THESIS

NON-LINE-OF-SIGHT ELECTRO-OPTIC LASER COMMUNICATIONS IN THE MIDDLE ULTRAVIOLET

by

Dennis Michael Junge

Dec 77

Thesis Advisor: W.M. Tolles

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Acting Dean of Research
**Non-Line-of-Sight Electro-Optic Laser Communications in the Middle Ultraviolet**

Dennis Michael Junge
in conjunction with William M. Tolles

**Naval Postgraduate School**
Monteey, California 93940

December 1977

Naval Postgraduate School
Monteey, California 93940

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Electro-optic laser communication
middle ultraviolet
multiple scattering
non-line-of-sight

A Monte Carlo computer simulation was developed to model hypothesized electro-optic laser communication systems operating in the middle ultraviolet region of the spectrum called the solar blind. By assuming various source, propagation, and detector characteristics as well as certain performance parameters it is possible to predict the effective ranges and operating characteristics.
of such a system.
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Non-Line-of-Sight Electro-Optic Laser
Communications in the Middle Ultraviolet

by

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Lieutenant, United States Navy

Submitted in partial fulfillment of the
requirements for the degree of

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Thesis Advisor

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ABSTRACT

A Monte Carlo computer simulation was developed to model hypothesized electro-optic laser communication systems operating in the middle ultraviolet region of the spectrum called the solar blind. By assuming various source, propagation, and detector characteristics as well as certain performance parameters it is possible to predict the effective ranges and operating characteristics of such a system.
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I. INTRODUCTION

A. HISTORY

Historically optical communications systems were developed over line-of-sight paths due to the propagation characteristics of light. In the United States Navy line-of-sight communications became somewhat neglected with the advent of uhf radio technology. For years the only optical communication systems in the navy were the 12 inch (and for a time 24 inch) hand modulated signal lamps, yardarm blinkers, flag hoist, semaphore, and NANCY (a hand modulated infrared system). Some experimentation was conducted in the late 1940's and 1950's using cesium lamps operating in the near infrared (ir) and non-line-of-sight or over-the-horizon (OTH) communication was demonstrated. Interest waned, however, and progress stood still.

Technology has not stood still and today virtually any radio signal can be monitored and the transmitter localized using a direction finding receiver. While communication systems must be reliable and secure, sometimes it is important that they be covert (only detectable by friendly forces). Thus armed with a complete set of radio communication equipment, the unit commander nevertheless may find his forces in EMCON, talking to each other via flashing light.
B. VISIBLE LIGHT

In the 1960's several attempts were again made to demonstrate the feasibility of OTH and over-terrestrial-obstacles communication using atmospheric scattering [1-3]. These efforts utilized visible search lights, flash lamps, and lasers, and communication at night was successfully demonstrated over distances up to fifty kilometers.

Visible radiation is highly attractive because of the low extinction coefficients (depending on weather conditions). Unfortunately it is seriously degraded during daylight hours due to the background radiation from the sun which tends to interfere with detection.

C. INFRARED LIGHT

Recently, efforts have been made to study extended line-of-sight communications utilizing lasers operating in the near ir. One such effort utilized a 1.06 μ laser and employed remotely piloted vehicles (RPVs). Communications at this wavelength appeared quite promising and ranges from 25 to 150 nautical miles have been predicted [4].

In this region of the spectrum, however, there is noticeable background noise due to scattering of incident solar radiation and to emission by atmospheric particles heated by incident radiation. It should be noted that the daylight background visible radiance is due primarily to scattered radiation and night radiance is caused mainly by scattered radiation and moon light. In the near ir daylight
background radianc e is due mainly to atmospheric emission, while at night aurora and afterglow are the predominate sources.

D. ULTRAVIOLET LIGHT

There is a portion of the spectrum, however, that has been studied little and may offer much. This is the middle ultraviolet (uv). Atmospheric absorption in the .2 to .3 μ region is caused primarily by ozone [5-6]. The relatively dense layer of ozone (at approximately 22 km) prevents virtually all of the radiation in the 220 to 280 nm region from reaching the surface of the earth. Thus the limiting noise in an electro-optical communication system operating in the middle uv would be self-noise rather than atmospheric noise. Another critical feature to be considered is the attenuation properties of the intervening atmosphere. In the near uv (wavelengths between .3 and .4 μ) the primary attenuation mechanism at low altitude is scattering. This can be divided into Rayleigh scattering and particulate scattering by atmospheric aerosols. Rayleigh scattering, inversely proportional to the fourth power of wavelength, increases quite rapidly as the wavelengths become shorter. In practice, it has been difficult to determine the particulate scattering in the near uv. For clear sky conditions, however, a reasonable estimate of attenuation can be made by assuming that the entire attenuation is due to Rayleigh scattering [2].

In the middle uv particulate scattering is very significant. At high altitudes the large ozone concentration
gives rise to very large absorption coefficients. The concentration of ozone generally decreases as the altitude is decreased. Yet even at sea level the concentration is still high enough to have a significant effect. These propagation characteristics indicate that only short range communication systems operating in the middle uv are likely to be useful. Because of the inherent scattering characteristics of middle uv radiation, along with the minimal background problem in the solar blind region, short range non-line-of-sight communications utilizing broad band light sources have been developed [7-8].

Recently a number of reasonably efficient uv lasers have been developed with energies of .1 to one joule per pulse being reported [9-10]. These represent a new generation of radiation sources to be considered for use in the solar blind. A number of detectors and filters have been available for some time. It was these considerations that led to this study.

E. PURPOSE

The purpose of this effort is to develop an analytical tool for estimating the ranges for which optical communication is possible in the middle uv.
II. BACKGROUND

A. INTRODUCTION

A typical communication system consists of a transmitter, a receiver, and the intervening space between them. The characteristics of the transmitter and the receiver are highly dependent on each other and on the properties of the propagating medium. Each of the components may, however, be treated as a distinct and relatively separate entity. The radiation source, or transmitter, must be suitably modulated to carry information and must have sufficient power to overcome the attenuating mechanisms in the medium in which it operates. The characteristics of the propagating medium are not well defined and in general must be approximated. The detector, or receiver, must have sufficient sensitivity to detect and demodulate the attenuated signal that is embedded in background noise. Each of these components is present in an electro-optical communication system and is discussed below. It will be seen that 1) advances in sources dictate a new look at the problem; 2) uncertainties in propagation characteristics of the middle uv requires careful experimentation to gather suitable information such that a proposed system may be modeled; and that 3) certain aspects of detectors, such as filters, critically determine the performance parameters of such a system.
B. SOURCES

Ultraviolet sources suitable for communication purposes include omnidirectional broad band lamps such as xenon or mercury flash lamps, and lasers. Experiments utilizing omnidirectional sources have been carried out for several years at the Aeronautical Research Associates of Princeton (ARAP), Princeton, New Jersey [7-8]. Utilizing a pulsed source at 4810 pps and an average power level of ten to thirty watts, voice communication was demonstrated over a distance of several hundred meters. With suitable modifications, it is estimated that ranges of up to three kilometers may be achieved with the current system. With such a source, the conversion efficiency from input power to output power is about 0.1 percent.

New uv laser sources reported in the last year or so appear to have demonstrated orders of magnitude improvement in efficiency. Utilizing rare gas-halide eximer species, uv output with working efficiencies of up to ten percent are conceivable. Practical working models are already commercially available. Wavelengths in the range of 200 to 400 nm have been reported, indicating that various wavelengths in the middle uv may be attained with these sources. The power output of these devices typically runs from 0.1 joules upwards, depending on the size of the device. The high degree of collimation of the output suggests that the geometric patterns for communication will be highly directional. This is one aspect of the problem which is quite different from that observed for omnidirectional sources.
C. PROPAGATION

The absorption coefficient for middle uv radiation varies by orders of magnitude as a function of wavelength. Scattering by particulates at these wavelengths is more severe than in the visible region of the spectrum. The combined absorption and scattering cross-sections give rise to extinction coefficients which have been measured to be in the range of 0.1 to five per kilometer [11]. Further, variations of an order of magnitude in extinction coefficient over a spectral range of less than fifty nanometers indicates extreme sensitivity to wavelength, due largely to the absorption bands of species such as ozone which appear in this region.

At wavelengths less than 280 nm, the absorption coefficient of atmospheric ozone is sufficiently intense as to eliminate the large majority of solar radiation from the earth's surface [12]. This is largely due to an ozone concentration centered at 20 to 25 km above the surface of the earth. At sea level, the ozone concentration is typically an order of magnitude less than at the higher altitudes. This gives rise to reduced solar background levels without the high extinction coefficients found at higher altitudes. Such behavior suggests that communication in the middle uv may be feasible for relatively short distances at sea level, with a minimum of interference from solar background radiation.
Further major considerations for middle uv propagation are the effects due to multiple scattering. Although the primary light beam may be attenuated by an extinction coefficient which is readily measured, the scattered radiation, especially after multiple scattering, may contribute considerable intensity to the radiation density at an observer. Shettle and Green [12] present calculations for solar flux as a function of the sun's angle with respect to the zenith. Particularly at low angles of incidence, the diffuse radiation is found to be twenty to thirty orders of magnitude more intense than that due to the direct solar radiation. Such figures clearly emphasize the importance of properly treating the effects of multiple scattering when taking into account the effects of particulates in the atmosphere.

In general multiple scattering calculations are cumbersome to handle and yield limited information. Typically these calculations are done on a computer using either an analytical [13] or a Monte Carlo [14-15] approach. The Monte Carlo calculations are relatively easier to model, but utilize a significant amount of computer time and only approximate information is obtained. Some generally useful results have been presented by Bucher [14-15] again indicating orders of magnitude greater radiation densities from the multiple scattering effects than from transmission of direct radiation. The available results from such calculations do not readily answer questions such as 1) what is the
scattering intensity for non-line-of-sight communication given an obstacle with an assumed angle between transmitter-obstacle-receiver; and 2) what is the actual radiation density as a function of field of view and/or position of an observer located off from the axis of propagation at a given distance from a collimated source such as a laser.

D. DETECTION/DEMODULATION

The information available from a pulsed signal depends on the pulse width, repetition rate, and signal-to-noise ratio. Considerations such as these are included in papers by Kennedy [16-17] in which communication through optical scattering channels is discussed. Considerations leading to a suitable signal-to-noise ratio analysis are also presented by Yarif [18] for electro-optical systems. Whether the information is pulsed or continuous wave greatly affects the reception characteristics of the signal. The background light levels coming through the filters utilized before the detector critically determine the signal-to-noise ratio. The characteristics of the filter and detector thus are crucial in determining the information available in a communication system.
III. STATEMENT OF THE PROBLEM

One of the primary constraints on the development and deployment of any new system in the United States Navy is the limited fiscal resources available for such efforts. With the advent of new technology it is nevertheless impossible to utilize the technology to build a new system without first having a mission or demonstrated need for that system and second having a well-defined degree of confidence in the ability of that system to meet that need.

An electro-optical communication system utilizing a laser operating in the middle uv is such a system. (It requires little or no imagination to visualize the advantages of rapid short range, possibly non-line-of-sight, optical communications over the methods presently employed by the Fleet.) While it is beyond the scope of this project to develop such a system, it is within the scope of this effort to formulate a model of such a system capable of predicting the various parameters of that system.

The need for modeling is further enhanced by the limited amount of information available concerning the middle uv. By utilizing available information and by characterizing source, propagation, filter and detector characteristics, it is possible to formulate a model that will yield order of magnitude results for any proposed system. In the process of modeling and analysis the need for critical experiments
which would allow the modeling efforts to proceed with a greater degree of accuracy is sure to be revealed.
IV. LASER SOURCES

The past two or three years have seen a rapid development of lasers operating in the ultraviolet. The most notable recent advances have occurred with the rare gas-halide lasers in which an excited state eximer is formed. With species such as XeF*, the ground state is not an associated molecule, thus depopulation of the lower level is not a barrier to population inversion. Further, such rare gas-halide lasers have an inherently high quantum efficiency. Efficiencies relative to the energy deposited by an excitation electron beam are as high as ten percent, while overall working efficiencies of up to one percent have been reported [9,10,19]. With higher overall efficiencies anticipated, such lasers represent an attractive source of radiation for middle uv applications.

