OCEAN SURFACE WAVES: A BIBLIOGRAPHY AND SUMMARY OF RESEARCH.

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The surface of the ocean has a dual role in the research and development problems of the Navy Electronics Laboratory. In most naval problems it is the lower boundary for electromagnetic radiation and the upper boundary when one is concerned with underwater acoustics. It is natural, therefore, as the theories of propagation advance an interest arises as to the detailed character of the sea surface and the means of describing it mathematically. Interestingly, or perhaps not to the better informed, the wave lengths of electromagnetic radiation in air from 10 to 3000 megacycles are nearly the same as those of underwater sound of frequencies from 50 cycles to 24 Kc. Thus the information which is useful for the one case is probably applicable in the other.

This bibliography and brief literature survey was undertaken, specifically, to furnish information regarding the surface of the ocean, which could be used in the theory of reflection and scattering of electromagnetic radiation from the sea surface. It does not pretend to be a complete bibliography on ocean waves. Neither is it limited to the most recently published and accepted ideas. Research in ocean surface waves is a relatively new field, having received little impetus until World War II. Even at the present time deep sea observational data are insufficient for checking the latest theories on characteristics of ocean waves. Thus it seems that a brief historical review of research in ocean waves may be of value to the independent researcher as well as a summary of the latest results.
The bibliography is limited primarily to those publications which would have a bearing on the reflection, scattering, and absorption of radiation (electromagnetic or acoustic) impinging upon the surface. References to many fields of application of ocean waves have been omitted, such as,

- Forces of waves on structures
- Internal waves in the ocean
- Wave data derived from hindcasting techniques
- Relation of wave action and microseisms
- Transformation of waves from deep to shallow water, breaking waves, and other shoreline processes
- Long period waves—tides, seiches, momentum waves, ...
- Engineering of wave recorders
- Ship waves and wave resistance

Included in the bibliography are a few references, mainly observational, bearing upon the relation between sea states and underwater acoustic propagation. Conversely, there are none relating sea states to electromagnetic propagation, partially because apparently few exist, but mainly because unfamiliarity with electromagnetic theory leaves one with a feeling of inadequacy for sifting and selecting pertinent information.

SEA AND SWELL

"Sea" and "swell" are very broad terms used to differentiate between two general wave conditions, and the distinction should be noted at this time. "Sea" is used to describe waves resulting from the direct influence of the local winds. If the wind is low these may be only small wavelets as seen in a protected bay, but at higher wind speeds the waves become noticeably irregular. White caps appear (at winds over Beaufort Force 4),
heights vary rapidly, the distance between crests is short, a single crest cannot be followed parallel to the coast more than a few wavelengths (short crestedness), and waves do not appear to be traveling in just one direction but up to relatively large angles about a mean direction.

As the waves travel away from the generating area through a region of relative calm, they take on a more regular appearance, which is termed "swell" and is much more familiar to coastal residents of Southern California. With distance from the storm the waves appear to become longer and lower, and the heights do not vary rapidly, one wave closely resembling the next. Whitecaps are not a characteristic of swell.

Assuming an adequate knowledge of ocean waves, it is seen that swell can be predicted, or extrapolated, from known sea conditions. In establishing characteristics of the sea surface from easily observed parameters, we are therefore interested in the relation between wind and sea. The discussion to follow will pertain generally to the establishment of the characteristics of "sea" except as noted.

WAVE RESEARCH THROUGH WORLD WAR II

Classical studies of ocean waves are due primarily to Airy, Stokes, and Gerstner. Solutions to the wave equation were found for various types of waves, such as, infinitely low waves, waves of finite height, irrotational, and rotational, all involving a surface boundary condition of a simplified wave pattern of infinitely long-crested oscillatory waves. Details are to be found in Lamb's HYDRODYNAMICS and a summary of the theories of oscillatory waves was published by the War Department, Beach Erosion Board (Reference 11) at the beginning of World War II.

