A Theoretical Investigation of the Rectangular Microstrip Antenna Element

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A theoretical investigation of the rectangular microstrip antenna element

A theoretical treatment of the rectangular microstrip radiating element has been performed. The element has been modeled as a line resonator with radiation taking place at the open-circuited ends. With this simplified model, the input impedance and the far-fields have been calculated for different resonance modes. The interaction between the radiating ends will affect the input impedance and this has been considered by defining a mutual conductance. Also, a mutual conductance between microstrip elements has been expressed.
20. Abstract (Continued)

in far-field quantities and plotted as a function of spacing along the E- and H-planes. The directivity of an isolated element has been calculated as the directivity of one radiating end times the contribution due to the array factor.
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A Theoretical Investigation of the
Rectangular Microstrip Antenna Element

1. INTRODUCTION

The microstrip antenna consists of a radiating structure spaced a small frac-
tion of a wavelength above a ground plane. Antennas of this type have found appli-
cations where cost, weight, and ruggedness are important factors. Most of the
work reported so far has been experimental. However, the circular disc micro-
strip element has theoretically been treated as a cavity\(^1\) while the rectangular
patch has been modeled as a pair of slots separated by a transmission line.\(^2,3\)
A different approach has been presented\(^4\) where the radiating structure is modeled

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as a fine grid of wire segments. Some basic design formulas for microstrip patch antennas have also been reported.\(^5\), \(^6\)

In this paper, the rectangular microstrip radiating element is theoretically investigated. The input impedance and the resonant length are calculated by considering the element as a line resonator with the open-circuited terminations modeled as an RC network. From a radiation point of view the element is treated as two narrow slots, one at each end of the line resonator. The interaction between the two slots is considered by defining a mutual conductance. From the far-fields, the directivity of both a slot and a patch are calculated. The mutual conductance between patches is expressed in terms of far-field components alone.

2. LINE RESONATOR

A rectangular microstrip radiating element is schematically shown in Figure 1. The element is fed with a transmission line in the plane of the patch or from the back through the ground plane to give a field distribution which is uniform along the width. This is achieved either by making the width of the element shorter than a half dielectric wavelength or by using many feed points separated by a wavelength. Therefore this structure can be treated as a transmission line which is open-circuited at both ends and supports quasi-TEM modes. The resonance frequencies must, to a first approximation, be a multiple of the half dielectric wavelength:

\[
f_n = \frac{n \cdot c_0}{2 \cdot L \cdot \sqrt{\varepsilon_{\text{eff}}}}
\]

where \(c_0\) is the velocity of light and \(\varepsilon_{\text{eff}}\) is the effective dielectric constant of a microstrip line of width \(W\) and length \(L\).

At the lowest resonance frequency, the fields at the ends are reversed as illustrated in Figure 1. The horizontal components of the fringing fields at either end are in phase and give a maximum radiated field normal to the patch. The vertical components and the fringing fields along the sides don't give any contribution at broadside but will have a minor influence on the far-fields for angles off boresight.


The fringing fields of the resonator can be viewed by means of a liquid crystal detector. By placing the detector above the patch, the fields in the plane of the detector can be visually observed. Three different detector displays of the resonance mode structures on same microstrip element are shown in Figure 2. The radiation takes place at the ends of the line resonator and the microstrip element, therefore, essentially behaves as two slots. At the higher order resonance modes, standing wave maxima are observed along the patch separated by a half dielectric wavelength.

