THE POSSI CODE - A COMPUTER PROGRAM FOR A TWO-DIMENSIONAL PROBLEM OF SOIL STRUCTURE INTERACTION-LINED CAVITY IN A BILINEAR MEDIUM AXISYMMETRIC CASE.

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A POSSI CODE—A COMPUTER PROGRAM FOR A TWO-DIMENSIONAL PROBLEM OF SOIL STRUCTURE INTERACTION—LINED CAVITY IN A BILINEAR AXISYMMETRIC CASE

**ABSTRACT**

POSSI is a FORTRAN Computer code for computing the response of an axisymmetric soil/structure interaction problem cylindrical \((r, z)\) coordinates. The configuration consists of a lined cylindrical cavity of finite length extending from the surface into a semi-infinite half-space, and loaded internally with a pressure \(p(z, \tau)\) or \(p(r, \tau)\) where applicable. The cavity lining is a thin cylindrical shell with bending stiffness, welded to an axisymmetric circular bottom plate with both bending and extensional motion. A right angle...
20. ABSTRACT (Continued)

may be maintained at the plate/shell connection. The surrounding material is bilinear, with hysteresis in volumetric response only.
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POSSI is a FORTRAN code for computing the response of an axisymmetric soil/structure interaction problem in cylindrical (r, z) coordinates. The acronym POSSI stands for Problem of Soil Structure Interaction. The basic two dimensional configuration of the problem, consisting of a lined cylindrical cavity of finite length extending from the surface into a semi-infinite half-space and loaded internally, is shown in Fig. 1. Modification of certain parameters in the program do, however, permit use of the code for one dimensional, or free field calculations.

Development of the computational approach for POSSI has been presented in Refs. [1]-[4]. This report is the final one in that series, and presents not only information about use of the code, but also adds certain theoretical features.

The structure which lines the cavity is considered as an axisymmetric thin cylindrical shell with bending stiffness, welded to an axisymmetric circular bottom plate. In the previous reports the plate was considered to have bending stiffness only. The work presented in this report includes extensional motion of the plate. Also presented is the development of equations governing the maintenance of a right angle at the joining of plate and shell, see Fig. 1. In addition to the equations for these new features the basic equations given in Refs. [3], [4] have been repeated here for convenience, and grouped according to the order of computation in the POSSI code.
FIG. 1
The material surrounding the structure, and interacting with it, is considered to be a bilinear material, with hysteresis in the response of volumetric effects, but linear response in shear. Provision is made for horizontal layering of the material, with different material properties for each layer.

The computational scheme used to describe the response of points in the field surrounding the structure is the pseudo-characteristic scheme combined with a fractional step method that was developed in Refs. [1]-[4]. As in Ref. [3] this scheme will be hereinafter referred to as the PC scheme.

The governing equations for the motion of the structural lining were obtained from a method of the variational calculus applied to the finite difference form of the total strain energy. The right-angle condition at the joining was introduced as an equation of constraint with the addition of a Lagrangian multiplier. The resulting equations are first order finite-difference in nature.

A number of options are built into the POSSI code. They include: location and type of applied load; dimensional or non-dimensional output; lined or unlined cavity; right-angle at plate-shell connection maintained or not maintained; horizontal layering in the field, or no horizontal layering; graphic or printed output, or both; one-dimensional configurations in either r-t or z-t coordinates as special cases; the shell and the plate may have different material properties or thicknesses. These, and other features, are discussed in Section III of this report.
In order to illustrate use of the POSS1 code typical output from three sets of computations are presented. The first case is for an unlined cavity in a layered material, subjected to a time dependent, and location dependent internal pressure $p(z, t)$. Two one-dimensional subcases are also very briefly discussed. The second case is for a structurally lined cavity, in a layered material with a right angle maintained at the shell-plate connection, subjected to an internal pressure $p(z, t)$. Results in this case are presented in non-dimensional form. The third case is a rerun of the second, with results presented in dimensional form.
A. Field

The material surrounding the structure acts linearly in shear (i.e., $C$, the shear modulus, remains constant), but bilinearly in volumetric response. That is, the bulk modulus $K$ has two possible values: $K = K_{LD}$ for $(-J_1)$ in virgin loading, $K = K_{UN}$ for $(-J_1)$ less than or equal to the previous compressive maximum. Figure 2 shows how this restricts the dependency of $e_v$ on $J_1$. The model continues onto the tensile side of the $J_1, e_v$ curve along a straight line of essentially unloading slope, unless tension is experienced initially, in which case the loading slope is used. See Fig. 2.

The basic differential equations which describe the material motion are the same as those for a linear elastic continuum. Only the parameters $K$ (bulk modulus) and $v$ (Poisson ratio) must be appropriately chosen, according to the criteria mentioned above, for conditions of loading or unloading. $C$ (the shear modulus) remains constant throughout the computation. In non-dimensional terms these equations are

\[
\frac{\partial r}{\partial t} = \frac{\partial r}{\partial r} + \frac{\partial r}{\partial z} + \frac{\partial r}{\partial z}
\]

\[
\frac{\partial z}{\partial t} = \frac{\partial z}{\partial r} + \frac{\partial z}{\partial z} + \frac{\partial z}{\partial z}
\]

\[
\frac{\partial r}{\partial t} = \mu \lambda^2 \left[ \frac{\partial r}{\partial r} + \frac{\partial r}{\partial z} \right] + \frac{\partial r}{\partial z} + \frac{\partial r}{\partial z}
\]

\[
\frac{\partial z}{\partial t} = \mu \lambda^2 \left[ \frac{\partial z}{\partial z} + \frac{\partial z}{\partial z} \right] + \frac{\partial z}{\partial r} + \frac{\partial z}{\partial r}
\]

\[
\frac{\partial r}{\partial t} = \mu \lambda^2 \left[ \frac{\partial z}{\partial z} + \frac{\partial z}{\partial z} \right]
\]
FIG. 2
The symbols are defined in Table 1, as are all the parameters, the stresses, velocities and displacements which appear in the basic equations for both field and structure. The quantities appearing in Table 1 are given in both dimensional and non-dimensional form.

Using the method of fractional steps the basic equations, with r, z and t dependencies are separated into two problems: in r, t coordinates; in z, t coordinates. Each is then a one-dimensional wave propagation problem with its own set of P-, S- and "zero"-characteristics. Integration is set up along these characteristics resulting in the equations which are summarized in Table 2. (The superscripts refer to points in the space time meshes, shown in Figs. 3a and 3b.)

In order to obtain values of stress and velocity at points intermediate to the mesh points (i.e. at the R and Q pts. shown in Figs. 3a and 3b, between L-1, L and L, L+1, respectively) a three point interpolation scheme was used leading to a "second order" integration scheme which exhibits errors of $\Delta r^2$ and $\Delta z^2$ in r- and z-directions respectively.

As an example of this interpolation scheme, if $F_{R^p}$, is the value of the quantity to be found from known quantities $F_L$, $F_{L-1}$, $F_{L+1}$ (Fig. 3a) then

$$F_{R^p} = (1-\alpha^2)F_L + \frac{\alpha}{2} (\alpha-1)F_{L+1} + \frac{\alpha}{2} (\alpha+1)F_{L-1}$$