Early visual observations of sea state were reported by one of two very crude coding systems numbering from zero to nine. The first system
combined lengths of short, average, or long with a height description of low, medium, or high, e.g. high swell average. The second scale which is still widely used classifies the waves according to height ranges. The inadequacy of this type of observation for detailed analytical work is apparent.

A notable exception to the subjective observations was the work of German scientists in making stereophotographs of the sea surface. The thirty plates due to Schumacher (Reference 42) showing contours of height at one-half meter intervals over an area about 150 by 250 meters is one of the notable early contributions in presenting "3-D" data on the ocean surface. Ten plates were taken later by Weinblum and Block (Reference 43) using Schumacher's equipment. One plate from this paper is reproduced in figure 1. From this it is easy to recognize the scope of the problem of characterizing the sea surface.

During World War II wave research received a large impetus because of the problems of mine warfare and amphibious landings. The research into the whole field of ocean waves from generation by wind, through propagation, to transformation and breaking in shallow water culminated in the Sverdrup-Munk procedures for forecasting sea, swell, breakers and surf (Reference 9, 53, 54, 57). Here the concept of "significant waves" was adopted. The "significant height and period" were defined as the average height and period of the one-third highest waves, and were those waves which an observer tended to record. Imperical relationships were established between the significant wave and wind speed, duration time of the wind, length of generating area (fetch), distance from storm, travel time, and decrease in wave height during travel for the purpose of forecasting sea, swell, breakers, and surf conditions.
Although it is recognized as insufficient in itself, the "significant wave" is still used as one of the parameters to describe waves, along with other averages, such as, the average of the highest one-tenth waves, especially in dealing with operational problems. The use of the above procedures and the attempts to refine them, with the aid of studies from the field of random noise, have led to the presently accepted concepts and methods of characterizing the sea surface.

CUMULATIVE WAVE RESEARCH (POST 1949)

It is only within the past several years that methods of measurement and analyses for describing the surface have accounted for its great variability. It is now agreed that at least two other parameters besides some average of wave height and period are necessary to describe the surface of the sea. The first is the frequency distribution of wave heights within a time sample and the second is the wave spectrum, energy distribution versus frequency (most often expressed as period in wave studies). Other properties of waves that have yet to be investigated fully are the slope, curvature, and beam width (directionality).

Under a given wind condition, when the time record of the sea surface at a fixed point is analyzed, it is found (1) there exist both a frequency distribution of wave heights and a continuous spectrum of periods and (2) there is no unique relationship between the height and period (or wave length).

Distribution of wave heights.

Longuet-Higgins (Reference 5) has investigated analytically the case of a single narrow frequency band of waves assumed to have originated at many places over a wide area and having random phase, so that the probability distribution of wave amplitudes can be given by the Rayleigh "random walk"
distribution. Average amplitudes for various high fractions of the total
number of waves are expressed by

$$a^{(p)} \overline{a} = {x \over \bar{a}} + C \int_0^\infty \frac{e^{-y^2}}{y} dy$$

Eq. (1)

where $\bar{a}$ rms amplitude

$p$ proportion of $a$'s which exceed $r$

$a^{(p)}$ mean value of all waves of amplitude greater than $r$

A graph of $a^{(p)} \overline{a}$ versus $p$ is shown in figure 2.

Although derived for a single narrow frequency band, the ratios found
by Longuet-Higgins agree fairly well with those obtained from observed wave
records, which are certainly not limited to cases meeting these restrictions.
Some of these are compared in Table I. Probably the most satisfactory
comparison with the theoretical values is obtained when the waves originate
from a single distant storm.

Note in equation (1) that as $p$ approaches zero $a^{(p)} \overline{a}$ approaches
infinity logarithmically, so that "for the validity of this result it is
essential that the sample containing $pN$ wave amplitudes shall not be too
small." (N being the total number of waves in the record). The relation-
ship of the expected maximum amplitude and most probable maximum amplitude
to the rms amplitude is treated further by Longuet-Higgins, with special
reference to ship motion.