The fields at the ends of the line resonator can be represented by an equivalent uniform magnetic current along the Z-axis. This approximation is reasonable as long as the ground plane spacing is very small; that is, \( k_0 h \ll 1 \) where \( k_0 \) is the propagation constant. The far-fields of a uniformly illuminated narrow slot expressed in standard spherical coordinates are:

*The microwave liquid crystal detector was supplied by James Sethares.
Figure 2. Liquid Crystal Displays of a Rectangular Microstrip Element Excited in Different Modes. (a) Microstrip element, (b) first mode, 3.10 GHz; (c) second mode, 6.15 GHz; (d) third mode, 9.15 GHz
\[
\begin{align*}
\mathbf{E}_\Phi &= -j \cdot \frac{V_0}{\pi} \cdot e^{-jk_r r} \cdot \sin \left(\frac{\pi W}{\lambda_0} \cdot \cos \theta\right) \cdot \sin \theta \\
\mathbf{H}_\theta &= -\sqrt{\frac{\varepsilon}{\mu}} \cdot \mathbf{E}_\Phi
\end{align*}
\]

where \(V_0\) is the voltage across the slot and \(\lambda_0\) is the wavelength in free space. The far-fields are linearly polarized and independent of the ground plane spacing \(h\), as long as this is a small fraction of a wavelength.

3. NETWORK MODEL.

The radiation at the open-circuited ends of the line resonator can be represented by a radiation conductance. This is defined as a conductance which will dissipate the same power as that radiated by the slot:

\[
G = \frac{1}{\pi} \sqrt{\frac{\varepsilon}{\mu}} \int_0^\pi \frac{\sin^2 \left(\frac{\pi W}{\lambda_0} \cdot \cos \theta\right)}{\cos^2 \theta} \cdot \sin^3 \theta \cdot d\theta.
\]

This integral has been solved numerically and a plot of the radiation conductance as a function of the width can be found in Derneryd.\(^3\)

The effect of the fringing fields at the ends of the line resonator can be represented either as a shunt capacitance or as a lengthening of the line. In both cases, the end effect can be modeled as an equivalent susceptance. The microstrip element can, therefore, be modeled as a network with two admittances separated by a transmission line of width \(W\) and length \(L\) as shown in Figure 3.

![Figure 3. Network Model of the Microstrip Radiating Element](image-url)
The resonant length of the element is defined as the length at which the input admittance is pure real. The input admittance is found by transforming the slot admittances to the feed point and adding them in parallel. The resonant length is given by:

$$\tan \beta L = \frac{2 \cdot B \cdot Y_c}{G^2 + B^2 - Y_c^2}$$

(4)

where $\beta$ is the propagation constant in the dielectric. The resonant length depends on the width of the element, the ground plane spacing, and the dielectric constant but it is independent of feed point location as long as quasi-TEM modes are excited. An effective dielectric constant has to be defined taking into account the fringing fields along the sides before the propagation constant can be determined.

In most practical cases, the characteristic admittance of the transmission line, $Y_c$, is much greater than both the radiation conductance and the equivalent susceptance. Therefore, by solving Eq. (4) the resonant lengths can be expressed as:

$$L = n \cdot \frac{\lambda_e}{2} - 2 \cdot \Delta L$$

(5)

That is, the resonant length of an element is a multiple of a half dielectric wavelength corrected by a term $\Delta L$, which represents the extension of the terminal plane from the physical open end due to the end capacitance $C$. The line extension is expressed as:

$$\Delta L = \frac{v \cdot C}{Y_c}$$

(6)

where $v$ is the velocity in the dielectric. The resonance frequency of the microstrip element is thus decreased a small fraction if the effective length is used in the calculations.

4. RADIATION PATTERN

The radiation pattern of a rectangular microstrip element modeled as two slots separated by a distance $L$ is approximately found by pattern multiplication. The array factor for the odd mode excitations is
\[ A F_0 = 2 \cdot \cos \left( \frac{nL}{\lambda_0} \cdot \sin \theta \cdot \cos \phi \right) . \] (7)

For the even modes, the horizontal components of the fringing fields are out-of-phase and, therefore, the array factor becomes

\[ A F_e = 2 \cdot \sin \left( \frac{nL}{\lambda_0} \cdot \sin \theta \cdot \cos \phi \right) . \] (8)

The radiation pattern of a microstrip element along a vertical cut in the E-plane is determined by the array factor alone since the far-fields of a slot are constant in this plane. Along the H-plane, the array factor for odd mode excitation is constant while it is zero for the even modes. Therefore, the radiation patterns of the odd modes are completely determined by the element pattern of the slot.

