, between 0 and 1 to represent how much he will value the more expert of the judges over the less expert.

2. Based upon the assumptions listed in step 1, the principal decision
maker determines the number of levels of expertise he will consider (c),
and assigns a numerical weight between 0 and 1 to each level \(W_L\), where
\(L = 1,2,\ldots,c\). In addition to each judge's rank order, an expertise
level from 1 to c is established for each of the objectives ranked,
utilizing either a judge's self-rating as a measure or a decision maker's
rating as a measure. As an alternative to these measures of expertise,
the judge's peers might rate him.

3. Arrow's Method of Majority algorithm, (see Appendix B), is modified
by substituting the following for step 7:

For each of the m judge's rank orders, establish an \(n \times n\) preference
matrix \(A_k\), where \(n\) is the number of objectives being ranked, \(k =
1,2,\ldots,m\) is the particular judge and \(a_{ij} = W_L\), if objective \(i\) is
preferred to objective \(j\) (where \(L\) is the lowest level of expertise,
comparing levels of \(i\) and \(j\)) and 0 otherwise. If objective \(i\) is equal
to objective \(j\) then \(a_{ij} = W_L/2\). Generate a group frequency matrix \(B\)
by summing these matrices as \(\sum_{k=1}^{m} A_k\); thus, each \(b_{ij}\) will represent
the number of judges that prefer objective \(i\) to objective \(j\).

Example problem.
Consider the earlier problem, but now assume the six judges, although
"expert" in the general area of concern, are not equally conversant
with each of the four specific technical objective areas. The decision
maker then asks them to indicate whether they consider themselves to be:
(1) vaguely familiar, (2) knowledgeable, or (3) expert in each of the objective areas. The results of their rankings and levels of expertise are as follows:

(1) Rank order B > A > C ~ D 
   Expertise 2 2 1 3

(2) Rank order A > B > D > C 
   Expertise 3 2 1 2

(3) Rank order A > D > B > C 
   Expertise 2 3 1 3

(4) Rank order B > A > D > C 
   Expertise 3 2 2 1

(5) Rank order A ~ B > D > C 
   Expertise 3 3 1 2

(6) Rank order B > A > C ~ D 
   Expertise 2 1 3 3

The decision maker must weight the levels of expertise.

Assume he does so as follows:

<table>
<thead>
<tr>
<th>Expertise Level</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Vaguely familiar</td>
<td>0.2</td>
</tr>
<tr>
<td>2. Knowledgeable</td>
<td>0.6</td>
</tr>
<tr>
<td>3. Expert</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Preference matrices are established for each judge:
<table>
<thead>
<tr>
<th>JUDGE 1</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>0</td>
<td>.2</td>
<td>.6</td>
</tr>
<tr>
<td>B</td>
<td>.6</td>
<td>-</td>
<td>.2</td>
<td>.6</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>.1</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>.1</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>JUDGE 2</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>.6</td>
<td>.6</td>
<td>.2</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>-</td>
<td>.6</td>
<td>.2</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>.2</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>JUDGE 3</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>.2</td>
<td>.2</td>
<td>.6</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>-</td>
<td>.2</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>.2</td>
<td>.2</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>JUDGE 4</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>0</td>
<td>.2</td>
<td>.6</td>
</tr>
<tr>
<td>B</td>
<td>.6</td>
<td>-</td>
<td>.2</td>
<td>.6</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>.2</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>JUDGE 5</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>.5</td>
<td>.6</td>
<td>.2</td>
</tr>
<tr>
<td>B</td>
<td>.5</td>
<td>-</td>
<td>.6</td>
<td>.2</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>.2</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>JUDGE 6</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>0</td>
<td>.2</td>
<td>.2</td>
</tr>
<tr>
<td>B</td>
<td>.2</td>
<td>-</td>
<td>.6</td>
<td>.6</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>.5</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>.5</td>
<td>-</td>
</tr>
</tbody>
</table>
The group frequency matrix is generated:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.3</td>
<td>2.0</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1.9</td>
<td>2.4</td>
<td>2.2</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.0</td>
<td>0.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>0.2</td>
<td>1.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The group preference matrix is generated:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

The final weighted group ranking is B > A > D > C, which in this case is identical to the Arrow's Method of Majority unweighted group rank order.
Algorithm for Thurstone's Law (Cases 2 through 5).