Because other measures of wave height may be reliably related to the
rms amplitude the problem of forecasting the sea condition becomes a problem
of relating this parameter to the wind speed, duration time, and fetch.
<table>
<thead>
<tr>
<th>Author</th>
<th>Source</th>
<th>$a_{m}/a$</th>
<th>$a^{(1/10)}$/a</th>
<th>$a^{(1/3)}$/a</th>
<th>$a_{rms}$/a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longuet-Higgins (1952)</td>
<td>Analytical</td>
<td>1.77</td>
<td>1.27</td>
<td>1.60</td>
<td>0.886</td>
</tr>
<tr>
<td>(Also Swann and Barber (1950)</td>
<td>Ref. 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seiwell (1948) Ref. 20</td>
<td>Bermuda and Cuttyhunk</td>
<td>1.57</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wiegal (1949), Ref. 23</td>
<td>Pt. Sur, Calif.</td>
<td>1.85</td>
<td>1.27</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hecate Head, Ore.</td>
<td>1.91</td>
<td>1.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pt. Arguello, Calif.</td>
<td>1.85</td>
<td>1.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>1.87</td>
<td>1.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Range of Individual</td>
<td></td>
<td></td>
<td>±20%</td>
<td>±10%</td>
</tr>
<tr>
<td></td>
<td>Values</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Johnson (1950) Ref. 15</td>
<td>Abbott’s Lagoon</td>
<td>1.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Darbyshire (1952) Ref. 27</td>
<td>Fendeen and Perranporth, England</td>
<td>1.49</td>
<td>1.61</td>
<td>0.83</td>
<td></td>
</tr>
<tr>
<td>Putz (1952) Ref. 32</td>
<td>Average for 25 records***</td>
<td>1.62</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Range in 25 records</td>
<td></td>
<td></td>
<td>1.50-1.83</td>
<td></td>
</tr>
</tbody>
</table>

*Observed data are from pressure records corrected for depth of recorder.

**$a_{m}$ = Maximum amplitude, $a^{(1/10)}$ = average amplitude of highest one-tenth, $a^{(1/3)}$ = average amplitude of highest one-third, $a_{rms}$ = average amplitude.

***Records from Oceanside, California (4); Pt. Sur, California (15); Hecate Head, Ore. (2); Guam, M.I. (3); Pt. Arguello (1).
Wave Spectrum

Periodogram analyses of wave records, first initiated by the British in 1945 (Reference 34), indicate that ocean waves from a single source area can be considered as having a continuous frequency spectrum with a single maximum. A spectrum analysis of waves with both swell and a local wind sea present is more complex and may show a double energy maximum, see figure 3. The continuous spectrum is considered to be the resultant of the generation at different points in the storm area of a large number of small waves of random phase.

For predicting wind waves Neumann (Reference 6) using visually observed data and dimensional considerations has proposed a spectrum of waves for a fully developed sea, as follows

$$W_0 = \frac{C_{\text{const}} \cdot \lambda^2}{\pi^3 \sigma^3 \gamma^{-6}} + \frac{\frac{2}{\gamma^2}}{\gamma^2}$$  
Eq. (2)

where  
$W_0$ - the relative energy distribution  
$\lambda$ = frequency  
$\nu$ = wind velocity

The Neumann wave spectrum is shown in figure 4 for three different wind speeds.

The properties of the Neumann spectrum should be noted, (1) the product of the frequency of the energy maximum, $\lambda_{\text{max}}$, and the wind velocity is a constant

$$\lambda_{\text{max}} \cdot \nu = \sqrt{\frac{2}{g}}$$  
Eq. (3)
and (2) by integration "the total energy accumulated in the composite wave motion of a fully arisen sea increases in proportion to the fifth power of the wind speed."

The rms amplitude, \( \bar{a} \), is related to the total energy, \( U \), by

\[
\bar{a}^2 = \frac{2U}{g} \int \int_{P_0} W_0 \, d\omega
\]

Eq. (4)

From observations of wave height the constant in the total energy equation is determined by Neumann, so that both the spectrum and the wave heights of a fully developed sea are expressable as functions of the wind velocity. It follows from the total energy equation and was verified experimentally by observations of Neumann that the height parameter is proportional to the 2.5 power of the wind speed.

