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NAVY ELECTRONICS LABORATORY
San Diego, California

CONFORMAL/PLANAR ARRAY SONAR SYSTEM

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Conclusions and Recommendations

We cite several salient conclusions:

1) Turbulent boundary-layer (TBL) noise appears to dominate the noise measured by flush elements of 5-in. diameter in the forward array (HF) through substantially all the frequency range 0.2 to 10 KHz at speeds 15 to 30 kt. The same appears generally true of the further aft array (LF) at speeds 20 to 30 kt. As an indication of the absolute noise levels, at 20 kt on HF-3 at 2.5 KHz it is -20 db re (1 µbar)$^2$/Hz.

2) Where TBL noise on these large flush elements is thought predominant:

a) At fixed distance aft the scaling of the noise spectrum $P(w/2\pi)$ is fairly well represented by

$$P(w/2\pi) = \rho \frac{2\delta_* U^3}{2F(\omega \delta_*/U_\infty)}.$$  

With distance aft fixed, however, the appearance of $\delta_*$ in this form is arbitrary.

b) At differing distances aft (but at fixed $R_o$) the scaling is fairly well represented, especially in an intermediate range of frequency, by

$$P(w/2\pi) = \rho \frac{2\delta_* U_\infty (\omega R_o/U_\infty)^{-2}}{2F(\omega \delta_*/U_\infty)}.$$  

$P(w/2\pi)$, however, varies roughly as $w^{-3}$ in this range; hence this form, so far as $\delta_*$ is concerned, merely reflects approximate independence of $\delta_*$, and therefore its use in the scaling is not uniquely justified.

c) Relative to the measured noise on small elements (radius $r_o$), in roughly the lower third of the log frequency range 0.2 to 10 KHz, where the small elements in question are not seriously affected by area averaging, the noise spectra on the large elements diverge from those on the small according roughly to a reduction factor due to area averaging given by $(\omega R_o/0.6U_\infty)^{-S}$, where $R_o$ is the

*Here $U_\infty$ denotes ship speed and $\delta_*$ boundary-layer displacement thickness at the element.
large element radius, with a slightly larger than two.
At somewhat higher frequency, however, instead of be-
coming parallel, with a fixed db separation
$10 \log (R_0 / r_0)$, the spectra for large and small ele-
ments somewhat converge toward one another. Neverthe-
less, the spectra for the large elements in this region
continue to scale with speed as may be expected where
TBL noise predominates.

3) Regarding the G elements at the sea chest 2 location, the
element flush-mounted in the fiberglass window, and espe-
cially the element recessed in a flooded cavity behind,
display large speed-dependent humps in their noise spectra
in a broad high-frequency range. These humps are tenta-
vively attributed to effects of bubbles at this location.

4) Recession of elements in the water-filled cavity may have
reduced flow noise more than it reduced sensitivity to a
signal.

5) Recession of elements (D) behind rubber layers of varying
thickness may have moderately reduced flow noise. The
extent of reduction did not depend systematically on layer
thickness from 59/64 in. to 5-1/32 in.

6) The noise spectra of the layer-shielded elements above
1.5 KHz varied widely and erratically from element to ele-
ment. This effect may have been due to splash or bubble
noise.

7) The measured in-situ calibration curves for the window-
mounted and layer-covered elements in many cases varied in
an unexplained fashion with time, frequency, and element
identity.

On the basis of the present report we offer the following
recommendations regarding further processing and analysis
of the PURVIS II data.
a) Study carefully the indicated paths of natural bubbles as they depend on speed and maneuver and try to relate to the measured noise spectra for the G elements.
b) Process and analyze spectra for other G elements, including C9 or C10.
c) Investigate the variability of noise spectra for D and G elements and the dependence on turns and headings, and listen to audio noise reproductions to the end of identifying possible noise sources.
d) Further reduce and analyze in-situ calibrations.
e) With reference to the HF and LF spectra, investigate other proposed TBL noise scaling laws.
f) Analyze spectra for more HF and LF elements at various speeds and give particular consideration to the tendency of spectra to level off in the middle frequency range.
3. Analysis and Interpretation of Noise Data

3.1 Flush Wall-Mounted Elements

The hydrophones of this category are comprised by the groups designated HF, LF, and H. Flush elements G9 and G10 are considered with the other G elements in Section 3.2.

3.1.1 Autospectra

HF Elements

Noise spectra measured for several HF elements are shown in Figs. 3.1.1-1 to 3.1.1-8.

One objective of the analysis of the noise measurements is to infer the character of the noise on an element as a function of the pertinent variables. The types of noise contributing may be broadly classified as produced by an acoustic field or directly by pressure fluctuations associated with the turbulent boundary layer (TBL). The former, however, may include a component due to excitation of non-rigid boundaries by the turbulent flow; the locally generated part of this component may not be additive but correlated with the direct TBL contribution. While the acoustic noise will depend on the specific ship and element configuration, the direct TBL noise spectra at a given speed, distance aft, and element size will be substantially universal.

Referring first to the HF elements, we see from the figures that at speeds 15 kt to 30 kt, the noise spectra typically increase with speed nearly as $\omega^6$. At these speeds, the frequency dependence, on the average, is roughly as $\omega^{-2.7}$ to $\omega^{-3.7}$. Below 15 kt, though the frequency dependence is not much different, the speed dependence weakens. These features accord with tentative expectation if TBL flow noise is the dominant noise source at 15 to 30 kt.

The extent of variability of measured noise spectra from one HF element to another is rather small. A typical comparison is that between HF-5 and HF-12 in Figs. 3.1.1-3 and 4.
(Since these elements are separated by 10.5 in. circumferentially, the comparison indicates an increased noise for the element nearer the water surface, if attributed to this cause rather than to mere statistical deviation, by about 3 db at 20 kt.) At the higher speeds, the small variation among elements reinforces the presumption that the dominant noise is that due directly to TBL pressure fluctuations.

According to the "outer" scaling law for the TBL pressure spectrum, which corresponds to the usual type of dimensionless plot of measured spectra, the TBL noise spectrum has the functional form

\[ P(\omega/2\pi) = \rho^2 \frac{\delta}{U_\infty} \pi \frac{\omega \delta}{U_\infty}, \]

where \( \pi \) is a function of the dimensionless argument \( \Omega_1 \), \( \omega \delta / U_\infty \), \( \rho \) is the fluid density, \( U_\infty \) the asymptotic flow speed, and \( \delta \) displacement thickness. Hence, if this form is valid, in the absence of other appreciable noise sources the measured spectra for different speeds will coalesce on plots of

\[ 10 \log [P(\omega/2\pi)/\delta U_\infty^3] \text{ vs. } 10 \log \Omega_1. \]

Even if the form (3-1) is valid for the TBL noise on a point element, however, it will be valid for a large element, i.e. one where \( \omega \delta / U_\infty \gg 1 \), only at fixed \( R_o / \delta \), since \( \pi \) in this regime depends on \( R_o / \delta \) as well as on \( \Omega_1 \). The dependence of \( \delta \) on \( U_\infty \) is sufficiently weak (see App. 1) that at fixed distance aft and fixed \( R_o \) a test of the outer scaling law (3-1) is meaningful also for large elements.

The coupled speed and frequency dependence of measured noise spectra for the HF elements is tested explicitly against the scaling law (3-1) in Figs. 3.1.1-7 and 8. The measured data agree fairly well with the suppositions that in the speed range 15 to 30 kt the principal contribution to noise spectra on the HF elements is due to the TBL and that this
Contribution, at fixed $\delta_3/\rho_0$, satisfies the scaling law (3-1) at least in an extensive middle range of frequency. According to these suppositions, at the lower speeds 5 and 10 kt, on the other hand, the TBL noise contribution is small relative to other noise, being lower than the total noise in the case of element HF-3, for example, by about 3 to 14 dB, depending on frequency, at 10 kt and by about 18 to 25 dB at 5 kt.

Alternative possible scaling laws for TBL pressure that differ from (3-1) can be proposed (e.g. see Ref. 3). Applied to the instance of a large element, these would yield speed dependences that differ only moderately from that given by (3-1). Hence, while our conclusion regarding the relative magnitude of the TBL noise contribution is not subverted, we must maintain an open mind as to whether (3-1) correctly represents the TBL scaling. Further pertinent comparisons will be made below.

Regarding the dependence of noise spectra of HF elements on heading and turning conditions, Fig. 3.1.1-5 shows results for HF-3 at 20 kt for headings 0°, 90°, and 270°. The result at 180° is nearly identical with that at 0° and is therefore omitted. At this speed the noise is evidently nearly independent of heading except for local peaks in the high-frequency range in the instance of 90° heading. Fig. 3.1.1-6 shows results also for HF-3 at 20 kt at heading 0° and during a full right rudder turn. The level in the turn is about 1 to 3 dB above the other. In a full left rudder turn the result for this element is nearly identical with that for full right rudder.

**LF Elements and Comparison with HF Elements**

Measured spectra for various LF elements are shown in Figs. 3.1.1-9 to 13.

The variation of noise level among LF elements is perhaps somewhat greater than among HF elements. Measured noise spectra for LF-6, for the most part, lie below those for the
other processed LF elements.

Figs. 3.1.1-9 and 10 show that the noise spectra for LF-6 and LF-7 first rise appreciably above the zero-speed level at 15 kt. (Indeed, the measured spectra at 10 kt in the high-frequency region unaccountably lie below those at 0 kt.) Nevertheless, no appreciable increase in noise spectra is measured between 15 and 20 kt. At 25 kt and especially 30 kt the measured increments in noise, however, are large.

Noise spectra for LF-6 and LF-7 are compared with those for a typical HF element, HF-3, in Figs. 3.1.1-14 to 16. At zero speed the levels for the LF's are higher than that for the HF, which has a more secluded, forward position, by about 18 db. At 20 kt, by which the LF levels have surmounted their high values for zero speed, LF-7 lies several db above HF-3 and LF-6 at moderate frequencies, and all three differ only modestly above 3 kHz. At 30 kt the level for LF-7 lies above HF-3 by a few db and by more above 5 kHz. LF-6, on the other hand, lies somewhat below HF-3 up to 3 kHz. In Fig. 3.1.1-16a, spectra for both LF-7 and HF-3 at 20 kt are compared with that measured on the 3-in. diameter element 5E111 flush-mounted at Sea Chest 2 in the PURVIS I test, a location comparable with that of the LF's. The spectrum for 5E111, which is somewhat less reduced by area averaging, still lies for the most part between the others.

Dimensionless plots of the type (3-2) are given at 10, 20, and 30 kt for LF-7 in Fig. 3.1.1-18 and LF-6 in Figure 3.1.1-17. On these plots the spectrum for 10 kt lies above the others by a substantial frequency-dependent margin in the case of LF-6 and by a larger but less variable amount of the order of 15 db in the case of LF-6. The spectra for 20 and 30 kt in each instance do not closely approach coalescence, though the curves for LF-6 remain close in the interval up to 10 log (ωδ/R/u∞) = 13, and the 20-kt curves do not lie systematically above the 30-kt curves nor hence suggest a large acoustic component at 20 kt. Similar dimensionless plots are given for LF-4 at 10, 20 and 30 kt and for LF-12 at 5, 10, 15, 20, and
30 kt in Figs. 3.1.1-19 and 20. For IF-12 at 5 kt the curve lies far above the others, as would be expected, but for speeds 15 kt and above the curves, though again not coalescing in detail, do display comparable average levels and slopes.

To compare HF and LF elements, corresponding as they do to different values of $R_o/\delta_x$, we must suitably generalize the conjectured outer scaling law (3-1). In App. 1 it is pointed out that for large elements ($\omega R_o/U_\infty > 2 \pi$, $R_o > \delta_x$) the analog of the outer scaling law (3-1) for point elements consists in the following functional form for the TBL noise spectrum:

\[(3-3) \quad P(\omega/2\pi) = (R_o/\delta_x)^{-s} \rho_1^2 \xi^3 \frac{U_\infty}{2} \frac{U_\infty}{\delta_x} \Pi_1 (\omega \delta_x/U_\infty),\]

where $\rho_1$ is a function not necessarily simply related to $\Pi_1$ in Eq. (3-1) for the point spectrum (if the latter holds at all) and, so far as the indicated area dependence of TBL noise for large elements corresponds to $R_o^{-2}$, we must have $s = 2$. The scaling law (3-3) would imply coalescence of plots for different $R_o/\delta_x$, as well as different $U_\infty$, provided the plot is taken as

\[(3-4) \quad 10 \log [P(\omega/2\pi)/\delta_x^3 U_\infty^3] + 10 s \log (R_o/\delta_x) = 10 \log \Omega_1 \]

in place of (3-2). If in some range of $\omega \delta_x/U_\infty$ the spectrum $P(\omega/2\pi)$ should vary as $\omega^{-3}$ while conforming to (3-3) with $s = 2$, we note, then the spectrum would be independent of $\delta_x$ and hence of distance aft.

Dimensionless plots of the type (3-4) with $s = 2$ are given for elements HF-3 at 15 and 20 kt and LF-7 at 20 and 30 kt in Fig. 3.1.1-21 and again with LF-6 in place of LF-7 in Fig. 3.1.1-22. In the instance of HF-3 and LF-7 the coalescence is quite good in the middle frequency range $8 < 10 \log (\omega \delta_x/U) < 13$. At lower and especially higher values of $\omega \delta_x/U_\infty$ the coalescence is poor. In the instance of HF-3 and LF-6 the coalescence is poorer in the middle range and better in the extreme ranges,
being fairly good, in fact, in the lower range. A similar dimensionless plot for HF-3 and HF-6 is made in Fig. 3.1.1-23 on assumption that $s = 3$ instead of 2. For this value of $s$, the coalescence of the curves for the two elements in the middle range becomes very poor.

