A Corrected Parabolic-Equation Program Package for Acoustic Propagation

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**Title:** A CORRECTED PARABOLIC-EQUATION PROGRAM PACKAGE FOR ACOUSTIC PROPAGATION

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**Abstract:**
This report documents a package of computer programs which solves the parabolic equation for acoustic propagation using a split-step algorithm. These programs are implemented on the Texas Instruments ASC at NRL, and make use of its pipelining and vectorizing capabilities. They include the ability to partially correct for the errors induced by the parabolic assumptions. The report contains a summary of the theory and information on how to run the programs.
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A CORRECTED PARABOLIC-EQUATION PROGRAM PACKAGE FOR ACOUSTIC PROPAGATION

I. INTRODUCTION

The Helmholtz Equation for two-dimensional acoustic wave propagation in the ocean is

\[ \nabla^2 \psi(r,z) + k_o^2 n^2(r,z) \psi(r,z) = 0, \]  

(1)

where

- \( k_o \) = reference wave number = \( 2 \pi f / c_o \),
- \( n(r,z) \) = refraction index = \( c_o / c(r,z) \),
- \( \psi(r,z) \) = time independent acoustic pressure,

with

- \( F \) = source frequency,
- \( c_o \) = reference sound speed,
- \( c(r,z) \) = sound speed,
- \( r \) = cylindrical range coordinate,
- \( z \) = depth coordinate.

We write the solution of (1) as

\[ \psi(r,z) = \frac{ik_0 r}{\sqrt{k_0 r}} \phi(r,z) \]  

(2)

where \( \phi \) satisfies

\[ \phi_{rr} + 2ik_0 \phi_r + \phi_{zz} + k_o^2 [n^2(r,z) - 1 - \frac{1}{4(\kappa_o r)^2}] \phi = 0. \]  

(3)

We assume that \( \phi / r^2 \) is negligible in the far field and that

\[ |\phi_{rr}| \ll |2k_0 \phi_r| \]  

[1].

The physical assumptions of this approximation have been discussed in the literature [2–5]. Thus, we approximate \( \phi \) by a function \( \varphi \) which satisfies the parabolic equation

\[ 2ik_0 \varphi_r + \varphi_{zz} + k_o^2 [n^2(r,z) - 1] \varphi = 0. \]  

(4)

Note: Manuscript submitted December 23, 1977.
We approximate the solution of (4) by using the "split-step" algorithm [6] which marches in range:

\[
\hat{p}(r+\Delta r, z) = \exp \left( ik_0(n^2-1) \Delta r/2 \right) \\
\times \int \left\{ \exp \left( i \Delta r s^2/2k_0 \right) \right\} \}
\]

(5)

\[
\hat{f}(r, z) \}
\]

In (5), \( \hat{f}(r, z) \) is the Fourier Transform of \( f(r, z) \), \( a \) is the transform variable and \( \hat{p} = p \).

In order to meet periodicity requirements, the transform is taken over the finite region shown in Figure 1.

In the shaded regions, \( n^2 \) is given an exponentially increasing imaginary part which has the effect of causing the pressure to die off as the boundaries at \( D+\delta \) and \(-D-\delta \) are approached. This prevents energy from the periodically repeated regions containing image sources from entering the transform region. The boundary condition \( p=0 \) at the ocean surface is replaced by an image source at \( S' \). In practice, this boundary condition allows us to use a discrete sine transform [7] over the smaller region indicated in the figure.

The speed of this algorithm is due to the use of a sine transform which is essentially the Fast Fourier Transform (FFT).

The error which occurs from solving equation (4) by the split-step algorithm can be reduced by taking small range steps [8]. In our implementation we have included the option to partially correct [9,10] for the "parabolic" approximations (those made preceding equation (4) above). Analytically, we can express these corrections by

\[
\psi(r, z) = \frac{e^{ik_0 r}}{\sqrt{k_0 r}} \left\{ \hat{p}(r, z) + ir \frac{2}{k_0} \right\}
\]

where the second derivative term represents the corrections and dropping this term yields the usual parabolic equation approximation.