In recent literature by Pierson et al. (References 7, 8, 61) on wave theory and analysis and forecasting procedures the rms amplitude is designated as \( \sqrt{E} \). The wave forecasting procedures involve prediction of a point on a "co-cumulative spectrum" which establishes the range of wave periods and the quantity, \( \sqrt{E} \), from the wind velocity, fetch, and duration time. These procedures are described fully in reference 61, along with empirical modifications which are made when wind conditions do not generate a fully developed sea.

That there is not full agreement on the forecasting of wave spectra is evidenced by the fact that Darbyshire (Reference 27) proposes a sharply peaked spectrum composed of two intersecting curves with the total energy proportional to only the third power of the wind speed, as opposed to the fifth power due to Neumann. Darbyshire's results agree with Neumann in that the period of the maximum amplitude is a constant times the wind speed for a fully developed sea.
where \( T_s \) is the period of the highest waves in seconds, \( U \) is the gradient wind in knots, and \( \lambda \) is the fetch length in nautical miles, so that the exponential term is negligible when \( \lambda > 100 \) N. miles.

**Autocorrelation**

A third type of presentation, the autocorrelation function of a wave record, deserves mentioning although it has not been widely used. One may argue that the same information is available through the more easily obtained spectrum. On the other hand, some physical properties of the record may be more readily recognized from the autocorrelation function (Tukey, Reference 65). In particular, the repeatability or forecastability can be measured by the autocorrelation function. This application is discussed by Seiwell (Reference 40) who considers that the fluctuations of the ocean surface may often be represented as having two general parts, (1) one or more cyclic components related to swell from distant storms and (2) an autoregressive component, due to the action of local winds.

**Slope of the Sea Surface**

By using photographs of the sun's glitter on the sea surface Cox and Munk (References 25, 26) have determined that the rms slope is linearly related to the wind speed. The maximum rms slope found over the range of wind speeds encountered (up to 30 knots) was about 15°. The slopes appear normally distributed crosswind with a slight skewness appearing in the distribution parallel to the wind, the greater slopes being more frequent on the downwind side of the waves. It can be inferred from the Neumann spectrum that the period of these waves is about 2 to 3 seconds. However,
this should be considered with caution until further evidence is obtained, because it is suspected that the Neumann spectrum is deficient in the high frequency region, particularly, the region of low frequency capillary waves. Research anticipated by Cox and Munk on the curvature of the waves will assist in answering this question.

DEFICIENCIES

Three deficiencies in the knowledge of ocean waves which are yet to be overcome, are receiving increased attention of investigators, the directional properties, empirical data from generating areas, and theory of generation of waves.

Directional Properties

With the exception of one recent unpublished set of observations and excluding the slope investigations which pertain to short periods, all records of ocean waves or swell have been made with a recorder at a single point versus time, or along a line as with a radio altimeter recording from aircraft. Thus, the directional properties of the various components of the waves are not considered. Aside from the "beam width" of sea from a given storm, the directional properties of waves at a point arriving from two or more distinct generating areas may be an important factor in underwater acoustic scattering and reflecting properties of the sea surface.

Empirical data

Good observational data on the height distribution and spectrum of ocean waves in the generating areas are still lacking, especially for higher wind speeds. These are required to verify the theoretical spectra and establish the constants in them. The need for these is apparent when one considers that operationally the sea surface character
must be established from readily available parameters, such as meteorological charts and weather observations, as opposed to the difficult observation and lengthy analysis of direct wave measurements.

Theory on the Generation of Waves

There remains the need for a theory relating the physical forces generating the waves to the observable physical properties (Eckart, Reference 1). The interpretation of dependent or related events, such as scattering from the surface, may lie in a full understanding of the basic phenomenon.

Organization of Bibliography

The bibliography which follows has been divided into several sections according to subject material.

I. General Theory. Papers on the solution of wave equations under differing boundary conditions, theory of generation, theoretical distributions and spectra.

II. Observational Data and Analysis.

A. Simplified Parameters: Data which are presented in the form of average or significant heights and periods, as opposed to distributional.

