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NEAR FIELD PATTERNS OF SEISMIC RADIATION

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Near field displacement, velocity and acceleration ground motion are computed for a buried finite strike slip fault using generalized multipolar ray theory to study the effects of rupture velocity, rise time of particle displacement, direction of rupture relative to observing station and the presence of the free surface. Computed displacement seismograms demonstrate that the rise time and rupture velocity can be traded off to produce similar wave shapes emphasizing the difficulty of separating the effects of rise...
time and rupture velocity can be traded off to produce similar wave shapes emphasizing the difficulty of separating the effects of rise time and rupture velocity. On the other hand, synthetic accelerograms exhibit much character as the rise time becomes a small part of the rupture duration and individual contributions to the acceleration signals such as the P- and S-wave stopping phase can be seen. Near field synthetic accelerograms hold promise for the study of fault rupture parameters. For the strike-slip model studied allowance for the presence of a free surface by doubling of the amplitude of infinite space signals seems to be approximately correct even in the case of computed vertical component accelerograms. However, this result is not generally true for the residual static displacements. The known result of the appearance of Rayleigh waves at horizontal distances greater than five times the source depth is confirmed for dislocation sources. A portion of the near-field coda can be attributed to the presence of crustal layers which must be accurately known before the source radiation can be completely known.
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PREFACE

The research discussed in this report had several objectives: 1) an examination of the effect of the free surface on the near-field seismic radiation for seismic sources imbedded in an elastic half-space; 2) an examination of the interrelation of various parameters such as source dimension, rise time and frequency and 3) an examination of the effects of fault length and rupture velocity in the near-field together with a study of the near-field coda.
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FIGURE 18. Theoretical point source seismograms.

a) Homogeneous half-space.

b) Layered half-space. Source depth 12.5 km, epicentral distance 5 km.
INTRODUCTION

Observations have recently been made with broadband displacement transducers recording within a few source depths, source dimensions or wavelengths of central California earthquakes (Johnson and McEvilly, 1974). At these short distances, where hopefully the effects of scattering and attenuation are minimized, one hopes to gain insight into details of the faulting mechanism, such as rupture velocity and the time function of displacement.

Our motivation for computing theoretical seismograms is to gain some insight into the effects of the free surface, and rupture parameters for a propagating strike-slip fault in the near-field. Specification of a working model and its resultant theoretical seismogram, which can be compared to observed data, is an important facet in our ability to predict strong ground motion in the near field of potentially damaging earthquakes.

Near-field displacements have been examined using dislocation modeling in an infinite space (Haskell, 1969; Kanamori, 1972; Trifunac, 1974; Trifunac and Udwadia, 1974; Anderson, 1974; Anderson and Richards, 1975). The effect of a free surface on near-field displacements has been examined by Kawasaki et al. (1973) and Kawasaki and Suzuki (1974). Anderson (1976) has examined the near-field motions due to a shallow rupturing fault using a Green's function which is a solution to Lamb's problem. Anderson's paper critically discusses the conditions under which the free surface may be reasonably accounted for by doubling the amplitude of motions in an infinite space. In particular, the SH motions from a strike-slip source and the P-SV motions from a dip-slip source were examined. Archuleta and Frazier (1976) have
utilized a three-dimensional finite element approach to examine particle
displacements and velocities in the near field for propagating shear frac-
tures.

In this research, we have made use of the generalized multipolar ray
theory (GMRT) advocated by Ben-Menahem and Vered (1973) to examine the
near-field motions of a propagating strike-slip fault. GMRT is a comprehen-
sive theory adaptable to the computation of both near-field and far-field
theoretical seismograms utilizing surface or buried dislocation sources.
The theory also permits, under certain conditions, the computation of ground
velocity and acceleration.
THEORY

We adopt a cartesian coordinate system in which the free surface of a homogeneous half space is represented by the xy plane. In the xz plane we define a rectangular fault surface:

\[ 0 < x < L \]
\[ H_1 < z < H_2 \]

Displacement occurs such that all points of the rectangle having the same coordinate x move simultaneously, i.e. a line source with a width \( h = H_2 - H_1 \) moves along the xz plane.

It has been shown by Ben-Menahem and Vered (1973) that the Laplace transformed surface displacements of each component due to a dislocation point source located at \((0, 0, H)\) are given by expressions of the form

\[
\bar{U} = \frac{1}{r^N} \left( \cos \frac{\pi \phi}{2} \right) \sin \left( \frac{\pi \phi}{2} \right) \sum_{n} \int_{0}^{\infty} J_m(\text{sur}) u^{2\ell + m + 1} f(u) \exp\left[-s\left(u^2 + \frac{1}{c^2}\right)\right] du \quad (1)
\]

\( \ell, n, m, N \) and \( M \) are source dependent integer constants. \( r \) is the epicentral distance, \( \phi \) is the azimuthal angle, \( c \) is the elastic wave propagation velocity, \( s \) is the Laplace parameter, \( u \) is an integration variable and \( f(u^2) \) is a function of \( u^2 \). A step-like source time function is initially assumed at the source, i.e., \( U_0(s) = U_0 s^{-1} \).

The depth of the source, \( H \), appears only in the exponent in (1). Thus, using the principle of superposition, the Laplace-transformed displacement generated by a line source which is composed of point sources continuously distributed along an interval \( h = H_2 - H_1 \), is given by the expression
\[
\bar{U}_{H_1} = \frac{1}{r} \sum_{n=1}^{\infty} \int_0^{\infty} J_m(\text{sur}) u^{2\ell+m+1} \frac{f(u^2)}{(u^2 + \frac{1}{c^2})^{1/2}} du
\]

\cdot \{\exp[-s(u^2 + \frac{1}{c^2})^{1/2} - \frac{1}{H_1}] - \exp[-s(u^2 + \frac{1}{c^2})^{1/2} - \frac{1}{H_2}]\} du \tag{2}

We denote by \( \bar{U}_z, \bar{U}_r, \) and \( \bar{U}_\phi \) the vertical, radial and azimuthal components of displacement. From Table 2 in Ben-Menahem and Vered (1973) and (2) we obtain

\[
\bar{U}_z = \frac{dx \sin 2\phi}{4\pi} U_0 \int_0^{\infty} \frac{\Gamma_2}{\Delta} \left( \frac{b'}{a} E_a - a E_b \right) du \tag{3}
\]

\[
\bar{U}_\phi = \frac{dx \cos 2\phi}{2\pi} U_0 \int_0^{\infty} \left\{ \frac{\Gamma_2}{\Delta} \left( \frac{b}{a} E_a + \frac{P}{b^2} E_b \right) - \frac{1}{b} E_b \right\} du \tag{4}
\]

\[
\bar{U}_r = \frac{dx \sin 2\phi}{4\pi} U_0 \int_0^{\infty} \left\{ \frac{2\Gamma_2}{\Delta} \left( \frac{b}{a} E_a + \frac{P}{b^2} E_b \right) + \frac{1}{\Delta} (b' E_b - \frac{u^2 b}{a} E_a) \right\} du \tag{5}
\]

where

\[
E_a = \begin{bmatrix} -saH_1 & -saH_2 \\ e & -e \end{bmatrix}
\]

\[
E_b = \begin{bmatrix} -sbH_1 & -sbH_2 \\ e & -e \end{bmatrix}
\]

\[
\Gamma_m = u^{m+1} J_m(\text{sur})
\]

\[
P = \begin{bmatrix} b' - 2ab \end{bmatrix}
\]

\[
\Delta = b'^2 - u^2 ab
\]
\[ a = (u^2 + 1/\alpha^2)^{1/2} \]
\[ b = (u^2 + 1/\beta^2)^{1/2} \]
\[ b' = (u^2 + 1/2\beta^2) \]

The longitudinal and shear wave velocities are denoted by \( a \) and \( \beta \) respectively.