The observed wavelengths of several rare gas-halide lasers are presented in Table I. It should be noted that the laser output from these sources may be Raman shifted with reasonable high conversion efficiencies [20]. By passing the output from a KrF laser through ten atmospheres of hydrogen, conversion to the first stokes wavelength of 25 percent and to the second stokes wavelength of ten percent was observed [20]. At eighty atmospheres pressure, more than fifty percent conversion efficiency to the first stokes wavelength has been observed.
TABLE I

Characteristic Wavelengths of Several Rare Gas-Halide Lasers Along with Some Stokes Shifted Wavelengths Observed with Hydrogen Gas

<table>
<thead>
<tr>
<th>Eximer</th>
<th>First Stokes Wavelength (nm)</th>
<th>Second Stokes Wavelength (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KrF</td>
<td>249</td>
<td>279</td>
</tr>
<tr>
<td>XeF</td>
<td>351</td>
<td></td>
</tr>
<tr>
<td>XeCl</td>
<td>308</td>
<td></td>
</tr>
<tr>
<td>XeBr</td>
<td>282</td>
<td>322</td>
</tr>
</tbody>
</table>

The practical performance of rare gas-halide lasers is illustrated by the commercial availability of a model which will produce 0.1 joule pulses at 15 pps. One table top model is reported to produce 1.5 joule pulses with a 125 nanosecond pulse width. The development of these lasers is in its infancy. Sources with higher repetition rates and greater reliability are to be expected.

Due to the rapidly expanding availability of middle uv lasers sources, no single presently available source is assumed, but rather the effort in the succeeding sections considers the possibility of utilizing a middle uv source at representative wavelengths and with an output power consisting of millijoule to joule pulses. Thus the behavior of any unforeseen laser or coherent beam may be predicted.
V. PROPAGATION PROPERTIES OF THE ATMOSPHERE AND THE MULTIPLE SCATTERING PROBLEM

A. INTRODUCTION

A laser signal propagating through the atmosphere is subjected to various attenuation mechanisms. The time spread of a pulse can be broadened due to scattering, while the amplitude can be decreased due to absorption and scattering by atmospheric constituents. The inherent directionality of a laser source indicates that the position of the detector relative to the source should also affect the reception of the signal. Since a large fraction of the total extinction process is due to scattering, multiple scattering should be an important consideration affecting the propagation characteristics for distances greater than one or two extinction lengths.

A complete description of all of the effects of the atmosphere including inhomogeneities (clouds, terrestrial objects, surface effects, etc.) is beyond the scope of this paper. A relatively concise treatment of the atmosphere and the multiple scattering problem entails characterizing the atmosphere, simulating atmospheric effects, and tabulating the results of the simulation.

B. CHARACTERISTICS OF THE ATMOSPHERE

1. Attenuation

In order to characterize the propagation of optical information in the atmosphere it was first necessary to
determine the extinction, absorption, and scattering coefficients. Table II was constructed based primarily on the information reported by Dunkelman [21]. The Rayleigh scattering coefficients were adapted from Penndorf [22].

For the purpose of this investigation the following definitions and relationships apply:

\[ \lambda \] is the wavelength
\[ K_A \] is the absorption coefficient
\[ K_{SR} \] is the Rayleigh scattering coefficient
\[ K_{SP} \] is the particulate scattering coefficient
\[ K_S \] is the total scattering coefficient
\[ K \] is the total extinction coefficient
\[ K_S = K_{SR} + K_{SP} \]
\[ K = K_S + K_A \]
\[ R_S \] is the ratio of \( K_S \) to \( K \) or \( (K_S/K) \)
\[ R_P \] is the ratio of \( K_{SP} \) to \( K_S \) or \( (K_{SP}/K_S) \)

From Table II it can be seen that \( K_A, K_{SR}, \) and \( K \) tend to decrease as wavelength increases in the middle UV. The relationship of \( K_{SP} \) and consequently \( K_S, R_S, \) and \( R_P \) to wavelength is not so easily described.

2. **Angular Distribution of Scattered Photons**

Complete characterization of the propagation of optical information required the angular distribution or phase function, \( P(\theta) \), of the scattered photons for the wavelengths of interest. Figure 1 [23] was the primary source for phase function information.
<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>$K_A$ (km$^{-1}$)</th>
<th>$K_{SR}$ (km$^{-1}$)</th>
<th>$K_S$ (km$^{-1}$)</th>
<th>$R_P$</th>
<th>$R_S$ (K/K)</th>
<th>$R_{SP}$ (K/K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>230</td>
<td>2.58-3.5</td>
<td>.4934</td>
<td>.03-2.9</td>
<td>3.1-6.0</td>
<td>0.6-5.9</td>
<td>.6-5.9</td>
</tr>
<tr>
<td>240</td>
<td>1.4-3-2.8</td>
<td>.4061</td>
<td>.2-6-3.6</td>
<td>2.1-5.0</td>
<td>.32-7.2</td>
<td>.32-7.2</td>
</tr>
<tr>
<td>250</td>
<td>.68-2.6</td>
<td>.382</td>
<td>.18-2.4</td>
<td>1.2-3.4</td>
<td>.43-7.9</td>
<td>.43-7.9</td>
</tr>
<tr>
<td>260</td>
<td>.43-2.4</td>
<td>.2842</td>
<td>.29-2.2</td>
<td>1.0-2.9</td>
<td>.57-8.6</td>
<td>.57-8.6</td>
</tr>
<tr>
<td>270</td>
<td>.23-1.6</td>
<td>.2404</td>
<td>.33-2.1</td>
<td>1.2-3.4</td>
<td>.57-8.6</td>
<td>.57-8.6</td>
</tr>
<tr>
<td>280</td>
<td>.13-1.8</td>
<td>.2055</td>
<td>.33-2.1</td>
<td>1.2-3.4</td>
<td>.57-8.6</td>
<td>.57-8.6</td>
</tr>
<tr>
<td>290</td>
<td>.046-1.32</td>
<td>.1765</td>
<td>.065-1.9</td>
<td>1.2-3.4</td>
<td>.57-8.6</td>
<td>.57-8.6</td>
</tr>
<tr>
<td>300</td>
<td>.012-1.05</td>
<td>.1525</td>
<td>.056-1.7</td>
<td>1.2-3.4</td>
<td>.57-8.6</td>
<td>.57-8.6</td>
</tr>
<tr>
<td>310</td>
<td>.003-1.32</td>
<td>.1826</td>
<td>.044-1.6</td>
<td>1.2-3.4</td>
<td>.57-8.6</td>
<td>.57-8.6</td>
</tr>
</tbody>
</table>
ABSOLUTE PHASE FUNCTION
vs SCATTERING ANGLE

7/15/77

FIGURE 1. Absolute Phase Function vs. Scattering Angle [23]
In order to mathematically model this information, the Henyey-Greenstein function, Eq. 1, and some of its modifications were studied. The Neer-Sandri function [7], Eq. 2, was a logical choice; however, the function used by Zachor [24], Eq. 3, was chosen for simplicity in modifying the author's initial phase function to properly account for backscattering.

\[
P(\theta, g) = \frac{1 - g^2}{4\pi} \left[ \frac{1}{(1 + g^2 - 2g \cos \theta)^{3/2}} \right] \tag{1}
\]

\[
P(\theta, g) = \frac{1 - g^2}{4\pi} \left[ \frac{1}{(1 + g^2 - 2g \cos \theta)^{3/2}} + \frac{g(3 \cos^2 \theta - 1)}{2 |\cos \theta_0| (1 + g^2 - 2g |\cos \theta_0|)^{5/2}} \right] \tag{2}
\]

where \( |\cos \theta_0| = 1/7 \)

\[
P(\theta, g) = \frac{1 - g^2}{4\pi} \left[ \frac{1}{(1 + g^2 - 2g \cos \theta)^{3/2}} + \frac{0.5(3 \cos^2 \theta - 1)}{(1 + g^2)^{3/2}} \right] \tag{3}
\]

Fitting the data reported by \( \text{Prer} \) [23] with Eq. 3 resulted in Figures 2 and 3. These figures represent the absolute and normalized phase functions, while Table III lists the coefficients, utilized for this simulation.
FIGURE 2. Absolute Phase Function vs. Scattering Angle
Utilizing a Modified Henyey-Greenstein Function to Approximate the Data by Neer, et al. [23]
FIGURE 3. Normalized Phase Function vs. Scattering Angle Utilizing a Modified Henyey-Greenstein Function to Approximate the Data by Neer, et al. [23]
TABLE III

Coefficients Used in the Simulation

<table>
<thead>
<tr>
<th>λ</th>
<th>$K_A$ (km$^{-1}$)</th>
<th>$K_{SR}$ (km$^{-1}$)</th>
<th>$K_{SP}$ (km$^{-1}$)</th>
<th>$K_S$ (km$^{-1}$)</th>
<th>$K$ (km$^{-1}$)</th>
<th>$R_S$</th>
<th>$R_P$</th>
<th>$K_S/K$</th>
<th>$K_{SP}/K_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>.9372</td>
<td>.3382</td>
<td>.7187</td>
<td>1.0569</td>
<td>1.9941</td>
<td>.53</td>
<td>.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>280</td>
<td>.2185</td>
<td>.2055</td>
<td>.6165</td>
<td>.8220</td>
<td>1.0405</td>
<td>.79</td>
<td>.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>.0289</td>
<td>.1525</td>
<td>.5407</td>
<td>.6932</td>
<td>.7221</td>
<td>.96</td>
<td>.78</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For all three cases the values of $g$ and $f$ utilized were 0.75 and 0.5, respectively. The values for 250 nm appear to be optimistic while the values for 300 nm appear to be somewhat pessimistic.
C. DESCRIPTION OF THE CHARACTERISTICS OF THE SIMULATION

The purpose of this simulation was to provide approximate results for the multiple scattering problem. By assuming various source, propagation, and detector characteristics, a user could in principle predict the effectiveness of a proposed system.

The Monte Carlo method described in Appendix A was employed to generate information detailing characteristic patterns of photon scattering as a function of wavelength, position of an observer, pulse spreading, and field of view of the detector being employed. The observer may be located at any arbitrary angle off from the axis of propagation and at any arbitrary distance from the source (measured in extinction lengths).

Of primary interest were the characteristic patterns of multiply scattered radiation. The simulation provided for obtaining diagrams of photon flux at a given position as viewed by either the source or the receiver. In addition, the patterns received by the detector were capable of being limited by field of view restrictions. By measuring the photon flux at a given position, contours of equal photon flux were generated. These contours provided a simple means of visualizing the radiation patterns.

Detector field of view considerations indicate the field of view required to receive a given threshold of photons at a fixed position as well as the maximum range at a given angle that a threshold number of photons can be
detected for a fixed field of view. Finally, pulse spreading information due to the time delay caused by multiple scattering was also available.

D. CHARACTERISTIC PATTERNS OF MULTIPLY SCATTERED RADIATION

Figures 4 and 5 represent the pattern of photons received by a 180 degree field of view detector located zero to nine and 36 to 45 degrees off from the axis of propagation respectively at one extinction length from a 300 nm laser source. As the detector approached the axis of propagation more photons were received. The photons along the phi prime equal to zero axis represent single scattered radiation. Many of the photons received were, however, multiply scattered.

Figures 4 and 5 are similar to Figures 6 and 7 only in the latter figures the receiver is located five extinction lengths from the source. The percentages of multiply scattered photons increases greatly as the receiver is moved either off from the axis of propagation or away from the source. At five extinction lengths, virtually none of the radiation received is single scattered. This is a strong indication of the hazards of single scattering models in the middle uv.

