AN INVESTIGATION OF THE EFFECTS OF DENSITY, SIZE, AND SHAPE UPON THE AIR CLASSIFICATION OF MUNICIPAL TYPE SOLID WASTE

JUNE 1977

FINAL REPORT

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CIVIL AND ENVIRONMENTAL ENGINEERING DEVELOPMENT OFFICE

(AIR FORCE SYSTEMS COMMAND)
TYNDALL AIR FORCE BASE
FLORIDA 32403
An Investigation of the Effects of Density, Size and Shape Upon the Air Classification of Municipal Type Solid Waste

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Air Classification
Solid Waste
Resource Recovery
Solid Waste Management

The objective of this research was to determine the feasibility of separating municipal solid waste type materials into more than two fractions by passing the material through a vertical air classifier. This feasibility was demonstrated by suspending specimens of varying densities, sizes, and shapes in a vertical air classifier and noting the terminal velocities of the materials. Since most shredded solid waste approximates flat plates of varying sizes and shapes, flat plates of six different materials in aspect ratios (length over width) from one to four and in four different sizes from 0.0625 to 1.000...
square inches (0.4032 to 6.4516 square centimeters) were evaluated to determine terminal velocity. The materials studied included steel, aluminum, balsa wood, cardboard, paper, cloth, and glass. The theoretical development, the experimental results, and the analysis of variance statistical tests indicate that municipal solid waste type material does exhibit a difference in terminal velocity as a function mostly of density and only slightly of the size and shape parameters tested. This indicates that municipal solid waste may be separable into several fractions provided the proper air classification equipment is used.
PREFACE

This report was prepared through the Air Force Institute of Technology, Wright-Patterson AFB, Ohio. It was accomplished under Job Order 21036W82. Lt Col Patrick J Sweeney was principal investigator. Captain Robert F Olfenbuttel was project monitor.

This report has been reviewed by the Information Officer and is releasable to the National Technical Information Service (NTIS). At NTIS it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

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CHAPTER I

INTRODUCTION TO THE PROBLEM

Statement of the Problem

Americans throw away in a single year: 80 billion metal cans, 34 billion glass bottles, over 50 million tons of paper, 7 million old cars and trucks, nearly 8 million television sets, and more than 4 million tons of plastic materials (1:40; 2:1).

This amounts to over 135 million tons of municipal solid waste (MSW) per year and is expected to triple by the year 2000 (2:1-1). This growing problem is complicated by the dwindling number of landfill disposal sites available, the energy "crisis," the depletion of other natural resources, and the public's concern about ecology and waste (3:1).

Recent legislation has stimulated an interest in resource recycling, recovery, and reuse. Of particular interest to engineers and scientists are those inexpensive schemes that not only recover valuable resources but also convert generally nonrecyclable products into a fuel supplement fraction (4:21). Because of the nonhomogeneity of MSW, separation of the individual components for reuse or
sale is a capital intensive operation (5:52-54; 6:16-19; 7:29-31). However, since the nation's annual trash pile contains, for example, about 400 million dollars worth of aluminum cans, 190 million dollars in ferrous metal (8:277-283) and 10 million BTUs per ton (MSW has about half the heat content of coal) (9:Vol.4,Sec.3.4), advancing the technology is desirable.

As stated above, one of the major difficulties that arise in many recovery systems is the separation of the components (10:1). Currently, most systems utilize some type of air separation or classification device to assist in the component separation (11; 22; 46). Generally, the MSW is passed over or into a moving air stream in such a manner that the heavy particles drop out and the lighter ones are forced into a holding area. This is defined as two fraction separation, with the light fraction normally used as a fuel supplement and the heavier fraction receiving further processing (12:59). If the additional cost of multiple processing could be eliminated by a single pass separation device, recycling systems would be more economically competitive with incineration and land fill.

This study investigates the feasibility of a multiple component separation of MSW type components in a variable cross-sectional vertical air classification device.
The Solid Waste Problem

"Everyone wants us to pick up their garbage but no one wants us to put it down [13:9]." This quote summarizes the general attitude toward solid waste held by the vast majority of American citizens. The problem of solid waste disposal has been neglected in the past but, as the magnitude of the waste increases, this issue can no longer be put aside (14:4-5). Our dwindling natural resources, our environmental concerns, and our desire to improve the quality of life should provide impetus for improved and economically-feasible, alternate solutions to the solid-waste disposal problem.

Solid waste has been increasing nationally due both to an increasing population and an increasing per capita generation of waste (15:1,4). In 1920, approximately 2.8 pounds per person per day of solid waste were generated in the United States (16:1). Current estimates indicate a daily per capita refuse production in the U.S. to be nearly 3.5 pounds and increasing (16:10). The quantity of waste is enormous and so are the problems associated with acceptable disposal methods. The present U.S. pattern for disposal of solid waste indicates that the use of open dumps is declining while the use of sanitary landfill and resource recovery is increasing (17:423).

The increasing rate of solid waste generation is resulting in a "dwindling availability of disposal sites within economic hauling distances of major population..."
centers [18:9]." In addition, there are potential problems associated with the disposal in many sanitary landfills which could degrade the environment by improper control of leaching, etc. (19:583). Public attitudes about the environment and our depletion of natural resources help focus attention on the solid waste problems (10:1; 19:580).

J. C. Kennedy has defined resource recovery as the controlled disposal of solid wastes by the alteration of the solid waste into a reusable material which can be repossessed by the community (20:149-152). Resource recovery offers a solution to the solid waste problems of dwindling landfill sites and depleting natural resources (19; 20; 21; 22; 23). As noted in the SCS Engineers' report titled "Survey of Solid Waste Handling Unit Operations,"

The constituents of solid waste themselves have intrinsic resource value. They can be recovered for use, rather than discarded, thereby serving two desirable functions: (1) the quantity of waste requiring disposal is reduced, and (2) resources are conserved [21:136].

Public Environmental Concern

"Two years ago, no one was interested in getting energy from garbage, but now things are booming. Everyone has a system to demonstrate [24:29]." The American public is demonstrating an ever-increasing awareness and concern for the quality of the environment and the protection of our limited natural resources (25:VII-VX; 26:108-112). The United States Congress has long been aware of the public
concern about waste disposal and recently has significantly increased the amount of legislation in this area. Legislation enacted since 1886 includes:

1. Section 3 of the Harbor Act of 1886 declared it illegal to place rubbish in the New York Harbor (27).

2. The Refuse Act of 1899 prohibited the disposal of pollutants into any of the nation's navigable waters (28).

3. The Solid Waste Disposal Act of 1965 was the first significant action aimed directly at solving the solid waste management problems (29).

4. The Resource Recovery Act of 1970, Public Law 91-512, provided added incentive to resource recovering programs and subsequently provides additional assistance in the area of solid waste management (9). This legislation includes the following statement:

The Congress finds (1) that the continuing technological programs and improvement in method of manufacturing, packaging, and marketing of consumer products has resulted in an ever-mounting increase, and in the change in the characteristics of the mass of material discarded by the purchases of such products; (2) that inefficient and improper methods of disposal of solid waste result in scenic blight, create serious hazards to public health, including pollution of air and water resources, accident hazards, and increase in rodent and insect vectors of disease; (3) have an adverse effect on land values; (4) create public nuisances, and otherwise interfere with community life and development; (5) at the failure or inability to salvage and reuse such materials economically results in the unnecessary waste and depletion of our natural resources (30).

5. The Clean Air Act of 1970 provided further legislation and legal support for the establishment and
enforcement of clean air standards which curtails open pit burning and most incineration systems (17:423; 31:4.40).

Perhaps the most widely known of all the recent environmental legislation is the National Environmental Policy Act (NEPA) of 1969, PL 91-190. The NEPA established a broad national policy for the protection of the environment. Section 101(b) of the NEPA states that consistent with the national policy the United States may "enhance the quality of renewable resources and approach the maximum attainable recycling of depletable resources [32]."

Objective

The primary objectives of this study were: (1) to determine the feasibility of separating municipal solid waste type material into more than two fractions by a single pass through a variable cross-sectional area vertical air classifier, and (2) to investigate some of the underlying basic principles of air classification.

Determining the effects of the size, shape, and density parameters upon separation has not heretofore been demonstrated experimentally, but it is theorized to be significant. This information is important to those designing waste recovery systems employing shredders and air classification systems. Terminal velocity information for separation of components of MSW will also aid in the design of multiple separating air classifiers.
Research Approach

It was believed that multiple separations of MSW in a single pass is possible if a variable velocity can be maintained within the separation device. Also, it was believed that the distribution of the suspended MSW components as a function of the terminal velocity could be determined.

In order to maintain a variable and controlled velocity within the air classifier, a variable cross-sectional area device was designed and constructed. A motor and compressor combination was fitted to the top of the funnel- or V-shaped classifier to create a fluid flow. Figure 1 is a sketch of this general setup.

Inside the device the air velocity diminishes with vertical height and therefore results in a segregation of particles of different terminal velocities. Then as the air flow is reduced, the position of each category of particles in the unit is lowered until it finally drops from the bottom. Exit from the classifier is a function of the terminal or floating velocities of the different components (See Figure 2).

As the material drops, the static pressure of the throat area was observed on a manometer and recorded. Several runs were conducted for each material, and each different size and shape. The mean values of the recorded drop (terminal) velocities were then evaluated for each combination of density, size, and shape. Analysis
FIGURE 1. SKETCH OF ORIGINAL SET UP

FIGURE 2. CHANGE IN PARTICLE DISTRIBUTION POSITION AS A FUNCTION OF VELOCITY
of variance techniques were used to identify the significant factors and a regression model was constructed for use in predicting terminal velocities of all components found in MSW.

Research Hypotheses

1. Shredded municipal solid waste type material can be separated into several distinct component fractions by one pass through an air classification device with a variable cross-sectional area.

2. The terminal velocity for different MSW type materials can be accurately predicted.

3. The variability of the terminal velocity of a particle of MSW type component is reduced by increased shredding operations.

4. The significant factors affecting the separation of MSW type components are density, size, and shape.
CHAPTER II
LITERATURE REVIEW

Introduction

An in-depth review of the appropriate literature indicates that although many articles have mentioned air classification few actually deal with the theoretical reasons why it works. Apparently either the authors are unconcerned with the underlying principles or prefer not to devote the time necessary to investigate the ramifications of air classification. Several researchers have tested different types of classifiers and have provided some data, but few have correlated their results to underlying first principles. It is the intent of this section to review those articles in the literature that discuss the principles of air classification.

Included in this review are an examination of aerodynamic principles, the basic principles of air classification, descriptions of current air separation equipment, and current resource recovery systems that utilize air classifiers.
Aerodynamics

Subsonic fluid flow is usually classified as laminar, transitional, or turbulent and this classification is a function of the velocity variations within the boundary layer of the test specimen or model. In laminar flow, the velocity varies almost linearly whereas in turbulent flow this variation is exponential (33:346). The transitional flow pattern is between laminar and turbulent. The laminar flow patterns usually exist at Reynolds's Numbers below 2100 (34).

When the velocity of the flow past a body is slow enough (laminar) Stokes' Law will apply and the resistance, \( R \), of a sphere is

\[
R = 6\pi\mu rv
\]

(Ref. 35) \( \text{(1)} \)

where \( \mu \) is the fluid viscosity, \( r \) is the sphere radius, and \( v \) is the fluid velocity.

As the flow velocity increases the Stokes' equation begins to be invalid and the Newtonian equation becomes a better approximation of the resistance. The Newtonian resistance of a sphere is

\[
R = \frac{\pi}{2} \Delta' r^2 v^2
\]

(2)

where \( \Delta' \) is the specific gravity of the fluid (36:169). Unfortunately, Newton's equation must be modified by inserting a coefficient of resistance, \( Q \), in order to equate the theoretical and experimental data. Present
practice calls for the use of $C_D$, drag coefficient, in place of Newton's original $Q$.

In some cases, neither the Stokes nor the Newtonian equations can be applied, if the area of interest is in a transitional flow pattern. The Oseen formula has been widely used for these cases. Although this formula can be thoroughly supported mathematically, it does not exactly match experimental results (37). The Oseen formula is usually expressed as

$$R = \pi \mu v m (1 + \frac{3}{8} \Delta r \frac{v_m}{\mu})$$

where $v_m$ is the terminal velocity (38).

A number of other formulas have also been proposed with better approximations. Wadell has proposed the best fitting formula; unfortunately, it contains a fractional exponent and is wholly empirical.

In order to compare the resistance of solids in moving fluids, a nondimensional number, the Reynold's Number $(RN)$, is used and is defined as

$$RN = \frac{\rho V L}{\mu}$$

where $\rho$ is the density of the fluid, $V$ is the velocity, $L$ is the "descriptive" length, and $\mu$ is the fluid viscosity. Figure 3 shows the coefficient of resistance versus $RN$ data reported by Allen, et al. (35:177).
Since experiments are conducted in many different wind tunnels, a correction factor should be used in order that equivalent information be obtained at all facilities. The most widely used formula was developed by Ladenburg-Faxen and is usually employed in conjunction with the Stokes' resistance, $R$. The correction factor is

$$R = (1 + 2.1 \frac{r}{r'})$$  \hspace{1cm} (Ref. 39) \hspace{1cm} (5)

where $r$ and $r'$ are the radius of the sphere and the enclosing cylinder, respectively.

In Newtonian flow, the Monroe correction factor for wall effects is utilized. Excellent experimental results are obtained with this formula in both the transitional and turbulent regions of fluid flow. Like the Ladenburg-Faxen formula, the Monroe correction is used as
a multiplier to the Newtonian resistance equation:

\[ R = \frac{\pi}{2} r^2 v^2 \left( \frac{1}{1 - \left( \frac{r}{r'} \right)^3} \right)^2 \]  
(Ref. 40)  
(6)

Several authors have attempted to mathematically represent equal settling velocities, that is, equal terminal velocities, for different particles in the same fluid. Generally, this is referred to as a free settling ratio, \( P_f \), and is defined as

\[ P_f = \left( \frac{\Delta_1 - \Delta'}{\Delta_2 - \Delta'} \right)^{m_2} \left( \frac{m_1}{m_2} \right) \]  
(7)

where \( \Delta_1 \) and \( \Delta_2 \) are the specific gravities of the particle, the exponents, \( m \), range from 1/2 in laminar flow to unity in turbulent flow. There is only general agreement between theory dealing with spheres and experimental data dealing with different shaped particles due to the large effects of rotation of nonspherical shapes (35:187).

Many authors have attempted to quantify effects of particles moving simultaneously in a fluid. The interactions of the particles on each other creates collisions and varies the air flow over all of the particles. The Monroe and Francis corrections are considered only first approximations, and Monroe's work was the only one found that considered the turbulent regime (35:189). The work of Einstein, Guthand, Gold, Kermack, McKendrick,
and Ponder basically consider the effects of changing the overall viscosity of the fluid by adding the particles (41; 42; 43; 44). The latter group's formula approaches the experimental results the best:

\[ \nu = \frac{2}{9} (1-\gamma^{2/3})(1-\gamma)(1-2.5\gamma)g \frac{(A-A')}{\mu} \]  

where \( \gamma \) is the volumetric fraction occupied by the solid in the suspension.

Wadell has attacked the problem associated with determining terminal velocities for nonspherical bodies by introducing the parameter \( \psi \), which is defined as

\[ \psi = \frac{s}{S} \]  

(Ref. 37:34)  

where \( s \) is the surface area of a sphere which has the same volume as the particle versus the actual surface \( S \) of the particle. Figure 4 compares drag coefficients versus Reynolds' Number for nonspherical bodies.

For thin flat plates Oberbeck's work showed that at low velocities resistance is equal to \( 16\mu rv \) for plates normal to the flow and if the plate is parallel to the fluid flow resistance is equal to \( 10.67\mu rv \). Additional work by Richards indicates that at low speeds particles tend to orient parallel to flow, whereas, in turbulent high speed flow the particles tend to orient cross-wise to the flow. He also noted that an axis of symmetry passing through the center
of gravity will aid this phenomena and a lack of symmetry increases the vibration, rotation, and wobbling (46).

Only rigid shapes have been considered up to this point. Unfortunately not all solid waste is truly rigid and many components such as paper, leaves, and plastic wrap are quite flexible. In addition, several materials such as cloth are also porous. These nonrigid materials constitute a significant portion of the MSW.

These nonrigid items will require special analytical techniques in order to estimate appropriate drag coefficients. The air resistance of nonrigid bodies such as parachutes, wind socks, and flags is a function of fabric weight and weave, the aspect ratio (flags), and flutter. The total drag on these porous materials includes the skin friction and the flow separation components. Experimental
data shows that drag coefficients rarely exceed 0.30 for most cloth-like materials. Also of note is the fact that the drag increases exponentially as a function of fabric weight and linearly as a function of the aspect ratio (horizontal over vertical flag dimensions) (47:3-24).

The MSW components in an air classifier will rotate, wobble, and tumble. Mr. E. A. Smith has conducted experiments on autorotating wings that may provide some insight into the aerodynamics of air classifying MSW. His work on freely falling wings at Reynolds Numbers of about 4000 reflected average lift and drag coefficients compared to those observed in his fixed axis test. He noted drag coefficients increased nearly linearly with the logarithm of the Reynolds Number. Between Reynolds Numbers of $10^4$ and $10^5$ the maximum drag coefficient varied from about 2.3 to 0.3. In the same Reynolds Number range the average $C_D$ varied from 1.0 to 1.3. His experiments also show that the drag followed a sinusoidal curve and was maximum at approximately $10^\circ$ and $190^\circ$ ($0^\circ$ means normal to the flow and $90^\circ$ is streamlined) (48).