The time domain solution is obtained by the method of Cagniard-Pekeris for dislocation sources. As an example we will treat the radial component of motion. We first write (5) in the form

\[ \bar{U}_{r} = \bar{U}_{r,p_{1}} + \frac{1}{s} \bar{U}_{r,p_{2}} + \bar{U}_{r,s_{1}} + \frac{1}{s} \bar{U}_{r,s_{2}} \] (6)

where

\[ \bar{U}_{r,p_{1}} = \frac{dx \sin 2\phi}{4\pi} U_{0} \int_{0}^{\infty} \frac{\Gamma_{u^2b}}{a\Delta} E_{a} du \] (7)

\[ \bar{U}_{r,p_{2}} = \frac{-dx \sin 2\phi}{2\pi r} U_{0} \int_{0}^{\infty} \frac{\Gamma_{b}}{a\Delta} E_{a} du \] (8)

\[ \bar{U}_{r,s_{1}} = \frac{-dx \sin 2\phi}{4\pi} U_{0} \int_{0}^{\infty} \frac{\Gamma_{b'}}{1/\Delta} E_{b} du \] (9)

\[ \bar{U}_{r,s_{2}} = \frac{-dx \sin 2\phi}{2\pi r} U_{0} \int_{0}^{\infty} \frac{\Gamma_{2\beta^2}}{b^2\Delta} E_{b} du \] (10)
We next perform the time domain inversion of (7)-(10) using equations (42) and (53)-(57) from Ben-Menahem and Vered (1973).

\[
U_{r, p_1} = \frac{\sin 2\phi}{2\pi r} \frac{U_0}{0} \frac{dx}{dt} \frac{H(t-t_p)}{p} \frac{\text{Im}}{\text{Im}} \int_{\tau_p}^{t} \frac{b v^3 (t-\tau+\nu r) d\tau}{[(\tau_p+\tau)(t-\tau+2\nu r)]^{1/2}[(\tau_p-\tau)(t-\tau)]^{1/2} \Delta}
\]

\[
H_1
\]

\[
U_{r, p_2} = \frac{\sin 2\phi}{2\pi r^3} \frac{U_0}{0} \frac{dx}{dt} \frac{H(t-t_p)}{p} \frac{\text{Im}}{\text{Im}} \int_{\tau_p}^{t} \frac{v[2(t-\tau)(t-\tau+2\nu r) + \nu^2 r^2] d\tau}{[(\tau_p+\tau)(t-\tau+2\nu r)]^{1/2}[(\tau_p-\tau)(t-\tau)]^{1/2} \Delta}
\]

\[
H_1
\]

\[
U_{r, s_1} = \frac{\sin 2\phi}{2\pi r} \frac{U_0}{0} \frac{dx}{dt} \frac{H(t-t_{bs})}{bs} \frac{\text{Im}}{\text{Im}} \int_{\tau_{bs}}^{t} \frac{v b b' (t-\tau+\nu r) d\tau}{\Delta[(\tau_s+\tau)(t-\tau+2\nu r)]^{1/2}[(\tau_s-\tau)(t-\tau)]^{1/2} \Delta}
\]

\[
H_1
\]

\[
U_{r, s_2} = \frac{\sin 2\phi}{2\pi r^3} \frac{U_0}{0} \frac{dx}{dt} \frac{H(t-t_{bs})}{bs} \frac{\text{Im}}{\text{Im}} \int_{\tau_{bs}}^{t} \frac{v p[2(t-\tau)(t-\tau+2\nu r)+\nu^2 r^2] d\tau}{b b\Delta[(\tau_s+\tau)(t-\tau+2\nu r)]^{1/2}[(\tau_s-\tau)(t-\tau)]^{1/2} \Delta}
\]

\[
H_1
\]
where

\[ \nu^2 = -u^2 \]

\[ \nu = \frac{\tau r}{R^2} - \frac{H}{R^2} \left( \frac{R^2}{C^2} - \frac{r^2}{C^2} \right)^{1/2}, \quad \tau < \frac{R}{C} \]

\[ \nu = \frac{\tau r}{R^2} + \frac{iH}{R^2} \left( \frac{r^2}{C^2} - \frac{R^2}{C^2} \right)^{1/2}, \quad \tau > \frac{R}{C} \]

\[ R^2 = r^2 + h^2 \]

\[ C = \alpha \text{ or } \beta \]

\[ \tau_{bs} = \min\left[ \frac{R}{\beta}, \frac{r}{\beta} + H \left( \frac{1}{\beta^2} - \frac{1}{\alpha^2} \right)^{1/2} \right] \]

\[ \tau_p = \frac{R}{\alpha} \]

\[ \tau_s = \frac{R}{\beta} \]

\[ f(H) \bigg|_{H_1}^{H_2} = f(H_1) - f(H_2) \]

From (6) we thus obtain

\[ U_r(t) = U_{r,p_1}(t) + \int_0^t U_{r,p_2}(\tau)d\tau + U_{r,s_1}(t) + \int_0^t U_{r,s_2}(\tau)d\tau \tag{15} \]

Equation (15) gives the ground displacement due to a step function \( U_0 \).

For a continuous source function \( U_0 \delta(t) \) we denote the ground displacement by \( G_r(t) \). \( G_r(t) \) is given by
\[ G_r(t) = \int_0^t U_r(t-\tau)g'(\tau)d\tau \] (16)

\[ G_r(t) = \int_0^t U'_r(\tau)g(t-\tau)d\tau \] (17)

From (15)-(17) we obtain

\[ G_r(t) = \left\{ \left[ U_{r,p_1}(\tau) + U_{r,s_1}(\tau) \right] g'(t-\tau) + \int_0^t \left[ U_{r,p_2}(\tau) + U_{r,s_2}(\tau) \right] g(t-\tau) \right\}d\tau \] (18)

By assuming that \( g(t) \) has first and second continuous derivatives we can obtain ground velocity and acceleration by differentiating (18):

\[ G'_r(t) = \left\{ \left[ U_{r,p_1}(\tau) + U_{r,s_1}(\tau) \right] g''(t-\tau) + \int_0^t \left[ U_{r,p_2}(\tau) + U_{r,s_2}(\tau) \right] g'(t-\tau) \right\}d\tau \] (19)

\[ G''_r(t) = \left\{ \left[ U_{r,p_1}(\tau) + U_{r,s_1}(\tau) \right] g'''(t-\tau) + \int_0^t \left[ U_{r,p_2}(\tau) + U_{r,s_2}(\tau) \right] g''(t-\tau) \right\}d\tau \] (20)