In order to implement the algorithm of equation (5), the sound speed field \( c(r, z) \) must be known (in order to calculate \( n(r, z) \)), and an initial pressure field must be given (which will be marched out in
range). We have separated these tasks from the range-stepping program called CSPLIT by writing two preliminary programs called PROFIL and START. PROFIL reads the environmental data, interpolates sound-speed profiles, writes bathymetry and a list of sound-speed profiles on a file for use by START and CSPLIT, and draws a plot of these profiles and the ocean bottom. START creates a file containing the initial pressure field corresponding to $\psi$ in equation (2). The user can choose from three alternatives: (1) a normal mode calculation [11], (2) a functional form which is Gaussian in depth, or (3) supply a complex FORTRAN Function, which will override the Gaussian.

A flow chart indicating the relationships between these programs is shown in Figure 2.

The direct outputs from CSPLIT are a listing and/or plot of transmission loss vs. range for each input depth. If the user wishes, CSPLIT will produce a file containing the parabolic solution at every depth grid point for every range step. This file can be used as input to two auxiliary programs VCTT and INTCON; VCTT creates a transmission loss vs depth listing and/or plot for each input range, and INTCON draws intensity contours.

Sections II, III, and IV describe how to access and use PROFIL, START, and CSPLIT respectively. The inputs for each program are explained and their formats are given. The various outputs are described in more detail. The auxiliary programs VCTT and INTCON are described in appendices A & B respectively.
Fig. 1 — Periodically repeated transform region. In the shaded regions, \( n^2 \) is given an exponentially increasing imaginary part which has the effect of causing the pressure to die off as the boundaries at \( D+\delta \) and \(-(D+\delta)\) are approached.
Fig. 2 — Flow chart indicating the relationships between the main programs, PROFIL, START, and CSPLIT
II. PROFIL: GENERATION OF SOUND SPEED PROFILES

PURPOSE:

Read environment, interpolate sound speed profiles, create a list of profiles on FTO2FO01 for later use, and plot these profiles. Careful consideration should be given to the interpolation, since each profile is in effect until the range of the following profile is reached.

USE:

/ JOB EXAMPLES PROFIL,acct. no.,user code,LOC=RTE7,OPT=(R)
/ LIMIT BAND=20,SEC=30
/ PD F,USERCAT/D81/L60/ PARA
/ PD YOURPATH,USERCAT/......
/ ASG SYS.OMOD,F/PROFIL/OBJ,USE=SHR
/ ASG LIBSUBS,P/LIBSUBS/OBJ,USE=SHR
/ LNK
LIBRARY LIBSUBS
/ FXQT
..... DATA.....
/ CAT YOURPATH/FTO2FO01,ACNM=FTO2FO01
/ FOSYS FT59FO01,TYPE=PLOT,FORM=00
/ EOJ

INPUTS:

NBOT: Number of bottom points. 1<NBOT<101.

NP: ABS(NP) is the number of input profiles. NP<0 implies that the bottom is flat.

IPLOT: 0 or 1. 0 implies no plot, 1 creates FT59FO01 which may be "FOSYSed" (TYPE=PLOT) or plotted offline.

RMAX: Maximum range in km. Can override when running CSPLIT.

BR(I),BD(I), I=1,NBOT: BD(I) is the depth at range BR(I). Depths must be in meters, and ranges in kilometers.

NC: Number of points on current input profile. 1<NC<50.

NSPFYD: Number of connections specified. NSPFYD<50.

IRGN: Number of profiles to generate between current input profile and the next input profile. Unless R2 > 0, these profiles will be equally spaced between the two input profiles.

RI: Start Range (in km) for current input profile.
R2: If R2 > 0 and IRGN > 0, then the interpolated profiles will be equally spaced between range R2 and the range of the next input profile. Specify R2 in kilometers.