Steps 1 through 8 of Arrow's Method of Majority Algorithm, (see Appendix B), are utilized, if, after step 6, a Chi-Square Goodness of Fit Test indicates the numerical values reflecting the judge's preference for different objectives are normally distributed. After step 6, a Chi-Square Goodness of Fit Test, described as follows, must be inserted in the sequence which is otherwise identical to the steps in Arrow's Method of Majority:

The null hypothesis that the observed values are not different from normally distributed values is tested by computing the Chi-Square test statistic from the relation

\[ \chi^2 = \sum_{i=1}^{k} \frac{(x_i - mp_i)^2}{mp_i}, \]

where, \( x_i \) = observed frequency of values in the \( i \)-th interval
\( m \) = total number of values in the set
\( p_i \) = the probability that a given value will occur in the \( i \)-th interval
\( k \) = total number of intervals over the
range of the data; thus, $m_p = \text{expected frequency of values in the } i\text{-th interval.}$

\[ \chi^2_{\text{calc}} \text{ is compared with } \chi^2_{\text{table}} (k-3 \text{ d.f.}) \]

*(One degree of freedom is lost in summing the frequencies over k intervals, and two are lost by the requirement to estimate the mean and variance of the data set.)*

If $\chi^2_{\text{calc}} > \chi^2_{\text{table}}$, the null hypothesis is rejected, and it is concluded the distribution of the joint order is not normal.

If $\chi^2_{\text{calc}} \leq \chi^2_{\text{table}}$, the null hypothesis is not rejected, and it is concluded that there is no reason to believe the data is derived from other than a normal distribution. In general, the number of judges exceeds the number of objectives being ranked. The Chi-Square Goodness of Fit Test is applied to the set of data corresponding to each objective over the range of all judges. For instance, in this example, there are four objectives, A through D. A separate test would be applied to each of the four data sets. It has been found in practice (29:44) that sets at the extreme ends of the joint order are often non-normal, while the middle sets are normal. This does not invalidate the amalgamation process, and is merely a reflection of some distortion introduced through truncation in transforming the rankings to ordinal values and real numbers. When there are a relatively large number of objectives being ranked, an exception to
the usual procedure may be performed. In this case, where there are at least 20 objectives which have been ranked, the date set of \(R_j\)'s may itself be tested for normality. When this is done, \(n\) is substituted for \(m\) in the relation. No matter which procedure is used, the level of confidence \(\alpha\) (Greek letter Alpha) and the number of intervals \(k\) must be selected arbitrarily before conducting the test. In practice, a sensitivity analysis is conducted \textit{a priori} to determine an appropriate level of confidence. The number of intervals will be selected so that the expected number or frequency of values in the \(i\)-th interval, either \(n\pi_i\) or \(np_i\), will be at least 5 and preferably be 10. (For example, if there are 60 judges, \(k\) could be chosen as 6, and since \(m\) is 60, then \(np_i = 10\) and \(\pi_i = .1667\). Because the degrees of freedom lost is three, the number of intervals must be at least four. Most statistical texts discuss this test in considerable detail. For instance Guenther (8:185-87) covers this test in a clear, concise manner.

If Case 1 applies, only the ordering need be accomplished and the algorithm may commence at this point without the statistical tests being performed.

The steps for using Case 2 will now be listed, and then comments will be made concerning use of Cases 1, 3, 4, and 5.

Steps 1 - 8, plus a test for normality having been accomplished, the following should be done:
9. Calculate the probabilities of differences that exist between the adjacent objectives, as ordered in step 8, using the relation

\[ p(i > j) = f(i > j) / [f(i > j) + f(j > i)]. \]

10. Obtain \( z \) values from a table of the standard normal distribution from any statistical text book.

11. Calculate the unbiased estimate of variance for the \( m \) values of the \( i \)-th objective using the relation

\[ s_i^2 = \left[ 1 / (m - 1) \right] \sum_{k=1}^{m} (x_{ik} - \bar{x}_i)^2, \]

where \( x_{ik} \) is the value of the \( i \)-th objective as ranked by the \( k \)-th judge, and \( \bar{x}_i \) is the mean of the values from \( m \) judges.

12. Calculate the coefficient of correlation \( r_{ij} \), expressing the degree of association between the \( i \)-th and \( j \)-th objectives, by using the relation

\[ r_{ij} = \frac{\text{cov}_{ij}}{s_i s_j}, \]

where \( \text{cov}_{ij} \), the covariance, is computed from the equation

\[ \text{cov}_{ij} = \frac{\sum_{k=1}^{m} (x_{ik} - \bar{x}_i) (x_{jk} - \bar{x}_j)}{(m - 1)}. \]

13. Calculate the difference of values between pairs, \( v_i - v_j \), using the complete form of Thurstone's equation

\[ y_i - v_j = z_{ij} \sqrt{s_i^2 + s_j^2 - 2r_{ij} s_i s_j}, \]
where \( v_i \) and \( v_j \) are the values of the
i-th and j-th objectives.