Table 1 gives the necessary change of variables to evaluate the integrals in (11)-(14). Calculated seismograms for the infinite media can be accomplished by a slight change of the integrands in (11)-(14).
Table 1

Integral Transformations

\[
U_{r, p_1}, U_{r, p_2}
\]

\[
q = \sin^{-1}\left(\frac{t - \tau_s}{t - \tau_p}\right)^{1/2}
\]

\[
U_{r, s_1}, U_{r, s_2}
\]

I \quad r < r_c

\[
r_c = \frac{R}{(\alpha^2 - \beta^2)\beta^2}^{1/2}
\]

\[
q = \sin^{-1}\left(\frac{t - \tau_s}{t - \tau_s}\right)^{1/2}
\]

II \quad r > r_c

\[
t < \tau_s
\]

\[
q = \sin^{-1}\left(\frac{t}{\tau_s}\right)^{1/2}
\]

III \quad r > r_c

\[
t > \tau_s
\]

\[
\text{write} \quad \int_{\tau_{bs}}^{t} \int_{\tau_{bs}}^{\tau_s} = \int_{\tau_{bs}}^{\tau_s} + \int_{\tau_{bs}}^{t}
\]

\[
q = \sin^{-1}\left(\frac{\tau_s}{\tau_s}\right)^{1/2}
\]

\[
q = \sin^{-1}\left(\frac{t - \tau_s}{\tau_s}\right)^{1/2}
\]

\[
q = \sin^{-1}\left(\frac{t - \tau_s}{\tau_s}\right)^{1/2}
\]
To obtain seismograms for a finite source, we next divide the length of the fault L into equal subintervals $\delta L_i$ ($i = 1, \ldots, r$). For every subinterval, the corresponding time function $g(t)$ derived from expression (1) or (2) is computed for the selected azimuthal angle $\phi$ and epicentral distance $r$. The final signal is calculated by adding the signals corresponding to all the subintervals $\delta L_i$, taking into account the appropriate time delay. A source time function

$$g(t) = \begin{cases} 
0 & t < 0, \\
\frac{t}{\tau} - \frac{1}{2\pi} \sin \left( \frac{2\pi}{\tau} t \right) & 0 \leq t \leq \tau, \\
1 & t > \tau,
\end{cases}$$

(21)

where $\tau$ is the adopted rise time is assumed. The total time of rupture is equal to $(L/v) + \tau$ where $L$ is the fault length and $v$ is the rupture velocity. The assumed source time function is basically a step function with rounded shoulders and it will be useful for purposes of discussion to define an effective rise time $\tau_e$ equal to $0.6 \tau$.

The effective rise time $\tau_e$ is simply the rise time of a ramp step function positioned along the time axis to approximate the response of the rounded step function.

The numerical results were checked by independently computing the static displacements for a buried strike-slip point source. In an elastic half-space the static displacements are given by the expressions (Ben-Menahem and Singh, 1968):

$$\bar{U}_z = \frac{\Omega_1 \sin 2\phi}{R^2} \tan^2 \psi$$

$$\bar{U}_r = \frac{\Omega_1 \sin 2\phi}{R^2} \left[ A \omega - B \right] \csc \psi \tan^2 \frac{\psi}{2}$$

(22)
\[
\bar{U}_\phi = \frac{\Omega_1 \cos 2\phi}{R^2} [4\sigma - 2] \csc \psi \tan^2 \frac{\psi}{2}
\]

where \( \Omega_1 = U_0 \frac{dS}{4\pi} \),

\( U_0 \) = amount of dislocation,
\( dS \) = fault area,
\( \psi \) = angle between vertical axis through source and radius vector to point of observation,
\( W = \cos \psi \)
\( \sigma \) = Poisson's ratio,
\( A = [3(1 + W)^2W - (1 - 2\sigma)(2 + W)] \),
\( B = [3(1 + W)^2 - 4(1 - 2\sigma)] \).

The static displacements in an infinite space can be derived by making use of the Laplace transform relation

\[
\lim_{t \to \infty} U(t) = \lim_{s \to 0} s \bar{U}(s)
\]

\[
\bar{U}_z = \frac{3\Omega_1 \sin 2\phi}{4(1 - \sigma)R^2} \cos \psi \tan^2 (\psi/2)(1 + \cos \psi)^2,
\]

\[
\bar{U}_r = \frac{\Omega_1 \sin 2\phi}{4(1 - \sigma)R^2} (3 \cos^2 \psi + 4\sigma - 5) \csc \psi \tan^2 (\psi/2)(1 + \cos \psi)^2
\]

\[
\bar{U}_\phi = \frac{\Omega_1 \cos 2\phi}{2(1 - \sigma)R^2} (2\sigma - 1) \csc \psi \tan^2 (\psi/2)(1 + \cos \psi)^2.
\]

All displacement components for all sources tend to a limit, the so-called "residual deformation." In the near-field this limit is reached very soon after the arrival of the S-wave. Evaluation of the ratio of the half-space static displacements to those of the infinite space for
various angles $\psi$ points out that the static displacements for an elastic half-space cannot in all cases be allowed for by simply doubling the infinite space displacements. This conclusion is in agreement with the results of Anderson (1976). A further check on the infinite space calculations was made using results obtained using the computer program developed by Boatright and Boore (1975).
The coordinate system for the model assumed in this paper is shown in Figure 1. A P-wave velocity $a = 5.5$ km/sec and an S-wave velocity $b = 3.2$ km/sec has been used throughout. A rectangular fault surface having an area of $3 \text{ km}^2$ is imbedded in an elastic half-space at a depth of $5 \text{ km}$. 5 km can be taken as a representative depth for California earthquakes. The fault plane is arbitrarily taken to lie in a N-S direction with rupture initiating at the northern end and propagating in a southerly direction ($-x_1$ axis). We shall be examining the $U_1$, $U_2$ and $U_3$ components of the derived displacements, velocities and accelerations at various station locations. Stations $A_1$ and $A$ lie along the strike of the fault in the direction of rupture and stations $B_1$ and $B$ at an angle of $45^\circ$ to the strike (azimuth $235^\circ$). Stations $C_1$ and $C$ are located at an azimuth of $325^\circ$ and stations $D_1$ and $D$ are along the strike but behind the direction of rupture. In the results which follow seismograms are computed for rupture velocities of 2, 2.5 and 3 km/sec and source rise times $\tau$ of 0.5 and 1 seconds. The computed seismograms are given in units of $U_0/1000$ cm where $U_0$ is the assumed displacement on the fault.

We first proceed to examine the effect of the inclusion of a free surface. In Figure 2 we show theoretical seismograms for the transverse component of motion ($U_3$) at station $A$ and $D$ with and without the presence of the free surface. As can be seen the pulse shapes are similar and the peak to peak amplitudes are doubled when the free surface is included. This agrees well with theory for an incident SH component of motion. Figure 3 compares the transverse component of displacement at stations $A_1$ and $D_1$ located closer to the ends of the fault. At these locations the peak to peak amplitudes are only approximately doubled, ranging from a factor of 1.7 to 2, when the free
surface is included. Stations Al and Dl are located at distances less than the critical distance for the existence of the SP head wave.