Gaudin and Marchildon investigated the behavior of circular cylinders moving singly through water at low (below 2400) Reynolds Numbers. They determined that some $C_D$ differences do exist for cylinders of different densities and that oscillation is a function of the fluid forces and the cylinder inertia (49).
In the high Reynolds Number region Barker and Christiansen determined a drag coefficient for several different shapes. They also noted slight differences in $C_D$ for specimens of different densities. They report that the following equation is within ±10 percent of their experimental results.

$$C_D = \frac{2V(\rho_p - \rho_f)g(d_{\text{min}}/d_{\text{max}})^m(\rho_p/\rho_f)^n(\rho_f/\rho_{\text{water}})^{1/6}}{\rho_f \mu^2}$$  (Ref. 50) (10)

where:

- $\rho_p$ = density of specimen,
- $\rho_f$ = density of fluid,
- $d_{\text{min}}$ = minimum dimension, and
- $d_{\text{max}}$ = maximum dimensions.

An Overview of Current Resource Recovery Systems

The Resource Recovery Act of 1970 and the energy crisis have stimulated an increasing interest in resource recycling, recovery, and reuse of our solid wastes (9). Of particular interest to engineers and scientists are those systems that not only recover the individual glass and metal fractions but also recover the organic fraction for use as a fuel.

For centuries mankind has been winnowing wheat from chaff using the same basic principles that are employed by air classification devices today (51:1). Most of the Refuse Derived Fuel (RDF) systems employ an air classifier in its processing operation. Generally, the MSW is passed over or
into a moving air stream in such a manner that the heavy particles (normally the nonburnables) drop out and the lighter or burnable fraction is forced into a holding area and subsequently collected and used as a fuel.

Along with utilizing the light fraction of the MSW as a fuel, some also continue to process the heavy fraction for further resource recovery purposes. Generally, the ferrous metal is recovered by magnet separation due to the minimum costs involved, the ease of removal, and the fact that the ferrous is usually salable locally. In addition, recovery of other components can also be accomplished. However, these multiple component removal systems frequently are not economically feasible. This can be due to the high cost of equipment, the low concentration of valuable components, the fluctuating market value, and the relatively low resale value of some components. A once through air classification device with multiple separations not only permits a better fuel with a higher heating value to be extracted but also facilitates separation of some of the metal and glass fractions.

A review of the literature indicates that the National Center for Resource Recovery, the U.S. Bureau of Mines, and the Environmental Protection Agency (EPA) have either conducted or sponsored analysis of air classification systems (51; 52; 53; 54; 55; 56; 57; 58). The most comprehensive study of air classification was conducted by the Stanford Research Institute under the aegis of the EPA.
The three main factors that affect the separation of particles in an air stream are particle size, shape, and specific gravity (47:16-17). The vertical, horizontal, and inclined separator designs (Figure 5) utilize these parameters to separate the different MSW fractions.

![Diagram of three types of classifiers: Vertical, Horizontal, and Inclined](image)

**FIGURE 5. THREE TYPES OF CLASSIFIERS**

It is obvious from Figure 5 that with the vertical and inclined classifiers only one separation is possible. It would appear that for multiple separation the horizontal classifier would prove to be more feasible. However, the Bureau of Mines has experimented with a unit of this type and reports little success in using horizontal classifiers for multiple separation. Unfortunately, the material...
separations do not occur at the designated separation stations and rarely does a clean separation result (56:14).

Note also that in all of these designs the diameter (cross-sectional area) remains constant. For example, the intake and exhaust ports for the vertical unit are equal in cross-sectional areas. Assuming a frictionless fluid flow pattern, this would indicate equal buoyant force at each end of the column and would therefore preclude the possibility of single pass multiple separation.

**The Zig Zag Classifier**

By definition classification is an operation in which granular particles of different sizes and densities are allowed to settle through a fluid. If a rising fluid is contained in a vertical chamber and the fluid speed is also controlled, then the lighter particles will be transported out the top and the heavier particles will fall to the bottom.

The vertical separator is the most widely used. However, in nearly all cases a slight modification to the vertical chamber is incorporated. This change consists of bending the vertical column into a "zig zag" shape (see Figure 6) (58:15). This modification creates a turbulent air flow and material tumbling action which increases the mixing and provides for a better separation of the materials. This mixing, in effect, prevents a large light fraction from carrying a heavy one into the light fraction repository.
Although many interactions such as buoyancy and interparticle collision are possible, only the terminal velocity need be considered as important when developing separation strategies (58:13). According to R. A. Boettcher of SRI,

Expressions have been developed for terminal velocity under turbulent, streamline, and transitional conditions. These expressions generally apply to spherically shaped particles and involve the particle's diameter, its specific gravity, and the density and viscosity of the gas. Constants in these equations must be determined experimentally and can, therefore, be determined for irregular fragments as well as spherical particles. In all cases, the terminal velocity increases with increasing particle density and particle size. Particle shape exerts a great deal of influence on this velocity, particularly for lightweight fibrous materials. When the flow is confined, electrostatic forces on smaller sizes of these materials can become as important as gravitational forces. The air velocity required to float a particle when the current as a whole is vertically upward is usually
different from the velocity with which the particle settles in still air and both are different from the velocity necessary to transport the particle, as for pneumatic conveying, when a major component of the current direction is horizontal. Although related to terminal velocity and floating velocities, fluidizing velocities for the zig zag classifier, as reported herein, are not directly comparable [58:13].

In order to accomplish satisfactory separations, the following are required:

1. Suitable feed speed
2. Particle dimensions no greater than three-fourths of the throat dimensions
3. Granular flow
4. Proper feed preparation (shredding of MSW)(40:5)

Mr. Boettcher considers the advantages of air classification to be its: "dry processing capability; sharp, clean separation capability; high capacity throughput; low power requirements; low operating manpower requirements; and dust-free operation [8:7]." His list of limitations include the feeder and throat size limitations, pre-shredding and multiple column operation for more than two-component separation (8:7). This last limitation would appear to be based more upon the type of classifier used than on an in-depth study of the feasibility of multiple separation with a single unit.

A review of the experimental results reflected the intuitively obvious fact that higher fluid velocities increase the amount of the light fraction that enters the light fraction holding area. At very high speeds all of
the light fraction entered the light fraction holding area, but so also did many of the heavy fractions. At very low speeds a similar but opposite result was observed for the two fractions. The percentage of the desired fraction over the total fraction entering a holding area can be varied by altering the classifier speeds.

The National Center for Resource Recovery

A 1973 National Center for Resource Recovery Bulletin (NCRR) lists the factors affecting air classification and their interrelationships to be:

1. Particle Size: Affects material bulk density and air column loading. Also aids in determining air speed.
2. Particle Shape: Aids in determining separation air speed.
3. Particle Density: Affects material bulk density and air column loading. Aids in determining separation air speed.
5. Moisture Content: Affects material density.
6. Air Column Loading: Determines capacity and separation efficiency and affects air speed.
7. Air Speed: Determines the point of separations (59:18).
The Light Fraction Model

While working as a 1974 Summer Fellow (National Science Foundation, Faculty Participation Program) with the NCRR, Dr. Dah-Nien Fan developed a mathematical model for the air classified light fraction (LF) of shredded MSW. This LF model can be used to compute moisture content, calorific value, and ash value of the light fraction as a function of air classification velocity. The equations of the model are:

\[
\% \text{ Moisture Content of LF} = (0.08 + \frac{15.2}{65.8 + 36V^{1.3}}) \times 100 \quad (11)
\]

\[
\% \text{ Ash Content of LF} = (1 - \frac{62.7}{65.8 + 0.36V^{1.3}}) \times 100 \quad (12)
\]

Heating Value in \(10^6\) J/Kg of LF = \[
\frac{842.4}{65.8 + 0.36V^{1.3}}
\]

(13)

Note: The air classifier velocity, \(V\), is in meters per second. It is interesting to also note that a one meter per second increase in air speed amounts to approximately a one percent increase in ash content, a 1/4 percent decrease in moisture content, and 150,000 J/Kg decrease in heating value (60).

Dr. Fan's equations enable system designers to conduct sensitivity analysis on the moisture, ash content and the heating value of the light fraction as a function of the velocity of the air classifier.
The Bowerman Tank

Dr. Frank R. Bowerman designed a fluid settling tank classifier which will perform comprehensive fraction-ization of mixed solid wastes (61). The tank (Figure 7) contains a flowing medium, usually water, and a series of collection baskets on the bottom. The material is placed into a tank at one end. The heavy particles fall out at the bottom in the buckets marked in Figure 7 as HVY. The lighter particles travel downstream to the other baskets.

![Flow Diagram](image)

**FIGURE 7. BOWERMAN TANK**

Assuming that the fluid flow is slow enough to satisfy Stokes Law, Bowerman determined that similar sized and shaped particles would fall out according to their relative densities. He stated that

\[
\frac{V_{s1}}{V_{s2}} = \left(\frac{\rho_{s1} - \rho}{\rho_{s2} - \rho}\right)^{1/2}
\]

(14)
where \( V_{s1} \) and \( V_{s2} \) are the settling velocities of particles of densities \( \rho_{s1} \) and \( \rho_{s2} \), respectively. The actual distance traveled and the specific settled position of the particles is determined by the vector sum of the settling velocity and the horizontal velocity of the fluid. Bowerman's experimental work indicated that shape is indeed a significant factor for he noted that sheets or flat plates required considerably more settling time than did spheres of the same densities (61:37).

**Shredded Automobile Component Separation**

K. C. Dean, C. J. Chindgren, and LeRoy Peterson used both a horizontal and a vertical air classifier to recover nonferrous metals from shredded automobile nonmagnetic reject scrap. They reported that

Ninety-six percent of the metal was recovered in a 74 percent metal concentrate. Tandem operation of both classifiers recovered 92 percent of the metal in an 80 percent concentrate while rejecting 87 percent of the nonmetallics. Heavy media (water) separations of air-classified concentrate produced an overall recovery of 91 percent of the metal in the form of a 99 percent metal concentrate [62:1].

Later using the same equipment they investigated the possibilities of separating metals and nonmetals without the prior separation of the nonferrous materials. They reported that they recovered 97 percent of the combustibles and rejected 88 percent of the nonmetal noncombustibles from auto scrap (63:1).
Multistage Separators

P. M. Sullivan and Harry V. Makar of the U.S. Bureau of Mines have been operating a continuous mechanical separation pilot plant for MSW. This facility relies primarily on multistage processing (64:116,128). They have determined that by using many different processes in series a cleaner separation will be achieved and that "80 to 90 percent of the combustibles are collected in the cyclones [64:138]." The use of multiple stage separation equipment in series has long been recognized as a useful but rather expensive method of separating materials. This can be accomplished by using different types of equipment such as sieves, classifiers, etc., or by cycling materials through a series of similar devices.

A short but quite good theoretical development of settling velocities of particles falling in a low velocity flow is included in a 1972 paper by the Great Lakes Research Institute. Several different materials of different shapes were studied in a liquid medium to determine if separation could be effected with a single pass. As with many other systems, including air separators, the authors basically agree that a good clean separation would entail multiple passes (61).

Dr. D. E. Wilson of the Massachusetts Institute of Technology (MIT) reports that MIT is using a cyclonic type classifier in their recovery system. This patented classifier uses a radially inward-flowing vortex of air or water
to provide the separating medium. This system utilizes air for light fraction separation and water as a medium for further separation of the heavies. Their work indicates that in this design the particles are more affected by the density of each particle than on their drag coefficients. At this time, however, MIT has limited their investigations to known shapes and have not yet studied the patterns of irregular shapes (55:212a).

The "Vibrolutriator"

Rodgers M. Hill of Triple/S Dynamics describes the "Vibrolutriator" as a process that uses both mechanical vibration and air to purify, separate, or remove differing particles from a stream of products (65:1). Figure 8 shows a sketch of the "vibrolutriator."

![Diagram of Triple/S Dynamics Classifier](image-url)
The separation of the light fraction is accomplished by the combination of three actions. The first is vibration which stratifies the material bed into heavies and lights. This conveying agitation tends to settle the heavier (denser) particles to the bottom of the material bed as it is conveyed down the length of the elutriator. The second action on the material is an inertial effect. Lighter particles are required to follow a U-shaped path with the airstream, while heavy particles are discharged at the lower end after traveling in a relatively straight line. The third action which completes the function of the elutriator is fluidizing air in two or more high-velocity, low mass flow curtains through the material bed. This fluidizing air changes the direction of the lighter particles and moves them into a position to be picked up and conveyed by the exhaust air.

Air pulled through the feed inlet gives the initial acceleration of the lighter particle as the material bed is agitated by the vibration. A final stripping of light particles is accomplished at point C in figure 8 as the heavy fraction discharges from the elutriator.

The resulting separation is less sensitive to particle size than a conventional vertical air classifier, either straight or zigzag design.

Particles of a similar size but different gravities tend to follow different paths through the elutriator. This is due to the stratification caused by the vibration, the inertial effect (i.e. denser particles tend to follow a straight line path more closely), and the effect of the fluidizing air on the lighter particles. The result is that the heavy particle is never exposed to the full exhaust air flow.

Mr. Hill also states that

In a typical refuse recycling installation, the primary objectives are: a fuel product with as low an ash content as practical; a clean ferrous fraction with minimal organic contamination; and a heavy fraction relatively free of putrescibles [65:3].

He further states that in conventional systems (air classifiers) often light particles trap heavier ones and transport them into the exhaust or light fraction. He states that because of the prestratification in the "Vibrolutriator" this problem is nearly eliminated (65:8). He agrees with other experimenters when he states that the
terminal velocity of the particles will be determined by its size, shape, and specific gravity (65:8). In conclusion, he states that

... the Triple/S Dynamics "VibroLutriator" will produce a combustible fraction, and a heavy fraction, from material that has been through a primary shredder only, that will conform to the following specifications:

1. The aspirated fraction will contain no more than 5% inerts. Inerts defined as free, available inert material which is retained on a square mesh screen of 1/4" clear opening.

2. The heavy fraction will contain less than 3% fibrous material. This fibrous material being defined as objects whose terminal velocity in still air is equal to or less than 600 feet per minute.

3. The heavy product will contain 98% of the metals in the feed, except aluminum. We defined metals, except aluminum, as follows: metals, except aluminum, which are not physically attached to light materials.

4. Finally, we state that the heavy product will contain 98% of the aluminum in the feed other than foil. We define foil as all aluminum products fabricated from aluminum stock which is thinner than 28 gauge W and M scale.

In general, the types of classification equipment currently being used do not lend themselves to clean separations without resorting to multiple passes. Current practitioners seem more interested in developing equipment and separating schemes than in developing a theoretical base. This work is an attempt to study the underlying principles of separation and the development of a new classifier design.

Summary

Even though the basic principles of air classification have been known for centuries and volumes of aerodynamic information are available only a few researchers
have developed an expertise that is directly convertible to MSW separation systems. Most aerodynamic data is not based upon free falling and tumbling objects of unknown dimensions. Unfortunately MSW components in a separator are free falling, wobbling objects of unknown dimensions in the air classification process. Data concerning free falling objectives of various shapes is also quite sketchy.

Due to the variability of the many parameters that will affect a clean MSW separation by air classifications, it appears that theoretical analysis will require extensive test data in order to quantify many unknown values. For example, at Reynolds' Numbers from $10^3$ to $10^5$ the drag coefficient can vary from 0.3 to 10.0 depending upon the shape of the specimen. Obviously this variance will significantly affect the terminal velocities and thus the separation capability of an air classifier. The meager amount of drag data on flexible specimens and the extreme difficulty of developing a meaningful theoretical approach indicates that experimental data will be absolutely required.

Current air classification systems have been unsuccessful in attaining clean separations with a single pass. To date only by using multiple passes of the MSW through the separators have clean separations been observed. This suggests that a new design of air classifiers is required. A new design concept for air
classifiers was used for all experiments reported in this dissertation.
CHAPTER III

THEORETICAL DEVELOPMENT

**Introduction**

It was assumed that (1) the size, shape, and density of the different components in municipal solid waste (MSW) would be the controlling factors in the air classification process, and (2) the distribution of the MSW components as a function of terminal velocity will indicate which materials can be separated by air classification. For example, note in Figure 9 the different positions on the horizontal axis which could be obtained for three hypothetical MSW components. This figure reflects the case where three components would stratify at different terminal velocities and subsequently could be easily air separated.

![Figure 9. Frequency of occurrence versus \( V_t \) for three different materials](image)

**FIGURE 9. FREQUENCY OF OCCURRENCE VERSUS \( V_t \) FOR THREE DIFFERENT MATERIALS**
Based on first principles the effect of density, size, and shape upon the terminal velocity for MSW type materials are theorized. It is proposed that if density is the primary factor and the distribution of the terminal velocity is small, then effective air separation is possible. If, however, the size or shape has a significant influence, then separation may be complicated due to the varying sizes and shapes of shredded MSW.

Because of the ease of evaluating the aerodynamics of spheres and because so much theoretical and experimental work is available, the analysis began with spheres and then considered flat plates and other nonspherical shapes more typical of shredded refuse.