Radiation patterns of a rectangular microstrip element with the dimensions 10 mm \( \times \) 30.5 mm have been recorded for the first three resonance modes. The vertical cuts along the E-plane are shown in Figures 4 through 6. As comparison, the theoretical radiation patterns are also included in the figures as dotted lines. The minor fluctuations in the recorded patterns are due to the finite ground plane (0.61 m \( \times \) 0.61 m). A vertical cut of the radiation pattern along the H-plane of the lowest mode is shown in Figure 7. The two slot model of the microstrip radiating element gives very good agreement between theory and practice except in the region close to endfire.
Figure 5. Radiation Pattern Along the E-plane of the Second Mode, 6.15 GHz

Figure 6. Radiation Pattern Along the E-plane of the Third Mode, 9.15 GHz
5. INPUT IMPEDANCE

The input impedance for different feed point locations can be calculated with the network model. At resonance, the input impedance at an arbitrary feed point, a distance $x$ from one end of the microstrip element is pure real. By transforming the slot admittances to the common point and adding them together, the input impedance at resonance is found to be:

$$Z_{in}(x) = \frac{1}{2} \cdot G \left( \cos^2 \beta x + \frac{G^2 + \frac{B^2}{Y_c} \cdot \sin^2 \beta x - \frac{B}{Y_c} \cdot \sin 2 \beta x}{Y_c} \right)$$

(9)

Usually $\frac{G}{Y_c} \ll 1$ and $\frac{B}{Y_c} \ll 1$ and therefore (9) simplifies to:

$$Z_{in}(x) = \frac{1}{2} \cdot G \cdot \cos^2 \beta x$$

(10)

except close to the center of the element.

The mutual effect between the two slots is not included in the above formulas. The behavior of two coupled antennas can be described by network concepts. The relation between the voltages and the currents at the input terminals is then represented by an admittance matrix. The corresponding network for two identical
radiators is drawn in Figure 8. At resonance, the voltages at the ends of the microstrip element are equal in magnitude and the phase difference is a multiple of $180^\circ$. Taking into account the mutual conductance $G_{12}$, the expression for the input impedance has to be modified to:

$$Z_{in}(\chi) = \frac{1}{2(G_{11} \pm G_{12})} \cdot \cos^2 \beta \chi$$  \hspace{1cm} (11)

where $G_{11}$ is the radiation conductance of an isolated slot. The plus sign corresponds to the odd modes while the minus sign refers to the even modes. The denominator is simply the radiation conductance of a patch at resonance.

![Figure 8. Network Representation of a Pair of Coupled Antennas](image)

6. MUTUAL CONDUCTANCE

A mutual conductance between two antennas can be derived by considering the total radiated power in a manner similar to that used to define the radiation conductance for an isolated antenna. If the antennas are excited by equal voltages, the mutual conductance expressed in far-field components is:

$$G_{12} = \frac{1}{|V_o|^2} \cdot \text{Re} \int \overline{E_1 \times H_2^*} \, ds$$  \hspace{1cm} (12)

where $\overline{ds}$ is a vector normal to a sphere of large radius and the asterisk denotes the complex conjugate.
6.1 Slots

Consider two slots of length $W$ placed in the $X$-$Z$ plane and aligned in the $Z$-direction. The mutual conductance when radiating into half space is:

$$
G_{12} = \frac{1}{\pi} \sqrt{\frac{E}{\mu}} \int_{0}^{\pi} \frac{\sin^{2} \left( \frac{\pi W}{\lambda_{0}} \cos \theta \right)}{\cos^{2} \theta} \cdot \sin^{3} \theta \cdot J_{0} \left( \frac{x}{\lambda_{0}} \cdot 2\pi \cdot \sin \theta \right) \cdot \cos \left( \frac{Z}{\lambda_{0}} \cdot 2\pi \cdot \cos \theta \right) \, d\theta
$$

where $J_{0}$ is the zero-order Bessel function of the first kind. At zero spacing, Eq. (13) gives the radiation conductance (3) of an isolated slot.

