DOPPLER EFFECT ON BACK-SCATTERED FAR-FIELD INTENSITY FOR COHERENT LIGHT ON A ROTATING CYLINDER.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The average intensity and contrast of the back-scattered light from a distant, coherently illuminated, rotating cylinder is examined from a theoretical viewpoint. Although the inclusion of the Doppler effect drastically affects the speckle pattern, it does not significantly alter its statistics for realistic values of target size and angular velocity. Therefore, measurements of average intensity and contrast are practically unaffected.
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I. INTRODUCTION

When a distant cylindrical target is illuminated with coherent light, the back-scattered field is a complex speckle pattern which depends on the reflectivity and roughness of the target. The dependence of contrast on the roughness has been examined by Smith\(^1\). The characteristics of high spatial frequency speckle which sweeps across the detector as a rough cylinder rotates have been studied by George\(^2\). Such investigations have particular relevance to the laser radar applications.

Although the frequency spectrum and speckle configuration at any one instant are very much affected by the Doppler effect, the statistical averages of intensity and contrast are not significantly affected for realistic values of angular velocity (a few radians per second), size (1 m or so), and distance (a few hundred km). The Rayleigh-Sommerfeld diffraction formulation\(^3\) is employed in this work.

II. THEORETICAL FOUNDATION

The electric field at the surface of the cylinder is given by

\[
a_2(\ell, \theta, \theta' - \alpha) \approx a(\ell, \theta, \theta' - \alpha) \exp(j\phi(\theta, \theta' - \alpha, \ell)) \exp(jk\Delta r),
\]

where

\[
\Delta r = \rho - \rho \cos \theta,
\]

and

\[
a(\ell, \theta, \theta' - \alpha) = a_0 \bar{p}_r(\ell, \theta, \theta' - \alpha),
\]

---


in which $a_\circ$ is a scalar representing the incident electric field amplitude and $p_r(\ell, \theta, \theta-\alpha)$ is the position-dependent surface reflectivity (see Figure 1). The symbol $\phi$ represents a phase change of light upon reflection due to roughness at length position $\ell$ and angular position $\theta$ for the surface orientation $\theta-\alpha$. The angle $\alpha$ is, of course, the reference angle with respect to $\theta = 0$ for the surface structure. For small values of $\theta$, $\phi = -2kh$ where $h$ is the deviation of the surface from strictly cylindrical. For larger values of $\theta$, $\phi$ depends on $\theta$ somewhat (see Appendix). The reflectivity $p_r$ is assumed to vary slowly compared to $\phi$ so that the two values are essentially independent.

\[ dA(\ell, \theta-\alpha) = \frac{1}{j\lambda} a_2(\ell, \theta-\alpha) \frac{1}{r} \exp(2\pi r \cos\theta) \, ds \]  

Figure 1. Geometry of illuminated cylinder.

The Rayleigh-Sommerfeld diffraction theory gives the far-field backscatter amplitude for a surface element $ds$ as

\[ dA = \frac{1}{j\lambda} a \exp(j\phi) \exp(2\pi r \cos\theta) \, ds \]  

where

\[ k' = k (1 - \nu_\ell/c) = (1 - \rho \omega \sin\theta/c) \]  

is the Doppler-modified wave vector. The symbols $\omega$ and $c$ represent the angular velocity and speed of light, respectively. Combining Equations (1) and (4) gives

\[ dA = \frac{1}{j\lambda} a \exp(j\phi) \exp(2\pi r \cos\theta) \, ds \]
Since \( r = R + \Delta r \) (\( \Delta r \ll R \)) and, for reasonable values of \( \omega \), \( k' \Delta r \) may be replaced by \( k \Delta r \), then

\[
dA = \frac{1}{j\lambda R} \exp(j\phi) \exp(j2k\Delta r + k'R) \cos \theta \, ds , \tag{7}
\]

or

\[
dA = \frac{1}{j\lambda R} \exp(jkR) \exp(jk\left[2\rho(1-\cos \theta) + \left(\frac{k'}{k} - 1\right) R\right]) \cos \theta \, ds . \tag{8}
\]

Defining the shape factor

\[
F(\theta, \gamma) = \exp(jk[2\rho(1-\cos \theta) - \gamma] \cos \theta) , \tag{9}
\]

where

\[
\gamma = R(1 - k'/k) = \rho \omega R \sin \theta / c , \tag{10}
\]

the back-scattered field amplitude at the detector is

\[
A(\alpha) = \frac{\rho}{j\lambda R} \exp(jkR) \int_{0}^{\pi} a \exp(j\phi) F(\theta, \gamma) \, d\theta \, dl . \tag{11}
\]

The real part of the shape factor is symmetrical about zero for \( \gamma = 0 \) and is near unity for small \( \theta \), but as \( \theta \) increases, it oscillates with decreasing period. For \( \gamma = 0 \), the imaginary part is asymmetric and is zero at \( \theta = 0 \). As \( \theta \) increases, it also oscillates with decreasing period. To be rigorous, it should be mentioned that \( F \) does not contain all the effect of shape. The roughness-phase \( \phi \) also depends to some extent on shape (through \( \cos \theta \)) and not surface position \( \theta - \alpha \) alone. However, \( F(\theta, \gamma) \) is the factor that depends on the Doppler effect.

