THE DYNAMICS OF THE BOTTOM DENSE LAYER OF THE DEEP OCEAN

by

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The dynamics of the bottom boundary layer of the deep ocean.

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THE DYNAMICS OF THE BOTTOM BOUNDARY LAYER OF THE DEEP OCEAN*

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ABSTRACT

Profiles of salinity and temperature from the center of the Hatteras Abyssal Plain have a signature that is characteristic of mixing up a uniformly stratified region: a well-mixed layer above the bottom, bounded by an interface. The penetration height of the mixed-layer varies from about 10 m to 100 m and has been correlated by Armi and Millard (1976) with the one day mean velocity, inferred from current meters located above the bottom boundary layer.

Here the dynamics of such layers is discussed. A model of entrainment and mixing for a flat bottom boundary layer is outlined; this model is however incomplete because we find too little known of the structure of turbulence above an Ekman layer. An alternate model is suggested by the estimate, from the correlation of penetration height with velocity of the internal Froude number of the mixed layer, $F \approx 1.7$. This value indicates that the large penetration height may be due to the instability of the well-mixed layer to the formation of roll waves.

INTRODUCTION

In a recent study of the bottom boundary layer of the deep ocean Armi and Millard (1976) have described aspects of this layer as observed with a CTD profiler. An example of a salinity and potential temperature profile taken over the smooth Hatteras Abyssal Plain is shown in figure 1. Here the well-mixed region extended to about 55 meters above the bottom and was bounded by a sharp interface across which the salinity, potential temperature and potential density changed. Above the interface there was a nearly uniformly stratified region.

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Figure 1. A salinity, potential temperature profile (from Armi and Millard, 1976) in the middle of the Hatteras Abyssal Plain. Dotted line indicates structure could have formed by mixing up the stratified region above. The traces from both the lowering and raising of the profiler are shown.

The signature of the bottom boundary layer, with its well-mixed region bounded by an interface and a stratified region above, is distinctive. A signature like that of Fig. 1 is also seen in laboratory experiments in which a uniformly stratified fluid in a tank is mixed up by stirring with a grid as in the experiments of Cromwell (1960) and Linden (1975), or a surface stress as in the experiments of Kato and Phillips (1969). The signature is typical of a mixing process, the penetration of which is bounded by the interface which forms as a result of mixing the uniformly stratified fluid.
The thickness of the characteristic well-mixed region of the bottom boundary layer on the smooth abyssal plain was correlated by Armi and Millard (1976) with the one-day mean velocity measured just above the layer to show that the layer is of dynamic origin. The region does not form a pool or have a distinctive water mass characteristic. The thickness of the layer is large compared with estimates of the turbulent Ekman layer height.

Some quantitative relationships. Quantitative data characterizing the observed bottom mixed layers and the adjacent stratified fluid are as follows. A correlation of penetration height and velocity for the smooth Hatteras Abyssal Plain gives

\[ \frac{h}{U} \approx 1.2 \times 10^3 \text{ sec}; \] (1)

typical values of \( h \) and \( U \) being 50 m and 4 cm/sec. The celerity of long interfacial waves can be calculated knowing the Brunt-Väisälä frequency in the stratified region above the mixed layer. The Brunt-Väisälä frequency,

\[ N = \sqrt{g \frac{\partial \rho}{\partial z}} \] (2)

is .4 c.p.h. \((7 \times 10^{-5} \ \text{sec}^{-1})\) and is nearly constant for all the profiles taken over the center of the Hatteras Abyssal Plain. By continuity of mass the reduced gravitational acceleration for the mixed layer is given by

\[ g' = g \frac{\Delta \rho}{\rho} = \frac{1}{2} N^2 h. \] (3)

Neglecting the stratification above the interface, the celerity of a long wave on the interface is

\[ c = \sqrt{g' h}, \] (4)

about 2.5 cm/sec for the typical penetration height of 50 meters. An internal Froude number, \( F \), can be defined for the bottom-mixed layer, c.f. Turner (1973, p. 12):

\[ F = \frac{U}{c} \] (5)

Using (3) and the ratio of penetration height to velocity we find \( F \approx 1.7 \).
Turbulent Ekman layer and stratified Ekman layer models. Theories for the bottom boundary layer have considered the effects of rotation alone and in combination with the stabilizing effect of the sharp interface formed by entrainment of the stratified fluid above. We will now review these theories for the bottom boundary layer, limiting the review primarily to those theories for a flat bottom.