(6)
<table>
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<tr>
<th>TABLE I PARAMETERS AND NON-DIMENSIONAL PARAMETERS</th>
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<tr>
<td>1. PARAMETERS</td>
</tr>
<tr>
<td>COORDINATES</td>
</tr>
<tr>
<td>$R_0$ Reference radius. Usually the nominal</td>
</tr>
<tr>
<td>radius of the cavity</td>
</tr>
<tr>
<td>$R, Z$ Radial and depth coordinates</td>
</tr>
<tr>
<td>$t$ Time</td>
</tr>
<tr>
<td>$L$ Length of the cavity</td>
</tr>
<tr>
<td>2. PARTICLE VELOCITIES AND DISPLACEMENTS</td>
</tr>
<tr>
<td>$\hat{U}_r$ Radial particle velocity</td>
</tr>
<tr>
<td>$\hat{V}_r$ Longitudinal particle velocity</td>
</tr>
<tr>
<td>$\hat{U}_z$ Radial displacement</td>
</tr>
<tr>
<td>$\hat{W}_r$ Longitudinal displacement</td>
</tr>
<tr>
<td>$\hat{V}_z$ Longitudinal displacement</td>
</tr>
<tr>
<td>$W_r$ Radial displacement</td>
</tr>
<tr>
<td>$W_z$ Longitudinal displacement</td>
</tr>
<tr>
<td>3. FORCES AND STRESSES</td>
</tr>
<tr>
<td>$\bar{p}_0$ Reference stress from applied load</td>
</tr>
<tr>
<td>$\sigma_r$ Radial stress</td>
</tr>
<tr>
<td>$\sigma_z$ Longitudinal stress</td>
</tr>
<tr>
<td>$\sigma_\theta$ Hoop stress</td>
</tr>
<tr>
<td>$\sigma_{rz}$ Shear stress</td>
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<tr>
<th>PARAMETERS</th>
<th>NON-DIMENSIONAL PARAMETERS</th>
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<tr>
<td>4. ( \bar{J}_1 ) First invariant of stress ( \bar{J}_1 = \bar{\sigma}_r + \bar{\sigma}_z + \bar{\sigma}_0 )</td>
<td>( J_1 = \frac{\bar{J}_1}{\rho_0} )</td>
</tr>
<tr>
<td>( \bar{N}_s, \bar{N}_p ) Longitudinal forces in the shell and plate, respectively</td>
<td>( N_s = \frac{\bar{N}_s}{\rho_0 \bar{D} \bar{R} \bar{R}_0 \rho C^2} ), ( N_p = \frac{\bar{N}_p}{\rho_0 \bar{D} \bar{R} \bar{R}_0 \rho C^2} )</td>
</tr>
<tr>
<td>( \bar{Q}_s, \bar{Q}_p ) Shear forces in the shell and plate, respectively</td>
<td>( Q_s = \frac{\bar{Q}_s}{\rho_0 \bar{D} \bar{R} \bar{R}_0 \rho C^2} ), ( Q_p = \frac{\bar{Q}_p}{\rho_0 \bar{D} \bar{R} \bar{R}_0 \rho C^2} )</td>
</tr>
<tr>
<td>( \bar{M}_s, \bar{M}_p ) Bending moments in the shell and plate, respectively</td>
<td>( M_s = \frac{\bar{M}_s}{\rho_0 \bar{D} \bar{R} \bar{R}_0 \rho C^2} ), ( M_p = \frac{\bar{M}_p}{\rho_0 \bar{D} \bar{R} \bar{R}_0 \rho C^2} )</td>
</tr>
</tbody>
</table>

5. **MATERIAL PROPERTIES**

| \( \rho \) Reference density | \( \mu = \frac{\rho_1}{\rho} \) |
| \( \rho_i \) Material density of field (a different \( \rho_i \) will be specified for each layer) |  |
| \( C \) Reference wave speed |  |
| \( C_{LD}^2 = \frac{K_{LD} + \frac{4}{3} G}{\rho} \) Loading \( P \)-wave speed | \( \lambda_{LD} = \frac{C_{LD}}{C} \) |
| \( C_{UN}^2 = \frac{K_{UN} + \frac{4}{3} G}{\rho} \) Unloading-Reloading \( P \)-wave speed | \( \lambda_{UN} = \frac{C_{UN}}{C} \) |
| \( C_s^2 = \frac{G}{\rho} \) Shear wave speed | \( \lambda_s = \frac{C_s}{C} \) |
| \( \nu_{LD} = \frac{3K_{LD} - 2G}{6K_{LD}} \) Loading Poisson ratio | \( \nu_{LD} \) |
| \( \nu_{UN} = \frac{3K_{UN} - 2G}{6K_{UN}} \) Unloading-Reloading Poisson ratio | \( \nu_{UN} \) |
| \( C_{LD}, C_{UN}, \) \( C_s \) will be specified for each layer |  |
6. STRUCTURE PROPERTIES

<table>
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<tr>
<th>PARAMETERS</th>
<th>NON-DIMENSIONAL PARAMETERS</th>
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<tr>
<td>$\Omega_s^2 = \frac{E_s}{(1-\nu_s^2)\rho_s R_s^2}$, $\Omega_p^2 = \frac{E_p}{(1-\nu_p^2)\rho_p R_p^2}$</td>
<td>$\omega_s = \frac{\Omega_s R_s^0}{C}$</td>
</tr>
<tr>
<td>$\alpha_s = \frac{1}{12} \left(\frac{h_s}{R_s^0}\right)^2$, $\alpha_p = \frac{1}{12} \left(\frac{h_p}{R_p^0}\right)^2$</td>
<td>$\omega_p = \frac{\Omega_p R_p^0}{C}$</td>
</tr>
<tr>
<td>$\beta_s = \frac{\rho_R}{\rho_s h_s}$, $\beta_p = \frac{\rho_R}{\rho_p h_p}$</td>
<td>$\alpha_s, \alpha_p$</td>
</tr>
<tr>
<td>$D_s = \frac{E_s h_s}{R_s^0(1-\nu_s^2)}$, $D_p = \frac{E_p h_p}{R_p^0(1-\nu_p^2)}$</td>
<td>$\beta_s, \beta_p$</td>
</tr>
</tbody>
</table>

Fundamental frequency for shell and plate, respectively

$h_s, h_p$ Shell, plate thickness
### Table 2: Equations for Computation of Stresses and Velocities in the Field

<table>
<thead>
<tr>
<th>P-waves</th>
<th>S-waves</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_p = \sigma_r + \lambda V_r$, $M_p = \sigma_r - \lambda V_r$</td>
<td>$P_s = \sigma_{rz} + \lambda V_z$, $M_s = \sigma_{rz} - \lambda V_z$</td>
</tr>
<tr>
<td>$p^N_p = \frac{\lambda \Delta t}{\rho} \left[ \sigma_r - \sigma_0 + \frac{\nu}{1-\nu} \lambda V_r \right] + p^0_p$</td>
<td>$p^N_s = \frac{\lambda \Delta t}{\rho} \left[ \sigma_{rz} - \sigma_0 + \frac{\nu}{1-\nu} \lambda V_z \right] + p^0_s$</td>
</tr>
<tr>
<td>$M^N_p = \frac{\lambda \Delta t}{\rho} \left[ \sigma_r - \sigma_0 - \frac{\nu}{1-\nu} \lambda V_r \right] + p^0_p$</td>
<td>$M^N_s = \frac{\lambda \Delta t}{\rho} \left[ \sigma_{rz} - \sigma_0 - \frac{\nu}{1-\nu} \lambda V_z \right] + p^0_s$</td>
</tr>
<tr>
<td>$\sigma^N_r = \frac{p^N_p + M^N_p}{2}$</td>
<td>$\sigma^N_s = \frac{p^N_s + M^N_s}{2}$</td>
</tr>
<tr>
<td>$\nu^N_r = \frac{p^N_p - M^N_p}{2\lambda}$</td>
<td>$\nu^N_s = \frac{p^N_s - M^N_s}{2\lambda}$</td>
</tr>
<tr>
<td>$\sigma^N_r = \frac{1}{z} \left[ 1 - \frac{\nu}{1-\nu} (\sigma^N_r - \sigma^L_r) + \lambda^2 \frac{\nu^N_r}{r} \frac{1}{1-\nu} (1 - 2\nu) \right]$</td>
<td>$\sigma^N_s = \frac{1}{z} \left[ 1 - \frac{\nu}{1-\nu} (\sigma^N_s - \sigma^L_s) + \lambda^2 \frac{\nu^N_s}{r} \frac{1}{1-\nu} (1 - 2\nu) \right]$</td>
</tr>
<tr>
<td>$\bar{P}_p = \sigma + \lambda V$</td>
<td>$\bar{P}<em>s = \sigma</em>{rz} + \lambda V$</td>
</tr>
<tr>
<td>$\bar{M}_p = \sigma - \lambda V$</td>
<td>$\bar{M}<em>s = \sigma</em>{rz} - \lambda V$</td>
</tr>
<tr>
<td>$\bar{P}^N_p = \bar{P}_p$</td>
<td>$\bar{P}^N_s = \bar{P}_s$</td>
</tr>
<tr>
<td>$\bar{M}^N_p = \bar{M}_p$</td>
<td>$\bar{M}^N_s = \bar{M}_s$</td>
</tr>
<tr>
<td>$\sigma^N_z = \frac{\bar{P}^N_p + \bar{M}^N_p}{2}$</td>
<td>$\sigma^N_z = \frac{\bar{P}^N_s + \bar{M}^N_s}{2}$</td>
</tr>
<tr>
<td>$\nu^N_z = \frac{\bar{P}^N_p - \bar{M}^N_p}{2\lambda}$</td>
<td>$\nu^N_z = \frac{\bar{P}^N_s - \bar{M}^N_s}{2\lambda}$</td>
</tr>
<tr>
<td>$\sigma^N_r = \sigma_r + \frac{\nu}{1-\nu} (\sigma^N_z - \sigma^L_z)$</td>
<td>$\sigma^N_s = \sigma_s + \frac{\nu}{1-\nu} (\sigma^N_z - \sigma^L_z)$</td>
</tr>
<tr>
<td>$j^N_r = j^L_r + \frac{1}{1-\nu} (\sigma^N_r - \sigma^L_r)$</td>
<td>$j^N_s = j^L_s + \frac{1}{1-\nu} (\sigma^N_s - \sigma^L_s)$</td>
</tr>
</tbody>
</table>