Figures 8, 9, and 10 represent the contours of equal photon flux for 250, 280, and 300 nm wavelengths. The label for each contour is
RECEIVER LOCATED ONE EXTINCTION LENGTH FROM THE SOURCE. $\theta = 0$ TO 9 DEGREES
$\lambda = 300$ NM. FIELD OF VIEW = 180 DEGREES

FIGURE 4. Photon Scattering Diagram ($\theta'$ vs $\phi'$) as Viewed by a Receiver Located $\theta$ Degrees from the Axis of Propagation.
RECEIVER LOCATED ONE EXTINCTION LENGTH FROM THE SOURCE. $\theta = 36$ TO $45$ DEGREES
$\lambda = 300$ nm. FIELD OF VIEW = 180 DEGREES

FIGURE 5. Photon Scattering Diagram ($\theta'$ vs $\phi'$) as Viewed by a Receiver Located $\theta$ Degrees from the Axis of Propagation.
RECEIVER LOCATED FIVE EXTINCTION LENGTHS FROM THE SOURCE. \( \theta = 0 \) TO 9 DEGREES
\( \lambda = 300 \) NM. FIELD OF VIEW = 180 DEGREES

FIGURE 6. Photon Scattering Diagram (\( \theta' \) vs \( \phi' \)) as Viewed by a Receiver Located \( \theta \) Degrees from the Axis of Propagation
FIGURE 7. Photon Scattering Diagram ($\theta'$ vs $\phi'$) as Viewed by a Receiver Located $\theta$ Degrees from the Axis of Propagation.
FIGURE 8. Contours of Equal Photon Flux as a Function of the Position of the Observer
Figure 9. Contours of equal photon flux as a function of the position of the observer.
FIGURE 10. Contours of Equal Photon Flux as a Function of the Position of the Observer
Relative Flux = \(-\log_{10}\left[\frac{N}{N_0 \cdot 2\pi R^2 (\cos \theta_1 - \cos \theta_2)}\right]\)

(4)

where \(2\pi R^2 (\cos \theta_1 - \cos \theta_2)\) is the area of that portion of a hemisphere located \(R\) extinction lengths from the source subtended by \(\theta_1\) and \(\theta_2\), \(N\) is the number of photons penetrating that area, and \(N_c\) is the number of photons leaving the source. Each successive contour of photon flux represents a decrease of one order of magnitude in the relative photon flux. Thus, for example, the contour labeled 2 gives the location at which the photon flux is \(10^{-2}\) photons per unit area, where unit area is one extinction length squared.

E. FIELD OF VIEW CONSIDERATIONS

Figures 11 and 12 represent the relative number of photons detected at a receiver located zero to nine and 36 to 45 degrees off from the axis of propagation at one and five extinction lengths respectively from a 250 nm laser source. A sixty degree (full cone angle) field of view detector would receive seventy-five to eighty percent of the incident radiation for the \(\theta_1\) (zero to nine degrees) and the \(\theta_2\) (36 to 45 degrees) cases at one extinction length. At five extinction lengths these percentages tend to remain stable. Thus, given a minimum required photon flux at a detector, it is possible to select the proper field of view for that detector. Conversely, utilizing a fixed
FIGURE 11. Relative Number of Photons vs Field of View for an Observer Located \( \theta \) Degrees from the Axis of Propagation

- Observer located one extinction length from the source.
- \( \theta_1 \) = 0 to 9 degrees
- \( \theta_2 \) = 36 to 45 degrees
- \( \lambda \) = 250 NM
FIGURE 12. Relative Number of Photons vs Field of View for an Observer Located 36 Degrees from the Axis of Propagation.

OBSERVER LOCATED FIVE EXTINCTION LENGTHS FROM THE SOURCE

- $\theta_1 = 0$ TO 9 DEGREES
- $\theta_2 = 36$ TO 45 DEGREES
- $\lambda = 250$ NM
field of view detector, it is possible to determine the maximum effective range for a minimum photon flux. (The results for 280 and 300 nm were substantially the same as the 250 nm simulation).

F. PULSE SPREADING CONSIDERATIONS

Figures 13 and 14 represent the pulse spreading as received by a detector located zero to nine and 36 to 45 degrees off from the axis of propagation at one and five extinction lengths from a 250 nm laser source, respectively. The pulse spreading function is a function of the position of the observer and time dispersion caused by multiple scattering. The relative time dispersion, $T(R,D)$ is defined as follows:

$$T(R,D) = \frac{D}{R} - 1$$

where $D$ is the total distance traveled by each photon and $R$ is the distance from the source to the shell penetrated by each photon.

The results of pulse spreading at other distances and angles was examined. The pulse spreading function is nearly constant for values of $R$ from one to ten extinction lengths, thus one figure suffices as an approximation to any distance in this angular range. The increase in pulse spreading with increasing angle is illustrated with one additional plot at 36 to 45 degrees off from the
FIGURE 13. Pulse Spreading for an Observer Located \( \theta \) Degrees from the Axis of Propagation
FIGURE 14. Pulse Spreading for an Observer Located Five Extinction Lengths from the Source

\[ \theta_1 = 0 \text{ TO } 9 \text{ DEGREES} \]
\[ \theta_2 = 36 \text{ TO } 45 \text{ DEGREES} \]
\[ \lambda = 250 \text{ NM} \]
axis of propagation. Reproduction of the results at each cone angle introduces a cumbersome presentation of this information, hence only one representative sample is included.

G. SUMMARY

The absorption and scattering coefficients as well as the phase functions from previous experiments were utilized and the behavior of multiple scattered radiation was simulated using the Monte Carlo method. This model was used to study the characteristics of the source beam at a receiver located an arbitrary angle off from the axis of propagation at a given distance from the source. With such information available, the geometric pattern of possible communication links can be determined. A photon density as a function of receiver field of view and position of the observer was determined as well as the effects of multiple scattering on pulse spreading.
VI. SIGNAL-TO-NOISE CONSIDERATIONS

A. INTRODUCTION

The photon flux information presented in the previous section may be utilized to estimate the signal-to-noise ratio (S/N or SNR). This information, however, must be used in conjunction with the characteristics of the detector. Additional considerations therefore include the characteristics of the optical filter, quantum efficiency of the detector, background level of radiation, etc. This section considers the characteristics of available devices and of known atmospheric flux levels in order to estimate the flux level necessary for reliable communication.

B. FUNDAMENTAL RELATIONSHIPS

The detector current generated by a light signal of power $P_S$ and of frequency $\nu$ is

$$i_S = \frac{GneP_S}{h\nu}$$  \hspace{1cm} (6)

where $G$ is the gain of the detector, $n$ is the quantum efficiency of the detector, $e$ is the electronic charge, and $h$ is Planck's constant [18].

The dominant noise mechanism for detectors operating in the visible and ultraviolet region of the spectrum is shot noise. The mean square of the noise current due to this noise mechanism is
\[
\langle i_n^2 \rangle = 2eG^2 \left[ i_d + \frac{ne}{h\nu} (P_S + P_B) \right] B \tag{7}
\]

where \( G \) is the gain of the detection device (typically a photomultiplier), \( i_d \) is the dark current, \( P_B \) is the power of the background radiation present due to sources other than that from the desired signal, and \( B \) is the bandwidth of the receiver [18, 25, 26, 27].

The desired expression for \( S/N \) is thus the ratio of the square of Equations (6) and of (7).

\[
\frac{\langle i_n^2 \rangle}{\langle i_n^2 \rangle} = \frac{\frac{neP_S}{h\nu}^2}{2e[i_d + \frac{ne}{h\nu} (P_S + P_B)]B} \tag{8}
\]

In several specific cases the dark current \( i_d \) is negligible, whereupon Equation (8) reduces to Equation (9).

\[
S/N \approx \frac{\eta P_S}{2h\nu B} \left( \frac{P_S}{P_S + P_B} \right) \tag{9}
\]

For pulsed signals having a time duration of \( \tau \), the bandwidth \( B \) is effectively \( 1/2\tau \). The relationship for \( S/N \) thus reduces to the number of photons converted \( \rightarrow \) electrons in a pulse, degraded by the ratio of the signal power to total power received over the duration of the pulse.

Each of the expressions for \( S/N \) may be modified slightly by consideration of a duty factor, introduced to account for the fact that a signal is not constant over the duration.
of a pulse. Consideration of this introduces a multiplicative factor equal to Equation (10).

\[
\frac{D_S}{D_N} = \int_{-\tau/2}^{\tau/2} \left( \frac{i_S(t)}{i_N(t)} \right)^2 dt
\]

(10)

where \(D_S/D_N\) is the ratio of the duty factor of the signal to that of the noise over the duration of the pulse, \(\tau\).

C. BACKGROUND LEVELS OF RADIATION

The expressions for S/N involve critically the background levels of radiation due to other sources. In communication experiments this is typically due to scattered sunlight. Reliable flux levels at wavelengths longer than 300 nm are reported by Valley [28]. The flux levels at shorter wavelengths are a result of both direct and diffuse transmission. These levels are critically dependent on the thickness of the ozone layer, and have been calculated for various atmospheric conditions [12].

The photon flux calculated by Shettle and Green is presented in Table IV. The dramatic drop in background radiation levels at around 280 nm is evident. The signal that would be transmitted through a narrow band (10 nm) multilayer dielectric filter at 300 nm would include up to \(10^{13}\) photons/cm\(^2\) as background noise photons, whereas at
wavelengths below 280 nm this background radiation level may be considered to be quite small.

TABLE IV
Photon Flux Due to Direct and Diffuse Transmittance through the Earth's Atmosphere [12]

A solar angle of ninety degrees is assumed, along with a 0.32 cm ozone thickness.

<table>
<thead>
<tr>
<th>λ (nm)</th>
<th>Flux (watts/m² nm)</th>
<th>Flux (photons/cm² nm sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>1.07 X 10⁻²</td>
<td>1.6 X 10¹²</td>
</tr>
<tr>
<td>295</td>
<td>5.15 X 10⁻⁴</td>
<td>7.8 X 10¹⁰</td>
</tr>
<tr>
<td>290</td>
<td>2.14 X 10⁻⁶</td>
<td>3.2 X 10⁸</td>
</tr>
<tr>
<td>285</td>
<td>9.50 X 10⁻¹¹</td>
<td>1.4 X 10⁴</td>
</tr>
<tr>
<td>280</td>
<td>9.09 X 10⁻¹⁹</td>
<td>1.4 X 10⁻⁴</td>
</tr>
</tbody>
</table>

D. DETECTOR CHARACTERISTICS

The detector characteristics which relate to a given S/N expression include filter characteristics, quantum efficiency, detector area, and field of view. Using a combination of multilayer dielectric filters and absorption filters, it is possible to construct a device having composite characteristics which will pass ten percent of the incident flux at 265 nm and reject the ambient solar flux in the visible to a degree sufficient to reduce the overall background count level to approximately ten counts per
second [29]. The transmission characteristics of this filter in the 200 to 400 nm region are indicated in Figure 15. It is thus expected that somewhat greater than one percent but less than ten percent of the photons in the 250 to 280 nm region will pass through the filter. Once the photons reach the photocathode of a typical photomultiplier, the conversion efficiency is usually greater than ten percent but less than 25 percent. Thus, for purposes of estimating S/N, the combined effects of filter transmission characteristics plus quantum efficiency of the photocathode may be accounted for by choosing a value of \( \gamma \) to be greater than \( 10^{-3} \) but less than \( 10^{-2} \). With unforeseen developments it is possible to expect a combination having a composite value of \( 10^{-1} \), however such a device is not presently available.

The area of the detector used in the simulation was chosen to be nominally one square inch, typical of currently available photomultiplier tubes.