**Basic Concepts**

In order to suspend a particle in an air stream the drag must equal the weight.

\[
\text{Weight} = \text{Drag} \quad (15)
\]

\[
(Vol)(\rho_w)g = \frac{1}{2} \rho_a C_D V_t^2 S \quad (16)
\]

- \( Vol \) = Volume (Ft\(^3\))
- \( \rho_w \) = Density (Slugs/Ft\(^3\))
- \( \rho_a \) = Density of Air (Slugs/Ft\(^3\))
- \( C_D \) = Drag Coefficient
- \( V_t \) = Terminal Velocity (Ft/Sec)
- \( S \) = Area (Ft\(^2\))
- \( g \) = 32.174 (Ft/Sec\(^2\))

Algebraically transpose equation (16) so that the effects of the different parameters on the terminal
velocity can be determined by:

\[ V_t = \left( \frac{2 \rho_w (\text{Vol})g}{\rho_a C_D S} \right)^{1/2} \]  

(17)

Equation (17) shows that the terminal velocity is inversely proportional to the square root of \( \rho_a \), \( C_D \), and \( S \). Therefore, as either the air density, the drag coefficient, or the area increase the terminal velocity decreases. Similarly, the terminal velocity is directly proportional to the square root of the density and the volume. Heavier or more voluminous components will require higher terminal velocities than less dense or smaller components. This assumes that size does not vary enough to change the Reynolds Number and subsequently the drag coefficient of the materials being evaluated.

**Sphere Drag and Terminal Velocity**

To evaluate the effects of the various parameters on the terminal velocity a spherical particle is considered:

\[ V = \text{Vol of a sphere} = \frac{4}{3} \pi r^3 \]

\[ S = \text{Projected area of a sphere} = \pi r^2 \]

\[ V_t = \left[ \frac{2g}{\rho_a C_D} \left( \rho_w \right) \frac{4}{3} \frac{\pi r^3}{\pi r^2} \right]^{1/2} \]  

(18)

\[ V_t = \left[ \frac{8 (\rho_w)rg}{3 \rho_a C_D} \right]^{1/2} \]  

(19)
Equation (19) indicates that for spheres of the same radius tested under aerodynamic conditions of equal air density and Reynolds Numbers (and subsequently equal drag coefficients) the terminal velocity will be a function of the square root of sphere density.

**Sphere Drag—Theoretical Approach**

Stokes has shown that in a laminar flow field the resistance of a sphere (drag) is

\[
D = 6\pi\mu rv
\]  
(Ref. 35:36)  
(20)

\[
\mu = \text{viscosity} \\
r = \text{radius} \\
v = \text{velocity}
\]

Experimental results have shown this to be a good predictor of small (less than 50 microns) sphere performance and can be used in estimating terminal velocities for small spheres at low speeds and thus low Reynolds Numbers.

Newton's second law of motion (F=ma) when applied to a sphere falling in a laminar flow results in

\[
m \frac{dv}{dt} = mg - m'g - D
\]  
(21)

\[
m = \text{mass of sphere (slugs)} \\
m' = \text{mass of displaced fluid (slugs)} \\
g = \text{gravity} \\
t = \text{time} \\
v = \text{velocity} \\
D = \text{Drag}
\]
Therefore:

\[
\frac{4}{3} \pi r^3 g (\rho)_S \frac{dv}{dt} = \frac{4}{3} \pi r^3 [(\rho)_S - (\rho)_A] g - 6 \pi \mu rv
\]  

(22)

\[\text{Density of Sphere (Slugs/Ft}^3)\]

\[\text{Density of Air (Slugs/Ft}^3)\]

\[
\frac{dv}{dt} = \left[\frac{(\rho)_S - (\rho)_A}{(\rho)_S}\right] g - \frac{9}{2} \frac{\mu V}{(\rho)_S r^2}
\]  

(23)

Letting \(\frac{dv}{dt} = 0\) as in the case for terminal velocity gives:

\[
v_t = \frac{2}{9} \left[\frac{(\rho)_S - (\rho)_A}{\mu}\right] r^2 g
\]

(24)

The terminal velocity is proportional to the difference in densities of the sphere and air and increases as the square of the radius.

The Stokes equations are significantly limited by the laminar flow constraint. Newton's work is an attempt to fill this void and he defines the resistance to motion (drag) in turbulent flow as:

\[
D = C_D \frac{\pi}{2} (\rho)_A r^2 v^2
\]

(25)

\[C_D = \text{Drag coefficient}\]

For turbulent flow the combination of Newton's equation and the second law of motion gives:
\[ m \frac{dv}{dt} = mg - m'g - D \]  
(Ref. 35:172)  \hspace{1cm} (26)

\[ \frac{4}{3} \pi r^3 g(\rho) \frac{dv}{dt} = \frac{4}{3} \pi r^3 [ (\rho)_S - (\rho)_A ] g \]

\[ - C_D \frac{\pi}{2} (\rho)_A r^2 v^2 g \]  \hspace{1cm} (27)

\[ \frac{dv}{dt} = \frac{[(\rho)_S - (\rho)_A]}{(\rho)_S} g - \frac{3C_D}{8} \frac{(\rho)_A v^2}{(\rho)_S r} \]  \hspace{1cm} (28)

At terminal velocity \( \frac{dv}{dt} = 0 \), and:

\[ v_t = \left[ \frac{8}{3C_D} \frac{g [ (\rho)_S - (\rho)_A ] r}{(\rho)_A} \right]^{1/2} \]  \hspace{1cm} (29)

The terminal velocity is a function of the differences in specific weights divided by the specific weight of air and the square root of the sphere radius. Note that when \( \rho_s \gg \rho_a \), equation (29) is the same as equation (19), which it should be.

It is interesting to note that for laminar flow the terminal velocity of small size spheres varies as the square root of the radius. Therefore, the Newtonian range agrees with the basic concept presented in equation (19).

Osborne Reynolds developed a method of presenting data on all solids and in all fluids by uniting the drag coefficient to a nondimensional number. Reynolds' Number (RN) is defined as:

\[ RN = \frac{\rho v r}{\mu} \]  \hspace{1cm} (Ref. 33:3)  \hspace{1cm} (30)
In 1937 the Committee on Sedimentation of the National Research Council published the now famous Reynolds Number versus coefficient of resistance (drag) curve (36: 176), which was presented in Figure 7. This figure has been verified experimentally by many aerodynamicists. The Stokes region applies quite accurately up to RN=0.6 and the Newtonian range is valid from RN=800 to 200,000.

**Nonsphere Drag**

Since most shredded MSW is not spherical but plate-like, it is important to investigate the effects of nonsphericity upon the terminal velocities of the MSW components.

The basic formula for terminal velocity which was developed earlier is repeated:

\[
V_t = \left[ \frac{2\rho_w (Vol) g}{\rho_a c_D S} \right]^{1/2}
\]  

(31)

The farthest departure from a spherical shape is a flat plate. When introduced into an air stream a flat plate can be orientated either normal or parallel (streamlined) to the flow or any position in between. When the flow is normal to the flat plate the terminal velocity equation becomes:

\[
V_t = \left[ \frac{2\rho_w}{\rho_a c_D} g \frac{L \cdot W \cdot t}{L \cdot W} \right]^{1/2}
\]  

(32)
Equation (33) shows that the terminal velocity of a plate normal to the air flow is a function of the square root of the plate thickness. This is not unreasonable for as the thickness increases so does the weight of the material and therefore the terminal velocity.

Similarly, the terminal velocity of a square plate with the edge normal to the air flow (streamlined) is a function of the square root of the length of a side (L)

\[ V_t = \left[ \frac{2 \rho_w t g}{\rho a C_D} \right]^{1/2} \]  

(33)

Note as before that increasing L also increases the weight and subsequently the terminal velocity. In summary, the terminal velocity of a flat plate is a function of the square root of the density and one dimension of the plate. This assumes that the \( C_D \) remains relatively constant in the area of interest.

Because the MSW components are suspended in an air stream they will be subject to rotation about their three axes. This rotation will affect the terminal velocity and will cause the MSW components to oscillate over a rather wide range of angular velocities.

For a flat plate rotating about an axis normal to the air flow (see Figure 10) the following model can be considered:
At $\theta = 0^\circ$, equation (33) is appropriate; at $\theta = 90^\circ$ equation (34) describes its relation to $V_t$. At any instant during the rotation the "exposed" surface area, $S$, will be equal to the $L \cos \theta + t \sin \theta$. Therefore equation (32) becomes:

$$V_t = \left[ \frac{2\rho_w}{\rho_a c_D} \frac{L \cdot W \cdot t}{W(L \cos \theta + t \sin \theta)} \right]^{1/2}$$

(35)

Since we are unable to predetermine a rotation speed, it is necessary to assume an average value for $\sin \theta$ and $\cos \theta$. This will be compatible with the assumption that $\frac{dv}{dt} = 0$, for instantaneous periods of time.

The average absolute value for $\sin \theta$ between $0$ and $180^\circ$ is $\frac{2}{\pi}$ and for $\cos \theta$ the average absolute value is $\frac{2}{\pi}$.

Using these average values equation (35) becomes:
\[ V_t = \left[ \frac{2\rho_w}{\rho a C_D} \frac{gL \cdot W \cdot t}{2W(L+t)/\pi} \right]^{1/2} \]  \hspace{1cm} (36)

If \( t \ll L \) then \( (L+t) \approx L \), and

\[ V_t = \left[ \frac{\rho_w C_D}{\rho a C_D} \right]^{1/2} \]  \hspace{1cm} (37)

Equation (37) shows that a rotating plate exhibits a higher terminal velocity than a streamlined plate, but less than a plate normal to the flow.

In an air stream of varying velocities and physically limiting boundaries, the rotating plate can be defined within a specified envelope. Assuming that the air stream velocity decreases with increasing height, then the maximum height or therefore the minimum terminal velocity for a flat plate will be determined by equation (28). The lowest position and subsequently the highest terminal velocity will be determined by equation (34).

Due to the side load caused by air striking the plate at an angle during rotation the plate will be forced to one side or the other until it contacts either another plate or the wall. These collisions will alter the rotation pattern but in the final analysis may have little, if any, effect upon the limiting terminal velocities.

It is more realistic to assume rotation of a plate about two axes normal to the air flow. This requires a more complex theoretical development (see Figure 11).
Rotation about the Y-axis will affect the "exposed area" by relationship:

\[ W \cos \alpha + t \sin \alpha \]

Rotation about both the X and Y axis of the exposed area will be:

\[ S = (L \cos \theta + t \sin \theta)(W \cos \alpha + t \sin \alpha) \] (38)

Equation (38) should be valid for all angles of \( \theta \) and \( \alpha \).

An evaluation of equation (38) for a flat plate both normal and parallel to the stream flow provides a limit check.
When the plate is normal to the flow, $\theta^\circ = 0$ and $\alpha^\circ = 0^\circ$.

Therefore:

\[ S = (L \cos \theta^\circ + t \sin \theta^\circ)(W \cos \alpha^\circ + t \sin \alpha^\circ) \]  
\[ = [L(1)+t(0)] [W(1)+t(0)] \]  
\[ S = LW \]  

(39)  
(40)  
(41)

When the plate normal to the flow is rotated about the $Y$-axis $90^\circ$, the exposed area will be:

\[ S = (L \cos \theta^\circ + t \sin \theta^\circ)(W \cos \alpha^\circ + t \sin \alpha^\circ) \]  
\[ = [L(1)+t(0)] [W(0)+t(1)] \]  
\[ = Lt \]

(42)  
(43)  
(44)

This relationship describes the "exposed" edge of a flat plate aligned parallel to the flow and also agrees with the previous work.

Using the average "exposed" area for terminal velocity calculations provides a value somewhere between the maximum and minimum values. It must be mentioned that transient forces are being evaluated by a "stop action" process and although not the perfect approach it provides insight into the dynamic solution.

As before, the absolute average exposed area for a rotating plate about one axis is $\frac{2}{\pi}$. 
\[ S = [L \cos \theta + t \sin \theta][W \cos \alpha + t \sin \alpha] \quad (45) \]

\[ = [L \left( \frac{2}{n} \right) + t \left( \frac{2}{n} \right)][W \left( \frac{2}{n} \right) + t \left( \frac{2}{n} \right)] \quad (46) \]

\[ = \left( \frac{2}{n} \right)^2 [L+t][W+t] \quad (47) \]

Again if \( t \ll L \) and \( t \ll W \) then \( (L+t) \approx L \) and \( (W+t) \approx W \) and finally

\[ S = \left( \frac{2}{n} \right)^2 WL \quad (48) \]

The terminal velocities are

\[ V_t = \left[ \frac{2 \rho_w t g}{\rho a C_D} \right]^{1/2} \]

Flow Normal to Plate (49)

\[ V_t = \left[ \frac{2 \rho_w L g}{\rho a C_D} \right]^{1/2} \]

Flow Streamlined with Plate (50)

\[ V_t = \left[ \frac{\tau' \rho_w t g}{2 \rho a C_D} \right]^{1/2} \]

Flow with Rotating Plate (51)

The assumption that \( t \ll L \) will yield a value in (51) less than (49) but greater than (50). That is, the terminal velocity will be more than that of a stationary plate streamlined to the flow and less than the same plate positioned normal to the flow. The above development although not a proof does support the conclusions that rotation of flat plates about two axes simultaneously will affect the terminal velocities. The third axis is parallel to the stream flow and is insignificant to the above
development. These results show that the changing orientation of the flat plate will cause both the mean \(\langle \vec{v} \rangle\) terminal velocity and the standard deviation \(\sigma\) to vary as a function of the dimensions \((L, W, t)\) of the flat plate.

The above analysis has been based upon the assumption of a constant drag coefficient, \(C_D\), for all of the various plates. Although this assumption simplifies the analysis and is fairly reasonable for spheres in Reynolds Number ranges from 1000 to 100,000 the relative nonspherical shape of flat plates should be considered to more accurately explain what happens to free floating flat plates in an air stream. A review of the original assumption that \(t \ll L\) and \(W\) is in order. As \(L\) and \(W\) become smaller and smaller the afore assumed relationships of \(L+t=L\) and \(W+t=W\) becomes less acceptable and therefore,

\[
S = \left(\frac{1}{\pi}\right)^2 \frac{2}{[L+t][W+t]} \tag{52}
\]

Substituting this expression for the average exposed areas into a modified equation (31) results in the relationship:

\[
V_t = \left[ \frac{2g}{\rho_a C_D} \left(\frac{\rho_w}{\rho_a}\right) \frac{LWt}{\left(\frac{L}{\pi}\right)^2 (L+t)(W+t)} \right]^{1/2} \tag{53}
\]

A rearrangement yields

\[
V_t = \left[ \frac{2\rho_w g}{2\rho_a C_D} \frac{LWt}{(L+t)(W+t)} \right]^{1/2} \tag{54}
\]
This indicates that as the length and width are reduced the terminal velocity will increase slightly. This reduction is limited to when \( L=W=t \) and a cube is formed.

As presented earlier, Wadell and others have investigated the effects of particle shape both theoretically and experimentally (37:291). Their work is based upon a ratio of the surface area of a sphere, \( s \), which has the same volume as a particle to the actual surface, \( S \), of the particle. This ratio is defined as

\[
\psi = \frac{s}{S}
\]  

(55)

Figure 4 shows the relationship of Reynolds Number to the coefficient of resistance for nonspherical shapes for a series of nonspherical free floating bodies (\( \psi = .125 \) to \( \psi = 1 \)). Table 1 shows the sphericity calculations for various geometric bodies.

**TABLE 1. SPHERICITY OF VARIOUS GEOMETRIC BODIES**

<table>
<thead>
<tr>
<th>Body</th>
<th>Sphericity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>1.000</td>
</tr>
<tr>
<td>Cube</td>
<td>0.806</td>
</tr>
<tr>
<td>Prism a X a X 2a</td>
<td>0.767</td>
</tr>
<tr>
<td>Prism a X 2a X 2a</td>
<td>0.761</td>
</tr>
<tr>
<td>Prism a X 2a X 3a</td>
<td>0.725</td>
</tr>
<tr>
<td>Disk h = r</td>
<td>0.827</td>
</tr>
<tr>
<td>Disk h = 1/3r</td>
<td>0.594</td>
</tr>
<tr>
<td>Disk h = 1/10r</td>
<td>0.323</td>
</tr>
<tr>
<td>Disk h = 1/15r</td>
<td>0.220</td>
</tr>
<tr>
<td>Cylinder h = 3r</td>
<td>0.860</td>
</tr>
<tr>
<td>Cylinder h = 10r</td>
<td>0.691</td>
</tr>
<tr>
<td>Cylinder h = 20r</td>
<td>0.580</td>
</tr>
</tbody>
</table>
In order to quantify the effects of sphericity, Reynolds numbers at 1000 and 10,000 were selected from Figure 4 and tabulated in Table 2.

**TABLE 2. COEFFICIENTS OF DIFFERENT SHAPED PARTICLES AT TWO REYNOLDS' NUMBERS**

<table>
<thead>
<tr>
<th>Re却olds Number</th>
<th>1,000</th>
<th>10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>ψ</td>
<td>1</td>
<td>.45</td>
</tr>
<tr>
<td></td>
<td>.806</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>.704</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>.531</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>.237</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>.220</td>
<td>9</td>
</tr>
</tbody>
</table>

From Table 2 it is clear that terminal velocity is significantly affected by shape change. In equation (14) it is noted that terminal velocity is inversely proportional to the square root of $C_D$. It has been previously noted that the volume to surface area is a weak function.

To evaluate the effects of increasing plate size while maintaining a constant plate thickness the results of Figure 4 are combined with equation (17):

$$V_t = \left[ \frac{2\rho_w \text{(Vol)} g}{\rho_a C_D S} \right]^{1/2}$$

(56)
As the size of the plate increases (smaller $\psi$ values) the $C_D$ value increases faster than the volume over area relationship resulting in larger terminal velocities.