Can the bottom mixed layer be treated as a classical turbulent Ekman layer? Such a treatment has been suggested by Wimbush and Munk (1971) and Weatherly (1972, 1975). The experiments of Caldwell, Van Atta, and Helland (1972) and Howroyd and Slawson (1975) give the height of the turbulent Ekman layer, $h_e$, as

$$h_e = \frac{4}{f} \frac{u^*}{f}.$$  

(For the experiments of Caldwell et al. $h_e = \delta_{95}$, the height at which the velocity is 99% of the geostrophic velocity; for the experiments of Howroyd and Slawson $h_e$ is defined as the height at which the velocity is parallel to the geostrophic velocity.) Biscaye and Eittreim (1974) report that photographs on the Matteras Abyssal Plain display a monotonous, flat, mud bottom showing only some "lebenspuren" with relief of about 1 cm. With the Coriolis parameter $f = 7 \times 10^{-5}$ sec$^{-1}$ and the friction velocity $u^*$ for a smooth bottom given by

$$u^* = (1/30)U,$$  

(c.f. Csanady, 1967)

the Ekman layer height to velocity ratio is

$$\frac{h_e}{u^*} \cong 2 \times 10^2 \text{ sec.}$$  

Using the empirical result of (1), we see the penetration height of the mixed layer is about six times the turbulent Ekman layer height. The Ekman layer can thus constitute only the lower sixth of the well-mixed region of the bottom boundary layer. For the typical velocity of 4 cm sec$^{-1}$ the Ekman height is only 8 meters.

The effects of unsteadiness are discussed by Wimbush and Munk (1971) who note the time scale for the entire Ekman layer is $2\pi/f$; therefore those features of the boundary layer, in particular the logarithmic layer, with time scales very much less than $2\pi/f$ can be approximated by steady-state theory. As suggested by Munk, Snodgrass and Wimbush (1970), the simplest
procedure is to reinterpret the turbulent Ekman height (6) with $u_a$ dependent on the local mean current. Even if $u_a$ were dependent on the maximum value of the velocity, the Ekman layer height given by (8) would be larger by at most 6 m since the most energetic inertial or tidal velocities are $\approx 3$ cm/sec.

We must also be cautious about using an Ekman layer to model even the lower portion of the bottom boundary layer on a slope of only one in a hundred. Then an advective term, say $\partial u / \partial y$, in the mean value equations of motion, will scale in the Ekman layer like $\partial U^2 / h_e$, where $\alpha$ is the slope. The Rossby number in the turbulent Ekman layer can then be found using (6) and (7). It is of order $10^2\alpha$. The Rossby number is therefore of order unity for a slope of only $10^{-2}$; inertial effects then must be included in a model of this layer. Nonetheless, the vertical length scale defined by (7) is probably still important. It is the length scale for which the Rossby number of the most energetic bottom generated turbulence is of order unity. At this length scale the effects of rotation will be felt by the largest turbulent "eddies".

A model, of the bottom boundary layer, combining the effects of stratification and rotation, has been proposed by Thompson (1973). This is a slab model of the homogeneous well-mixed region, with the penetration height given by a bulk Richardson number closure assumption based on the height of the layer and jumps across the sharp interface of density and velocity. The velocity difference is that between the geostrophic velocity above and the mean velocity in the layer. This bulk Richardson number closure is also used by Pollard, Rhines and Thompson (1973) to whom Thompson refers for a detailed explanation. The penetration height of 8 meters, for $u \approx 5$ cm sec$^{-1}$, as suggested by Thompson is much too small.

Csanady (1974) finds the parameterization of interfacial stability, used by Pollard et al. (1973) and Thompson (1973), unattractive because the control of the mixed layer depth may be independent of the mechanism that maintains the stability of the interface. Csanady suggests that a limit to entrainment is set by the turbulent length and velocity scales in the mixed layer and the density difference across the interface. This asymptotic limit for the entrainment is deduced from the experiments of Kato and Phillips (1969). The limit used by Csanady is given by

$$ \frac{\delta \rho}{u_a^2} = \frac{500}{g'} $$

where $g'$ is the reduced gravitational acceleration at the interface,
h is the layer thickness, and $u_*$ is the friction velocity or turbulent velocity scale. We note however that with $u_*^2 = 10^{-1} U^*$, used by Csanady, equation 9 is equivalent to a Froude number closure of $F = 1.4$, not very different from the value chosen by Pollard et al. (1973) of unity or the empirical result found here of $F \approx 1.7$. The existence of an asymptotic entrainment limit, such as expressed by (9) above and used by Csanady, is certainly not established and must be questioned in light of the experiments of Turner (1973, p. 291) in which no tendency towards a limit is actually observed.