In Figures 4 and 5 we examine the effect of the presence of a free surface on the three components of displacement at stations B and C. As before the overall pulse shapes are quite similar and the main effect of the presence of the free surface is to increase the amplitude of the wave pulse. As before a factor of 2 increase in amplitude is a good approximation although in some instances, particularly for the vertical component of displacement $U_2$, the difference in amplitudes is closer to a factor of 2.5. It thus appears, that for the strike-slip example presented here, allowance for a free surface by doubling the amplitude of the wave pulse computed for an infinite space is valid to within ±25% or so. Anderson (1976) has pointed out that the SV contribution to the displacement is strongly influenced by the presence of the free surface, depending on the angle of incidence, and is particularly emphasized with a dip-slip source.

As can be seen in Figures 2-5 the static displacements computed with the presence of a free surface differ by a factor of 1 to approximately 2 times the infinite space static displacements depending on the component of motion and the station location. It can be demonstrated using the expressions given by (22-23) that this is indeed the expected result, i.e., the static displacements cannot in all cases be allowed for by simply doubling the infinite space displacements.

Figures 6, 7 and 8 show displacement seismograms computed at stations B, C, Bl and Cl which compare the effects of varying the rupture velocity for fixed source rise times of 0.5 and 1.0 seconds. The effect of a free surface has been included. For a fixed source rise time increasing the rupture
velocity from 2 to 3 km/sec produces a small increase in amplitude and a slight decrease in the width of the wave pulse. This is to be expected inasmuch as the width of the pulse should be a function of the total time interval in which the fault is rupturing. If we define $R_a$ as the distance from the edge in which rupture begins to the observing point the P-wave initial motion begins at $R_a/\alpha$. The radiation from the source is completed at a time $L/v + \tau_e$; S-waves initiating at this moment will arrive at the point of observation at a time $L/v + \tau_e + R_b/\beta$ where $R_b$ is the direct distance from the edge in which rupture is completed. The time

$$T = \frac{L}{v} + \tau_e + \frac{R_b}{\beta} - \frac{R_a}{\alpha}$$  \hspace{1cm} (24)

seems to control the pulse width of the near-field seismograms computed here. The theoretical seismograms presented here all possess pulse widths equal to $T$. Equation 24 describes the dependence of rise time, rupture velocity and the direction of rupture. The effect of the direction of rupture may be demonstrated by comparing seismograms at stations A and D and A1 and D1. For example, the difference in the value of $T$ for stations D and A is about 1 second and for stations D1 and A1 is about 0.6 second. This difference can be seen on the computed seismograms. The reason that the pulse width is simply $T$ appears to result from the fact that the static displacement approximately achieves its final value soon after the arrival of the S-wave contributed from the segment in which the fracture is terminated. This will in general, however not always be true; for example, at larger distances where the emerging Rayleigh wave will contribute to a larger pulse width than that given by (24).

*For a point source the pulse width is equal to $\tau_e + R\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$.  

19
It may be noted that the terms \( \frac{L}{v} \) and \( \tau_e \) in (24) are independent of the azimuth to the station. Thus, one may expect that different kinematical fault models possessing the same \( \frac{L}{v} + \tau_e \) will result in the same pulse width. This implies that rupture velocity is not separable by examining the pulse width at stations of varying azimuths unless some connection that ties \( \frac{L}{v} \) and \( \tau_e \) is assumed.

The displacement seismograms reveal that both the rise time and rupture velocity can be traded off to produce very similar wave shapes emphasizing that it would be difficult to separate the effects of rise time and rupture velocity using displacement seismograms. Any differences in displacement seismograms for various rupture parameters appear to be subtle except for an infinite rupture velocity where differences in rise time produce noticeable differences in the pulse shape, as shown in Figure 8. We therefore examined the vertical component of particle velocities and accelerations at stations B and C to evaluate their appropriateness for learning about details of the rupture process. One intuitively expects that more insight into the details of the faulting are carried in the high frequencies which are essentially filtered from displacement seismograms.

Figure 9 shows the vertical component of the particle velocities generated at station B for various combinations of rise time and rupture velocity. Here it can be noted that for a source rise time of 0.5 seconds increasing the rupture velocity from 2 to 3 km/sec produces significant changes in the amplitude and wave shape of the particle velocity pulse. In particular, the amplitude increases and the wave period (here taken as the peak to peak interval between wave maxima) decreases as the rupture velocity approaches that of the shear velocity (3.2 km/sec) of the medium.
For a slower rise time of 1.0 seconds the particle velocity pulses are somewhat simpler in character although the same gross effect can be seen. The comparison of particle velocities computed with and without the presence of a free surface reveal that even though the gross character of the velocity pulse appears similar one can note differences especially in the amplitude of the first positive upswing relative to the second upswing. However, if one compares peak to peak maximum particle velocities inclusion of a free surface produces an increase ranging from a factor of 2 to 2.3.

Figures 10 and 11 show the particle accelerations at station B for three different rupture velocities and source rise times of 0.5 and 1.0 seconds. The synthetic accelerograms exhibit much character and individual contributions to the acceleration signals such as the S-wave stopping phase (Savage, 1965) can be seen. For a source rise time of 0.5 seconds we have indicated the expected arrival time of the S-phase and the stopping phase of the S-wave. A small P-wave stopping phase can be recognized when the rupture velocity is 3 km/sec but cannot be noted for slower rupture velocities because its time of arrival occurs after the onset of the S-phase.

The accelerograms are much simpler in character when the rise time is increased to 1.0 second (Figure 11). In this case individual stopping phases are not obvious inasmuch as the rise time is comparable to the fault length divided by the rupture velocity, i.e., the fault is still radiating energy from near the initial point of rupture as the rupture reaches the end of the fault. Figure 12 shows accelerograms computed at station C located in the backward quadrant to the direction of rupture. In these accelerograms distinct P and S wave stopping phases can be noted.

The accelerograms presented here indeed emphasize that the adopted rise time of the source and the rupture velocity do play an important part in controlling the amplitude and overall shape of computed near-field accelerograms.
Whether synthetic accelerograms are justified in practice for comparison with observed accelerograms, however, would seem to depend on the azimuthal coverage of observations available, an assessment of the importance of layering and attenuation and whether enough fault parameters are independently known so that an appropriate kinematic model can be defined.

The near-field seismograms presented here have $r/h$ ratios much less than 5 and there is no evidence of a Rayleigh wave. Pekeris and Lifson (1957) have shown for a buried point source that the Rayleigh pulse begins to emerge at $r/h = 5$ and is clearly recognizable at $r/h = 10$. To examine the applicability of this conclusion for a buried strike-slip source we computed the vertical and radial displacements at increasing epicentral distances (Figures 13 and 14). In these calculations a point source and a source rise time of 0.5 seconds were assumed together with a source potency $U_0 dS$ of $1 \times 10^{-6}$ unit faults. (One unit fault = 1000 m $- km^2$ where $dS$ is the fault area). As can be seen, a Rayleigh pulse is noticeable at $r/h = 5$. Similar results were obtained for a dip-slip* source (Figure 15). Figure 16 shows the effect of decreasing the depth of focus $h$ for various distances $r$. Decreasing the depth of the source emphasizes the higher frequency components of the Rayleigh wave and a large Rayleigh pulse is prominent on the seismogram when $r/h \geq 50$. The conclusions of Pekeris and Lifson (1957), and amplified upon by Mooney (1974, 1976), thus also appear applicable to buried dislocation sources.