E. STATISTICAL CONSIDERATIONS

The number of photons converted into an electrical signal follows Poisson statistics:

\[
P(r) = e^{-n} \frac{n^r}{r!}
\]

(11)

where \( P(r) \) is the probability that \( r \) photons are observed in a single pulse which has an average number of \( n \) photons per pulse.
FIGURE 15. Transmission Characteristics for a Typical Composite Filter
The probability of observing an event with a number of photons above the threshold value $r_{th}$ is

$$P(r > r_{th}) = \sum_{r=r_{th}}^{\infty} \frac{e^{-n} n^r}{r!} \quad (12)$$

Graphs of this function are readily available [30]. Given an average number, $n$, equal to ten and a threshold level of two (to distinguish from the random single photon events which occur at the detector with a frequency of occurrence equal to ten or twenty photons per second), a probability of detection of such pulses is .9995. If the average number $n$ is five with the threshold number equal to two, the probability of detection is 0.96. Thus an average value of ten photons per pulse in the absence of background radiation is chosen as a desired level to avoid excessive error. This means that a desirable value of S/N is ten.

F. SUMMARY OF SIGNAL-TO-NOISE CONSIDERATIONS

For the case of minimal background radiation, Eq. (13) gives the relationship for the number of signal photons required.

$$\frac{P_s}{h \nu} = \frac{1}{n} \left( \frac{S}{N} \right) \quad (13)$$

If background radiation dominates the power incident on the detector (as would be the case at wavelengths longer than 300 nm), then the number of signal photons required is given by Eq. (14).
\[
\frac{P_B \tau}{h \nu} = \left( \frac{1}{n} \frac{S}{N} \frac{P_B \tau}{h \nu} \right)^{1/2}
\]  

(14)

Table V gives a summary of the necessary photon flux external to the detector in order to obtain a S/N equal to ten. These flux levels may be used in conjunction with the figures of Section V to generate the geometric patterns over which signal transmission may be expected.

<table>
<thead>
<tr>
<th>(\lambda) (nm)</th>
<th>(n)</th>
<th>(\frac{P_B}{h \nu \text{photons/sec}})</th>
<th>(\frac{P_S \tau}{h \nu \text{photons}})</th>
<th>Required flux (photons/cm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>(10^{-3})</td>
<td>---</td>
<td>(10^4)</td>
<td>(1.6 \times 10^3)</td>
</tr>
<tr>
<td>265</td>
<td>(10^{-2})</td>
<td>---</td>
<td>(10^3)</td>
<td>(1.6 \times 10^2)</td>
</tr>
<tr>
<td>280</td>
<td>(10^{-3})</td>
<td>---</td>
<td>(10^4)</td>
<td>(1.6 \times 10^3)</td>
</tr>
<tr>
<td>300</td>
<td>(10^{-2})</td>
<td>(10^{13})</td>
<td>(10^5)</td>
<td>(1.6 \times 10^4)</td>
</tr>
</tbody>
</table>

TABLE V

Photon Flux Required in order to Give a S/N Equal to Ten

Assumed parameters:

Filter characteristics for Figure 15
Quantum efficiency of ten percent
\(\tau = 10^{-6}\) seconds (significant pulse spreading)
Area of detector is one square inch

Note: Filter characteristics at 300 nm have been assumed for a narrow band (ten nm) multilayer dielectric filter.
VII. DISCUSSION

Although not currently available, three millijoule laser sources of different wavelengths (250, 280, and 300 nanometers) capable of being pulsed at an appropriate rate (200 pulses per second for 2400 bit per second data rate using 12 bits per pulse [4]) were assumed. (The navy has used vocoders with 2400 bits per second capability since the 1940's). Table V lists the assumptions leading to the required photon flux in order to achieve a S/N equal to ten. By utilizing the contours of equal photon flux in Figures 8, 9, and 10, the ranges for which pulse position modulation communication is possible were predicted. Table VI lists the results of these calculations.

Predicted performance for lasers operating at 250 and 280 nm are comparable for sixty degree or greater off axis detection. For detection angles less than sixty degrees the 280 nm laser is superior. The performance of the 300 nm laser does not appear to be degraded by the background radiation as much as might be expected. The performance for this laser from zero to twenty degrees off from the axis of propagation is similar to the 280 nm laser. Beyond thirty degrees the ranges decrease markedly, indicating very little backscattering. It cannot be overstressed that the results above are for a simulation of assumed laser sources with assumed atmospheric propagation
### TABLE VI

Predicted Ranges or Range Bands for Communication in the Middle UV for Pulsed Millijoule Lasers

(*θ* is the angular location of the receiver with respect to the axis of propagation.)

\[
\begin{align*}
\lambda &= 250 \text{nm} \ (1.6 \times 10^3 \ \text{Photons cm}^{-2} \ \text{req'd}) \\
\lambda &= 280 \text{nm} \ (1.6 \times 10^3 \ \text{Photons cm}^{-2} \ \text{req'd}) \\
\lambda &= 300 \text{nm} \ (1.6 \times 10^4 \ \text{Photons cm}^{-2} \ \text{req'd})
\end{align*}
\]

<table>
<thead>
<tr>
<th>θ (Degrees)</th>
<th>Range (km)</th>
<th>Range (km)</th>
<th>θ (Degrees)</th>
<th>Range (km)</th>
<th>θ (Degrees)</th>
<th>Range (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.4</td>
<td>3.4</td>
<td>0</td>
<td>5.5</td>
<td>0</td>
<td>5.2</td>
</tr>
<tr>
<td>30</td>
<td>1.1</td>
<td>2.1</td>
<td>30</td>
<td>2.9</td>
<td>10</td>
<td>3.6</td>
</tr>
<tr>
<td>60</td>
<td>0.52</td>
<td>1.7</td>
<td>60</td>
<td>1.6</td>
<td>20</td>
<td>2.2</td>
</tr>
<tr>
<td>90</td>
<td>0.34</td>
<td>1.2</td>
<td>90</td>
<td>1.0</td>
<td>30</td>
<td>0.93</td>
</tr>
<tr>
<td>120</td>
<td>0.30</td>
<td>1.0</td>
<td>120</td>
<td>0.92</td>
<td>40</td>
<td>0.24</td>
</tr>
<tr>
<td>150</td>
<td>0.33</td>
<td>1.1</td>
<td>150</td>
<td>0.94</td>
<td>90</td>
<td>0.10</td>
</tr>
<tr>
<td>180</td>
<td>0.49</td>
<td>1.2</td>
<td>180</td>
<td>1.2</td>
<td>180</td>
<td>0.10</td>
</tr>
</tbody>
</table>

(Ranges for *λ* = 250 nm should be approximately half-way between the ranges listed.)

(Ranges for *λ* = 280 nm should be on the order of the range listed.)

(Ranges for *λ* = 300 nm should be somewhat less than the range listed.)
characteristics and assumed detector characteristics. Clearly, in order to make accurate predictions for real systems it is necessary to obtain accurate atmospheric characteristics as well as other system parameters.
VIII. CONCLUSION

Lasers operating in the middle uv do not appear to be suitable for long range communications; however, they present definite possibilities for short range (several km) applications. Since radiation in this region is absorbed exponentially, rather than as the inverse of distance squared, the potential for covertness is highly accentuated. Current lasers, however, having a power output on the order of 0.1 joules or greater with low pulse rates (15 to 20 pulses per second), are not suitable for voice communication due to the low repetition rates. The pulsed mode of operation appears to hold definite advantages over amplitude modulation of a continuous wave laser from simple signal-to-noise considerations. Good possibilities for successful communication are offered by pulse position modulation. Millijoule laser sources operating with 200 or more pulses per second are desirable but not currently available.

The contours of equal photon flux presented in this paper are useful for determining the communication patterns of possible laser sources. The pulse spreading and field of view figures are likewise useful for predicting the characteristics of proposed receivers. A multiple scattering model is definitely required for proper prediction of photon flux over expected distances of communication.
APPENDIX A

MONTE CARLO SIMULATION OF THE MULTIPLE SCATTERING PROBLEM

A. FUNDAMENTAL RELATIONSHIPS

1. Weighting Factors for Monte Carlo Calculations

The following calculations served as a model for the derivation of the various weighting factors:

PROBLEM: Given a random number generator that provides numbers within a specified interval along the x-axis (with equal probability for each interval dx) obtain random numbers with a probability \( p(y) \). I.e., find \( y(x) \) such that \( p(y) \, dy = dx \).

SOLUTION: Let Random Variable \( Y \) be distributed in \( y_o \leq Y \leq y_2 \) and its density function be given by

\[
p(y) = \begin{cases} 
p(y) & y_o \leq y \leq y_2 \\
0 & \text{otherwise}
\end{cases}
\]

The distribution function, \( P(y) \), is given by

\[
P(y) = P(Y \leq y) = \int_{-\infty}^{y} p(y) \, dy
\]

Therefore, \( P(y) = \int_{y_o}^{y} p(y) \, dy \), since \( p(y) = 0 \) for \( y < y_o \).

Now,

\[
p(y) \, dy = dx
\]
Integrating,

\[ y(x) = y_0 + \int P^{-1}(\Delta x) \]

where

\[ y_0 = y(x_0) \]

and \[ \Delta x = x - x_0 \]

This concept applied to specific cases yields the following fundamental relationships for weighting factors:

a. Exponential

\[ p(y) = \frac{1}{\tau} e^{-y/\tau} \quad (0 \leq y \leq \infty) \]

\[ y(x) = -\tau \ln (1 - \Delta x) \]

b. Henyey-Greenstein

\[ p(\theta) = \frac{(1-g^2)}{2(1+g^2-2g \cos \theta)^{3/2}} \quad (0 \leq \theta \leq \pi) \]

where

\[ g = \langle \cos \theta \rangle \]

\[ \theta(x) = \cos^{-1}\left( \frac{1+g^2}{2g} - \frac{1}{2g} \left[ \frac{1-g^2}{2g(1+g^2-\Delta x)} \right]^2 \right) \]
c. Modified Henyey-Greenstein [Zachor 1977]

\[ p(\psi) = \frac{1-g^2}{2} \left[ \frac{1}{(1+g^2-2g \cos \theta)^{3/2}} + \frac{f (3 \cos^2 \theta - 1)}{2 (1+g^2)^{3/2}} \right] \]

\[ 0 \leq \theta \leq \pi \]

where

\[ g = \langle \cos \delta \rangle \quad \text{and} \quad f \text{ is a weighting factor.} \]

\[ \omega(x) = \cos^{-1} \left( 1 + \frac{g^2}{2g} - \left[ \frac{(g^2-1)^2}{8g^3} \right] \frac{1}{[\Delta x - F(\theta) - \frac{1+g^2}{2g}]^2} \right) \]

where

\[ F(\theta) = \left[ \frac{f \frac{(1-g^2)}{4 (1+g^2)^{3/2}}}{(1+g^2)^{3/2}} \right] (\cos \theta - \cos^3 \theta) \]

d. Uniform

\[ p(\psi) = \frac{1}{2\pi} \quad 0 \leq \psi \leq 2\pi \]

\[ \psi(x) = 2\pi \Delta x \]

e. Rayleigh

\[ p(\psi) = \frac{3}{8} (1 + \cos^2 \psi) \quad 0 \leq \psi \leq \pi \]
\[ \theta(x) = \cos^{-1}(A + B) \]

where

\[ A = \sqrt{\frac{b}{2} + \sqrt{\frac{b^2}{4} + 1}} \]
\[ B = \sqrt{\frac{b}{2} - \sqrt{\frac{b^2}{4} + 1}} \]

and

\[ b = 8 \Delta x - 4 \]

2. Position and Coordinates in Photon-Fixed Coordinate System

a. Axis Rotation

Axis rotation is accomplished by the following transformation:

\[
\begin{bmatrix}
    x' \\
    y' \\
    z'
\end{bmatrix} =
\begin{bmatrix}
    \cos \Delta \theta \cos \Delta \phi & -\cos \Delta \theta \sin \Delta \phi & -\sin \Delta \theta \\
    \sin \Delta \phi & \cos \Delta \phi & 0 \\
    \sin \Delta \theta \cos \Delta \phi & -\sin \Delta \theta \sin \Delta \phi & \cos \Delta \theta
\end{bmatrix}
\begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix}
\]

b. Axis Translation

Axis translation is accomplished by the following transformation:
\[
\begin{bmatrix}
x'' \\
y'' \\
z''
\end{bmatrix} = \begin{bmatrix}
x' \\
y' \\
z'
\end{bmatrix} + \Delta r \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\]

3. **Angles \((\mu, \phi, \nu, \;'; \;')\) for a Photon Passing Through a Shell**

A laser is located at the origin of the \(\hat{i}_o, \hat{j}_o, \hat{k}_o\) coordinate system. The initial direction of propagation of the collimated beam is along the \(-\hat{k}_o\) axis. \(\theta\) is the angle between the position vector \(\vec{R}\) (pointing from the location where the photon penetrates a shell toward the laser source) and the vector \(-\hat{k}_o\). \(\phi\) is the angle between the projection of \(-\vec{R}\) onto the \((\hat{i}_o, \hat{j}_o)\) plane (denoted by \(-\vec{R}_p\)) and the vector \(\hat{i}_o\).