14. Set the value of the lowest ranked objective equal to 1.00, where
such assignment of this value to the least preferred objective is
arbitrary. The value could have been 0, 10, or 100, but 1.00 is a
convenient convention.

15. Add the difference in value to each ascending objective.

16. Normalize the assigned values using the relation

\[
\frac{i_{\text{N}}} {N} = \frac{i_a} {\sum_{i=1}^{n} i_a},
\]

where \( i_{\text{N}} \) is the normalized value of the
i-th objective, \( i_a \) is the assigned value
of the i-th objective, and \( \sum_{i=1}^{n} i_a \) is the
sum of the assigned values over the set of
n objectives.

17. Now that the values are on the same scale, rank them
cardinally.

Looking at the other four cases, we see the following:

Case 1 - The equation is used in its complete form. This case does not
apply to group opinion; it is used only for a single judge making
multiple rankings.

Case 3 - The equation can be simplified by dropping the last term
under the square root if the i-th and j-th distributions are uncorrelated,
\( r_{ij} = 0 \). This may be verified using the test statistic \( t \) computed from
58
the relation
\[ t = r_{ij} \sqrt{\frac{m-2}{1-r_{ij}^2}} \]

The calculated value of \( t \) is compared to the tabulated \( t \), (see any statistical text for a table of the "Student's" \( t \) distribution), at \( m - 2 \) degrees of freedom and a specified level of significance, \( \alpha \) (Greek letter Alpha). If the calculated value exceeds the tabulated value, the null hypothesis that the distributions are uncorrelated is rejected. The conclusion is that the distributions are correlated, \( r_{ij} \neq 0 \). Conversely, if the calculated value does not exceed the tabulated value, the null hypothesis is not rejected, \( r_{ij} = 0 \).

Case 4 - The equation reduces to
\[ v_i - v_j = 0.707 z_{ij} (s_i + s_j) \]

The same hypothesis test to support this case also supports the even more simplified version, Case 5. Therefore, this case is not recommended for use in group opinion amalgamation.

Case 5 - The equation reduces to
\[ v_i - v_j = 1.414 z_{ij} \sqrt{s_{ij}} \]

The \( F \) - test, where the null hypothesis is that the variances are equal, is applied by computing the \( F \) statistic from the relation
\[ F = s_i^2 / s_j^2, \]

where the larger variance is the numerator.
This calculated value of F is compared to the tabulated value, (see any statistical text for a table of the F distribution) at a specified level of significance, entering the table at \( m_1 - 1 \) (column) and \( m_j - 1 \) (row) degrees of freedom. If the calculated F exceeds the tabulated value, the null hypothesis is rejected. The conclusion is that the variances are not equal. If the calculated value of F does not exceed the tabulated value, the null hypothesis is not rejected. The conclusion is that the variances are equal.

Example problem.
Since the meaningful comparison of techniques can be made using the same problem for all techniques, and since it is necessary that it be designed with a relatively small number of judges and objectives for ease of manipulation and understanding; the example we are using should not pass a normality test. This step is omitted, and we will proceed directly from the joint order we obtained in the Arrow's example to step 9 in our sequence of operations.

The joint order is

\[
B > A > D > C
\]

Writing the set of adjacent differences:

\[
(B > A), \ (A > D), \ \text{and} \ (D > C).
\]

The probabilities that the \( i \)-th objective is preferred to the \( j \)-th objective is calculated:
\[ p(B > A) = \frac{f(B > A)}{f(B > A) + f(A > B)} = \frac{3.5}{3.5 + 2.5} = 0.5833 \]
\[ p(A > D) = \frac{6}{6 + 0} = 1.0 \]
\[ p(D > C) = \frac{5}{5 + 1} = 0.8333 \]

Using the table of the standard normal distribution, the value of the standard normal deviate, \( Z \), corresponding to the \( p(i > j) \), is found.

- \( Z(B > A) = 0.2103 \)
- \( Z(A > D) = 3.5999 \)
- \( Z(D > C) = 0.9660 \)

Now we need an unbiased estimate of standard deviation.

