ATMOSPHERIC-ABSORPTION ADJUSTMENT PROCEDURE
FOR AIRCRAFT FLYOVER NOISE MEASUREMENTS

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Final Report

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The method described in this report for adjusting measured aircraft flyover noise levels for differences in the atmospheric absorption of sound occurring under test and reference meteorological conditions has not been approved by the Federal Aviation Administration for use in aircraft noise type certification under Part 36 of the Federal Aviation Regulations.
Atmospheric absorption of sound
Aircraft noise
Sound propagation
Computer program

An analytical method was developed for adjusting measured aircraft noise levels for differences in atmospheric absorption between test and reference meteorological conditions along the sound propagation path. The method is based on the procedure in the proposed American National Standard ANSI S1.26 for calculating pure-tone sound absorption as a function of the frequency of the sound and the temperature, humidity, and pressure of the air. Measured aircraft noise levels are assumed to be 1/3-octave-band sound pressure levels. A computer program was written in FORTRAN IV to carry out the calculations. The operation of the computer program, the required input data, and all symbols and terms used in the program are described. A program listing of source statements is provided. Recommendations are given for applying the method to routine processing of aircraft noise measurements.
### METRIC CONVERSION FACTORS

#### Approximate Conversions to Metric Measures

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*°F in = 1.8°C (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 266: Units of Weight and Measures. Price 12.25, SD Catalog No. C13.11.266.*
PREFACE

The author gratefully acknowledges contributions from several helpful discussions with Professor Henry E. Bass of the Department of Physics and Astronomy at the University of Mississippi and Mr. Louis C. Sutherland at Wyle Laboratories.
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INTRODUCTION

Working Group S1-57 of the Standards Committee S1 on Acoustics developed a new procedure for calculating the absorption of sound by the atmosphere. In 1977, the new procedure was in the process of being approved and published by the American National Standards Institute (ANSI) and the Acoustical Society of America as an American National Standard Method for the Calculation of the Absorption of Sound by the Atmosphere. When issued, the new American National Standard will have the ANSI designation S1.26 and hence will be referred to in this report as an ANS S1.26. The technical basis and the experimental verification for the new calculation procedure are documented in reports prepared by Shields and Bass and by Sutherland.

The new procedure for calculating atmospheric absorption is based on fundamental physical principles. Extensive laboratory tests have validated the method for pure tone sounds over a wide range of air temperature and humidity. Although the primary purpose of the new ANSI Standard is to specify methods for determining absorption of discrete-frequency pure-tone sounds by the atmosphere, the proposed Standard also presents a general description of the application of the calculation procedure to bands of noise, e.g., sounds analyzed by constant-percentage-bandwidth filters such as 1/3-octave-band filters.

Techniques applicable to each particular class of sound propagation problems are not necessarily provided in ANS S1.26 because the new Standard is intended to be basic and general in accordance with the policy of the Acoustical Society of America. Thus, it may be necessary for a specific user to develop a method to apply the proposed procedures to a particular problem, such as analysis of aircraft flyover noise measurements.

The existing procedure for adjusting aircraft flyover noise levels for differences in atmospheric absorption losses is based on a combined theoretical, experimental, and empirical method developed by the Aircraft Noise Measurement Committee A-21 of the Society of Automotive Engineers (SAE).
The latest version of the SAE procedure is contained in SAE Aerospace Recommended Practice ARP 866A. The procedures of SAE ARP 866A are incorporated directly in Part 36 of the Federal Aviation Regulations (FAR 36), in Annex 16 to the Standards and Recommended Practices of the International Civil Aviation Organization, and in International Standard IS 3891 of the International Standards Organization as part of a procedure for describing noise around an airport.

The empirical adjustments to laboratory data and available theory that were used in the development of SAE ARP 866 were derived from aircraft noise levels and meteorological data measured a short distance above ground level. A basic assumption in analysis of the flyover noise test results was that the meteorological measurements near the ground represented conditions all along each sound path. Based on a study reported by McCollough and True, the FAA has proposed modifying the original procedures of FAR 36 to require application of atmospheric absorption adjustments by the SAE ARP 866A method along segments of the sound propagation paths. The FAA 'layered-weather' or stratified-atmosphere approach is intended to extend the use of SAE ARP 866A to varying meteorological conditions aloft.

This report describes a computer program for applying the proposed ANSI Standard ANS S1.26 to the problem of adjusting measured aircraft flyover noise levels for differences in atmospheric absorption losses between test and reference meteorological conditions along the propagation path connecting the source and the receiver. The computer program is written in the FORTRAN IV language.

The following Sections (a) describe the physical characteristics of the problem of determining adjustments for atmospheric absorption losses, (b) give reference meteorological conditions, (c) describe the procedures for determining atmospheric absorption losses by the methods of FAR 36 and the proposed ANS S1.26, (d) describe the development of the computer program, (e) demonstrate the application of the procedure of ANS S1.26 for actual aircraft noise measurements, (f) outline the general requirements for adapting the computer program to routine processing of aircraft noise measurements, and (h) present recommendations for investigations that should be conducted prior to considering use of the new ANSI procedures for aircraft noise type.
certification.

Appendices contain the analytical basis for the adjustment procedures for discrete-frequency pure-tone sounds and for bands of noise. A listing of the computer program source statements and definitions of all symbols are also provided in Appendices.

In consonance with stated Government policy, all physical quantities are in the international system of units. Conversion to U.S. customary units, if required, may be made by reference to the Metric Conversion Factors given in the prefatory material to this report.

DESCRIPTION OF THE PROBLEM

The problem of determining atmospheric absorption loss adjustments from test to reference meteorological conditions is illustrated schematically in figure 1. Measured sound pressure levels are shown by the solid line in the idealized 1/3-octave-band spectrum plot on the right. The measured levels represent data recorded at some instant of time as the airplane flew overhead with airspeed $V_a$ and Mach number $M_a$; the airplane's position on the flight path (shown on the left in figure 1 as a level-flight flyover) is denoted by the angle $\phi$ between the flight path and a line to the microphone. The location of the sound source, at the time of emission, is at an angle $\psi$.

Meteorological conditions at the time of the flyover noise recording could be as indicated by the measured profiles of temperature, relative humidity, and pressure. The example in figure 1(a) indicates a strong ground-based temperature inversion with a weak inversion aloft at a height above the height of the aircraft. The humidity profile indicates substantially-lower relative humidity aloft than at ground level over most of the sound propagation path. The low upper-air humidity could result in significant atmospheric absorption effects on sound signals propagating from the airplane to the microphone. The air pressure measurements show the expected trend of a reduction in pressure with increasing height. Height is assumed to be measured above the elevation of mean sea level.

During data reduction, the recorded sound signals are analyzed by some type of filter system and averaged by some kind of detector circuit. Filter
Figure 1. - Schematic illustration of determination of atmospheric absorption adjustments to measured aircraft noise levels.
systems can be analog or digital. As indicated by the sketch in figure 1(a), the transmissibility function (filter frequency response) of practical analog filters will have some finite transmission in the stopbands below and above the cutoff frequencies at the limits of the passband. These filter skirts will transmit some energy and increase the indicated band level above that which would be indicated by an ideal filter.

The essence of the problem is shown in figure 1(b) — namely, to determine what the sound pressure levels at the microphone would have been had the flyover noise test been conducted with reference rather than test meteorological conditions. Reference meteorological conditions are indicated by defined values for temperature, relative humidity, and pressure at some reference height along with reference lapse rates. The reference height is indicated by the square symbols.

The frequency response of the filter under reference conditions is identical to the frequency response under the test meteorological conditions.

The result of applying atmospheric absorption corrections generally yields an increase in high-frequency sound pressure levels as indicated by the dashed line in the spectrum sketch.

The balance of this report describes development of a computer program that applies the procedure of the proposed ANSI S1.26 to adjust aircraft flyover noise measurements. The specific purpose of the calculation procedure is to adjust 1/3-octave-band sound pressure levels, measured at a point near the ground during an aircraft flyover noise test, for the difference between the atmospheric absorption losses that occurred at the time of the test and those that would have occurred if the aircraft noise signal had propagated through a reference atmosphere having specified temperature, humidity, and pressure as a function of height.
REFERENCE METEOROLOGICAL CONDITIONS

FAR 36 Definitions

As in all earlier versions, the October 1977 version of FAR 36 defines reference meteorological conditions in paragraph A36.3(c)(1) of Appendix A as follows:

(a) sea level pressure of 2116 psf (76 cm Hg) (101.325 kPa)
(b) ambient temperature of 77°F (25°C)
(c) relative humidity of 70 percent
(d) zero wind.

Appendix A of FAR 36 does not explicitly state a height above ground level for the reference pressure, temperature, and relative humidity. Also, no mention is made of corresponding reference lapse rates with height for pressure, temperature, relative humidity, or wind.

The reference meteorological conditions were probably meant to apply everywhere along the sound propagation paths because the original development of SAE ARP 866 only used 'surface' weather measurements to adjust from test to reference conditions. Surface weather data were sometimes measured at a height above ground level of about 1.5 m somewhere in the vicinity of the microphones, and were sometimes obtained from airport facilities where the sensors were usually about 10 m above ground level.

No specification of reference lapse rates was required in the original version of FAR 36 because atmospheric absorption adjustments used the 'surface' data exclusively. It was always considered, however, that the 'surface' meteorological measurements, wherever measured, should be representative of the conditions near the microphones at the time of the aircraft noise recordings. Paragraph A36.1(d)(4) requires that the airport tower, or another facility, must be approved for use as the location at which measurements are made of atmospheric parameters representative of conditions existing over the geographical area in which aircraft noise measurements are made. However, the surface wind velocity and temperature must be measured near the microphone at the approach, sideline, and takeoff measurement locations.
The general meteorological test conditions defined in paragraph A36.1(b)(3) for acceptable test conditions include limits on ambient temperature and wind at a height of 10 m above ground level. No height is specified for the acceptable limits on relative humidity. It has become general practice, with FAA approval, to measure 'surface' meteorological test conditions of pressure, temperature, relative humidity, and wind at a height of 10 m.

The only limitation in FAR 36 on meteorological conditions aloft is the statement in A36.1(b)(3)(v) that there should be no temperature inversion or anomalous wind conditions that would significantly affect the noise level of the aircraft when the noise is recorded at the measuring points defined in Appendix C. The wording of this paragraph was the basis for the investigations conducted by the FAA in 1974 and reported in reference 7. The extensive changes that were proposed in NPRM 76-218 for the requirements for meteorological test data and the data-adjustment procedures also stem from the varying interpretations that were permitted by A36.1(b)(3)(v).

Reference Lapse Rates

In order to calculate differences in atmospheric absorption losses under test and reference conditions, it is necessary to define reference atmospheric conditions. Specification of reference atmospheric conditions along the sound propagation paths requires specification of reference lapse rates and reference conditions at some reference height.

Lapse rates for temperature and pressure have been developed and standardized. Reference 9 defines a standard atmosphere for Northern midlatitudes (45° north) and average annual conditions. References 10 and 11 provide atmospheric data at other latitudes and for seasonal variations.

The standard atmosphere of reference 9 is dry. Relative humidity lapse rate information is given in references 10 and 11.

The lapse rate for temperature is $-6.5 \text{ K/km}$ to a height above mean sea level of 10 km. The relative humidity lapse rate is approximately $-6.5$ percentage points per km to a height above mean sea level of 4 km. For pressure in standard atmospheres (i.e., the ratio of the air pressure $P$ to the standard sea-level atmospheric pressure $P_0$), an approximate lapse rate
for the logarithm of \((P/P_0)\), derived from the tabulated values in reference 9, is \(-5.393 \times 10^{-5}\) log \((P/P_0)\) per meter to a height above mean sea level of 4 km.

To be consistent with the definitions of FAR 36 for reference meteorological conditions and with current practice for a measurement height for surface meteorological data, the following reference conditions were defined at a reference height (altitude) of 10 m above mean sea level:

- air temperature: 298.15 kelvins (25.0°C);
- relative humidity: 70.0 percent; and
- air pressure: one standard atmosphere or 101.325 kPa or 29.921 in. Hg.

These definitions for reference meteorological conditions at the 10-m reference height are not the same as those in references 9 and 11 where, at the reference height of mean sea level, the standard air temperature is 288.15 kelvins (15°C), the standard relative humidity is 75 percent, and the air pressure is one standard atmosphere.

Equations

For purposes of this study, reference meteorological conditions were specified by the following equations:

\[
RTK = RTKO + RTKL (H - RH); \quad (1)
\]
\[
RHR = RHRO + RHRL (H - RH); \quad (2)
\]

and

\[
RPA = 10^{LPA} \quad (3)
\]

where

\[
LPA = RPAL (H - RH). \quad (4)
\]

The symbols have the following meanings. \(RTK\) is the reference temperature, in kelvins, and \(RHR\) is the reference relative humidity, in percent, at any height \(H\). \(RTKO\) and \(RHRO\) are the reference values at the reference height (i.e., 298.15 K and 70 percent). \(RTKL\) and \(RHRL\) are the reference lapse rates (i.e., \(-0.0065\) K/m and \(-0.0065\) percentage points per meter). \(H\) is height, in meters, above mean sea level; \(RH\) is the reference height of 10 m. \(RPA\) is the reference air pressure in standard atmospheres at any height. \(LPA\) is the log of the pressure in standard atmospheres at any height and \(RPAL\) is its lapse rate of \(-5.393 \times 10^{-5}\) per meter.
Note that uniform meteorological conditions could, if desired, be generated by setting RTKL, RHRL, and RPAL equal to zero.

**FAR 36 ATMOSPHERIC-ABSORPTION ADJUSTMENT PROCEDURE**

Paragraph A36.3(d)(1) of FAR 36 requires that measured noise data be adjusted from the test meteorological conditions [measured in accordance with the requirements of paragraph A36.1(d)(4)] to the reference meteorological conditions of paragraph A36.3(c). The adjustments must be made using the methods of the latest version of SAE ARP 866 by the general procedures given in paragraph A36.5. Detailed data-correction procedures are given in paragraph A36.6.

The detailed procedures of A36.6 determine a correction factor to be added to the measured value of the test-day effective perceived noise level (EPNL). The correction factor is intended to account (1) for the change in atmospheric sound absorption over the distance from the airplane flight path to the microphone at the time associated with the maximum value of the tone-corrected perceived noise level (i.e., at the time of occurrence of the test-day PNLT), and (2) for the change in atmospheric sound absorption caused by differences in the sound propagation pathlength between the location of the microphone and the locations of the effective sound source on the test and the reference airplane flight paths.

The correction factor that is defined in paragraph A36.6(a) to account for differences between test-day and reference-day atmospheric absorption is 
\[ \Delta L = \text{PNLT}_{\text{ref}} - \text{PNLT}_{\text{test}} = \Delta \text{EPNL} = \text{EPNL}_{\text{ref}} - \text{EPNL}_{\text{test}}. \]

The value of tone-corrected perceived noise level under acoustic reference-day conditions (PNLT\text{ref}) is determined by adjusting the measured 1/3-octave-band sound pressure level spectrum associated with PNLT\text{test}. The adjustment procedure accounts for differences in atmospheric absorption and acoustic pathlength by means of separate formulas given in paragraph A36.6(d) for the takeoff, sideline, and approach noise measurements.

Changes in the duration correction factor are determined from the ratio of the minimum distances from the microphone to the test and the reference flight paths.
Microphones are required by FAR 36 to be placed at a height of 1.2 m above the ground surface. Interference effects between sound waves propagating directly from the source to the microphone and sound waves reflected from the ground surface introduce peaks and nulls in the spectra at frequencies that depend on the geometry of the test setup and the speed of the source. The magnitude of the spectral irregularities caused by ground reflection effects can be several decibels. The usual industry practice is to not remove irregularities caused by ground reflections before calculating atmospheric absorption adjustments to the measured noise levels.

The spectrum of the aircraft noise signal at the microphone, compared with the spectrum of the noise at the source, is shifted to higher or lower frequencies by an amount that depends on the airplane Mach number \( M \) and the sound propagation angle \( \psi \). This Doppler effect causes an apparent shift of the source spectrum to higher frequencies as the airplane approaches the microphone [by \( 1/(1-M \cos \psi) \)] and to lower frequencies after the airplane passes the microphone [by \( 1/(1+M \cos \psi) \)]. For the range of airplane Mach numbers and propagation angles normally encountered in flyover noise testing, the frequency shift is on the order of 10 to 25 percent.

For broadband random sounds, it is not necessary to consider Doppler frequency shifts to the observed spectra. For discrete-frequency signals, however, one should consider the effect of the source motion when attempting to relate the frequency of the source (e.g., a fundamental fan-blade-passage frequency) to the frequency of a signal identified as a pure tone in the passband of some 1/3-octave-band filter. Knowledge of the true frequency of the pure-tone source is needed to determine the pure-tone atmospheric-absorption adjustment factor at the Doppler shifted frequency.