Also of interest is the variability of terminal velocity as a function of plate size. As noted in Figure 9, the variance of particular components will have a significant bearing upon whether or not these differing components can be adequately separated.

The relationship developed earlier concerning the "exposed area" as a function of $L$, $W$, $t$ and rotation angles is

$$S = (L \cos \theta + t \sin \theta) (W \cos \alpha + t \sin \alpha)$$

(57)

If $W$ and $L$ are assumed to be much larger than $t$, this equation reduces to

$$S = (L \cos \theta) (W \cos \alpha)$$

(58)

In the interest of simplicity and ease of understanding let $\theta$ and $L$ remain constant. As before it must be noted that the value of $\cos \theta$ or $\cos \alpha$ can not be negative and therefore only absolute values will be considered. As $W$ increases a significant increase in $S$ is observed. Although Figure 12 is only representative for values of $W=1$ and $W=3$, it does show the relative changes of $S$ as a function of $W$. Increasing $W$ will result in wider variation in $S$ and subsequently in $V_t$. Variation in $L$ and $\alpha$ will merely increase the relative differences and thus will increase this
variation. Although the $S$ is inversely proportional to $V_t$, as the surface area increases the variance in the terminal velocity will also increase.

**Summary**

This theoretical development began with spheres and continued through flat plates and other nonspherical shapes. In the Reynolds Number area of interest (turbulent region) it was noted in equation (19) that for spheres the terminal velocity varies as the square root of the radius.

\[ V_t = \left[ \frac{8 \rho_w r g}{3 \rho a C_D} \right]^{1/2} \tag{59} \]

The flat plate development shows that the terminal velocity is a function of the square root of the density and a representative length. Similarly, the terminal velocity
is inversely proportional to the square root of the air
density and drag coefficient.

\[ V_t = \left[ \frac{2 \rho_w L g}{\rho_a C_D} \right]^{1/2} \]  

(60)

The analysis also shows that rotation of flat plates will
increase the variability of the terminal velocity and
increasing the plate size may cause even more variability.
The relative sphericity of the specimens will significantly
affect the terminal velocity of all of the above observa-
tions.

The analysis supports the hypotheses that: (1) MSW
type material should be separable into several component
fractions by air classification, (2) equations can be
developed to predict the terminal velocities, (3) increased
shredding (size reduction) should reduce the variability in
terminal velocity, (4) density is a significant factor
affecting air separation, and (5) size and shape may also
have significant effect.
CHAPTER IV

DESCRIPTION OF EQUIPMENT AND TEST SPECIMENS

Introduction

The equipment used during the experimental phase of this research included both commercially available and specially designed component parts. The early experiments were conducted using equipment that was borrowed from the National Center for Resource Recovery (NCRR) and the later work was accomplished using equipment borrowed from the United States Air Force at Wright-Patterson Air Force Base, Ohio.

The NCRR equipment included nearly all of the equipment that was used by R. A. Boettcher of the Stanford Research Institute during his experiments which resulted in the publication titled "Air Classification of Solid Wastes" (58). In addition to the NCRR equipment the experimenter designed and had built a special vertical wind tunnel of variable test chamber size. This tunnel was constructed in such a manner that the test area opening could be varied during the experiments throughout a significant range of opening angles from 0° to 90°.
The Air Force equipment and a modified test chamber were used during the later and most critical stage of the experiments. Included in the equipment is a 40 horsepower motor, compressor, ducting, valves, and several banks of manometers. The tunnel was modified for the final experiments by including a flow channelizer and a fixed angle opening of approximately 6°.

The materials studied in the test chamber included steel, aluminum, balsa wood, paper, cloth, cardboard, and glass. Each component was cut into specific sizes and shapes and then weighed. One hundred samples were prepared for each different size and shape of each material. The shape parameter was evaluated by using square and elongated flat plates from samples of all of the different materials except glass which was too brittle to cut to special sizes with the available equipment.

Original Equipment

The original equipment included a compressor, motor, dust collector, and valves from the National Center for Resource Recovery plus a vertical wind tunnel, ducting, and a bank of manometers. Figure 13 shows a sketch of the original equipment set up.

The specification of the NCRR equipment follows:

Compressor: Positive Pressure Blower
Buffalo Forge Company
Size 2; Type RE
Wheel Diameter 18 1/8" S.O. M248
FIGURE 13. ORIGINAL EQUIPMENT SET UP

Motor: Induction Type
Wagner
Model C56-50100-62
1 HP 3 Phase
60 Cycle 3450 RPM
Volts 208-220-240
Amp 2/2/1.5 Cont. 40°C
50 Cycle 2860 RPM
Amp 2.6/2.6/1.8 Cont. 50°C

Valve: Slicing Gate Type

Dust Collector: Cyclone Separator
Carter-Day Company
Diameter 3'
Height 6'

Manometer: The Merriam Company
Cleveland, Ohio
50° U Tube

The ducting consisted of 26 gauge sheet metal that was fastened with metal screws and air sealed with plastic electrical tape. Duct #1 (tunnel to the dust collector) was 8 inches in diameter. Duct #2 (dust collector to compressor) was 12 inches in diameter.
The wind tunnel was constructed of wood, metal, and plexiglass. The design was such that varying the tunnel dimensions was accomplished by rotating side mounted cranks. The entire back of the tunnel was instrumented with static pressure probes located on center line and spaced horizontally and vertically 3 inches apart. Figure 14 is a sketch of the classifier working section and Figure 15 shows the static port locations and identification numbers. A box-like chute was located on the rear face of the tunnel with double sliding gate valves which was used for placing samples into the tunnel during a few experimental runs.

Final Equipment

The final equipment set up included a 40 horsepower motor, compressor, ducting, valves, V-device, a bank of manometers, and an oil filled slant manometer. Figure 16 depicts the final equipment set up. The specifications of specific equipment follow:

Compressor: Spencer Gas Booster
The Spencer Turbine Company
Hartford, Connecticut
Lot No. 33575
Serial No. 39576
3000 CFM (maximum)

Motor: 40 HP
220/440 Volt
3 Phase
60 Cycle
3540 RPM
96/48 Amp
Serial No. 523493
Squirrel Cage Motor
The Lewis Allis Company
Milwaukee, Wisconsin
FIGURE 14. THE CLASSIFIER WORKING SECTION
FIGURE 15. STATIC PORT LOCATIONS AND IDENTIFICATION NUMBERS
A - Flow Channelizer
B - Nozzle/Diffuser
C - Vent
D - Motor and Compressor
E - Butterfly Valve
F - Slant Manometer
G - U Tube Manometers
H - Specimen Catch Tray
I - Screen

FIGURE 16. FINAL EQUIPMENT ARRANGEMENT
Valves: Spencer "Blast Gate" Butterfly (Exhaust) Manufacturer Butterfly

Ducting: 12" Diameter 26 Gauge Sheet Metal

Manometer: The Meriam Company Cleveland, Ohio 50" U Tube

Slant Manometer: Meriam Instrument Company Cleveland, Ohio Type GP-6 Model A-434 Serial No. G-6 Range 4" (Meriam Red Oil Specific Gravity 0.827)

In order to reduce the turbulence and separation in the tunnel encountered during the original runs, a flow channelizer was placed downstream of the tunnel, the opening angle was fixed at 6°, and a nozzle was also instrumented with static pressure probes which were located on centerline at 1/2 inch intervals. The improvements are annotated in Figure 16.

Test Specimens

The materials evaluated included steel, aluminum, balsa wood, cardboard, paper, cloth, and glass. From each material, except glass, one hundred 1/4", 1/2", 3/4", 1" and 1 1/2" square flat plate samples were prepared. A random sampling of 10 from each size and different material was weighed, and the mean weights were computed.

In addition to the square plates, rectangular plates with aspect ratios from 2 to 4 and with areas equal to particular square samples were prepared. Similar weighings and statistical data were obtained for the
rectangular plates. The different sizes and shapes prepared are described in Table 3. The calculated densities and measured thicknesses of the different materials are included in Table 4. Spheres and shattered fragments were used in the glass experiments.

**TABLE 3. SPECIMEN SIZES AND SHAPES**

<table>
<thead>
<tr>
<th>Shape (Aspect Ratio)</th>
<th>.0625 in²</th>
<th>.250 in²</th>
<th>.5625 in²</th>
<th>1.000 in²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.250</td>
<td>.500</td>
<td>.750</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>.250</td>
<td>.500</td>
<td>.750</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>.177</td>
<td>.354</td>
<td>.503</td>
<td>.707</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>.354</td>
<td>.707</td>
<td>1.060</td>
<td>1.414</td>
</tr>
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<td>3</td>
<td>.144</td>
<td>.289</td>
<td>.433</td>
<td>.577</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>.433</td>
<td>.866</td>
<td>1.300</td>
<td>1.732</td>
</tr>
<tr>
<td>4</td>
<td>.125</td>
<td>.250</td>
<td>.375</td>
<td>.500</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>.500</td>
<td>1.000</td>
<td>1.500</td>
<td>2.000</td>
</tr>
</tbody>
</table>

**TABLE 4. DENSITY AND THICKNESS OF MATERIALS**

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (lbs/ft³)</th>
<th>Sample Thickness (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>487.1</td>
<td>0.02625</td>
</tr>
<tr>
<td>Aluminum</td>
<td>171.4</td>
<td>0.0310</td>
</tr>
<tr>
<td>Balsa Wood</td>
<td>12.5</td>
<td>0.0615</td>
</tr>
<tr>
<td>Cardboard</td>
<td>36.2</td>
<td>0.0267</td>
</tr>
<tr>
<td>Paper</td>
<td>33.8</td>
<td>0.0038</td>
</tr>
<tr>
<td>Glass</td>
<td>144.4*</td>
<td>--</td>
</tr>
</tbody>
</table>

*Computed from measurements and weighings of glass spheres.
CHAPTER V

EXPERIMENTAL PROCEDURES

Introduction

The initial set of experiments were concerned with developing an understanding of the effects of varying the inlet angles and velocities on the flow pattern within the V-shaped classifier. The flow and subsequent pressure patterns were compared with theoretical calculations and evaluated for turbulence and consistency. The second phase of tests investigated the effect of different sized and weighted spheres in the classification device. During this phase of the study it became obvious that there was a distinct relationship between terminal velocity, sphere weight and size. The abundant aerodynamic data for spheres in the literature provided an effective means for comparing the observed terminal velocity data with theoretical values.

Phase three experiments were conducted on flat plates to determine the effects of size, shape, density, and classifier loading on the terminal velocity of the samples. In order to determine the effect of the number of test specimens in the device at any one time several
test runs were conducted by varying the numbers of samples initially placed into the device.

The fourth phase of experiments dealt with determining the terminal velocities of the different test materials by varying sizes and shapes. More specifically, this phase consisted of determining the terminal velocity of 5 different materials of 4 different sizes and 4 different shapes. Glass was treated separately due to the fact that glass normally enters the air classifier in irregular shapes.

Phase five consisted of evaluating the terminal velocities of glass spheres of different sizes and batches of mixed materials of varying sizes and shapes. The batches of mixed materials (metal, paper, etc. of different sizes) were run to see if the terminal velocities measured for the individual material would be affected due to the presence of materials of different composition, size, and shape.

**Phase One: Calibration of Classifier**

For the first phase of this study the equipment was set up as shown in Figure 13 (Chapter IV). The first series of runs were limited to measuring the centerline static pressure and subsequently the velocity within the device. During Run 1 the device was opened to 19.73° at row 1, see Figure 15, and for Run 2 it was opened 32.81°. While the motor was
operating static readings of the centerline pressure ports were read from the bank of water-filled manometers.

Run 3 included pressure readings of the centerline ports and one port on each side of centerline. Run 4, like Run 3, was conducted with the device open 12 inches at row 1, but during this run as many as two ports on each side of centerline were used. In addition, during Run 4 two screens were added to the bottom of the device in hopes of reducing the turbulence in the device.

During Runs 6 to 16 centerline pressures were recorded in all ten rows while; (1) varying the air stream velocity within the device, (2) adding and removing screens, and (3) changing the opening of the device from 2.85 to 18 inches at row 1.

Runs 17 to 32 were conducted with a top row opening at 10 inches and varying air stream velocities.

The primary purpose of this phase of the study was for the experimenter to check out the equipment, to develop an understanding of this equipment, and to test experimental procedures.

A tabular presentation of these early experiments is included in Appendix A.

Phase Two: Analysis of Spheres in the Air Classifier

With the flow pattern established within the V-shaped classifier the measured air velocities were used
to estimate the terminal velocities of various sized and weighted spheres.

For these experiments color coded spheres of different sizes and weights were studied. Both the drop out pattern and the distribution within the operating V-device were observed. The heavier spheres were coded blue, the lightest were yellow and the intermediate density spheres were coded red.

Batches of spheres of a single color and batches of spheres of differing colors were run. The sphere distribution pattern in the device was noted as a function of color. The drop out velocities and pattern observed were recorded.

In order to observe the effects of density independently, a dozen ping pong balls were injected with water in varying amounts from 1.5 to 24.5 grams. These spheres were then placed into the device and drop out velocities were recorded.

In order to investigate the effects of collisions and air flow disruption with more than one specimen in the device, several experiments were conducted with 1, 2, 3, 5, and 6 spheres of equal density in the unit. The velocities at which the first sphere dropped out in each test run was recorded.

Phase Three: Aerodynamics of Flat Plates

The effects of particle interaction, wall effects, and choking are discussed in Chapter II. Generally
equations concerning these factors are based upon a ratio of the volume occupied by the specimens versus the total affected volume. This affected volume in the classifier is that portion of the flow pattern that is changed because of the suspended material.

From the literature review it is known that increasing the number of flat plate specimens in the device will alter the air flow and subsequently the terminal velocities. This effect was evaluated by placing different numbers of different sized square flat plate specimens into the device and recording the velocity at which the first plate dropped out. Each combination was repeated five times to reduce the effects of random errors.

From the lot of 100 steel specimens of size 1/4 by 1/4 inches (1/16 square inches) batches of 1, 2, 3, 5, and 10 were selected. Then each batch was placed in the device. The air flow was reduced until a specimen dropped out of the classifier. The manometer reading for this condition was recorded. This test sequence was repeated five times for each batch of samples. This procedure was also repeated for steel specimens of sizes 1/2 by 1/2 inch, 3/4 by 3/4 inch, 1 by 1 inch and 1 1/2 by 1 1/2 inch.

After the runs with steel specimens were completed the entire sequence was repeated with the aluminum plates. In addition, aluminum specimens of sizes 1/4 by 1/2 inch, 1/4 by 1 inch, and 1/4 by 1 1/2 inches were tested in
sample sizes 1 to 10 exactly as above. These experiments were conducted using aluminum to evaluate the effects of changing shape on the terminal velocity.

The next group of experiments were conducted in order to determine the effects of density, size and shape on the terminal velocity of different materials. From the specimen population of 100 for each different sized material 4 random samples of 25 units were selected with replacement. The 25 units were put into the device, the velocity was reduced incrementally, and the manometer reading and number that dropped out at each velocity were recorded. This was repeated 4 times for each different sized and shaped specimen. Following the runs the mean and standard deviation values of the terminal velocity were computed using statistical methods described in Chapter VI.

**Phase Four: The Experimental Design**

This phase consisted of evaluating the terminal velocity of each different material in its different sizes and with its different aspect ratios (length over width). This entire experimental design consisted of 5 materials of 4 different sizes and 4 aspect ratios each repeated 4 times for a total of 320 experimental tests. When completed, the experimental results obtained were sufficient to calculate the mean and standard deviation of the terminal velocities for each combination of density, size, and shape tested. The means were then compared to values computed using
the equations developed in Chapter III. In addition, the statistical significance of each parameter on terminal velocity was evaluated.

Prior to each test the date, barometric pressure, temperature, and manometer zero reading were recorded on the data sheet. The arrangement and purpose of the test were also noted. Then from a specific material population of 100 specimens of the particular size and aspect ratio a random sample of 25 was selected. After the motor/compressor was activated and the velocity within the device had reached the proper speed (high enough to support the specimens within the viewing position of the device but not so high as to cause the specimens to be forced into the top screen) the samples were thrust upward into the device from below. The velocity was then reduced incrementally in 15 second intervals until all of the specimens had dropped from the device. As noted in the schematic drawing of the final arrangement a tray was located directly below the bottom opening. This tray was moved along under the device and was used to catch the falling specimens. A different part of the tray was located under the device for each different velocity. This permitted continuous operation and a means of identifying the samples from each velocity reduction during the experiment. After shutting off the motor/compressor the number of specimens in each tray was recorded on the data sheet opposite the appropriate manometer readings. This procedure was repeated 320 times so
that each possible combination of density, size, and shape would be tested 4 times. This replication was necessary to reduce the effects of experimental error.

It should be noted that the 15 second detention time for each different drop velocity (pressure reading) was determined to be optimum given the equipment limitations and desired experimental results. The objective of the research was to determine a mean and variance of the drop or terminal velocity. When the time between velocity changes exceeded 15 seconds either none dropped as the velocity was too high or more dropped out because the velocity was too low. The 15 seconds is a compromise and was held constant for this series of experiments. After each test the 25 specimens were returned to the proper bin and another random sample of 25 was selected for the next test.

Throughout this experimental phase the turbulence and interparticle action were observed.