A photon is located at the origin of the \(V_x, V_y, V_z\) and the \(U_x, U_y, U_z\) coordinate systems. As the photon passes through the shell, \(\theta'\) specifies the angular direction of photon travel (along \(\hat{U}_z\)) relative to \(\vec{R}\) (the line-of-sight position vector). \(\phi'\) is the angle between the projection of \(\hat{U}_z\) (denoted by the vector \(\vec{W}\)) onto the \((V_x, V_y)\) plane and \(V_x\).

The vectors \(\vec{R}_i, \vec{R}_j, \text{ and } \vec{R}_k\) are position vectors in the \(U_x, U_y, U_z\) coordinate system denoting the location of the points \((1,0,0), (0,1,0),\) and \((0,0,1)\) in the \(i_o, j_o, k_o\) system.

---

1See Figure 16.
FIGURE 16. Diagram of the Photon Fixed Coordinate System
\( \hat{k}_o \) coordinate system. (These points are used for orientation purposes.)

a. \[
\hat{\phi} = \cos^{-1} \left( \frac{\vec{R} \cdot \hat{k}_o}{|\vec{R}| |\hat{k}_o|} \right)
\]

\[
\hat{\phi} = \cos^{-1} \left[ \frac{\vec{R} \cdot (\vec{R}_k - \vec{R})}{r} \right]
\](0 \( \leq \theta \leq \pi \)

b. \[
\hat{\phi} = \cos^{-1} \left( \frac{\vec{R} \cdot \hat{i}_o}{|\vec{R}|} \right)
\]

\[
\hat{\phi} = \cos^{-1} \left[ \frac{\vec{R} \cdot (\vec{R}_i - \vec{R})}{r} \right]
\]

where

\[
r_p = \sqrt{r^2 - [(\vec{R}_K - \vec{R}) \cdot \vec{R}]^2}
\](0 \( \leq \phi \leq 2\pi \)

c. \[
\hat{\theta}' = \cos^{-1} \left( \frac{\hat{U}_z \cdot \vec{R}}{|\vec{R}|} \right)
\]

\[
\hat{\theta}' = \cos^{-1} \left( \frac{\hat{U}_z \cdot \vec{R}}{r} \right)
\](0 \( \leq \theta' \leq \pi \)

d. \[
\hat{\phi}' = \cos^{-1} \left( \frac{\hat{V}_x \cdot \vec{W}}{w} \right)
\](0 \( \leq \phi' \leq 2\pi \)

where

\[
w = 1 - (\hat{V}_z \cdot \hat{U}_z)^2
\]
4. **Mapping Spherical Coordinates onto Polar Coordinates and Coordinate Transformation**

In order to map a point on a sphere onto a circle(s) such that there is a one-to-one mapping the following transformations were utilized:

**Case I.** $0 \leq \theta \leq \pi/2$

$$r = R \sqrt{1 - \cos \theta}$$

$$\theta = \cos^{-1}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

**Case II.** $\pi/2 \leq \theta \leq \pi$

$$r = R \sqrt{1 + \cos \theta}$$

$$\theta = \cos^{-1}\left[\left(\frac{r}{R}\right)^2 - 1\right]$$

In order to plot $(\theta, \phi)$ the following coordinate transformation was used:

I.e. - Given $(\theta, \phi)$, find $(x, y)$.

$$r = \begin{cases} R \sqrt{1 - \cos \theta} & 0 \leq \theta \leq \pi/2 \\ R \sqrt{1 + \cos \theta} & \pi/2 \leq \theta \leq \pi \end{cases}$$

$$x = r \cos \phi$$

$$y = r \sin \phi$$
5. **Phase Functions**

A factor of \( \frac{1}{2\pi} \) arises from the normalization process for the phase function.

\[
\int_0^{2\pi} \int_0^\pi p(\phi) \sin \theta \, d\theta \, d\phi = 1
\]

(a) Particulate

\[
P(\phi) = \frac{1-g^2}{4\pi} \left[ \frac{1}{(1+g^2-2g \cos \theta)^{3/2}} \right.
\]

\[
+ \left. \left( \frac{3}{2} \frac{\cos^2 \theta - 1}{(1+g^2)^3/2} \right) \right]
\]

(b) Rayleigh

\[
P(\phi) = \frac{3}{16\pi} \left( 1 + \cos^2 \theta \right)
\]
APPENDIX B

DEVELOPMENT OF THE FUNDAMENTAL RELATIONSHIP
FOR RAYLEIGH SCATTERING

\[ p(\theta) = \frac{3}{8} (1 + \cos^2 \theta) \quad 0 \leq \theta \leq \pi \]

\[ x = \int_{0}^{\theta} p(\theta) \, d\cos \theta \]

\[ = \int_{0}^{\theta} \frac{3}{8} (1 + \cos^2 \theta) \, d\cos \theta \]

\[ = \left. -\frac{3}{8} (\cos \theta + \frac{\cos^3 \theta}{3}) \right|_{0}^{\theta} \]

\[ x = \frac{3}{8} (\cos \theta + \frac{\cos^3 \theta}{3}) + \frac{1}{2} \]

Now solve for \( \cos \theta \).

\[ \cos^3 \theta + 3 \cos \theta + (8x - 4) = 0 \]

Let

\[ y = \cos \theta \]

\[ b = 8x - 4 \]

\[ a = 3 \]
\[ y^3 + ay + 6 = 0 \]

(See Reference 31 for the solution to a cubic equation.)

\[ y = A + B \]

or

\[ y = -\frac{A + B}{2} + \frac{A - B}{2} \sqrt{3} \]

or

\[ y = -\frac{A + B}{2} - \frac{A - B}{2} \sqrt{3} \]

where

\[ A = \sqrt[3]{-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}} \]

and

\[ B = \sqrt[3]{-\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}} \]

Now

\[ \frac{b^2}{4} + \frac{a^3}{27} = \frac{b^2}{4} + \frac{3^3}{27} = \frac{b^2}{4} + 1 \]

\[ \frac{b^2}{4} > 0; \quad \frac{b^2}{4} + 1 > 0 \]
Therefore, only one real root and two conjugate imaginary roots exist.

Since \( y = \cos \theta \), only one real root is considered.

\[
\theta = \cos^{-1}(A + B)
\]

where

\[
A = \sqrt[3]{\frac{-b}{2} + \sqrt{\frac{b^2}{4} + 1}}
\]

\[
B = \sqrt[3]{\frac{-b}{2} - \sqrt{\frac{b^2}{4} + 1}}
\]

and

\[
b = 8x - 4
\]
APPENDIX C

SAMPLE CALCULATION FOR RANGE PREDICTION

PROBLEM: Given a single channel 280 nm laser utilizing pulse position with power output per pulse equal to one millijoule, determine the communication pattern for a S/N equal to ten. Assume the detector characteristics presented in Section VI.

SOLUTION: From Table V, the minimum photon flux for a S/N of ten is $1.6 \times 10^3$ photons per cm$^2$.

Now,

$$1 \text{ watt} = 5.0345 \times 10^{18} \frac{\text{photons}}{\text{sec}}$$

where

$$\lambda = .280 \quad (\lambda \text{ is measured in } \mu)$$

$$1 \text{ joule} = 1 \text{ watt sec}$$

$$= 1.40966 \times 10^{18} \text{ photons}$$

$$1 \text{ millijoule} = 1.40966 \times 10^{15} \text{ photons}$$

From Figure 9, at contour 2 the relative flux is $10^{-2}$ of the initial photons per unit area where the unit area is one extinction length squared. From Table III one extinction length is
or .9611 kilometers. The relative flux is $1.0826 \times 10^{-12} \text{ cm}^{-2}$. The irradiance is given by the product of $1.40966 \times 10^{15}$ and $1.0826 \times 10^{-12}$ or 1526 photons per square cm. By measuring the distance from the source to the detector located on contour 2 (Figure 9) and recalling that one extinction length for this case is .9611 km, the ranges for an irradiance of 1526 photons/cm$^2$ may be determined. Table VI lists the results of this calculation.
APPENDIX D
CHECKS ON POSSIBLE ERRORS

Each computational procedure involved in writing this program was verified and an effort was made to ensure the entire program functioned properly. The procedures utilized to document the correct behavior of this routine are described in this appendix.

The various weighting functions in the program were verified by comparing the actual distribution of photons generated by the computer to the analytical solution for the distribution of random numbers obtained using a programmable calculator. (The statements required for this generation remain in the program, but the output of the results is suppressed.)

Verification of axis transformation computations was simple and straightforward. The axes were rotated about the Z axis, the Y axis, and finally about both axes to ensure proper behavior. (No documentation steps of this nature have been left in the program.) The output of $\theta$, $\gamma$, $\varphi$, and $\phi'$ for each photon at each shell was ordered by a computer sort routine in order to facilitate the preparation of a scatter diagram indicating the pattern formed by the simulated photons. Inspection of such a pattern did in fact reveal the presence of a fault which was corrected.
The results of Duntley [32] in describing multiply scattered radiation in the propagation of laser light underwater are most useful. Flux as a function of cone angle is presented for distances of up to 19 attenuation lengths. The scattering phase function for light underwater is a sharply peaked function which may be approximated with a Henyey-Greenstein function with a g value equal to 0.96 [33]. Calculations of photon flux as a function of cone angle were carried out with the computer program developed. Total attenuation of the beam down to a level of $10^{-3}$ could be calculated with some measure of accuracy (only 10,000 simulated photons were utilized for these calculations.) The results of these calculations agreed with the behavior observed in these underwater scattering measurements, simulating correctly the flux levels out to 16 attenuation lengths (the limit calculated).

Further verification of the accuracy of the program was obtained by comparison with calculations by Zachor and Green [24]. In this article, photon flux as a function of viewing angle was calculated for several distances from the source. The same conditions were assumed with the model used here, resulting in satisfactory agreement with the information published by Zachor for distances of 1.43, 2.73, and 4.95 extinction lengths for 300 nm radiation.
APPENDIX E

PROGRAM DESCRIPTION AND DOCUMENTATION

A. COMPONENTS OF THE PROGRAM

1. Weighting Function Subprograms

RANEXP is a function subprogram that generates random numbers weighted exponentially. It is used for distance calculations.