ATMOSPHERIC-ABSORPTION ADJUSTMENT PROCEDURE BY ANS S1.26

Each 0.5-s sample of the aircraft noise signal is assumed to be separated, if required, into broadband and discrete-frequency spectral components. An algorithm is assumed to be available to search the measured spectra and to determine the separate components. Atmospheric-absorption adjustments are determined separately for each spectral component. Details of the analytical basis for the pure-tone and the broadband adjustment procedures are given in
Appendices A and B, respectively. The discussion here gives only the general outline of the methods.

**Pure Tones**

The propagation path is assumed to be divided into a number of segments over which an average temperature, humidity, and pressure can be established. The atmospheric absorption at the frequency of the tone (i.e., at the Doppler-shifted frequency) is calculated first for the meteorological conditions of the atmosphere at the time of the test and then for the meteorological conditions of the reference atmosphere. The difference is added to the measured pure-tone sound pressure levels to obtain the sound pressure levels adjusted to reference meteorological conditions.

The pure-tone adjustment factor, $\Delta L_{T, TR}$, is defined as

$$\Delta L_{T, TR} = 10 \log A$$

where the adjustment factor $A$ is determined from

$$A = 10 \left[ (a_{1,1} - a_{1,2})(\Delta \xi_1 /10) + \cdots + (a_{n,1} - a_{n,2})(\Delta \xi_n /10) \right].$$

The lengths of the various segments of the propagation path are $\Delta \xi_1$, $\Delta \xi_2$, $\cdots$ to $\Delta \xi_n$. The pure-tone absorption coefficients $a$ for the different meteorological conditions on each path segment are calculated by the method of the proposed ANSI S1.26 at the specified frequency. Equations (A2) to (A6) from Appendix A are used to determine the absorption coefficient $a$ in nepers per meter. The absorption coefficient $a$ in decibels per meter is then determined from $a = 8.686 a$. The lengths of the propagation path segments are determined by dividing the length of the total sound propagation path into intervals according to the heights at which meteorological data are measured.

**Bands of Noise**

Development of the analytical basis for determining atmospheric absorption losses for bands of wideband random noise and the corresponding computational procedure were the main technical efforts of the study reported here. The general procedure involves the evaluation of two integrals over the
frequency range of the passband of each of the contiguous filter bands.

The adjustment factor, $\Delta L_{B,TR}(f_1)$, for a band of noise at center frequency $f_1$ with passband lower and upper frequencies $f_L$ and $f_U$ is determined from

$$\Delta L_{B,TR}(f_1) = 10 \log \left\{ \frac{\int_{f_L}^{f_U} A \, df}{\int_{f_L}^{f_U} C_1 \, df} \right\}$$

where $A$ is the absorption factor of equation (6) over the propagation path and $C_1$ is the spectral density of the sound pressure signal at the microphone before passage through the data-acquisition and data-processing system. Equation (7) assumes that the frequency response of the real filters can be adequately represented by the response of ideal filters.

The central problem in evaluating equation (7) is how to determine a reasonable approximation to $C_1$ knowing only the measured sound pressure levels after passage through the data-acquisition and data-processing system. For each passband, the approximation chosen for the analysis in Appendix B is

$$G_1(f) = G_1(f_1) \left( \frac{f}{f_1} \right)^{\ell'}$$

where $G_1(f_1)$ is the pressure spectral density at the band center frequency and $\ell'$ is the slope of the spectrum. On a plot of log $G_1$ vs log $f$, the approximation of equation (8) is a straight line with slope $\ell'$ over the frequency range from $f_L$ to $f_U$.

With equation (8), equation (7) becomes

$$\Delta L_{B,TR}(f_1) = 10 \log \left\{ \frac{\int_{f_L}^{f_U} A \, f^{\ell'} \, df}{\int_{f_L}^{f_U} f^{\ell'} \, df} \right\}$$

The numerator term in equation (9) is evaluated as the summation of a series of consecutive integrals. The number of integrals in the sum is determined by dividing the filter passband into SE segments or constant-percentage-bandwidth frequency intervals.

To determine an integrable expression for $A$, it is assumed that the absorption, over any of the segments $\Delta f$ which make up the sound propagation path, is a function of frequency only. Over the narrow range of frequency of one of the integrals in the sum, the absorption is assumed to be
proportional to some power of the frequency $f$. This assumption leads to the expression

$$ A(f) = A(f_j) \left[ \left( \frac{f}{f_j} \right)^K \right] $$

(10)

where $A(f_j)$ is the pathlength absorption factor at the frequency $f_j$ at the beginning of the frequency range for one of the integrals and $K$ is an effective absorption-spectrum slope parameter averaged over the pathlength and over the differences between absorption-spectrum slopes under test and reference meteorological conditions.

The slope $\xi'$ of the sound pressure spectral density is estimated from the band-level noise-spectrum slope $\xi$ using

$$ \xi' = \xi - 1. $$

(11)

With equations (10) and (11), evaluation of equation (9) leads to four possible cases.

When $\xi' > 0$, the band-loss adjustment factor is

$$ \Delta L_{B,TR}^{SE}(f_j) = 10 \log \sum_{j \neq 1} ^{SE} BL_{1j} $$

where

$$ BL_{1j} = BL_{1j} \text{ when } K+\xi \neq 0 $$

(13)

and

$$ BL_{1j} = BL_{2j} \text{ when } K+\xi = 0. $$

(14)

When $\xi' = 0$ and $K \neq 0$, then

$$ \Delta L_{B,TR}^{SE}(f_j) = 10 \log \sum_{j \neq 1} BL_{3j}. $$

(15)

When $K = 0$, then there is no difference between the test and reference conditions and

$$ \Delta L_{B,TR}^{SE}(f_j) = 0.0 $$

(16)

regardless of noise slope $\xi$.

Appendix B gives the specific expressions for the terms $BL_{1j}$, $BL_{2j}$, and $BL_{3j}$ as well as specific procedures for determining the band center frequencies.
the passband limiting frequencies, and a choice for the number of passband frequency intervals \textit{SE}. Specific recommendations are also given for determining the noise slope \& for each band.

\textbf{General Limitations}

In agreement with the recommendations of the proposed \textit{ANS S1.26}, applicability of the procedures described here for pure tones or bands of noise is limited to air temperature in the range from 0.0° to 40.0° celsius, relative humidities from 10.0 to 100.0 percent, and nominal 1/3-octave-band center frequencies from 50 Hz to 10 kHz. The procedures are also limited to air of standard composition and normally encountered air pressures.

\textbf{APPLICATION TO ADJUSTMENT OF AIRCRAFT FLYOVER NOISE MEASUREMENTS}

The absorption-loss adjustment factors can be applied to measured spectra associated with any data sample. The procedure is thus applicable to the FAR 36 method of adjusting only the spectrum associated with the maximum value of the test-day tone-corrected perceived noise level as well as to adjustment of every 0.5-s data sample throughout the duration of a flyover — as might be desired for research investigations of aircraft noise propagation.

\textbf{General Assumptions}

The following general assumptions were made in the calculation of absorption-loss adjustment factors.

1. For each frequency of interest, the effective source of sound from the airplane is at a single point on the flight path that can be determined, for each instant of time during a flyover, from the airplane's position and airspeed. The airplane's position is defined by the coordinates of an airplane reference point. The coordinates include position along the flight track, lateral deviation, and height above ground level (height AGL).

2. For each data sample throughout the flyover noise recording, the direction of sound propagation from the source to the receiver (microphone) is constant and defined by a propagation angle whose sides are the flight path and a straight line (sound ray) between the source and the receiver. Bending of the sound rays caused by refraction effects occurring during
propagation through regions or layers of different temperature (different speeds of sound) is negligible. Bending of the sound rays caused by propagation through winds of varying speeds and direction is also negligible because the wind is calm at all points along every sound propagation path.

3. Attenuation and scattering effects caused by sound propagation through atmospheric turbulence are negligible for each sound ray.

4. Aircraft noise signals are filtered into twenty four 1/3-octave bands with nominal band center frequencies ranging from 50 to 10,000 Hz.

5. Nonlinear propagation effects caused by high sound pressure levels at the source are negligible.

6. No atmospheric absorption adjustments are to be made when the aircraft noise signal is not 5 dB or more above the level of the corresponding ambient noise in any 1/3-octave band.

Assumptions Regarding Input Data

The following assumptions were made for the data required as input to the calculations. The assumptions apply to the general problem of adjusting measured aircraft noise signals for FAR 36 or research purposes as well as the particular test cases selected for demonstration.

Meteorological Parameters. — Temperature and humidity data are measured at regular intervals of height AGL at several times throughout each test day. Values of temperature and humidity, applicable to each set of test sound pressure levels from a flyover noise measurement, can be interpolated from the available measurements. The unit for temperature is kelvins; for relative humidity it is percent. Temperature and relative humidity profiles are available to a height greater than the airplane height. Height is measured in meters.

Station barometric pressure at a height of 10 m is measured periodically throughout each test day. The pressure is applicable to the actual runway elevation and not corrected to sea level. A value for barometric pressure applicable to the time of the flyover noise test can be interpolated from the available measurements. The unit for barometric pressure measurements is the kilopascal (or inches of mercury).
(If air pressure measurements as a function of height AGL are available, it is preferable to use the measured pressure-profile data, interpolated to the time of the flyover noise test, in conjunction with the measured temperature and relative humidity profiles rather than the single 10-m pressure measurement.)

**Airplane Tracking.** The coordinates of an airplane reference point are available as a function of time throughout the duration of, and in synchronization with, the recording of the airplane's noise signal. Airplane coordinates are relative to an origin point on the ground and consist of x, y, z data. The x-coordinate is in the ground plane and along the nominal flight track. The z-coordinate is height AGL. The y-coordinate is in the ground plane and is a measure of lateral deviation from the nominal flight track. The unit for distance measurements is the meter.

**Noise Source.** The parameters that determine the acoustical characteristics of the airplane noise source are constant throughout the duration of each sampling interval, i.e., engine power setting, airspeed, airplane configuration, and attitude are all constant over each time sample.

**Sound Pressure Levels.** Sound pressure levels in 1/3-octave bands are available at 0.5-s intervals throughout the duration of each flyover noise recording. Sound pressure levels represent true mean-square sound pressures, are in decibels (dB) referenced to 20 micropascals, and are corrected for all known sources of error including: windscreen corrections, microphone response corrections, recording and processing-system frequency-response corrections, system sensitivity, and interference by ambient noise.

**ambient Noise Corrections.** Corrections for ambient noise interference are made when the aircraft noise signal is 5 dB or more above the corresponding ambient noise signal in each 1/3-octave band. When the aircraft noise signal is not more than 5 dB above the ambient noise signal, the data processing system does not attempt to make a correction and sets the 1/3-octave-band sound pressure level to 0.0 dB or some other negligible value.

**Averaging Time.** Other than integration over the 0.5-s duration of the sampling interval, no additional smoothing of the sound pressure level measurements is included in the data, e.g., no running averaging of the data
samples is made to simulate sound pressure levels that would have been measured on a sound level meter with high damping.

Identification of Data Sample Time. — The instant of time associated with the flyover noise data samples is the midpoint of the 0.5-s sample period because that time is most representative of the aircraft's position during the 0.5-s period.

The assumptions described above for averaging time and identification of the data sample time may not comply with the data-processing techniques used by all applicants for aircraft noise type certification under FAR 36. Similarly, not every applicant may use the 5-dB rule for rejecting data samples contaminated by ambient noise; some may use, for example, a 3-dB rule. The specific and the general assumptions made here, however, are considered to be a consistent interpretation of the proper application of atmospheric absorption corrections.

Special Considerations

There are three important considerations that should be taken into account in determining atmospheric absorption loss adjustments to aircraft flyover noise measurements. Development of techniques to overcome the problems arising from the considerations in actual applications was beyond the scope of the project.

Ground Reflection Effects. — Absorption loss adjustments should be applied to sound spectra that represent equivalent free-field sound pressure levels. Measurements from microphones located above a ground plane will have ground-reflection effects caused by interference between direct and reflected waves. Unless the microphone height is large with respect to the longest wavelength of interest or unless the microphone is mounted flush in an acoustically large and rigid surface, the measured spectra will have interference peaks and nulls in the frequency range of interest. Spectral irregularities should be removed before attempting to apply atmospheric absorption corrections because the methods of the proposed ANS S1.26 cannot be used to account for ground attenuation effects on reflected waves.

If the acoustical impedance of the ground surface is known (or can be reasonably estimated), then available analytical techniques can be applied to remove the ground reflection effects. If significant spectral irregularities
from ground-reflection effects are present in the measured data, it will be necessary to remove them in order to avoid wrongly identifying an interference peak as a discrete-frequency sound from the airplane.

**Separation of Discrete and Broadband Components.** — After ground-reflection effects have been removed from the measured sound pressure levels to yield a spectrum representative of free-field sound pressure levels, the spectra should then be examined to identify the frequencies and levels of discrete-frequency components. Pure-tone absorption loss corrections are applied to the discrete-frequency components; band-loss corrections are applied to the filtered broadband levels. The tone identification procedure of reference 4 could be used to establish the levels of the separate discrete and broadband spectral components.

**Missing Band Levels.** — To avoid attempting to apply atmospheric absorption corrections to invalid data, it is necessary for the data processing system to identify the bands irretrievably contaminated by ambient noise and to eliminate them from determination of noise slopes or absorption losses. If band levels are missing from only the beginning or end of the set of 24 bands with center frequencies between 50 and 10,000 Hz, then the calculation of absorption losses could proceed by limiting the analysis to those bands for which valid data exist. If data are not available for bands scattered between 50 and 10,000 Hz or if there are two or more adjacent (but not initial or final) bands with missing levels, then either special rules based on additional assumptions would need to be invoked (because of the requirement to have a value for the slope of the noise spectrum as a parameter in computing the band losses) or no attempt should be made to determine absorption losses in these cases.

**DESCRIPTION OF COMPUTER PROGRAM**

Adjustment of measured aircraft noise levels for differences between test and reference meteorological conditions is based on the analytical approach described above and in Appendices A and B. For any sample of data throughout the duration of a measured aircraft noise signal, the calculation proceeds by determining the atmospheric absorption loss along the calculated propagation path from the microphone to the source of sound using atmospheric conditions existing at the time of the test and then from the source to the microphone using reference meteorological conditions. The difference between the atmospheric absorption losses along the path determines the magnitude and sign
of the adjustment required for each 1/3-octave-band sound pressure level. The computer program that was prepared to calculate adjustment factors for bands of noise is called BANDLOSS and is described in Appendices C and D.

Figure 2 indicates the steps involved in implementing a method to calculate band-loss atmospheric absorption adjustment factors by means of three sequentially stepped DO loops and one subroutine. The DO loops consist of one main loop and two inner or nested loops.

After reading in, calculating, or defining the necessary input information, the program enters the outer calculation loop. The outer loop steps through the set of 24 bands comprising the measured input sound pressure levels. The first inner loop, loop 2, performs the numerical integration on frequency $f_j$ over the frequency range of the passband of each filter and returns the value of $BL_j$ or $BL_j$ to the outer loop.

At each increment of $f_j$ from $j = 1$ to $j = SE$, the first inner loop calls the second inner loop, loop 3, for a calculation of the effective absorption-slope parameter $K$ and the exponent $AE$ of the absorption factor $A(f = f_j) = AJ$. The third loop calculates the quantities $AE$ and $K$ for the test and reference
meteorological conditions applicable to each segment of the sound propagation paths, i.e., at increments in height from $H = 1$ to $H = IA$ where $IA$ is an index counter representing the height for the meteorological measurements that is just less than the height of the aircraft at the time when the sound was emitted from the aircraft noise source. At each step in height, loop 3 calls subroutine ANSAB to calculate the sound absorption coefficient for specified frequency, temperature, humidity, and pressure.

The total number of calculations required by the three loops is the product of the ranges of the DO loops or $24 \times SE \times IA$. For typical aircraft noise measurements, the index $IA$ could have a value ranging from 15 to 30. The number of frequency segments is $SE = 20$. Thus, the three loops require up to 15,000 calculations. Subroutine ANSAB is called four times at each height step by loop 3.

The BLI or BL3J sum for each band is passed back to loop 1 by loop 2. The band loss adjustment factor $\Delta L_{BT,TR}$ is the output of loop 1. Adjusted sound pressure levels are obtained by adding the calculated $\Delta L_{BT,TR}$ values to the input sound pressure levels.

Input information consists of the various quantities outlined in figure 3. The measured inputs are the noise, meteorological, and airplane data. Calculated input data consist of the heights for the meteorological measurements, the height at the midpoint of each height interval and the corresponding air temperature, relative humidity, and atmospheric pressure in standard atmospheres. Temperature, relative humidity, and pressure under reference conditions are also calculated. Sound propagation path lengths and the differences in the levels of the input 1/3-octave-band sound pressure levels are determined. The precise values of the band center frequencies are calculated.

The reference meteorological conditions, the band frequency ratio, the number of frequency-interval segments in a filter passband, and the International Standard Band Numbers for use in calculating the band center frequencies are all defined.

