FOREIGN TECHNOLOGY DIVISION

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by

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R. O. Soroka

It is known, that the lift force coefficient for a small wing elongation and small angles of attack is proportional \( \alpha \), and that Johnson (1) equation is correct for the lift force.

With increased \( \alpha \) this dependence becomes complicated. Betz was the first to examine similar problems. He showed that when the wing chord is much larger than the swing, the transverse flow that appears with this actually affects its aerodynamic characteristics. Betz introduced a definition of the transverse flow resistance coefficient and obtained following dependence for a normal plate force:

\[
C_a = 2 \sin^2 \alpha. \tag{1}
\]

Other formulas obtained later are known:

\[
C_a = \frac{\pi}{2} \lambda \alpha + \frac{\pi}{2} \alpha^3 \quad \text{(Kukhemann)},
\]

\[
C_a = \frac{\pi}{2} \lambda \alpha + \frac{\pi}{2} \alpha^{3/2} \quad \text{(Veber)} \tag{2}
\]

Further examinations of discontinuous wing streamline was conducted by Brown and Michel [2], Mangier and Smit [3] and other authors.
More detailed bibliography on this subject is in the work [4]. The experiments with the visualization of streamline behind the delta-wing with small elongation performed at the Institut of Hydromechanics, Academy of Science, Ukrainian SSR [AH UYRP], show that two vortex lines are coming from the front edge of the wing down at some angle to the direction of stream (Figure 1).

Let us apply a method of plain section.

Then, in the perpendicular plain stream \( \zeta \) (Figure 2) the problem will be reduced to the finding of potential of velocities \( \psi \), conditioned by the presence of two concentrated intensities of vortexes \( \Gamma \), which are present above the plate at a distance \( ah \) (\( h \) is a positive number). Another words, for each cross-section of \( \psi \) from the outcome of the Laplace equation there is

\[
\Delta \psi = \frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial^2 \psi}{\partial \eta^2} = 0 \tag{3}
\]

with limiting conditions

\[
\psi \to 0 \quad \text{at} \quad \sqrt{\xi^2 + \eta^2} \to \infty; \quad \frac{\partial \psi}{\partial \eta} = 0 \quad \text{at} \quad \eta = 0 \quad \left(-a < \xi < a\right).
\]

Since the complex potential for a disc is determined very simply, then by means of a conforming transformation \( \zeta = \frac{a}{2} \left( z + \frac{1}{z} \right) \) we shall reflect the inside of the plate on the outside of a single disc. The reversed transition, accounting for the correlation of infinite points in the plain \( z \) and \( \zeta \) will be
The Complex potential

\[ w(z) = \frac{\Gamma}{2\pi i} \left[ \ln(z - z_A) - \ln(z - z_B) + \ln\left(z - \frac{1}{z_B}\right) - \ln\left(z - \frac{1}{z_A}\right) \right]. \] (5)

and the velocity

\[ \vec{v} = u_x - iu_y = \frac{dw}{dz} = \frac{\Gamma}{2\pi i} \left[ \frac{1}{z - z_A} - \frac{\bar{z}_A}{\bar{z}_A - 1} + \frac{\bar{z}_B}{\bar{z}_B - 1} - \frac{1}{z - z_B} \right]. \] (6)

Here, the lines above indicate the established magnitudes. Let us introduce the polar coordinates: \( z = Re^{i\theta}, z_A = Qe^{i\theta}, z_B = -Qe^{-i\theta}. \)

then

\[ \frac{dw}{dz} = \frac{\Gamma}{2\pi i} \left[ \frac{e^{i\theta}}{K^2e^{2i\theta} + e^{2i\theta} - R^2 + R\left(\theta + \frac{1}{\theta}\right)e^{i(\theta + \theta)}} \right]. \] (7)

On the plate

\[ \frac{dw}{dz} = \frac{dw}{dz} : \frac{dz}{dz} \bigg|_{R=1} = -\frac{\Gamma}{2\pi i \sin \theta} \times \frac{4 \cos \theta \cdot \cos \theta_1 \left(\frac{q - 1}{q}\right)}{4 \cos^2 \theta \cdot \cos^2 \theta_1 - 2 \sin \theta \cdot \sin \theta_1 - q - 1}. \] (8)

It is seem from this that the limiting conditions are realized on the plate. Let us express \( \rho, \theta \) by the coordinates of an area \( \xi \)
Including that \( \xi_A = a, \eta_A = ah \), we shall have

\[
x_A = 1 + \sqrt{\frac{h \sqrt{4 + h^2} - h^2}{2}},
\]
\[
y_A = h + \sqrt{\frac{h \sqrt{4 + h^2} - h^2}{2}} \quad \phi_a^2 = x_A^2 + y_A^2.
\]