As a final example we examine observed near-field seismograms for a $M_L = 4.6$ earthquake recorded at SAGO-East and SAGO-Central as presented by Johnson and McEvilly (1974). In Figure 17 the signals shown represent actual recordings of ground displacement recorded at epicentral distances of 2.3

*Results for a dip-slip source can be derived using Table 2 in Ben-Menahem and Vered (1973).
km and 5.5 km. We do not attempt a detailed study of the source parameters of this earthquake but rather only demonstrate a fit of amplitudes and pulse widths based on several assumptions. A detailed study of this earthquake from a dynamic point of view is given by Israel and Vered (1977). First, the epicentral location was shifted by 0.75 km in the direction S 32° W in order to achieve agreement with the observed amplitudes at all stations. An adjustment of this amount is within the uncertainty of the epicenter location. Secondly, a fault area of 3 km² is assumed and rupture is taken to initiate at the focus and propagate to the southeast. A kinematic model is taken with a constant fault dislocation and a constant rupture velocity of 3 km/sec. The particle displacement is given by (21) and the rise time depends on the distance from the start of rupture according to

$$\tau = \frac{L - L}{L} \left( \frac{L}{v} \right)$$

where $L$ is the fault length, here taken to be 3 km. The choice of fault kinematics is quite arbitrary and the theoretical seismograms shown in Figure 17, although quite satisfactory, are not unique.

A better azimuthal station distribution together with a model which incorporates the geological complexity of the area could possibly reduce the possible solutions. However, details of the rupture process are not resolvable from examination of only near field displacements.

As pointed out by McEvilly and Johnson (1974) the comparison of synthetic seismograms with actual seismograms provides a method for estimating the source moment in the time domain. The average source potency determined by comparison of the amplitude of the S-wave pulse with observations at SAGO-Central and SAGO-East is 185 cm-km². Converting to seismic moment,
assuming a rigidity of $2.73 \times 10^{11}$ dynes/cm$^2$, yields a value of $5 \times 10^{23}$ dyne-cm. With an assumed fault area of $3 \text{ km}^2$ this leads to an average dislocation $\delta_0$ of 61.7 cm. The computed seismic moment of $5 \times 10^{23}$ dyne-cm agrees well with the value determined by Johnson and McEvilly (1974) assuming a point dislocation and the values obtained by Israel and Vered (1977) for a dynamical model of rupture, and points to the difficulty in extracting details of the rupture process from only examining near-field displacement data (Anderson and Richards, 1975). Near field synthetic accelerograms, however, hold some promise for the study of fault rupture parameters.

An attempt has also been made to examine what produces the coda in the seismic near field. Theoretical seismograms have been computed for a point source imbedded in a layered medium. An example of such a calculation is shown in Figure 18. Displayed are the seismograms for a vertical point source placed in a homogeneous half-space and in a layered half-space with parameters appropriate for the crustal structure near the San Andreas fault. The source depth was taken to be 12.5 km and the epicentral distance 5 km. Our computed seismograms demonstrate the general computational result that neither pulse amplitudes, nor pulse durations, are sensitive to crustal layering but that a portion of the near-field coda can be attributed to the presence of crustal layers and hence the receiver structure must be accurately known before the source radiation can be completely known.
CONCLUSIONS

Theoretical seismograms have been presented for a buried, propagating strike-slip fault to examine the effect of various rupture parameters and the presence of the free surface on the resulting particle displacement, velocity and acceleration at various azimuthal near-field locations. Rupture velocity and source rise time usually cannot be independently assessed on displacement seismograms inasmuch as the near-field pulse width is a function of fault length, rupture velocity, effective source rise time and the distances from the beginning and end of the rupture plane to the observing station. However, given a judicious (or fortuitous) azimuthal choice of observation points relative to a propagating strike-slip fault near-field synthetic accelerograms suggest some promise for the evaluation of fault rupture parameters. This could be of some importance in studying ground motions in the near-field of potentially damaging earthquakes.

For the strike-slip model discussed here, allowance for the presence of a free surface by doubling of the displacements, velocities and accelerations computed in an infinite space appears to be valid to within 25% or so. This conclusion is not generally valid in the case of the residual static displacements. A portion of the near-field coda can be directly attributed to the presence of crustal layers which must be accurately known before precise estimates of source parameters can be made.
REFERENCES


PUBLISHED PAPERS
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\[ \alpha = 5.5 \text{ km/sec} \]
\[ \beta = 3.2 \text{ km/sec} \]

**PLAN**

- \( D \) to \( D_1 \) distance: 2.5 km
- \( B \) to \( B_1 \) distance: 2.5 km
- \( C \) to \( C_1 \) distance: 2.5 km

**SECTION**

- \( X_1 \) to \( X_2 \) distance: 5 km
- \( X_2 \) to \( X_3 \) distance: 1 km
- \( X_3 \) to \( X_4 \) distance: 5 km

**FIGURE 1**
U_3 DISPLACEMENT

INFINITE SPACE

HALF SPACE

X_1 = -5Km
V = 2Km/sec
2.5Km/sec
3Km/sec
V = 2Km/sec
2.5Km/sec
3Km/sec

A

X = 5Km
\tau = 0.5 \text{ sec}

D

\tau = 1.0 \text{ sec}

U_{0}/10000 \text{ (cm)}

0 1.95 3.90 5.85 7.80 9.75
SECONDS

FIGURE 2
$U_3$ DISPLACEMENT

--- INFINITE SPACE
--- HALF SPACE

$X_1 = -2.5\text{ Km}$

$V = 2\text{ Km/sec}$
$3\text{ Km/sec}$
$2.5\text{ Km/sec}$

$T = 1.0\text{ sec}$
$T = 0.5\text{ sec}$

$U_0/1000$ (cm)

$V = 3\text{ Km/sec}$

FIGURE 3
FIGURE 4

- INFINITE SPACE
- HALF SPACE

\[ B \]

\[ U_0 / 1000 \text{ (cm)} \]

\[ U_1 \]

\[ U_2 \]

\[ \tau = 0.5 \text{ sec} \]

\[ \tau = 1.0 \text{ sec} \]

\[ V = 2 \text{ Km/sec} \]

\[ 2.5 \text{ Km/sec} \]

\[ 3 \text{ Km/sec} \]

\[ V = 2 \text{ Km/sec} \]

\[ 2.5 \text{ Km/sec} \]

\[ 3 \text{ Km/sec} \]

\[ 0 \ 1.95 \ 3.90 \ 5.85 \ 7.80 \ 9.75 \]

