Millstone Quarry, Figure 1, situated on Millstone Point in Waterford, Connecticut, was operated for the extraction of granite construction materials from the late 1700's to the early 1950's. In 1943 the lower reaches of the quarry were abandoned and subsequently began to fill with brackish water, the salt being supplied by seepage along the southern and western walls. From 1943 to the final operations, rock was quarried above the rising water line.

The manner in which the quarry developed its present hydrological conditions is uncertain. However, it is certain that Millstone Quarry has characteristics not evident in other closed bodies of water within the Southeastern Connecticut region (1). This is one of the reasons for establishing at Millstone Quarry the USL Millstone Quarry Research Facility. The facility, which performs underwater acoustics testing, is housed on a barge roughly centered over the deepest portion of the quarry. In order that the sound velocity structure might be calculated, the vertical salinity profile was determined and assumed to be invariant in time. Also a vertical string of thermocouples was installed to obtain the time variations of the vertical thermal profile. The temperature data discussed in this report were obtained for just such sound velocity calculations and were made available through J. G. Navin, Acting Branch Head, Millstone Quarry and H. J. Wilms, Acting Division Head, Acoustics Measurements Division, USL.

C. F. Gerber (2) (3) brought the unique hydrological conditions of Millstone Quarry to the attention of the Oceanography Branch of the Ocean Sciences Division, USL, and subsequent investigation have resulted in detailed summaries of the unique scientific value of Millstone Quarry (4) (5).
This report is a first attempt at rationalizing the time variations of the vertical thermal profile with existing theories on thermal mixing in the oceans. Application of present theories on thermal mixing require evaluating various physical constants. This report deals with the time and space variations of one of those physical constants - the vertical kinematic thermal eddy diffusivity, $K_z^*$.

STATEMENT OF INVESTIGATION

When one cares to make an investigation on the inter-relationship between several inputs to the same system, the obvious application of scientific principle is to observe the system output for all possible controlled permutations of the several inputs. However, for investigations in which environmental weather data are system inputs, one can not obtain in a controlled manner all possible permutations of the several inputs. Therefore, either exact measurements must be made on each of the several environmental inputs at a rate which is greater than that observed for changes in the system output, or assumptions must be made at a level capable of justifying any conclusions concerning the system output.

It is quite obvious that the previous statement leaves two approaches open for planning investigations for the generation of new data; yet for any treatment of historical data one is forced to make assumptions about any possible system inputs which were not recorded.

The geometry of Millstone Quarry is presented in Diagram 1 and since the thermocouples occupy fixed points in that geometrical space, one must assume that the data are adequate to completely represent the horizontal and vertical thermal field. Also, since temperature data were not obtained in a continuous manner, one must assume that at no time between any two data samples did the temperature have a value which is not bracketed by the temperature corresponding to the two data samples. The thermocouples measured the temperature at a fixed depth below the surface, but a rise in water level of two feet occurred during the test period and caused a change in the absolute measurement geometry. Therefore one must also assume that the vertical thermal changes were rapid compared to the rise in water level.

It should be noted that while no data are available to validate the preceding assumptions for the period July 1965 to August 1966, investigations subsequent to that time indicate that the assumptions are realistic for the period August 1966 to the present. It is in this context that figure 2 presents the time variations of the vertical thermal profile for Millstone Quarry.
Knowledge is obtainable on ocean mixing by application of the conservation laws of mass, momentum, and energy to the data which one observes. The present discussion is confined to the exchange of thermal energy across the Millstone Quarry air/sea interface and its subsequent distribution in the vertical column. The continual exchange of energy across the boundaries of the quarry require the water column to continually re-establish equilibrium by internal redistribution of heat and matter.

For any given set of equilibrium conditions (temperature, salinity, and pressure) one may define the media by the equation of state for sea water. Small perturbations of equilibrium conditions will require changes in the media to re-establish equilibrium. Changes in the media will be such that the entropy of the new equilibrium state will be at a maximum and by examining the entropy of all possible final states having the same total energy and mass, one can define the new equilibrium conditions. The only additional information required to describe the changing media
is the rate which the system approaches equilibrium for any specified change in conditions (temperature, salinity, or pressure).