D(I),C(I),I=1,NC : C(I) is the sound speed at depth D(I). Sound speeds should be in m/sec, and depths in meters. Note that D(I) must be 0.00.

N(I),M(I), I=1,NSPFYD: specifies that the N(I) th point on the current profile is to be connected to the M(I) th point on the next profile. (Regions between input profiles are partitioned into isogradient triangular sectors and interpolated profiles within regions are defined by the sector boundaries. The connections provide a way for the user to influence the partitioning. In many cases no connections are needed.)

KMPERI: Range scale for plot: km/inch.

MPSPI: Scale for plotting profiles: (m/sec)/inch.

FORMATS:

CARD 1:
NBOT,NP,IPLLOT,RMAX (3I4,F10.3)

CARD 2:
BR(I),BD(I),I=1,NBOT (10F8.2)

CARD 3:
NC,NSPFYD,IRGN,R1,R2 (3I5,5X,2F10.2)

CARD 4:
D(I),C(I),I=1,NC (10F8.2)

CARD 5:
N(I),M(I),I=1,NSPFYD (16I5)
(Omit card 5 if 0 is input for NSPFYD)

(Repeat cards 3,4 and 5 for each profile to be input.)

CARD 6:
KMPERI,MPSPI (2I3)

OUTPUT:

1. Input profiles on print file FT06F001.
2. Plot of profiles on FT9F001 (if IPOINT=1). A delta or a
nabla will mark the range for each profile. Nablins indicate
that the profile was an input profile and deltas indicate
that the profile is an interpolated profile. Each profile is
positioned with respect to its symbol so that the symbol
marks a 1500 m/sec reference point for plotting the profile.

3. Profiler and bottom on FT02F001. Suggest that you catalogue
FT02F001.
III. START: GENERATION OF INITIAL PRESSURE FIELD

PURPOSE:

Create initial pressure field on FT18F001.

USE:

/ JOB EXAMPLES,START,acct. no.,user code,LOC=LTE,OPT=(R)
/ LIMIT BAND=????,MIN=????
/ PD P,USERCAT/......
/ ASGP FT02F001,YOURPATH/FT02F001,USE=SHR
/ ASG SYS.MOD,P/START/OBJ,USE=SHR
/ ASG LIBSUBS,P/LIBSUBS/OBJ,USE=SHR
/ LNK
LIBRARY LIBSUBS
/ WAIT
/ FXQT CPTIME=????,ADDMEM=????
...... DATA ......
/ CAT YOURPATH/FT18F001,ACNM=FT18F001
/ EOJ

BANDS: If ISTART=1 or ITAPE=0, 20 bands should be sufficient, otherwise use ((Guess at number of modes)*NN + 5*NN)/16384 + 20, where NN is the number of points in depth (NN = 2**NPOW).

CPTIME: If the normal mode calculation is used, it is not unusual to need more than the default. To do a problem with NPOW=9, NINT=4, and 164 modes, 45 seconds are needed. Another problem with NPOW=11, NINT=4, and 325 modes required 430 seconds.

ADDMEM: MAX(8*NN + 4000,8000).

INPUTS:

The file containing the profile(s) created by profil must be assigned with the access name FT02F001.

ISTART: 0 OR 1. If ISTART=0, the initial field will be created by a normal mode calculation. If ISTART=1, a real Gaussian will be used, or you may supply your own initial field by writing a COMPLEX FORTRAN FUNCTION with the name "STFTN", which will return for a given depth the complex pressure at that depth. The following FORTRAN statements may be used:
COMPLEX FUNCTION STFTN(Z)
DOUBLE PRECISION RO,C0,F,BSS,SOURD
COMMON /OUT/ RO,C0,F,BSS,SOURD

STFTN= ...
RETURN
END

The normal mode start should be more accurate, but can be very expensive at higher frequencies.

NPOW: The number of points in depth on the initial field will be 2**NPOW . NPOW < 13. NPOW should be large enough so that (4./3.)*(maximum depth along track)/2**NPOW is approximately one-half wavelength.