**Comparison of HF Elements with Small Elements**

The noise spectra measured for HF-3 are compared in a dimensionless plot with those measured in other tests on certain small elements in Fig. 3.1.1-24. The small elements chosen are the 1/8-in. diameter FS-1 number 1666 of the PURVIS I test series and two 0.11-in. diameter elements of Franz's Albacore noise measurements (Ref. 9). The FS-1 was located in Sea Chest 1 of the PURVIS I test at a distance aft nearly the same as HF-3 (27 ft), and the two chosen elements of the Albacore test were located at distances 21 and 46 ft. aft, which straddle the distance aft of HF-3.

The 5-in. diameter element HF-3 constitutes a large element, i.e. $\omega R_0/U_\infty \gg \pi$, throughout the frequency range in question; the FS-1 and Albacore elements constitute small elements, i.e. $\omega R_0/U_\infty \ll 1$, at 20 kt up to about 1 kHz, or in the case of the FS-1 up to about 10 log $(\omega R_0/U_\infty) = 7$ independently of $U_\infty$. At sufficiently high frequency that even the smaller elements would constitute large elements, the spectrum for the HF-3, if the area-averaging effect is as $R_0^{-2}$, should lie below the spectrum on the smaller elements by an amount increasing asymptotically to 32 db. In the lower range where the smaller elements behave as point sensors there is some prior empirical basis to think the spectrum on the large element of radius $R_0$ would be equal to that on the smaller multiplied by $(\omega R_0/\eta U_\infty)^{-2}$, where $\eta \sim 0.6$ (Ref. 10); this relation has also a theoretical basis if the dominant contributing wavenumber range in the TBL pressure spectrum is of the order of the convective value $\omega/U_\infty$ for both large and small elements.
The relation between the HF-3 and FS-1 spectra in Fig. 3.1.1-24 is seen roughly to correspond to this supposition. On the other hand, at larger $\omega d U/\omega$, the spectrum for HF-3 begins to decrease less, rather than more rapidly with frequency than the FS-1, and the difference in the spectra is reduced to a fairly small one at the higher frequencies. This circumstance does not accord with prior expectation for TBL noise spectra.

The Albacore spectra, we note, lie further above the spectra for the large elements than do the FS-1 spectra. These too, however, decline as least as rapidly as the large-element spectra in a wide frequency range. Digressing to the question of the difference between the measured FS-1 and Albacore noise levels, we observe that the FS-1 used for Fig. 3.1.1-24 is quieter than most; in Fig. 3.1.1-25 we give dimensionless spectra for this element and another FS-1, number 1638, along with the two Albacore spectra.

H Elements

Spectra are available for one of the far-aft, flush, hull-mounted H elements, namely H-5, and are shown in Fig. 3.1.1-26 and compared with HF and LF elements in Figs. 3.1.1-27 to 30. At 0, 10, and, in run 340B, 20 kt the spectra for H5 are nearly coincident. It is concluded that, on account of the proximity of this element to machinery spaces, the levels up to this speed are dominated by this noise source. At 30 kt and, in disagreement with run 340A, also to a lesser degree in run 340B at 20 kt, the spectra lie higher. In the comparison of these levels with those for HF and LF elements, however, the H levels lie above the latter. It thus appears that spectra for this element, except possibly at 30 kt, cannot be used in making inferences concerning the dependence of TBL noise on distance aft.
3.1.2 Cross-spectra

The magnitude and phase of the normalized cross-spectral density of noise measured on various pairs of HF elements are shown in low and high-frequency bands for a number of different runs and ship speeds in Figs. 3.1.2-1 to 82, Vol.1.

Regarding the magnitude \( \gamma (\vec{f}, \omega) \) of the cross-spectra, where \( \vec{f} = (f_1, f_3) \) denotes the center-to-center separation vector between elements, we make the following observations.

At zero knots, the magnitudes, apart from relatively narrow-band irregularities and a pronounced dip as frequency decreases toward 0.5 kHz, generally are substantial (e.g. \( \zeta > 0.3 \)) up to a frequency \( \Omega = 2 \pi f \) whose value is roughly consistent with the formula \( \omega \gamma/\omega \sim 2 \) to 3 for the upper frequency limit above which an acoustic noise field incident with a distribution in azimuth is expected to yield a substantially decreased cross-spectral magnitude [see Eq. (A3-8) and related discussion in App. 3]. Superimposed on several of the plots of cross-spectral magnitude is the function \( |J_0(\omega \gamma/\omega)| \) that would apply (App. 3) if the pressure were due to an acoustic field that is isotropic, uncorrelated in direction, and propagating in the plane of the elements. Detailed correspondence does not ordinarily occur, nor should be expected, but some similarity is noted for element pairs HF5-HF12, HF12-HF6, and HF12-HF8.

At 10 kt the cross-spectral magnitudes are generally reduced relative to zero kt, especially in the lower frequency interval (\( \lesssim 2 \) kHz). In some instances the pattern of variation remains somewhat similar, in most rather changed. At 20 kt the magnitudes are decidedly further reduced (except for pairs and frequencies where the magnitude at 10 kt was already low). An exception occurs for the adjacent-element pairs HF5-HF12 and HF5-HF4 at high frequency; for the former pair at frequencies...
≥ 5.5 kHz the magnitude is noticeably higher than at 10 kt, rising to a peak at 8 kHz, and for the latter there is a band of substantial magnitude at ~8.5 kHz.

At 30 kt, however, the magnitudes, at least in the high-frequency range (≥5 kHz), are generally somewhat higher than at 20 kt. The magnitude in this high range, in fact, on the average is higher than in the frequency range just below. In narrow frequency intervals, peaks of relatively high magnitude occur at such high frequencies also in some instances at lower speed.

We offer the following inferences from the described behavior of the cross-spectral magnitudes.

At zero speed the pattern broadly corresponds to what might be expected for an acoustic field with a fairly complex and frequency-dependent angular distribution. We are unable, however, to account for the dip in the magnitude at frequencies ≤0.5 kHz.

The observed decrease in magnitude with increasing ship speed can reasonably be attributed only to an increased relative contribution to the noise autospectra of the element pair due to a pressure field whose individual normalized cross-spectral magnitude at the frequencies and separations in question is much smaller than that of an acoustic field. In terms of Eq. (A3-14) of App. 3, some \( \Psi (\omega) \) in the denominator becomes increasingly large relative to the others, but the numerator is not correspondingly affected, since the \( Y_2 \) multiplying this particular \( \Psi \) is supposed relatively small. The only likely such contribution to the pressure field is that due to the turbulent boundary layer. Assuming this in fact to be the cause, we conclude that the TBL contribution to the noise autospectra of the HF elements is roughly comparable with the acoustic noise at 10 kt and predominates over the latter at 20 kt at least up to a frequency ~3 kHz. This conclusion is based mainly on the evidence of the pairs of adjacent elements, since the acoustic field itself yields a relatively lower cross-spectral magnitude when the elements are further separated.
At the same time, at frequencies \( \gtrsim 5 \text{kHz} \), in certain bands at least, it appears that the acoustic contribution to autospectra exceeds the TBL at 20 kt, and in fact that acoustic sources are present in this range that are absent or weaker at 0 and 10 kt. Such a source seems especially strong at \( \sim 8 \text{kHz} \); referring to the measured autospectra on HF elements at 20 kt, we note a corresponding hump at about this frequency. (On the other hand, the pronounced hump in the autospectra at \( \sim 0.9 \text{kHz} \) for 20 kt seems correlated, if at all, with a dip and not a rise in the cross-spectral magnitude.)

At 30 kt the apparent increase of acoustic noise in the high-frequency range becomes much more pronounced. The humps in the measured autospectra in this range, however, are no more pronounced than at 20 kt, and indeed similar humps occur also for the lower speeds.

The additional high-frequency acoustic noise inferred from the increase in cross-spectral magnitude at 20 kt and especially 30 kt appears to derive from sources in a relatively well-defined direction. We conclude this from the fact that \( \omega \gamma/c \gg 1 \) in the range in question, and an azimuthally isotropic distribution, for example, would therefore yield a magnitude \( |J_0(\omega \gamma/c)| \ll 1 \), whereas a unidirectional wave, at the other extreme, would yield a magnitude unity. It is suggested that this increase in the cross-spectral magnitude at 20 kt and especially 30 kt may be due to some incipient cavitation.

Thus far in this discussion of cross-spectra we have considered only the magnitude and ignored the phase. If only an acoustic field is present, as noted in App. 3, we expect a relatively slow variation of phase with a change of \( 2\pi \) occurring only over frequency intervals in which \( \omega \gamma/c \) changes by at least roughly \( 2\pi \), except that abrupt changes in phase by as much as \( \pi \) can occur at intervals in \( \omega \gamma/c \) equal to \( \sim 2.4 \) to \( \pi \) [e.g., see App. 3, Eq. (A3-16)]. If another noise field is present,
notably one due to the TBL, but has a normalized cross-spectral magnitude small compared to that of the acoustic noise, as noted in App. 3, the phase will still be approximately that for the acoustic field alone, and this may be so even if the contribution of the TBL to the noise autospectra exceeds the acoustic one. Hence the type of phase variation just described may be expected to apply to the measured spectra in question at most speeds and frequencies.

In actuality, the measured phase variation is seen to be more rapid and erratic in many instances than suggested by this account. This irregular behavior is largely confined to frequency intervals where the cross-spectral magnitude is small, but is scarcely less noticeable at zero speed, where the TBL contribution is absent, than at higher speed. It is tentatively suggested that the true phase is well behaved, as expected, but that the numerical computation procedure produces spurious and rather random results when the cross-spectral magnitude is as low as it frequently is. This conjecture is susceptible to investigation, but no answer can be given in the present report. In addition, it is worth pointing out, the phase curves as drawn to connect adjacent computed values are presumably frequently unjustified when these adjacent values differ by nearly $2\pi$; i.e., since the phase values $\pi$ and $-\pi$ must be identified, the connecting line from a computed phase $\pi - \delta_1$ to $-\pi + \delta_2$, say, where $\delta_1 \ll 1$, $\delta_2 \ll 1$, should not be the direct line, but rather one leading from $\pi - \delta_1$ to $\pi$ and then one leading from $-\pi$ to $-\pi + \delta_2$. Some violent apparent phase variations in the plots are thus eliminated.

In any case, we have not been able to employ the measured phase plots for these large-element cross-spectra of the Purvis II tests for any useful inference.
The magnitude and phase of the normalized cross-spectral density of noise measured on various pairs of LF elements are shown in Figs. 3.1.2-83 to 164, Vol. 1.

We may compare these cross-spectral magnitudes with those for pairs of HF elements discussed above. Pairs LF3-LF2 and LF12-LF11, for example, have the minimum spacing of 30.5 in. of the LF array, and this separation is comparable with (slightly smaller than) the spacing of the pair HF12-HF8. For the most part, the cross-spectral magnitude at the various speeds have much the same character for these comparable pairs of elements in the LF and HF arrays. On a few of the plots of cross-spectral magnitude for zero speed is superposed the function \(|J_0(\omega \gamma /c)|\) of App. 3, Eq. (A3-8). A few particularities of the LF pairs may be cited.

For several LF pairs a pronounced peak in the cross-spectral magnitude appears at about 7.5 kHz and persists through speeds up to 20 kt. A corresponding hump in the autospectra of most of the LF elements likewise occurs near this frequency. It is inferred that an acoustic noise source with relatively well-defined effective direction exists at this frequency.

The perceptible though oscillatory increase in the cross-spectral magnitude at high frequency at 30 kt which was exhibited by the HF element pairs even when the separation was comparable to that of these LF pairs does not occur for these latter. If that rise was due to cavitation, then either the cavitation source was more acoustically accessible to the HF than the LF elements or else its effect on the LF's remains masked by other acoustic noise.

In two 10-kt runs with no change in controlled run variables, the cross-spectral magnitudes for LF pairs in Run 345 were substantially lower than in Run 858. This difference is
attributed to the fact that the movable strut near the LF array was extended during the 800-series runs but not during the 300-series. As indicated by comparison of autospectra and confirmed by these cross-spectral magnitudes, the strut produced spurious added acoustic noise on the LF elements. Hence it is more appropriate to use the 300-series results. On the basis of Run 345 it would appear that TBL noise predominates through most of the frequency range already at 10 kt.

Concerning the variation among results for the various runs at 20 kt, we note, pronounced peaks in the cross-spectral magnitude for LF3-LF2 appear at 0.8 and 1.7 kHz in Run 543 at 180° heading which are not prominent in the other 20-kt runs.

Figs. 3.1.2-165 to 184 show cross-spectra for a pair of elements, HF2 and H5, which are located at widely separated positions along the hull. The results give some indication of results to be expected when the crossed elements have more or less the least possible correlation. We note, however, in Run 340B at 20 kt, a local peak in the magnitude at ~8 kHz, presumably corresponding to an acoustic wave at this frequency propagating roughly along the length of the ship.