Superscripts refer to points in Figs. 3a and 3b
CHARACTERISTICS FOR THE $r$-DIRECTION

FIG. 3a

CHARACTERISTICS FOR THE $z$-DIRECTION

FIG. 3b
where the appropriate value of $\lambda$ determines $\alpha$ as

$$\alpha = \frac{\lambda \Delta t}{\Delta r} \quad (7)$$

In the POSSI Code this interpolation scheme has been used at all points in the field in both the $r$- and $z$-directions.

The PC-scheme as outlined above does not provide for following or maintaining shock fronts. Computational results from a loading with a very short rise time would therefore be dispersed.

**B. Structure**

The structure lining the cavity is a cylindrical shell with bending stiffness, welded at the bottom to a flat circular plate which deforms both by bending and extension. The equations of motion in both radial and longitudinal directions for both structural components have been obtained by a variational procedure applied to the total strain energy expressed in finite difference form. The expressions for potential energy of the shell (including bending energy, extensional energy and work done by the loads) plus the kinetic energy of the shell are given in Appendix B of Ref. [4]. Repeated here, for convenience, in terms of the non-dimensional quantities defined in Table 1, the total energy for the shell is

$$U_s = \frac{\pi D}{2} \left[ \frac{R_0}{\omega_s} \left( \frac{R_0}{\omega_s} \right) \right] \left\{ \frac{1}{2} \frac{w_s^2}{w_s} \left[ \frac{\partial W_r}{\partial z} + \frac{\partial W_z}{\partial r} \right] + 2 \frac{w_s^2}{w_s} \frac{\partial W_r}{\partial z} \right\} \frac{\partial W_r}{\partial z} + \frac{1}{2} \frac{w_s^2}{w_s} \left[ \frac{\partial W_r}{\partial r} \right]^2 + \frac{1}{2} \frac{w_s^2}{w_s} \left[ \frac{\partial W_z}{\partial r} \right]^2 \right\} \frac{\partial W_r}{\partial z} \frac{\partial W_z}{\partial z}$$

$$- \beta_s \left[ \left( \rho + \sigma \right) \frac{\partial W_r}{\partial r} + \sigma \frac{\partial W_z}{\partial z} \right] \left[ \left( \rho + \sigma \right) \frac{\partial W_r}{\partial r} + \sigma \frac{\partial W_z}{\partial z} \right] + \frac{1}{2} \left[ \frac{\partial W_r}{\partial z} + \frac{\partial W_z}{\partial z} \right] \frac{\partial W_r}{\partial z} \frac{\partial W_z}{\partial z} \right\} dz \quad (8)$$
A similar expression for the plate motion includes the energy of extensional plate motion which was excluded from the plate energy expression given in Ref. [4]. The derivatives in these expressions are written in finite-difference form at every point on the shell. Central differences are used for non-boundary points, forward or backward differences, as appropriate, are used for points in the neighborhood of the corner. A variational procedure is then applied to the total energy, first with respect to radial displacement, then with respect to longitudinal displacement. See Appendix B, Ref. [4]. The result is two equations of motion, one in the \( r \)-direction, one in the \( z \)-direction at each point of the structure. These equations may be written symbolically as

\[
V^N_m = V^L_m + A^L_m A^I
\]

The subscript "\( m \)" is written as \( r_s, z_s, r_p, z_p \) depending upon the direction of motion being computed, and whether the term comes from shell or plate effects. The quantities \( A^L_m \) are

\[
A^L_{z_s} = \frac{\beta_s}{s} \alpha_{rz} + \frac{\alpha_s}{s} [d_2 z w + \psi_s d_1 z w + \alpha_s d_3 z w]
\]

\[
A^L_{r_s} = \beta_s (r_s + \alpha_s) - \frac{\alpha_s}{s} [d_2 z w + \psi_s d_1 z w + \alpha_s (d_4 z w - d_3 z w)]
\]
for the shell, and

\[ A^L_{zp} = \beta \frac{1}{p} \left( p^L_z + \omega^2 \right) \frac{1}{p} [D4RWZ + 2(D3RWZ) - D2RW + D1RWZ] \]  

(13)

\[ A^L_{rp} = \beta \frac{1}{p} \frac{1}{rz} + \omega^2 \frac{1}{p} [D2RW + D1RWR + D0RWR] \]  

(14)

for the plate. The superscript "L" refers to a point on the structure, see Fig. 4. The superscript "N" refers to the new value of the quantity at point L. The symbol D2RWZ stands for the second derivative with respect to r of W, D1ZWR stands for the first derivative with respect to z of W, etc. The finite difference expressions for these symbols are tabulated in Table 3a and 3b (for the shell), Tables 4a and 4b (for the plate).

Equations (10)-(14) are expressions for velocities. The displacements are found as

\[ w^N_z = w^L_z + \Delta t w^N_z \]  

(15)

\[ w^N_r = w^L_r + \Delta t w^N_r \]  

(16)

The POSSI code also provides for computation of forces and moments within the structure. For the shell portion the normal, shear and moment quantities per unit of circumferential length are

\[ \bar{N}_s = \frac{p_0 D_R}{2 \rho C^2} \left[ \frac{\partial W}{\partial z} + v \frac{W}{r} - \alpha \frac{\partial^2 W}{\partial z^2} \right] \]  

(17)

\[ \bar{Q}_s = \frac{p_0 D_D}{2 \rho C^2} \alpha \left[ \frac{\partial W}{\partial z} \right] \left[ \frac{3}{2} \frac{\partial^2 W}{\partial z^2} - \frac{1}{2} \frac{\partial^2 W}{\partial z^2} \right] \]  

(18)

\[ \bar{M}_s = \frac{p_0 D_R}{2 \rho C^2} \alpha \left[ \frac{\partial W}{\partial z} \right] \left[ \frac{3}{2} \frac{\partial^2 W}{\partial z^2} - \frac{1}{2} \frac{\partial^2 W}{\partial z^2} \right] \]  

(19)
FIG. 4   NUMBERING OF POINTS ON STRUCTURE
TABLE 3a EQUATIONS OF MOTION FOR THE SHELL

Longitudinal Motion: \( V_{n}^{u} = V_{z}^{L} + \Delta t \left[ \frac{2\gamma^{L} L_{z}}{s z} + \omega^{2}_{s} \left[ D_{22}^{L} W_{z} + \omega D_{12}^{L} W_{r} - \alpha_{s} D_{32}^{L} \right] \right] \)