B. Distributions: Frequency distributions of wave heights, periods, and slopes, for "individual waves" within a given observational or sampling period.

C. Spectra and Autocorrelation: Wave spectra, and autocorrelations for a given wave record.

D. Stereophotogrammetry: Contour charts of the sea surface from stereophotos.
III. Ocean Waves and Underwater Acoustics

A. Observations: Observed phenomena in underwater acoustics related to sea state, correlation between propagation and sea states, reflection coefficients.

B. Theoretical and experimental: Theories and laboratory experiments relating underwater acoustic scattering or reflection to sea state.

IV. Miscellaneous: Summaries of wave research, model studies of waves, analytical procedures, recording and analysis systems, wave forecasting procedures.

Not all papers, of course, are limited to a single subject. It is attempted to show those papers which contain a significant amount of information in more than one subject by listing them under the primary subject and making a notation under the secondary subject.

ACKNOWLEDGMENT

Appreciation is expressed to Drs. R. S. Arthur and W. H. Munk, Scripps Institution of Oceanography for reading the summary. The author, however, must assume full responsibility for the choice of material to be included and to be omitted.
BIBLIOGRAPHY

1. **GENERAL THEORY:**


See also reference 27

II. OBSERVATIONAL DATA AND ANALYSIS:

A. Simplified Parameters:


B. Distributional:


See also references 19 and 36.

C. Spectral and Autocorrelative:


III. OCEAN WAVES AND UNDERWATER ACOUSTICS:

A. Observations:


45. Raitt, R. W. "Reflection Coefficients of Surface and Bottom," UCDWR Int. Rept. No. A8, 22 May 1944. Confidential


B. Theoretical and Experimental:


IV. MISCELLANEOUS:


STEREOPHOTOGRAMMETRIC PLOT
[FROM WEINBLUM & BLOCK, 1949; REFERENCE 43]

CONTOUR INTERVAL; 0.5 METER

LOCATION: 49° 50' N, 13° 30' W
WIND SPEED: TO 30 METERS/SEC
WAVE HEIGHT: 13 METERS
WAVE LENGTH: ~ 200 METERS
MAX. SLOPE: 0.40

FIG. 1
GRAPH OF $\frac{a^{p^2}}{a}$ AS A FUNCTION OF $p$

FOR A NARROW FREQUENCY BAND

(EQUATION 1)

[FROM LONGUET-HIGGINS, 1952; REF. 5]

FIG. 2
WAVE SPECTRA

FROM BARBER AND URSELL, 1948

REFERENCE 34

14 - 15 MARCH, 1945
PENDEEN, ENGLAND

16 - 17 MARCH, 1945
PENDEEN, ENGLAND

17 MAY, 1946
PERRANPORTH, ENGLAND

THE ANALYSES FOR 15 MARCH 23:00 HRS AND ONWARDS ARE AT A REDUCED SENSITIVITY (1/2 OF PREVIOUS RECORDS)

NARROW BAND SWELL FROM DISTANT STORM IN NORTH ATLANTIC, NO LOCAL SEAS

BROAD BAND SWELL FROM NEARBY STORM WITH LIGHT LOCAL SEAS

DOUBLE MAXIMUM SPECTRUM, SWELL FROM STORM IN THE SOUTH ATLANTIC, MODERATE LOCAL SEAS

NOTE: METHOD OF ANALYSIS PRODUCES SERIES OF PEAKS AT SUBMULTIPLE INTERVALS OF TOTAL RECORDING TIME. ENVELOPE OF PEAKS WOULD BE REPRESENTATIVE OF THE CONTINUOUS SPECTRUM.

FIG. 3
Wave spectra for fully arisen sea at a wind speed of 20, 30, and 40 Knots, respectively.

\[ U, C_{40} \] Knots

\[ (I) \]

FIG. 4

[SPECTRAL ENERGY IN ARBITRARY UNITS]

\[ v = \frac{1}{T} \]

[FROM NEUMAN, 1953; REFERENCE 6]