Recalling the conclusions based on HF and LF autospectra, we note that the cross-spectra appear to indicate that TBL noise begins to dominate acoustic noise at somewhat lower speed than inferred previously. In view of the complex influences on the cross-spectra we tentatively regard this discrepancy as not serious and consider the indications from autospectra as more reliable.
3.2 Flush and recessed elements in sea chest 2

Among the G elements, auto- or cross-spectra are available involving G5 through G10. The pair G5 and G6 are recessed behind a flooded cavity covered by the fiberglass window of sea chest 2 and are mounted flush in the air-backed rear face of this cavity, which is likewise fiberglass. The front window is 9/16 in. thick and the rear window 3/4 in. thick. The lateral dimensions of the cavity are about 36 in. longitudinally and 21 in. circumferentially. The pair of elements G7 and G8 are set flush in the outer fiberglass window of this sea chest. Over some area surrounding this pair there is a 1/2-in. steel doubler plate attached to the inside of the window. The pair G9 and G10 are flush-mounted in the hull above the window. The center spacing of the pairs is 8 in.

3.2.1 Autospectra

Measured spectra are presently available only for elements G5 and G7 and are shown in Figs. 3.2.1-1 to 5 together with comparisons with a neighboring LF element. Spectra presently available for the G elements are limited, including only speeds 15 and 25 kt for the flush window-mounted element G7.

The speed and frequency dependence for the recessed element G5 differs strikingly from the dependence observed for flush hull-mounted elements. In the range of frequency up to about 1.5 kHz, the spectra decrease regularly and similarly, apart from minor humps, at an average rate given roughly by $\omega^{-2}$ to $\omega^{-3.4}$. In this region
the spread among the various speeds from 0 to 30 kt is \( \lesssim 11 \text{ db} \), and the variation with speed is not monotonic. At frequencies ranging from 1.4 to 3 kHz depending on speed, the spectra for 10, 15, 20, and 25 kt turn upward and reach broad peaks of varying height at about 7.5 kHz with a tendency to a secondary peak in the same broad rise at about 4 kHz. This broad high-frequency peak develops and declines in a rather regular way with speed. Thus, at 0 kt no such peak occurs, at 10 kt the peak is pronounced, at 15 kt decidedly higher, at 20 kt somewhat lower again, at 25 kt still lower, and at 30 kt the peak has degenerated to a minor leveling trend. At the 7.5-kHz peak, the measured noise level at 15 kt is actually 20 dB above that at 30 kt, while the latter is 16 dB above that at 0 kt.

Relative to a typical LF element, the spectra for G5 in the range of lower frequency distinguished above are much lower, especially at the higher speeds. In the range of the high-frequency maxima, however, the G5 spectra rise well above the LF spectra at 10 through 25 kt, but the G5 spectrum at 30 kt is again much below the LF.

As for the flush window-mounted element G7, the spectra at 15 and 25 kt are substantially higher than for G5, especially at 15 kt in the valley preceding the high-frequency rise. The spectrum approaches more nearly the spectrum for an LF element in the lower frequency range but, as with G5, greatly exceeds it in the higher range. The broad humps at high frequency still occur but are less pronounced than for G5. Fig. 3.2.1-2 shows also that the position
and height of the high-frequency peak for 15 kt was considerably
different for runs at 70° and 90° headings, but at 25 kt no such
difference was observed.
3.2.2 Cross-spectra

The magnitude and phase of the normalized cross-spectral density of noise measured on three pairs of G elements are shown in Figs. 3.2.2-1 to 60, Vol. 1. These pairs, namely G5-G6, G7-G8, and G9-G10, represent pairs of G elements of the three types, respectively recessed behind a flooded cavity behind the fiberglass window, flush-mounted in this window, and flush-mounted in the hull above the window. All pairs have the same 8-in. center spacing.

On the plot of the magnitude for the G9-G10 pair at zero speed, the function $|J_0(\omega \gamma/c)|$ of App. 3, Eq. (A3-8) is once more superposed. We may compare the magnitude for G9-G10 at the various speeds with that for the pair of adjacent HF elements HF5-HF4 shown previously. At zero speed the magnitude for the former pair is substantially higher than for the latter in the high-frequency band above 2 kHz. The pairs are more similar at 10 kt, apart from a strong peak for the G pair at 5 kHz. For the G pair at 20 kt, unlike the HF pair, there is no rise at frequencies $\gtrsim$ 8 kHz, and at 30 kt the average magnitude is considerably below that for the HF pair.

The magnitude for the flush, window pair G7-G8 at zero speed is lower than for the flush, hull pair G9-G10. At 30 kt, for G7-G8, unlike G9-G10, there is a gradual rise to a peak at $\sim$1.6 kHz.

For the recessed pair G5-G6 at zero speed, the average magnitude in the high and low bands is of the same order as for HF5-HF4. There is, however, a notable failure of the computed magnitudes in the two frequency bands to join continuously at $\sim$1.4 kHz. Though the data used for one band is only a subset of the data used for the other, a joining discrepancy of the order computed should be grossly unlikely statistically. (The joining is rather poor also for G7-G8 at zero speed and very poor for G5-G6 again at 20 kt.) At 10 kt, the average magnitude for G5-G6...
is somewhat higher than for G9-G10 or HF5-HF4, with a build-up to a high-frequency peak at \( \approx 8 \) kHz. At 20 kt the magnitude is much reduced, though still higher than for G7-G8 and G9-G10 or even HF5-HF4. At 30 kt, the magnitude drastically increases for frequencies \( \approx 2 \) kHz, where the curve greatly resembles that for zero speed. This increase at 30 kt suggests increased relative prominence of some acoustic field that affects the recessed elements and to a lesser degree the flush window-mounted elements, but not the flush hull-mounted elements.
3.2.3 Analysis

We may seek an explanation for the measured C5 and C7 spectra, employing also the measured G5-G6 and G7-G3 cross-spectra. If we regard only the lower frequency range where the spectra lie below the LF spectra and depend weakly and irregularly on speed, we might suppose the following: The direct flow noise due to the TBL is drastically reduced for the recessed G5 element by the acoustic wavelength-filtering action of the intervening water between the element and the flow (e.g., see App. 2). Thus the high-wavenumber noise will be attenuated to a negligible level and white low-wavenumber noise will be reduced, at frequencies \( \lesssim 1.2 \text{ kHz} \), by a factor \( \sim R_o^2/8L^2 \), i.e., with depth \( L = 6 \text{ in.} \) and \( R_o = 2.5 \text{ in.} \), by \( \sim 16 \text{ dB} \).* This reduction of flow noise then leaves the element dominated by the residual acoustic noise at its location.

Even without confronting the high-frequency behavior, this explanation is challenged by consideration of the cross-spectral magnitude for the pair G5-G6 measured at 0, 10, and 20 kt. In the low-frequency range under discussion there is, in fact, a striking similarity among the cross-spectral magnitudes at 0, 10, and 30 kt, and the magnitude is rather high. On the other hand, at 20 kt the cross-spectral magnitude is quite different, being much lower in

*This estimate neglects the difference between the impedance of the window and that of an equal thickness \( h_o \) of water; this neglect is valid at the pertinent low wavenumbers, since \( \left( \frac{\omega h_o}{c} \right) \left( \frac{\rho_0}{\rho} - 1 \right) \ll 1 \), where \( \rho_0/\rho \) is the relative density of the windown material.
average magnitude. (The runs for which the cross-spectra are available, it is noted, are unfortunately not the same as the runs at the corresponding speeds for which the autospectra are available, but one would hope that the behavior in question is reproducible between runs.)

In addition, the supposition that flow noise on G5 is reduced in a more or less expected way in the lower frequency range without further anomaly would suggest that the spectrum on the flush G7 should have nearly the same speed dependence and roughly the same level as for neighboring flush hull-mounted elements. From the data at 15 and 25 kt, there is little speed dependence for G7 through much of this range. At the same time, the contribution of the local acoustic field is rather uncertain, so that there is no clearcut discrepancy. The level for G7 and 25 kt is appreciably below that for most LF elements at this speed, whereas at 15 kt from ~ 0.4 to ~ 1.5 kHz the level is nearly as high as for the LF's, suggesting some possible acoustic cancellation of low-wavenumber flow noise on account of low baffle impedance.

In particular, despite the 1/2-in. steel doubler plate attached to the window surrounding element G7, the effective acoustic impedance of the baffle may be substantially less than for the hull-mounted elements. Some indication should be given by comparison of in situ calibration measurements for the two types of mounting. Comparison of Figs. 4-4 and 4-6 referring respectively to the LF and the flush G elements shows, in fact, that the apparent sensitivities in situ for the LF elements is greater than the measured absolute
sensitivity by an amount approaching the theoretical limit of 6 db for an infinite rigid baffle at normal incidence, except at very low frequency, whereas the sensitivities in situ for the 6 elements are lower than the absolute sensitivities by a few db at most frequencies. It is not clear, however, why this result obtains also for elements G9 and G10, inasmuch as these elements are hull-mounted.

From theoretical considerations we may point out the expected character of the effect of a low-impedance air-backed baffle on the noise measured by a flush element (e.g., see Ref. 4). First, the high-wavenumber part of the direct TBL noise on a flush element in a rigid baffle will remain nearly the same for the low-impedance baffle, provided \( \omega R_0/U_\infty \gg 1 \). By virtue of non-rigidity, however, there will exist a resonant acoustic field due to excitation by TBL pressure on the baffle, or by mechanical shaking, of modes with wave numbers \( k_n \simeq k_{\nu}(\omega) \), where \( k_{\nu}(\omega) \) is the wavenumber for free bending waves in the water-loaded plate (baffle) at frequency \( \omega \). If the

* For a thin plate with water loading on one side \((m=1)\) or two sides \((m=2)\), if the mechanical damping is small, this wavenumber of free bending waves is given approximately by \( k_{\nu}(\omega) \simeq k_0 (\omega/k_0)^{1/3} \) for \( k_0 h \ll 1 \) or by \( k_{\nu}(\omega) \simeq k_0 \) for \( k_0 h \gg 1 \), where \( k_0 \) is the wavenumber of the bending waves in the absence of water loading and \( h \left[ = (\rho_f/\rho)h \right] \) is the fluid-equivalent plate thickness. \( k_0 \) is given by \( k_0 = (\omega/a_0)^{1/2} \), where \( a_0 = c_t h / [ 6(1-\sigma) ]^{1/2} \) with \( c_t \) the shear sound velocity in the bulk plate material, \( \sigma \) its Poisson ratio, and \( h_0 \) the plate thickness.
baffle stiffness is rather low, as for the fiberglass window proper, the modes excited in the frequency range of concern are high modes, and the \( k_n \) are closely spaced relative to the effective width of the resonance peak in wavenumber. If \( k_r(\omega)R_o \gg 1 \), the contribution of these resonant modes to noise is substantially reduced by area averaging. Also, for \( \omega/U_{\infty} \gg k_r(\omega) \), excitation of these modes by the high-wavenumber part of the TBL pressure spectrum produces modal pressures that tend to cancel one another alternately.

With regard to non-resonant acoustic contributions resulting from excitation of the baffle locally by TBL pressure, the contribution due to the high-wavenumber part of the TBL spectrum, in some indicative regime at least, is of the order of the high-wavenumber contribution to the TBL noise on a flush element of area equal to the free (unsupported) area of the baffle plate, and hence is small. As for the contribution due to the low-wavenumber part of the TBL spectrum, if \( k_r(\omega)R_o \gg 1 \), the baffle impedance there is mass-dominated and, where \( \omega R_o/c \gg 1 \), this contribution depends mainly on \( h/R_o \), where \( h \) is the effective water-equivalent plate thickness of the baffle. This contribution is opposite in sign and proportional to the low-wavenumber direct TBL pressure, and tends to cancel the latter; in the extreme limit of \( h/R_o \ll 1 \) and \( \omega R_o/U_{\infty} \ll 1 \), for example, this part of the acoustic field just cancels the corresponding part of the TBL pressure field, i.e. in the limit where the baffle is a pressure-release surface and the radiation impedance of the element is negligible, the element face is likewise nearly a pressure release surface. This
tendency toward cancellation for low baffle impedance naturally applies also to a radiated acoustic noise field transmitted to the element directly through the water.

Thus the effect of finite baffle impedance on the noise measured by a flush element depends not only on complex frequency-dependent relations involving the resonance parameters and boundary conditions of the loaded and supported baffle, but also on the size of the element and the incompletely established character of the wave-number spectrum of TBL pressure at low wavenumbers.

We return now to the question of the speed-dependent increase in measured noise on C5 (and C7) at high frequency. The character of this effect strongly suggests some type of resonance phenomenon. Referring to C5, we recall that the water-filled cavity separating the pair of elements C5 and C6 from the flow is covered by a fiberglass window of 9/16-in. thickness, and the opposite face in which the elements themselves are flush-mounted is an air-backed fiberglass plate of thickness 3/4 in.

Since the noise spectra near the maxima do not increase at all monotonically with flow speed, if the effect to be explained is real, we apparently have to deal with a response of the interior acoustic field that depends at each frequency not only on the magnitude of the source spectrum, but also on some other speed-dependent property of the source, unless we suppose that the speed-dependent noise source in question, e.g. bubble noise, decreases in some interval with increasing speed. It is natural to think of the condition
of hydrodynamic coincidence, at which the wavenumber of free bending waves in the cover bounding the flow coincides with the wavenumber where the TBL spectrum for a given frequency has a maximum, i.e. 
\[ k_x(\omega) \approx \frac{\omega}{u_c} \]
where \( u_c \) is a large fraction of \( U_\infty \). For the window, however, the stiffness would have to be unreasonably low to yield coincidence at frequencies so high and flow velocities so low as those involved. Moreover, coincidence should occur for each speed but at a changing frequency, whereas the maxima in the spectra occur at a relatively speed-independent frequency for speeds where they appear at all.