<table>
<thead>
<tr>
<th>Points ( L )</th>
<th>( D_{22}^{L} )</th>
<th>( D_{12}^{L} )</th>
<th>( D_{32}^{L} )</th>
<th>( \tilde{c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L = L_{1} )</td>
<td>( \frac{w_{L+1}^{L} - 2w_{L}^{L} + w_{L-1}^{L}}{h_{z}^{2}} )</td>
<td>( \frac{w_{L+1}^{L} - w_{L-1}^{L}}{h_{z}^{2}} )</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( L )</td>
<td>( \frac{w_{L+1}^{L} - 2w_{L}^{L} + w_{L-1}^{L}}{h_{z}^{2}} )</td>
<td>( \frac{w_{L+1}^{L} - w_{L-1}^{L}}{h_{z}^{2}} )</td>
<td>( \frac{w_{L+2}^{L} - 2w_{L+1}^{L} + 2w_{L-1}^{L} - w_{L-2}^{L}}{2h_{z}^{2}} )</td>
<td>1</td>
</tr>
<tr>
<td>( L = L_{C} - 2 )</td>
<td>( \frac{w_{L}^{L} - 2w_{L}^{L} + w_{L-2}^{L}}{h_{z}^{2}} )</td>
<td>( \frac{w_{L}^{L} - w_{L-2}^{L}}{h_{z}^{2}} )</td>
<td>( \frac{w_{L}^{L} - 3w_{L}^{C-2} + 3w_{L}^{C-4} - w_{L}^{C-4}}{2h_{z}^{2}} )</td>
<td>1</td>
</tr>
<tr>
<td>( L = L_{C} - 1 )</td>
<td>( \frac{w_{L}^{C-1} - 2w_{L}^{C-1} + w_{L}^{C-2}}{h_{z}^{2}} )</td>
<td>( \frac{w_{L}^{C-1} - w_{L}^{C-2}}{h_{z}^{2}} )</td>
<td>( \frac{w_{L}^{C-1} - 2w_{L}^{C-1} + 2w_{L}^{C-2} - w_{L}^{C-3}}{2h_{z}^{2}} )</td>
<td>1</td>
</tr>
</tbody>
</table>
| \( L = L_{C} \) | \( \frac{w_{L}^{C-1} - w_{L}^{C}}{h_{z}^{2}} \) | \( \frac{-w_{L}^{C}}{h_{z}^{2}} \) | \( \frac{-w_{L}^{C} + w_{L}^{C-1} - w_{L}^{C-2}}{2h_{z}^{2}} \) | \( \frac{1}{1 + \gamma} \) **

* See Fig. 4
** See Eq. (27) for definition of \( \gamma \)
TABLE 3b EQUATIONS OF MOTION FOR THE SHELL

<table>
<thead>
<tr>
<th>Points</th>
<th>D1ZWZ</th>
<th>D4ZWR</th>
<th>D3ZWR</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>L=1</td>
<td>(-\frac{\omega_s}{1 - \alpha_s} \frac{\omega_L}{r})</td>
<td>0</td>
<td>0</td>
<td>1.</td>
</tr>
<tr>
<td>L</td>
<td>(\frac{L+1 - \omega_{L-1}}{z})</td>
<td>(\frac{\omega_{L+2} - 4\omega_{L+1} + 6\omega_{L-1} - 4\omega_{L-2}}{\delta z})</td>
<td>(\frac{\omega_{L+2} - 2\omega_{L+1} + \omega_{L+2}}{2\delta z})</td>
<td>1.</td>
</tr>
<tr>
<td>L=LC-2</td>
<td>(\frac{\omega_{LC-2} - \omega_{LC-3}}{\delta z})</td>
<td>(-\omega + \delta_{LC-1} - 3\omega_{LC-3} + \omega_{LC-4})</td>
<td>(-\omega_{LC} + 2,5\omega_{LC-1} - 3\omega_{LC-2} + \omega_{LC-3})</td>
<td>1.</td>
</tr>
<tr>
<td>L=LC-1</td>
<td>(\frac{\omega_{LC-1} - \omega_{LC-2}}{\delta z})</td>
<td>(-\omega_{LC} + 3\omega_{LC-1} - 3\omega_{LC-2} + \omega_{LC-3})</td>
<td>(-\omega_{LC} + \omega_{LC-1} + \omega_{LC-2})</td>
<td>1.</td>
</tr>
<tr>
<td>L=LC</td>
<td>(\frac{\omega_{LC} - \omega_{LC-1}}{\delta z})</td>
<td>(\frac{\omega_{LC} - 2\omega_{LC-1} + \omega_{LC-2}}{\delta z^2})</td>
<td>(\frac{\omega_{LC-1} - \omega_{LC}}{\delta z^3})</td>
<td>1.</td>
</tr>
</tbody>
</table>

*See Fig. 4

** See Eq. (27) for definition of \(\gamma\)
**TABLE 4a  EQUATIONS OF MOTION FOR THE PLATE**

<table>
<thead>
<tr>
<th>Points*</th>
<th>D4RWZ</th>
<th>D3RWZ</th>
<th>D2RWZ</th>
<th>D1RWZ</th>
<th>( \omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>L=LC</td>
<td>( \frac{(w_{LC+2} - 2w_{LC+1} + w_{I}C)}{\Delta r^2} )</td>
<td>( \frac{(w_{LC+2} - 3w_{LC+1} + 2w_{I}C)}{\Delta r^2 \cdot r_{LC}(\Delta r)^3} )</td>
<td>( \frac{w_{I}C + 1 - w_{I}C}{\Delta r^2 \cdot r_{LC}(\Delta r)^3} )</td>
<td>0</td>
<td>( \frac{\gamma}{1+\gamma} )</td>
</tr>
<tr>
<td>L=LC+1</td>
<td>( \frac{(w_{LC+3} - 3w_{LC+2} + 3w_{LC+1} - w_{LC})}{\Delta r^4} )</td>
<td>( \frac{(w_{LC+3} - 3.5w_{LC+2} + 4w_{LC+1} - 1.5w_{LC})}{\Delta r^4 \cdot r_{LC}(\Delta r)^3} )</td>
<td>( \frac{w_{I}C + 2 - 1.5w_{LC+1} + 5w_{LC}}{\Delta r^4 \cdot r_{LC}(\Delta r)^3} )</td>
<td>( \frac{-w_{LC+2} + w_{I}C}{\Delta r^4 \cdot r_{LC}(\Delta r)^3} )</td>
<td>( \frac{(-w_{LC+2} + w_{I}C)}{\Delta r^4 \cdot r_{LC}(\Delta r)^3} )</td>
</tr>
<tr>
<td>L=LC+2</td>
<td>( \frac{(w_{LC+4} - 4w_{LC+3} + 6w_{LC+2} - 3w_{LC+1} + w_{LC})}{\Delta r^4 \cdot r_{LC+2}(\Delta r)^3} )</td>
<td>( \frac{(w_{LC+4} - 4w_{LC+3} + 6w_{LC+2} - 3.5w_{LC+1} + 5w_{LC})}{\Delta r^4 \cdot r_{LC+2}(\Delta r)^3} )</td>
<td>( \frac{w_{I}C + 3 - 2w_{LC+2} + w_{LC}}{\Delta r^4 \cdot r_{LC+2}(\Delta r)^3} )</td>
<td>( \frac{-w_{LC+3} + w_{LC}}{\Delta r^4 \cdot r_{LC+2}(\Delta r)^3} )</td>
<td>( \frac{-w_{LC+3} + w_{LC}}{\Delta r^4 \cdot r_{LC+2}(\Delta r)^3} )</td>
</tr>
<tr>
<td>L</td>
<td>( \frac{(w_{L}L^2 - 4w_{L}L^1 + 6w_{L}L^0 - 4w_{L}L^{-1} + w_{L}L^{-2})}{\Delta r^4} )</td>
<td>( \frac{-w_{L}L^2 + 2w_{L}L^1 - 2w_{L}L^0 + w_{L}L^{-2}}{\Delta r^4 \cdot r_{L}(\Delta r)^3} )</td>
<td>( \frac{w_{L}L^1 + 1 - 2w_{L}L^0 + w_{L}L^{-1}}{\Delta r^4 \cdot r_{L}(\Delta r)^3} )</td>
<td>( \frac{-5w_{L}L^1 + 5w_{L}L^{-1}}{\Delta r^4 \cdot r_{L}(\Delta r)^3} )</td>
<td>( \frac{-5w_{L}L^1 + 5w_{L}L^{-1}}{\Delta r^4 \cdot r_{L}(\Delta r)^3} )</td>
</tr>
<tr>
<td>L=IC</td>
<td>( \frac{18w_{IC} - 24w_{IC-1} + 6w_{IC-2}}{\Delta r^4} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.</td>
</tr>
</tbody>
</table>