As previously stated, the data for this report were not obtained for the specific purpose of determining the rates of mixing. The data at hand are inadequate to describe the transfer of salt within the media except to say that it is small and will be disregarded as a possible contribution to the maximizing of entropy (6) (7). Likewise, pressure changes are neglected and the equation of state is reduced to changes in density as a function of temperature. See Figure 3 (8). The salinity profile presented in Figure 4 must be consulted to determine the quiescent density for any given temperature and depth.

Heat flux at the upper boundary of Millstone Quarry alters the temperature at that boundary, and then the change in temperature spreads into the interior. The temperature changes spread away from the boundary by a combination of molecular diffusion and turbulent diffusion. Turbulent diffusion, like molecular diffusion, is proportional to the product of specific heat and vertical thermal gradient but generally with a much larger proportionality constant (9).

Molecular diffusion will always be present, but turbulent diffusion depends on the presence of turbulence. Turbulence may be generated at the upper surface by wind stress or may be generated due to the conversion of potential energy into kinetic energy as is the case when a negative density gradient is present. Turbulence may be decreased by the buoyancy effect and viscosity in regions where the density gradient is positive. Diagram 2 represents the relationships of the preceding discussion.

HEAT FLUX

WIND STRESS

Equation of state for sea water

Increase of system potential energy
STABILITY

Increase of system kinetic energy
TURBULANCE

Diagram 2 Energy flow diagram for Millstone Quarry vertical mixing
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The high quarry walls will reduce the effects of wind on the upper surface (10) and in the absence of wind stress a positive heat flux will cause the system to tend toward vertical stability. If the heat flux is negative, the system will tend toward vertical turbulence.

The thermal spreading is represented as

\[ \frac{d \Theta}{dt} = \nabla \cdot \Theta \] (1)

For the quarry one must assume that the substantial derivative is equivalent to the local derivative.

Where

\[ \frac{d \Theta}{d t} = \frac{d}{d z} \left( K_z \frac{d \Theta}{d z} \right) \] (2)

\[ \Theta = C_p T \]
\[ C_p = \text{specific heat at constant pressure} \]
\[ T = \text{temperature} \]
\[ K_z = \text{vertical exchange coefficient for heat} \]

If one assumes that in time the specific heat at any depth is constant and that \( K_z \) and \( C_p \) are constant over the depth interval between two measurements, one may express the kinematic eddy diffusivity as

\[ K_z = \frac{\frac{dT}{dt}}{\frac{\partial^2 T}{\partial z^2}} \] (3)

By extension to Proudman (11), one may consider the determination of \( K_z \) from the time variations of isotherms at a given station. See Diagram 3. Draw lines parallel to the time axis; one for each depth at which

Diagram 3 Graphical Determination of \( K_z \)
a measurement was obtained. Let denote the increment of temperature separating isotherms, the increment of average temperature from Z to \((Z- z)\) and denote the increment of average temperature from Z to \((Z+ z)\).

Let denote the time separation between two isotherms at the same depth and let all derivatives be evaluated at P.

Therefore, in time at any given depth \(Z\),

\[
\delta_t T = \frac{\partial T}{\partial t} \delta t
\]

Also from the curve for the average temperature as a function of depth

\[
\delta_+ T = \frac{2}{\delta x} \delta z + \frac{1}{\delta z} \frac{\partial^2 T}{\partial x^2} (\delta z)^2 + \frac{1}{6} \frac{\partial^3 T}{\partial x^3} (\delta z)^3
\]

\[
\delta_- T = \frac{2}{\delta x} \delta z - \frac{1}{\delta z} \frac{\partial^2 T}{\partial x^2} (\delta z)^2 + \frac{1}{6} \frac{\partial^3 T}{\partial x^3} (\delta z)^3
\]

Therefore

\[
\delta_+ T - \delta_- T = \frac{\partial^2 T}{\partial x^2} (\delta z)^2
\]

\[
K_z = \frac{\delta T}{\delta t} \frac{\delta^2 T}{\delta x^2} = \frac{\delta T}{\delta t} \frac{\delta_+ T - \delta_- T}{(\delta z)^2}
\]

\[
K_z = \frac{[\delta_+ T - \delta_- T]}{\delta^2 T} \frac{(\delta z)^2}{\delta t}
\]

\(P\;PP^*\) is considered to represent the average thermal variations within the neighborhood bordered by the two isotherms separated by \(\delta T\) and the two depth limits separated by \(2\delta z\). The evaluation of equation 6 is graphically determined as per diagram 3.

\[
K_z = \frac{2 \left[ \frac{\delta z}{D'} - A + C - B \right]}{[D' - A + C - B]} \]

6
which is independent of the temperature difference between the two isotherms. 