Effects of convection. The effects of convection due to geothermal heat flux on the ocean bottom have been estimated using the Monin-Obukov length, $L_M$, by Wimbush and Munk (1971). With a geothermal heat flux $H = 1.5 \times 10^{-6}$ cal cm$^{-2}$ sec$^{-1}$ and $u_* = 0.1$ cm sec$^{-1}$, $L_M = 10^2$ m. This scaling must however be approached with some caution since the dimensional argument used to derive the Monin-Obukov length contains neither the effects of the smaller length scale, $h_0 = ku_*/f$, due to rotation, nor the existence of a well-mixed layer bounded by an interface.

We note also that the time required to raise the temperature of a well-mixed layer 50 m in height by 1 m$^2$C due to geothermal heating alone is about 40 days. Temperature variability due to the mesoscale variation on a time scale of 40 days can be as large as 40 m$^2$C.

One might argue that convection due to the variability of 25 m$^2$C of the background temperature gradient moving across a constant temperature bottom might create the 50 m thick layers by penetrative convection into the ambient 0 m$^2$C present. However such convection due to a constant ambient temperature and varying temperature due to advection of the mixed layer by mesoscale motions will also fail to explain the unusually large penetrative heights of the mixed layers because of the short diffusive length scale and hence small heat content within the sediments, over the time scale of the eddy. The available maximum temperature contrast due to variability of the background temperature field will only penetrate the sediments to a depth scale $L \approx \sqrt{CT}$ or about 50 cm in 20 days. Indeed a small depth compared with a typical layer depth of 50 m. The signatures of the profiles also do not support such an argument.

Differential advection. We can estimate the slope of the interface that would be associated with any differential advection within the mixed layer and above the bottom Ekman layer, assuming such differential advection is a geostrophic flow and no interfacial stress exists. A change in height of the interface, on the order of the mixed-layer depth, $h$, will then occur over
a distance, $x$, given by

$$x = \frac{g^2 h}{f v^*} \quad (10)$$

If the assumed differential velocity is only one tenth the geostrophic velocity, $U$, we can use typical values $c = 2.5 \text{ cm sec}^{-1}$, $h = 50 \text{ m}$, $U = 4 \text{ cm sec}^{-1}$ and find $x \approx 2 \text{ km}$ and $h/x \approx 1/50$. Variations over this short a length scale were not observed, and we conclude that any geostrophic differential velocity above the Ekman layer must be slower than about $.4 \text{ cm sec}^{-1}$.

Some preliminary thoughts on modeling the bottom boundary layer. The treatment, of the entrainment by turbulence at an interface in terms of estimated turbulence velocity and length scales at the interface (c.f. Turner, 1973, chap. 9) is attractive. The idea is particularly attractive if a strong feedback mechanism exists between the entrainment mechanism and the generation mechanism for the turbulence. It is useful therefore to explore the possibility of characterizing the turbulence in the well-mixed region of the bottom boundary layer and testing the feasibility of establishing either an entrainment velocity that is small but finite, or even a hard entrainment limit based on the turbulence reaching the interface.

The well-mixed region of the bottom boundary layer has been shown to consist of a lower part, about one sixth the total height, which can be modeled as a turbulent Ekman layer. The turbulence level in a turbulent Ekman layer has been measured in the experiments of Howroyd and Slawson (1975) and Caldwell et al. (1972). But what of the turbulence above the height of the Ekman layer and below the interface? Here the experiments of Howroyd and Slawson indicated that the turbulence velocity fluctuations approach a nearly constant value of $u^* \approx .025 U$; Tatro and Mollo-Christensen (1967) and more recently Ingram (1971) have reported similar results. Some controversy regarding these experiments has been pointed out by Cerasoli (1975). Tatro and Mollo-Christensen attribute the dominant frequency of the fluctuations, which is just less than the inertial frequency, $f$, to inertial oscillations in the interior region. They suggest that an instability in the Ekman boundary layer, which always has a frequency higher than the inertial frequency, can excite a first subharmonic with less than the inertial frequency, and the resulting inertial wave is found to propagate throughout the region above the Ekman layer. It should be noted that in a turbulent Ekman layer the largest turbulent fluctuations may also have the correct velocity and length scales to excite radiating inertial
waves. With a turbulence frequency, \( \omega = 0 \left( \frac{u_\infty}{h_e} \right) \), and the length scale, \( h_e = 0 \left( \frac{u_\infty}{\tau} \right) \), the turbulence frequency is given by \( \omega = 0(f) \); the turbulence could thus excite inertial waves and it will become difficult to distinguish between such waves and true turbulence.

