Interim Report on
Research Grant No. 76-2879, entitled
Structure of Materials,
Electromagnetic Scattering
and Fluid Dynamics*

*October 1, 1976 - October 31, 1977
Principal Investigator: Robert P. Gilbert
Interim report on research Grant AFOSR 76-2879 covering work completed, and in progress. Includes papers written and published during the period October 1, 1976 to October 31, 1977.
R. P. Gilbert and colleagues researched the Riemann-Hilbert boundary value problems for elliptic systems of $2n$ equations in the plane $[1,2]$. These problems are of interest in the study of composite laminates. Work on composite elastic materials is in progress by Gilbert and Wendland. Their approach depends heavily on the Begehr-Gilbert representations for solutions. A first report should be available shortly.

Gilbert and A. Jeffrey began joint work on systems of mixed-type (see also, Gilbert-Schneider [3]). These equations appear, for example, in the study of compressible fluid flow in the transonic region.

Gilbert and Hsiao completed a research effort on using finite element methods in connection with integral equations of the first kind to solve boundary value problems associated with elliptic equations [4]. This extends the results developed by Hsiao and Wendland [i,j].

G. C. Hsiao continued his investigations of singular perturbation problems. These researches are listed as manuscripts [9,10] and the book monograph.

Further results concerning numerical and constructive approaches to boundary value problems were made by Gilbert-Vidic [8] who developed an ALGOL program for the accurate computation of Gilbert's G-kernel. Gilbert and Goodrich [5] have investigated Bergman's
kernel function in $\mathbb{R}^3$. To do this the regularity of the $\ell$-kernel had to be investigated in the closure of the Cartesian product of the respective domain. The $\ell$-kernel is not regular on the boundary but is seen to lie in a suitable trace-class.

Other results concerning constructive approaches for meta-parabolic equations may be found in [6] and [7].

The Coinvestigator, David Colton, has continued his research on the use of transformation operators to investigate the solutions of parabolic equations with variable coefficients, in particular, these modeling heat conduction in a non-homogeneous medium. In [12] these operators were used to obtain an improved version of Colton's earlier results on reflection principles for parabolic equations. In connection with Lewy's results on reflection laws for elliptic equations, this work now provides a complete picture of reflection properties of solutions to partial differential equations in two independent variables. In [13] Colton's method of transformation operators was used in conjunction with double layer heat potentials to provide a new method for solving initial-boundary value problems for parabolic equations in two independent variables with variable coefficients defined in domains with moving boundaries. Numerical experiments on the use of transformation operators to solve heat conduction problems in a nonhomogeneous medium with moving boundaries were presented in [14] and it was demonstrated that such methods provide an efficient numerical method for solving heat conduction problems of this type.
The Coinvestigator has also continued his investigations on the theory of wave propagation in a spherically stratified medium. In [15] the Coinvestigator's method of transformation operators was used to provide a new method for solving the Fock initial-boundary value problem modeling radiowave propagation around the earth under the assumption of a spherically stratified atmosphere. This new approach leads to the solution of the problem by means of the minimization of a certain quadratic functional, and is based on results on asymptotic analysis due to Erdelyi and the approximation of entire functions due to Levinson. The approximations obtained are valid not only in the shadow and illuminated regions, but in the penumbra region as well. In [16] the problem of the scattering of acoustic waves by an unbounded spherically stratified nonhomogeneous medium was considered, and approximations were obtained which agree with the Born approximation in their common region of validity, but are also applicable for values of the wave number such that the Born approximation is no longer valid. The transformation operator constructed in this paper also strengthens the earlier results of Colton and Wendland on the scattering of acoustic waves by an obstacle in a spherically stratified medium, in the sense that the assumption that the spherically stratified medium had compact support can now be removed.

Surveys of some of the above work can be found in the Coinvestigator's invited paper at the Third Scheveningen Conference on Differential Equations [17] and the invited lecture series at the University of Delaware [18].
During the period October 1, 1976 - October 1, 1977, the following papers were written by R. P. Gilbert:


During the same period G. C. Hsiao wrote (in addition to Nos. 4 and 6 above) the following papers:


Manuscripts completed
on Grant No. 76-2879 (cont.)


The following manuscripts appeared in print during this time:


Manuscripts completed on AFSOR Grant No. 76-2879


Summary Rept.
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Work in preparation includes a monograph text:

"Introduction to Singular Perturbation Problems for Partial Differential Equations"