SECONDS
FIGURE 6

B $U_1$

$V = 2 \text{Km/sec}$

$V = 2.5 \text{Km/sec}$

$V = 3.0 \text{Km/sec}$

$\tau = 0.5 \text{ sec}$

$\tau = 1.0 \text{ sec}$

C

$U_3$

$U_1$

$U_3$

$0 \ 1.95 \ 3.90 \ 5.85 \ 7.80$

SECONDS

$U_o/1000 \ (\text{cm})$
FIGURE 8

VERTICAL COMPONENT $U_2$

$\tau = 0.5$ sec  $\tau = 1.0$ sec

$\text{---} V = \infty$
$\text{---} V = 3\text{km/sec}$
$\text{---} V = 2\text{km/sec}$
Figure 9

Station B Vertical Component

τ = 0.5 sec

τ = 1.0 sec

3.0 Km/sec

2.5 Km/sec

2 Km/sec

u°/1000 cm/sec

INFINITE SPACE

HALF SPACE

30

20

10

0

-10

-20

-30

5 sec
STATION B VERTICAL COMPONENT

$\tau = 0.5 \text{ sec}$

\[ \ddot{u} / 1000 \text{ cm/sec}^2 \]

- INFINITE SPACE
- HALF SPACE

2 Km/sec  2.5 Km/sec  3 Km/sec

5 sec
STATION C VERTICAL COMPONENT

$\tau = 0.5 \text{ sec}$

\[ \frac{\ddot{u}_0}{1000}, \text{ cm/sec}^2 \]

- 100

0

100

2 Km/sec

2.5 Km/sec

3 Km/sec

--- 5 sec ---

INFINITE SPACE

HALF SPACE

FIGURE 12
Vertical Displacement, Strike Slip

$h = 10 \text{ km}$, $U_o d S = 1 \times 10^{-6}$ unit faults

$T = 0.5$ seconds

$\alpha = 6.1 \text{ km/sec}$  $\beta = 3.6 \text{ km/sec}$

FIGURE 13
Radial Displacement, Strike Slip Source
h = 10 km \( \nu \alpha dS = 1 \times 10^{-6} \) unit faults
\( T_0 = 0.5 \) seconds
\( \alpha = 6.1 \) km/sec, \( \beta = 3.6 \) km/sec \( \sigma = 0.023 \)

FIGURE 14
Radial Displacement, Dip Slip

\( h = 10 \text{Km} \), \( U_0 \text{dS} = 1 \times 10^{-6} \) unit faults

\( T_0 = 0.5 \) seconds

\( \alpha = 6.1 \text{ km/sec} \), \( \beta = 3.6 \text{ km/sec} \), \( \sigma = 0.23 \)
Radial Displacement Strike Slip

- $h = 1\text{ km}$
- $U_d S = 1 \times 10^{-6}$ unit faults
- $T_s = 0.5\text{ sec}$  Step source, rounded shoulders
- $\alpha = 6.1\text{ km/sec}$, $\beta = 3.6\text{ km/sec}$

**FIGURE 16**
SAGO-CENTRAL \( A\bar{U} = 159 \text{ cm-km}^2 \)

2847 \( \mu \)

E \( A\bar{U} = 210 \text{ cm-km}^2 \)

3117 \( \mu \)

SAGO-EAST

3690 \( \mu \)

SECONDS

0 5 10 15 20

27 OCT 69 10h 59m 42.8s \( h = 12.5 \text{ km} \)

\( \Delta = 5.5 \text{ km } A\bar{z} = 240^\circ \)

\( \Delta = 2.3 \text{ km } A\bar{z} = 274^\circ \)

INITIATION OF RUPTURE TO SE AT EPICENTER

\( V = 3 \text{ km/sec} \)

\( \alpha = 5.5 \text{ km/sec}; \beta = 3.18 \text{ km/sec} \)

FIGURE 17
FIGURE 18

(a) Displacement (\( \mu \)) vs. time (sec)

(b) Displacement (\( \mu \)) vs. time (sec)
APPENDIX A

SAMPLE COMPUTER LISTING

There are two programs to compute the vertical component of motion for a propagating strike-slip fault. Program one should be submitted first, the numerical results of this program being stored on disc. This information is used when program two is submitted. The programs are used to compute ground displacements, velocities or accelerations. The source may be either a line source or a rectangular source. Computations may be done for an infinite or half space model.

The most time consuming computations are the functions $u_{z,p_1}$, $u_{z,s_1}$ (see eqns. (15), (16)). Program one computes the numerical values of these functions once and for all. The values of $u_{z,p_1} + u_{z,s_1}$ are stored on disc for all subrectangular sources required. Program two uses these values several times to compute the acceleration, velocity, or displacement, for any required rise time or rupture velocity.
PROGRAM ONE

INPUT (see figure 1)
ALFA: P velocity (km/sec)
BETA: S velocity (km/sec)
H: Source depth in km (depth to top of fault plane)
D: Horizontal range in km
DT: time increment in \( u_{z,p1} \) computations.

Time: the required computational time of ground motion (sec)
E"": the absolute permitted error in \( u_{z,p1}, u_{z,s1} \)
EE"": the relative permitted error in (same as above)

DELH: fault width (km)
DELX: fault length (km)
PHI: azimuth to station in degrees (see figure 1)
NH: number of subrectangles required

INDX1: 1 denotes a half space model; 2 an infinite space model
INDX2: 1 denotes a rectangular source; 2 a line source at depth H, length DELX.

OUTPUT
1/ Arrivals time of P, SP, S from depth H and H + DELH for all subrectangles.
2/ The numerical values of \( u_{z,p1}, u_{z,s1} \) for all rectangles.

These values are printed and stored on the disc in unit 8.

++The error less than the absolute error or the relative error.
PROGRAM TWO

INPUT

NV: The number of different rupture velocity cases to be computed.

INDEX: 1 for displacement ground motion, 2 for velocity ground motion, and 3 for acceleration.

TZ: rise time in seconds (2 in equation (21))

VZ: the first value in rupture velocity (km/sec)

DV: increment in rupture velocity

OUTPUT

A/ The values of rupture velocity and rise time are printed as well as the values of ground motion. Displacement amplitude is given in terms of $U_0$, velocity in terms of $U_0/\text{(sec)}$, and acceleration in terms of $U_0/(\text{sec}^2)$.

B/ A plot for each required rupture velocity. The values of displacements, velocities and accelerations were multiplied by 1000 before plotting.
PROGRAM ONE