Indeed, even apart from the speed dependence of the maxima, we are unable to propose a reason why such a high broad maximum should occur, with a level there greatly exceeding the level on a flush hull-mounted element. We cannot suppose, for example, that the peak is associated with coincidence of the bending-wave resonance wavenumber with a modal wavenumber, i.e. \( k_n \approx k_x(\omega) \), since, among other reasons, for the applicable magnitudes of parameters, such resonances, if observable, would be narrow and closely spaced in frequency. Likewise, the interior acoustic field due to modes that propagate normal to the baffle and are excited by the pressure field outside the window cannot be substantially more intense than the corresponding exciting exterior field, since a resonant build-up of this sort can occur only to the extent that \( (\omega h_o/c)(\rho_o/\rho) \gg 1 \), where \( h_o \) is the thickness of the outer window and \( \rho_o/\rho \) are respectively the mass density of this window and of water. In the subject regime this parameter is instead less than unity, and the effective "Q" of the supposed resonant cavity is small.
In addition, the inner fiberglass face in which elements G5 and G6 are mounted is air-backed, and, having moderate impedance itself at the predominant wavenumbers of the interior acoustic field, it constitutes a baffle of relatively low impedance. Since this impedance is lower than that of the baffle containing the flush G7 and G8, we might expect lower noise and also lower in situ sensitivity. From Figs. 4-7 and 4-6 we see that the in situ sensitivity of G5, for example, at the higher frequencies is rather less than that of G7 relative to the respective absolute sensitivities, but the opposite is true in the frequency range from 1 kHz to 2 kHz. In attempting to account for the noise measured by the window-mounted G elements, it may be pertinent to bear in mind the wider variations from element to element and the more erratic frequency dependence found in the in situ calibrations for these elements relative to those for hull-mounted elements. Likewise, the in situ calibrations of G elements, though mutually consistent, are often unrepeatable.

We may attempt to correlate the speed-dependent high-frequency behavior of the G5 autospectra with indications of the G5-G6 cross-spectra at 0, 10, 20, and 30 kt (Figs. 3.2.2.41 to 60). The rises in the autospectra, we recall, occurred among these speeds at 10 kt and especially 20 kt. The cross-spectral magnitudes at high frequency are targets at 10 and 30 kt, with a build-up for 10 kt to a moderate peak at 8 kHz. At 20 kt the magnitude at high frequency is somewhat lower with no notable broad maxima. We thus derive from these cross-spectra no clue to an account of the G5 and G7 autospectra at high frequency.
In the absence of any other likely explanation of the high-frequency behavior of the G5 and G7 autospectra, it naturally suggests itself that the increased high-frequency noise is due to bubbles. These elements are so located that indeed the bubble stream may have passed over them, and furthermore the proximity of the stream might depend on ship speed in a nonmonotonic fashion related to that of the magnitude of the high-frequency humps in the autospectra. The pronounced dependence of the high-frequency hump in the G7 spectrum at 15 kt on heading reinforces this supposition.

On the other hand, earlier analog data for autospectra of G9 and G10, the nearby elements flush in the hull above the window, indicated no anomalous high-frequency behavior, whereas it would seem likely that these too would be affected by bubbles at some ship speeds. Also, there is no tie-in between an account of the anomalous G5 and G7 spectra by bubble noise and the erratic results of the in-situ calibrations for these window elements.

Nevertheless, bubbles seem most likely to be implicated, and this hypothesis should be thoroughly pursued in further analysis. In this regard it will be desired to examine more of the G spectra, including G9 and G10, when available, investigate the variation of the high-frequency humps between runs under nominally identical conditions, study carefully the photographic evidence regarding bubble paths as a function of ship speed, and listen to audio reproductions of the recorded noise.

Apart from the region of the high-frequency humps, the G5 noise spectra at the higher speeds are well below those for the flush LF elements. Relative to LF-7, for example, at 25 kt. the noise was lower by at least 10 db up to 3 KHz. Hence, it appears that recession of elements from the flow does decrease flow noise. As pointed out, however, this effect in the present

* A new correct spectrum for G5 at zero speed shows a noise level which, from 0.8 to 2 KHz, lies well above that at 15 kt. Hence more than mere attenuation of flow noise by the dome is involved.
configuration was attended by a partially compensatory decrease in the in-situ sensitivity of the recessed element to a signal.

3.3 Layer-shielded Elements in Sea Chest 1

The D elements of sea chest 1 were covered, and thus shielded from direct TBL pressure fluctuations, by rubber layers of maximum thicknesses given in the table below. The layers covering elements D1, D5, D6, and D10 were roughly planar and integral with a rubber window covering the entire sea chest. The layers covering the other elements were limited in lateral extent to a cone diverging from the element periphery at 60° from the normal but merging into the window layer of thickness ~1/2 in. The D elements had sensing faces of diameter 1.5 in.

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>D5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness (in.)</td>
<td>1-5/32</td>
<td>1-47/64</td>
<td>2-51/64</td>
<td>4-59/64</td>
<td>5/8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>D10</th>
<th>D9</th>
<th>D8</th>
<th>D7</th>
<th>D6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness (in.)</td>
<td>59/64</td>
<td>5-1/32</td>
<td>2-25/32</td>
<td>1-23/32</td>
<td>13/16</td>
</tr>
</tbody>
</table>

3.3.1 Autospectra

Measured noise spectra for D elements are shown in various groupings and compared with flush element HF-3, which occupied a similar position aft, in Figs. 3.3.1-1 to 16. Results of in-situ calibrations of the D elements are shown in Figs. 4-9, 4-10.

It is a conspicuous fact, for non-zero speed, that in no appreciable frequency interval do the measured noise spectra lie nearly in the inverse (or direct) order of thickness of the layers interposed over their faces.

At zero speed the noise spectra are comparable except in a range ~4 to 7 KHz and near 1.2 KHz. This result obtains
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despite the fact that the apparent sensitivities of the elements given by the in-situ (overside) calibration differ greatly among elements. These apparent sensitivities likewise bear no notable relation to the thickness of the respective layers. In some instances and frequency intervals the measured in-situ sensitivities lie above, in others below, the free-field sensitivities. The order of the calibration curves undergoes decided reversals with changing frequency. The noise spectra at zero speed lie well above that for element HF-3, and, in fact, the spectra do not emerge appreciably from this zero-speed level until a speed of 20 kt.

The noise spectra on the various D elements do not differ greatly from one another in the low-frequency range up to ~1.3 kHz at any speed through 30 kt. In this range, at 20 kt the spectra for the most part are a few db below that for the flush element HF-3, and somewhat further below at 30 kt.

In the higher frequency range the spread among the spectra is larger and increases with speed at 20 and 30 kt. As stated, the relative levels are not simply related to the order of layer thickness or type of layer installation (planar or conical). At 30 kt, from ~1.3 kHz to ~3.5 kHz the spectra lie 10 db or more below that for HF-3.

We may wish to consider the consequences of the supposition that the intrinsic sensitivities of some of the D elements were altered in the process of installation, or the similar supposition that the installation produced some peculiarity of the in-situ condition, e.g., air bubbles between sensitive face and covering layer, such that the apparent sensitivity of an element to both noise and to a calibration signal differed from its value in a satisfactory final configuration of interest. In such event it would be more appropriate to use the in-situ sensitivities in place of the nominal absolute sensitivities actually employed for the reduction of the noise data. (The measured sensitivities must be corrected for the angle of incidence and distance from the calibration source.) Inspection of the noise spectra with this point in mind, however, does not produce a more likely or sensible ordering of levels. If we take the same approach but discount the overside calibrations, we might employ relative
noise spectra measured at zero speed to adjust the relative spectra at high speeds; this course also produces unconvincing results.

Possibly the erratic variation among noise spectra at the higher frequencies is attributable to splash or bubble noise. The D elements are at least more accessible to these sources than are the HF elements, though they would seem still relatively remote from the main bubble stream. As in the case of the C elements, further analysis of this possibility is warranted.

If we look apart from the high-frequency anomalies of the noise spectra and the erratic character of the in-situ calibrations, we may suggest rather uncertainly on the basis of the comparisons at 20 and 30 kt that the layers on the D elements have somewhat reduced the flow noise relative to flush elements.

We recall once more the character of the anticipated effect of a covering layer on noise (App. 2), where three components of the noise are distinguished: (1) high-wavenumber, (2) low-wavenumber, (3) acoustic. If the layers were all planar and of sufficiently large extent, we would expect component (1) to be greatly reduced at all frequencies >0.2 KHz for all the layer thicknesses considered, since even at \( U_\infty = 30 \text{ kt} \) we have \( uL/U_\infty > 2 \) for \( u/2\pi > 0.2 \text{ KHz} \) if \( L > 1 \text{ in} \). In the instances of those elements where a large portion of the thickness \( L \) has only the lateral extent of a conical plug, the reduction of this component, however may be much less than for a planar layer of the full thickness.

The component (2), on assumption that it is roughly white noise in wave number, would be reduced somewhat as shown in the accompanying sketch, the maximum reduction being 25 db for \( L = 5 \text{ in} \). In the instances of a conical plug the reduction factor may be somewhat less than for an equally thick planar layer. The component (3) would not be appreciably reduced at all.

These expectations based on a fluid layer, however, are questionable in the present instance of a cover made of rubber, which has a low shear sound velocity. With reference to components
(1) and (2), it was assumed, in particular, that $V_s/c < c$, where $c$ presumably should be considered of the order of the shear sound velocity in the case of a solid layer. If the sketch pertaining to reduction of components (2) does apply, the upper frequency limit for appreciable reduction, we note, is given by $\omega/2\pi \sim c/\kappa R_o$, which decreases with decreasing $c$.

Though the design plan for the D elements called for a thinnest layer of 1/4 in. thickness, in fact the thinnest was of 5/8 in. thickness. It is unfortunate that there was not also a flush-mounted element of the same type.

Spectra of wavenumber-white noise beneath planar fluid layers of various thicknesses.
3.3.2 Cross-spectra

The magnitude and phase of the normalized cross-spectral density of noise measured on three pairs of D elements are shown in Figs. 3.3.2-1 to 52, Vol. I. These pairs, namely D2-D1, D3-D8, and D4-D9, consist of elements at roughly the same depth, increasing in the order given. Among these elements, we recall, the covering layers diverge conically from the peripheries of all but D1. The center spacings of the respective pairs are 20, 12, and 43 in.

There is no conspicuously uniform change in the behavior of the cross-spectral magnitude with increasing element depths (and spacings) from pair to pair. The average magnitude in the high-frequency range (≈ 2 kHz) at 10 kt does increase with this progression, and the magnitude is very high for the pair D4-D9. The magnitudes in various instances display strong peaks and dips. In several of the plots the continuity at the joining of the curves computed for the low- and high-frequency ranges is poor. Overall, the cross-spectral magnitudes are somewhat higher than for the flush pair HF5-HF4, for example.

It is not unambiguously clear what effect the recession of the elements ideally should have on the cross-spectra. Certainly the high-wavenumber TBL noise, which has a cross-spectral magnitude appreciable over a relatively limited extent in ω [e.g., see App. 3, Eq. (A3-3)], will contribute less to the autospectra on recessed elements, so that the magnitude of the normalized total cross-spectral density might be higher for such elements. This result should hold at least if true acoustic noise and not low-wavenumber TBL noise is the chief additional contributor to the noise autospectra.

For the D3-D8 pair at zero speed, the cross-spectral phase, it is noted, has an unusually regular linear dependence in two frequency intervals extending from 4 to 10 kHz. An effective phase velocity can be computed from the slope and corresponds to an acoustic wave with wave vector at a particular angle relative to the element separation.
REFERENCES

APPENDIX I

DIMENSIONLESS SPECTRA AND POSSIBLE SCALING LAWS FOR TURBULENT BOUNDARY-LAYER PRESSURE

It has become customary, with reference both to laboratory measurements of TBL noise and to ship measurements of self-noise on flush elements at non-vanishing ship speed, to present noise autospectra in dimensionless form vs. frequency in dimensionless form where the forms used are those defined by taking boundary-layer displacement thickness $\delta_x$ as the length parameter and asymptotic flow speed $U_\infty$ as the velocity parameter. Presentation in this form is obviously equivalent in information value to the raw measured spectra, entailing no assumptions, but the utility and appropriateness of this form is measured by the extent to which noise spectra due to the TBL scale in such a way that spectra for different values of $U_\infty$ and $\delta_x$ (i.e. of speed and distance aft) tend to coalesce on such a plot. The degree of coalescence, however, may well suffice to render this method convenient for presentation of data referring to different speeds without inspiring confidence in the optimality or even adequacy of the associated scaling law for extrapolation of results to an unmeasured regime.\(^8\)

If one is assured that TBL noise spectra do coalesce on some form of dimensionless plot, we note, then if measured spectra, e.g. sea-test data, are so plotted, they must lie above the basic TBL curve by an amount that measures the contribution of noise sources other than the TBL.