Values of this integral have been calculated for two special cases. The normalized mutual conductance between two narrow slots along the $E$-plane is shown in Figure 9 as a function of spacing. The longer the slot, the stronger is the coupling. The corresponding curves for coupling along the $H$-plane are plotted in Figure 10. As expected, the coupling decreases much more rapidly.

6.2 Patches

The mutual conductance between two rectangular microstrip elements is found from Eq. (2) and (12) together with (7) or (8). Along the $E$-plane, the mutual conductance between patches with length $L$ and width $W$ excited in the odd modes is:

$$
G_{12} = \frac{1}{\pi} \sqrt{\frac{E}{\mu}} \int_{0}^{\pi} \frac{\sin^{2} \left( \frac{\pi W}{\lambda_{0}} \cos \theta \right)}{\cos^{2} \theta} \cdot \sin^{3} \theta \cdot \left\{ 2 \cdot J_{0} \left( \frac{x}{\lambda_{0}} \cdot 2\pi \cdot \sin \theta \right) + J_{0} \left( \frac{x + L}{\lambda_{0}} \cdot 2\pi \cdot \sin \theta \right) + J_{0} \left( \frac{x - L}{\lambda_{0}} \cdot 2\pi \cdot \sin \theta \right) \right\} \, d\theta
$$

The first term in Eq. (14) is twice the mutual conductance between two slots along the $E$-plane spaced by a distance $x$. The second and the third term are the coupling between two slots separated by distances $x + L$ and $x - L$ along the $E$-plane. The approximation used to derive (14) is that the far-fields of a patch are determined by the element factor times the array factor.

The integrals in (14) have been computed numerically and the results are shown in Figure 11. The mutual coupling between two patches along the $H$-plane for odd modes is:
Figure 9. Normalized Mutual Conductance Between Two Narrow Slots Along the E-plane

Figure 10. Normalized Mutual Conductance Between Two Narrow Slots Along the H-plane
Figure 11. Normalized Mutual Conductance Between Two Rectangular Microstrip Elements Along the E-plane

Figure 12. Normalized Mutual Conductance Between Two Rectangular Microstrip Elements Along the H-plane
The first term is identified as twice the mutual coupling between two slots spaced a distance $Z$ along the H-plane. The second term is twice the coupling between two slots separated a distance $L$ along the E-plane and a distance $Z$ along the H-plane. The normalized mutual coupling has been computed and the result is drawn in Figure 12. As with the slots, the coupling decays faster along the H-plane than along the E-plane. An interesting point is that the coupling is stronger along the E-plane for wider elements while the opposite is true along the H-plane. Patches excited in the even mode resonances can be analyzed in a similar way.

7. DIRECTIVITY

The directivity of an antenna is defined as the ratio between the maximum power density and the average radiated power. The radiation conductance is also expressed in far-field components. Therefore, there exists a relation between the directivity and the radiation conductance of an antenna.

7.1 Slots

The directivity of an isolated slot can be expressed as:

$$D_o = \frac{1}{30 \cdot G} \left( \frac{w}{\lambda_o} \right)^2.$$  \hspace{1cm} (16)

The radiation conductance for short slots, $w/\lambda_o \ll 1$, is proportional to the square of length. The directivity of a short slot therefore approaches a constant value 3 which is equivalent to 4.8 dB. The radiation conductance for a long slot is proportional to the length of the slot and Eq. (16) simplifies to:

$$D_o = 4 \cdot \frac{w}{\lambda_o} \quad \frac{w}{\lambda_o} \gg 1.$$  \hspace{1cm} (17)
The directivity of an isolated slot is plotted in Figure 13 as a function of the length. Subject to the restrictions on h mentioned earlier, the directivity is independent of the ground plane spacing.