F: The source frequency in Hertz.

RO: The range in km where the initial field is desired. Suggest that RO be at least several times the wavelength.

BSS: The speed of sound in the bottom (m/sec).

SOURD: The depth of the source in meters.

( The remaining inputs are not needed if ISTART=1 . )

ITAPE: 0 or 1. If ISTART=0 and ITAPE = 1, the eigenvalues and eigenfunctions are written on FT17F001 (a QDAM file), and some unformatted information is on FT03F001 which could be used to open FT17F001. Unless you have some use for the eigenvalues and eigenfunctions, you should use ITAPE = 0.

NINT: When calculating the modes, the program actually uses NINT*(2**NPOW) points in depth. NINT must be a power of two, and NINT*(2**NPOW) must be no greater than 16384. NINT=6 is good in most cases.

MAXMOD: This is the maximum number of modes that will be used. If you want all modes included, MAXMOD must be at least as large as the actual number of modes. If MAXMOD is less than the actual number of modes, then only the lowest modes (those with 0 to MAXMOD-1 turning points) are included.
RH01: The water density in gm/cc.

RH02: The density in the bottom in gm/cc.

EPSILN: A convergence parameter, suggest EPSILN = .001.

FORMATS:

CARD 1:
ISTART,NPOW,F,RO,BSS,SOURD (213,4F10.3)

CARD 2:
ITAPE,NINT,MAXMOD,RH01,RH02,EPSILN (314,3F10.3)

(CARD 2 not needed if ISTART=1)

OUTPUT:

1. The eigenvalues and the initial pressure field on FT06F001. III, JJJ, and KKK are listed for each eigenvalue. III and JJJ indicate how many of each of two types of iterations were needed. If KKK=1, this indicates that this mode was zeroed near the surface since it did not sufficiently converge to the surface boundary conditions.

2. The initial pressure field is on FT18F001. Suggest that you catalogue FT18F001.

3. Files FT03F001 and FT17F001. (Only if ITAPE = 1. See description of input ITAPE for explanation of these files.)
IV. CSPLIT: MARCH PARABOLIC SOLUTION OUT IN RANGE

PURPOSE:

Use the split step algorithm to calculate the solution to the parabolic equation approximation to the Helmholtz equation.

USE:

/ JOB EXAMPLE$CSPLIT,acct. no.,user code,LOC=RITE7,OPT=(R)
/ LIMIT BAND=????,MIN=????
/ PD P,USERCAT/D81/L60/PARA
/ PD YOURPATH,USERCAT/......
/ ASGP FT02FO01,YOURPATH/FT02FO01,USE=SHR
/ ASGP FT18FO01,YOURPATH/FT18FO01,USE=SHR
/ ASG SYS.OMOD,P/CSPLIT/OBJ,USE=SHR
/ ASG LIBSUBS,P/LIBSUBS/OBJ,USE=SHR
/ LNK
LIBRARY LIBSUBS
/ WAIT
/ FXQT CTIME=????,ADDMEM=????
..... DATA ..... 
/ CAT YOURPATH/FT66FO01,ACNM=FT66FO01,DTYP=TAPE
/ POSYS FT59FO01,TYP=PLLOT,FORM=00
/ EQJ

BANDS: ITAPE*(2*NN*(# of steps + 1)) where NN=2**NPOW.

CTIME: Approximately proportional to number of points in depth and number of range steps. To do a problem with 1024 points in depth and 1250 range steps 20 seconds are needed. If ICORR=1, the run time will approximately double.

ADDMEM: MAX(6*NN + 4000,8000).

INPUTS:

The file containing the profile(s) created by PROFIL must be assigned with the access name FT02FO01.

The file containing the initial pressure field created by START must be assigned with the access name FT18FO01.

IBFLAG: 0 or 1. 0 implies that the bottom is flat.
IPFLAG: 0 or 1. 0 implies that this is a problem with only one sound speed profile.