Let the condition that \( \gamma / \sin \theta \equiv \rho \omega R / c \ll \rho \) (or \( \omega R / c \ll 1 \)) be assumed. Therefore part of the exponent in Equation (9) may be written

\[
-2 \rho \cos \theta - \gamma = -2 \rho \left( \cos \theta + \gamma / (2\rho) \right) .
\]

Since \( \gamma / 2\rho = \omega R \sin \theta / (2c) \) [see Equation (10)], then

\[
-2 \rho \cos \theta - \gamma = -2 \rho \left[ \cos \theta + \omega R \sin \theta / (2c) \right]
\]

or, since \( \omega R / (2c) \ll 1 \), and thus \( \cos[\omega R / (2c)] \approx 1 \) and \( \sin[\omega R / (2c)] = \omega R / (2c) \),
Use of trigonometric equations further allows one to write

\[- 2 \rho \cos \theta - \gamma \approx - 2 \rho \left[ \cos \left( \frac{\omega R}{2c} \right) \cos \theta + \sin \left( \frac{\omega R}{2c} \right) \sin \theta \right].\]

Therefore Equation (9) becomes

\[F(\theta, \gamma) \approx \exp j2\rho \left[ 1 - \cos(\theta - \omega R/2c) \right] \cos \theta.\]

Since \(\omega R/2c \ll 1\), the \(\cos \theta\) factor at the end may be replaced by \(\cos(\theta - \omega R/2c)\) for values of \(\theta\) not near \(\pi/2\). Hence

\[F(\theta, \gamma) \approx F(\theta - \omega R/2c, 0).\]

Thus

\[A(\theta) \approx \frac{\rho}{j\lambda R} \exp jkR \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp j \Phi F(\theta - \omega R/2c, 0) \, d\theta \, dl.\]

### III. CONCLUSION

The Doppler effect has merely shifted the center of \(F\) by a small amount for \(\omega R/2c \ll 1\). This will have a pronounced effect on any measurement of \(A(\theta)\), but not its statistics. This is because the shape factor has displaced only slightly with respect to the average value of \(\theta\) over the surface.

For an example, let \(\omega = 2\pi \text{ rad s}^{-1}\), \(R = 500 \text{ km}\), \(\rho = 1 \text{ m}\). Thus \(\omega R/2c \approx 5 \times 10^{-3} \text{ rad}\), which certainly satisfies the condition \(\omega R/c \ll 1\).

Since for \(\omega R/c \ll 1\), the statistics of the backscatter is not much changed, average intensity \(\langle I \rangle = \langle A^2 \rangle\) and contrast \(\langle I^2 \rangle^{1/2}/\langle I \rangle\) are practically unchanged. This does not mean, however, that the frequency spectrum spread of the backscatter is insignificant.
Appendix. DEPENDENCE OF PHASE $\phi$ ON $\theta$ AND $\theta - \alpha$

Assume the second derivative $(1/\rho^2) d^2h/d\theta^2$ is very small and the fractional change of $h$ over the distance $\rho \Delta \theta$ is small. Also assume that the reflectivity $r_\rho(l, \theta - \alpha)$ varies slowly over distance $\rho \Delta \theta$, so that

$$a(l, \theta, \theta - \alpha) = a(l, \theta - \Delta \theta, \theta - \Delta \theta - \alpha).$$

Examination of Figure A-1 allows one to write

$$\frac{\phi}{k} \approx - [H \cos(2\theta - 2\Delta) + H], \quad \text{(A-1)}$$

where

$$H \approx \frac{h(l, \theta - \alpha)}{\cos(\theta - 2\Delta)}. \quad \text{(A-2)}$$

Since

$$\Delta(l, \theta - \alpha) \approx (1/\rho) \frac{dh}{d\theta} \equiv D(l, \theta - \alpha), \quad \text{(A-3)}$$

then

$$\frac{\phi}{k} \approx - h(l, \theta - \alpha) \frac{2 \cos^2(\theta - \Delta)}{\cos(\theta - 2\Delta)}. \quad \text{(A-4)}$$

For small $\Delta$,

$$\frac{\phi}{k} \approx - 2 h(l, \theta - \alpha) \cos \theta. \quad \text{(A-5)}$$

Because $\phi$ depends not only on $\theta$, but on $h$ and $D$ which, in turn, depend on $l, \theta - \alpha$, one can write

$$\phi \approx \phi(l, \theta, \theta - \alpha).$$
Figure A-1. Sketch of roughness detail.
The perforated line is the constructed surface from which the actual rough surface deviates.
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