Appendix C lists the FORTRAN source statements for program BANDLOSS, including subroutine ANSAB, for computation of atmospheric absorption adjustment.
MEASURED DATA

NOISE : SPL, TS, TN, ZM
METEOROLOGICAL : TC, HR, IMAX, BP
AIRPLANE : AH, AM, AS, TOH

CALCULATED DATA

METEOROLOGICAL : HM, AHM, ATK, AHR, PA,
RTK, RHR, AND RPA
SOUND PROPAGATION
DISTANCES : D
NOISE BAND LEVEL
DIFFERENCES : DL
GEOMETRIC MEAN
FREQUENCIES : FI

DEFINITIONS

REFERENCE METEOROLOGICAL PARAMETERS: RHM, RTKO, RTKL,
RHRO, RHRL, RPAL, RBPO
FREQUENCY RATIO: RF
NUMBER OF FREQUENCY-INTERVAL SEGMENTS: SE
INTERNATIONAL STANDARD BAND NUMBERS: ISBN

Figure 3.-Input information needed for calculation of $\Delta L_B, TR$ and SPLA.

Factors in accordance with the analyses in Appendices A and B. Appendix D describes and defines the symbols and terms used in program BANDLOSS.

The statement line numbers in Appendix C for subroutine ANSAB are sequential to those of the main part of program BANDLOSS because the subroutine and the main program were merged together for calculating the test cases. The statements for the input data were developed specifically for the demonstration test cases described in the next section. Relatively minor modifications would be required to adapt the procedures described here to a specific flyover-noise data-acquisition/data-reduction system. The main calculation loops could be used without alteration. The program was written for direct on-line submittal from an interactive terminal to a large-scale digital computer. No major changes would be required for batch-mode submittal through a terminal, or for conversion to input through a card reader, or for incorporation in a data-processing program.
DEMONSTRATION OF APPLICATION OF COMPUTER PROGRAM

To demonstrate application of the computation procedures to aircraft noise test data, we consider the situation where the following assumptions are valid.

1. The airplane flew a level flight path in a straight line over the microphone (i.e., \( y = 0.0 \)) such that the airplane height was constant throughout the duration of each noise recording (i.e., \( z = \) constant). The airplane position was synchronized with the noise recording and the time when the airplane was directly over (or closest to) the microphone is known.

2. The airplane flew over a runway whose elevation was approximately equal to that of mean sea level.

3. The spectrum of the aircraft noise signal for each time sample is equivalent to a free-field spectrum. No corrections are required for spectral irregularities caused by ground reflection effects. The calculation procedure does not need to contain a method to search the spectrum and identify and remove spectral irregularities caused by ground reflections.

4. The noise spectra used for demonstration of the atmospheric-absorption correction method represent broadband sound sources only. The calculation procedure does not need to contain a method to search the spectrum and identify the frequency and level of pure-tone components and the remaining broadband components.

5. The demonstration spectra are not contaminated by interference from ambient noise. Valid sound pressure levels are available for all 24 bands with nominal center frequencies from 50 to 10,000 Hz. The calculation procedure does not need to contain a method to search the spectrum to identify, count, and locate bands where the corresponding ambient noise levels were too high and the aircraft noise signal was set to the negligible value.

6. There were 31 heights (\( \text{IMAX} = 31 \)) at which air temperature and relative humidities were measured. Air pressure was measured only at a height of 10 m.

With these assumptions the geometrical relationship between the airplane and the microphone is as shown in figure 4. The airplane is at a height \( \text{AH} \) above ground level; the microphone is at a height \( \text{ZM} \). The height between the flight path and the microphone is \( \text{AMH} = \text{AH} - \text{ZM} \).
When the airplane is at a location on the flight path defined by angle $\phi$ to the microphone, the sound was emitted from an effective source with emission angle $\psi$ between the flight path and the ray between the source and the microphone. The time $t_R$ when the sound was received at the microphone is measured relative to the time when the aircraft was over, or closest to, the microphone, i.e., where $\phi = 0$ and $\psi = \cos^{-1} M_a$ with $M_a$ the airplane Mach number ($M_a = V_a/c$ where $V_a$ is the airspeed and $c$ is the speed of sound). The time $t_R$ is referenced to the time of the midpoint of the 0.5-s sample of data that was digitized to determine the set of measured input sound pressure levels.

If the time at overhead (TOH at $\phi = 90^\circ$) and the time at the start of digitizing the 0.5-s samples of data (TS) are known, and the time of the selected noise data sample relative to the start time (TN) is also known, then the relative time $t_R$ can be found from

$$t_R = TS + 0.25 + TN - TOH \quad (17)$$

in seconds if TS and TOH are in seconds on a consistent basis. The 0.25-s term is included to shift the origin to the middle of the 0.5-s period since TS starts at the beginning of the first data sample for this demonstration.
From geometrical considerations, the time \( t_R \) can be shown to be related to the height, airspeed, Mach number, and propagation angle through

\[
t_R = \left(\frac{A M H}{V_a}\right) \left(M_a \csc \psi - \cot \psi\right). \tag{18}
\]

Note that for \( t_R = 0 \), equation (18) requires that \( \cos \psi = M_a \).

We solve equation (18) for \( \cos \psi \) using some algebraic manipulation and trigonometric identities to find

\[
\cos \psi = \left[M_a \left(\frac{A M H}{V_a t_R}\right)^2\right] \left[\left(\frac{A M H}{V_a t_R}\right)^2 + 1\right]^{-1} \\
\pm \left[\left(\frac{A M H}{V_a t_R}\right)^2 + 1\right]^{-1} \\
\sqrt{M_a^2 \left(\frac{A M H}{V_a t_R}\right)^4 - \left[\left(\frac{A M H}{V_a t_R}\right)^2 + 1\right] \left[\left(\frac{A M H}{V_a t_R}\right)^2 M_a^2 - 1\right]} \tag{19}
\]

where the + sign applies when \( t_R < 0 \) (i.e., for \( 0^\circ < \psi < \cos^{-1} M_a \)) and the - sign applies when \( t_R > 0 \) (i.e., for \( \cos^{-1} M_a < \psi < 180^\circ \)).

With a value for \( \psi \) from equation (19), the total length of the propagation path is

\[
PD = \frac{A M H}{\sin \psi}. \tag{20}
\]

The length \( D_j \) of any segment along the propagation path defined by heights \( H_M \) where meteorological data were measured can be determined, in general, from

\[
D_j = \frac{(H_M_{j+1} - H_M_j)}{\sin \psi} \tag{21}
\]

except for the first and last segments.

For the first segment, we sort through the \( H_M \) array to find the first height value that is greater than the microphone height \( Z_M \) and call that value \( H_{M_{II}} \). Then, the length of the first segment is

\[
D_1 = \frac{(H_{M_{II}} - Z_M)}{\sin \psi}. \tag{22}
\]

For the last segment, we sort through the \( H_M \) array and find that height value which is just less than the airplane height \( A_H \) and call that value \( H_{M_{IA}} \). The length of the last segment is then

\[
D_{IA} = \frac{(A_H - H_{M_{IA}})}{\sin \psi}. \tag{23}
\]
The input data described in figure 3 are read or calculated. With the array of propagation distances computed using equations (21), (22), and (23), calculation of atmospheric absorption adjustments can proceed.

Two test cases were selected. The measured, or test, meteorological data for the two cases are shown in figure 5 as vertical profiles of air temperature and relative humidity. There were 31 measurements at heights between 1.2 and 915 m. The station barometric pressure $BP$ is given in the subcaptions. Reference profiles of temperature and relative humidity are shown by the dashed lines. The airplane height was 629 m for test case 1 and 154 m for test case 2 as indicated by the arrows marked AH. Winds were always calm.

For test case 1 the temperature above 200 m was not a great deal different than the reference temperature, below 200 m the air was colder than the reference temperature. Relative humidity was everywhere significantly less than the reference relative humidity.

For test case 2, the temperature was everywhere less than the reference temperature, especially below 200 m. The relative humidity was close to the reference value above 150 m and greater than the reference value below 150 m.

For test case 1, the molar concentration of water vapor ranged from 1.0 to 0.45 percent as height increased and from 1.03 to 0.93 percent for test case 2. Thus the air was always significantly drier than at the reference condition of 25°C and 70 percent relative humidity at 10 m where the molar concentration of water vapor is approximately 2.2 percent. Atmospheric absorption under the test conditions would therefore be expected to be substantial for the higher frequencies and the adjustments to reference conditions would be large for the higher frequency bands, especially over the long propagation distance of test case 1.

The results of running program BANDLOSS for test case 1 are shown in table 1 in the format provided by the program. Table 2 lists the temperature and relative humidity profile data for test case 1. Tables 3 and 4 provide similar data for test case 2.
Figure 5.-Profiles of temperature and relative humidity; reference profiles indicated by dashed lines; airplane height at AH.
Table 1. Results from program BANDLOSS for test case 1.

| Band (kHz) | Freq (kHz) | SPL at REC (dB) | SPL at REC (dB) | A correction factor
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<td>103.9</td>
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</table>

\[ A_h = 29.6 \text{ m/s}, \quad A_S = 81.5 \text{ m/s}, \quad A_{AM} = 0.24, \quad Z_M = 1.2 \text{ m}, \quad P_D = 894.1 \text{ m} \]

\[ T_S = 30444.95 \text{ ms}, \quad T_1 = 9.05 \text{ s}, \quad T_{DH} = 30463.55 \text{ s}, \quad \text{PSI} = 44.3 \text{ deg} \]

AH = airplane height above ground level; AS = airspeed; AM = airplane Mach number; ZM = height of microphone above ground level; PD = sound propagation distance; TS = time for start of data analysis; TN = time after TS for noise data sample; TOH = time when airplane was over microphone; and PSI = sound propagation angle relative to flight path.
Table 2.-Measured and reference profile data for air temperature and relative humidity applicable to test case 1.

<table>
<thead>
<tr>
<th>HEIGHT AT MIDPOINT (M)</th>
<th>MEAN TEMP AT MIDPOINT (°C)</th>
<th>MEAN REL. HUM. AT MIDPOINT (%)</th>
<th>HYP. TEMP. AT HEIGHT (K)</th>
<th>HYP. REL. HUM. AT HEIGHT (%)</th>
<th>PCT.</th>
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<td>70.9</td>
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</tr>
</tbody>
</table>

*a 'Measured' temperatures and relative humidities listed above are values interpolated from the actual measured values to represent temperatures and relative humidities at heights midway between the heights where the temperatures and relative humidities were actually measured. The midpoint values represent average values over the pathlength segments.*
Table 3. Results from program BANDLOSS for test case 2.

<table>
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<tr>
<th>BAND CENTER FREQUENCY</th>
<th>MEAS. SPL AT REF.</th>
<th>ADJ. SPL AT REF.</th>
<th>ABSORPTION CORRECTION FACTOR, DB</th>
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<td>84.0</td>
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<tr>
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<tr>
<td>1000</td>
<td>04.0</td>
<td>84.0</td>
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</table>

AF=154.6M VSz 74.4M/S AM= .22 ZM= 1.2M PD= 377.2M
TS=26kHz US TN=16.0S TUM=26kHz US PSI=156.1DEG
Table 4.-Measured and reference profile data for air temperature and relative humidity applicable to test case 2.

<table>
<thead>
<tr>
<th>HEIGHT AT MIDPOINT OF INTERVAL (M)</th>
<th>MEAS. TEMP. AT MIDPOINT HEIGHT (°C)</th>
<th>MEAS. RH. AT MIDPOINT HEIGHT (%)</th>
<th>REF. TEMP. AT MIDPOINT HEIGHT (°C)</th>
<th>REF. RH. AT MIDPOINT HEIGHT (%)</th>
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<td>50.1</td>
<td>56.7</td>
<td>288.6</td>
<td>64.2</td>
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</table>
The results for the test cases are shown graphically in figure 6. The subcaptions give the airplane height, airspeed, Mach number, microphone height, time of the midpoint of the data sample relative to the overhead time, total propagation distance, and sound emission angle. Test case 1 was for a time prior to overhead; test case 2 was for a time after overhead.

For test case 2, all 24 band levels represent actual measured data. For test case 1, however, the levels in the last five bands were provided by extrapolation and thus the input spectrum for test case 1 is partially artificial.

Referring to tables 1 and 3, we note that the calculated adjustment factors are actually negative numbers for several of the 'midfrequency' bands — 630 to 1250-Hz for test case 1 and 400 to 3150-Hz for test case 2. The negative values indicate less atmospheric absorption under the test conditions than under the reference conditions, in contrast to the usual situation where the minimum absorption occurs near the reference conditions.

In the low frequency bands, the adjustment factors are all small to negligible, as expected, even for the 899-m pathlength of test case 1.

In the high-frequency bands, the adjustment factors are quite large, especially for test case 1. For test case 2, the adjusted spectrum shape looks plausible. For test case 1, however, the relatively flat spectrum of the adjusted sound pressure levels may be unreasonable, although its appearance is distorted in figure 6(a) because of the need to use a 20 dB/division scale for test case 1 instead of the standard 10 dB/division scale as used for test case 2 in figure 6(b).

For test case 1, moreover, the last five band levels (4000 to 10,000 Hz) had to be estimated as noted earlier because the measurement system noise floor was equivalent to a sound pressure level of about 40 dB. The true spectral rolloff rate might very well have been more rapid than estimated for the meteorological conditions of figure 5(a) and the propagation distance of 899 m. If a more rapid rolloff had been estimated, the calculated values of the adjusted sound pressure levels would have been lower. More-rapid rolloff rates, however, would probably have been no more accurate than the arbitrary rates selected for test case 1.
Figure 6.-Results of using program BANDLOSS to determine atmospheric absorption adjustments from test to reference meteorological conditions.
Test case 1 was included deliberately even though some band levels were missing and had to be supplied by arbitrary extrapolation. The results indicate the difficulty of obtaining reasonable estimates of the true pressure spectral density at the microphone on the basis of band levels when the spectral slopes are large.

Large spectral slopes likely mean that the single straight-line approximation of Appendix B is not realistic for the pressure spectral density function over the entire passband of the filters. Large spectral slopes probably also mean that significant energy is contributed by either the lower or upper stopbands. Thus the fundamental assumption that the actual frequency response of the real filters can be reasonably approximated by that of ideal filters is probably not valid for spectral slopes as steep as those for the estimated high-frequency sound pressure levels of test case 1. If the filter response cannot be represented by that of ideal filters, then the analysis of Appendix B would have to be modified to include integration over that portion of the filter's stopbands which contribute 'significant' energy. A definition of 'significant' would have to be supplied and a test for convergence of the integral would have to be developed.

The conclusions are (1) that extrapolation to supply missing low- or high-frequency band levels may give misleading results, especially extrapolation to obtain high-frequency sound pressure levels where atmospheric absorption effects are largest, and (2) that there probably is some maximum spectral slope for which the assumptions of Appendix B are valid. It was beyond the scope of the project to determine the range of spectral slopes for which the method of Appendix B was valid.

**DATA PROCESSING AND ANALYSIS**

Most existing systems for processing a high-volume of aircraft flyover noise measurements should already have provisions to supply almost all the input data needed by program BANDLOSS for calculating atmospheric absorption loss adjustments for bands of noise. Some minor modifications may be needed to provide data in the measurement units required by the program. A routine may be required to calculate the lengths of the sound propagation paths as is done in lines 130 to 137 of program BANDLOSS in Appendix C. The level
differences (DL in lines 176 to 179) and the definitions for RF, SE, and ISBN would have to be added. The calculations of reference meteorological conditions would also have to be added.

The statements in lines 191 to 233, with subroutine ANSAB (lines 278 to 297), comprise the essence of the program statements needed to determine band-loss adjustment factors $\Delta L_{B,TR}$. The results of the calculations could be applied to any 0.5-s sample of wide-band data included in the analysis of a flyover noise signal.

The three special considerations listed earlier should be taken into account before attempting to calculate absorption adjustments to measured sound pressure levels. These were: (1) determining which of the input sound pressure levels are missing because of interference from high ambient or background noise levels, (2) removing spectral irregularities caused by ground reflection effects, and (3) separating the input spectrum into discrete and broadband components.

Special rules will have to be developed to determine reasonable estimates for the slope of the input spectrum when it is determined that there are missing band levels between the first and last bands. A separate computational routine would have to be created to compute the adjustment factors for the discrete-frequency components of the spectrum, $\Delta L_{T,TR}$, based on the procedures of Appendix A.

RECOMMENDATIONS

This report has described the development of a method to calculate the atmospheric absorption experienced by broadband sound propagating in a quiescent atmosphere and analyzed by fractional octave band filters. The analytical method for bands of noise is based on several assumptions. Some additional analyses and experimental data are needed to verify the assumptions before adopting the procedures in routine applications.