C PROGRAM TO COMPUTE SURFACE MOTION OF HALFSpace, DUE TO BURIED
C SOURCE VERTICAL MOTION
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION U(1008),US(1008),QS(1008),QK(1008)
COMMON/PLQ/US,DT,TZ
COMMON/PHI/ALFA,BETA,
COMMON/AAA/INDX1,INDX2
READ 70,ALFA,BETA,H,D,DT,TIME,E,EE
READ 70,DELH,DELX,PHI
READ 73,INDX1,INDX2
PRINT 104,ALFA,BETA,H,D,DT,TIME,E,EE
PRINT 74,DELH,DELX,PHI
READ 75,NH
PRINT 75,INDX1,INDX2
70 FORMAT(8F10.5)
73 FORMAT(3I5)
DO 207 I=1,NH
207 Q(I)=0.
H=H+DELH/2.0D0
WRITE(8) H,J,DT,TIME,DELH,DELX,PHI
WRITE(8) H,ALFA,BETA
74 FORMAT(1X,5E15.7)
PI=PI/1.0D0
PHI=PHI*PI/1.0D0
DO 40 DI=1.00
RNS=DI*DI
RS=RNS+(H-DELH/2.0D0)*(H-DELH/2.0D0)
R=SQRT(RS)
T=R/ALFA
IP=1.0/DT+1
NI=TIME/DT+IP
75 FORMAT(1X,5I5)
H1=M
ANH=NH
DXT=1.0D0/ANH
ALC=0.0D0
JS=0.
DO 27 II=1,NH
AI=II
TS=TS+DXT
AL=DELX*(TS)
ST=AL-ALC
SP=(AL+ALC)/2.0D0
ALC=AL
RNS=DI*DI*SP-2.0D0*DI*SP*DCOS(PHI)
DIM=DSQRT(RNS)
H=H1+5.0D0*DELH
IF(INDX2.EQ.2)H=H1+0.500*DELH
D=DIM
MNH=2
IF(INDX2.EQ.2)MNH=1
DO 2 II=1,1008
2 QQ(I)=0.0D0
DO 1 J=1,MNH
AJ=1.
IF(INDX2.EQ.2)AJ=1.
IF(J.EQ.2)*AJ=1.
H=H-DELH
301 CALL VRM(H,J,N1)
DO 1 I=1,1008
QQ(I)=QQ(I)+Q(I)*AJ
1 CONTINUE
DO 3 I=1,1008
3 QQ(I)=QQ(I)
WRITE(8) (Q(I),I=1,N1)
PRINT 104,(Q(I),I=1,N1)
27 CONTINUE
104 FORMAT(2X,10E12.4)
STOP
END
SUBROUTINE VR$(H,DIM.N1)
IMPLICIT REAL*8 (A-H,O-Z)
EXTERNAL FUPS,FUNSP,FUNSS
DIMENSION Q(1008),QS(1008),QS1(1008),QS2(1008),QS3(1008)
COMMON/PLO/ U,QS1,DT,TZ
COMMON HR2,DR2,SA,SB,SC,RA,rb,TPB,TBS,OT,TBSR
COMMON/PP1/ALFA,BETA,TIME,E,EE,IPLOT
COMMON/AAA/INDX1,INDX2
C
C D=01M
DO 999 I=1,1008
Q(I)=0.D0
QS(I)=0.D0
QS2(I)=0.D0
QS3(I)=0.D0
999 Q(I)=0.D0
PIK=0.5D0*CARCOS(-1.D0)
PRINT 73,H,D
73 FORMAT (10X,'VERTICAL DISPLACEMENT',18X,'SOURCE DEPTH',18X,'FAC3',
1 'KM',18X,'HORIZONTAL RANGE',18X,'FAC3',18X,'KM')
RS=H+D+D
DR2=H/RS
HR2=H/RS
R =DSQRT(RS)
FA =1.00/ALFA
EB =1.00/BETA
SA =FA*FA
SB =EB*EB
HCR=H/DSQRT(SB/SA-1.D0)
SC =0.5*SB
RA =RS*SA
RB =RS*SE
TB P =RS*FA
PRINT 74,TBP
74 FORMAT (10X,'P ARRIVAL TIME','F10.5',' SEC.')
IP =TB P/DT +1
TB =1P*DT
T=TB-DT
N=NI
DO 1 I=IP,N
T=1+DT
Q(I)=QUAD(0.00,PIK,FUNP,E,EE,4)
TB SR =R*EB
TBSR =0.00*H/DSQRT(SB-SA)
IF(I+EX=2)GO TO 3
IF(O=HCR)3,3,4
3 TBS =TBSR
75 FORMAT (10X,'SP HEAD WAVE ARRIVAL TIME','F10.5',' SEC.')
PRINT 75,TBSR
TS =I BS/DT +1
J =IS-IP +1
TS=IS*DT
T =TS-DT
DO 2 I=IS,N
T=1+DT
Q(I)=QUAD(0.00,PIK,FUNSP,E,EE,4)
2 Q(I)=Q(I)+Q(S(I))
GO TO 10
4 TBS =TBSR
PRINT 76,TBSR,TBSR
76 FORMAT (10X,'SP HEAD WAVE ARRIVAL TIME','F10.5',' SEC.')
IS =TBS/DT +1
TS=IS*DT
J =IS-IP +1
T =TS-DT
PDRH=DARSIN (DSQRT(TBSR/TB SR))
DO 18 I=IS,N
T=T+DT
IF(T.GT.TBSR)GO TO 9
PCR =DARSIN (DSQRT(TBSR/T))
Q5(I)=QUAD(PCHR,PIK,FUNSP,E,EE,4)
Q(I)=Q(I)+QU1(I)
GO TO 18
9 Q52(I)=QUAD(0.00,PIK,FUNSP,E,EE,4)
Q53(I)=QUAD(PDRH,PIK,FUNSS,E,EE,4)
Q(I)=Q(I)+Q53(I)+Q52(I)
18 CONTINUE
10 FACTOR=1.
RETURN
END
FUNCTION FLNP(X)
IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 C,V,VR,VSQ,P,TVR,D,VR2,AV2,BV2,CDSQRT
COMMON HR2,DR2,SA,SB,SC,RA,RB,TBP,TBS,RANGE,TT
COMMON/AAA/INDX1,INDX2
C=([0.00,1.00])
T=TT-(TT-TBP)*DSIN(X)**2
IN THIS SUB T MEANS TAU
A= T*T-RA
IF (A.LT.0.00) A=0.00
A= DSQRT(A)
V=DR2*T+HR2*A*C
VSQ= V *V
P =SC - VSQ
AV2=SA-VSQ
BV2=SB-VSQ
VR=V*RANGE
DT=TT-T
TVR=DT+VR*VR
DT=DT+DT
VR2=VR*VR
D=CDSQRT(AV2)
C=0
IF (INDX1.EQ.2) GO TO 1
D=V*P*D/*((P*P+VSQ*DSQRT(BV2))*CDSQRT(TVR*(T+TBP)))*((DT+VR+VR2)
GO TO 2
1 D=V*C *(DT+TVR+VR2) *(DSQRT(TVR*(T+TBP)))*((DT+VR+VR2)
2 IF (INDX2.EQ.1) D=D/C
FUN=D
FUNS =FUN
RETURN
END