A theoretical discussion can be advanced (e.g. Ref. 3) suggesting that at sufficiently high frequency (perhaps for $\omega \nu / \nu_\kappa > 0.1$) the appropriate TBL pressure scaling parameters may be not $\delta_x$ and $U_\infty$ but rather the length and speed parameters characteristic of the inner part of the boundary layer, namely $\nu / \nu_\kappa$ (proportional to the thickness of the viscous sublayer) and $\nu_\kappa$, where $\nu$ denotes kinematic viscosity and $\nu_\kappa$ the friction

\(^8\)This is particularly true with regard to dependence of TBL noise on distance aft.
velocity. We refer to the pressure scaling form corresponding to parameters $v_0 \nu$ and $v_1 \nu$ as the inner scaling law and to that corresponding to parameters $\delta_0 \nu$ and $U_0 \nu$ as the outer.

For the Purvis data processing reported here, some noise autospectra were plotted in both inner and outer dimensionless forms, defined specifically below. The form taken in each case refers to point pressure and, even if the pertinent scaling law for TBL pressure is valid, TBL spectra measured by elements of radii $R_0 \nu$ therefore cannot be expected to coalesce unless $aR_o / U_0 \nu$ $<$ 1. Since the appropriate area correction factor, at fixed $R_0 \nu$ and $\delta_0 \nu$, may depend almost solely on $aR_o / U_0 \nu$, however, the outer form of dimensionless plot (though not the inner), if valid for $aR_o / U_0 \nu$ $<$ 1, may yield coalescing plots for varying $U_0 \nu$ but fixed $R_0 \nu$ even when $aR_o / U_0 \nu$ $>$ 1. Accordingly, the outer form of plot is useful and has been computed for elements of all sizes.

Various alternative possible scaling forms may be conjectured also for large elements ($aR_o / U_0 \nu$ $\gg$ $\mu$) (e.g. Ref. 3), which would take account of the area-averaging factor. Thus far these have not been computed in the Purvis data processing, apart from hand-computed results in Sec. 2 based on Eq. (A1-10) below.

The outer and inner forms of dimensionless plot are respectively

\[(A1-1) \quad 10 \log n_1 \nu \text{ vs. } 10 \log \Omega_1 \]
\[10 \log n_2 \nu \text{ vs. } 10 \log \Omega_2 \nu \]

with $n_1 \nu, \Omega_1$ defined as follows. Let $P(f)$ denote the spectrum of noise pressure on a given element, so normalized that the mean squared noise pressure is given by

\[(A1-2) \quad \langle p_f \rangle^2 = \int_0^\infty df P(f) = (2\pi)^{-1} \int_0^\infty dw P(w) / 2\pi. \]

*Since $\delta_0 \nu$ depends weakly on $U_0 \nu$, and strongly on distance aft, coalescence for given large $R_0 \nu$ at differing $U_0 \nu$ would be only approximate, and no coalescence would be expected for elements at varying distances aft. The appropriate generalization for dependence on $R_0 / \nu \delta_0$ is considered further on.*
The dimensionless frequency and pressure-spectra variables are defined by

(Al-3) \[ \Omega_1 = \frac{\omega \delta \xi \gamma}{U_\infty^2}, \quad \eta_1 = \frac{P(\alpha/2\alpha)}{\rho \delta \xi U_\infty^3}, \]
\[ \Omega_2 = \frac{\omega \nu}{\nu \xi}, \quad \eta_2 = \frac{P(\alpha/2\alpha)}{\rho^2 \nu \xi \nu^2}, \]

in which \( U_\infty \) is identified with ship speed. These definitions may be rewritten as

(Al-4) \[ \Omega_1' = \frac{(\omega \nu/U_\infty^2) R_\xi}{\eta_1}, \quad \eta_1' = \frac{P(\alpha/2\alpha)}{\rho^2 \nu U_\infty^2 R_\xi}, \]
\[ \Omega_2 = \frac{\eta_1}{g^2 R_\xi}, \quad \eta_2 = \eta_1 (R_\xi/g^2), \]

where

(Al-5) \[ R_\xi = \frac{U_\infty \delta \xi}{\nu}, \quad g = \frac{\nu \xi}{U_\infty}. \]

\( R_\xi \) and \( g \) are considered to be given adequately by the corresponding quantities for the TBL on a flat plate (at zero angle of attack) at a distance \( x \) from the leading edge equal to distance aft of the bow (taken as Frame 0). \( R_\xi \) and \( g \) have been computed from the following formulas, applicable at the high Reynolds numbers of concern, as discussed below:

(A1-6) \[ R_\xi = 1.30(10^5)(R_\xi/10^8)^{0.842}, \]
\[ g = 0.03023(R_\xi/10^8)^{-0.066}, \]

where

\[ R_\xi = \frac{U_\infty x}{\nu}. \]

Expression (A1-6) for \( g \) represents a power-law fit over the interval \( 0.97(10^7) < R_\xi < 0.77(10^9) \) to the values tabulated in Table 21.1, p. 540, of Ref. 6, with the fit made exact at \( R_\xi = 10^8 \). That Table was computed from a logarithmic velocity profile with constants determined empirically and provided an excellent fit to \( g \) up to the
maximun value $R_x=0.45(10^9)$ for which a measured value is available from Fig. 21.2 of Ref. 6. Eq. (Al-6) for $g$ fits the tabulated values very closely for $10^7<R_x<10^9$.

Expression (Al-6) for $R_x$, giving the displacement thickness $\delta_x$ as a function of distance aft $x$ and flow speed $U_m$, was obtained as follows. The momentum equation was integrated to give the increase in momentum thickness $\delta_m$ with downstream distance from $x_o$ to $x$. In the integrand, the friction coefficient was taken to be that yielded by formula (Al-6) for $g$. The lower limit $x_o$ was taken to correspond to $R_{x_o}=10^7$, and the corresponding value of $\delta_m$ was computed from the relation $R_m=U_m\delta_m/\nu=0.045R_x^{0.79}$ given by Ref. 7, Eq. (7-80), p. 487, as valid in this range. $R_x$ was computed from the $R_m$ obtained by integration by the relation $R_x=H R_m$; the shape coefficient $H$ was taken as $H=(1.61g)^{-1}$, given in Ref. 7, Fig. 7-6, p. 483, as a fit to experimental data (at somewhat lower Reynolds numbers, however, than those in question), and $g$ was once more computed by (Al-6).

Finally, as in the instance of $g$, a power law was fitted to the resulting computed $R_x$ over the interval $10^7<R_x<0.77(10^9)$, with the fit made exact at $R_x=10^8$.

We extend consideration of the outer pressure scaling law to the case of large elements ($\omega R_o/U_o>1$) to permit a comparison with measured noise spectra on elements at differing distance aft. If the point pressure spectrum scales according to the outer law (i.e., if the measured $\pi_1$ defined by (Al-3) depends only on $\Omega_1$), and possibly even if it does not, for $R_o\gg\delta_x$ the spectrum of area-averaged TBL pressure on a large element is expected to have approximately the form (Ref. 3)

$$P(\omega/2\pi) = (\omega R_o/U_o)^{-2} \delta_x^2 U_o^3 N(\delta_x/R_o,\omega \delta_x/U_o), \quad (Al-7)$$

where $N$ is some function of the indicated dimensionless arguments.

If, in addition, the wavenumber spectrum of TBL pressure varies little in most of the low-wavenumber range defined by $K \lesssim 2\delta x/R_o^{-1}$,
which can be true (if at all) only for $\omega R_0/U_\infty > 2n > \omega R_0/c$.

then (Al-7) reduces to the more restricted form

$$P(\omega/2n) = (\omega R_0/U_\infty)^{-2} \frac{2}{\rho \sigma^2 U_\infty^3} F(\omega R_0/U_\infty),$$

where $F$ is another function. This form, unlike (Al-7), implies a unique area dependence, namely as $R_0^{-2}$. If the wavenumber spectrum does not satisfy the restriction leading to (Al-8) but increases more or less smoothly with wavenumber, the general outer form (Al-7) may nevertheless hold and be approximated by the form

$$P(\omega/2n) = (\omega R_0/U_\infty)^{-s} \frac{2}{\rho \sigma^2 U_\infty^3} F(\omega R_0/U_\infty),$$

where $s$ is regarded as roughly constant and lies between 2 (as in (Al-8)) and 3; (Al-9) may be rewritten as

$$P(\omega/2n) = (R_0/\sigma^2)^{-s} \frac{2}{\rho \sigma^2 U_\infty^3} G(\omega R_0/U_\infty),$$

where $G(\chi) = \chi^{-s} F(\chi)$. Now we define a modified dimensionless spectrum $\tilde{\pi}_1$ by

$$\tilde{\pi}_1 = \pi_1 (R_0/\sigma^2)^{s},$$

with $\pi_1$ still defined in terms of a measured spectrum $P(\omega/2n)$ by (Al-4). Hence, if TBL pressure dominates the measured noise on a large element in some regime and this pressure satisfies the outer scaling law (Al-10), then plots of $10 \log \tilde{\pi}_1$ vs. $10 \log \theta_1$ will coalesce, and, unlike the instance of plots employing the unmodified $\pi_1$, the comparison of these plots has a basis even where the values of $R_0/\sigma^2$ differ, since the form (Al-10) takes proper account of this dependence on $R_0/\sigma^2$. 
APPENDIX 2.

REDUCTION OF FLOW NOISE BY A COVERING LAYER

We review briefly the basic points relevant to the question of noise reduction by a layer or a fluid dome with negligible cover impedance.** At a given frequency, three contributions to the noise on a large flush element (radius \( R_o \), \( \omega R_o / U_\infty \gg \pi \)) are distinguished. The first two are due directly to pressure fluctuations associated with the turbulent boundary layer (TBL). Of these, the first is a high-wave number part (\( K > \omega / U_\infty \)) which is the only kind that would be present if the pressure were generated by "frozen" eddies convected down-stream at velocities not exceeding the ship speed \( (U_\infty) \). This part varies with radius as \( R_o^{-3} \). Any additional pressure fluctuations due to surface roughness may be predominantly also of this high-wavenumber character. The second is a low-wavenumber component (\( K \ll 2\pi R_o^{-1} \)); the amplitude of this component is no doubt much smaller than that of the former, but it is more heavily weighted in the average pressure on the element, since its contribution is much less reduced by area averaging. The third contribution to noise is understood to include all other sources; it is presumed to have the character of a radiated sound field (modified by interaction with the flow-bounding hull and including any sound due to compressibility of the fluid of the TBL).

Shielding the given element by a layer of depth \( L \) is expected to have the following effects on the three contributions. The first (high-wavenumber) part will be reduced to negligibility provided roughly \( L \gg U_\infty / \omega \) and the lateral dimensions of the layer

* This appendix is based on Refs. 3 and 4.

** The explicit mathematical analysis that has been done pertains to a fluid, not a solid, layer. We expect, subject to experimental test, that an elastic solid behaves similarly provided the transverse sound velocity is of the order of the sound velocity for the fluid analog and large compared to the ship speed.
are large compared to the element diameter (and perhaps larger than the wave length \( \lambda (= 2\pi c/\omega) \) of sound in the layer material). In some parameter regime, more specifically, this part is reduced rather as though averaged, not over the element area, but over the lateral area of the layer. The second (low-wavenumber TBL) part will be reduced to an extent depending mainly on the ratios \( R_o/\lambda \) and \( R_o/L \). For example, if the wavenumber spectrum of the TBL pressure at frequency \( \omega \) is constant in the range in question (whence this part of the average-pressure spectrum for the flush element would vary as \( R_o^{-2} \)), this part for the shielded element is reduced as if averaged, not over the element area, but over an area \( \pi R_e^2 \), if \( R_e \geq R_o \), where:

\[
R_e^{-2} = (\pi/\lambda)^2 + 1/8L^2
\]

i.e., roughly over an area of radius equal to the smaller of three times the layer thickness or one third the sound wave length in the material; if \( R_e < R_o \), however, (as becomes true at sufficiently high frequency) this part is not appreciably reduced. The third (radiative) part will not be substantially reduced.

* When the lateral sheath size (nominal radius = \( \alpha \)) is sufficiently large and the depth is not too small, i.e., roughly when

\[
\pi/\alpha < \omega/U_\infty - (\omega^2/c^2 + L^-2)^{1/2},
\]

for sheathed elements the contribution corresponding to this high-wavenumber component may be composed approximately as the sum of a component arising from sheath modes with \( k_n \sim K \) and nearly equal to the entire high-wavenumber contribution for an infinite sheath (\( \alpha = \infty \)), and a component arising from relatively unattenuated sheath modes \( [k_n \lesssim (\omega^2/c^2 + L^-2)^{1/2}] \); these may be designated the infinite-sheath and the low-mode components, respectively.