Phase Five: Materials Mixed Experiments

A mixture of sizes and shapes of each material was put into the device after the appropriate suspension velocity had been attained. The velocity was then reduced incrementally as described in phase four. The drop velocities and pattern (number that dropped, size and shape) were recorded. This was repeated several times for batches of each material.
Next the materials were mixed and a random sample of approximately 100 specimens was thrust into the device running at a speed high enough to suspend all specimens. This was repeated several times and the results were recorded.

Three different sized glass spheres were also tested to measure terminal velocity, for comparison with theoretical values, and to determine the stratification within the operating device. In subsequent tests broken glass fragments were placed into the running device and the drop out patterns were recorded versus terminal velocities. The theoretical values for glass were next compared to predicted terminal velocities as determined by a regression model.

Cloth specimens of the sizes previously run were also tested in the device. In each test 25 specimens of the same size were placed into the running device. This was repeated several times for all of the different sizes of cloth. Finally a mixture of different cloth sizes was tested.
CHAPTER VI

EXPERIMENTAL RESULTS

Phase One: Calibration of Classifier

In order to evaluate the performance of the classifier design, the terminal velocities and the velocity distribution for a number of different materials were determined. The aerodynamic effects of turbulence, separation, etc. may preclude the stratification of the materials and subsequently defeat the objective of this research.

The original device consisted of a variable opening chamber that could be positioned from 0° to 90° of opening angle (see Figure 14). Pressure taps were spaced on 3 inch centers both vertically and horizontally (see Figure 15). The different velocities within the device are shown in Figure 17 as LV and HV.

Mathematically, this velocity relationship under laminar flow conditions is represented as:

\[ A_n V_n = A_p V_b \] (61)

\[ V_n = \frac{A_p}{A_n} V_b \] (62)

A = Horizontal Area
V = Vertical Velocity (see Figure 17)
FIGURE 17. SKETCH OF FUNNEL SHAPED CLASSIFIER AND THEORETICAL VELOCITIES

Table 5 shows the areas, $A_n$, as a function of the V-shaped device opening angle, $\theta$, and position, and also shows the $A_b A_n$ ratio (see Figure 15). Note that $A_b$ is a plane at the lowest position and is designated line number 10 (area of 8.25 square inches).

The results of run #1 are included in Table 6. The results of the run obviously indicate that the flow within the V-device does not behave in a manner predicted by equation (62). This was a result due to the large opening angle of 19.4° and the subsequent separation of the flow and the inherent turbulence.

Runs #2 through #5 reflect similar disparities. It was believed that reducing the opening angle would provide a closer agreement of the theoretical and experimental results. This reduced opening was accomplished on run #6 and is tabulated in Table 7.
<table>
<thead>
<tr>
<th>Line</th>
<th>Opening 0°</th>
<th>$A_n/A_b$</th>
<th>Opening 4.25°</th>
<th>$A_n/A_b$</th>
<th>Opening 6.90°</th>
<th>$A_n/A_b$</th>
<th>Opening 11.70°</th>
<th>$A_n/A_b$</th>
<th>19.73°</th>
<th>$A_n/A_b$</th>
<th>32.81°</th>
<th>$A_n/A_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.75</td>
<td>1.0</td>
<td>4.7500</td>
<td>.5789</td>
<td>6.0</td>
<td>.45833</td>
<td>8.2500</td>
<td>.3333</td>
<td>12.000</td>
<td>.2292</td>
<td>18.000</td>
<td>.1528</td>
</tr>
<tr>
<td>2</td>
<td>2.75</td>
<td>1.0</td>
<td>4.5276</td>
<td>.6074</td>
<td>5.6388</td>
<td>.4877</td>
<td>7.6388</td>
<td>.3600</td>
<td>10.9724</td>
<td>.2506</td>
<td>16.3052</td>
<td>.1687</td>
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<td>1.0</td>
<td>4.3054</td>
<td>.6387</td>
<td>5.2777</td>
<td>.5211</td>
<td>7.0277</td>
<td>.3913</td>
<td>9.9446</td>
<td>.2765</td>
<td>14.6108</td>
<td>.1882</td>
</tr>
<tr>
<td>4</td>
<td>2.75</td>
<td>1.0</td>
<td>4.0832</td>
<td>.6735</td>
<td>4.9166</td>
<td>.5593</td>
<td>6.4166</td>
<td>.4286</td>
<td>8.9168</td>
<td>.3804</td>
<td>12.9164</td>
<td>.2129</td>
</tr>
<tr>
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<td>2.75</td>
<td>1.0</td>
<td>3.8610</td>
<td>.7123</td>
<td>4.5555</td>
<td>.6037</td>
<td>5.8055</td>
<td>.4737</td>
<td>7.8890</td>
<td>.3486</td>
<td>11.2220</td>
<td>.2451</td>
</tr>
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<td>2.75</td>
<td>1.0</td>
<td>3.6388</td>
<td>.7557</td>
<td>4.1944</td>
<td>.6556</td>
<td>5.1944</td>
<td>.5294</td>
<td>6.8612</td>
<td>.4008</td>
<td>9.5276</td>
<td>.2886</td>
</tr>
<tr>
<td>7</td>
<td>2.75</td>
<td>1.0</td>
<td>3.4166</td>
<td>.8049</td>
<td>3.8333</td>
<td>.7174</td>
<td>4.5833</td>
<td>.6000</td>
<td>5.8334</td>
<td>.4714</td>
<td>7.8332</td>
<td>.3511</td>
</tr>
<tr>
<td>8</td>
<td>2.75</td>
<td>1.0</td>
<td>3.1944</td>
<td>.8609</td>
<td>3.4720</td>
<td>.7921</td>
<td>3.9722</td>
<td>.6923</td>
<td>4.8056</td>
<td>.5722</td>
<td>6.1388</td>
<td>.4480</td>
</tr>
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<td>9</td>
<td>2.75</td>
<td>1.0</td>
<td>2.9722</td>
<td>.9252</td>
<td>3.1111</td>
<td>.8839</td>
<td>3.611</td>
<td>.7615</td>
<td>3.778</td>
<td>.7279</td>
<td>4.4444</td>
<td>.6188</td>
</tr>
<tr>
<td>10</td>
<td>2.75</td>
<td>1.0</td>
<td>2.7500</td>
<td>1.000</td>
<td>2.7500</td>
<td>1.000</td>
<td>2.7500</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

*The opening at the top is at line 1.*
### Table 6. Run #1 Results for $\theta = 19.7^\circ$

<table>
<thead>
<tr>
<th>Line</th>
<th>Experimental Velocity</th>
<th>Theoretical Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>129.75</td>
<td>37.39</td>
</tr>
<tr>
<td>2</td>
<td>130.62</td>
<td>40.89</td>
</tr>
<tr>
<td>3</td>
<td>130.62</td>
<td>45.11</td>
</tr>
<tr>
<td>4</td>
<td>133.21</td>
<td>62.06</td>
</tr>
<tr>
<td>5</td>
<td>133.21</td>
<td>56.87</td>
</tr>
<tr>
<td>6</td>
<td>134.91</td>
<td>63.39</td>
</tr>
<tr>
<td>7</td>
<td>136.58</td>
<td>76.91</td>
</tr>
<tr>
<td>8</td>
<td>141.49</td>
<td>93.35</td>
</tr>
<tr>
<td>9</td>
<td>148.55</td>
<td>118.76</td>
</tr>
<tr>
<td>10</td>
<td>163.15</td>
<td>163.15</td>
</tr>
</tbody>
</table>

### Table 7. Run #6 Results for $\theta = 32.81^\circ$

<table>
<thead>
<tr>
<th>Line</th>
<th>Experimental Velocity</th>
<th>Theoretical Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>97.62</td>
<td>75.54</td>
</tr>
<tr>
<td>2</td>
<td>97.65</td>
<td>79.26</td>
</tr>
<tr>
<td>3</td>
<td>98.81</td>
<td>83.35</td>
</tr>
<tr>
<td>4</td>
<td>101.08</td>
<td>87.89</td>
</tr>
<tr>
<td>5</td>
<td>102.20</td>
<td>92.95</td>
</tr>
<tr>
<td>6</td>
<td>104.40</td>
<td>98.61</td>
</tr>
<tr>
<td>7</td>
<td>107.61</td>
<td>105.03</td>
</tr>
<tr>
<td>8</td>
<td>110.73</td>
<td>112.34</td>
</tr>
<tr>
<td>9</td>
<td>118.65</td>
<td>120.73</td>
</tr>
<tr>
<td>10</td>
<td>130.49</td>
<td>130.49</td>
</tr>
</tbody>
</table>
The results of run #6 more closely reflect the desired results; however, this angle appears to still be too large causing the turbulence to remain high.

The effects of screens were evaluated during runs 7 and 8 to determine if these could be used to reduce the turbulence. Note also that run #8 was accomplished at a slower velocity than previous runs in order to determine if velocity changes would impact on the desired results (see Table 8).

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Run #7 ($\theta = 4.25^\circ$)</th>
<th>Run #8 ($\theta = 4.25^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_{\text{exp}}$</td>
<td>$V_{\text{theo}}$</td>
</tr>
<tr>
<td>1</td>
<td>99.81</td>
<td>76.04</td>
</tr>
<tr>
<td>2</td>
<td>99.95</td>
<td>79.79</td>
</tr>
<tr>
<td>3</td>
<td>102.20</td>
<td>83.90</td>
</tr>
<tr>
<td>4</td>
<td>103.30</td>
<td>88.47</td>
</tr>
<tr>
<td>5</td>
<td>105.48</td>
<td>93.57</td>
</tr>
<tr>
<td>6</td>
<td>108.66</td>
<td>99.27</td>
</tr>
<tr>
<td>7</td>
<td>110.73</td>
<td>105.73</td>
</tr>
<tr>
<td>8</td>
<td>113.76</td>
<td>113.09</td>
</tr>
<tr>
<td>9</td>
<td>119.60</td>
<td>121.53</td>
</tr>
<tr>
<td>10</td>
<td>131.36</td>
<td>131.36</td>
</tr>
</tbody>
</table>

Screening or screening with reduced velocities shows improved results.

Experiments #9 through #13 are similar to previous runs and the results are also similar.

Runs #14 through #22 were used to evaluate the velocity patterns both vertically and horizontally. Although the velocities do not vary according to $A_n V_n = A_b V_b$
vertically, the velocities of the different lines did remain relatively constant. This indicates that stratifications of MSW type material would probably occur within the device.

**Phase Two: Analysis of Spheres in the Classifier**

Runs #23 and #24 were conducted to check the terminal velocity of a single sphere (ping pong ball) in the classifier. The results of these runs were compared to runs #25 through #29 to determine if choking was occurring. The weights of the spheres ranged from 1.7314 to 1.9409 grams and average terminal velocity for the two runs ranged from 15.99 to 19.87 feet per second. The overall average terminal velocity was 18.98 feet per second. Table 9 shows the results of runs #23 through #29.

<table>
<thead>
<tr>
<th>Run No.</th>
<th>No. of Spheres</th>
<th>Pressure Δh(in.)</th>
<th>Velocity V_t(Ft/Sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>23 &amp; 24</td>
<td>1</td>
<td>.0775</td>
<td>18.98</td>
</tr>
<tr>
<td>25</td>
<td>2</td>
<td>.10</td>
<td>21.56</td>
</tr>
<tr>
<td>26</td>
<td>3</td>
<td>.12</td>
<td>23.61</td>
</tr>
<tr>
<td>27</td>
<td>4</td>
<td>.14</td>
<td>25.51</td>
</tr>
<tr>
<td>28</td>
<td>5</td>
<td>.15</td>
<td>26.40</td>
</tr>
<tr>
<td>29</td>
<td>6</td>
<td>.16</td>
<td>27.27</td>
</tr>
</tbody>
</table>

Data in Table 9 indicates that increasing the number of spheres in the device will affect the terminal velocity of the first drop. Obviously a choking effect is present.
in the V-device and the more spheres the higher the required terminal velocity. However, in order to evaluate the aerodynamic effects of multiple specimens it is necessary to evaluate first the effects of single spheres in the V-device.

The experiments numbered 30 to 33 were used to check the calculated velocity of individual spheres in the V-device with experimental results. New ping pong balls were partially filled with water by using a hypodermic needle. The hole made by the needle was then covered with a small (1/4" x 1/4") piece of tape. The balls were weighed individually and then suspended in the V-device. The terminal velocity was determined from the static pressure port in the throat of the device. The theoretical terminal velocity of the spheres was also computed and the two velocities were compared. During these runs the temperature remained at 77°F (298°K) and the barometric pressure was 29.19 inches of mercury. The computed density was 0.0022406 slugs per cubic foot.

Since the size of the spheres (1 inch diameter) was significant when compared in the cross-sectional area of the throat (3 in. x 2.75 in.), correction for choking was included in the analysis of the data.

As noted in Chapter II the Kermack, McKendrick, and Ponder correction factor is usually considered the best available estimator for choking. As shown in equation (8):
Since the throat of the classifier has a rectangular cross-section \((\gamma)\) is computed using the following approximation for the effective radius:

\[
\frac{r_e}{r_{\text{Effective}}} = \frac{(2.75+3.00)/2}{2} = 1.4375 \text{ inches}
\]

The effective volume one inch in height \((h)\) is therefore:

\[
\text{Volume} = \pi r_e^2 h = \pi (1.4375)^2 (1) = 6.4918 \text{ in.}^3
\]

The volume of a one inch sphere is equal 0.5236 in\(^3\) and \(\gamma\) is computed as follows:

\[
\gamma = \frac{\text{VolSphere}}{\text{VolDevice}} = \frac{.5236}{6.4918} = 0.08065
\]

and the correction factor is:

\[
f = (1-\gamma^{2/3})(1-\gamma)(1-2.5\gamma) = 0.5969
\]

With this factor the theoretical terminal velocity can be determined from
As well, the experimental drag coefficient, \( C_D \), can be computed from

\[
C_D = \frac{2f^2 \rho \omega (\text{Vol}g)}{\rho a S V_t^2}
\]  

(69)

Both \( V_t \) and \( C_D \) are tabulated in Table 12. (Note: for \( V_t \), the \( C_D \) of 0.50 was approximated using Figure 3).

### Table 10. Terminal Velocity and \( C_D \) for Spheres of Selected Densities

<table>
<thead>
<tr>
<th>Ball Number</th>
<th>Weight (Grams)</th>
<th>( V_t^* ) (Ft/Sec)</th>
<th>( V_t ) (Ft/Sec)</th>
<th>( C_D )</th>
<th>( (4-3)/3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.3649</td>
<td>30.47</td>
<td>30.39</td>
<td>.507</td>
<td>+0.3%</td>
</tr>
<tr>
<td>2</td>
<td>3.7401</td>
<td>31.96</td>
<td>32.04</td>
<td>.513</td>
<td>-0.3%</td>
</tr>
<tr>
<td>3</td>
<td>5.5615</td>
<td>40.31</td>
<td>39.08</td>
<td>.480</td>
<td>+3.1%</td>
</tr>
<tr>
<td>4</td>
<td>6.2018</td>
<td>42.55</td>
<td>41.27</td>
<td>.480</td>
<td>+3.0%</td>
</tr>
<tr>
<td>5</td>
<td>8.6451</td>
<td>48.18</td>
<td>48.72</td>
<td>.522</td>
<td>-1.1%</td>
</tr>
<tr>
<td>6</td>
<td>11.0188</td>
<td>54.51</td>
<td>55.01</td>
<td>.520</td>
<td>-0.9%</td>
</tr>
<tr>
<td>7</td>
<td>12.8108</td>
<td>56.60</td>
<td>59.32</td>
<td>.560</td>
<td>-4.8%</td>
</tr>
<tr>
<td>8</td>
<td>13.9766</td>
<td>61.70</td>
<td>61.95</td>
<td>.515</td>
<td>-0.4%</td>
</tr>
<tr>
<td>9</td>
<td>15.9747</td>
<td>64.28</td>
<td>66.23</td>
<td>.542</td>
<td>-3.0%</td>
</tr>
<tr>
<td>10</td>
<td>18.2597</td>
<td>68.13</td>
<td>70.81</td>
<td>.551</td>
<td>-3.9%</td>
</tr>
<tr>
<td>11</td>
<td>19.5959</td>
<td>73.70</td>
<td>73.34</td>
<td>.505</td>
<td>-0.5%</td>
</tr>
<tr>
<td>12</td>
<td>29.9701</td>
<td>83.45</td>
<td>84.46</td>
<td>.523</td>
<td>-1.2%</td>
</tr>
</tbody>
</table>

Ave \( C_D = 0.518 \)

*\( V_t^* \): computed from experimental data.
An analysis of Table 10 shows a range of percent error from -4.8 to +3.1 percent, a $C_D$ range from 0.480 to 0.560 and average 0.518. The percent error is very small and well within the expected limits when considering the experimental equipment and accuracy of manometer readings. The average $C_D$ of 0.518 is an excellent value for the drag coefficient for the spheres tested in the Reynolds' Number area of interest.

With the average drag coefficient as noted in Table 10 the choking factor for multiple spheres can now be determined by trial and error using the following formula:

$$V_t = f \left[ \frac{2\rho_s g(Vol)}{\rho a SC_D} \right]^{1/2} \quad (70)$$

$$f = V_t \left[ \frac{\rho a SC_D}{2\rho_s g(Vol)} \right]^{1/2} \quad (71)$$

Note from Table 9 the increase in $V_t$ as the number of spheres increases and since $\rho_a$, $C_D$, $S$, and $\rho_s$ are constant, $f$ will necessarily be different in each case. This varying $f$ demonstrates that different sized volumes within the device are affected by changing the number of spheres in the classifier. Table 11 depicts the changing $f$ values as a function of $V_t$ and was computed using equation (71).