*See Fig. 4*

** See Eq. (27) for definition of \( \gamma \)
<table>
<thead>
<tr>
<th>Points*</th>
<th>D2WR</th>
<th>D1WR</th>
<th>DORWR</th>
<th>( \bar{c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L = LC )</td>
<td>( \frac{w_{LC+1}^{r}}{(\Delta r)^2} )</td>
<td>( \frac{w_{LC+1}^{r} - w_{r+1}^{r}}{(\Delta r)^2} )</td>
<td>( \frac{w_{LC+1}^{r}}{\Delta r} ) + ( \frac{w_{LC}^{r}}{\Delta r} )</td>
<td>( \frac{w_{LC}^{r}}{r_{LC}^2} )</td>
</tr>
<tr>
<td>( L = LC+1 )</td>
<td>( \frac{w_{LC+2}^{r} - 1.5w_{LC+1}^{r} + 0.5w_{LC}^{r}}{(\Delta r)^2} )</td>
<td>( \frac{w_{LC+2}^{r} - 2w_{LC+1}^{r} + w_{LC}^{r}}{(\Delta r)^2} )</td>
<td>( \frac{w_{LC+1}^{r}}{\Delta r} ) + ( \frac{w_{LC}^{r}}{\Delta r} )</td>
<td>( \frac{w_{LC}^{r}}{r_{LC+1}^2} )</td>
</tr>
<tr>
<td>( L = LC+2 )</td>
<td>( \frac{w_{LC+3}^{r} - 2w_{LC+1}^{r} + w_{LC}^{r}}{(\Delta r)^2} )</td>
<td>( \frac{w_{LC+3}^{r} - 2w_{LC+2}^{r} + w_{LC+1}^{r}}{(\Delta r)^2} )</td>
<td>( \frac{w_{LC+2}^{r}}{\Delta r} ) + ( \frac{w_{LC+1}^{r}}{\Delta r} )</td>
<td>( \frac{w_{LC+2}^{r}}{r_{LC+2}^2} )</td>
</tr>
<tr>
<td>( L = L-1 )</td>
<td>( \frac{w_{L+1}^{r} - 2w_{r}^{r} + w_{L}^{r}}{(\Delta r)^2} )</td>
<td>( \frac{w_{L+1}^{r} - w_{L}^{r}}{\Delta r} )</td>
<td>( \frac{w_{L}^{r}}{2r_{L}^2} )</td>
<td>( \frac{w_{L}^{r}}{r_{L}^2} )</td>
</tr>
</tbody>
</table>

* See Fig. 4
** See Eq. (27) for definition of \( \gamma \)
For the plate portion the normal, shear and moment quantities per unit of circumferential length are

\[ \bar{N} = \frac{D_p R^2}{\rho c^2} \left[ \frac{3W}{r} + \frac{W}{r^2} \right] \]  
\[ \bar{Q} = \frac{D_p R^2}{\rho c^2} \alpha \left[ -\frac{3}{r^3} + \frac{1}{r} \frac{\partial^2 W}{\partial r^2} - \frac{1}{r^2} \frac{\partial W}{\partial r} \right] \]  
\[ \bar{M} = \frac{D_p R^2}{\rho c^2} \alpha \left[ -\frac{1}{r^2} \frac{\partial^2 W}{\partial r^2} + \frac{1}{r} \frac{\partial W}{\partial r} \right] \]  

See Table 1, item 4 for definition of the corresponding non-dimensional and dimensional quantities.
1. Shell-Plate Connection

Equations (10)-(14) describe velocities at structure points providing no restriction is made concerning the relative angle between the shell wall and the plate bottom. If a right angle is to be maintained at this joining then the equation is modified for points near the corner.

The condition of a right angle at the shell-plate interface is stated by the constraint condition

\[ G = \left[ \frac{\partial W}{\partial z} + \frac{\partial W}{\partial r} \right]_{z=L} = 0 \]  

(17)

the subscript "LC" defines the point at the interface, see Fig. 4. This is introduced into the variational procedure by the Lagrangian multiplier \( \lambda \) such that the total variational expressions are

\[ \delta (U_s + U_p) = \lambda \frac{\partial G}{\partial m_L} = 0 \]  

(18)

where \( W_{m_L} \) indicates displacement at point "L" in either the r-direction, or the z-direction, and \( U_s, U_p \) are the total energy expressions given in Eqs. (8) and (9). For convenience the energy variation is written as

\[ \frac{\partial (U_s + U_p)}{\partial \lambda_{m_L}} = -\lambda_{m_L} + \frac{\partial W_{m_L}}{\partial m_L} \]  

(19)

[where the quantities \( \lambda_{m_L} \) are given in Eqs. (11)-(14)] and the constraint, Eq. (17), is written in finite difference form.
Then Eqs. (18), (19) and (20) combine to give

\[
\frac{\Delta V_{r}^{LC-1}}{\Delta t} = \frac{\Delta r}{\Delta z} \Lambda + \Lambda_{r}^{LC-1}
\]
(21)

\[
\frac{\Delta V_{r}^{LC}}{\Delta t} = - \frac{2}{1 + \gamma} \frac{\Delta r}{\Delta z} \Lambda + \Lambda_{r}^{LC} + \Lambda_{r}^{LC}
\]
(22)

\[
\frac{\Delta V_{z}^{LC}}{\Delta t} = - \frac{2}{1 + \gamma} \frac{\Delta z}{\Delta t} \Lambda + \Lambda_{z}^{LC} + \Lambda_{z}^{LC}
\]
(23)

\[
\frac{\Delta V_{r}^{LC+1}}{\Delta t} = \frac{r_{LC+1}}{r_{LC+1}} \frac{\Lambda + \Lambda_{r}^{LC+1}}{\Lambda_{z}^{LC+1}}
\]
(24)

A fifth equation, which allows for solution of \( \Lambda \) is obtained from the second derivative with respect to time of Eq. (19)

\[
\frac{\partial^2 G}{\partial t^2} = \frac{\Delta V_{r}^{LC}}{\Delta t} - \frac{\Delta V_{r}^{LC-1}}{\Delta t} + \frac{\Delta V_{r}^{LC}}{\Delta t} - \frac{\Delta V_{r}^{LC+1}}{\Delta t} = 0
\]
(25)

Combining Eqs. (20)-(24) yield

\[
\Lambda = \Lambda_{r}^{LC} + \Lambda_{z}^{LC} - \Lambda_{r}^{LC-1} + \Lambda_{r}^{LC} + \Lambda_{z}^{LC} + \Lambda_{z}^{LC} - \Lambda_{z}^{LC+1}
\]
\[
\frac{\Delta r}{\Delta z} \Lambda_{r}^{LC} + \Lambda_{z}^{LC} + \Lambda_{z}^{LC} + \Lambda_{z}^{LC} - \Lambda_{z}^{LC+1}
\]

where

\[
\gamma = \frac{D_{s}}{D_{p}} \frac{\Delta r}{\Delta z} \frac{\omega}{s} \frac{2}{r_{LC}}
\]
(27)

See Fig. 4 for points corresponding to superscripts LC, LC+1, LC-1, etc.

In a layered medium it is understood that the \( \Delta z \) term appearing in equations related to the maintenance of a right angle at the corner
should be the value of the $\Delta z$ in that layer in which the plate sits.

For a structure in which there is no right angle restriction at the corner the equations of motion for all points are given by Eq. (10). For a structure with a right angle restriction Eqs. (21)-(24) must be used as equations of motion for points LC-1, LC and LC+1. All other points are described as before.
III THE POSSI CODE

A. Conditions for use

Among the options built into the Possi Code is a choice of location and type of loading. There are three possibilities; loading on the upper boundary, see Fig. 5a; loading on the interior of the cavity (or structure, if it is present), Fig. 5b; loading on both the upper boundary and the cavity interior, Fig. 5c. In each case a time variation may be combined with an appropriate space variation, as shown. The input parameter controlling which of the three loading types is to be used is TLOAD. See card #2, Table 7. The actual loading function is to be input directly into program functions APSGZ and SIGAP as described in the next section. If no load is applied on either the upper or cavity boundary conditions of zero stress are assumed. It should be noted that boundary conditions at maximum grid lines are the type of transmitting boundary obtained from mirror image values of stress and velocity. Conditions along a line through the origin, but below the cavity are, of course, those of zero velocity in the radial direction.