IPLOT: 0 or 1. If IPLOT = 0, no plot files will be created.

ITAPE: 0 or 1. If ITAPE = 1, the parabolic solution at each range will be written on FT66FOOI. See outputs for uses of this file.

NPOW: The number of points in depth is 2**NPOW. NPOW must be the same as it was for START.

NRD: The number of receiver depths. 0<NRD<16.

DELR: The range step in kilometers.

RMAX2: Maximum range. If RMAX2 is not positive then the maximum range which was input to PROFIL will be used.

STARTR: If STARTR > 0.00, then the pressure field at the first range larger than STARTR km is written on FT19FOOI. This can be used to "restart" CSPLIT by releasing FT18FOOI, and renaming FT19FOOI to FT18FOOI.

RECD(I), I=1,NRD: The receiver depths in meters.

ICORR: The number of steps to take between corrections. If ICORR = 0, then no corrections will be made.

ABCOEF: A parameter effecting the absorption in the bottom. Suggest ABCOEF=.01.

DBPKM: Attenuation in the water. If DBPKM is greater than or equal to zero, then the attenuation will be DBPKM dB per kilometer. If DBPKM < 0, then the attenuation will be calculated as a function of the frequency.

AKMPI: Scale for transmission loss plots in km/inch.

FORMATS:

CARD 1:
IBFLAG, IPFLAG, IPLOT, ITAPE, NPOW, NRD   (4I2,2I3)

CARD 2:
DELR, RMAX2, STARTR   (3F10.3)
CARD 3:
RECD(I),I=1,NRD  
(8F10.3)

CARD 4:
ICRR,ABCOEF, DBPKM  
(I4,2F10.5)

CARD 5:
AKMP  
(F6.2)

OUTPUT:

1. Transmission loss at all ranges for each receiver depth on FT06F001.

2. Plots of transmission loss vs range at each receiver depth on FT39F001. (Only if IPLIT = 1)

3. The pressure field at each range on FT66F001. (If ITAPE = 1)
   This file can be used to draw transmission loss vs depth plots at several ranges (see program VCUT), or to draw intensity contours (see program INTCON). We also have a way to get "shade" plots of the intensity using this file.

4. A restart field on FT19F001. (If STARTR > 0)
REFERENCES


7. We are indebted to H. Brock of the Naval Research Laboratory, Washington, D.C., for providing this subroutine.


APPENDIX A - VCUT: TRANSMISSION LOSS VS DEPTH

PURPOSE:

To draw plots of transmission loss vs depth at several ranges using the data file created by CSPLIT.

USE:

/ JOB EXAMPLE$VCUT,acct. no.,user code,LOC=RT7,OPT=(R)
/ LIMIT BAND=????,MIN=1
/ PD P,USERCAT/D81/L60/PARA
/ PD YOURPATH,USERCAT/......
/ ASCP FT66F001,YOURPATH/FT66F001,USE=SHR
/ ASC SYS.OMOD,P/VCUT/OBJ,USE=SHR
/ LNK
/ WAIT
/ FXQT
...... DATA ......
/ FOSYS FT59F001,TYPES=PLOT,FORM=00
/ EOJ

BANDS: Space for the data file FT66F001 which was created by CSPLIT must be provided.

CPTIME: Default normally sufficient for less than ten ranges.

ADDMEM: MAX(6*NN+4000,8000)

INPUTS:

The data file created by CSPLIT must be assigned with the access name FT66F001.

NPOW: Must be the same as for START and CSPLIT.

NSTP: The number of steps taken by CSPLIT.

NR: The number of ranges to be input. 0<NR<51.

RANGE(I),I=1,NR: A transmission loss vs depth plot will be generated for each range input. The plot for RANGE(I) will be at the first range step which is at or beyond RANGE(I). Input ranges in kilometers.
D1: Start depth in meters for plots.

D2: Final depth in meters for plots.

DMAX: Maximum depth of the water in meters.