Function subprogram RANTH generates random values for theta and theta prime. Since the interaction may be either Rayleigh scattering or particulate scattering, either calculation may be performed. A uniform random number (generated by RANDU) within the interval [0,1] is obtained and compared to $R_p$ (the ratio of particulate to total scattering). If the random number is less than $R_p$, the particulate scattering calculation is made. (Otherwise the Rayleigh calculation is performed.) Due to the complexity of the particulate phase function it is difficult to obtain an exact solution; therefore an iterative method is employed. An exact solution is available when $f$ is set to zero and a more efficient calculation is utilized.

RANPH generates a random value of phi or phi prime uniformly weighted on the interval $[0,2\pi]$. The steps in this function subprogram are trivial.
2. LITE Subroutine

The main subroutine of the Monte Carlo simulation, called LITE, simulates a photon which randomly interacts with atmospheric constituents. (In order to make the most efficient use of computer time, non-interaction was not considered.) Initially the photon travels along the $-K_0$ axis (see Figure 16, Appendix A), a distance $\Delta R$ (determined by function subprogram RANEXP). A uniform random variable in the interval $[0,1]$ is compared to $R_S$ (the ratio of the total scattering coefficient, $K_S$, to the total extinction coefficient, $K$). If the random number is smaller than $R_S$, the photon is scattered. (Otherwise it is absorbed and the photon is lost.) If it is scattered, the distance of the photon from the laser is updated and a new value of $R$ is calculated. Since the photon has been scattered, new angles and distances must be computed. This is accomplished by generating values for $\Delta \theta$, $\Delta \phi$, and $\Delta R$ (using RANTH, RANPH, and RANEXP, respectively). The photon coordinate system is transformed (translated and rotated) to a new coordinate system. The distance from the source is calculated and compared with the radius of a shell. (A series of concentric shells with radii measured in extinction lengths, may be formed in order to determine the photon flux or angular distribution of photons as a function of distance from the source.) Various calculations are then made to determine whether a photon has penetrated through a shell, passed in and out of a shell, or penetrated an outer shell. These calculations are necessary to ensure proper accounting of
the photon flux. When a photon passes through a shell, the position of the source with respect to the penetration point is obtained. Theta, phi, and the radius of the shell (measured in extinction lengths) are stored in a three dimensional array for future use. Theta prime and phi prime are also determined as a function of distance (using RANTH and RANPH). These too, are stored in a three dimensional array and specify the direction of incident photon flux as observed by a receiver located a given distance from the source at a given angle off from the axis of propagation. This process is repeated until all of the photons have been used to generate information. (It should be noted that many extra steps incorporated to debug the program, remain in to facilitate future implementation by another user.)

3. DRLITE Routine

The routine DRLITE is used to drive the subroutine LITE. Information is read into the computer and the results are written out using this routine. The spatial distribution of photons as a function of position and detector field of view is calculated as well as the total number of photons in each shell. The output of other available information is optional and is controlled by IPRT and METHOD statements. This information includes pulse spreading (or photon time of arrival) as a function of position and field of view, the relative photon flux at each annular ring in a shell,
and a least square curve fitting of a modified Henyey-Greenstein function to the phase function information generated by the computer.

4. Miscellaneous Subroutines and Function Subprograms

LSTHG, LEAST, GAUSS3, and RANDU are miscellaneous subroutines used in the program. LSTHG is used to least squares fit a modified Henyey-Greenstein function to the information generated. LEAST and GAUSS3 are used in conjunction with LSTHG. EQN is a function subprogram supporting LSTHG. RANDU is a uniform random number generator that provides numbers in the interval [0,1]. It is located in the computer library. The variable IX is the seed and may be any odd integer with nine digits or less. It may not be zero. In order to use RANDU the following FORTRAN statements are used:

\[
\begin{align*}
IX &= 948752759 \\
\text{CALL RANDU (IX, IY, Y)} \ \\
IX &= IY
\end{align*}
\]

The seed IX is used in the calculation and a new seed IY is generated for future use. A uniform random number, Y, is in the interval [0,1] and is also available for future calculations. The last statement (\(IX = IY\)) resets the seed IX to a new value.
THIS ROUTINE IS INTENDED TO CARRY THE MONTE CARLO MULTIPLE
CALLED LITE, WHICH CALCULATES THE DISTRIBUTION OF PHOTONS
FROM A UNIDIRECTIONAL LIGHT SOURCE. IT IS USED TO READ
INFORMATION INTO THE PROGRAM AND TO CONTROL THE OUTPUT.
THE DISTRIBUTION OF PHOTONS AND THE TOTAL NUMBER OF PHO-
TONS IN EACH SHELL ARE STANDARD OUTPUT INFORMATION.
OPTIONAL OUTPUT IS CONTROLLED BY METHOD AND IPRT STATE-
MENTS. (METHOD CONTROLS THE SHAPE OF THE THETA BINS,
WHILE IPRT CONTROLS THE ACTUAL OUTPUT).

IPRT = 0, 1, 2, 3, 4, 5, 6, 7, 8, OR 9 YIELDS THE STANDARD
OUTPUT.
IPRT = 1, 2, 3, 4, OR 5 YIELDS THE NEGATIVE LOG OF
RELATIVE FLUX AT EACH ANNUAL RING.
IPRT = 3, 4, 5, 6, 7, OR 8 YIELDS THE TIME OF ARRIVAL
OR PULSE SPREADING INFORMATION.
IPRT = 5, 6, 7, 8, OR 9 YIELDS A LEAST SQUARE FIT TO
A MODIFIED FENVEY-GREENSTEIN FUNCTION TO THE
PHASE FUNCTION INFORMATION.

REAL*8 GIN
INTEGER*4 BINS(20,50,1), BINDIST(20,50,20)
REAL*4 RFLUX(20,50,1)
EQUVALENCE (BINS,RFLUX)
DATA PI/3.1415926536/

1 WRITE(7,500)
500 F CORMAT('NPHOT,NTHTA,NFLDVM,NSHLS,METHOD,IX,DISTSH,RATIO,
* 6 RPT, RACK, GIN ',*,15,12,12,12,12)
*10 5F5.4,F5.2,*
READ(5,505) NPHOT, NTHTA, NFLDVM, NSHLS, IFRT, METHOD, IX,
* DISTSH, RATIO, G, RPT, RACK, GIN
505 F CORMAT(15,12,212,211,110,5F5.4,F5.2)
IF(METHOD.EQ.1) METHOD = 1
IF(NPHOT.EQ.0) STOP
WRITE(6,110)
110 F CORMAT(1H1)
CALL LITE(NPHOT,NTHTA,NFLDVM,NSHLS,DISTSH,RATIO,G,RPT,RACK,
* IX,BINS,BINDIST,METHOD,IPRT)
WRITE(6,515)
515 F CORMAT('/DISTRIBUTION OF PHOTONS IS GIVEN BY')
DCL 50 I = 1,NSHLS
ISUM = 0
CC 44 J = 1,NTHTA
DC 44 K = 1,NFLDVM
ISUM = ISUM + BINS(I,J,K)
44 CONTINUE
WRITE(6,520)
520 F CORMAT(IX)
WRITE(6,440) I,ISUM
44C FCRMAT1X,*'TOTAL NUMBER IN SHELL *,12,* IS *,16)
   IF(NFLOD=.EQ.1) GC TO 47
   DC 46 J=1,NTHETA
   ISUM = 0
   DC 45 K=1,NFLOD
   45 ISUM = ISUM + BINS(I,J,K)
   WRITE(6,441) J,ISUM
   441 FORMAT(' NO. IN BIN ',*,12,* IS *,16)
   WRITE(6,521)(BINS(I,J,K),K=1,NFLOD)
   46 CONTINUE
   GC TO 48
   47 WRITE(6,521)(BINS(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   48 CONTINUE
   IF(IPRT.LE.2) GO TO 50
   WRITE(6,442)
   442 FORMAT('/* TIME OF ARRIVAL BINS GIVEN BY*,/)
   CC 49 J = 1,NTHETA
   45 WRITE(6,522)(BINDST(I,J,K), K = 1,20)
   522 FORMAT('X,2C16)
   50 CONTINUE
   IF(IPRT.GE.6) GO TO 399
   IF(IPRT.LT.1) GO TO 1
   WRITE(6,450)
   450 FORMAT('/* THE NEGATIVE LOG CF RELATIVE FLUX AT EACH*,
   * ANNUAL RING IS*')
   CC 60 I = 1,NSHLS
   R = DISTSHI
   DC 55 J = 1,NTHETA
   TH1 = (PI*(J-1))/NTHETA
   IF(METHOD.EQ.3) TH1 = (TH1*(J-1)/NTHETA
   IF(METHOD.EQ.3) TH2 = (TH2*J)/NTHETA
   AREA = 2.*PI*R**2*(COS(TH1)-COS(TH2))
   WRITE(7,999) J,AREA
   599 FORMAT('X,12,* THE AREA IS*,F10.6!)
   DC 55 K = 1,NFLOD
   IF(BINS(I,J,K).EQ.0) RFLUX(I,J,K) = 0.0
   IF(BINS(I,J,K).EQ.0) GO TO 55
   RFLUX(I,J,K) = -ALCG10(BINS(I,J,K)/(AREA*NPICT))
   55 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
   452 FORMAT(1X,2C06.2)
   60 CONTINUE
   WRITE(6,452)(RFLUX(I,J,K), K = 1,NFLOD), J = 1,NTHETA)
   521 FCRMAT1X,*2C16)
FUNCTION HANXP(IX, IY, TAU)
C**** THIS FUNCTION GENERATES A RANDOM NUMBER WEIGHTED EXPONENTIALLY
CALL RNDFN(IY, RN)
IX = IX
RHANexp = -TAU*ALG(1.0 - RN)
RETURN
END
C
FUNCTION RANTH(IX, IY, HENB, HENC, HEND, RPT, FBACK)
C**** THIS FUNCTION GENERATES A RANDOM VALUE OF THETA
C SINDICALLY WEIGHTED
CALL P1/2.1.570795326/.
CALL RNDFN(IY, RN)
IX = IX
IF(RN*GT.RPT) GC TC 20
IF(FBACK.EQ.1.0) GC TO 10
CALL RNDFN(IY, RN)
IX = IX
CRANM1 = 0.3
CC 5 I = 1.0
CRANTH = HENB/IRN- HENC-HENCFBACK*CRANM1*(1.0-CRANM1**2)**2
CP WRITETF(6,999) I,CRANTH,RN
999 FCMAT(IY,14.2F12.6)
IF(ABS(CRANTH-CRANM1).LT.0.001) CC TO 8
CRANM2 = CRANM1
5 CONTINUE
CRANTH = 0.25*(CRANM2*CRANTH)+0.*CRANM1
CP WRITETF(7,998) CRANTH,RN
998 FCMAT(IY,210.6)
E CONTINUE
IF(ABS(CRANTH).GT.1.0) CRANTH = CRANTH/(ABS(CRANTH) + .00001)
RANTH = ARCOS(CRANTH)
RETURN
C**** CALCULATE THETA USING FCMRATE SCATTER FUNCTION HERE
10 CALL RNDFN(IY, RN)
IX = IX
C WRITETF(6,990) HENB, HENC, RN
C 99C FCMAT(IY,4F12.6)
RANTH = ARCOS(HENB/HENC)*2.0
RETURN
C**** CALCULATE THETA ASSUMING RAYLEIGH SCATTERING HERE
20 CALL RNDFN(IY, RN)
IX = IX
RLB = 4.0*RN-2.0
RLBS = SQRT(RLB**2-1.0)
CAPA = (-RLB+RLBS)**(1./3.)
CAPB = -(RLB+RLBS)**(1./3.)
FUNCTION
FUNCTION RANPH(IX, IY)
C*** THIS FUNCTION GENERATES A RANDOM VALUE OF PHI
DATA TP1/6.233183C72/
CALL RANUD(IX, IY, RN)
IX = IY
RANPH = TP1*RN
RETURN
END
THIS SUBROUTINE SIMULATES A PHOTON WHICH RANDOMLY COLLIDES WITH PARTICLES, SCATTERED AT ANGLES WEIGHTED BY VARIOUS FUNCTIONS, AND DETERMINES LOCATION OF INTERSECTION WITH VARIOUS SPHERES.

