### Report Documentation Page

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<td>13. SUMMARY OF RESULTS</td>
<td>The following theories were systematically developed: (1) Harmonic classification theory of Riemannian manifolds, (2) Quasiharmonic classification theory of Riemannian manifolds, (3) Theory of bounded biharmonic functions on Riemannian manifolds, (4) Dirichlet finite biharmonic functions, (5) Bounded Dirichlet finite biharmonic functions, (6) Riesz representation of biharmonic functions, (7) Green's functions of simply supported bodies, (8) Green's functions of clamped bodies. These theories were published in the 83 papers listed in Sections V-V.</td>
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FINAL REPORT


Title of project: Biharmonic and potential functions and their applications.

Grant numbers: DA-ARO-D-31-124-71-G20,
    DA-ARO-D-31-124-71-G181,

Name of institution: University of California, Los Angeles.

Author of report: L. Sario.

This report is divided into six sections:

I. Main achievements.
II. Papers published.
III. Papers accepted.
IV. Papers submitted.
V. Research monograph published.
VI. Ph.D. training.
I. Main achievements

A. Research. The following theories were systematically developed:

(1) Harmonic classification theory of Riemannian manifolds.
(2) Quasiharmonic classification theory of Riemannian manifolds.
(3) Theory of bounded biharmonic functions on Riemannian manifolds.
(4) Dirichlet finite biharmonic functions.
(5) Bounded Dirichlet finite biharmonic functions.
(6) Riesz representation of biharmonic functions.
(7) Green's functions of simply supported bodies.
(8) Green's functions of clamped bodies.

These theories were published in the 83 papers listed in Sections II - IV, and the research monograph described in Section V. (The totals of our publications under all our Army grants is 153 papers and 6 research monographs.)

B. Training. The doctoral dissertations of 7 students, listed in Section VI, were directed. (The total number of doctoral dissertations directed under all our Army grants is 35.)

II. Papers published


(14) "A remark on classification of Riemann surfaces with respect to $\Delta u = Pu$" by M. Nakai, Bull. Amer. Math. Soc. 77 (1971), 527-530.


(22) "Royden's algebra on Riemannian spaces" by J. Chang and L. Sario, Math. Scand. 28 (1971), 139-158.


(25) "The equation \( \Delta u = Pu \) on \( E^m \) with almost rotation free \( P \geq 0 \)" by M. Nakai, Tôhoku Math. J. 23 (1971), 413-431.


(33) "The equation \( \Delta u = Pu \) on the unit disk with almost rotation free \( P \geq 0 \)" by M. Nakai, J. Diff. Eqs. 11 (1972), 307-320.


(40) "Positiveness of the reproducing kernel in the space PD(R)" by I. Singer, Nagoya Math. J. 48 (1972), 67-72.

(41) "Generators of the space of bounded biharmonic functions" by L. Sario and C. Wang, Math. Z. 127 (1972), 273-280.


(49) "Existence of Dirichlet finite biharmonic functions on the Poincaré 3-ball" by L. Sario and C. Wang, Pacific J. Math. 48 (1973), 267-274.

(50) "Quasiharmonic functions on the Poincaré N-ball" by L. Sario and C. Wang, Rend. Mat. 4 (1973), 1-14.


(60) "Counterexamples in the biharmonic classifications of Riemannian manifolds" by L. Sario and C. Wang, Pacific J. Math. 50 (1974), 159-162.


III. Papers accepted

(73) "A nonexistence test for biharmonic Green's functions of clamped bodies" by M. Nakai and L. Sario, Math. Scand.

(74) "Existence of negative quasiharmonic functions" by M. Nakai and L. Sario, Jubilee volume dedicated to the 75th birthday of Academician I. Vekua.


(76) "A strict inclusion related to biharmonic Green's functions of clamped and simply supported bodies" by M. Nakai and L. Sario, Ann. Acad. Sci. Fenn.

(77) "Existence relations between harmonic and biharmonic Green's functions" by M. Nakai and L. Sario, Rend. Ist. Mat. Univ. Trieste.


(79) "Duffin's function and Hadamard's conjecture" by M. Nakai and L. Sario, Pacific J. Math.


(81) "Harmonic and polyharmonic degeneracy" by L. Chung, L. Sario, and C. Wang, Math. Scand.

IV. Papers submitted

(82) "Green's functions of the clamped punctured disk" by M. Nakai and L. Sario, J. Austral. Math. Soc.

(83) "One point clamping and supporting" by M. Nakai and L. Sario, Rend. Mat.
V. Research monograph published


VI. Ph.D. training

The doctoral dissertations of the following students were directed:

5. N. Mirsky, November 1972.