** i.e., when averaged over wave-vector direction in the boundary, constant per unit area in two-dimensional wave-vector space.
reduced for any L, except that if the material is such as to introduce an acoustic impedance mismatch with respect to the water outside, both a signal and this part of the noise will be reduced similarly (such mismatch is thus not desired). The reduction factors, $S_1$, say, representing the effect of the layer on these (1) high-wavenumber, (2) low-wavenumber, and (3) acoustic radiative components of the average-pressure spectrum, each normalized to unity for $L = 0$, are thus expected to depend on thickness L at fixed frequency somewhat as shown in Figure A2-1. It is assumed in Figure A2-1 that $R_o \lesssim \lambda/\pi$. The asymptotic level approached by $S_2$ decreases with decreasing $R_o/\lambda$, and, in fact, if the low-wavenumber spectrum is constant, we have approximately $S_2 \to (R_o/\lambda)^2$. $S_1$ at large L depends mainly on the superficial extent of the sheath (e.g., on $\omega$) but also somewhat on $\omega$ and L.
Figure A2-1. Schematic dependence of noise-reduction factors on layer thickness for
(1) high-wavenumber, (2) low-wavenumber,
(3) acoustic components.
APPENDIX 3

NORMALIZED CROSS-SPECTRAL DENSITY OF NOISE FOR ACOUSTIC
AND TURBULENT BOUNDARY-LAYER PRESSURE FIELDS AND
FOR UNCORRELATED SUPERPOSED FIELDS

A.3.1 Introduction

The measured noise cross-spectra between flush elements
ordinarily involve a frequency-dependent mixture of pressure fields
with characteristic and differing properties, namely a turbulent
boundary-layer (TBL) field, an acoustic field, and perhaps a
bending-wave field. Here we shall note the cross-spectra expected
for each of these individually, and draw inferences about normal-
ized cross-spectra associated with the superposed fields in per-
tinent idealized examples. We thereby provide a basis for analysis
and interpretation of the measured cross-spectra.

A.3.2 Cross-Spectrum for TBL Noise

Let \( \tilde{v}_I(\vec{s},w) \) denote the frequency transform of the cross-
correlation of the average pressures on a pair of flush elements,
where \( \vec{s} = (s_1, s_2) \) is the center-to-center separation vector, with
\( s_1 \) streamwise, and the subscript 1 on \( \tilde{v}_I \) is used to distinguish
the TBL as the pressure field being considered. Likewise, let
\( \psi_1^{(1)}(w), \psi_1^{(2)}(w) \) denote the respective average-pressure spectra
due to the TBL on the elements individually. The normalized
cross-spectral density is then given by

\[
(A3-1) \quad \psi_1(\vec{s},w) = \frac{\tilde{v}(\vec{s},w)}{[\psi_1^{(1)}(w)\psi_1^{(2)}(w)]^{1/2}}.
\]

For small elements, i.e., for \( u_R / U_c \ll \) where \( U_c \) is the
asymptotic flow velocity and \( R_o \) the radius of either element,
laboratory measurements on the TBL of a flat plate have yielded,
in a broad regime, the similarity form

\[
(A3-2) \quad \psi_1(\vec{s},w) = \exp(i\omega s_1/u_c) F(\eta_{sp}/u_c),
\]
where $\mathcal{F}$ is a positive function of the indicated dimensionless argument [with $\mathcal{F}(0) = 1$] and the effective convection velocity $u_c$ decreases from $-0.8u_\infty$ at low frequency to $-0.58u_\infty$ at relatively high frequency (e.g., see Ref. 1). The measurements have determined $\mathcal{F}$ for $\zeta = (\zeta_1, 0)$, $\zeta = (0, \zeta_3)$, and for some intermediate values of $\zeta_3/\zeta_1$. Similar results have been obtained in sea tests on the TBL of a ship's hull (e.g., Ref. 2 and Purvis 1 phase of the tests reported here). As a simple approximation that roughly matches these results, for indicative purposes we may take

\[
(A3-3) \quad \mathcal{F}(\omega \zeta/u_c) = \exp(-\alpha_1 \omega \zeta_1/u_c) \exp(-\alpha_3 \omega \zeta_3/u_c),
\]

provided $\alpha_1 \zeta_1 > \alpha_3 \zeta_3$ or $\alpha_1 \zeta_1 < \alpha_3 \zeta_3$, with

$$\alpha_1 = 0.105, \quad \alpha_3 = 0.714.$$

In App. 4 a rough estimate is made of the change in $\mathcal{F}(\zeta_0, \omega)$ from that given for small areas by $(A3-2)$ when the elements (assumed equal and circular) are not small ($\omega R_0/u_\infty > 1$). It is indicated that the phase factor in $\mathcal{F}$ is relatively insensitive to this effect and that the magnitude $|\mathcal{F}|$ may be increased in some regime by a factor $-2$ but does not likely differ in order of magnitude or general behavior.

**A.3.3 Cross-Spectra for Acoustic Noise**

We consider now pure acoustic noise having the wave number $\omega/c$ and spatial coherence of a radiated field, and incident on the pair of correlated elements with some angular distribution but no correlation of the noise from different directions. Let $N(\theta, \phi, \omega)$ denote the spectral density of net noise pressure per unit solid angle due to waves incident from azimuth $\phi$ relative to

*For arbitrary $\alpha_1 \zeta_1/\alpha_3 \zeta_3$ we may reasonably presume

\[
(A3-4) \quad \mathcal{F}(\omega \zeta'/u_c) = \exp\left\{-(\alpha_3 \omega/u_c)\left[\zeta_3^2 + (\alpha_1/\alpha_3)^2 \frac{\zeta_1^2}{\zeta_3^2}\right]^{1/2}\right\}
\]

(see also App. 4).
the $\zeta_j$ direction and polar angle $\theta$ relative to the normal to the plane of elements. We continue consideration to a regime where the elements are small compared to the sound wavelength $(aR_0/c \ll 1)$, where $c$ is the speed of sound in water. Using a subscript 2 to denote the acoustic field being considered and forming the normalized cross-spectrum $\psi_2(\zeta, \omega)$ as at (A3-1), we obtain

$$ \psi_2(\zeta, \omega) = [d\Omega(\theta, \phi, \omega)]^{-1} \iint d\Omega(\theta, \phi, \omega) \times \exp[i(\omega/c)\sin(\zeta_1 \cos \phi + \zeta_3 \sin \phi)], $$

where $d\Omega = d\theta d\phi \sin \theta$.

For orientation, we may examine special cases. First, if the noise derives from a unique direction $\theta_o, \phi_o$, we have simply

$$ \psi_2(\zeta, \omega) = \exp[i(\omega/c)\sin \theta_o(\zeta_1 \cos \phi_o + \zeta_3 \sin \phi_o)] = e^{i\zeta \cdot \zeta_c} $$

where $\zeta_c$ is the incident wave vector ($|\zeta_c| = \omega/c$). In this instance we have $|\psi_2| = 1$, and the phase of $\psi_2$ advances by $2\pi$ only as $\omega/c$ changes by an amount $>>2\pi$ depending on the angle between $\zeta_c$ and $\zeta$. A second special case is that of hemispherical isotropy; from (A3-5) we then obtain

$$ \psi_2(\zeta, \omega) = \sin(\omega \zeta/c)/(\omega \zeta/c) $$

($\zeta^2 = \zeta_1^2 + \zeta_3^2$). A further case is that of azimuthal independence with the wave vectors constrained to parallel the plane of $\zeta$ (two-dimensional isotropy, $\theta = \pi/2$); we obtain

$$ \psi_2(\zeta, \omega) = J_0(\omega \zeta/c). $$

In the cases of (A3-7) and (A3-8), unlike (A3-6), $|\psi_2|$ is oscillatory and small $(\ll 1)$ for $\omega \zeta/c \gg \pi$; likewise, the phase of $\psi_2$ is sectionally constant, jumping from zero to $\pi$ and back again as $\psi_2$ changes sign with increasing $\omega \zeta/c$.
A.3.4 Cross-Spectra for Bending Waves

Along with the acoustic radiation field from sources well removed from the elements in question, the total acoustic pressure will be affected by the field due to such sources but mediated by the hull or other bounding surface with bending-wave response, and likewise due to more local excitation of bending-waves by the TML pressure field. At a given frequency, many wave numbers (or wave speeds) will contribute to this bending-wave field. The principal dependence on wave number will be inversely as the impedance of the water-loaded plate. The distribution of wave numbers and wave directions will not be continuous but rather spaced in accord with the modes of the finite local mechanical structure.

The previous discussion of the cross-spectrum for an incident radiation field may be regarded as roughly applicable also to the present field where, however, there is a distribution of wave speeds c with a peak near the speed of free bending waves in the loaded plate (and \( \sin \theta = 1 \)). These waves may produce jumps in the phase similar to those associated with (A3-8), but more closely spaced in frequency, since the pertinent c will ordinarily be much smaller than the speed of sound in water. The magnitude of this pressure field and hence its influence on the actual measured normalized cross-spectra will depend on the impedance of the local plate or window surrounding the elements.

A.3.5 Cross-Spectra for Shielded Elements

We consider briefly the implication for the cross-spectra of component fields discussed above where the elements are not flush but shielded from the flow boundary by a layer or dome.

The TML cross-spectrum, given for a flush element by (A3-2), is drastically altered; for a layer of large lateral extent the high-wavenumber part of the pressure on each element, which largely determines the form of the cross-spectrum for flush-mounted elements, is greatly attenuated for depths \( L \gg \frac{U_o}{a} \), as
discussed in App. 2. Some contribution from TBL pressure at low wave numbers may remain, modified by the impedance of the dome cover, if any. Assuming a white wavenumber spectrum, which may be roughly valid in the pertinent range if the cover impedance is small, we can readily find the normalized cross-spectrum for such a pressure field on shielded elements in a rigid baffle at depth \( L \) in a fluid of sound speed \( c \) in the particular regime \( \omega L/c \gg 1, (\omega L/c)^{-1}(\xi/L) \ll 1 \); we obtain

\[(A3-9) \quad \hat{S}_1(\xi, \omega) = 2J_1(\omega \xi/c) / (\omega \xi/c). \]

The fluid dome has been treated as laterally infinite in deriving this result, which is strictly justified only if the lateral dimensions, say \( a_2 \), satisfy \( (\omega L/c)(\omega a/c)^{-1/2} \ll 1 \). The result in the more interesting regime where \( \omega L/c \leq 1 \) and \( \xi \sim L \) is less readily evaluated and depends sensitively on the assumed wavenumber spectrum.

The results for cross-spectra of acoustic fields given above apply equally to fluid-shielded elements, but the actual effective angular distribution no doubt is conditioned greatly by the local structure and geometry.

A.3.6 Characteristic Normalized Cross-Spectra for Mixed Pressure Fields

We consider now the behavior of the magnitude and phase of the normalized cross-spectrum where the pressure on either element is due to a superposition of fields of the idealized types discussed above. We assume that the component fields considered are uncorrelated with one another. This is likely to be a good, though imperfect, approximation in the application in question. Accordingly, the (unnormalized) cross-spectrum of area-averaged noise pressure on a pair of elements, \( \hat{V}(\xi, \omega) \), may be written as a sum of the cross-spectra, \( \hat{S}_1(\xi, \omega) \), due to
The component pressure fields $i$:

$$\psi_i(\zeta, \omega) = \sum_i \psi_i(\zeta, \omega).$$

The autospectrum of average pressure for the $v$-th element of the pair, $\psi^{(v)}(\omega)$, may likewise be written as a sum of the component autospectra:

$$\psi^{(v)}(\omega) = \sum_i \psi_i^{(v)}(\omega) \quad (v = 1, 2).$$

We denote the magnitude and phase of the normalized cross-spectrum due to the total field by $\gamma(\zeta, \omega)$, $\phi(\zeta, \omega)$, respectively, choosing $-\pi < \phi < \pi$; we denote the magnitudes and phases for the normalized cross-spectra due to the individual component fields similarly by $\gamma_i(\zeta, \omega)$, $\phi_i(\zeta, \omega)$ ($-\pi < \phi_i < \pi$). Hence we have

$$\psi_i(\zeta, \omega) = \gamma_i(\zeta, \omega) \exp[i\phi_i(\zeta, \omega)]$$

$$(A3-12) \quad \psi_i(\zeta, \omega) = \bar{\psi}_i(\zeta, \omega) / [\psi_i^{(1)}(\omega) \psi_i^{(2)}(\omega)]^{1/2}$$

and by (10) and (11)

$$\psi(\zeta, \omega) = \gamma(\zeta, \omega) \exp[i\phi(\zeta, \omega)]$$

$$= \psi(\zeta, \omega) / [\psi^{(1)}(\omega) \psi^{(2)}(\omega)]^{1/2}$$

$$= \sum_i \bar{\gamma}_i(\zeta, \omega) / [\sum_i \psi_i^{(1)}(\omega) \psi_i^{(2)}(\omega)]^{1/2} / [\sum_j \psi_j^{(1)}(\omega) \psi_j^{(2)}(\omega)]^{1/2}$$

$$(A3-13) \quad \psi(\zeta, \omega) = \sum_i \gamma_i(\zeta, \omega) / [\sum_i \psi_i^{(1)}(\omega) \psi_i^{(2)}(\omega)]^{1/2} \exp[i\phi_i(\zeta, \omega)] / [\sum_j \psi_j^{(1)}(\omega) \psi_j^{(2)}(\omega)]^{1/2}$$

In the special case of equal autospectra, $\psi_i^{(1)}(\omega) = \psi_i^{(2)}(\omega) = \psi_i(\omega)$, (13) becomes

$$\psi(\zeta, \omega) = [\sum_i \psi_i^{(1)}(\omega)]^{-1} \sum_i \gamma_i(\zeta, \omega) \psi_i(\omega) \exp[i\phi(\zeta, \omega)].$$

(A3-14)
This form holds also if \( b_i^{(2)}(\nu) = \psi_i^{(1)}(\nu) = \psi_i(\nu) \), where \( b \) is independent of \( i \).