Kz as determined from equation 7 are presented in Figure 5.

RESULTS AND CONCLUSIONS

A comment on dimensional units is in order. The isotherms in Figure 2 are presented in the English system and are the units in which the measurements were taken. It has been previously stated that the data were not taken specifically for this study. The depth units on Figure 4 and 5 were reported in feet to allow ease of comparison with Figure 2. This is contrary to scientific convention, however, the necessary conversions were made to allow Kz to be reported in the conventional units of cm 2/sec. The combinations of mixed units presented in this report in no way detract from the subject at hand. There was however a serious misleading artifact introduced into Figure 5. The decimal positions are not indicative of the number of significant figures. The values are only accurate to the nearest one-tenth cm 2/sec.

The Kz in figure 5 are heavily averaged in time, first because the data were obtained at most once a day which immediately places a lower limit on the Kz. Furthermore, data were available for only 198 days of the 385 days interval covered in this report. Additional time averaging was necessary to eliminate scatter resulting from the evaluation of the individual thermal profiles.

The accuracy of the individual thermal profiles is ±0.5° and the precision of any point on a given profile is ±0.1 ° F.

For those portions of Figure 2 where the isotherms approach vertical and Kz is very large, the maximum Kz that can be reliably determined from the figure is 1.2 cm 2/sec for the measurements at three foot separation and 3.3 cm 2/sec for the five foot intervals. For those near vertical isotherm portions of Figure 2, one may return to the individual thermal profiles and determine Kz provided the temperature differentials exceed the precision and accuracy of the measurement system. At the system limit, the Kz would be five times that determined from Figure 2. For those portions of Figure 2 where the isotherms approach horizontal and for the region below the 56F isotherm the Kz is drastically reduced. The minimum Kz that could be detected from Figure 2 is 8.6 x 10^-3 cm 2/sec.

There are many combinations of comparisons which one might employ in evaluating Figure 5. Only three will be discussed. The time variations of Kz in the depth interval between 33 and 38 feet serve to demonstrate the general
pattern of increasing during Fall and Winter and decreasing in Spring and Summer. This pattern is evident to a great extent in the regions above the halocline. The vertical variations of $K_z$ are quite different for the various seasons but the important observation is that starting very rapidly in the Fall and continuing through the Winter, large portions of the vertical water column obtain the same degree of turbulence. This indicates that essentially the entire hyperlimnion will become thoroughly mixed. This is the same conclusion one might have obtained from the salinity profile of Figure 4.

Another important feature of Figure 5 is that during the Spring and Summer the lower region of the hyperlimnion lag behind the upper regions in the decrease of turbulence. This is because warming of the surface layers which decreases the turbulence has thereby decreased the rate at which heat can be transported to an underlying water mass.

The third point to be discussed is the almost complete isolation of the hypolimnion from the thermal variations produced by heat flux on the quarry’s upper surface. This isolation is due to the large positive density gradient resulting from the thermal and salinity gradients. The buoyancy effect is sufficient to damp out turbulence thereby preventing thermal exchange between the hyperlimnion and hypolimnion. This depression of turbulence is sufficient to allow the hypolimnion to be anoxic. (7)

In summary the energy relationship of Diagram 2 is substantiated by the temporal and spatial variations of turbulence as demonstrated in Figure 5.

Despite the limitations imposed on this report by the nature of the data, it is clearly evident that within the limits of Millstone Quarry the vertical kinematic eddy diffusivity varies over three orders of magnitude. This range is in excess of that normally observed in the oceans and on the lower end is only six times that attributed to molecular diffusion. (12)

It would therefore appear that one might within the limits of Millstone Quarry, obtain detailed information on the properties of thermal mixing provided one was induced to make a closer observation.

LLOYD C. HUFF
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Fig. 2 Isotherms for Millstone Quarry

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Fig. 3 Sigma T for sea water as a function of temperature and salinity. — density maximum; —— freezing point.

Fig. 4 Salinity Profile Millstone Quarry
Fig. 5 - Temporal and Spatial Distribution of $K_z$ (cm$^2$/sec)

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