The other options, such as non-dimensional versus dimensional computations, number of layers, lined or unlined cavity, right angle at shell-plate interface, graphic or printed output, are also controlled by input parameters which are listed in detail in Table 7, Appendix A.

The options of one dimensional (r-t or z-t) computations are activated by choice of grid dimensions. For a one dimensional r-t computation choose a long slender cavity with loading on the interior vertical surface as p(t), and a wide grid. For a one dimensional z-t computation...
FIG. 5a LOADING ON UPPER BOUNDARY

FIG. 5b LOADING ON INTERIOR OF CAVITY

FIG. 5c COMBINED LOADING

LOADING OPTIONS FOR POSSI CODE
choose a p(t) loading on the upper boundary, a long slender cavity with no loading and a deep grid.

The numbering system used to identify points in the field and points on the shell is shown in Fig. 6. Special note should be made of the fact that field points are numbered (in the r-direction) from the first point away from the cavity, rather than from the origin, except at points below the cavity where numbering does begin from the origin. Regardless of grid dimensions chosen there should be at least 5 points on the vertical boundary of the cavity and at least 5 points on the horizontal boundary from r=0 to r=r_{LC} at the bottom of the cavity.

Without precise stability criteria, it is recommended that time and space increments be chosen such that the quantity \( \frac{\lambda_{max} \Delta t}{\Delta r} \) is well below 1.0 when there is no structure, and below when a shell-plate structure is present.

Typical compile time on a CDC 6600 is approximately 11 seconds. Typical run time for a 60 by 40 point grid, with three horizontal layers and no structure, running 300 time steps with printed output every 10th time step, and output for time history plots every 2nd time step is about 380 seconds. Typical run time for a 60 by 40 grid with one layer and a structural lining, running 200 time steps with the same frequency of output as above is about 220 seconds. Additional output will, of course, increase running times noticeably.

It may also be noted here that all input is on TAPE 2, all printed output is on TAPE 3, and all output for graphing is on TAPE 4.
TYPICAL NUMBERING OF POINTS
ON THE STRUCTURE

TYPICAL NUMBERING OF POINTS
IN THE FIELD

LAYER #1

LAYER #2

LAYER #3

FIG. 6
There are a few programming features which users of the POSSI code may wish to modify for certain types of computations. For instance, the way the code is presently written a velocity cutoff of \( V = \left| \sqrt{v_x^2 + v_y^2} \right| = 0.001 \) controls the extent of the computational field for a disturbance moving outwardly from the cavity. See comments in the next section, Description of Subroutines, Subroutine LOCATE. Also, currently the code allows for only three cycles of load, unload, reload. Three cycles were sufficient for the computations done thus far. See comments in the next section under Subroutine TEST, and see Fig. 8.
B. DESCRIPTION OF SUBROUTINES

The following describes the essential function of each subroutine in the POSSI Code. The Subroutines are listed in the order in which they are compiled in the code.

**RZ2D - MAIN**

Increments time. Activates integration first in the r-direction (CALL RDIR), then in the z-direction (CALL ZDIR). If a structure is present with a right angle restriction at its corner, RZ2D - MAIN corrects the shell corner motion for that restriction. Integrates velocities into displacements, Eqs. (15), (16) at grid points on the structure. Calls for check of loading, unloading, reloading conditions at all field points (CALL TEST). Initiates graphic (CALL GRAPHS) or printed (CALL SLOT) output as required.

**LOCATE**

Computes arrays LR, LRMX which control the extent of integration in the r-direction, and arrays LZ, LZMX which control the extent of integration in the z-direction. At each time step the value of

\[ V = \sqrt{\frac{V_r^2}{r} + \frac{V_z^2}{z}} \]

is computed at each field point with coordinates I, J. As long as a velocity \( V > .001 \) is encountered the arrays are set as \( LR(I) = J + 1, LZ(J) = I + 1 \). The values LRMX, LZMX are then set as \( LRMX = \max LR(I), LZMX = \max LZ(J) \). This criterion was selected for disturbance expanding outward from the cavity, and for essentially non-dimensional computations where \( V = 1.0 \). For other shape disturbances or for some dimensional computations where very large or very small \( V \)'s are expected this criteria should be changed.
DATA

Requests input data according to specifications listed in Table 6. Initiates computation of constants (CALL CONST). Outputs all pertinent computational parameters as shown in the sample computations, Section IV.

CONST

Initializes arrays. All stresses and velocities are set to zero. Computes constants CNR1...CNR12, arrays CLD, CS, CUN as defined in the list of common variables in Appendix C. Computes other constants from input data which are basic to the code.

RDIR

Integrates equations of motion in the r-direction, at all field points, by evaluating the upper set of equations shown in Table 2. First integration is done in Region 1 as shown in Fig. 7a. Then Subroutine SHELL is called at the end of each 1\textsuperscript{th} line of integration to perform the integration of the shell equations of motion in both the r- and z-directions, (if there is a structure), or to compute the motion of field points at an applied pressure boundary, if there is no structure. If a structure is present, all its mass is considered to be distributed along the line defining the cavity, and all motions of points on that line are governed by the structural equations of motion. If no structure is present the line defining the cavity is all material and requires integration of the field equations in both the r- and z-directions. Therefore, in the case of no structure, r-direction integration in Region 1, Fig. 7a is followed by r-direction integration along the bottom boundary of the hole, (CALL PLATR), which is then followed by r-direction integration in Region 2, Fig. 7a.
Integrates equations of motion in the z-direction at all field points by evaluating the lower set of equations shown in Table 2. First the integration is done in Region 1, Fig. 7b, then the integration is done in Region 2. At the end of each J0th line of integration in Region 2 Subroutine PLATE is called to provide the integration of the plate equations of motion in both the r- and z-directions (if there is a structure), or to compute the field integration at points on an applied pressure boundary, if there is no structure. Then, if no structure is present, for the reasons stated above, integration of the field equations along the vertical cavity boundary is initiated by calling subroutine SHELZ.

UPBND

Integrates the equations of motion in the z-direction at field points along the upper, z = 0, boundary. Calls function APSCZ for applied stress boundary condition.

SHELL

Evaluates the equations of motion for the shell, as given in Tables 3a and 3b, if a structure is present. If the cavity is unlined, the field equations of motion are integrated in the r-direction for an applied stress boundary condition. The function SIGAP is called for application of $\sigma_r$ to the inner surface of the cavity or shell, ($\sigma_{rz} = 0$ is always assumed on this surface).

SHELZ

Integrates the field equations of motion in the z-direction along the vertical boundary of the cavity. This subroutine is activated only if the cavity is unlined.
PLATE

Evaluates the equations of motion for the plate as given in Tables 4a and 4b, if a structure is present. If the structure is unlined, the field equations of motion are evaluated in the z-direction for an applied stress boundary condition. The function SIGAP is called for an applied stress in the z-direction on the inner surface of the plate or the unlined cavity. (Again $\sigma_{rz} = 0$ is assumed on this surface.)

PLATR

Integrates the field equations of motion in the $r$-direction along the horizontal boundary of the cavity. This subroutine is activated only if the cavity is unlined.

COEFFS

Sets the coefficient arrays C and CS to the appropriate values for each layer. Sets the coefficient array C at each field point to loading or unloading values as determined by the value of the indicator KODES.

TEST

At each point in the field the new value of the first invariant $A_{11}$ is tested against the last previous compressive maximum $A_{11}^{\text{MAX}}$. Depending upon the outcome of this test the indicator KODES is given a numerical value to indicate conditions of loading, unloading/reloading as listed in Table 7, and shown graphically in Fig. 8. Providing for three cycles of load/unload was sufficient for the computations done so far. The user may find, for certain loadings, further cycling should be provided for.

NUVALS

The evaluation of equations given in Table 2 involves updating of quantities at each field point. $L$, from non updated quantities at $L-1$ and $L+1$. As the integration proceeds forward in either the $r$- or $z$-direction
Fig. 8 Values of Kodes
the updated quantities at "L" are stored in a set of temporary arrays 
(TSR, TSZ, TSRZ, TVR, TVZ, TAJ1) until the updated quantities at L+1 
are evaluated and stored temporarily. Then this subroutine NUVALS 
places the temporary arrays from point L into the final arrays (SGR, SCZ, 
SCRZ, VR, VZ, AJ1) of stress and velocity at point "L". This flip-flopping minimizes the amount of memory required in the integration.