The recommendations given below are divided into three categories: general, procedural and instrumentation considerations, and validation. Recommendations that are general in nature are for studies to help develop the method for use in a variety of applications, such as research investiga-
tions of aircraft noise propagation over long distances or through atmos-
pheric turbulence, prediction of aircraft flyover noise levels or noise ex-
posure level contours, and aircraft noise type certification. The procedural
and instrumentation considerations are relevant to all applications, but are
specifically appropriate to the aircraft noise measurement procedures of
Appendix A of FAR 36.

General

1. One of the fundamental assumptions in the analysis is that the slope
of the pressure spectral density vs frequency curve for the sound incident on
a microphone is not 'too' steep. This assumption permits replacement of the
frequency response of the actual filters used in processing the signal by the
frequency response of ideal filters.

A study should be conducted to determine the range of positive and nega-
tive spectral slopes for which the assumption of ideal filters gives valid
results. The study should consider the response characteristics of various
actual analog and digital filters as well as the response characteristics of
filters meeting the minimum transmission loss characteristics defined in
reference 12 for octave and 1/3-octave-band filters.

2. The magnitude of the calculated absorption depends on the spectrum
of the sound incident on the microphone. The actual sound spectrum is
approximated by a series of straight-line segments. A fundamental assump-
tion is that the slopes of the straight-line segments can be determined from the
average sound pressure spectral densities at the center frequencies of the
filter bands. For ideal filters, the bandwidth of the filters is defined
precisely. For real filters, an appropriate definition of bandwidth must
be developed.

The analysis in Appendix B develops a procedure for estimating the slopes
of the straight-line segments approximating the true spectrum on the basis of
the difference in the average pressure spectral densities in the frequency
bands above and below the particular band of interest. The validity of the
procedure needs to be critically examined, especially for measured sound
spectra which have large sound pressure level differences in adjacent bands
as often encountered in high-frequency data. The examination should be con-
ducted in conjunction with the study of the validity of approximating the response of real filters by that of ideal filters. The impact of alternate schemes for estimating the spectral slopes should be evaluated for typical spectra produced by a variety of aircraft types.

3. With the assumption of ideal filters, the procedure for determining the atmospheric absorption loss of broadband noise includes evaluation of an integral whose limits are the lower and upper bandedge frequencies of the corresponding ideal fractional-octave-band filter. The integration is performed by dividing the fractional octave band into several smaller frequency bands and approximating the integrand by a straight line (on logarithmic scales) over each sub-band. The examples in this report considered 1/3-octave bands which were then each subdivided into twenty 1/60-octave bands.

The accuracy of the integration improves as the fractional octave band is subdivided into an increasing number of sub-bands. The computing time, however, also increases. A study should be performed to evaluate the sensitivity and accuracy of the results to the number of sub-bands in the integration for a variety of assumed meteorological conditions and sound spectra.

4. In the demonstrations of the applications of the procedure developed here, a key assumption was that the noise produced by an aircraft flying over a microphone near the ground originated from a single acoustic point source. The source was assumed to be located at a point on the airplane flight path determined by the time of emission. The assumption of an acoustic point source is equivalent to assuming that the microphone is in the geometric and acoustic far field at all frequencies of interest.

For many cases of practical importance, the assumption of a single equivalent point source may not be reasonable because the airplane is a large, multi-engine type. The minimum distance between the microphone and the airplane reference point may be relatively short. Specific procedures need to be developed for application in situations where the aircraft cannot be considered to be equivalent to a single acoustic point source. The procedures should consider the necessity of defining apparent sound source locations as a function of frequency. For example, low-frequency jet noise sources might be located farther behind the jet exhaust nozzle than high-frequency jet noise sources.
5. The sound propagation path was assumed to be a straight line between the location of the effective acoustic point source and the microphone. Bending or curvature of the sound propagation path because of gradients in temperature or wind was ignored even though gradients were included in the profiles of test and reference temperature. A study should be performed to evaluate the need to incorporate a ray-tracing procedure in the determination of the lengths and directions of the various segments of the sound propagation path.

6. The demonstrations in this report of the procedure for calculating atmospheric absorption assumed that the nonlinear effects associated with propagation of high-amplitude sound waves could be neglected. It is known, however, that the spectrum of high-amplitude sound waves is modified by nonlinear effects as the waves propagate away from the source.

The sources of aircraft noise are obviously sources of high-amplitude sound. Research is needed to quantify the spectral effects introduced by nonlinear propagation of high-amplitude sound waves. Procedures should then be developed to incorporate these spectral effects into the procedures for determining atmospheric absorption adjustments. Since nonlinear effects modify the shape of the sound spectrum and since spectral slope is a factor in determining the atmospheric absorption of bands of noise, some method of simultaneously accounting for both effects may be required.

Procedural or Instrumentation Considerations

1. To determine an atmospheric absorption factor to adjust measured sound pressure levels from test to reference meteorological conditions, it was necessary to define an acoustic reference atmosphere with reference lapse rates for temperature, relative humidity, and pressure. Reference meteorological conditions at a height of 10 m above mean sea level (altitude) were specified to be those defined in FAR 36.

It is recommended that the acoustic reference atmosphere defined in this report be adopted for the purpose of calculating atmospheric absorption adjustment factors.
2. Since the atmospheric absorption correction for bands of noise depends on the sound spectrum which must be approximated from the measured sound pressure levels, a problem arises when some of the measured band levels are missing as a result of interference from high-level background noise.

Special rules for handling the problem of missing band levels need to be developed and tested. The rules should provide a procedure for examining the set of measured sound pressure levels and either estimating an appropriate spectral slope or deciding that no rational estimate can be made for the slope and that it is not feasible to determine corrections for differences in atmospheric absorption for that particular data sample.

3. A procedure to examine the measured set of sound pressure levels and remove ground-reflection spectral-interference effects should be developed, evaluated, and incorporated with the atmospheric absorption procedure. The evaluation should determine the magnitude of differences between removing or not removing the ground-reflection effects before calculating adjustments for atmospheric absorption. Microphone heights of 1.2 and 10 m should be evaluated. The analytical procedure for removing ground reflection effects could be based on the techniques of references 13 and 14.

4. Since different procedures are used to calculate atmospheric absorption losses for the discrete-frequency and the broadband components of an aircraft noise signal, a procedure for separating a given set of measured aircraft noise levels into the discrete and broadband components needs to be incorporated into an overall procedure for determining atmospheric absorption adjustments. The pure-tone identification procedure of FAR 36 would be a logical candidate for performing the spectral separation. Samples of aircraft noise signals with a mixture of discrete and broadband components should be evaluated to test the combined procedure.

5. The specific time (e.g., beginning, midpoint, or end) for synchronizing the 0.5-s data sample with the aircraft's position is a factor in determining the sound propagation distance. Data averaging time and the data averaging procedure are factors to be considered in relating the measured sound pressure levels to the average absorption experienced by sound waves incident on a microphone over the duration of a given data sample. For consistent
determination of atmospheric absorption adjustments, it is recommended that the midpoint of the duration of the data sample be used for synchronizing acoustic and aircraft data and that linear integration over the data-sample duration be used to obtain the time average of the filtered signals.

6. The current version of FAR 36 requires application of atmospheric absorption adjustments to only one of the 0.5-s data samples, the one associated with PNLT\textsubscript{test}, in order to determine PNLT\textsubscript{ref}. For most research and prediction purposes, it is recommended that absorption adjustments be determined for each data sample throughout the total duration of interest. For aircraft noise type certification, it is also recommended that consideration be given to requiring calculation of atmospheric absorption adjustments for all data samples within the duration of interest.

A study should be conducted to evaluate the impact of changing from applying absorption adjustments to just one data sample to applying them to all relevant data samples. The study should consider the impact on aircraft noise signals from a variety of aircraft types at various engine power settings and distances and under various test meteorological conditions. One purpose of the study would be to determine if the recommended procedure shifts the time of occurrence of PNLT under the acoustic reference meteorological conditions to a different time than under the test meteorological conditions.

It is also recommended that the study evaluate the impact of determining the duration correction factor under acoustic reference meteorological conditions from the set of PNLT\textsubscript{ref} values, calculated from the series of adjusted sound pressure levels, at the times corresponding to the PNLT\textsubscript{ref} values which are 10 dB less than the value of PNLT\textsubscript{ref}. The current FAR 36 procedure of accounting for atmospheric absorption for just one data sample does not specifically consider the change in duration that would be associated with the time variation of tone-corrected perceived noise level under acoustic reference meteorological conditions.

Validation

1. The complexity and importance of the problem of adjusting measured data from test to reference conditions require that the total impact of the proposed new procedures be carefully evaluated. The evaluation should consider
the impact on the noise produced by different types of jet and propeller-powered aircraft tested at various power settings and distances and under a variety of meteorological conditions. Tests under meteorological conditions which are close to those of the acoustic reference atmosphere should be conducted to provide the baseline data against which to compare the adjusted data from tests under other meteorological conditions.

Differences between test-day and acoustic-reference-day sound pressure levels, perceived noise levels (with and without tone corrections), and effective perceived noise levels should be evaluated. Atmospheric absorption adjustment procedures should include the method described in this report and the method in FAR 36 which is based on that in SAE ARP 866A.

2. As part of the overall validation effort, it is recommended that consideration be given to determining whether the proposed new procedure can be simplified without significant loss of accuracy.

With the procedure of ARP 866A, there are essentially only four variables: distance, band center frequency (or lower bandedge frequency for the high-frequency bands), air temperature, and relative humidity. For pure tones, the proposed new procedure has these same four variables (but with the measured tone frequency rather than band center frequency), plus the pressure of the air.

For bands of noise, the proposed new procedure requires integration over the range of frequencies contributing significant energy to the indicated band level. The integration process requires determination of a rational estimate of the slope of the sound spectrum over the same range of frequency. If the assumption of ideal filters does not turn out to be acceptable, then the integration may also require incorporation of the response of the actual filters used for analyzing the measured sound pressure levels.

With only the variables of frequency, temperature, and relative humidity, it was feasible for SAE ARP 866A to include tables showing the dependence of sound absorption coefficients on the variables. With the proposed new procedure, it is not feasible to construct tables for the sound absorption coefficient unless the ranges of some of the variables are greatly restricted.
One potential simplification, however, that should be included in the validation effort is replacement of the integration by determination of the differences in atmospheric absorption at the band center frequency only.

CONCLUDING REMARKS

1. A computational procedure and corresponding computer program were developed to determine differences in atmospheric absorption under test and reference meteorological conditions. The procedure is based on calculation of pure-tone atmospheric absorption according to the experimentally verified method in the proposed American National Standard ANSI S1.26. The absorption adjustment procedure developed here is applicable to discrete-frequency and broadband components of an aircraft noise signal.

2. An acoustic reference atmosphere was defined for the variation of temperature, relative humidity, and pressure with height above mean sea level. Reference conditions from FAR 36 were specified at a height of 10 m.

3. Application of the data adjustment procedure was demonstrated for two sets of 1/3-octave-band aircraft noise levels. The adjustment procedure for bands of noise requires evaluation of an integral over the frequency range of each band. To perform the integration, an estimate must be supplied of the slope of the spectrum of the sound pressure incident on the microphone. The computational procedure includes a method for estimating the spectrum slope on the basis of the measured 1/3-octave-band sound pressure levels.

4. Recommendations were developed for several general and specific studies to provide information for validating or refining the assumptions made in developing the analytical procedure for calculating absorption adjustments for bands of noise. Other studies were recommended as part of a validation effort that should be conducted prior to considering incorporation of the proposed new procedure in FAR 36 as a replacement for the existing procedure that is based on SAE Aerospace Recommended Practice ARP 866A.

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30 December 1977
REFERENCES


In this Appendix we develop the analytical expressions needed to calculate the magnitude of the atmospheric-absorption adjustment factor between test and reference conditions for discrete-frequency pure-tone signals. Absorption losses are determined in accordance with the procedures of the proposed American National Standard ANS S1.26.

The amplitude of the pressure in a sound wave decreases exponentially in the direction of propagation because of absorption processes. References 1 and 2 describe the physical mechanisms for these processes.

Ignoring attenuation of a sound wave's amplitude caused by spreading losses as the distance from the source increases and assuming a quiescent atmosphere, the decrease in pressure from point 1 to point 2 over a distance \( \xi \) can be expressed as

\[ p_2 = p_1 e^{-\alpha \xi} \]  
(A1)

where \( \alpha \) is the sound attenuation coefficient due to atmospheric absorption, or, for short, the absorption coefficient in units of nepers/m when distance \( \xi \) is in meters.

The absorption coefficient \( \alpha \) is a function of the frequency \( f \) of the sound wave and the temperature, humidity, and pressure of the air. From the proposed ANS S1.26, the formula giving the relation between \( \alpha \) and the physical variables is

\[ \alpha = f^2 \{1.84 \cdot 10^{-11} (P/P_0)^{-1} (T/T_0)^{1.2} \\ + (T/T_0)^{-5/2}[1.278 \cdot 10^{-2}(\exp(-2239.1/T)/(f_{r,0} + (f^2/f_{r,0}))) \\ + 0.1068(\exp(-3352/T))/(f_{r,N} + (f^2/f_{r,N}))]\} \]  
(A2)

where \( P \) is atmospheric pressure, \( P_0 \) a reference atmospheric pressure, \( T \) the
APPENDIX A

temperature of the air, $T_0$ a reference air temperature, $f_{r,0}$ the vibrational relaxation frequency for oxygen, and $f_{r,N}$ the vibrational relaxation frequency for nitrogen.

The expressions for the two vibrational relaxation frequencies are

$$f_{r,0} = \left(\frac{P}{P_0}\right) \left(24 + 4.41 \cdot 10^4 h \left[\frac{(0.05 + h)}{(0.391 + h)}\right]\right)$$  \hspace{1cm} (A3)

and

$$f_{r,N} = \left(\frac{P}{P_0}\right) \left(\frac{T}{T_0}\right)^{-1/2} \left[9 + 350h \exp\left(-6.142\left(\frac{T}{T_0}\right)^{-1/3} - 1\right)\right]$$  \hspace{1cm} (A4)

where $h$ is the molar concentration of water vapor in a given sample of moist air.

The value of the molar concentration is determined from the relative humidity and atmospheric pressure using

$$h = h_r \left(\frac{P_{\text{sat}}}{P_0}\right) / \left(\frac{P}{P_0}\right)$$  \hspace{1cm} (A5)

where $h_r$ is the relative humidity and $P_{\text{sat}}$ is the saturation vapor pressure of water (over liquid water) at the pressure and temperature of the moist air.

The ratio $P_{\text{sat}}/P_0$ is a function of temperature only and, as in proposed ANS Sl.26, is determined from

$$\log \left(\frac{P_{\text{sat}}}{P_0}\right) = 10.79586\left[1 - \frac{T_0}{T}\right] - 5.02808 \log \left(\frac{T}{T_0}\right) + 1.50474 \cdot 10^{-4} \left(1 - 10^{-8.29692\left(\frac{T}{T_0}\right)-1}\right) + 0.42873 \cdot 10^{-3} \left(10^{4.76955\left[1 - \frac{T_0}{T}\right]} - 1\right) - 2.2195983$$  \hspace{1cm} (A6)

where $T_0$ is the triple-point isotherm temperature with the exact value of 273.16 kelvins.

Equations (A2) to (A6) are combined to determine a value for $\alpha$ for given values of $f$, $T$, $h_r$ and $P$ and suitable values for $P_0$ and $T_0$. In the procedure described in this report, values of $\alpha$ are calculated by a subroutine called ANSAB for American National Standard ABsorption. Reference pressure and temperature values are $P_0 = 101.325$ kPa and $T_0 = 293.15$ K.
APPENDIX A

Over any given path of length $\xi$ where the absorption has the value $a$, the change, in decibels, in pure-tone sound pressure level $\Delta L_T$ between that at the beginning and end of the path (in the direction of propagation) is determined from the squared pressures and equation (A1) as

$$\Delta L_T = L_1 - L_2 = -10 \log \left( \frac{p_2^2}{p_1^2} \right) = -10 \log e^{-2a_\xi}$$  

$$= 20 a_\xi \log e$$  

$$= 8.686a_\xi$$  

$$= a_\xi$$  

where $a = 8.686 \alpha$ is the pure-tone absorption coefficient in units of decibels per meter or dB/m and the symbol log stands for logarithm to the base 10. [Note that $a$ would have units of dB/(100m) if $\xi$ were in hundreds of meters or units of dB/km if $\xi$ were in kilometers.] Subroutine ANSAB also determines values for the absorption coefficient $a$.

If the temperature, relative humidity, and pressure are not constant over the propagation distance $\xi$, then the pure-tone absorption loss should be calculated from

$$\Delta L_T = 8.686 \int_{\xi_1}^{\xi_2} a \, d\xi$$  

(A10)

where $\xi_1$ and $\xi_2$ are the coordinates at the beginning and end of the propagation path.