Again using the works of Kermack, McKendrick, and Ponder the choking factor, $f$, can be used to determine $\gamma$ as follows:
\[ f = (1 - y^{2/3})(1 - y)(1 - 2.5y) \] (72)

**Table 11. Choking Factors as a Function of Terminal Velocity**

<table>
<thead>
<tr>
<th>No. of Spheres</th>
<th>( V_t ) (Ft/Sec)</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18.98</td>
<td>0.5301</td>
</tr>
<tr>
<td>2</td>
<td>21.56</td>
<td>0.6022</td>
</tr>
<tr>
<td>3</td>
<td>23.61</td>
<td>0.6595</td>
</tr>
<tr>
<td>4</td>
<td>25.51</td>
<td>0.7125</td>
</tr>
<tr>
<td>5</td>
<td>26.40</td>
<td>0.7374</td>
</tr>
<tr>
<td>6</td>
<td>27.27</td>
<td>0.7617</td>
</tr>
</tbody>
</table>

Since \( f \) is known, \( y \) can be determined and subsequently so can the effected volume within the device. Table 12 shows these values.

As noted in Table 12 the relationship between volume of the spheres in the device and the effected volume within the V-device is non-linear. Although not computed for this specific device and set of spheres, the basic relationship is of the following type:

\[ \text{Effective Vol}_{\text{Device}} = K(\text{Vol}_{\text{Materials}})^n \] (73)

Also noted is the fact that increasing the volume of materials will affect the height of the effected air stream in the device. Increasing the material loading within separators will require increasing the available volume. This may require full-scale separators to be rather tall devices.
TABLE 12. CHOKING AND AFFECTED VOLUMES

<table>
<thead>
<tr>
<th>No. of Spheres</th>
<th>f</th>
<th>γ</th>
<th>Vol ( \text{in.}^3 )</th>
<th>h** (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.5301</td>
<td>.0999</td>
<td>5.241</td>
<td>.8073</td>
</tr>
<tr>
<td>2</td>
<td>.6023</td>
<td>.0793</td>
<td>13.206</td>
<td>2.034</td>
</tr>
<tr>
<td>3</td>
<td>.6595</td>
<td>.0644</td>
<td>24.391</td>
<td>3.757</td>
</tr>
<tr>
<td>4</td>
<td>.7125</td>
<td>.0514</td>
<td>40.747</td>
<td>6.277</td>
</tr>
<tr>
<td>5</td>
<td>.7374</td>
<td>.0456</td>
<td>57.412</td>
<td>8.844</td>
</tr>
<tr>
<td>6</td>
<td>.7617</td>
<td>.0402</td>
<td>78.149</td>
<td>12.030</td>
</tr>
</tbody>
</table>

*Vol\( _1 \) is the computed affected volume within the device.

**h is the height above the throat of the device of the affected volume of air.

Phase Three: Aerodynamics of Flat Plates

Runs #34 through #58 were conducted in order to evaluate the effects of shape and choking upon a representative square plates. For these runs steel plates were used. Table 13 shows the mean terminal velocities for different quantities of varying sized specimens.

An evaluation of Table 13 indicates that:

1. The mean terminal velocity generally increases with increased numbers of specimens in the classifier.
2. The mean terminal velocity generally increases with increased size.
3. The larger squares (1 1/2 x 1 1/2 in.) may be too large for the device as noted by the nonconformity of the measured terminal velocities of these squares.
TABLE 13. TERMINAL VELOCITY AS A FUNCTION OF SIZE AND NUMBER OF STEEL SPECIMENS IN THE CLASSIFIER

<table>
<thead>
<tr>
<th>No. in</th>
<th>1/4 x 1/4 (inch)</th>
<th>1/2 x 1/2 (inch)</th>
<th>3/4 x 3/4 (inch)</th>
<th>1 x 1 (inch)</th>
<th>1 1/2 x 1 1/2 (inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>42.98</td>
<td>43.84</td>
<td>43.84</td>
<td>46.50</td>
<td>41.23</td>
</tr>
<tr>
<td>2</td>
<td>42.34</td>
<td>44.88</td>
<td>45.49</td>
<td>49.57</td>
<td>42.77</td>
</tr>
<tr>
<td>3</td>
<td>46.89</td>
<td>45.90</td>
<td>46.30</td>
<td>49.01</td>
<td>47.87</td>
</tr>
<tr>
<td>5</td>
<td>47.67</td>
<td>50.50</td>
<td>47.67</td>
<td>51.76</td>
<td>53.17</td>
</tr>
<tr>
<td>10</td>
<td>46.89</td>
<td>50.13</td>
<td>48.06</td>
<td>51.76</td>
<td>53.17</td>
</tr>
</tbody>
</table>

Runs #59 through #98 were accomplished with aluminum in order to substantiate the results obtained in runs #34 through 58. Table 14 shows these results.

TABLE 14. TERMINAL VELOCITY AS A FUNCTION OF SIZE AND NUMBER OF ALUMINUM SPECIMENS IN THE CLASSIFIER

<table>
<thead>
<tr>
<th>No. in</th>
<th>1/4 x 1/4 (inch)</th>
<th>1/2 x 1/2 (inch)</th>
<th>3/4 x 3/4 (inch)</th>
<th>1 x 1 (inch)</th>
<th>1 1/2 x 1 1/2 (inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.39</td>
<td>26.58</td>
<td>24.77</td>
<td>24.77</td>
<td>25.14</td>
</tr>
<tr>
<td>2</td>
<td>25.87</td>
<td>27.27</td>
<td>25.51</td>
<td>27.27</td>
<td>24.01</td>
</tr>
<tr>
<td>5</td>
<td>27.27</td>
<td>28.27</td>
<td>29.24</td>
<td>27.61</td>
<td>24.39</td>
</tr>
<tr>
<td>10</td>
<td>30.18</td>
<td>28.60</td>
<td>29.24</td>
<td>28.92</td>
<td>29.87</td>
</tr>
</tbody>
</table>

The aluminum plates behaved similar to the steel plates, however, the nonconformal behavior started with the 1 x 1 rather than 1 1/2 x 1 1/2 inch plates.

Runs #84 through #98 were conducted in order to investigate the effects of changing aspect ratios, of length over width. Table 15 shows the terminal velocities of varying quantities of specimens versus aspect ratios.
An evaluation of Table 15 indicates that:

1. The terminal velocity increases with increasing numbers of specimen in the device for sizes up to 1 inch in maximum dimensions. Specimens 1 1/2 inch in length displayed nonuniform behavior in the classifier.

2. The terminal velocity increases with increasing aspect ratio. As the aspect ratio increases so also does the size. However, an interaction of size and shape may exist; unfortunately, these experiments as conducted do not evaluate this possibility.

Experiments numbered 99 to 202 were conducted as a pilot study in order to develop some insight into the behavior of the different materials within the device. The unique terminal velocities for the different materials and sizes were evaluated with respect to the hypothesis developed in Chapter I.

The major conclusions from these studies were:

1. The theoretical terminal velocity of spheres varied from the experimental results by a constant factor.
It was believed that this was caused by the effects of choking within the device.

2. The terminal velocity increases slightly as the size of the specimens increases.

3. Increasing the size of the specimens increased the standard deviation of the terminal velocities.

4. A definite relationship between specimen density and terminal velocity does exist with higher density.

5. Air separation of materials of different densities was demonstrated in a V-shaped classifier.

These experiments also provided the author with the data and experimental proficiency necessary to design and complete the experimental work required to answer the hypothesis developed in Chapter I.

Phase Four: The Experimental Design

This phase of the experimentation was designed to answer the research questions posed in Chapter I. The experimental design was constructed so that the effects of size, shape, and density could be independently evaluated. This was accomplished by the use of the Three-Way Analysis of Variance (ANOVA) design.

This particular model consisted of 320 experimental tests and Table 16 shows the general arrangement of this particular design. Note that for each of the four sizes (0.0625, 0.250, 0.5625, and 1.000 square inches) four different aspect ratios (1 to 1, 2 to 1, 3 to 1, and 4 to 1)

85
TABLE 16. EXPERIMENTAL DESIGN

<table>
<thead>
<tr>
<th>SIZE</th>
<th>0.0625 (in.²)</th>
<th>0.250 (in.²)</th>
<th>0.5625 (in.²)</th>
<th>1.000 (in.²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHAPE *</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>Steel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cardboard</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Balsa Wood</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paper</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Shape: Ratio of length to width

A: 1 to 1  
B: 2 to 1  
C: 3 to 1  
D: 4 to 1  

were tested for each material four times. Table 17 presents an enlargement of a portion of the experimental test plan. It shows how the computed average experimental terminal velocities \(A_1, A_2, A_3, A_4\) for the four tests on each material were recorded. For this example steel specimens of size 0.0625 square inches of aspect ratio 1.000 is used. All of this data provided the input for the statistical analysis.

TABLE 17. AN ENLARGEMENT OF A PORTION OF THE EXPERIMENTAL DESIGN

<table>
<thead>
<tr>
<th>SHAPE (Aspect Ratio)</th>
<th>0.0625 (in.²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A (1-1)</td>
</tr>
<tr>
<td>STEEL</td>
<td>(\frac{A_1}{A_2})</td>
</tr>
<tr>
<td>ALUMINUM</td>
<td></td>
</tr>
</tbody>
</table>
The actual experimental results are included as part of Appendix A (pp. 215-218). A description of the ANOVA is included as part of Appendix B.

Utilizing the equations developed in Chapter III and the drag coefficients for nonspherical bodies presented in Chapter II the terminal velocities were computed and compared with experimental results (see Figure 19).

The theoretical terminal velocities were calculated using equation (60):

\[ V_t = \left( \frac{2 \rho_w L g}{\rho_a C_D} \right)^{1/2} \]  

(60)

Although \( \rho_w \) and \( \rho_a \) were known, the appropriate values for \( L \) and \( C_D \) had to be determined. This was accomplished by using Figure 4, \( C_D \) Versus RN for Different \( \psi \) Values. Unfortunately, in this type of an approach drag coefficients are dependent upon the Reynolds Number which is dependent upon the appropriate length and the flow velocity. Therefore the experimental average velocities for each material, shape, and size combinations was utilized in the Reynolds Number determination. This procedure is justified since the drag coefficient is a relatively weak function of Reynolds Number in the region of operation. The drag coefficient, \( C_D \), varies only slightly within the Reynolds Number range from \( 10^3 \) to \( 10^6 \).
The representative length dimensions were determined during the computations of the ψ values when determining equivalent sphere sizes for the different samples. This was accomplished by:

1. determining the area for each of the different plate sizes.
2. computing the radius of a sphere of equal area to the flat plate.
3. calculating L, the length, which is equal to the diameter of the calculated sphere (as referenced in Figure 3).

The ψ values were determined as described in Chapter II, equation (9). Figure 4 was used to determine drag coefficients. However, it should be recognized that the drag coefficient values were selected from a graph and are approximate. The effects of these approximations are not significant since the square root of this value is used for terminal velocity determinations.

The Reynolds Numbers, Ψ values, appropriate lengths, drag coefficients, and terminal velocities obtained are presented in Table 18.

After the 320 experimental tests had been completed the pressure data were used to calculate velocities using the formula described in Chapter III. These data were then grouped into 5 feet/second intervals for the 1600 specimens tested. As noted before, both in the literature and during
## TABLE 18. DATA DEVELOPED FOR CALCULATING THEORETICAL VELOCITIES

<table>
<thead>
<tr>
<th>Material</th>
<th>Size--in²</th>
<th>.0625</th>
<th>.250</th>
<th>.5625</th>
<th>1.000</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>STEEL</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ=15.14 slugs/ft³</td>
<td>RN</td>
<td>1606</td>
<td>2780</td>
<td>3825</td>
<td>5193</td>
</tr>
<tr>
<td></td>
<td>ψ</td>
<td>.436</td>
<td>.304</td>
<td>.241</td>
<td>.202</td>
</tr>
<tr>
<td></td>
<td>CD</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>.0122</td>
<td>.0194</td>
<td>.025</td>
<td>.032</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>31.1</td>
<td>39.2</td>
<td>44.5</td>
<td>50.35</td>
</tr>
<tr>
<td><strong>ALUMINUM</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ=5.33 slugs/ft³</td>
<td>RN</td>
<td>875</td>
<td>1425</td>
<td>2072</td>
<td>2712</td>
</tr>
<tr>
<td></td>
<td>ψ</td>
<td>.471</td>
<td>.333</td>
<td>.266</td>
<td>.224</td>
</tr>
<tr>
<td></td>
<td>CD</td>
<td>6.0</td>
<td>6.0</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>.0129</td>
<td>.0200</td>
<td>.026</td>
<td>.032</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>17.6</td>
<td>21.9</td>
<td>25.0</td>
<td>27.7</td>
</tr>
<tr>
<td><strong>BALSA WOOD</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ=0.389 slugs/ft</td>
<td>RN</td>
<td>190</td>
<td>389</td>
<td>759</td>
<td>1040</td>
</tr>
<tr>
<td></td>
<td>ψ</td>
<td>.614</td>
<td>.470</td>
<td>.387</td>
<td>.232</td>
</tr>
<tr>
<td></td>
<td>CD</td>
<td>6.5</td>
<td>6.5</td>
<td>6.5</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td>L</td>
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<td>.026</td>
<td>.033</td>
<td>.041</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>5.1</td>
<td>6.5</td>
<td>7.3</td>
<td>8.1</td>
</tr>
<tr>
<td><strong>CARDBOARD</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ=1.125 slugs/ft³</td>
<td>RN</td>
<td>173</td>
<td>308</td>
<td>514</td>
<td>769</td>
</tr>
<tr>
<td></td>
<td>ψ</td>
<td>.440</td>
<td>.307</td>
<td>.244</td>
<td>.204</td>
</tr>
<tr>
<td></td>
<td>CD</td>
<td>7.0</td>
<td>7.0</td>
<td>7.0</td>
<td>7.0</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>.0122</td>
<td>.0195</td>
<td>.025</td>
<td>.030</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>7.3</td>
<td>9.2</td>
<td>10.4</td>
<td>11.4</td>
</tr>
<tr>
<td><strong>PAPER</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ=1.051 slugs/ft³</td>
<td>RN</td>
<td>18</td>
<td>26</td>
<td>37</td>
<td>47</td>
</tr>
<tr>
<td></td>
<td>ψ</td>
<td>.144</td>
<td>.092</td>
<td>.071</td>
<td>.058</td>
</tr>
<tr>
<td></td>
<td>CD</td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>.0064</td>
<td>.011</td>
<td>.013</td>
<td>.016</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>2.0</td>
<td>2.6</td>
<td>2.9</td>
<td>3.2</td>
</tr>
</tbody>
</table>

L = Feet

V = Ft/Sec

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the experiments, a size limitation on the specimens does exist due to the small size of the experimental apparatus. Data on pages 215—218 show specimens of 1 1/2 inch and longer exhibiting a significantly higher terminal velocity than did the smaller specimens. During the tests flat plate specimens of high aspect ratios tended to rotate more about the longer axis than the shorter axis. Also when these high aspect ratio specimens became vertical and streamlined with the air flow they tended to drop out immediately. The collisions with the walls often caused large samples to streamline and subsequently drop. Therefore, in order to eliminate the effects of the oversized specimens, these data were eliminated in the calculations. The 1200 experimental points for each material were then normalized so that the distribution of each material could be plotted as a function of terminal velocity. These data are included as Tables 19, 20, and 21 and are graphically presented in Figure 18. From Figure 18 a "smooth" curve is fitted to the data to derive Figure 19. This may be more representative of the actual data since interval measures were used in determining Figure 18. Figure 19 also shows the values for the theoretical terminal velocities computed in Table 18.