We note the idealized example of perfect correlation in the sense that \( \gamma_1(\zeta, \nu) = 1 \), \( \phi_1(\zeta, \nu) = 0 \). In this instance, (13) reduces to

\[
(A3-15) \quad \psi(\zeta, \nu) = \Sigma_i \left[ \psi_i^{(1)}(\nu) \psi_i^{(2)}(\nu) \right]^{1/2} \left[ \Sigma_i \psi_i^{(1)}(\nu) \right]^{1/2} \left[ \Sigma_j \psi_j^{(2)}(\nu) \right]^{1/2}.
\]

By Schwartz's inequality, this yields \( \psi(\zeta, \nu) \leq 1 \). If \( b_i^{(2)}(\nu) = \psi_i^{(1)}(\nu) \), then the equality holds, but if instead the mixture of noise spectra differs for the two elements, then the magnitude \( \gamma_1(\zeta, \nu) \) is less than unity even though \( \gamma_1(\zeta, \nu) = 1 \) for each \( i \).

Let us imagine a superposition of an azimuthally isotropic acoustic field, with normalized cross-spectrum \( \psi_2(\zeta, \nu) \) of (A3-8) and a unidirectional acoustic field, with cross-spectrum \( \psi_3(\zeta, \nu) \), say, given by (A3-6). Eq. (A3-14), then yields

\[
(A3-16) \quad \psi(\zeta, \nu) = (\psi_2 + \psi_3)^{-1} \left[ \psi_2 \mathcal{J}_0(\omega \zeta/c) + \psi_3 e^{i\omega \zeta/c} \right],
\]

where \( \psi_i \) denotes the autospectra \( \psi_i(\nu) \) and \( \zeta_p \) is the projection of \( \zeta \) along the wave vector of the field \( i = 3 \). The magnitude \( \gamma(\zeta, \nu) \) of \( \psi \) has a form intermediate between \( \gamma_2 = \mathcal{J}_0(\omega \zeta/c) \) and \( \gamma_3 = 1 \); where \( \mathcal{J}_0(\omega \zeta/c) = 0 \), in particular, we have \( \gamma = \psi_3/(\psi_2 + \psi_3) \). In a range of \( \omega \zeta/c \) such that \( \mathcal{J}_0(\omega \zeta/c) < 0 \), if \( \psi_2/\psi_3 \) has an appropriate value, the phase \( \varphi \) of \( \psi \) may assume any value; so long as \( \omega \zeta/c < 2.4 \) so that \( \mathcal{J}_0(\omega \zeta/c) > 0 \), however, \( \varphi \) remains a small angle.

Let us now consider a superposition of a TBL pressure field \( i = 1 \) and an acoustic field \( i = 2 \), where we no longer specify a particular idealized form for the latter. We regard the possible frequency dependence of \( \gamma \) and \( \varphi \) at fixed \( \zeta \). Suppose
the elements are sufficiently small that the magnitude of the TBL contribution dominates the autospectra, i.e., \( \psi_1(\omega) \gg \psi_2(\omega) \). In the low-frequency range where \( a_4 \omega \xi / u_c < 1 \) and \( a_3 \omega \xi / u_c < 1 \), according to (A3-3) for \( \psi_1(\omega) \), we will then have also \( \psi_1(\xi, \omega) \psi_1(\omega) \gg \psi_2(\xi, \omega) \psi_2(\omega) \); by (A3-14) the magnitude of the phase \( \psi \) will there be given approximately by the corresponding TBL functions \( \psi_1, \psi_2 \) of (A3-3) and (A3-2). In view of the rapid decrease of the TBL cross-spectral magnitude \( \psi_1(\omega) \) implied by (A3-3) relative to the decrease of the acoustic magnitude \( \psi_2 \) exemplified by (A3-6) and (A3-8), we see however, that in some frequency range where \( \omega \xi / u_c \gg 1 \) but \( \omega \xi / c \leq 1 \) we will have \( \psi_1(\xi, \omega) \psi_1(\omega) \ll \psi_2(\xi, \omega) \psi_2(\omega) \). In this regime Eq. (A3-14) yields rather

\[
(A3-17) \quad \psi(\xi, \omega) = \psi_2(\xi, \omega)
\]

So long as \( \psi_1(\omega) \gg \psi_2(\omega) \), therefore, the magnitude \( \psi \) of the normalized cross-spectrum will be small, but the phase \( \psi \) will be approximately the phase \( \psi_2 \) of the acoustic field, which varies slowly and remains small for \( \omega \xi / c \leq 1 \). It may happen that the acoustic contribution to the autospectra decreases less rapidly than the TBL contribution and, at some higher frequency, becomes comparable with and then larger than the latter \( \psi_2(\omega) \gg \psi_1(\omega) \). In such a range, according to (A3-17) the magnitude \( \psi \) will again increase and approach the acoustic value \( \psi_2 \) while the phase will continue to be nearly equal to \( \psi_2 \).
APPENDIX 4

CROSS-SPECTRAL DENSITY OF TURBULENT BOUNDARY-LAYER PRESSURE FOR NON-VANISHING AREAS

We indicate here to what degree the normalized inter-element cross-spectral density of area-averaged TBL pressure depends on element area. We take the elements to be circular and of equal radii \( R_0 \) and denote the normalized cross-spectral density of area-averaged pressure by \( \psi_R (\zeta, \omega) \) (identifiable with \( \psi_1 (\zeta, \omega) \) of App. 3). In general, this normalized cross-spectrum is related to the (unnormalized) cross-spectrum for point pressure, \( \tilde{\psi}_o (\zeta, \omega) \),

\[
(A4.1) \quad \psi_R (\zeta, \omega) = \frac{\int d^2 \bar{R}_1 \int d^2 \bar{R}_2 \tilde{\psi}_o (\bar{R}_2 - \bar{R}_1, \omega)}{\int d^2 \bar{R}_1 \int d^2 \bar{R}_2 \tilde{\psi}_o (\bar{R}_1 - \bar{R}_1, \omega)},
\]

where \( \int d^2 \bar{R}_1 \) and \( \int d^2 \bar{R}_2 \) run over the area of a single element and \( \int d^2 \bar{R}_2 \) over that of the other. We denote the autospectra of average pressure on either element by \( Q_R (\omega) \) (identifiable with \( \psi_1 (\omega) \) of App. 3), whence \( Q_o (\omega) \) is the spectrum of point pressure. In terms of the wavenumber-frequency spectrum of TBL pressure, \( P(\bar{R}, \omega) \), we have

\[
(A4.2) \quad \tilde{\psi}_o (\zeta, \omega) = \int d^2 \bar{R} e^{i \bar{R} \cdot \bar{k}} P(\bar{R}, \omega) \{ \bar{k} = (k_1, k_3) \}.
\]

(Thus we may write \( Q_o (\omega) = \tilde{\psi}_o (0, \omega) \).) Inserting this in \((A4.1)\) and performing the area integrals we obtain

\[
(A4.3) \quad \psi_R (\zeta, \omega) = \frac{\int d^2 \bar{R} e^{i \bar{R} \cdot \bar{k}} [2J_1 (KR_0) / KR_o] \int d^2 \bar{R} [2J_1 (KR_0) / KR_o] P(\bar{R}, \omega)}{\int d^2 \bar{R} [2J_1 (KR_0) / KR_o] P(\bar{R}, \omega)}.
\]

In the approximation of point elements \( (\omega R_0 / U_0 < 1) \), we replace the area-averaging factor \( [2J_1 (KR_0) / KR_o]^2 \) by unity in both numerator and denominator [recall App. 3, Eq. (A3-2)].

Suppose first that \( P(\bar{R}, \omega) \) contains a non-convective, low-wavenumber contribution, say \( P_\omega \), that, with reference to

\* This appendix is an edited version of App. A of Ref. 8.
non-vanishing $R_\omega$, may be regarded as independent of wavenumber $\vec{k}$. Such a component is found to yield a vanishing contribution to the numerator in (A4.3), reflecting the fact that a flat wavenumber spectrum corresponds to zero correlation distance and hence no cross-correlation between elements. We therefore expect that the major contribution to the numerator, if not to the denominator ($Q_\omega(\omega)$), in (A4.3) arises from the region of the convective peak (where $k_1' \sim \omega/\nu_e$) in $P(\vec{k},\omega)$. Accordingly, we assume a form for $P(\vec{k},\omega)$ that is thought to represent correctly the character of this peak, though it cannot be expected to have validity in the "tail" where $k_1' \ll \omega/\nu_e$. In this way, we probably overestimate, if anything, the normalized cross-spectrum $\hat{\phi}_0(\vec{k},\omega)$, a course preferable to underestimating it. We may expect also that the error from our crude representation of $P(\vec{k},\omega)$ in the region of its peak will in part be cancelled on account of its appearance in both numerator and denominator of (A4.3).

For reasons we shall not pursue*, we assume that $P(\vec{k},\omega)$ may be taken for present purposes as

$$P(\vec{k},\omega) = B^2\nu_e^{-1}\left[\frac{k^2}{k_1^2} \frac{(\omega - u_0 k_1')^2}{v_e^2 + \omega^2}ight]^{-\nu(1/2)},$$

$$(k^2 = k_1^2 k_2^2),$$

where $u_0$ is a certain average convection velocity (not to be identified precisely with $u_e$ in Eq. (A3-2) of App. 3; $v_e$ is an effective rms local convection velocity associated with turbulence and shear, of the order of $3v_\kappa$, or perhaps somewhat larger, with $v_\kappa$ the usual friction velocity; $B^2$ is a suitably dimensional factor regarded as constant; the constant $\nu$ may be related to the high frequency dependence of the convective contribution to the point spectrum given by (A4.4) according to $Q_\omega(\omega) \propto \omega^{-2\nu+1}$ (see Eq. (A4.6) below), and we presume $\nu \sim 5/3$; $a^{-1}$ represents a mean spatial scale of the pressure fluctuations, perhaps $a^{-1} \approx 6\kappa$ at moderate frequencies.

Referring first to point areas ($\omega R_\omega / u_0 \ll 1$), from (A4.4)

---

*The explicit form (A4.4) has its origin in a treatment of homogenous turbulence (Ref. 5), but in general character is thought correct for the present purpose.
and (A4.2) we find:

\[
\tilde{\psi}_0(\xi,\omega) = 2^{-\nu+3/2} \pi \Gamma(\nu+1/2) \left( \frac{\omega^2}{u_e^2 + a^2} \right)^{-1/2} B_{-1}(\omega^2 / u_e^2 + a^2)^{-\nu+1/2} \left( \frac{\omega^2}{u_e^2 + a^2} \right)^{-\nu+1/2} / 2
\]

\[
(A4.5) \quad x \exp(i \omega \xi_1 / u_e) \left( \xi_3^2 + \mu^2 \xi_1^2 \right) (\nu-1/2) / 2
\]

where

\[
u^2 - u_o^2 + v_e^2, \quad u_e = u_o^2 / u_o, \quad \mu = v_e / u_e,
\]

and \(K_\nu\) denotes a modified Bessel function of the exponentially decreasing sort. Likewise, we obtain

\[
(A4.6) \quad Q_0(\omega) = \tilde{\psi}_0(0,\omega) = \pi (\nu-1/2)^{-1} \frac{2}{u_e^2 + a^2} (\omega / u_e^2 + a^2)^{\nu-1/2}.
\]

Division of (A4.5) by (A4.6) yields the normalized cross-spectrum \(\psi_o(\xi,\omega)\) for point areas implied by (A4.4). We write \(\psi_o(\xi,\omega)\) explicitly in the approximation valid for \(\left( \omega^2 / u_e^2 + a^2 \right)^{1/2} (\xi_3^2 + \mu^2 \xi_1^2)^{1/2} \gtrless 1\) when \(K_{\nu-1/2}\) is replaced by its asymptotic form:

\[
(\xi_3 / \xi_1)^{1/2} (\nu-1/2) 2^{1-\nu} \Gamma(\nu+1/2)^{-1} \exp(i \omega \xi_1 / u_e)
\]

\[
x \left[ (\omega^2 / u_e^2 + a^2)^{1/2} (\xi_3^2 + \mu^2 \xi_1^2)^{1/2} \right] (\nu-1/2)^{1/2} \exp \left[ - (\omega^2 / u_e^2 + a^2)^{1/2} (\xi_3^2 + \mu^2 \xi_1^2)^{1/2} \right].
\]

For \(\xi_3 = 0\) or \(\xi_1 = 0\), we note, with appropriate \(\nu\), this form is closely similar to form (A3-2) and (A3-3) or A3-4) in App. 3.