With $\Delta \xi = \xi_2 - \xi_1$ and an average absorption coefficient $\bar{\alpha}$ defined by

$$\bar{\alpha} = \frac{1}{\Delta \xi} \int_{\xi_1}^{\xi_2} a \, d\xi$$,

equation (A10) can be written as

$$\Delta L_T = 8.686 \bar{\alpha} \Delta \xi$$  

(A11)

where $\bar{\alpha}$ is the mean value of $a$ over pathlength $\Delta \xi$.

To determine the difference in atmospheric absorption loss between test and reference meteorological conditions along a sound propagation path, we let $L_1$ in equation (A7) represent the sound pressure level at the receiver after passage through the test atmosphere, or atmosphere 1. The level $L_2$ is the
APPENDIX A

sound pressure level at the receiver (from the same source at the same location) after passage through the reference atmosphere, or atmosphere 2.

The pure-tone atmospheric-absorption adjustment factor between test and reference conditions, \( \Delta L_{T,TR} \), is, by analogy to equation (A7),

\[
\Delta L_{T,TR} = L_2 - L_1 = 10 \log \left( \frac{p_{R2}^2}{p_{R1}^2} \right)
\]

where \( p_{R1}^2 \) and \( p_{R2}^2 \) are the squared pure-tone sound pressures at the receiver and \( \Delta L_{T,TR} \) is the number of decibels to be added algebraically to the measured levels, \( L_1 \), to get the adjusted levels \( L_2 \).

If the squared sound pressure at the source is \( p_S^2(f,0) \) and if the propagation path consists of segments \( \Delta \xi \) long, then the squared sound pressure at the receiver, \( p_{R1}^2 \), can be written by repeated application of equation (A1), as

\[
p_{R1}^2(f,\xi) = [p_S^2(f,0)](e^{-2\alpha_{1,1}\Delta \xi})(e^{-2\alpha_{2,1}\Delta \xi}) \cdots (e^{-2\alpha_{n,1}\Delta \xi})
\]

where the subscripts indicate the path segment number and the conditions of atmosphere 1. There are \( n \) segments along the total length of the path. The absorption coefficients would be those calculated using equations (A2) to (A6) for the (constant) temperature, humidity, and pressure appropriate for each path segment. The path length segments are assumed to be short enough that the calculated absorption coefficient is a good representation of the average absorption over the segment.

For sound propagation from the same source and over the same path, the squared sound pressure at the receiver after propagation through atmosphere 2, \( p_{R2}^2 \), can be written as

\[
p_{R2}^2(f,\xi) = [p_S^2(f,0)](e^{-2\alpha_{1,2}\Delta \xi})(e^{-2\alpha_{2,2}\Delta \xi}) \cdots (e^{-2\alpha_{n,2}\Delta \xi})
\]

Substituting equations (A13) and (A14) in equation (A12) gives
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\[ \Delta L_{T,TR} = 10 \log \left\{ e^{2(\alpha_{1,1} - \alpha_{1,2}) \Delta \xi_1} + 2 (\alpha_{2,1} - \alpha_{2,2}) \Delta \xi_2 + \cdots \right\} \]

or

\[ \Delta L_{T,TR} = 10 \log A \]

where \( A \) is the absorption factor defined by the exponential term on the right hand side of equation (A15).

To apply equation (A15), the absorption coefficients \( \alpha_{n,1} \) and \( \alpha_{n,2} \) are calculated using subroutine ANSAB for each pathlength segment \( \Delta \xi_n \) for specified values of temperature, humidity and pressure appropriate to that segment. The separate terms in the exponent are calculated and summed to determine the value of \( A \) and hence \( \Delta L_{T,TR} \).

An alternate expression for the absorption factor in equation (A15), which is more useful for some applications, can be obtained from the mathematical identities

\[ e^{-2\alpha \Delta \xi} = e^{-2a_{/10}} \Delta \xi \]

and

\[ e^{-2\alpha \Delta \xi} = e^{-2(a_{/10}) \Delta \xi} \]

With equations (A17) and (A18), the absorption factor becomes

\[ A = 10^{2(\alpha_{1,1} - \alpha_{1,2}) \Delta \xi_1/10} + 2 (\alpha_{2,1} - \alpha_{2,2}) \Delta \xi_2/10 + \cdots \]

In applying these pure-tone atmospheric absorption adjustment factors to 1/3-octave-band sound pressure levels containing discrete-frequency components, it is assumed that the tone frequency is always within the essentially zero-loss portion of a filter's passband.
APPENDIX B
ATMOSPHERIC ABSORPTION ADJUSTMENT FOR BANDS OF NOISE

Analysis

Reference 1 contains a discussion of the application of the proposed new technique of ANSI S1.26 for determining pure-tone sound absorption coefficients to calculation of the atmospheric absorption losses for noise analyzed by constant-percentage-bandwidth or fractional-octave-band filters. The procedure in reference 1 for determining band-loss adjustment factors is not directly applicable to the problem of correcting for differences in sound absorption losses occurring during propagation through a horizontally stratified atmosphere. The analysis described in this Appendix uses some of the methods presented in reference 1 to develop a procedure applicable to adjustment of measured aircraft noise levels from test to reference meteorological conditions. The basic concepts for the analysis procedure for bands of noise were developed by Louis C. Sutherland for the S1-57 Working Group that prepared the proposed new American National Standard on atmospheric absorption.

The sound source is assumed to produce a noise signal having a continuous pressure spectral density over the complete range of frequencies included in the analysis. The pressure spectral density of the noise signal at the source is denoted by $G_s(f, 0)$ at any frequency $f$. After propagation from the source to the receiver over distance $\xi$, the pressure spectral density of the noise signal is $G_1(f, \xi)$ for propagation through atmosphere 1 with test meteorological conditions and $G_2(f, \xi)$ for propagation through atmosphere 2 with reference meteorological conditions.

By analogy to the development of equations (A13) and (A14) in Appendix A, the pressure spectral densities $G_1(f, \xi)$ and $G_2(f, \xi)$ can be related to $G_s(f, 0)$ by

$$G_1(f, \xi) = G_s(f, 0) \left( e^{-2\alpha_{1,1} \Delta \xi_1} \right) \left( e^{-2\alpha_{2,1} \Delta \xi_2} \right) \cdots \left( e^{-2\alpha_{n,1} \Delta \xi_n} \right) \quad (B1)$$

and

$$G_2(f, \xi) = G_s(f, 0) \left( e^{-2\alpha_{1,2} \Delta \xi_1} \right) \left( e^{-2\alpha_{2,2} \Delta \xi_2} \right) \cdots \left( e^{-2\alpha_{n,2} \Delta \xi_n} \right). \quad (B2)$$
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We relate $G_2$ to $G_1$ by solving equation (B1) for $G_s$, substituting the result into equation (B2), and combining related terms in the exponent to obtain

$$G_2(f,\xi) = G_1(f,\xi) \left\{ e^{2(a_{1,1} - a_{1,2}) \Delta \xi_1} + e^{2(a_{2,1} - a_{2,2}) \Delta \xi_2} + \cdots ight\}$$

$$+ 2(a_{n,1} - a_{n,2}) \Delta \xi_n \right\}$$

(B3)

$$= G_1(f,\xi) A(f,\xi)$$

(B4)

where we have used the same symbol $A$ for the absorption factor defined previously by equations (A15) and (A16) for pure-tone sound signals.

The set of 1/3-octave-band sound pressure levels after propagation through atmosphere 1 and analysis by filters having response characteristics 1 is called $L_1$.

The set of 1/3-octave-band sound pressure levels at the receiver location after propagation through atmosphere 2 and analysis by 1/3-octave-band filters having transmission response 2 is called $L_2$. We want to find $L_2$ knowing the values of $L_1$, the characteristics of the atmosphere, and the characteristics of the filters.

The measurement process that results in the measured levels $L_1$ and that would result in the levels $L_2$ is shown schematically in figure B1. The sound pressure signal incident on the microphone is a broadband signal whose amplitude $p_1(t)$ varies randomly with time. The frequency spectrum of the signal $p_1(t)$ is a smooth, continuous function as indicated by the sketch of $G_1(f)$ vs $f$.

The sound pressure signal is recorded and processed by some data-acquisition/data-processing system. The system contains a set of 1/3-octave-band filters each of which has a power transfer function $|H_1(f;f_1)|^2$ defined by the square of the absolute value of its frequency response. The response curve for one of the filters with band center frequency $f_1$ is shown. Except for the filter, the rest of the entire data-acquisition/data-processing system is assumed to have unity gain at all frequencies of interest.
Figure B1.-Illustration of steps involved in determining 1/3-octave-band sound pressure levels representing the spectrum of a broadband random noise incident on a microphone.
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Depending on the bandwidth of the filter, the signal at the output of the filter has an approximately sinusoidal waveshape but with a random amplitude as indicated by the sketch of p(t) vs t. The spectral density of the voltage equivalent of the p(t) signal is shown by the sketch of $S_1(f;f_1)$ vs f around the band center frequency $f_1$. The function $S_1(f;f_1)$ is related to the pressure spectral density of the input signal $G_1(f)$, after conversion from a pressure signal to a voltage signal and passage through a filter, by

$$S_1(f;f_1) = G_1(f) \left| H_1(f;f_1) \right|^2.$$  \hspace{1cm} (B5)

For the example shown in figure B1, the $S_1(f;f_1)$ function is equal to $G_1(f)$ in the flat-response (passband) region of the filter around the center frequency $f_1$. In the frequency region below the lower bandedge frequency $f_L$ (i.e., in the lower stopband), the $S_1(f;f_1)$ function decreases more slowly than it does in the frequency region above the upper bandedge frequency $f_U$ (i.e., in the upper stopband) because the $G_1(f)$ function increases at frequencies less than $f_L$ and decreases at frequencies greater than $f_U$. Unless the rolloff rate of $\left| H_1(f;f_1) \right|^2$ is steeper than the slope of $G_1(f)$, the $S_1(f;f_1)$ function will be asymmetric and, depending on the shape and slope of $G_1(f)$, there can be significant contributions from either the lower or upper stopband frequency regions. If the filter were an ideal filter with unity gain in the passband and infinite rejection in the stopbands, then $S_1(f;f_1)$ would just equal $G_1(f)$ between the passband limiting frequencies $f_L$ and $f_U$ and be zero at all other frequencies.

The final step in the process is the detection of the equivalent p(t) signal and the display (or output) of the sound pressure level for the band. Detection involves squaring and time averaging to find the mean-square (or root-mean-square) value of p(t) or $< p^2(t)>$ where the <> signs indicate time averaging and it is understood that suitable conversion or scaling factors would be included to provide proper units.

The sound pressure level $L_1(f_1)$ in the band at $f_1$ for the filtered equivalent sound-pressure signal p(t) is
APPENDIX B

\[ L_1(f_1) = 10 \log \left( \frac{\langle p^2 \rangle}{p_{ref}^2} \right) \]  
\[ \text{(B6)} \]

where \( p_{ref}^2 \) is the square of the reference pressure (\( p_{ref} = 20 \mu Pa \)).

The mean-square value of a signal is equal to the integral over frequency of the pressure spectral density of the signal. Thus, equation (B6) can be written as

\[ L_1(f_1) = 10 \log \left\{ \int_0^\infty S_1(f;f_1) df \right\}/p_{ref}^2 \]  
\[ \text{(B7)} \]

or, with equation (B5), as

\[ L_1(f_1) = 10 \log \left\{ \int_0^\infty [G_1(f)][|H_1(f;f_1)|^2] df \right\}/p_{ref}^2 \]  
\[ \text{(B8)} \]

Similarly, for the sound pressure levels \( L_2(f_1) \) after propagation through the reference atmosphere, we can write

\[ L_2(f_1) = 10 \log \left\{ \int_0^\infty S_2(f;f_1) df \right\}/p_{ref}^2 \]  
\[ \text{(B9)} \]

or

\[ L_2(f_1) = 10 \log \left\{ \int_0^\infty [G_2(f)][|H_2(f;f_1)|^2] df \right\}/p_{ref}^2 \]  
\[ \text{(B10)} \]

As in the initial statement of the problem, we assume that the same set of filters would be used to measure both \( L_1 \) and \( L_2 \) and thus set

\[ |H_1(f;f_1)|^2 = |H_2(f;f_1)|^2 = |H(f;f_1)|^2 \]  
\[ \text{(B11)} \]

We also use equation (B4) to replace \( G_2(f) \) in equation (B10) to obtain

\[ L_2(f_1) = 10 \log \left\{ \int_0^\infty G_1(f) A(f) |H(f;f_1)|^2 df \right\}/p_{ref}^2 \]  
\[ \text{(B12)} \]

and

\[ L_1(f_1) = 10 \log \left\{ \int_0^\infty G_1(f) |H(f;f_1)|^2 df \right\}/p_{ref}^2 \]  
\[ \text{(B13)} \]

The band-loss atmospheric-absorption adjustment factor from test to reference meteorological conditions, \( \Delta L_{B,TR} \), is the quantity we seek. It is determined for the band with center frequency \( f_1 \) from

\[ \Delta L_{B,TR}(f_1) = L_2(f_1) - L_1(f_1) \]  
\[ \text{(B14)} \]
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and is to be added to the measured levels \( L_1(f_1) \) to obtain the adjusted levels \( L_2(f_1) \).

For any band, using equations (B12) and (B13), we can write

\[
\Delta L_{B,TR} = 10 \log \left\{ \int_{0}^{\infty} G_1 A |H|^2 df / \int_{0}^{\infty} G_1 |H|^2 df \right\}
\]

where the notation showing the functional dependence on frequency has been omitted.

We now assume that the slope of the pressure spectral density \( G_1 \) is not 'too steep' (neither too positive nor too negative) and that the response \( |H|^2 \) of each of the real filters can be adequately represented by that of ideal filters with the same set of band center frequencies. In other words, we assume that there is negligible contribution to the integrals in equation (B15) at frequencies outside the filter passband and replace the infinite frequency range by an integration from \( f_L \) to \( f_U \) with \( |H|^2 = 1.0 \).

Determination of the range of pressure spectral density slopes for which the response curves of practical filters could be safely approximated by the response of an ideal filter was beyond the scope of this project. This matter should be investigated in actual applications of the techniques discussed here because sound spectra with large positive or large negative slopes can result in significant energy being transmitted by the stopbands of certain practical filters. Large negative slopes are often encountered, for example, in the high-frequency part of aircraft noise spectra at long distances because of atmospheric absorption effects.

With the assumptions described above, equation (B15) becomes

\[
\Delta L_{B,TR} = 10 \log \left\{ [\int_{f_L}^{f_U} G_1 A df] / [\int_{f_L}^{f_U} G_1 df] \right\}.
\]

To integrate equation (B16) we require an expression for the pressure spectral density function \( G_1(f) \). We assume that we can find a reasonable approximation to the true pressure spectral density function, over the frequency range of a filter's passband, by means of a line segment of slope \( \ell' \) as shown in the sketch in figure B2. At the band center frequency \( f_1 \), the
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Figure B2.-Example of approximation to pressure spectral density $G_1(f)$ by line segment of slope $\ell'$ over frequency range from $f_L$ to $f_U$ of a 1/3-octave-band filter with band center frequency $f_1$.

The line segment approximation has the value $G_1(f_1)$.

Along the line segment in figure B2, the slope between $G_1(f)$ at any general frequency $f$ and $G_1(f_1)$ at band center frequency $f_1$ can be written as

$$\ell' = \frac{\log G_1(f) - \log G_1(f_1)}{\log f - \log f_1}. \quad (B17)$$

From equation (B17) we can write the general expression for the straight-line approximation to $G_1(f)$ as

$$G_1(f) = G_1(f_1)\left(\frac{f}{f_1}\right)^{\ell'} \quad (B18)$$

or

$$G_1(f) = G_1(f_1)\left(\frac{f_1}{f}\right)^{-\ell'}. \quad (B19)$$

Substituting equation (B19) into equation (B16) yields, after pulling the constant $G_1(f_1)f_1^{\ell'}$ outside the integrals and canceling,

$$\Delta L_{B, TR} = 10 \log \left\{ \frac{\int_{f_L}^{f_U} A f^{\ell'} df}{\int_{f_L}^{f_U} f^{\ell'} df} \right\} \quad (B20)$$
To evaluate equation (B20) we begin with the denominator term. When $\xi' \neq -1$, we have
\[
\int_{f_L}^{f_U} f^{\xi'} \, df = \frac{1}{1/(\xi'+1)} \left( f_U^{\xi'+1} - f_L^{\xi'+1} \right).
\] (B21)
When $\xi' = -1$, we have
\[
\int_{f_L}^{f_U} f^{-1} \, df = \ln \left( \frac{f_U}{f_L} \right)
\] (B22)
where $\ln$ represents natural logarithms.

To integrate the numerator term in equation (B20) we replace the single integral over the frequency range of the passband by a summation of integrals over a number of equally-spaced constant-percentage-bandwidth segments $SE$. This method of integration, instead of a straightforward numerical integration technique available from a number of standard scientific computational subroutines, was selected to minimize computer time (and cost) for a given level of accuracy.