Although the data depicted in Figure 19 does exhibit differences as a function of density for the different materials it does not reflect the effects of size or shape. The size effects are graphically portrayed in Figure 20 which shows the distributions of the four sizes of steel specimens as a function of terminal velocity.
### TABLE 19. EXPERIMENTAL DATA--STEEL

<table>
<thead>
<tr>
<th>Size</th>
<th>Shape</th>
<th>Terminal Velocity - Ft/Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>31-35</td>
</tr>
<tr>
<td>.0625 in²</td>
<td>1/4 x 1/4</td>
<td>5</td>
</tr>
<tr>
<td>.177 x .354</td>
<td>5</td>
<td>54</td>
</tr>
<tr>
<td>.144 x .433</td>
<td>5</td>
<td>57</td>
</tr>
<tr>
<td>.125 x .500</td>
<td>5</td>
<td>51</td>
</tr>
<tr>
<td>Number</td>
<td>5</td>
<td>167</td>
</tr>
<tr>
<td>Percent</td>
<td>1%</td>
<td>42%</td>
</tr>
<tr>
<td>.250 in²</td>
<td>1/2 x 1/2</td>
<td>1</td>
</tr>
<tr>
<td>.354 x .707</td>
<td>2</td>
<td>32</td>
</tr>
<tr>
<td>.289 x .866</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>.250 x 1.00</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>Number</td>
<td>4</td>
<td>63</td>
</tr>
<tr>
<td>Percent</td>
<td>1%</td>
<td>16%</td>
</tr>
<tr>
<td>.5625 in²</td>
<td>3/4 x 3/4</td>
<td>1</td>
</tr>
<tr>
<td>.530 x 1.061</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>.433 x 1.30</td>
<td>13</td>
<td>39</td>
</tr>
<tr>
<td>Number</td>
<td>14</td>
<td>99</td>
</tr>
<tr>
<td>Percent</td>
<td>5%</td>
<td>33%</td>
</tr>
<tr>
<td>1.00 in²</td>
<td>1 x 1</td>
<td>9</td>
</tr>
<tr>
<td>No. &amp; %</td>
<td>9</td>
<td>59</td>
</tr>
<tr>
<td>Total</td>
<td>Number</td>
<td>9</td>
</tr>
<tr>
<td>Percent</td>
<td>0%</td>
<td>21%</td>
</tr>
<tr>
<td>Size</td>
<td>Shape</td>
<td>Terminal Velocity - Ft/Sec</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
<td>---------------------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11-15</td>
</tr>
<tr>
<td>.0625 in²</td>
<td>1/4 x 1/4</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td>.177 x .354</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>.144 x .433</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>.125 x .500</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Number</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td>Percent</td>
<td>17%</td>
</tr>
<tr>
<td>.250 in²</td>
<td>1/2 x 1/2</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>.354 x .707</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>.289 x .866</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>.250 x 1.00</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Number</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>Percent</td>
<td>19%</td>
</tr>
<tr>
<td>.5625 in²</td>
<td>3/4 x 3/4</td>
<td>46</td>
</tr>
<tr>
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<td>.530 x 1.061</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>.433 x 1.30</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Number</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>Percent</td>
<td>18%</td>
</tr>
<tr>
<td>1.00 in²</td>
<td>1 x 1</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>No. &amp; %</td>
<td>36</td>
</tr>
<tr>
<td>Total</td>
<td>Number</td>
<td>231</td>
</tr>
<tr>
<td></td>
<td>Percent</td>
<td>19%</td>
</tr>
<tr>
<td>Size</td>
<td>Shape</td>
<td>Cardboard Terminal Velocities (Ft/Sec)</td>
</tr>
<tr>
<td>--------</td>
<td>----------</td>
<td>----------------------------------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>.0625 in²</td>
<td>1/4 x 1/4</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>.177 x .354</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>.144 x .433</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>.125 x .500</td>
<td>59</td>
</tr>
<tr>
<td>Number</td>
<td></td>
<td>214</td>
</tr>
<tr>
<td>Percent</td>
<td></td>
<td>54%</td>
</tr>
<tr>
<td>.250 in²</td>
<td>1/2 x 1/2</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>.354 x .707</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>.289 x .866</td>
<td>61</td>
</tr>
<tr>
<td></td>
<td>.250 x 1.00</td>
<td>32</td>
</tr>
<tr>
<td>Number</td>
<td></td>
<td>156</td>
</tr>
<tr>
<td>Percent</td>
<td></td>
<td>39%</td>
</tr>
<tr>
<td>.5625 in²</td>
<td>3/4 x 3/4</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>.530 x 1.061</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>.433 x 1.30</td>
<td>40</td>
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<tr>
<td>Number</td>
<td></td>
<td>98</td>
</tr>
<tr>
<td>Percent</td>
<td></td>
<td>33%</td>
</tr>
<tr>
<td>1.00 in²</td>
<td>1 x 1</td>
<td>4</td>
</tr>
<tr>
<td>No. &amp; %</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>Totals</td>
<td>Number</td>
<td>471</td>
</tr>
<tr>
<td>Percent</td>
<td></td>
<td>39%</td>
</tr>
</tbody>
</table>
FIGURE 18. MATERIAL DISTRIBUTION AS A FUNCTION OF TERMINAL VELOCITY
Note that larger sizes tend to exhibit higher terminal velocities; however, it is obvious from Figure 20 that air classification by size alone is not possible. This figure shows that size will have some bearing upon separation but is a much weaker function than density.

Data in Table 19 was also used to determine the effects of shape upon terminal velocities. This information is depicted in Figure 21.

Figure 21 shows that shape effects are not evident in the experimental results. This indicates that shape may not affect terminal velocities for flat plates as long as the specimen's longest dimension is less than one half the classifier "diameter."
The Three-Way Analysis of Variance showed that all main, two-way, and three-way effects are statistically significant at the 0.01 alpha level. This means that each term contributes to the overall effectiveness of the model (see Table 22). However, there are factors which may have caused the results to be significant.

One factor is that the explained variation accounts for 99.8 percent of the total variation, leaving a very small residual. Another factor is that the main effect of material accounts for 96.53 percent of the explained variation while size accounts for only 1.29 percent and shape accounts for only 0.23 percent of the explained variation. Because the residual variation was small and its number of
degrees of freedom (240) was large, the mean square of the residual was less than one. Consequently, the main effects of all interaction effects were significant.

To support the above conclusions a Multiple Classification Analysis (MCA), which shows the pattern of effects of each independent variable by showing the mean of each level as a deviation from the grand mean, was performed.

Table 23 shows that for material the range of deviation of velocities (Ft/Sec) is from 31.84 to -16.64, for size -2.47 to 2.93, and for shape from -1.14 to 1.16. The greater the range the stronger the variable is in describing the variation. This table also provides a descriptive statistic, $\eta^2$ (a measure of the strength of the effect of material, size, and shape on terminal velocity). Thus material explains $(.98)^2 = 96.04$ percent, size explains $(.11)^2 = 1.21$ percent, and

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F-Test</th>
</tr>
</thead>
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<tr>
<td>Main Effects</td>
<td></td>
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<td>103,814</td>
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<td>10,381</td>
<td>13,177</td>
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<td>25,551</td>
<td>32,434</td>
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<td>Shape</td>
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<td>102</td>
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<tr>
<td>2-Way Interactions</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Material - Size</td>
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<td>50</td>
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<td>44</td>
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<td>3-Way Interaction</td>
<td>409</td>
<td>36</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>Material - Size -Shape</td>
<td>409</td>
<td>36</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>Explained</td>
<td>105,876</td>
<td>79</td>
<td>1,340</td>
<td>1,701</td>
</tr>
<tr>
<td>Residual</td>
<td>189</td>
<td>240</td>
<td>.8</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>106,065</td>
<td>319</td>
<td>332</td>
<td></td>
</tr>
</tbody>
</table>
TABLE 23.  MULTIPLE CLASSIFICATION ANALYSIS (SIMPLIFIED)

<table>
<thead>
<tr>
<th>Variable and Category</th>
<th>Deviation from Grand Mean</th>
<th>Eta</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Material</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel</td>
<td>31.84</td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td>7.27</td>
<td></td>
</tr>
<tr>
<td>Balsa Wood</td>
<td>-11.81</td>
<td></td>
</tr>
<tr>
<td>Cardboard</td>
<td>-10.65</td>
<td></td>
</tr>
<tr>
<td>Paper</td>
<td>-16.65</td>
<td>0.98</td>
</tr>
<tr>
<td><strong>Size</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0625 in.²</td>
<td>- 2.47</td>
<td></td>
</tr>
<tr>
<td>0.250</td>
<td>- 1.29</td>
<td></td>
</tr>
<tr>
<td>0.5625</td>
<td>0.83</td>
<td></td>
</tr>
<tr>
<td>1.0000</td>
<td>2.93</td>
<td></td>
</tr>
<tr>
<td><strong>Shape</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Square</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td>2 by 1</td>
<td>- 1.14</td>
<td></td>
</tr>
<tr>
<td>3 by 1</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>4 by 1</td>
<td>1.16</td>
<td></td>
</tr>
</tbody>
</table>

shape explains \((.05)^2 = 0.25\) percent of terminal velocity variation.

The ANOVA in Table 22 is predicated upon the assumption of additivity of the individual effects of the "treatments." Since the known equation of terminal velocity (equation 17) is of the multiplicative type, it was deemed advisable to investigate the significance of a multiplicative ANOVA. In order to apply ANOVA technique to the data base and obtain a multiplicative result, a logarithmic function was utilized. This was accomplished by using the logarithm of the terminal velocities
found in the basic experimental design. The results of this transformed ANOVA are included as Table 24.

The results of the multiplicative model (Table 24) show that all main effects and interactions are significant. Just as in Table 22 the density (material) factor has a rather large F value when compared to either the main effects of size or shape. Also the interactions both two-way and three-way show relatively similar F values. These results seem to indicate, that even though there is strong evidence (equation 17) that a multiplicative model may be better than the additive type, there is little evidence that the model tested improves the analysis.

### Table 24. THREE-WAY ANOVA \( \log_{10} \text{VEL} \)

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>F-Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main Effects</td>
<td>105.499</td>
<td>10</td>
<td>10.550</td>
<td>1703.073</td>
</tr>
<tr>
<td>Material</td>
<td>103.756</td>
<td>4</td>
<td>25.939</td>
<td>4187.334</td>
</tr>
<tr>
<td>Size</td>
<td>1.533</td>
<td>3</td>
<td>0.511</td>
<td>82.488</td>
</tr>
<tr>
<td>Shape</td>
<td>0.210</td>
<td>3</td>
<td>0.070</td>
<td>11.310</td>
</tr>
<tr>
<td>2-Way Interactions</td>
<td>1.834</td>
<td>33</td>
<td>0.056</td>
<td>8.973</td>
</tr>
<tr>
<td>Material - Size</td>
<td>1.147</td>
<td>12</td>
<td>0.096</td>
<td>15.433</td>
</tr>
<tr>
<td>Material - Shape</td>
<td>0.313</td>
<td>12</td>
<td>0.026</td>
<td>4.217</td>
</tr>
<tr>
<td>Size - Shape</td>
<td>0.374</td>
<td>9</td>
<td>0.042</td>
<td>6.701</td>
</tr>
<tr>
<td>3-Way Interactions</td>
<td>0.648</td>
<td>36</td>
<td>0.018</td>
<td>2.906</td>
</tr>
<tr>
<td>Material - Size - Shape</td>
<td>0.648</td>
<td>36</td>
<td>0.018</td>
<td>2.906</td>
</tr>
<tr>
<td>Explained</td>
<td>107.981</td>
<td>79</td>
<td>1.367</td>
<td>220.651</td>
</tr>
<tr>
<td>Residual</td>
<td>1.487</td>
<td>240</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>109.468</td>
<td>319</td>
<td>0.343</td>
<td></td>
</tr>
</tbody>
</table>
The relatively large F value for density in both models is an indication of the high significance of the density factor and the smaller effects of size and shape. These conclusions are supported by the MCA of Table 23.

The statistical significance of the interactions is an interesting phenomenon, but not wholly unexpected. A review of equation 17 shows that terminal velocity is a function of density, volume, and a "representative" area. A definite relationship between volume and area does exist. This relationship may be turning up as an interaction of all three factors. In addition the different thicknesses of the specimens may also be contributing to these interactions. The researcher has been aware of the thickness differences and the fact that these differences could impact upon the ANOVA results. However, as noted in the analysis of the MCA, Table 23, more than 97 percent of the variability is explained by the model, which is an indication that the effects of thickness differences may be small in the range examined in these experiments.

These analyses provide an answer to the fourth hypothesis raised in Chapter I (The significant factors affecting the separation of MSW type components are density, size, and shape). Material density has shown to have a significant effect upon the terminal drop out velocity while size and shape have only a minor effect. The more dense the material becomes, the greater the terminal velocity and changes in size and/or shape affect the terminal velocity of the flat plates only slightly.
The statistical results are shown graphically in Figure 22. This shows that separation of different materials is a function of the density difference. This result answers the first hypothesis raised in Chapter I.

The second hypothesis in Chapter I considers the possibility for designing a mathematical model that can be used to accurately predict the terminal velocity of MSW type components. In the following paragraphs four different regression models are presented and discussed.
The staff at the University of Chicago created a computer package titled the Statistical Package for the Social Sciences (SPSS) which contains the step-wise multiple linear regression model used in this research. Material, size, and shape were treated as independent variables and terminal or drop out velocity \( V_t \) was the dependent variable. Two different type models were constructed. The first is the discrete type and utilized nominal data. The other three are of the continuous type.

The SPSS Program provided the coefficient of determination \( (R^2) \) for each multiple regression attempted. This \( R^2 \) is the total amount of variation that can be explained by the model. The "F" test was used to determine significance for each successive regression model. Each individual coefficient of regression was similarly evaluated using the Student's "t" test.

The final nominal data model (NDM) has an \( R^2 = 0.97878 \) and contains 10 significant factors plus a constant. The model is:

\[ V_t = -0.38294 + 48.49375(MD1) + 23.91422(MD2) + 5.40600(SZD4) + 5.99734(MD4) + 4.83953(MD3) + 3.29687(SZD3) - 2.3000(SHD2) - 1.61762(SHD3) + 1.18350(SZD2) - 0.72950(SHD1) \]  

(74)

where

\[ \begin{align*}
    MD1 &= 1 \text{ if steel, 0 otherwise} \\
    MD2 &= 1 \text{ if aluminum, 0 otherwise} \\
    SZD4 &= 1 \text{ if 1 square inch, 0 otherwise}
\end{align*} \]
MD4 = 1 if cardboard, 0 otherwise
MD3 = 1 if balsa wood, 0 otherwise
SZD3 = 1 if 0.5625 square inch, 0 otherwise
SHD2 = 1 if 1 by 2 rectangle, 0 otherwise
SZD3 = 1 if 1 by 3 rectangle, 0 otherwise
SZD2 = 1 if 0.25 square inch, 0 otherwise
SHD1 = 1 if square, 0 otherwise

The classical hypothesis tests substantiate the significance of the model.

F Test

\[ H_0: B_1 = B_2 = \ldots = B_n \quad \text{B = Mean Value in Feet/Sec} \]

\[ H_1: B_i \neq B_j \text{ for } i \neq j \]

\[ F_{\text{model}} = 1425.38122 \]

\[ F_{10, 309, .95} = 1.86 \]

therefore, reject \( H_0 \) and conclude that at the 95 percent level the regression is significant.

Similarly, the significance of each entering coefficient of regression can be determined by

\[ H_0: B_i = 0 \quad \text{B = Normalized Mean Value in Feet/Sec} \]

\[ H_1: B_i \neq 0 \]

The test statistic, \( t_{309, .95} \) is equal to 1.645 which is less than the \( t \) value of all samples. Therefore the \( H_0 \) is rejected and the coefficients are significant at the 95 percent level.
The first continuous data model is

\[ V_t = 1.90101 + 0.09255 \text{Material} + 5.79895 \text{Size} \]

\[ = 1.90101 + 0.09255 \rho_w + 5.79895 A \]  

(75)

where

\[ \rho_w = \text{Material} = \text{the density of the material in pounds per cubic foot, and} \]

\[ A = \text{Size} = \text{the size, length times width, of the specimen in square inches.} \]

Equation (75) is graphically shown in Figure 23. The size limits are 0.0625 and 1.00 square inches, the same as used during the experiments.

---

The theoretical results are shown in the graph, with different materials plotted as follows:

- Balsawood
- Cardboard
- Paper
- Aluminum
- Steel

The graph compares the model limits (EQ. 75), experimental results (Tables 19, 20, 21), and theoretical results (Table 13).

**FIGURE 23.** A COMPARISON OF MODEL, THEORETICAL, AND EXPERIMENTAL RESULTS AS A FUNCTION OF DENSITY
The validity of the model is reflected in the F test below:

\[ F_{\text{Model}} = 2965.96180 \]

\[ F_{317, .95}^2 = 3.03 \]

Similarly, the validity of each coefficient is reflected in the value, \( t_{317, .95} = 1.645 \), which is lower than either of the \( t_{\text{sample}} \) in the model. Again reject the \( H_0 \) and conclude that the coefficients are significant at the 95 percent level. The \( R^2 \) is equal to 0.94895 for this model.

As noted earlier in equation (17) the terminal velocity is a function of the square root of the density.

\[ V_t = \left( \frac{2 \rho_w (\text{Vol}) g}{\rho_a C_D S} \right)^{1/2} \]  

(17)

Since \( \rho_w \), g, and \( \rho_a \) can be considered to be known quantities, equation (17) could be reduced to

\[ V_t = K_1 \sqrt{\frac{\rho_w (\text{Vol})}{C_D S}} \]  

(76)

Then if \( C_D \) is relatively constant in the Reynolds Number area of interest for similarly shaped specimens and the \( (\text{Vol})/S \) ratio could be represented by a "typical length," 1, then

\[ V_t = K_2 \sqrt{\rho_w} \]  

(77)
And finally if it could be assumed the $l$ varied only slightly then

$$V_t = K_3 \sqrt{\rho_w}$$  \hspace{1cm} (78)

Using this relationship as the basis a new continuous regression model was constructed. This model uses size and the square root of the density as independent variables. This resulted in

$$V_t = -11.24643 + 2.58251 (\rho_w)^{1/2} + 5.79895A$$  \hspace{1cm} (79)

Since the theory shows that for a material of zero density the terminal velocity would be zero, the following force fitted linear regression model was constructed using the square root of the density as the only independent variable. This model which passes through the vertex exhibited a $R^2$ of 0.937 and resulted in

$$V_t = 1.90953(\rho_w)^{1/2}$$  \hspace{1cm} (80)

Equations (79) and (80) are plotted as Figure 24 which also includes the experimental and theoretical results derived earlier.