We now get to the point of treating non-vanishing \(R_o\) in certain tractable regimes. In conceiving such regimes we are cognizant that, since \(\xi > 2R_o\), the factor \(e^{i K \cdot \xi}\) has a shorter period of oscillation in \(K\) than does the area-averaging factor except along a direction of \(K\) nearly normal to \(\xi\). First, we assume \(\omega R_o / u_e \ll 1\) and consider corrections in \(\psi R_o(\xi,\omega)\) to lowest order in \(R_o\). We therefore use the first two terms of the expansion.
\[
\left[2J_1^2(\kappa R_o)/\kappa R_o\right]^2 = 1 - (1/4) (\kappa R_o)^2 + \ldots
\]

and assume \( \nu > 3/2 \) in (A4.4), in which case the contribution of the second term to the numerator and denominator of (A4.3) converges and yields the correction asymptotically in the subject limit. The numerator may be written as

\[
\left[(1/4)R_o^2(\eta^2/\beta \xi_1^2+\delta^2/\beta \xi_3^2)\right] R_o(\xi_1, \omega)
\]

with \( R_o \) given by (A4.5). We assume \( u = v_c/u_c < 1 \) (a reasonable approximation) but \( \omega \xi_1 u_c \ll 1 \) (which implies \( \xi_1 \gg R_o \)). We restrict consideration to \( \xi_3 = 0 \) for simplicity, as the case of greatest concern. To the order considered, we find

\[
\frac{\delta_{R_o}}{R_o}(\xi_1, 0, \omega) = \left\{1 - (1/4) (\omega R_o / u_c)^2 - (1/4) (R_o / \omega \xi_1)^2 (\omega^2 / u_c^2 + a^2)^{1/2} \right\} \xi_1 (\xi_1, 0, \omega).
\]

Similarly, for the denominator of (A4.3) we find

\[
Q_R_o(\omega) = \left\{1 - (1/4) (\nu - 3/2) \left[1 - R_o^2 (\nu - 1) \omega^2 / u_c^2 (1/2) a^2 \right]\right\} Q_o(\omega).
\]

The ratio yields, to the order considered,

\[
\psi_{R_o}(\xi_1, 0, \omega) = \left\{1 + (1/8) (\nu - 3/2) \left[1 - (\omega R_o / u_c)^2 (a R_o)^2\right] - (1/4) (R_o / \omega \xi_1)^2 (\omega^2 / u_c^2 + a^2)^{1/2} \right\} \psi_o(\xi_1, 0, \omega).
\]

This result indicates that the correlation is enhanced (\( \psi_{R_o} / \psi_o > 1 \)) as \( R_o \) increases within the domain of applicability. It indicates also that the phase of \( \psi_{R_o} \), i.e., the effective convection velocity, remains unaffected in this approximation. The range of \( R_o \) to which (A4.10) applies, however, is explicitly limited to that where the enhancement being investigated is small.
We can successfully consider a second, more interesting limit. This is defined by the conditions

\[(A4.11) \quad \omega R_0/u_o \gg 1, \mu(\omega R_0/u_o) \ll 1, \xi_3 = 0, \omega u_1/u_o \gg 1, \omega/u_e \gg a.\]

These conditions are crudely satisfied in much of the domain of interest. The second of conditions \((A4.11)\) ensures that \(J_1(KR_0)\) in \((A4.3)\) varies little as \(k_1\) varies through the convective peak, so that the \(k_1\) integration may be performed; the first of \((A4.11)\) then permits use of the asymptotic form for \(J_1((k_3^2+\omega^2/u_e^2)^{1/2}R_0^-3\pi/4);\) the first and third permit a local average value of 1/2 to be used for the resulting factor \(\cos^2((k_3^2+\omega^2/u_e^2)^{1/2}R_0^-3\pi/4);\) the fourth permits \(K_\nu\) to be replaced by \(K_{\nu+3}\), which has the same asymptotic form, with regard to a factor \(K_\nu((k_3^2+\omega^2/u_e^2)^{1/2}u_1/e_1)\) from the \(k_1\) integration in the numerator, and, when \(a\) is neglected by the fifth condition, the \(k_3\) integration can also be carried out. Ultimately we obtain for the normalized cross-spectrum

\[
\psi R_0(\xi_1,0,0) = \pi^{1/2}2^{1-v} \frac{\Gamma(\nu+1/2)}{\Gamma(\nu)} \exp(i\omega \xi_1/u_e) \tag{A4.12}
\]

Comparing with \(\psi_o(\xi_p,0,0)\) as given by \((A4.7)\) with \(a = 0\), we see that \((A4.12)\) differs only by having the constant coefficient \((v-1/2)/(v+1/2)\) of \((A4.7)\) replaced by \([((v+1/2)/v)\Gamma(\nu+1/2)/\Gamma(\nu)]^2.\) If \(v = 5/3\), the ratio of the latter to the former is 1.61. In the subject limits, the normalized cross-spectrum is thus increased relative to the instance of elements with vanishing area by 1.6 (and the phase is unchanged). The exact value computed may not be considered reliable in view of the many approximations. Nevertheless, the analysis suffices to indicate that the finite-area effect increases the inter-element cross-spectral density of average pressure by only a modest factor.

The denominator of \((A4.3)\) is given in the present approximation by

\[Q_{R_o}(a) = 4 \frac{\nu}{\Gamma(\nu+1/2)} \left(\frac{\Gamma(\nu)}{\Gamma(\nu+1/2)}\right)^2 B^2 R_o^{-3} u_e^{-1} (\omega/u_e)^{-2v-2}.\]
Element: HF-3

![Graph showing frequency vs. speed with different lines labeled A through G.]

- A: 0 Kts - Run 336 - Orange
- B: 0 Kts - Run 350 - Blue
- C: 10 Kts - Run 338 - Pink
- D: 15 Kts - Run 339A - Green
- E: 20 Kts - Run 340A - Purple
- F: 25 Kts - Run 331A - Black Dotted
- G: 30 Kts - Run 342 - Black

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Fig. 3.11-1
A. =0 KTS - Run 860
B. =10 KTS - Run 338
C. =15 KTS - Run 239A
D. =20 KTS - Run 340B
E. =22 KTS - Run 862
F. =30 KTS - Run 342

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CONFDENTIAL

Fig. 3.1.1-2
SPECTRUM DD REP 1 MICROBAR SQUARED TIMES SEC 6XX LOW BAND CPS 11
RUN 336 START TIME 14442410.0 SPEED 50 HEADING 360
TYPE SCR NO FT FROM ZCM
R68 HF-5 050 0671.3

A = HF-5 \{ \text{Run 336 - OKTS} \}
B = HF-12

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\text{Fig. 3.1.1-3}
A = HF-5
B = HF-12
\{\text{Run 340A - 20 Kts}\}

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\text{FIG. 3.1.1-4}
A = 20 knots - Run 340A, 0°
B = 20 knots - Run 436, 90°
C = 20 knots - Run 637, 270°
A. = Run 822 - 20 kts - Heading FRR
B. = Run 840A - 20 kts - Heading 0°

Note: Run 823 - FLR ≠ FRR
SPECTRUM DB REF 1 MICROBAR SQUARED TIMES SEC 0XX LOW BAND CF222 = 43D HIGH BAND CF222 = 45D
RUN 348 START TIME 1305420G.O SPEED 30 DEGREES OF HEADING 000
TYPE TKG
SAR 10 FT FROM GUN
H=6 D=71 11.75
A: 0 KTS, Run 348
B: 10 KTS, Run 345
C: 20 KTS, Run 347A
D: 30 KTS, Run 349

Fig. 3.1.1-9
Element LF-7

Spectrum: DB REL 1 MICRODRO SQUARE TIMES SSEC

- Low Band CF 222 - 440
- High Band CF 222 - 440

A = 0.015 RMS 843
B = 10 kHz RMS 845
C = 15 kHz RMS 848A
D = 20 kHz RMS 847A
E = 25 kHz RMS 848A
F = 30 kHz RMS 849

Fig. 3.1.1-10
SPECTRUM MD REF 1 MICROBAR SQUARED TIMES SEC 6 BY LOW BAND CF222 - 44A HIGH BAND CF222 - 44A
RUN 349 START TIME 135690.0 SPEED 30 HEADING 090
TYPE SRR NO FT FROM DRW
TAG LF-4 DFS 05.75

LF-4, RUN 349, 30 KTS
LF-6, RUN 349, 30 KTS
LF-7, RUN 349, 30 KTS

Fig. 3.1.1-13  CONFIDENTIAL
HF-3, RUN 340A, 20 KTS

LF-6, RUN 347A, 20 KTS

LF-7, RUN 347A, 20 KTS

Fig. 3.1.1-15
SPECTRUM DD REF 3 MICROBAR SQUARED TIMES SEC 60X LOW BAND CF36 - 550 - HIGH BAND CF56 - 610
RUN 342 START TIME 13467000.0 SPEED 30 HEADING 090
TYPE SER NO FT FROM EON
TRG HF-3 026 69-75

HF-3, RUN 342, 30 KTS
LF-6, RUN 349, 30 KTS
LF-7, RUN 349, 30 KTS

Fig. 3.1.1-16
A: LF-7, Run 347A
B: HF-3, Run 340A
C: SEIII, Run 331

20 KNOTS

Fig. 3.1.1-16A
ELEMENT: LF-7

A: 10 Kts, Run 345
B: 20 Kts, Run 347A
C: 30 Kts, Run 349
ELEMENT: LF-A

A: 10 KTS, Run 345
B: 20 KTS, Run 347A
C: 30 KTS, Run 349

Fig. 3.1.1-19. CONFIDENTIAL
A: HF-3, 15 kts, Run 859
B: HF-3, 20 kts, Run 856
C: LF-6, 20 kts, Run 3471
D: LF-6, 30 kts, Run 349

FIG: 3.1.1-22
A: 0 kts, Run 336
B: 10 kts, Run 338
C: 20 kts, Run 340A
D: 20 kts, Run 340B
E: 30 kts, Run 342

Fig. 3.1.1-26
SPECTRUM CD REF: MICROBAR TIPPED TIMES Sec 655 Low Band CPS 225 - 400 Band CPS 240
RUN NO START TIME 10/23/73 4:30 10 SPEED 10 HEADING 000
TYPE LWR NO FT FROM 200kip
125 02.75

A = HF-3 \{ RUN 338 \} 10 KPS
B = H-S \{ RUN 338 \}
C = LF-6 \{ RUN 345 \}

Fig. 3.1.1-28

CONFIDENTIAL
SPECTRUM DB REF 1 MICROMEGS SQUARED TIMES SEC 60 X LOW BAND CP524 546 X HIGH BAND CP526 526
RUN 340A START TIME 164.540.0 SPEED 20 BEARING 009
TYPE SER NO FT FROM DOW
TRG H-5 125 20.75

A = HF-3 \{Run 340A\} 20 KTS
B = H-5 \{Run 340A\}
C = LF-6 \{Run 340A\}

Fig 3.1.1-29
A = HIF-B
B = H-5
C = LF-6

Run 342
Run 349

30 KTS
SPECTRUM 60 ARM 1 MICROBAR SQUARED TIMES SEC 66X LOW BAND CF100 - 79A - HIGH BAND CF100 - 61A

RUN NO: START TIME 12:00:00:00 SPEED 20 HEADING 000
TYPE SER NO FT FROM BOW
THG 0-5 000 11.25

A: 0 KTS - BLUE - RUN 336
B: 10 KTS - GREEN - RUN 338
C: 15 KTS - RED - RUN 341A
D: 20 KTS - PURPLE - RUN 340A
E: 25 KTS - BLACK - RUN 348
F: 30 KTS - ORANGE - RUN 342

Fig 3.2.1-1
SPECTRUM - CURVES 1 MICROVARI SQUARE TIMES SEC OXY LOW BAND CF100 - 700 - HST BAND CF100 - 600
START TIME 1647+30.5 SPEED 45 MPH HEADING 600°
TYPE 578 NO FT FROM OXY
TAG 0718 11:35

A = 15 KTS 0° HEADING Run 346A
B = 15 KTS 90° HEADING Run 442
C = 25 KTS 0° HEADING Run 348A
D = 25 KTS 90° HEADING Not Shown
SPECTRUM OD REF 1 MICROBAR SQUARED TIMES SEC 0.0X LOW BAND CF 100 = 700 - HIGH BAND CF 100 = 600
RUN 346A START TIME 162700.0 SPEED 15 HEADING 000
TYPE SER. NO. FT FROM DOY
THE CIP. DOY 11.75

A = G-7
B = LF-7
C = G-S
SPECTRUM: DS REF 1 MICROBAR NAVAGED TIMES 500
RUN 304030:200 SPEED KB HEADING 000
TYPE
SEQ NO FT FROM B N
TAG
SPN 000 0078

A = LF-7 Run 347A
B = G-5 Run 340A

20 KTS
A = G-5 Run 348A
B = G-7 Run 348A (25 KTS)
C = LF-7 Run 348A

SPECTRUM BD REF 1 MICROBAR SQUARED TIMES SEC.
LOW BAND CF100 - 79A
HIGH BAND CF100 - 81A
RUN 348A START DATE 12491630.6 SPEED 25 HEADING 000

TAG: 09-9 190 11:25
FIG. 3.3.1-4

CONFIDENTIAL
D1H =  
D2H = ————
D3H =  
D4H = ————

Run 342 - 30KTS

Fig. 3.3.1-7  CONFIDENTIAL
DH ELEMENT - 30 KHz.

\[ \Delta q H^2 = \]  
\[ \Delta q H' = \]  
\[ \Delta 10H^2 = \]  

\{ Run 342 - 30 Kts \}

Fig. 3.3.1-8 CONFIDENTIAL
SPECTRUM OD REF 1 MICROHEL SQUARED TIMES SEC 60X LOW BAND CF56 - 70B - HIGH BAND CF56 - 74B
RUN 342 START TIME 1344700.0 SPEED 30 HEADING 400
TYPE SER NO FT FROM DOM
TAG DISH

A: 0 KTS, Run 336
B: 10 KTS, Run 338
C: 20 KTS, Run 340A
D: 30 KTS, Run 342

Fig. 3.3.1-14
CONFIDENTIAL