Thus, we write for the numerator term
\[
\int_{f_L}^{f_U} A f^{\xi'} \, df = \sum_{j=1}^{SE} \int_{f_j}^{f_{j+1}} A f^{\xi'} \, df.
\] (B23)

The number of segments $SE$ is chosen to be large enough that the frequency spectrum of the absorption, over the length $A\xi$ of each step along the propagation path, can be assumed to be represented by a series of elements over which the absorption is proportional to frequency raised to some power $k$. By absorption over a pathlength $A\xi$, we mean the ratio of the squared sound pressures at the beginning and end of the step, i.e., equations (A17) or (A18). The absorption spectrum is most conveniently represented using equation (A18) on logarithmic scales and, over each of the narrow bands of frequency in the filter passband defined by the number of segments $SE$, the absorption spectrum is approximated by a straight line with slope $k$. 

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Thus, for a given temperature, humidity, pressure, and pathlength distance, the slope $k$ of the straight-line approximation from a base or reference frequency $f_j$ to any general frequency $f > f_j$ can be written as

$$k = \frac{a(f) \Delta \xi/10 - a(f_j) \Delta \xi/10}{\log (f) - \log (f_j)} \quad \text{(B24)}$$

or

$$k = \frac{[a(f) - a(f_j)](\Delta \xi/10)}{\log (f/f_j)}. \quad \text{(B25)}$$

A general expression for any term in the absorption factor $A$ defined by equation (A19) can then be written, from equation (B24), as

$$a(f) \Delta \xi/10 = 10 a(f_j) \Delta \xi/10 \quad \text{(B26)}$$

Substituting the general expression from equation (B26) into equation (A19) for each of the separate meteorological conditions and pathlength segments yields

$$A = 10 \left[ [a_{1,1}(f_j) - a_{1,2}(f_j)] (\Delta \xi_1/10) \right.$$

$$+ \cdots + [a_{n,1}(f_j) - a_{n,2}(f_j)] (\Delta \xi_n/10)$$

$$\left. \times \frac{[(k_{1,1} - k_{1,2}) + (k_{2,1} - k_{2,2}) + \cdots + (k_{n,1} - k_{n,2})]}{(f/f_j)} \right] \quad \text{(B27)}$$

We introduce the abbreviations

$$A(f_j) = 10 \left[ [a_{1,1}(f_j) - a_{1,2}(f_j)](\Delta \xi_1/10) \right.$$

$$+ \cdots + [a_{n,1}(f_j) - a_{n,2}(f_j)](\Delta \xi_n/10) \right]$$

for the value of the effective absorption factor at $f = f_j$ and

$$K = (k_{1,1} - k_{1,2}) + (k_{2,1} - k_{2,2}) + \cdots + (k_{n,1} - k_{n,2}) \quad \text{(B29)}$$

for the effective slope parameter. The terms $A(f_j)$ and $K$ represent effective or average values over the total propagation path resulting from differences in atmospheric absorption between test and reference conditions.
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With equations (B28) and (B29), equation (B27) becomes

\[ A = A(f_j) \frac{((f/f_j)^K)}{\ldots}. \]  

(B30)

We can test the reasonableness of equation (B30) by noting that if the test and reference meteorological conditions were identical, the value of \( A(f_j) \) would be 1.0 and \( K \) would be 0.0 giving \( A = 1.0 \) as it should.

With equation (B30), one of the integrals in the sum on the right hand side of equation (B23) becomes

\[ \int_{f_j}^{f_{j+1}} A(f_j) \frac{((f/f_j)^K)}{\ldots} df \]  

(B31)

or

\[ A(f_j) \int_{f_j}^{f_{j+1}} \frac{f^{K+\ell}}{\ldots} df. \]  

(B32)

When \( K+\ell' \neq -1 \), we have

\[ A(f_j) \int_{f_j}^{f_{j+1}} \frac{f^{K+\ell}}{\ldots} df = A(f_j) \frac{f^{1/(K+\ell'+1)}}{\ldots}(f_{j+1}^{K+\ell'+1} - f_j^{K+\ell'+1}) \]

\[ = A(f_j) \int_{f_j}^{f_{j+1}} \frac{1/(K+\ell'+1)}{\ldots}(f_{j+1}^{K+\ell'+1} - f_j^{K+\ell'+1}) \]

\[ = A(f_j) \int_{f_j}^{f_{j+1}} \frac{1/(K+\ell'+1)}{\ldots}[(f_{j+1}/f_j)^{K+\ell'+1} - 1]. \]  

(B33)

When \( K+\ell' = -1 \), we have

\[ A(f_j) \int_{f_j}^{f_{j+1}} \frac{f^{-1}}{\ldots} df = A(f_j) \int_{f_j}^{f_{j+1}} \frac{1}{\ldots} ln(f_{j+1}/f_j). \]  

(B34)

With equations (B21), (B22), (B33), and (B34) there are thus four possible cases for the ratio of integrals in equation (B20).
When \( \ell' \neq -1 \) and \( K+\ell' \neq -1 \), equations (B33) and (B21) yield the ratio

\[
\frac{\sum \frac{A(f_j)}{\frac{f_j^{\ell'+1}}{1/(K+\ell'+1)}} \left[ \frac{1}{1/(K+\ell'+1)} \right] \left[ (f_{j+1}/f_j)^{K+\ell'+1} - 1 \right]}{\sum \frac{1}{\frac{f_j^{\ell'+1}}{f_L^{\ell'+1}}}}.
\]

(B35)

For convenience, we divide numerator and denominator of equation (B35) by \( f_i^{\ell'+1} \) and move the constant terms from the denominator to under the summation. This manipulation gives

\[
\frac{\sum \frac{A(f_j)}{(f_j/f_i)^{\ell'+1}} \left[ (f_i^{\ell'+1})/(K+\ell'+1) \right] \left[ (f_{j+1}/f_i)^{\ell'+1} - (f_L/f_i)^{\ell'+1} \right] - 1}{\sum \frac{1}{(f_i^{\ell'+1})}} = \sum \frac{B_{L1}(B36)}{j=1}
\]

and

\[
\Delta L_{B,TR} = 10 \log \sum \frac{B_{L1}(B37)}{j=1}
\]

when \( \ell' \neq -1 \) and \( K+\ell' \neq -1 \), or \( K \neq -(\ell'+1) \).

For the second case, we take \( \ell' \neq -1 \) and \( K+\ell' = -1 \), i.e., \( K = -(\ell'+1) \). Equations (B34) and (B21) yield

\[
\frac{\sum \frac{A(f_j)}{f_j^{-K}} \ln \left( \frac{f_{j+1}/f_j}{f_j} \right)}{\sum \frac{1}{(f_i^{\ell'+1})}} = \sum \frac{B_{L2}(B39)}{j=1}
\]

Employing the same technique as before and using \( K = -(\ell'+1) \) gives

\[
\sum \frac{A(f_j)}{(f_j/f_i)^{\ell'+1}} \left[ (f_i^{\ell'+1})/(\ell'+1) \right] \left[ (f_{j+1}/f_i)^{\ell'+1} - (f_L/f_i)^{\ell'+1} \right] - 1 \ln \left( \frac{f_{j+1}/f_j}{f_i} \right)
\]

\[
= \sum \frac{B_{L2}(B39)}{j=1}
\]
and
\[
\Delta L_{B,TR} = 10 \log \sum_{j=1}^{SE} BL2_j
\]  
(B40)

when \( \ell' \neq -1 \) and \( K+\ell' = -1 \), or \( K = -(\ell'+1) \).

For the third case, we have \( \ell' = -1 \) and \( K+\ell' \neq -1 \) or \( K \neq 0 \). Equations (B33) and (B22) yield
\[
\frac{SE}{\sum_{j=1}^{SE} A(f_j)(1/K) [(f_{j+1}/f_j)^K - 1]} \ln \left(\frac{f_u}{f_L}\right)
\]  
(B41)
or
\[
\frac{SE}{\sum_{j=1}^{SE} A(f_j)[1/K \ln(f_u/f_L)]([(f_{j+1}/f_j)^K - 1] = \sum_{j=1}^{SE} BL3_j
\]  
(B42)

and
\[
\Delta L_{B,TR} = 10 \log \sum_{j=1}^{SE} BL3_j
\]  
(B43)

when \( \ell' = -1 \) and \( K+\ell' \neq -1 \) (or \( K \neq 0 \)).

The final case is when \( \ell' = -1 \) and \( K+\ell' = -1 \), i.e., when \( K = 0 \). With \( K = 0 \), we have
\[
\Delta L_{B,TR} = 0.0.
\]  
(B44)

Note that when \( K=0 \) the adjustment factor is 0.0 regardless of the noise slope \( \ell' \). For \( \ell' \neq -1 \) and \( K=0 \) equation (B36) yields \( \sum_{j=1}^{SE} BL1_j = 1.0 \) and hence \( \Delta L_{B,TR} = 0.0 \) by equation (B37); for \( \ell' = -1 \) and \( K=0 \), equation (B44) applies.

To simplify equations (B36), (B39), and (B42) we note that the ratio of upper to lower bandedge frequencies, \( f_u/f_L \), is, by definition, a constant for any constant-percentage-bandwidth filter. We denote this frequency ratio by the symbol RF.

The upper bandedge frequency is related to the band center frequency by
\[
f_u = f_1 RF^{1/2}.
\]  
(B45)
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The lower bandedge frequency is related to the band center frequency by

\[ f_L = f_1 \cdot R_F^{-1/2}. \]  \hspace{1cm} (B46)

Since the passband is divided into \( SE \) equally spaced frequencies (on a log frequency scale), the ratio of any two consecutive frequencies is

\[ f_{j+1}/f_j = R_F^{1/SE}. \]  \hspace{1cm} (B47)

The choice of the value of the \( SE \) should, ideally, consider the slopes of the noise spectrum and the absorption spectrum. However, on the basis of experience, subdivision of the 1/3-octave bands into twenty constant-percentage intervals was estimated to be able to provide adequate accuracy for practical cases of interest to aircraft flyover noise measurements.

In order to provide a set of regularly spaced band center frequencies that do not allow irregular overlapping of the frequency response of the contiguous filters, the band center frequencies were defined to be the precise frequencies for 1/3-octave-band filters, as specified in reference 12, namely

\[ f_1 = 10^{N/30} = 10^N/10 \]  \hspace{1cm} (B48)

where \( N \) is any integer: positive, negative, or zero. The range of \( N \) for these 1/3-octave-band analyses is from 17 to 40 covering nominal band center frequencies from 50 to 10,000 Hz.

With the precise band center frequency defined by equation (B48) the band frequency ratio is

\[ RF = f_{i+1}/f_i = 10^{1/10} = 1.2589. \]  \hspace{1cm} (B49)

Note that equations (B48) and (B49) define a set of frequencies that coincide exactly with the internationally standardized and preferred band center frequencies at the important decade frequencies of 100, 1000, and 10,000 Hz.

Once a value has been established for \( RF \), then the frequencies \( f_j \) depend only on the choice of the number of segments \( SE \), and, as stated earlier,
acceptable accuracy can be obtained with

$$SE = 20. \quad (B50)$$

The size of the frequency steps between $f_j$ and $f_{j+1}$ is $RF^{1/SE} = 10^{1/200} = 1.0116$ for $SE = 20$. [If $RF$ were $2^{1/3}$, then the frequency interval would be $2^{1/60} = 1.0116$ or 1/60-th octave bands for $SE = 20$.]

The set of $f_j$ frequencies begin at $f_L$ and increment by the frequency spacing factor $RF^{1/SE}$. Thus, the $f_j$ frequencies are specified by

$$f_j/f_1 = (RF^{-1/2}) (RF^{(j-1)/SE}). \quad (B51)$$

When $j = 1$, $f_j/f_1 = f_L/f_1$. When $j = SE$, $f_j/f_1 = f_SE/f_1 = (RF^{-1/2}) (RF^{(SE-1)/SE})$ and $f_j/f_1 = f_{SE+1}/f_1 = RF^{1/2} = f_U/f_1$ at the end of the interval for the passband.

The only remaining item needed to determine the band-loss adjustment factors is specification of the spectral slope $r'$. Recall that $r'$ is the slope of the pressure spectral density of the sound pressure at the microphone before being analyzed by the 1/3-octave-band filters. This slope is not available since the pressure spectrum of the source sound pressure is not known; only the filtered band levels $L_1$ are available.

To proceed, we assume that the slope of a straight line between the average pressure spectral densities $W_1$ in the bands below and above the band of interest at center frequency $f_1$ is a reasonable approximation to the slope of the true pressure spectral density $C_1$. The average pressure spectral density over the passband of the filter at center frequency $f_1$ is

$$W_1(f_1) = \langle p^2(f_1) \rangle / \text{BW}_1 \quad (B52)$$

where $\langle p^2(f_1) \rangle$ is the mean-squared sound pressure in the band at $f_1$ and $\text{BW}_1$ is the bandwidth of the band or $(f_U - f_L)$. With equations (B45) and (B46) and equation (B6), we can express equation (B52) as

$$W_1(f_1) = [P_{ref}^{2} 10^{L_1(f_1)/10}]/[f_1^{(RF^{1/2} - RF^{-1/2})}]. \quad (B53)$$
Similarly,

\[ W_1(f_{i+1}) = \left[ \frac{L_1(f_{i+1})}{10} \right] / \left[ f_{i+1}(RF^{1/2} - RF^{-1/2}) \right] \] (B54)

and

\[ W_1(f_{i-1}) = \left[ \frac{L_1(f_{i-1})}{10} \right] / \left[ f_{i-1}(RF^{1/2} - RF^{-1/2}) \right] \] (B55)

The slope \( \xi' \) of the straight line over the band at \( f_i \) between \( W_1(f_{i+1}) \) at \( f_{i+1} \) and \( W_1(f_{i-1}) \) at \( f_{i-1} \) is shown by the sketch in figure B3. This slope is determined from

\[ \xi' = \frac{\log W_1(f_{i+1}) - \log W_1(f_{i-1})}{\log f_{i+1} - \log f_{i-1}} \] (B56)

With equations (B54) and (B55), equation (B56) becomes

\[ \xi' = \frac{[[L_1(f_{i+1}) - L_1(f_{i-1})]/10] - \log (f_{i+1}/f_{i-1})}{\log f_{i+1} - \log f_{i-1}} \]

or

\[ \xi' = \frac{[[L_1(f_{i+1}) - L_1(f_{i-1})]/10]}{\log f_{i+1} - \log f_{i-1}} - 1 \] (B57)

Figure B3.-Example of approximate determination of slope \( \xi' \) from average pressure spectral densities in bands below and above the band of interest at center frequency \( f_i \).
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The slope of measured sound pressure over the band at $f_i$, from the band above to the band below, is the band noise slope $\ell$ on a plot of the logarithm of the squared sound pressure versus the logarithm of band center frequency. Thus the band noise slope over the band at $f_i$ is

$$\ell = \frac{[L_1(f_{i+1}) - L_1(f_{i-1})]/10}{\log(f_{i+1}/f_{i-1})} \quad (B58)$$

where the factor of 10 is included to eliminate the decibels and provide a measure of the mean-square sound pressure in the band.

With equation (B58), the spectral slope $\ell'$ can be written as

$$\ell' = \ell - 1. \quad (B59)$$

With RF defining the frequency ratio for any set of band-spaced frequencies, we have $f_{i+1} = f_i RF$ and $f_{i-1} = f_i/RF$. Thus $f_{i+1}/f_{i-1} = RF^2$ and we can write the band noise slope of equation (B58) as

$$\ell = \frac{[L_1(f_{i+1}) - L_1(f_{i-1})]}{20 \log (RF)} \quad (B60)$$

for every band except for first and last band.

The slope $\ell$ over the first band is found from the difference in sound pressure level between that of the second band and that of the first band. The slope over the last band is found from the difference in sound pressure level between that of the last band and that of the next-to-last band.

Thus for the first band

$$\ell = \frac{[L_1(f_2) - L_1(f_1)]}{10 \log (RF)} \quad (B61)$$

and for the last band

$$\ell = \frac{[L_1(f_i) - L_1(f_{i-1})]}{10 \log (RF)}. \quad (B62)$$

Equations (B59), (B60), (B61), and (B62) define the spectral slopes $\ell$ and $\ell'$.