SRLNU

Performs the same function that NUVALS does, only for quantities 
being integrated along the cavity boundary in SHELZ and PLATR.

SIGAP

Function which sets the value of applied normal stress on the interior boundary of the cavity or the structure, as a function of time and location. Applied pressure is to be considered positive in this function. Each time a different applied pressure function is to be considered, the state-
ments in this function must be changed.

APSCZ

Function which sets the value of applied normal stress on the upper boundary, Z=0. See comments under SIGAP.

SLOT

Directs printed output, of computations, samples of which are shown in Appendix B. All output from SLOT is on TAPE 3.

GRAPH

Accumulates output for later use by plotting subroutines. All output from GRAPH is on TAPE 4.
Table 5 - Indicators for Loading, Unloading and Reloading

<table>
<thead>
<tr>
<th>KODES</th>
<th>CONDITION</th>
<th>COMPUTATIONAL PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Initial virgin loading</td>
<td>( C_{p_{LD}} ), ( C_{s} ), ( v_{LD} )</td>
</tr>
<tr>
<td>10</td>
<td>Unload from first compressive ( J_1 ) maximum, or reload up to that value ( -J_1 \leq (-J_1)_\text{max}^{(1)} )</td>
<td>( C_{p_{UN}} ), ( C_{s} ), ( v_{UN} )</td>
</tr>
<tr>
<td>11</td>
<td>Second virgin loading ( -J_1 &gt; (-J_1)_\text{max}^{(1)} )</td>
<td>( C_{p_{LD}} ), ( C_{s} ), ( v_{LD} )</td>
</tr>
<tr>
<td>100</td>
<td>Unload from second compressive ( J_1 ), or reload up to that value ( -J_1 \leq (-J_1)_\text{max}^{(2)} )</td>
<td>( C_{p_{UN}} ), ( C_{s} ), ( v_{UN} )</td>
</tr>
<tr>
<td>111</td>
<td>Third virgin loading</td>
<td>( C_{p_{LD}} ), ( C_{s} ), ( v_{LD} )</td>
</tr>
<tr>
<td>1000</td>
<td>Unload from third compressive ( J_1 ) or reload</td>
<td>( C_{p_{UN}} ), ( C_{s} ), ( v_{UN} )</td>
</tr>
</tbody>
</table>
IV  REPRESENTATIVE RESULTS

A. Case with layered medium, no structure.

Computations were done for the three-layered medium with no structure, shown in Fig. 9. The material parameters for this case are the same as those used for computations in a three-layered medium with an unlined cavity discussed in Ref. [3]. The loading on the interior of the cavity is a triangular pulse in time, Fig. 10a, but in this case has an exponential variation of $e^{-2z}$ (Fig. 10b) as opposed to the space independent pulse that was used in Ref. [3]. Quantities were computed in non-dimensional form, and graphic output was obtained for radial velocity near the mid-point of each layer at $r = 2.0$.

A sample of portion of POSSI code output is shown in Fig. 11. Listed in this figure are all the input quantities required for the computation of Case A results.

Radial velocity time histories at points near the mid-point of each layer, at $r = 2.0$ are shown in Fig. 12. The relative differences in arrival times are of course due to the different loading speeds in the three layers (1.0/1.091/5.145). The decreasing magnitude of velocity, with depth represents the response to the spatial distribution of the loading function. The relative length of time of a full pulse in each layer is due to the difference in unloading wave speeds in the three layers (2.4/3.325/6.417).

A stress profile of $\sigma_r$ versus $z$-direction is shown in Fig. 13. Here the existence of layers is evident, the discontinuity of radial stress at each interface shows only approximately because the interface line has been chosen, arbitrarily, to sit in the upper rather than the lower
FIG. 9 CONFIGURATION FOR COMPUTATIONS
\[ F(T) = T/6 \quad 0 \leq T \leq 0.6 \]
\[ F(T) = 2 - T/6 \quad 0.6 \leq T \leq 1.2 \]
\[ F(T) = 0 \quad T > 1.2 \]

**FIG. 10a** \( F(T) \)

\[ G(Z) = e^{-0.2Z} \]

**FIG. 10b** \( G(Z) \)
HILICAN SOLID, TWO-DIMENSIONAL OUT-GOING WAVE IN CYLINDRICAL COORDINATES WITH R-Z VARIATION ONLY
NO SHELL; 3 LAYERS; TRIANGLE LOAD WITH EXP(-Z²/R) DEPTH VARIATION

PARAMETERS FOR COMPUTATION

\[
\begin{align*}
\text{UN} &= 2.0000 & \text{UT} &= 0.1500 \\
\text{MAX RADIUS OF GRID} &= 7.000 & \text{MAX DEPTH OF GRID} &= 29.500
\end{align*}
\]

OUTPUT EVERY 10TH TIME STEP, UP TO A MAXIMUM OF 200 STEPS
NUMBER OF POINTS ON PLATE IN R-DIRECTION = 30
NUMBER OF POINTS ON SHELL IN Z-DIRECTION = 30
NUMBER OF POINTS IN FIELD IN R-DIRECTION = 40
NUMBER OF POINTS IN FIELD IN Z-DIRECTION = 40

OUTPUT POINTS ON SHELL AT \( l = 4, 10, 21, 28, 29, 30, 31 \)
\( 36, 39, \)

OUTPUT POINTS IN THE FIELD AT \((i, j) = (4, 5), (10, 5), (21, 5), (10, 10), (21, 10) \)

RESULTS COMPUTED IN NON-DIMENSIONAL FORM

LOADING ON INTERIOR OF CAVITY
- HOLLOW CAVITY, NO SHELL
  \( \text{CAVITY RADIUS} = 1.00 \)
  \( \text{CAVITY LENGTH} = 14.384 \)

MATERIAL PROPERTIES, FOR 3 LAYERS

<table>
<thead>
<tr>
<th>LAYER</th>
<th>NLYH</th>
<th>DZ</th>
<th>LAMBDA</th>
<th>LAMBDA</th>
<th>LAMBDA</th>
<th>NU</th>
<th>NU</th>
<th>Z MAX</th>
<th>DENSITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>500</td>
<td>1,000</td>
<td>2,000</td>
<td>3,500</td>
<td>.594</td>
<td>.594</td>
<td>223</td>
<td>1,500</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>500</td>
<td>1,141</td>
<td>3,325</td>
<td>4,500</td>
<td>.594</td>
<td>.594</td>
<td>386</td>
<td>2,500</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
</tbody>
</table>

TIME HISTORY TO BE PLOTTED EVERY 20TH TIME STEP:

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>LOCATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>POINTS ON SHELL</td>
<td>( i, j )</td>
</tr>
<tr>
<td>POINTS IN FIELD</td>
<td>( i, j )</td>
</tr>
</tbody>
</table>

SPACE PROFILES TO BE PLOTTED:

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>TIME</th>
<th>LINE</th>
</tr>
</thead>
<tbody>
<tr>
<td>IN R-DIRECTION</td>
<td>STEP</td>
<td>LOCATION</td>
</tr>
<tr>
<td>IN Z-DIRECTION</td>
<td>1</td>
<td>150</td>
</tr>
</tbody>
</table>

\( IT = 10 \) \( T = 0.150 \)

FIG. II
FIG. 12 VELOCITY TIME HISTORIES
CASE A - NO SHELL

\( z = 1.5, \, r = 2.0 \)

\( z = 4.5, \, r = 2.0 \)

\( z = 10.0, \, r = 2.0 \)
FIG. 13 STRESS PROFILE IN THE Z - DIRECTION AT R = 3.0 , T = 1.5

CASE A , NO SHELL
layer. The computation scheme permits only one medium to exist along any grid line.

B. Case with structure (non-dimensional)

For a second set of representative computations a structure was added to the material configuration of Case A. The same loading, Figs. 10a, 10b, was applied, this time to the interior of the structure. Output was obtained at the same space points as in Case A.

Figure 14 gives the initial POSSI code printout which lists all the parameters, both material and structural for this case. The structural parameters are the same as those used for the structure discussed in Ref. [4].