FUNCTION FUNS(X)
IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 C,V,VR,VSQ,P,TVR,D,VR2,AV2,BV2,CDSQRT
COMMON/AAA/INDX1,INDX2
C=([0.00,1.00])
T=TT-(TT-TBSR)*DSIN(X)**2
A= T*T-RR
IF (A.LT.0.00) A=0.00
A= DSQRT(A)
V=DR2*T+HR2*A*C
VSQ= V *V
P =SC - VSQ
AV2=SA-VSQ
BV2=SB-VSQ
VR=V*RANGE
DT=TT-T
TVR=DT+VR*VR
DT=DT+DT
VR2=VR*VR
D=CDSQRT(AV2)
C=CDSQRT(BV2)
IF (INDX1.EQ.2) GO TO 1
D=V*P*D/*((P*P+VSQ*DSQRT(BV2))*CDSQRT(TVR*(T+TBSR))
GO TO 2
1 D=V*C *(DT+TVR+VR2) *(DSQRT(TVR*(T+TBSR)))*((DT+VR+VR2)
2 IF (INDX2.EQ.1) D=D/C
FUN=D
FUNS =FUN
RETURN
END
FUNCTION FUNSP(X)
IMPLIED REAL*A-H,Q-Z*
COMPLEX*16 AV2,D,CSQRT
COMMON HR2,DR2,SA,SB,SC,RA,RB,TBP,TBS,RANGE,TT,TBSR
COMMON/AAA/INDX1,INDX2
T=TT **DSIN(X)**2
A=T*TT-TB
A=DSQRT(-A)
C
IN THIS SUB T MEANS TAU
V=DR2*T-HA2*A
VSQ= V *V
P =SC - VSQ
AV2=SA-VSQ
BV2=SB-VSQ
VR=VR*RANGE
DT=TT- T
TVR=DT+VR+VR
DT=DT+DT
VR2=VR*VR
D=CSQRT(AV2)
D=V*BV2*(DT+TVR+VR2)*D* DSQRT(T/(TVR*(T+TBSR)*(TBSR-T)))/(P*P+VSQ*
*DSQRT(BV2*(1.0,0,0.00)))/
IF (INDX2.EQ.1) D=D/CSQRT(BV2*(1.0,0,0.00))
FUN=D*(0.0,0.00-1.00)
FUNSP= FUN
RETURN
END

FUNCTION FUNSS(X)
IMPLIED REAL*A-H,Q-Z*
COMPLEX*16 AV2,D,CSQRT
COMMON HR2,DR2,SA,SB,SC,RA,RB,TBP,TBS,RANGE,TT,TBSR
COMMON/AAA/INDX1,INDX2
T=TBSR **DSIN(X)**2
A=T*T-TB
A=DSQRT(-A)
C
IN THIS SUB T MEANS TAU
V=DR2*T-HA2*A
VSQ= V *V
P =SC - VSQ
AV2=SA-VSQ
BV2=SB-VSQ
VR=VR*RANGE
DT=TT- T
TVR=DT+VR+VR
DT=DT+DT
VR2=VR*VR
D=CSQRT(AV2)
D=V*BV2*(DT+TVR+VR2)*D* DSQRT(T/(TVR*(T+TBSR)*(TBSR-T)))/(P*P+VSQ*
*DSQRT(BV2*(1.0,0,0.00)))/
IF (INDX2.EQ.1) D=D/CSQRT(BV2*(1.0,0,0.00))
FUN=D*(0.0,0.00-1.00)
FUNSS= FUN
RETURN
END
FUNCTION QUAD(A,B,FUN,ETA,MIN)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION Q(16)
H=B-A
FCNA =FUN(A)
FCNB =FUN(B)
TABS =DABS(H)*(DABS(FCNA)+DABS(FCNB)) *0.5
T = H *( FCNA + FCNB ) *0.5
NX =1
DO 12 N=1,15
H=H*0.5
SUM=0.DO
SCORR=0.DO
SUMABS=0.DO
DO 2 I=1,NX
XI=2.*DFLOAT(I)-1.
FCNXI = FUN(A+XI*H)
SUMABS=SUMABS + DABS(FCNXI)
FCNXI = FCNXI + SCORR
SS = SUM +FCNXI
SCORR= SUM-SS + FCNXI
2 SUM=SS
T= T*0.5+H*SUM
TABS= TABS*0.5 +CABS(H)*SUMABS
Q(N)= 2.*(T+H*SUM)*0.33333333
IF (N-2) 10,3,3
3 F =4.
DO 4 J=2,N
I=N+1-J
F=F*4.
4 Q(I)=Q(I+1)+(Q(I+1)-Q(I))/(F-1.)
IF (N-3) 9,5,5
5 IF (N-MIN) 9,6,6
6 X= DABS(Q(1)-QX2)+DABS(QX2-QX1)
IF (TABS) 7,8,7
7 IF (X/TABS-1.*(DABS(ETA)+0.14901161D-7)) 11,11,8
8 IF ( X-3.*DABS(EPS)) 11,11,9
9 QX1=QX2
10 QX2=Q(1)
12 NX=NX+1
11 QUAD = Q(1)
RETURN
END
PROGRAM TWO

```
10 A=0
20 B=-1
30 CALL MOCONDF(B,0,0,13)
40 CONTINUE
50 CALL PIVLRE
60 I=1
70 AN=0.035
80 CALL SSOLVE
90 FOR I=1,1000000
100 A=1.225+1.225*X
110 IF X>.001 THEN 200
120 X=.001
130 AM=AM+24
140 IF AM>1000 THEN 200
150 CALL SSOLVE
160 END
170 STOP
180 END
```
SUBROUTINE GRAPL(N,FAC,XMI,YMI,AMA,YMA)
COMMON/SP/YP(1,1005),XP(1,1005),BUFF(200)
COMMON/FP/N(1,1005)
DT=FAC
T=0.
DO 700 J=1,N
XP(J)=T
700 T=T+DT
DO 200 J=1,N
200 YP(J)=AY(J)+ICOO.
BUFF(3,1)=1.
BUFF(1,1)=1.
BUFF(1,17)=1.
CALL OBJECT(BUFF,XM1,YM1,AMA,YMA)
CALL SUBJECT(BUFF,0.1,J,0.3,DO).
CALL GRAFHP(BUFF,N,XP,YP,1.,X1,T1,0.,X V SB)
RETURN
END

SUBROUTINE SITCUT(U,T,UT,UT)
REAL*8 WLS,UTUB,UTUB
COMMON/XT/XT,UTUB,UTUB
COMMON/IN/INDEX.
DIMENSION A(1005),Q(1005),QST(1005)
EQUIVALENCE (A(I),A(I))
T=0.
PI=AUTH(-1.)
TP1=PI/10.
GM=TP1/10.
DO 11 I=1,N
11 A(1)=J.
DO 14 I=1,N
14 IF(T<UT,TZ)=J.
GO TO 1.
INDEX
GO TO 12.
INDEX
GO TO 12.
INDEX
12 A(I)=G*M*SIN(T)/TZ
GO TO 1
INDEX
GO TO 12.
INDEX
13 A(I)=G*M*CO(S(T))/TZ
T=T+DT
CONTINUE
GO TO 10.
10 FORMAT(A,1E0,2)
RETURN
END
SUBROUTINE CONV (N)
REAL*8 C,E,UTUB,TUDB
DIMENSION C(1000),U(1000),B(1000)
COMMON /PLC/ C,E,UTUB,TUDB
COMMON /SVSP/ A(1000),C(1000)
EQUIVALENCE (U(1),B(1))
DATA A(1),C(1),K=1.0
C=UTUB
T=UTUB
DO 1 I=1,N
1 A(I)=C(I)
CALL S1(T,UT,U)
DO 2 K=1,N
SUM=0.
KK=K+1
DO 3 J=1,K
JJ=KK-J
2 SUM=SUM+(J)*A(JJ)
3 U(I)=U(I)+SUM*C.*0.5*(B(I)=A(K)+B(K)*A(I))
DO 4 I=1,N
4 U(I)=C(I)
RETURN
END