With equations (B59), (B45), (B46), and (B47), the band loss adjustment factors can be written in a more convenient form as follows.
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When \( k' \neq -1, [\ell \neq 0], \) and \( k + k' \neq -1, [(k+\ell) \neq 0 \text{ or } K \neq -\ell, \) but \( K \neq 0], \) we have from equation (836)

\[
BL_1 = A(f_j)\left[\left(\frac{f_j}{f_1}\right)^{2/2} / (K+\ell)\right]\left[1/(RF^{2/2} - RF^{-2/2})\right][RF/(K+\ell)/SE - 1]. \tag{B63}
\]

When \( k' \neq -1, [\ell \neq 0], \) and \( k + k' = -1, [K + \ell = 0 \text{ or } K = -\ell, \) but \( K \neq 0], \) we have from equation (B39)

\[
BL_2 = A(f_j)\left[\left(\frac{f_j}{f_1}\right)^{2/2} / (SE)\right]\left[1/(RF^{2/2} - RF^{-2/2})\right] \ln (RF). \tag{B64}
\]

When \( k' = -1, [\ell = 0], \) and \( k + k' \neq -1, [(k+\ell) \neq 0 \text{ or } K\neq 0], \) we have from equation (B42)

\[
BL_3 = A(f_j)\left[1/(K \ln (RF))\right] (RF^{1/SE} - 1). \tag{B65}
\]

Since, by the definitions we have selected, the spectral slopes \( k' \) or \( k \) are constant over the passband of the filter, the three cases described by equations (B63), (B64), and (B65) really only define two options, assuming \( K\neq 0 \) at any frequency \( f_j \) in any passband. If \( K=0, \) then there is no adjustment factor and equation (B44) applies regardless of noise slope.

The two cases are for when \( \ell \neq 0 \) and when \( \ell = 0. \) When \( \ell \neq 0, \) then either equation (B63) or equation (B64) holds depending on whether \( K+\ell \neq 0 \) or \( K+\ell = 0 \) at a particular frequency step in the summation over a passband. We therefore define an index counter \( BL_1 \) to store contributions to the summation when \( \ell \neq 0 \) and where

\[
BL_1 = BL_j \text{ when } K+\ell \neq 0, \text{ eq. (B63)}, \tag{B66}
\]

and

\[
BL_1 = BL_j \text{ when } K+\ell = 0, \text{ eq. (B64)}. \tag{B67}
\]

The band loss adjustment factor when \( \ell \neq 0 \) is then

\[
\Delta L_{B, TR} = 10 \log \sum_{j=1}^{SE} BL_1. \tag{B68}
\]
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When $\ell = 0$, and $K+\ell \neq 0$ or $K \neq 0$, then only equation (B65) applies and no additional storage counter is needed. Thus

$$\Delta L_{B,TR} = 10 \log \sum_{j=1}^{\infty} BL_j$$

(B66)

when $\ell = 0$ and $K \neq 0$.

Recapitulation

The various expressions needed to carry out the calculations of band loss adjustment factors are repeated here for convenience.

When the band noise slope $\ell$ is nonzero ($\ell \neq 0$), then the band-loss adjustment factor $\Delta L_{B,TR}$ is found from

$$\Delta L_{B,TR} = 10 \log \sum_{j=1}^{\infty} BL_j$$

(B68)

where

$$BL_j = BL_{1j}$$

(B66)

when $K+\ell \neq 0$, or from

$$BL_j = BL_{2j}$$

(B67)

when $K+\ell = 0$, and where $K$ is the effective sound-absorption-factor slope over the entire sound propagation path.

When $\ell = 0$ and $K \neq 0$, then

$$\Delta L_{B,TR} = 10 \log \sum_{j=1}^{\infty} BL_j$$

(B69)

When $K = 0$, then

$$\Delta L_{B,TR} = 0.0$$

regardless of the band noise slope $\ell$. 
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The terms in the summations are defined by

\[ BL_j = A(f_j)[(f_j/f_i)^{\xi}][\xi/(K+\xi)][1/(RF^{\xi/2} - RF^{-\xi/2})][RF^{(K+\xi)/SE} - 1] \] (B63)

and

\[ BL_2 = A(f_j)[(f_j/f_i)^{\xi}][\xi/(SE)][1/(RF^{\xi/2} - RF^{-\xi/2})] \ln (RF) \] (B64)

and

\[ BL_3 = A(f_j)[1/K \ln (RF)] (RF^{K/SE} - 1). \] (B65)

The various expressions in the three sum terms are defined as follows.

The absorption factor \( A(f_j) \) at one of the frequencies \( f_j \) in the numerical integration over the frequency range of the filter passband from the lower \( f_L \) to the upper \( f_U \) bandedge frequency is

\[
A(f_j) = 10 \left\{ \left[ a_{1,1}(f_j) - a_{1,2}(f_j) \right] (\Delta \xi_1/10) + \left[ a_{2,1}(f_j) - a_{2,2}(f_j) \right] (\Delta \xi_2/10) + \cdots + \left[ a_{n,1}(f_j) - a_{n,2}(f_j) \right] (\Delta \xi_n/10) \right\}
\] (B28)

where the various values of absorption coefficient \( a \) are determined by equations (A2) to (A6) at frequency \( f_j \) for the temperature, humidity, and pressure conditions appropriate for each segment along the length of the propagation path (segments of length \( \Delta \xi \)) for either the test atmosphere (second subscript 1) or reference atmosphere (second subscript 2).

The effective absorption-factor slope term \( K \), over the frequency range from \( f_j \) to \( f_{j+1} \) and over the entire length of the propagation path, is

\[ K = (k_{1,1} - k_{1,2}) + (k_{2,1} - k_{2,2}) + \cdots + (k_{n,1} - k_{n,2}) \] (B29)

while the individual absorption slopes \( k \) are determined at the appropriate temperature, humidity, and pressure from

\[ k = \left\{ [a(f_{j+1}) - a(f_j)] (\Delta \xi/10) \right\} / \log (RF^{1/SE}) \] (B70)

after substituting \( f_{j+1} \) for \( f \) in equation (B25) and making use of equation (B47) for the frequency ratio \( f_{j+1}/f_j \).
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The 1/3-octave-band frequency ratio is

\[ RF = 10^{1/10} \]  \hspace{1cm} (B49)

and the number of segments over the frequency range of the filter passband is

\[ SE = 20. \]  \hspace{1cm} (B50)

The initial value of the \( f_j \) frequencies is the lower bandedge frequency

\[ f_L = RF^{-1/2} f_1 = 10^{-1/20} f_1 \]  \hspace{1cm} (B46)

where \( f_1 \) is the band frequency defined by

\[ f_1 = 10^{N/10} \]  \hspace{1cm} (B48)

and where \( N \) ranges from 17 to 40 for the 24 bands considered here.

The final value of the \( f_j \) frequencies is the upper bandedge frequency

\[ f_U = RF^{1/2} f_1 = 10^{1/20} f_1. \]  \hspace{1cm} (B45)

The increment in \( f_j \) frequencies is determined from

\[ f_{j+1}/f_j = RF^{1/SE} = 10^{1/200} \]  \hspace{1cm} (B47)

while a general \( f_j \) frequency is

\[ f_j = f_1 (RF^{-1/2}) (RF^{(j-1)/SE}). \]  \hspace{1cm} (B51)

For every band except the first and last bands, the band noise slope \( \lambda \) is found from

\[ \lambda = [L_1(f_{j+1}) - L_1(f_{j-1})]/20 \log (RF). \]  \hspace{1cm} (B60)

For the first band, we use

\[ \lambda = [L_1(f_2) - L_1(f_1)]/10 \log (RF) \]  \hspace{1cm} (B61)
APPENDIX B

and for the last band we use

$$\ell = \frac{L_1(f_i) - L_1(f_{i-1})}{10 \log (R)}.$$  \hspace{1cm} (B62)

The calculation procedures, symbols, and terminology given here in this recapitulation plus equations (A2) to (A6) from Appendix A are used directly for the computer program BANDLOSS, see Appendices C and D.
APPENDIX C
LISTING OF COMPUTER PROGRAM SOURCE STATEMENTS

Source statements, in FORTRAN IV, for program BANDLOSS are listed in this Appendix. The program was written to calculate atmospheric-absorption adjustment factors for a single 0.5-second sample of an actual aircraft flyover noise measurement. The aircraft noise spectrum is assumed to be that of a continuous broadband free-field signal with no discrete-frequency pure-tone components. The program determines 1/3-octave-band absorption adjustment factors $\Delta L_{B,TR}$. Subroutine ANSAB is called to calculate atmospheric absorption at specific frequencies within the passband of each 1/3-octave filter for each segment of the propagation path.

Input data are either defined in a data list or are read in. The data list defines the nominal center frequencies of the 1/3-octave bands from 50 Hz to 10,000 Hz for use in labelling the output data.

Input data are read in either directly from the terminal keyboard or from input files (if operating online). They could also be read in from punched cards or they could be supplied from another computer program which incorporated BANDLOSS as a subprogram. The terminal input data consist of: IMAX, TS, TN, TOH, AH, ZM, AS, and the set of 24 1/3-octave-band sound pressure levels, SPL. Measured meteorological data applicable to the input SPLs are read from the defined files as TC, HR, and BP.

Output data consist of tabulated values of the input 1/3-octave-band sound pressure levels, SPL, the adjusted 1/3-octave-band sound pressure levels, SPLA, and the 1/3-octave-band atmospheric-absorption adjustment factors, $BLTR = \Delta L_{B,TR}$. Associated airplane and geometric parameters are listed with SPL, SPLA, and BLTR. Meteorological parameters at the heights corresponding to the heights at the midpoint of each pathlength segment are also tabulated.

Several comments are included in the source statements to annotate the various steps in the program. All symbols and terms used in the computer program statements are defined in Appendix D.
SET F:5/TEMPHUM:IN (INPUT FILE FOR TEMPERATURE AND HUMIDITY)
SET F:5/PRESS:IN (INPUT FILE FOR STATION HYPOMETRIC PRESSURE)
SET F:6/MP: (WHEN RUNNING ON LINE)
SET F:6/LP: (FOR OFF-LINE LISTING)

*** PROGRAM NAME IS RANLUSS

*** PURPOSE OF THE PROGRAM IS TO DETERMINE ATMOSPHERIC ABSORPTION

*** LOSS ADJUSTMENTS BETWEEN TEST AND REFERENCE METEOROLOGICAL

*** CONDITIONS FOR AIRCRAFT FLYOVER NOISE MEASUREMENTS.

*** PROGRAM RANLUSS CALLS SUBROUTINE ANSAH

*** PROGRAM PREPARED BY DYTIC ENGINEERING, INC.

OF HUNTINGTON BEACH, CALIFORNIA; VERSION 17 DECEMBER 1977.

REAL K(1),&K2,K3,DLPA
DIMENSION HM(31),TC(31),HR(31),FTK(30),RHR(30),RPA(30),
1 ATK(30),AHR(30),D(30),DL(24),SPL(24),SPLA(24),
2 RTH(24),CF(24),HPI(30)
HEAD 2, IMAX,T5,TH,TON,AM,ZM,AM,AS
FORMAT (I2,F7.1,F4.1,F7.1,F5.1,F3.1,F4.2,F4.1)
READ 3,/SPL(1:1)=1,1,.12)
3 FORMAT (12F6.1)
HEAD 4,SPL(1:1)=13,24,
4 FORMAT (12F6.1)
DATA CF/50,60,80,100,125,150,200,250,315,400,500,630,
1,800,1000,1250,1500,2000,2500,3150,4000,5000,6300,8000,10000/

*** EKATUNE: RELATIVE HUMIDITY, AND PRESSURE FOR THE REFERENCE

*** ATMOSPHERIC CONDITIONS AT HEIGHTS WHICH ARE MIDWAY BETWEEN

*** THE HEIGHTS FOR WHICH METEOROLOGICAL MEASUREMENTS ARE

*** AVAILABLE.

Determine Heights for Meteorological Measurements.
(Note: Calculated Heights for Demonstration Exercise Only;
In General, height values would be part of input data.)

HM(1)=1.2; HM(2)=30.5
DU 5.1=3; IMAX
HM(1)=HR(1)+1)+30.5

Read input file that contains the measured values
of air temperature and relative humidity interpolated to the
Time of the Test.

Define File 5(31,10,E,INUM)
FORMAT (2F5.1)
DU 7=1,31
READ (5,1,6) IC(1),HR(1)

APPENDIX
NUM CALCULATE THE VALUES OF THE AVERAGE HEIGHTS AND THE
CORRESPONDING REFERENCE TEMPERATURES, RELATIVE HUMIDITIES,
AND PRESSURES AS WELL AS THE AVERAGE VALUES OF THE MEASURED
AIRM TEMPERATURES AND RELATIVE HUMIDITIES AT THE AVERAGE HEIGHTS.
MEASURED TEMPERATURES ARE CONVERTED FROM DEGREES CELSIUS
TO KELVINS.

IM=IMAX-1
DO 10 I=1,IM
AMH=AM(H(I)+1+HM(I))/2.
RHM=10.0
RTKU=298.15; FTKL=0.0065
RTK(I)=RTK+RTKL*(AMH-RHM)
RHRU=70.0; WHRU=0.0065
RHR(I)=RHR0+RHRU*(AMH-RHM)
RFAI=5.393*10.**(-5.)
LPA=APAL*(AMH-RHM)
RFT(I)=LPA+RFAI
ATK(I)=273.15*(TC(I)+1+TC(I))/2.
AMH(I)=(HP(I)+11+HM(I))/2.

NUM READ THE INPUT FILE THAT CONTAINS THE MEASURED, STATIOME
BARIOMETRIC PRESSURES INTERPOLATED TO THE TIME OF THE TEST.

DELETE FILE 511,7.E, JU(M)
READ (511,15) BP
RBP=29.921
PA=RP/RBP.

NUM DETERMINE THE TIME OF THE SOUND PRESSURE DATA SAMPLE.
RELATIVE TO THE TIME WHEN THE AIRPLANE WAS DIRECTLY OVERHEAD.
FACTOR OF 0.25S IS INCLUDED TO SHIFT TIME SCALE TO
MIDPOINT OF 0.5S DATA RECORD.

TR=TS+0.25+TN+TM

FIND THE HEIGHT OF THE AIRPLANE ABOVE THE MICROPHONE.

AMH=AH-2M

NUM DETERMINE SOUND PROPAGATION ANGLE CORRESPONDING TO THE TIME.

C1=AMH/(AS*TR)++2.
C2=C1*AM
C3=1.0+((AMH/(AS*TR))+2.
C4=AM+(1-C1)**(-1)
IF(TR<LT.0.0)PSI=ATAN((C1+SQR(T2(=C3-C4)))/C3)
IF(TR>LT.0.0)PSI=ATAN((AM)/C3)
IF(TR<LT.0.0)PSI=ATAN((C1-SQR(T2(=C3-C4)))/C3)
NOW FIND THE TOTAL LENGTH OF THE PROPAGATION DISTANCE FROM
THE SOURCE TO THE RECEIVER.

PU=AMH/SIN(PSI)

NOW DETERMINE THE INDICES FROM THE ARRAY OF HEIGHTS FOR THE
METEOROLOGICAL MEASUREMENTS THAT ARE ONE AND TWO LESS THAN
THE AIRPLANE HEIGHT.

DU 20 I=1,IMAX
IF(HM(I).GT.AH)GOTO 25
CONTINUE
I=I+1
IA=IA+1

NOW FIND THE LENGTHS OF THE PROPAGATION DISTANCES ALONG THE
PROPAGATION PATH CORRESPONDING TO THE INTERVALS OF HEIGHT
FOR THE METEOROLOGICAL MEASUREMENTS.

DU 30 I=1,IMAX
IF((HM(I)-ZM).GT.0.01)GOTO 35
CONTINUE
II=II+1
D(I)=(HM(I)-ZM)/SIN(PSI)
D(J)=(HM(J)-HM(I))/SIN(PSI)
D(I)=D(I)*D(J)

NOW SUM UP THE INDIVIDUAL PROPAGATION DISTANCES AND TEST TO
MAKE SURE THE TOTAL EQUALS THE PREVIOUSLY CALCULATED TOTAL
PROPAGATION DISTANCE.

TD=0.0
DU 45 J=1,IA
TD=TD+D(J)
IF((TD-PD).GT.0.1)GOTO 50
GOTO 60

WRITE (6,55)
FORMAT(15,PROPAGATION DISTANCES.)

NOW COMES THE CALCULATION OF ATMOSPHERIC ABSORPTION LOSSES.*****

ALTHOUGH NOT A PART OF THIS DEMONSTRATION PROGRAM, AT THIS TIME
IT WOULD BE APPROPRIATE TO EXAMINE THE INPUT SOUND SPECTRUM FOR
THE PRESENCE OF SPECTRAL IRREGULARITIES CAUSED BY GROUND-
PLANE REFLECTION EFFECTS. ANY INTERFERENCE PEAKS AND NULLS
SHOULD BE REMOVED BEFORE PROCEEDING.

AT THIS POINT IT WOULD ALSO BE APPROPRIATE TO EXAMINE THE
SPECTRUM TO DETERMINE IF THERE ARE ANY DISCRETE-FREQUENCY
TONES PRESENT. IF TONES ARE PRESENT, THEN THEIR LEVELS AND
FREQUENCIES SHOULD BE DETERMINED AND SEPARATE SPECTRA FOR
DISCRETE AND BROADBAND COMPONENTS SHOULD BE IDENTIFIED.
ALSO, AT THIS POINT, IT WOULD BE APPROPRIATE TO DETERMINE
HOW MANY, IF ANY, BAND LEVELS ARE MISSING BECAUSE OF
INTERFERENCE FROM AMBIENT NOISE.