Care must be exercised in using the nominal data model for this provides difference data from a known baseline. In the model under study the base lines are paper, size 0.0625 square inches, and a 1 by 4 rectangle. Paper was selected since it exhibited the lowest terminal velocity of all the materials tested.
EXPERIMENTAL RESULTS AS A FUNCTION OF THE SQUARE ROOT OF THE DENSITY

FIGURE 24: A COMPARISON OF MODEL THEORETICAL, AND EXPERIMENTAL RESULTS (EQ. 79, 80) WITH THEORETICAL LIMITS (TABLES 19, 20, 21) FOR TERMINAL VELOCITY (ft/sec)

Square Root of Density - (Pounds/ft$^3$)$^{1/2}$

- Steel
- Aluminum
- Paper Cardboard
- Balsa Wood

- Model (Eq. 80)
- Theoretical Results (Table 18)
- Experimental Results (Tables 19, 20, 21)
The continuous models are probably a more practical tool since only the material density and size are required as entering arguments and the third model, equation (80) may be the most practical. With caution this model can be used for evaluating materials other than those studied in these experiments.

These last models provide a means of accurately predicting the terminal velocities of MSW type materials other than those used in this study and answer the second research hypothesis in the affirmative.

The third hypothesis raised concerns the effect of increasing specimen size upon the standard deviation (SD) of the terminal velocity on MSW type material. The original data was combined into 100 observations per material, size, and shape combination. From this the standard deviations for all 80 combinations were computed using

\[ SD = \sqrt{\frac{\sum x^2 - (\sum x)^2}{N-1}} \]  

These standard deviations were used as inputs into the previously described additive ANOVA program. The Three-Way ANOVA indicates the significance of size. The MCA table reflected a deviation of -1.11 to 1.33 feet per second for the test specimens with the smaller sizes having smaller standard deviations of terminal velocities.
This statistical test shows that there is significant difference in the standard deviations of the terminal velocity as a function of size, which answers the third hypothesis in the positive. This result supports the graph of the experimental data shown in Figure 20.

Phase Five: Materials Mixed Experiments

A mixture of sizes and shapes of one particular material was put into the classifier. The material exhibited the same spinning, wobbling, etc. as observed previously. In fact even the large sized specimens (1.00 in\(^2\)) with high aspect ratios (1-4) again dropped out at slightly higher velocities than the other specimens. The terminal velocities were the same as those observed in the earlier experiments.

In addition, a mixture of all materials in all sizes and shapes was put into the classifier. Unfortunately the lighter material adhered to the top screen and trapped some of the smaller heavies. However, by varying the air speed it was possible to drop the steel into one tray, the aluminum into another, and the cardboard and balsa wood into a third tray. The separations were very "clean," particularly the steel and aluminum drops. Some small steel and aluminum specimens were dropped in with the cardboard and balsa wood.
These heavies were obviously trapped on the top screen and were observed falling out immediately after being released from the screen.

It was also noted that the average specimen weights in each catch tray decreased as the velocity was reduced. This indicates a slight difference in the terminal velocities as a function of specimen size. The larger items required slightly higher velocities. This is in part due to the small size of the equipment relative to the size of the larger specimens and the fact that larger items do exhibit slightly higher terminal velocities as was shown in Figure 20.

In order to determine if the velocities obtained by using the static pressure at the throat are compatible with previous experiments and the regression model, small glass spheres were tested. Experimental and theoretical velocities were compared. Three spheres of different sizes (0.881, 0.650, 0.607 in diameter) and weights (14.5256, 5.6423, and 4.7520 grams, respectively) were used for these tests.

Velocity was computed using the following equation:

\[ v = \left[ \frac{2\Delta h \rho g}{\rho_a} \right]^{1/2} \]  

(82)
During the runs the air temperature was 68°F, the measured pressure was 29.28 in Hg, the air density was 0.0022856 slugs/ft³, and the viscosity was 3.775 x 10⁻⁷ slugs per foot-second. The test velocities were based upon static readings of 4.0, 2.5 and 2.2 inches of water, respectively. These pressures equated to terminal velocities of 135, 107, and 100 ft/sec.

The Reynolds Numbers for the three spheres based upon their diameters were computed to be 60,007, 47,560, and 44,450 using the measured test velocities.

The theoretical terminal velocities were computed as follows:

\[ V_t = \left[ \frac{8 \rho \omega g}{3 \rho_a C_D} \right]^{1/2} \]  

(83)

The density of the glass spheres was computed to be approximately 4.4887 slugs/ft³.

It was deemed appropriate to let the theoretical terminal velocity equal the test velocity to solve for the drag coefficient, \( C_D \). The calculated values for the drag coefficients were 0.339, 0.399, and 0.426, respectively.

In order to consider the wall effects the Monroe correction factor was applied as follows:

\[ f = 1 - \left[ \frac{r}{r'} \right]^{3/2} \]

(84)
The corrected drag coefficients are equal to 0.2497, 0.3306, and 0.3597, respectively, which are near or beyond the critical Reynolds Number.

Note that the $C_D$s increase with decreasing size and decreasing Reynolds Number. A view of the classic $C_D$ versus RN chart, Figure 3, indicates that this relationship is valid near the critical RN. A review of the historical experimental procedures indicate that most experiments were accomplished with stationary and extremely smooth spheres of from 2 to 6 inches in diameter. These all exhibit a critical RN somewhere between $10^5$ and $10^6$.

The spheres used in this phase of the study may be slightly out of round; they are not as smooth as the spheres used in determining the curve shown in Figure 3; they wobble and spin. The sum of these effects may shift the critical RN, $C_D=0.30$, towards a value of $10^5$ (33:106).

If the arguments proposed are valid then the corrected drag coefficients do correlate well with accepted data and the terminal velocities measured. The static pressure readings therefore should be relatively accurate.

Using a density value of 4.4887 slugs per cubic foot for the glass the continuous data regression model is applied to predict the terminal velocity of the glass spheres. Unfortunately, the third term in the model is

\[ V_t = f \left[ \frac{8\rho_w v g}{3C_D \rho_a} \right]^{1/2} \]  

(85)
a size parameter in square inches and is not directly relative to the case of the sphere. Without this third term the model predicted a terminal velocity of 15.26 feet per second which is well below the measured speeds of 100, 107 and 135 feet per second. Including the surface area of the spheres increased the predicted terminal velocity to 24.69 feet per second. The model is unsurprisingly ineffective in predicting the terminal velocities of spheres since the measured velocities were recorded at up to 135 feet per second.

Flat broken glass fragments 0.070 inches in thickness were also studied in this phase. Four tests were conducted and the average terminal velocity of the test specimens was determined to be 22.11 feet per second. During the runs the glass behaved exactly as the other materials tested and it was noted that the larger pieces dropped at slightly higher velocities than did the smaller pieces. This dropping pattern (heavies first) significantly affected the average terminal velocity which would be lower if based upon the number of drops at different velocities rather than the weight of the specimens. The three continuous regression models using an average size of 3/4 square inches predicted terminal velocities of 23.04, 24.14, and 22.95 feet per second. The nominal data model cannot be used since glass was not considered in the model's construction.
The facts show that glass behaved similar to previous materials and the second continuous model predicted fairly well the terminal velocity supports the hypothesis of Chapter I.

Cloth specimens were also tested during this phase. Unfortunately, and as reported by others, these specimens agglomerated and dropped out in a single grouping during each replication of the experiments. The fibers cling to each other and the static electricity both of the device and of the specimens tends to eliminate the possibility for recording the terminal velocities of small cloth specimens.
CHAPTER VII

DISCUSSION OF RESULTS

The initial experimentation concentrated on developing a knowledge of the air flow within the classifier under differing conditions. Large opening angles caused separation of the flow, turbulence, and significant variations of flow velocities at all levels within the device. The use of flow channelizers or screens had little effect upon the flow at large opening angles.

Only when the classifier opening angle was reduced to 7° or less did the flow become streamlined. Stratifications of velocities were predictable under this condition and subsequently MSW separation appeared feasible.

In order to accurately calibrate the pressure readings with flow velocities, spheres of differing sizes and densities were placed into the classifier during the Phase Three experiments. The experimental results checked very well with theoretical calculations based upon the well established drag coefficients for spheres. This showed that the pressure readings observed during the experiments could be directly converted to air flow velocities during the successive phases of the study.
Choking, the effect of collisions of specimens within the classifier, and wall effects are extremely important factors that affect terminal velocities of the materials studied in the device. Using spheres of known sizes, densities, and observed terminal velocities the effects of choking were determined for this small separator. Unfortunately the effects of choking, collisions, and walls had to be determined as a single factor. The results were as theorized and the greater the number of specimens in the classifier the higher the measured average terminal velocity. This observation indicates that these factors would have to be considered in later phases of this study and that full size separators may have to be rather large devices. The opening angle limitation will dictate that in order to separate large volumes of many different components classifiers may have to be rather tall.

Phase Three was similar to Phase Two for the effects of choking were again evaluated. However, in this phase the specimens were flat plates, which not only are more typical of MSW components after shredding, but also are similar to specimens used in Phase Four testing.

These rather interesting tests showed that:

1. The mean terminal velocity of the specimens generally increases with increased numbers of specimens in the classifier.

2. The mean terminal velocity generally increases slightly with increased size.
3. The 1 1/2 inches long specimens did not react as expected in three out of five test series.

4. The terminal velocity increases slightly with increasing aspect ratio.

Observation 3 indicates that future air classifiers must be designed using the maximum dimension of the MSW as a design parameter. Some researchers have stated that the maximum dimension of the MSW should be not more than 2/3 the classifier throat diameter. Phase III results show that possibly 1/2 the throat diameter may be the upper limit for MSW components.

The first three test phases also served as a basis for the techniques and procedures utilized in the final two test series. The experimenter was extremely careful during these tests in assuring that random influences were minimized.

Phase Four experimentation was accomplished in such a manner that the research questions raised in Chapter I could be answered. This was accomplished by using an experimental design that enabled the researcher to evaluate the effects of density, size, and shape independently.

The data acquired during Phase Four was evaluated using the theoretical approach developed in Chapter III.

Figure 19 shows that separations of MSW components by air classification are indeed possible whenever the densities are significantly different. Similarly, when the
densities are nearly equal as is the case for balsa wood, cardboard, and paper clean separations may be difficult to impossible.

Figure 20 shows that increasing the size of a component specimen slightly increases the terminal velocity required. By using screening techniques that have been developed previously by others the distribution of the sizes could be reduced in actual practice and the variability of terminal velocity reduced markedly.

The theoretical, experimental, and statistical analyses all showed that density was the most significant factor that affects the terminal velocities of MSW components. The effects of size and shape, given the limitations of this study, are nearly insignificant. More specifically, the density function explains approximately 96 percent of the terminal velocity variation, whereas size and shape account for less than 2 percent. The analysis also showed that all materials could be separated except balsa wood and cardboard. Although paper and cardboard have nearly equal densities separation is still possible. This may have been due to the flexibility of the paper specimens or to the thinness of the samples. Unfortunately, with the experimental apparatus used, it is impossible to determine why balsa wood cannot be separated from cardboard.

Four multiple linear regression models were constructed to be used in predicting the terminal velocities
of the various components. This first model used nominal
data and explained over 97 percent of the variation. The
second model used continuous data based upon density and
size. This model explained nearly 95 percent of the variabil-
ity of the terminal velocity. The other regression
models were also of the continuous type; however, the
material factor was entered as the square root of the
material density. These are considered the best of the
four models since terminal velocity is a square root function
of density. These models explain 94 percent of the velocity
variability as a function primarily of density. All models
are excellent predictive tools; however, each has specific
limitations and must be applied with caution.

For each of the four tests of each combination of
density, size, and shape a standard deviation of the terminal
velocities was determined. The analysis of variance tests
were applied to these data. The results reflect that a sig-
nificant difference in terminal velocity variation as a
function of size does not exist. This conclusion is sup-
ported by both a theoretical analysis and a graphical pre-
sentation of the test data. Again, caution is recommended
in using this result for the specimen sizes were limited
to one inch and smaller.

In summary, the theory and the test results show
that MSW components can be separated by air classification,
given a significant difference in their densities.
Regression models can be constructed to accurately predict
the terminal velocities. The tests also demonstrate that size and shape may have only slight effects upon the terminal velocities for flat plate specimens. (The glass experiments show that spheres react differently than flat plates.)

Phase Five was an attempt at simulating actual MSW conditions by simultaneously inputting different materials of different sizes and shapes into the classifier. The materials behaved exactly as before and dropped from the classifier at the same terminal velocities as in Phase Four. The only difficulty observed was the fact that occasionally a light element would trap and carry a heavier one to the top screen. This caused a few specimens to drop out at lower than predicted terminal velocities. This problem is caused by equipment limitations and can be easily overcome with larger equipment.

Also during this phase broken glass fragments were tested in the classifier. Not unexpectedly these specimens performed exactly as had the other material. The average test value for the terminal velocity of glass was measured to be 22.11 feet per second and was estimated by the continuous regression models to be 23.04 feet per second.

These experimental results do show that refuse recovery systems that use this funnel- or V-shaped air classifier may be able to separate many of the MSW components and subsequently provide for more efficient and economical systems than the multiple pass systems.
CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The knowledge of municipal solid waste air classification was surprisingly meager considering the large numbers of facilities using the technique as part of their resource recovery systems. Little reliable quantitative methods were identified that had been developed sufficiently to be used in predicting the results of multiple separation of MSW. More often than not the experimental results were presented with only cursory mention of theoretical support. Most authors, however, do agree that the parameters of density, size, and shape are critical for air separation.

The objectives of this research were many-fold. First, it was intended to demonstrate that MSW type materials could be successfully air separated. The author observed that most shredded waste enters the air classifier in flat plate shapes and subsequently accomplished the experimentation on simulated MSW of measured sizes and flat plate shapes. The tests show that the different materials do stratify within rather narrow ranges of terminal velocity which is primarily affected by the density
of the materials. Further, the greater the differences in the density the easier the separation. Materials of nearly equal densities cannot be successfully separated by air classification.

The second hypothesis concerned the possibility of constructing a mathematical model that can be used for predicting the terminal velocities of MSW type materials. Both nominal and continuous type regression models were developed that can explain approximately 95 percent of the terminal velocity variations. The $R^2$ of 0.95 is truly excellent and models of this quality will provide reliable tools for estimations. However, as with most mathematical models one must be aware of the limitations and the initial constraints used in the formulation. In this case all specimens were flat plates and none measured over one inch in length. These are significant limitations of the models but do demonstrate the feasibility of modeling MSW separation systems.

The third hypothesis concerned the effect of size upon the standard deviation of the terminal velocities. The experiments do show that within the ranges of specimens used in these tests size does affect the standard deviations. The data shows a slight increase in variation as the size increases. Even with the limited size of the equipment and the limited range of specimen sizes the experiments demonstrated the variability caused by size.
The fourth hypothesis deals with the effect of size, shape, and density on the terminal velocity of different materials. Both the theoretical analysis, the statistical analysis, and the experimental runs show that material density is the most important factor of the three and that size and shape contribute very little to the changes in terminal velocity.

These experiments have shown that if the solid wastes can all report to the air classifier in small flat plate shapes that multiple separation is possible. Separating materials of significantly different densities should be quite "clean," but separating materials of nearly equal densities will be difficult to impossible with an air classifier.

**Recommendations**

During the preparation of this dissertation several ideas were developed that were beyond the scope of this paper. Additional work is suggested in the following areas.

**Material Analysis**

1. This research was limited to a few materials of limited sizes and shapes. Further studies should be conducted on the vast number of different materials found in MSW.

2. An analysis of the size and shape of shredded MSW should be undertaken. Of primary interest in this work
should be the shape distributions of the MSW components from different types of shredders.

3. Additional research in the area of economic tradeoffs of shredding versus "purity" of the air classification fractions should prove to be of value to those who design resource recovery systems.

4. Studies using actual MSW components in a larger classifier should be accomplished in order to evaluate the effects of moisture and static electricity on some components of varying sizes.

5. Since screening is an integral part of most resource recovery systems, optimum combinations of screening and classification should be developed. Determining the effect of screening upon the terminal velocity variation and, therefore, the purity of the separation would be extremely valuable.

System Analysis

As with most engineering developments, this study is one of the first steps in a series of developments that may result in a new and successful resource recovery system. It is suggested that the next step in this program should be the construction of a much larger classifier. The device should be of circular cross-section of increased size and height. The throat should be at least one foot square and the height high enough to provide simultaneously velocities from 1 to 150 feet per second within the device. This will assure a stratification of all MSW type components.
and equipment large enough at the throat to accept specimens up to six inches in size without any serious choking problems.

More advanced designs would integrate the drop out patterns with a variable positioned catch-tray arrangement. Two different designs are suggested and are included as Figures 25 and 26.

![Diagram](image)

**FIGURE 25. SINGLE CLASSIFIER WITH MOVING TRAY**

Figure 25 depicts a single funnel shaped air classifier with a moving component catch tray. The MSW is input after the classifier has reached the velocity high enough to suspend all components. As the air flow is reduced the tray is moved appropriately and catches the proper component in the correct portion of the tray.
Rotating Air Flow Restrictor

Opening Positions
A--High Velocity
High Density Drop
B--Medium Velocity (Typ)
Medium Density Drop
C--Low Velocity
Low Density Drop

SECTION A-A

Air Flow (Constant)

Variable Air Flow (Typ)

Rotating Tray

FIGURE 26. A MULTIPLE-CLASSIFIER DESIGN
A more sophisticated design is included as Figure 26. This multiple classifier design depends upon a constant air flow to the entire device but a variable flow to each individual classifier chamber. This is accomplished using a cam type arrangement that rotates in synchronization with the circular catch tray. Materials in the catch tray would be dumped when material is not dropping into the bin. This is the type of arrangement that would be required for municipal sized resource recovery systems.
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