- \( S_A^2 = \frac{1}{(m-1)} \sum_{k=1}^{m} (x_{ik} - \bar{x}_A)^2 \)
  \( \bar{x}_A = \frac{3.5}{6} = 1.5833 \)

- \( S_A^2 = \frac{[(2 - 1.58)^2 + (1 - 1.58)^2 + \ldots + (2 - 1.58)^2]}{5} = 0.2416 \)

The other variances are approximately

- \( S_B^2 = 0.6416 \)
- \( S_C^2 = 0.0666 \)
- \( S_D^2 = 0.3000 \)

- \( \text{COV}_{AB} = \frac{\sum_{k=1}^{m} (x_{ak} - \bar{x}_A)(x_{bk} - \bar{x}_B)}{m-1} \)
  \[ = \frac{[(2 - 1.58)(1 - 1.58) + (1 - 1.58)(2 - 1.58) + \ldots + (2 - 1.58)(1 - 1.58)]}{5} = 0.3114 \]
\[
S_A = \sqrt{S_A^2} = \sqrt{0.2416} = 0.4915 \\
S_B = \sqrt{0.6416} = 0.8009 \\
r_{AB} = \frac{Cov_{AB}}{S_A S_B} = \frac{0.3114}{(0.4915)(0.8009)} \\
= 0.7903 \\
V_B - V_A = Z_{(B > A)} \sqrt{S_B^2 + S_A^2 - 2 r_{AB} S_A S_B} \\
= 0.2103 \sqrt{0.6416 + 0.2416 - 2 (0.7903)(0.4915)(0.8009)} \\
= 0.1072
\]

The other difference of values are approximately:

\[
V_C - V_D = 1.3498 \\
V_D - V_C = 0.6598
\]

Setting \(C = 1.0000\) Normalizing: \(C = .1138\)

then \(D = 1.6598\) \(D = .1890\)

\(A = 3.0096\) \(A = .3425\)

\(B = 3.1168\) \(B = .3547\)

\(\Sigma = 8.7862\) \(\Sigma = 1.0000\)

Which now permits cardinal ranking as displayed on table 3.
TABLE 3 CARDINAL RANKING OF JOINT ORDER PREFERENCE

A .3425
B .3547
C .1138
D .1890

PREFERENCE
APPENDIX E

OTHER AMALGAMATING TECHNIQUES

The following techniques for amalgamating opinion were considered but rejected because they are less desirable than those described in Appendices B through D for accomplishing similar purposes.

Analysis of Variance (16:261-307), usually known simply as ANOVA, is customarily used in statistical analyses to determine if the observed differences among sample means, where more than two samples are being considered, is attributable to chance. In other words, it is used to show whether differences among several means are significant at some specified level of confidence. By employing the Duncan Multiple-Range Test (16:279-281), it is possible to extend the ANOVA to indicate whether a given mean (or group of means), differs significantly from another given mean (or group of means). It is then possible to rank order objectives based upon the ordering of judges. This technique is rejected because it is too laborious. It is also quite restrictive. Specifically, the joint order of the data must be normal with equal variances among the rankings.

A technique known as The Churchman and Ackoff Approximate Measure of Value (5:141-53) provides a collective decision by means of a majority rule on a succession of comparisons. The judges initially rank all objectives, and an arbitrary value of 100 is assigned the most preferred, with decreasing numerical values assigned the remaining ordered objectives.
The judge is then asked to compare the effect of the initial most preferred objective with that of the combined effect of varying combinations of the remaining objectives. A decision as to whether this objective is better, equal to, or worse than the combinations is made for each possible combination. The majority vote is then registered. Adjustments are then made as necessary in the rank order and the values ascribed to the objectives. This technique becomes extremely laborious for more than four or five objectives because of the large number of possible combinations. Churchman's Method is rejected because of the large requirement it imposes not only on the decision maker, but also on the individual judges.