With, as yet, so little known about the fluctuations which may occur in the mixed region above the turbulent Ekman height, it is perhaps premature to attempt to characterize an entrainment limit. Perhaps it would be more appropriate to characterize the stability of the radiated inertial waves. Nevertheless, if the fluctuations (either turbulent or radiated) scale with \( u_\infty \), and their vertical length scales with \( h_e \), the Ekman height, then the value of the turbulent Richardson number, \( \text{Ri}^\prime \), (c.f. Turner, 1973, p. 291)

\[
\text{Ri}^\prime = \frac{g^3 h_e}{u_\infty^4}
\]  

(11)

that corresponds to the observed empirical bulk Richardson number of \( \text{Ri} \approx 0.4 \), is \( \text{Ri}^\prime \approx 70 \). Turner (1973, fig. 9.3) finds that for this turbulent Richardson number the entrainment velocity has decreased to about \( 5 \times 10^3 \) times the unstratified entrainment velocity. Perhaps entrainment just matches the rate at which the mixed layer interface is eroded. We believe the erosion mechanism is likely to be internal waves breaking at the interface, a region of higher relative Brunt-Väisälä frequency.

The instability of the bottom well-mixed region to the formation of roll waves; a possible mechanism controlling the penetration height of the well-mixed layer. The immediate purpose of the arguments to be presented below is to suggest a possible mechanism by which a limiting vertical penetration height may be established. The appeal of the mechanism is that it does not depend on a detailed understanding of the entrainment and mixing mechanisms within the well-mixed layer; it was shown in the previous section that the details of this mixing are somewhat elusive. We are strongly attracted by the empirical correlation which yielded the result that the internal Froude number of the bottom boundary layer, \( F \approx 1.7 \). What kind of mechanism has an internal Froude number with a limiting critical value given by \( F \approx 1.7 \)?

We suggest that the mechanism may be the instability of the mean flow, in the well-mixed layer, to the formation of intermittent surges or roll waves. Such roll waves have been known for many years to occur in open channel flows on supercritical slopes; a classic picture of them can be found in the book of Cornish (1934). [See also Dressler (1949) and Stoker (1957,
Roll waves or intermittent surges form when the Froude number of the flow is sufficiently supercritical that the balance between slope, or pressure gradient, and friction is no longer stable; the critical Froude number for formation is \( F = 2 \) when the Chezy resistance law is assumed and \( F = 1.5 \) if the Manning formula is used (c.f. Lighthill and Whitham, 1955). Analyses of roll waves can also be found in many texts, for example Stoker (1957, p. 466) and Whitham (1974, p. 85). All of the analyses closely follow the original of Dressler (1949, 1952).

We tentatively picture the growth of the bottom well-mixed region proceeding as follows: The layer first grows to the Ekman layer height by normal turbulent mixing generated at the bottom. The internal Froude number of the well-mixed layer is given, using (3), (4) and (5) by

\[
F = \frac{\sqrt{g}}{N} \frac{U}{h}.
\]

At the penetration height of the Ekman layer, \( h \) is small, approximately one sixth of the final penetration height observed; yet above the Ekman layer height the velocity must always approach the geostrophic velocity. Therefore, the internal Froude number is large, \( F \approx 10 \). As long as the internal Froude number of the layer is larger than the critical value for the formation of roll waves, these intermittent surges or bores form. The bores have mixing associated with them which continues until the mixed layer is deepened sufficiently that the internal Froude number is just less than the critical value for the formation of the roll waves.

Although the instability of the mean flow to the formation of roll waves is a candidate for the mechanism which controls the unusually large penetration height of the bottom boundary layer, the instability does not provide an explanation for how a once-formed layer may decrease in height when the geostrophic velocity gradually decreases. The mechanism of roll wave formation only provides an explanation as the velocity increases or remains constant. We note however that any interface formed at the bottom of the ocean will be a region of relatively high Brunt-Väisälä frequency. It is at such regions of high relative Brunt-Väisälä frequency that internal waves will break; c.f. Turner (1973, p. 120) for a discussion of shear instability produced by internal waves at interfaces. The breaking of internal waves at the interface is an eroding mechanism which may always be present.
CONCLUSION

We have reviewed a number of existing models for the bottom boundary layer and find that none predict the large penetration height of the well-mixed region. This region extends about six times the height of what is considered to be a typical turbulent Ekman layer height. Because of the large penetration height, differential advection between the mixed layer and the water immediately above must be small; the layer is, we believe, advected over the flat Hatteras Abyssal Plain with the mesoscale motions. We have outlined how one might model mixing and entrainment in the homogeneous layer; unfortunately we find that too little is known about the structure of turbulence above a turbulent Ekman layer for us to complete such a model. The correlation of penetration height with velocity has allowed us to estimate, knowing the Brunt-Väisälä frequency of the stratification which was mixed to form the bottom layer, the value of the internal Froude number of this layer: $F \approx 1.7$. This value suggests that the penetration height may be controlled by the instability of the mean flow, in the bottom mixed layer, to the formation of roll waves or intermittent surges.

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