NOW FIND LEVEL DIFFERENCES FOR THE MEASURED NOISE SPECTRUM.
NOTE THAT DIFFERENT RULES ARE USED FOR THE FIRST AND
LAST BANDS THAN FOR THE OTHER BANDS.
NOTE: SPECIAL PROCEDURES WOULD BE REQUIRED TO HANDLE SPECTRA
WITH MISSING BAND LEVELS.

DU 65 1=2,23
DL(1)=(SPL(1+1)-SPL(1-1))/2.
DL(1)=SPL(2)-SPL(1).
DL(24)=SPL(24)-SPL(23).

DEFINE THE VALUE FOR THE FREQUENCY RATIO RF
RF=10,41

DEFINE THE NUMBER OF FREQUENCY-INTERVAL SEGMENTS SE IN
THE PASSBAND OF EACH FILTER.

SE=20.0
JSF=SE

*** THIS IS THE MAIN LOOP FOR CALCULATING BAND LOSS CORRECTIONS
DO 40 I=1,24
ISHN=I*4.
F1=10**(ISHN/10.)
L=DL(1)/10.510.1(I/RF))
BLJ=J0.0 HAL3=0.
DO 120 J=1,JSF
FJ=F1*(1./SQR(RF))*(RF**((J-1)/SE))
FJ1=FJ*RF**((1./SE))
AE=0.0
K=0.0

DU 70 H=1,IA
TKI=ATK(H)
HRT=AHR(H)
MAT=PA
CALL ANSAH(TKI,HRT,PA,FJ1,A1)
CALL ANSAH(TKI,HRT,PA,FJ1,A11)
TKR=RTK(H)
HRR=HRH(H)
PAR=RAPA(H)
CALL ANSAH(TK,RHR,PA,FJ1,A2)
CALL ANSAH(TKR,HRR,PA,FJ1,A21)
AE=AE+(A1-A2)*((H/H)/10.)
K1=(A11-A1)*((H/H)/10.)/(1./SE)*LUG010(RF))
K2=(A21-A2)*((H/H)/10.)/(1./SE)*LUG010(RF))
K=K+(K1+K2)*

IF(K1+K2<0)GOTO 85
AJ=10.*AE
FJ1=FJ/F1
IF(LHF.0,0.0 AND K*LHF.0,0.0)PL1J=1
AJ=(FJ1*L)*((L/(K+1))*1./(RF**((L/2.))))
2 \((RF**((L/2.)))*(1+RF**((K+L)/SE)))\)
BLJ=BLJ1+BLJ2
IF(L.NE.0.0)HALJ=1.0
1 AJ*(FJ)**1.X*(L/SE)**1.X*(RF**((L/2.)))=BLJ
2 PF**((L/2.)))**1.X*LOG(RF)
BLJ=BLJ1+BLJ2
80 IF(L.NE.0.0)HALJ=1.0
1 J=1.0**LOG(RF)**1.X**LOG((H/NP))
IF(L.NE.0.0)HALJ1=1.0
80 IF(L.EQ.0.0)HALJ1=1.0
80 RSLA(1)=RSLA(1)+HLTH(1)
PS10=PS10*57.3457935
C
C*** TABULATE THE RESULTS
C
WHITE (0,200)
200 FORMAT('1', '13', 'BAND', 'T24', 'MEAS.', 'T35', 'ADJ.', 'T45', 'ABSORPTION')
WHITE (0,205)
205 FORMAT('1', '12', 'CENTER', 'T23', 'SPL AT', 'T34', 'SPL AT', 'T44', 'CORRECTION')
WHITE (0,210)
210 FORMAT('1', '12', 'IFCN...', 'T24', 'IFCN...', 'T35', 'IFCN...', 'T46', 'FACT(R,)')
WHITE (0,215)
215 FORMAT('1', '12', 'BHZ', 'T25', 'DB', 'DM', 'T49', 'R,')
WHITE (0,217)
217 FORMAT ('/)
WHITE (0,220) (CF(N), SPL(N), SPLA(N), HLTR(N), N=1,24)
220 FORMAT('1', '15', 'T5, T23, F5, T34, F5, T47, F5, 1)
WRITE (0,225) AH, AS, AM, AM2, PO
225 FORMAT ('/T7', 'AH=', 'F5.1', 'H', '2X', 'AS=', 'F5.1', 'M/S=', '2X', 'AM=', 'F4.2',
12X, 'AM2=', 'F4.2', 'H', '2X', 'PO=', 'F6.1', 'M')
WRITE (0,230) T5, T6, T7, T8, T9, T10, T11, T12, T13, T14, T15, T16, T17, T18, T19, T20
230 FORMAT ('/T7', 'T5=', 'F5.1', 'R', '2X', 'T6=', 'F4.1', 'S=', '2X', 'T7=', 'F7.1', 'S=', '2X',
1'T8=', 'F7.1', 'S=', '2X', 'T9=', 'F5.1', 'M')
WRITE (0,235)
235 FORMAT ('/T6', 'HEIGHT AT', 'T20', 'MEAS.', 'TEMP.', 'T35', 'MEAS.', 'R.H.',
1'T6', 'REF. TEMP.', 'T60', 'PLF.', 'R.H.')
WRITE (0,240)
240 FORMAT ('1', '11', 'MIDPOINT LF', 'T20', 'AT MIDPOINT', 'T35', 'AT MIDPOINT',
1'T6', 'AT MIDPOINT', 'T60', 'AT MIDPOINT')
WRITE (0,245)
245 FORMAT ('1', '12', 'MIDPOINT HF', 'T20', 'AT MIDPOINT', 'T35', 'AT MIDPOINT',
1'T6', 'HEIGHT', 'T60', 'HEIGHT')
WRITE (0,250)
250 FORMAT ('1', '12', 'HEIGHT', 'T50', 'HEIGHT')
WRITE (0,255)
255 FORMAT ('/)
WHITE (0,260) (HMI(H), ATK(H), ANH(H), FTK(H), RHK(H), H=1,1M)
260 FORMAT (T8, T6, T1, T21, T6, T35, F5, T47, F6, T61, T51)
**APPENDIX C**

```plaintext
SUBROUTINE ANSWER (Tk,hr,PA,F,A)

REAL LPSUP0

T0=293.15
T01=273.15
LPSUP0=10.15966*(1.-(Tk/T0)) - 5.20%40H*LOG10(Tk/T01)

1 + 1.50474*%*H*(1.-(Tk/T01))

0.42873e-3*(4.76055*(1.-(Tk/T01)) - 1.)

3

PSPUP0=10.15*LPSUP0

PA=15.5*PA/PA

F*K2=PA*(24.44*41.04*H*(0.45+H)/(0.341+H))

F*K2=(PA/SQRT(Tk/T0))*(9.*

1 - 350.1*EXP(-0.142*(((Tk/T01)*-(1.*3.1)-1.)))

ALPHA=(F*K2.1*(((1.R+4.111)/(1./PA)*SLOT(Tk/T0))

1 + (Tk/T01)*-5.0))+(1.*2.0)+2*

2

0.1068*EXP(-335.2./Tk))/(THETA+(F*K2./F*K2.2)))

A=8.688*ALPHA

RETURN
END
```
APPENDIX D
SYMBOLS USED IN COMPUTER PROGRAM

A atmospheric sound absorption coefficient from subroutine ANSAB, dB/m
A1 atmospheric sound absorption coefficient along one segment of a propagation path, for test or measured meteorological conditions, and a frequency equal to \( f_j \), dB/m
A11 same as A1, but for a frequency equal to \( f_{j+1} \), dB/m
A2 same as A1, but for reference meteorological conditions, dB/m
A21 same as A2, but for a frequency of \( f_{j+1} \), dB/m
ACOS the FORTRAN library function arccosine
AE the exponent for the absorption term \( A(f_j) \) defined by equation (B28), dimensionless
AH airplane height above ground level at the time when the sound signal was emitted from the airplane noise source, m
AHM height values which are midway between the heights for the measurements of meteorological data, m
AHR average relative humidity at the heights above ground level corresponding to the heights in the array HMI, percent
AJ the absorption term \( A(f_j) \) in equation (B28) and equal to \( 10^{AE} \), dimensionless
ALPHA the atmospheric sound absorption coefficient from equation (A2) and subroutine ANSAB, nepers/m
AM airplane Mach number at the time when the sound was emitted from the airplane noise source, dimensionless
AMH height, or vertical distance, between the airplane and the microphone, m
ANSAB name of a subroutine for calculating pure-tone sound absorption coefficients given the temperature (in kelvins), relative humidity (in percent), and pressure (in standard atmospheres) of the air and the frequency (in hertz) of the sound wave
AS airspeed of the airplane at the time when the sound was emitted from the airplane noise source, m/s
APPENDIX D

ATK average air temperature at the height above ground level corresponding to heights in the array HMI, kelvins

BLJ1 or BL3J band loss adjustment factor for the j-th frequency segment or the sum over the SE frequency segments and calculated using equations (B66), (B67), or (B65), dimensionless

BLTR band loss adjustment for differences in atmospheric absorption between test and reference conditions, dB

BP measured station barometric pressure at the elevation of the runway where the noise tests were conducted, in. of Hg or mb or kPa

C1, C2, C3, C4 abbreviations for nondimensional expressions used in calculating the angle of sound propagation between the aircraft flight path and a sound ray from the aircraft noise source and the microphone

CF nominal frequencies used for designating the center frequencies of standard 1/3-octave-band filters, Hz

D lengths of the segments of the sound propagation path defined by the heights of the measurements of meteorological data (called Δz in equations (B28) and (B70), m

DL differences in noise levels across a 1/3-octave-band from equations (B60), (B61), or (B62), dB

.EQ. FORTRAN library function for 'is equal to'

EXP FORTRAN library function for the base e of natural logarithms and used to calculate FRN2 in subroutine ANSAB

F frequency of a sound wave in subroutine ANSAB, Hz

FI precise center frequency for a 1/3-octave-band filter by equation (B48), Hz

FJ the frequency at any one of the SE steps in the passband of a filter between f_L and f_U and calculated according to equation (B51), Hz

FJ1 the next higher increment in frequency, f_{j+1}, after f_j from equation (B47), Hz

FJ1 the ratio of FJ to FI (f_{j+1}/f_j) from equation (B51), dimensionless

FRN2 the vibrational relaxation frequency of nitrogen molecules from equation (A4) and subroutine ANSAB, Hz
APPENDIX D

FRO2 the vibrational relaxation frequency of oxygen molecules from equation (A3) and subroutine ANSAB, Hz

.GT. FORTRAN library function for 'is greater than'

H molar concentration of water vapor in a sample of moist air from equation (A5) and subroutine ANSAB, percent

HM height above ground level at which upper-air meteorological data were measured, m

HMI an array of heights above ground level that are midway between the heights at which upper-air meteorological data were measured and equal to the midpoint heights in AHM, m

HR relative humidity of the atmosphere measured at heights HM, percent

HRR relative humidity of the atmosphere under reference meteorological conditions at the midpoint heights in HMI and equal to the reference relative humidities in the array RHR, percent

HRT relative humidity of the atmosphere under the test (measured) meteorological conditions at the midpoint heights in HMI and equal to the test relative humidities in the array AHR, percent

IA an index counter from the array of heights HM that indicates the height for the set of meteorological data that is just less than the airplane height defined by AH

IA1 IA-1

IMAX the number of heights (or sets of meteorological data) in the array HM

ISBN International Standard Band Numbers for 1/3-octave-band filters

K the absorption slope parameter over the propagation path from equations (B29) and (B70), dimensionless

K1 an absorption slope term from equation (B70) for one segment of the propagation path and for the measured atmospheric conditions, dimensionless

K2 same as K1 but for reference atmospheric conditions, dimensionless

L the exponent from equations (B60), (B61), and (B62) for the slope of the measured 1/3-octave-band sound pressure levels, dimensionless

LOG FORTRAN library function for natural logarithms to base e

LOG10 FORTRAN library function for logarithms to base 10
LPA  the logarithm (base 10) of the atmospheric pressure (measured in standard atmospheres relative to 1.01325 x 10^5 pascals) under reference meteorological conditions at the midpoint heights in HMI, dimensionless

LPSOPO as used in subroutine ANSAB, the logarithm (base 10) of the ratio of the saturation vapor pressure to the reference atmospheric pressure, dimensionless

.LT. FORTRAN library function for 'is less than'

.NE. FORTRAN library function for 'is not equal to'

PA measured air pressure in standard atmospheres or the ratio of BP to an appropriate reference pressure RBPO, standard atmospheres

PAR air pressure in standard atmospheres under the reference meteorological conditions at the midpoint heights in HMI and equal to the reference air pressures defined by the array RPA, standard atmospheres

PAT air pressure in standard atmospheres under the test meteorological conditions and equal to PA, standard atmospheres

PD the total length of the sound propagation path as determined from PD = ANH/SIN(PSI), m

PSI sound propagation angle between the airplane's flight path and a ray from the source of sound to the microphone, radians

PSID same as PSI but in degrees

PSOPO antilog of LPSOPO in subroutine ANSAB, dimensionless

RBPO reference air pressure used to determine PA from the measured value of pressure BP (29.921 in. Hg or 1013.25 mb or 101.325 kPa)

RF frequency ratio for 1/3-octave bands and equal to 10^{1/10}, dimensionless

RHM a reference height above ground level for the reference values of temperature, relative humidity, and pressure and equal to 10.0 m

RHR the array of reference relative humidities at the midpoint heights in array HMI from equation (2)

RHRL the lapse rate of the reference relative humidity with height and equal to -0.0065 percentage points/m

RHRO the reference relative humidity at the reference height RHM and equal to 70.0 percent

RPA the array of reference air pressures at the midpoint heights in the array HMI from equations (3) and (4)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RPAL</td>
<td>the lapse rate of the logarithm of the reference air pressure with height and equal to $-5.393 \times 10^{-5} \text{ m}^{-1}$</td>
</tr>
<tr>
<td>RTK</td>
<td>the array of reference air temperatures at the midpoint heights in array HMI from equation (1)</td>
</tr>
<tr>
<td>RTKL</td>
<td>the lapse rate of the reference air temperature with height and equal to $-0.0065 \text{ kelvins/m}$</td>
</tr>
<tr>
<td>RTKO</td>
<td>the reference air temperature at the reference height RHM and equal to 298.15 kelvins</td>
</tr>
<tr>
<td>SE</td>
<td>the number of frequency segments in the passband of a filter and the number of steps in the numerical integration of equation (B23) using equations (B68) and (B69), with the value of 20 for this program</td>
</tr>
<tr>
<td>SPL</td>
<td>the measured 1/3-octave-band sound pressure levels in decibels re 20 $\mu\text{Pa}$, dB</td>
</tr>
<tr>
<td>SPLA</td>
<td>the 1/3-octave-band sound pressure levels after adjustment for differences in atmospheric absorption between test and reference meteorological conditions, dB</td>
</tr>
<tr>
<td>SQRT</td>
<td>FORTRAN library function for square root</td>
</tr>
<tr>
<td>TC</td>
<td>air temperature measured at heights HM, degrees celsius</td>
</tr>
<tr>
<td>TD</td>
<td>the total length of the sound propagation path as determined by summing the lengths of the segments D from 1 to IA, m</td>
</tr>
<tr>
<td>TK</td>
<td>air temperature used by subroutine ANSAB in the calculation of LPSOPO, FRN2, and ALPHA, kelvins</td>
</tr>
<tr>
<td>TKR</td>
<td>air temperature under reference meteorological conditions at the midpoint heights in HMI and equal to the air temperatures in the array RTK, kelvins</td>
</tr>
<tr>
<td>TKT</td>
<td>air temperature under the test (measured) meteorological conditions at the midpoint heights in HMI and equal to the air temperatures in the array ATK, kelvins</td>
</tr>
<tr>
<td>TO</td>
<td>a reference temperature used by subroutine ANSAB in the calculation of FRN2 and ALPHA and equal to 293.15 kelvins</td>
</tr>
<tr>
<td>TO1</td>
<td>a reference temperature used by subroutine ANSAB in the calculation of LPSOPO and equal to 273.16 kelvins</td>
</tr>
<tr>
<td>TN</td>
<td>time of the noise measurements represented by the array SPL and relative to the start time TS, seconds</td>
</tr>
<tr>
<td>TOH</td>
<td>time of day on a 24-hour clock (86,400 s) when the aircraft was over or closest to the microphone, seconds</td>
</tr>
</tbody>
</table>
APPENDIX D

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR</td>
<td>time of the midpoint of the noise data sample on a 24-hour (86,400-s) clock, relative to the overhead time TOH, seconds</td>
</tr>
<tr>
<td>TS</td>
<td>time of day on a 24-hour (86,400-s) clock at the start of digitizing the first sample of data from the original analog magnetic tape recording, seconds</td>
</tr>
<tr>
<td>ZM</td>
<td>height of the microphone above ground level, m</td>
</tr>
</tbody>
</table>