Since the intensity of vortexes on an infinite linear profile equals the velocity along the profile from positive ordinate, then at the end we obtain

\[
\tau(\xi) = \frac{4}{\alpha \pi} \frac{\xi y_A}{\alpha q_A (1 - \phi_A)} \left[ \frac{4 \xi x_A}{\alpha q_A (1 - \phi_A)} \right] \left[ 2 \frac{y_A}{\alpha q_A} \sqrt{1 - \left( \frac{\xi}{\alpha q_A} \right)^2} \right],
\]

where \( x_A, y_A, \) and \( \phi_A \) are expressed by formulas (10). According to the known law of division of vortexes, the lift force is easily obtained. Actually, the amount of motion in each elementary section is expressed by the formula

\[
dB = -\phi \int_{-}^{x} \xi \tau(\xi) \, d\xi,
\]

and its variation

\[
dP = \frac{dB}{dt} = -\phi \int_{-}^{x} \xi \tau(\xi) \, d\xi
\]
Figure 2. Diagram of streamline in a plain $\zeta$

Figure 3. Lift force coefficient of delta-wing with elongation $\lambda=0.7$ calculated on bases of theories: 1 - Brown and Michel; 2 - Edvards, 3 - Smit; 4 - Mangler and Smit; 5 - from (17), 6*

will be equal the lift force of the wing of the width $d\tau$. Here, $\tau$ is the coordinate in the direction of motion. At the stationary streamline

$$\frac{d}{dt} = \frac{d}{d\tau} \frac{d\tau}{dt} = v \frac{d}{d\tau}.$$  \hspace{1cm} (14)

Integrating according to $\tau$, $0 < \tau < L$ we obtain the wing lift force as a function of the distance of rear edge vortexes

$$P = - Q V \left[ \frac{d}{d\tau} \int_{-\infty}^{\tau} \xi (\xi) d\xi \right]_{\tau=0},$$  \hspace{1cm} (15)

*Flexon and Laurence, 7 Johnson, O Fink's measurements
same as at $\tau = L \ P = 0$. The intensity of isolated vortexes is determined according to the theory of Brown and Michel [2] from the condition of velocity limitation on the front edge. We find

$$\Gamma = \pi \nu \alpha \frac{h \sqrt{4 + h^2}}{\sqrt{h^4 + h^2 - h^2}}. \quad (16)$$

Then the lift force will be

$$P = -\rho V^2 \alpha^2 \cdot K(h). \quad (17)$$

$$K(h) = \frac{X_A \left( Q_A - \frac{1}{Q_A} \right) h l \sqrt{4 + h^2}}{\sqrt{h^4 + h^2 - h^2}} \times$$

$$\times \int \left( \frac{\xi^2 \frac{d\xi}{1 - \xi^2}}{\frac{2 X_A}{Q_A}} - \frac{2 y_A}{Q_A} \sqrt{1 - \xi^2} \right)^2. \quad (18)$$

The coefficient $K(h)$ is numerically calculated and is given below:

<table>
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<th>$-K(h)$</th>
<th>0</th>
<th>0.9</th>
<th>1.25</th>
<th>1.53</th>
<th>1.67</th>
<th>1.66</th>
<th>1.57</th>
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<tr>
<td>$h$</td>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
<td>0.7</td>
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<td>2.0</td>
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<td>10.0</td>
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We shall observe, that $P$ means nonlinear part of the lift force. The coefficient of the full lift force $C_{\alpha} = \frac{\lambda}{2} \left( 1 - K(h) \right)$. In a limiting case $\alpha \to 0$ we shall obtain $C_{\alpha} = \frac{\pi \lambda}{2}$, for $h \to 0$, $K(h) \to h^{1/2}$. The vortexes that move away from the front edges are inclined to the wing surface at an angle different from the angle of attack. As a
specific example we shall use Bolleya model [5], according to which the angle of inclination of the vortex cord equals one half of the angle of attack. In this, the relation between lift height of vortexes above the rear edge, its elongation and angle of attack will be \( h = \frac{2a}{\lambda} \).

Figure 3 shows the result of the calculation of the lift force coefficient \( \lambda = 0.7 \). For comparison, the presented graphs are taken from the work [4]. The obtained formula agrees will with the known theoretical and experimental data.

BIBLIOGRAPHY

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