AMPI: Depth scale for plots; meters/inch.

FORMATS:

CARD 1:
NPOW,NSTP,NR (3I4)

CARD 2:
RANGE(I),I=1,NR (8F10.3)

CARD 3:
D1,D2,DMAX,AMPI (4F10.3)

OUTPUT:

1. Depths and transmission losses for each input range on FT06F001.

2. Transmission loss vs depth plots for each input range on FT59F001.
APPENDIX B - INTCON: DRAWING INTENSITY CONTOURS

PURPOSE:

To draw intensity contours using the data file created by CSPLIT.

USE:

/ JOB EXAMPLE$INTCON,acct. no.,user code,LOC=RTE7,OPT=(R)  
/ LIMIT BAND=????,MIN=10  
/ PD P,USERCAT/D81/L60/para  
/ PD YOURPATH,USERCAT/......  
/ ASGP FT02F001,YOURPATH/FT02F001,USE=shr  
/ ASGP FT66F001,YOURPATH/FT66F001,USE=shr  
/ ASG SYS.OMOD,P/INTCON/OBJ,USE=shr  
/ ASG LIBSUBS,P/LIBSUBS/OBJ,USE=shr  
/ LNK  
LIBRARY LIBSUBS  
/ WAIT  
/ XFQT CPTIME=????,ADDMEM=????  
...... DATA ......  
/ FOSYS FT59F001,TYPE=PLOT,FORM=00  
/ EOJ

BANDS: Space for the data file FT66F001 which was created by CSPLIT must be provided.

CPTIME: 30 seconds for each 100 range steps should be sufficient.

ADDMEM: MAX(6*NN+4000,8000)

INPUTS:

The file containing the profile(s) created by PROFIL must be assigned with the access name FT02F001.

The data file created by CSPLIT must be assigned with the access name FT66F001.

NP0W: Must be the same as for START and CSPLIT.

NSTP: The number of steps taken by CSPLIT.
NCL: The number of contour levels to be plotted. 0<NCL<21.

LMIN: The minimum contour level to be drawn. If LMIN=0, then the minimum contour level will be calculated.

IDELCL: The difference between two consecutive contour levels.

AKPI: Range scale for the plot; km/inch.

XMIN: Start range in kilometers for the plot.

XMAX: End range in kilometers for the plot. If XMAX is more than 1000 range steps beyond XMIN, XMAX will be reduced by the program.

YMIN: Start depth in meters for the plot.

YMAX: End depth in meters for the plot. If YMAX is more than 400 depth points deeper than YMIN, YMAX will be reduced by the program.

FORMATS:

CARD 1:
NPOW,NSTP,NCL,LMIN,IDELCL,AKPI (5I4,F6.2)

CARD 2:
XMIN,XMAX,YMIN,YMAX (4F10.3)

OUTPUT:

1. Some information about the length and number of contour levels on FT06F001.

2. The contour plot on FT59F001.
APPENDIX C - SAMPLE OUTPUTS.

Figures 3, 4, 5, and 6 are some outputs from PROFIL, CSPLIT, VCU, and INTCON, respectively, for sample problem 1B from the AESD Workshop on Acoustic Propagation Modelling by Non-Ray-Tracing Techniques [12]. The frequency is 25 Hz and the source depth is 254 m. Since this is a single-profile and flat bottom problem, figure 3 is a particularly simple plot. The bottom is not apparent in figure 3, since it was drawn along the x-axis. The (m/sec)/inch scale indicated at the top of figure 3 applies to the original figure, before it was reduced for publication (the x-axis tick marks on the original were one inch apart). Figures 4 and 5 are transmission loss plots vs range and depth respectively. The flat bottom is apparent in figure 6, an intensity contour plot.
Fig. 3 — A sample plot from PROFIL
DEPTH (M) = 873.1

Fig. 4 — A sample plot from CSPLIT
RANGE (KM) = 60.150

Fig. 5 — A sample plot from VCU