There have been a number of authors who have recommended probabilistic approaches. Two of the most applicable of these are Winkler (28:61-B75) and Raiffa (17:104-56). Despite their arguments that subjective probabilities can be derived which assign numerical quantities to the intuitive feelings a judge has for the occurrence of some event, many other authors question the ease and the reliability of this process. Svestka (35:5-6) submits an argument against using subjective probabilities, stating that our actual belief at any instant is "fugitive and variable" and that it is extremely difficult to fix it in a manner sufficiently clear that it may be measured. He further elaborates his argument stating that it is not only difficult to ascribe numbers to intensities of feelings, but the things one feels most strongly about he frequently either takes for granted or is totally unable to give a probability for. The use of a probabilistic technique is rejected because of the difficulty in directly assigning subjective probabilities.
Black's Single Peaked Preferences (3:74-80) is a technique using an odd number of judges. Each judge orders the objectives individually utilizing a transformation curve on which there is a single point for which his utility is maximized and his utility decreases in either direction from the optimum. Taking the individual orderings representable by single-peaked preferences, the best joint order objective may be removed and then the one best among the remainder, (the one preferred by a majority of the individuals to any other objective remaining), is listed as being second best, etc. Since this technique provides results essentially the same as Arrow's Method of Majority, but is restrictive as to the number of judges, requires transitive orders, and requires every judge to order all the alternative objectives, it is rejected.

Considered also were Kendall's Method of First Differences (29:19-20) and Fishburn's Ordered Metric Method (29:20-21). They are similar in that they both enable the judge to cardinally rank objectives, which he is perhaps unable to rank directly. That is, instead of placing A, B, C--n in an ordered sequence and stating how much more one is preferred to another, the judge orders the objectives and then examines his preferences for each adjacent pair, ranking them in order of their differences from least to greatest using Kendall's Method. In Fishburn's Method, the same process is followed, but in addition, all other possible differences are listed. For example, Kendall's method examines the intensity of difference in preference the judge feels between A and B, B and C, etc., (assuming for convenience of explanation that the sequential ordering and
natural alphabetic lettering were identical), while Fishburn's method includes the adjacent differences, but also looks at A and C, A and D, etc., where each of the new differences spans an interval exceeding at least one of the adjacent differences already in the ranked set. Each new difference is compared to the next greatest difference in the ranked set until it is observed to be less than some difference. The new difference is placed into the ranked set so that it is greater than the difference below it and less than the differences above it. This process continues until all possible differences are ranked. When the judges have completed their ranking of objectives and of differences, the decision maker then assigns values to the spatial differences compatible with the relationships stated by the judges. The values of the differences are then used to compute the values of the objectives. The objective values are then normalized. Kendall's Method of First Differences appears to be a feasible technique; however, Thurstone's Law of Comparative Judgements accomplishes the same result, is more amenable to large numbers of judges and objectives, and is more compatible with the other techniques selected. If cardinal ranking is an overwhelming requisite and if the joint order fails to meet the normality requirement, Kendall's Method of First Differences might be considered. Fishburn's Higher Ordered Metric Measure is similar to the Churchman and Ackoff Approximate Method considered earlier, in that, for other than very minimal numbers of objectives it becomes too laborious for both the judges and the decision maker.

Certainly this paper would be remiss if it did not address the DELPHI Technique. It was developed to obtain the benefits accruing from an
interplay of ideas between a group of judges without being plagued with the attendant problem of staff meetings, conferences, panels, or brainstorming sessions. Its developers were attempting to avoid the pitfalls which may be associated with such group gatherings in their design of DELPHI. Inequality of the judges because of rank or experience and psychological factors such as a dominant personality, the desire for group approval, or a stubborn unwillingness to change from a previously expressed opinion are some of the alleged shortcomings of group meetings they attempted to design out of the DELPHI Technique (30:7 & 11:2).

Originally developed by the Rand Corporation in the late 1940's (11:2-4), it was widely used by universities, by the military services, by the Defense Intelligence Agency and the Central Intelligence Agency, and within civilian industry. It replaces direct debate with a carefully designed program of sequential individual interrogations (4:2-6). The judges receive information on the previous response which includes the median and the interquartile range of those rankings. They may also be provided one, or a few, relevant facts in addition to statistical feedback alone (6:28-30). An obvious limitation to the use of DELPHI is that the very nature of the feedback process necessitates the availability of considerable time. Sackman, in an indepth assessment of DELPHI (19:67-68), concludes that it is "basically an unreliable and scientifically unvalidated technique in principal" and that "its claim to represent valid expert opinion is empirically untenable and overstated." He further concluded that the allegedly designed-in features of the superiority of remote and private opinion over face-to-face encounter, as well as their counter-statements,
are unproven generalizations, and DELPHI anonymity reinforces unaccountability in method and findings. It also "discourages adversary process and inhibits exploratory thinking." Consequently, it was discarded as a recommended technique.
Cited References

Books and Reports


Articles


Unpublished Materials


Other References

Books and Reports


Articles


Unpublished Materials