INPUT PARAMETERS:
- NPHOT = NUMBER OF PHOTONS TO TRACE THROUGH
- NTHETA = THE NUMBER OF THETA "BINS" TO KEEP TRACK OF
- NFLOWS = THE NUMBER OF SHELLS OF "FLOW BINS" FOR EACH THETA BIN
- NSHELLS = THE NUMBER OF SHELLS TO BE INTERSECTED BY EACH PHOTON
- DISSH = THE DISTANCE BETWEEN EACH SHELL (AND FROM ORIGIN FCR FIRST)
- RATIC = RATIO OF SCATTERING CROSS-SECTION TO TOTAL EXTINCTION
- G = G VALUE FOR HENZY-GREENSTEIN FUNCTION
- RPT = RATIO OF PARTICLE TO TOTAL SCATTERING
- FEACK = RATIO OF BACKSCATTER TO TOTAL PARTICULATE SCATTERING
- IX IS A RANDOM NUMBER TO START THINGS

VARIABLES:
- X(I) = X COORDINATE; I=1 FOR ORIGIN, I=2 FOR X=S, I=3 FOR Z=1
  (X=1 MEANS (1,0,0) POINT FOR ORIGINAL COORD SYST, ETC.)
- Y(I) = Y COORDINATE; ITTO
- Z(I) = Z " "
- XT(I) = THE NEW COORDINATE AFTER SCATTERING BY THETA, PHI, AND TRANSLATION BY DISTANCE OR
  YT(I), AND ZT(I), LIKEWISE
- XT(I) IS A TEMPORARY COORDINATE POINT, AT THE INTERSECTION WITH THE SPHERE
- YS(I), ZS(I), LIKEWISE
- TP = VALUE OF THETA BY WHICH COORDINATE SYSTEM IS ROTATED
  AFTER SIMULATED COLLISION WITH PARTICLE
- PH = VALUE OF PHI, LIKEWISE
- TP = VALUE OF ANOTHER THETA PRIME, THETA RELATIVE TO PHOTON-FIXED COORDINATE SYSTEM
- PHP = VALUE OF PHI PRIME, IN PHOTON FIXED COORDINATE SYST
- BINS(I,J,K) = NO OF OCCURRENCES AT I TH SHELL WITH JTH BIN OF THETA AND KTH BIN OF THETA PRIME
- RSHL(I) = DISTANCE OF ITH SHELL FROM ORIGIN
- CTHETA = NEW VALUE OF THETA RELATIVE TO OLD DIRECTION
- DPHI = NEW VALUE OF PHI RELATIVE TO OLD DIRECTION
- HR = DISTANCE OF PHOTON TRAVEL AFTER A COLLISION
- IX, IY = FIXED-POINT RANDOM NUMBERS
- RICT = TOTAL DISTANCE A PHOTON TRAVELS
- DISSH = DISTANCE FROM ORIGIN OF PHOTON
- SCICT = DISTANCE FROM ORIGIN FCR PREVIOUS CALCULATION
- PCICT = DISTANCE ACTUALLY TRAVELED FROM ORIGIN TO SHELL
- ISAV = INTEGER CORRESPONDING TO NUMBER OF SHELL BEYOND WHICH THE PHOTON IS LOCATED
- XPCR = RATIO OF DISTANCE FROM POINT OF COLLISION TO A
SUBROUTINE LITE(NPHOT,NHTA,NFLDVW,NSHLS,DISTSH,PERTIC,
* G,RPT,FBACK,IX,INS,BINDT,ETHW,IPRT)
INTEGER*4 BINS(20,50),1, BINDT(2C,50,20)
DIMENSION X(4), Y(4), ZT(4), X1(4), Y1(4), Z(4)
CATA PI/3.14159265367, TP1/6.2831853072/, RCCTPI/1.772453851/
DATA EPSIL,1.00E-03/
WRITE(6,888)
888 FFORM(* ENTERED LITE*)
WRITE(6,8SC) NPHOT,NHTA,NFLDVW,NSHLS,IPRT,METHOD,CISTSH,
** RATIO,G,RPT,FBACK,IX,
8SC FFORM(I=1,*NPHOT = '*15,* NHTA = '*12,* NFLDVW = '*12,
** NSHLS = '*12,/* METHOD = '*',I5,' CISTSH = '*53, *
** RATIO = '*F7.5,' G = '*F7.5,' RPT = '*F7.5, *
** FBACK = '*F7.5,' IX = '*11C)
IENA = (1.0**G**2)/(2.0**G)
JXAB = (1.0**G**2)/(2.0**G)
HENC = (G+1.0)/(2.0**G)
HENC = (1.0**G**2)/(4.0*(1.0+G**2))**1.5
C**** INITIALIZE ARRAYS
DC 5 I = 1, NSHLS
CC 5 J = 1, NHTA
DC 4 K = 1,0
BINS(I,J,K) = 0
CE 6 K = 1, NFLDVW
BINS(I,J,K) = 0
C**** START LOOP FOR EACH PHOTCN
DC 100 NPH = 1,NPHET
IXS = IX
NSCA = 0
RXCT = 0.0
ISAV = 1
DC 15 I = 1,4
X(I) = 0.0
Y(I) = 0.0
Z(I) = 0.0
X(2) = 1.0
Y(2) = 1.0
Z(2) = 1.0
C**** CHOOSE DISTANCE OF INITIAL PHOTCN
DF = RANEXP(IY, IY, 1.0)
DC 17 I = 1,4
17 Z(I) = Z(I) + DR
16 IF(DR.LT.RSFL(ISAV+1)) GO TO 19
ISAV = ISAV+1
IF(ISAV.GT.RSFL) GC TO 100
GC TO 18
15 CONTINUE
RIOT = DR
SCLIST = RTOT
C**** DETERMINE IF PHOTCN IS ABSORBED OR SCATTERED
C CALL RANDUL(IY, IY, FN)
IX = IY
IF(RN.GT.RATIO) GC TO 100
C**** IF PHOTON IS SCATTERED, DETERMINE NEW ANGLES AND DISTANCES
NSCA = NSCA+1
CR = RANEXP(IY, IY, 1.0)
DTHETA = RANUL(IY, IY, HEN, HENB, ENC, HEND, RPT, FBACK)
DPhi = RANUL(IY, IY)
SIN = SIN(DTHETA)
CPhi = COS(DTHETA)
SPH = SIN(DPHI)
CPH = COS(DPHI)
CP = WRITE(6, 950) RTOT, CR, DTHETA, DPHI, STH, GTH, SPH, CPH
SSC FORMAT(*, RTOT, CR, DTHETA, DPHI, STH, GTH, SPH, CPH), /
1X, 8F10.5)
C**** ROTATE TO NEW PHOTON POSITION AND TRANSLATE
DC 30 I = 1,4
XT(I) = X(I)*CPhi-Y(I)*CTH-Z(I)*STH
YT(I) = X(I)*SPh+Y(I)*CTH
ZT(I) = X(I)*STH+Y(I)*SPh+Z(I)*CTH+ER
CP WRITE(6, 991) XT(I), YT(I), ZT(I)
991 FORMAT(*, COORDINATES*, 3F10.6)
C CONTINUE
DIST = SQRT(XT(I)**2+YT(I)**2+ZT(I)**2)
C**** CHECK TO SEE IF PHOTON HAS PENETRATED A SHELL
C**** CONSIDER IF PHOTON HAS PENETRATED IN AND OUT OF SHELl
C FF IS FRACTIONAL DISTANCE BETWEEN TWO POINTS GIVING
C THE DISTANCE OF CLOSEST APPROACH
FF = 1.0-ZT(I)/DR
IF(FF.LT.0.0) OR.(FF.GT.1.0) GC TO 35
DC = SQRT(XT(I)**2+YT(I)**2)
IF(DC.GT.RSFL(ISAV)) GO TO 35
ISAV = ISAV+1
CP WRITE(7, 920) DC, RSFL(ISAV), FF, IX
920 FORMAT(1X, 3F10.5, 1X, I10)
IF(DC.GT.RSFL(ISAV)) GO TO 50
C**** CONSIDER IF PHOTON HAS PENEGERATED CUTER SHELLS
25 IF((ISA1),GT,(NSHLS)) GO TO 100
30 IF((DIST,GT,RSHLS(ISA1))) GO TO 50
35 IaT((DIST,GT,RSHLS(ISA1))) GO TO 20
40 I% = ISA1 - 1
LC TO 27
C**** TIME TO UPDATE COORDINATES - GET READY FOR ANOTHER COLLISION
40 SCIST = DIST
50 iC 45 1 = 1,4
60 X(i) = X1(i)
65 Y(i) = Y1(i)
70 Z(i) = Z1(i)
45 CONTINUE
C**** GC TO 20 FOR ANOTHER COLLISION
GC TO 20
C**** CALCULATE COORDINATES OF POINT AT WHICH PHOTON PENEGERATES SHELL
50 I% = ISA1
CP WRITE(6,551) DR, ZT(1)
55 FFORMAT(**** DR., ZT(1) **", 2F10.5)
CP WRITE(6,552) SDIST, RSHLS(ISA1)
55 FFORMAT(**** SDIST, RSHLS(ISA1)**", 2F10.5)
CP WRITE(6,553) DIST, ROT1, ISA1
55 FFORMAT(**** DIST, ROT1, ISA1 **", 2F10.5, 2X, 12)
XPDW = (DR-ZT(1))**SQRT((ZT(1)-DR)**2-SDIST**2*RSHLS(ISA1)**2)/CR
CP WRITE(6,554) XPDR
55 FFORMAT(**** XPDR = *, F10.5)
DC 55 1 = 1,4
70 X(i) = X1(i)
75 Y(i) = Y1(i)
80 Z(i) = Z1(i) - (1.0-XPDR)*DR
55 CONTINUE
CP WRITE(6, 992) (XS(i),YS(i),ZS(i), i = 1,4)
CP WRITE(6,993) 'C**** CONCOPE AT SHELL: ', 12FS.5)
55 CODE OF THETA
VAL = ((XS(i)-XS(1)-XS(3)) + YS(i)-(YS(1)-YS(3)))
* + ZS(i) - (ZS(1)-ZS(3)))/RSHLS(ISA1)
55 IF(ABS(VAl) .GT. 1.) CALL PERP(1,1XS, VAL, XS, YS, ZS)
55 IF(ABS(VAl) .GT. 1.) WRITE(6,999) VAL, NPH, IXS
200 CODE OF ARCOS GT. 1.; VAL = '*,F10.5,' NPHET = '*, I5,' IX = '*
210 IF(VAL .GT. 1.0) VAL = 1.0
210 IF(VAL .LT. -1.0) VAL = -1.0
220 T9 = PI - ARCOS(VAL)
230 IF(TH .LT. 0.0) TH = 0.0
CP WRITE(6,995) RSHLS(ISA1)