Figure 13. Directivity of a Narrow Slot With Uniform Field Distribution

7.2 Patches

The microstrip element is considered as an array of two slots spaced a distance \( L \) along the \( E \)-plane. The directivity of a patch can thus be expressed as the directivity of a slot times the contribution due to the array factor. For odd mode excitations, the latter contribution at broadside is:

\[
D_{AF} = \frac{2}{1 + g_{12}}
\]  
(18)

where \( g_{12} \) is the normalized mutual conductance, as shown in Figure 9, between two slots along the \( E \)-plane.
The relative directivity of a patch is plotted in Figure 14 as a function of the length. As the length of the element increases, the directivity increases until a grating lobe appears in visible space which is the case for higher order mode resonances. A relative directivity of 3 dB is achieved when the mutual coupling between the slots is zero. The even modes always have a null in the radiation pattern at broadside.

Figure 14. Relative Directivity of a Rectangular Microstrip Element at Resonance

8. CONCLUSION

The rectangular microstrip element is treated as a line resonator. The radiation takes place predominantly from the fringing fields at the open-circuited ends. This has been verified by liquid crystal displays. These fields also represent an extension of the element, making the physical resonant length just shorter than a multiple of a half dielectric wavelength.
Calculations of the input impedance show that by varying the feed point along the transmission line, the element can be matched to all practical impedance levels. From a radiation point of view, the element is considered as a pair of slots. The mutual effect between the radiating ends generally decreases the input impedance.

The mutual coupling between elements can be seen as composed of coupling between the ends of one element and both ends of the other element. Calculations show that, as expected, the mutual effect is stronger along the E-plane than along the H-plane.
References

### Metric System

#### BASE UNITS:
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<th>Unit</th>
<th>SI Symbol</th>
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#### SUPPLEMENTARY UNITS:
| plane angle   | steradian    | sr        |         |
| solid angle   | radian       | rad       |         |

#### DERIVED UNITS:
| Acceleration  | metre per second squared | m/s²       |         |
| activity (of a radioactive source) | disintegration per second | (disintegration)/s |         |
| angular acceleration | radian per second squared | m/s²       |         |
| angular velocity | radian per second         | m/s        |         |
| density       | square metre            | m³         |         |
| electric capacitance | farad | F          |         |
| electrical conductance | siemens | S         |         |
| electric field strength | volt per metre | V/m        |         |
| electric inductance | weber | Wb         |         |
| electric potential difference | volt | V          |         |
| electric resistance | ohm | Ω          |         |
| electromagnetic force | joule | J          |         |
| energy        | joule per second         | J/s        |         |
| entropy       | joule per kelvin         | J/K        |         |
| force         | newton                   | N          |         |
| frequency     | hertz                    | Hz         |         |
| illuminance   | lux                      | lx         |         |
| luminous flux | lumen                   | lm         |         |
| magnetic field strength | ampere per metre | A/m        |         |
| magnetic flux | weber                   | Wb         |         |
| magnetic flux density | tesla | T          |         |
| magnetomotive force | ampere | A         |         |
| power         | watt                     | W          |         |
| pressure      | pascal                   | Pa         |         |
| quantity of electricity | coulomb | C         |         |
| quantity of heat | joule | J          |         |
| radiant intensity | watt per steradian | W/sr      |         |
| specific heat | joule per kilogram-kelvin | J/(kg·K) |         |
| stress        | pascal                   | Pa         |         |
| thermal conductivity | watt per metre-kelvin | W/m·K     |         |
| velocity      | metre per second         | m/s        |         |
| viscosity, dynamic | square metre per second | m²/s    |         |
| viscosity, kinematic | volt | V         |         |
| volume        | cubic metre             | m³         |         |
| wavenumber    | reciprocal metre        | 1/m        |         |
| work          | joule                   | J          |         |

#### SI PREFIXES:

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* To be avoided where possible.
MISSION
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