C WRITE . ACTION SUBPROGRAM TO CALCULATE THE FUNCTION TC EE LEAST SQUARES C A IS DIMENSIONED 10 AND CONTAINS THE PARAMETERS TO BE VARIED SUCH AS LS0020 C X(10). THE SUM OF SQUARES DISTRIBUTION X, XB, AND XC ARE THE THREE INDEPENDENT C PARAMETERS (YOU MAY WISH TO USE ONLY X) NOF IS THE FUNCTION NUMBER - LS0040 C BRANCHING TO DIFFERENT PARTS OF THE EQN SUBPROGRAM WHEN SEVERAL FUNCTION LS0045 C ARE TO BE FIT (FOR SEVERAL JOBS TO BE DONE). LS0046 C THE DIMENSIONED ARRAYS HAVE THE FOLLOWING MEANING-- LS0047 C T(10) CONTAINS THE PARAMETERS WHICH ARE TO BE VARIED LS0048 C X(200), XB(200), AND XC(200) CONTAIN THE INDEPENDENT PARAMETERS LS0049 C Z(200) CONTAINS THE OBSERVED VALUES OF THE FUNCTION (THE OBSERVED LS0050 C VALUES). LS0051 C F(10) CONTAINS THE MAGNITUDE OF THE INCREMENTS FOR THE PARAMETERS LS0052 C AT THE POINT WHERE THE PROGRAM TAKES NUMERICAL DERIVATIVES WITH REASONABLE ACCURACY LS0053 C R(200) CONTAINS THE DIFFERENCE BETWEEN OBSERVED Z(200) AND CALCULATED LS0054 C E(10) CONTAINS THE ESTIMATES OF THE ERRORS OF THE PARAMETERS LS0055 C ITERATION. LS0056 C INPUT = LS0057 C FIRST CARD - WILL BE REPRODUCED ON OUTPUT USED FOR LABELING. LS0058 C SECONDS CARD FORMAT(12,13,12,13,E10.2) LS0059 C IR = NUMBER OF PARAMETERS TO BE VARIED LS0060 C IS = NUMBER OF POINTS LS0061 C NOF = FUNCTION NUMBER (SEE ABOVE) LS0062 C NNP = NUMBER OF INDEPENDENT PARAMETERS LS0063 C EPSIL = IS USED AS A CRITERION FOR CONVERGENCE. IF THE RELATIVE VALUE LS0064 C OF THE RESIDUAL CHANGES BY LESS THAN EPSIL IN TWO SUCCESSIVE ITERATIONS LS0065 C CONVERGENCE IS REACHED. LS0066 C REAL*4 T(10), X(200), XB(200), XC(200), Z(200), FINCR(10), LS0067 C IR(200), E(10) C ALL ERRORS (208,0,-1,1,0,0) LS0068 C DC/ 2 = 1, ISLS0069 C 2 /I = IBIN(NN,1,1) LS0070 C SUM = 0.0 LS0071 C G0 5 = 1, 1, ISLS0072 C 5 SUM = SUM +2/(1) LS0073 C IF(SUM.LT.20) RETURN LS0074 C IR = 2 LS0075 C NNP = 1 LS0076 C EPSIL = .00C1 LS0077 C G0 TO (7,8,8), NGF LS0078 C CONTINUE LS0079 C A1 = SUM LS0080 C A2 = .90 LS0081 C A4 = 180.0, ISLS0082 C G0 TO 9 LS0083 C A1 = .8 LS0084
C 30 WRITE(6,105)
      WRITE(6,105)
      WRITE(6,105)
      WRITE(6,105)
      WRITE(6,105)
      WRITE(6,105)
      WRITE(6,105)
      WRITE(6,105)
      WRITE(6,105)
      RETURN
100 FORMAT (12,13,12,13,E10.2)
101 FORMAT (16F5.0)
102 FORMAT (16F5.0)
103 FORMAT (16F5.0)
104 FORMAT (16F5.0)
105 FORMAT (16F5.0)
106 FORMAT (80H)
107 FORMAT (99H NUMBER OF PARAMETERS NUMBER OF POINTS FUNCTION NUMBER OF INDEPENDENT PARAMETERS)
108 FORMAT (4X,12,24X,13,17X,12,19X,12,12X,E10.6)
109 FORMAT (1H)
110 FORMAT (26H THE PARAMETERS FED IN ARE)
111 FORMAT (1H,10F10.5)
112 FORMAT (1H,10F10.5)
113 FORMAT (1H,10F10.5)
114 FORMAT (1H,10F10.5)
115 FORMAT (1H,10F10.5)
116 FORMAT (1H,10F10.5)
117 FORMAT (1H,10F10.5)
118 FORMAT (1H,10F10.5)
119 FORMAT (34H THE OBSERVED VALUES TO BE FIT ARE)
120 FORMAT (1X,10F10.2)
121 FORMAT (1X,10F10.2)
122 FORMAT (1X,10F10.2)
123 FORMAT (1X,10F10.2)
124 FORMAT (31H THE INDEPENDENT PARAMETERS ARE)
125 FORMAT (1H,10F10.5)
126 FORMAT (1H,10F10.5)
127 FORMAT (1H,10F10.5)
128 FORMAT (1H,10F10.5)
129 FORMAT (38H THE BEST VALUES OF THE PARAMETERS ARE)
130 FORMAT (1H,10F10.5)
131 FORMAT (1H,10F10.5)
132 FORMAT (1H,10F10.5)
133 FORMAT (1H,10F10.5)
134 FORMAT (1H,10F10.5)
135 FORMAT (3H OBSERVED MINUS CALCULATED VALUES OF THE BEST FIT ARE)
1 (DEVIATION)
2CC STOP
1ENC
3C SUBROUTINE LEAST(IR,IS,A,X,XB,XC,Z,FINCR,EPSIL,NOITK,RR,L,NCF,R,E)
4C IMPLICIT REAL*8 (A-H,C-Z)
5C IR = NO. OF PARAMETERS, IS = NO. OF POINTS, A IS ARRAY OF PARAMETERS,
6C X IS INDEPENDENT VARIABLES, XB IS DEPENDENT, FINCR IS ARRAY OF INCREMENTS.
7C EPSIL IS A-FRACTIONAL-ERROR CRITERION, NOITR IS NO. OF ITERATIONS REQUIRED (UP TO 100).
8C NCF = NUMBER OF ESTIMATED ERRORS IN THE PARAMETERS:
9C DIMENSION A(10), X(200), XB(200), XC(200), Z(200), FINCR(10), R(200), 10C D(200,10), DT(13,203), DEL(1C), DS(10), EP(1C,1C), EP(1C,1C), LST01450
11C NCITR = 0
12CC DC 20 I = 1,IS
13CC 25 IF (NOITR-9) 130,130,4
14CC 15 NGITR = NOITR + 1
15CC 220 J = 1, IR
16CC 250 A(I,J) = A(I,J)+FINCR(J)
17CC 30 C(I,J) = EQN(A,X(I),X(I+1),XC(I),NCF)
18CC 220 A(I,J) = A(I,J)-FINCR(J)
19CC CONTINUE
20CC 30 I = 1,IS
21CC 35 J = 1, IR
22CC 30 C(I,J) = D(I,J)-CONST)/FINCR(J)
23CC 35 J = 1, IR
24CC 35 I = 1,IS
25C 958 WRITE(6,999) (DT(I,J), J = 1,IS)
26C 999 FORMAT(IX,12F10.5)
27CC DC 36 I = 1,IR
28CC 36 J = 1, IR
29CC 36 DPI(I,J) = 0.0
30CC 36 K = 1, IS
31CC 36 DPI(I,J) = DPI(I,J)+DT(I,K)*D(K,J)
32CC CALL GAUSS3 (IR,1.00E-30,DP,DPI,KER)
33CC IF(KER-1) 120,37,120
34CC 37 CC 40 I = 1,IS
35CC 38 J = 1, IR
36CC 38 DPI(I,J) = 0.0
37CC 38 K = 1, IR
38CC 38 CS(J) = DS(J)+DP(I,J)*DT(K,I)
39LST01450 LST01460 LST01470 LST01480 LST01490 LST01500 LST01510 LST01520 LST01530 LST01540 LST01550 LST01560 LST01570 LST01580 LST01590 LST01600 LST01610 LST01620 LST01630 LST01640 LST01650 LST01660 LST01670 LST01680 LST01690 LST01700 LST01710 LST01720 LST01730 LST01740 LST01750 LST01760 LST01770 LST01780 LST01790 LST01800 LST01810 LST01820 LST01830 LST01840 LST01850 LST01860 LST01870 LST01880 LST01890 LST01900 LST01910 LST01920
DC 39 L = 1,IR
29 DTL(I,1) = DEL(I)
40 CONTINUE
C DC 993 I = 1,IR
C 993 WRITE(*,992) (DP(I,J), J = 1,IR)
992 FORMAT(1X,12F10.5)
C DO 950 I = 1,IR
C 950 WRITE(6,991) (DT(I,J), J = 1,IS)
C WRITE(6,991) (R(I), I = 1,IS)
DC 110 I = 1,IR
DEL(I) = 0.0
DC 120 J = 1,IS
DEL(I) = DEL(I)+DT(I,J)*R(J)
DC 130 I = 1,IR
A(I) = A(I)+DEL(I)
IF(LOF.GE.4) GO TO 222
IF(LOF.GE.2) GO TO 216
IF(A(2).LT.0.99) A(2) = 0.99
GC TO 222
215 IF(A(1).LT.0.99) A(1) = 0.99
IF(A(2).LT.0.3) A(2) = 0.3
222 CONTINUE
DC 220 I = 1,IS
320 R(I) = Z(I) - EQN(A,X(I),X(I+1),X(I),NOF)
C C 50 I = 1,IS
50 RRC = RRC+R(I)**2
WRITE(6,102) (A(I), I = 1,IR), RRC
1C2 FORMAT(1H,10E12.4)
CRES = DABS(RRC-RRP) - EPSIL*RRP
RRP = RRC
IF(CRES) GE 100,100,2E
101 FORMAT(1H, CONVERGENCE FAILURE)
WRITE(6,101)
GC TO 100
12C WRITE(6,1001)
1001 FORMAT (16H MATRIX SINGULAR)
100 FIS = IS-IR
DO 150 I = 1,IR
E(I) = DSR[R(RRC*EPSPI(I,1))]
RETURN
END
C SLRoutines GAUSS3(N,EP,A,X,KER)
IMPLICIT REAL*8 (A-H,C-Z)
DIMENSION A(10,10), X(10,10)
REAL FUNCTION HG,*B(A0L,G,APN,X,DEL)
IMPLICIT REAL*8 (A-H, O-Z)
DATA CR/0.017453/;
TAT = 2.0*XFN-2.0
CP WRITE(6,991) AMPL,G,XPN,X
551 FCMAT(1X,4F10.6)
GS = G**2
TG = 2.0*G
OPGS = 1.0+GS
HG = AMPL*(1.0/(1.0/(1.0-G)**(TAT)))-1.0/(1.0-G)**(TAT))
1 = 1.0/(OPGS-TG*DOS(K*CR))**(XPN-1.0)
2 = 1.0/(OPGS-TG*DCS((X+DEL)*CR))**(XPN-1.0)
CP WRITE(6,999) AMPL,G,HG,X
995 FCMAT(1X,2F10.5,12.4,F10.4)
IF(INUM.GE.NCF) RETURN
END = 4
REAL FUNCTION EQN*B(A,X,XB,XC,NCF)
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION A(10)
GO TO (5,55,155,255), NOF
5 DEL = A(10)
EQN = HG(A(1),A(2),1.5 X,DEL)
RETURN
55 DEL = A(10)
ECN = HG(A(15),A(1),A(2),X,DEL)
RETURN
155 DEL = XB-X
ECN = HG(A(15),A(1),A(2),X,DEL)
RETURN
255 ECN = A(1)**X**2 + A(2)**X + A(3)
RETURN
ENC

SUBROUTINE PERR(ERRNC,I0S,PARAM,XS,YS,ZS)
INTEGER*4 ERRMO
DIMENSION XS(4), YS(4), ZS(4)
NUP = 4
IF(I0S.LT.4) NUP = IXS
WRITE(6,100) ERRNC,PARAM,IXS
100 FORMAT('ERROR DETECTED, LOCATION NO.' ,15,'* PARAM = '*
* E14.8, '1XS(1), YS(1), ZS(1), I = 1,NUP)
110 WRITE(6,110) (XS(I), YS(I), ZS(I), I = 1,NUP)
110 FORMAT('COORD. FROM PERR: '*12FS,5)
RETURN
ENC
LIST OF REFERENCES


29. Uros, N.M., personal communication.


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