Figure 15 presents radial velocity-time histories at the same points as for Case A. The effect of the mass of structure, one radii removed from it, is apparent from the reversal of the initially outwardly directed \( V_r \).

C. Case with structure (dimensional)

The input parameter chosen for this case were similar to those used for the runs in Ref. [4]. They are

\[
\begin{align*}
  p_0 &= 10.2 \# / \text{in}^2 \\
  R_0 &= 52000 \text{ inches} \\
  C &= 52000 \text{ inches/sec} \\
  \rho &= 0.001947 \frac{\# \sec^2}{\text{in}^4} (130 \# / \text{ft}^3) \\
  h_s &= h_p = 3380 \text{ inches} \\
  \Omega_s &= \Omega_p = 4.0 \text{ rps} \\
  \Delta \rho &= 0.2 \\
  \frac{R_s}{R_0} &= 46
\end{align*}
\]
LOADING ON INTERIOR OF CAVITY

CAVITY BOUNDARY WITH SHELL

LOAD HISTORY PLOTS EVERY 20TH TIME STEP

MAX VELOCITY LOCATION

MAX VELOCITY TIME LOCATION

EXIT MAX LOAD IN N gleich 47
FIG. 15 VELOCITY TIME HISTORIES
CASE B - WITH SHELL
\[ \Delta t = 0.01 \text{ sec} \]
\[ \frac{\Delta Z}{R_0} = 0.5 \]

Since the non-dimensionalizing factor \( \frac{pC}{p_0} = 1.0 \) the velocities shown in Fig. 15 may be interpreted as velocity output for this case. The units on the velocity scale in this case would be in inches/sec, and the time scale would be in seconds. A sample of the computer output of input information is shown in Fig. 16.
FIG. 16
V CONCLUSION

A description of a computer code known as POSSI has been presented. The code was written to provide a small, flexible tool for computing effects of a soil-structure interaction problem in two-dimensional (r, z) coordinates, where the cylindrical structure is buried in a type of dissipative material which may be layered. The POSSI code may be used as a check code for large finite element or finite difference computations, or it may be used to explore various soil-structure interaction problems (within the limitation of the type of material presented). It may also be used to efficiently run CIST type problems for cases in which the primary dissipation mechanism in the soil comes from pressure-volume hysteresis.

Results from two sample problems have been presented. The first being a long cylindrical hole, unlined, in a material of three layers. The second problem consisted of a structurally lined long cylindrical cavity in the same three layered material. In both cases results were present in non-dimensional form, input and output information from the code was provided for both. As a subcase to the second the results were also given in dimensional form.

In any computations using POSSI code there is certain care to be taken. Boundary and corner points present problems of approximation to real situations. Conclusions drawn about results at such points must be more carefully checked. Section III of this report lists some other points of care which should be taken when using the POSSI code.

Results from the two-dimensional r, \( \theta \) coordinate problem for this configuration have already been presented, Ref. [2]. A recommended step in this work is to combine the computer code that was written for
the results of Ref. [2] with the POSSI code to produce a three-dimensional code for \( r, \theta, z \) coordinates. Such a code would still be relatively small and flexible enough to be considered a check code. It would also provide a code for one of the few three-dimensional structure-medium interaction problems not restricted to being in an elastic material.
REFERENCES


APPENDIX A - INPUT INSTRUCTIONS FOR POSSI CODE

All input data required for the POSSI code is listed in Table 6 in the order in which it is required. Variable names, description and format for each card set are given. (Variable names are also defined in Appendix B.) Variables required for choice of options such as loading type, non-dimensional versus dimensional output, etc. are also listed.

Table 7 lists the numbering convention for each variable on the shell or in the field. If printed output or graphic output is desired the quantity to be output (stress, velocity or displacement) will be identified by this number system.
<table>
<thead>
<tr>
<th>CARD SET</th>
<th>VARIABLE</th>
<th>DESCRIPTION</th>
<th>FORMAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TITLE</td>
<td>Any alpha numeric description of the run being made. Total number of characters is 80.</td>
<td>8A10 one card</td>
</tr>
</tbody>
</table>
| 2        | NLAYR    | Number of layers  
= 1 for loading on cavity interior  
= 2 for loading on upper boundary  
= 3 for loading on both cavity interior and upper boundary | 1615 one card |
|          | ILOAD    |  
= 1 for loading on cavity interior  
= 2 for loading on upper boundary  
= 3 for loading on both cavity interior and upper boundary | 1615 one card |
|          | IR       | Maximum number of mesh points in the R-direction. See Fig.7b | 1615 one card |
|          | IZ       | Maximum number of mesh points in the Z-direction. See Fig.7a | 1615 one card |
| 3        | NOSHL    | = 0 if no structure lines the cavity  
= 1 if shell-plate structure lines the cavity | 1615 one card |
|          | IRSHL    | Maximum number of mesh points on the plate (in the R-direction). See Fig.7b | 1615 one card |
|          | IZSHL    | Maximum number of mesh points on the shell (in the Z-direction). See Fig.7a | 1615 one card |
| 4        | NPRT     | Output is printed every "NPRT" time step | 1615 one card |
|          | NPRSH    | Total number of points around the cavity at which printed output is desired | 1615 one card |
|          | NPAIR    | Total number of points in the field at which printed output is desired | 1615 one card |
|          | NORIT    | = 1 a right angle is preserved at the shell-plate connection  
= 0 no right angle is maintained | 1615 one card |
|          | NODIM    | = 0 all output is presented in non-dimensional form  
= 1 all output is in dimensional form | 1615 one card |
|          | NGRPH    | = 0 no graphed output is desired  
= 1 yes, some graphs are to be printed | 1615 one card |
<table>
<thead>
<tr>
<th>CARD SET</th>
<th>VARIABLE</th>
<th>DESCRIPTION</th>
<th>FORMAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>IF NODIM = 0 the following card is to be omitted</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>DENS, SPEED, RADIUS</td>
<td>Density of the material in the field. Initial loading speed of P-waves in the material. Nominal radius of the cavity. These are reference values for internal non-dimensionalization. These three quantities should have consistent units. That is, all inches or all feet. Whichever is used will determine whether output occurs in [inches, inches/sec, psi] or [feet, ft/sec, psf] units.</td>
<td>8F10 one card</td>
</tr>
<tr>
<td>6</td>
<td>ISHPR(NPR), NPR=1, NPRSH</td>
<td>Indices of mesh points on the cavity boundary where printed output is desired. See Fig. 6 for numbering order.</td>
<td>16I5 one card for NPRSH ≤ 16</td>
</tr>
<tr>
<td>7</td>
<td>IPR(NPR), JPR(NPR), NPR=1, NPAIR</td>
<td>I,J indices (in the Z and R-directions, respectively) of points in the field where printed output is desired. See Fig. 6 for the numbering order.</td>
<td>16I5 one card for (2^NPAIR) ≤ 16</td>
</tr>
<tr>
<td>IF NGRPH = 0 cards 9 through 11 are to be omitted. (That is, no graphed output is desired.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>ITGRF, MXGRS</td>
<td>Time history plots are made at every &quot;ITGRF&quot; time step. The total number of time histories of points on the structure for which graphs are desired.</td>
<td>16I5 one card</td>
</tr>
<tr>
<td></td>
<td>MXGRF</td>
<td>The total number of time histories of points in the field for which graphs are desired.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MXPRF</td>
<td>The total number of stress or velocity profiles in either the R- or Z-direction for which plots are desired.</td>
<td></td>
</tr>
<tr>
<td>IF MXGRS = 0 the following card is to be omitted. (That is, no time histories of points on the structure are requested.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CARD</td>
<td>VARIABLE</td>
<td>DESCRIPTION</td>
<td>FORMAT</td>
</tr>
<tr>
<td>------</td>
<td>----------</td>
<td>-------------</td>
<td>--------</td>
</tr>
<tr>
<td>9</td>
<td>NSHV</td>
<td>The number of the structure variable for which a time history is desired. See Table 7 for the number used to represent each variable.</td>
<td>1615 one card for each variable/point combination</td>
</tr>
<tr>
<td></td>
<td>NSHPT</td>
<td>The number of the mesh point on the structure at which a time history is to be plotted. See Fig. 6</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>If MXGRF = 0 the following card is to be omitted. (That is, no time histories for points in the field are requested.)</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>NFLV</td>
<td>The number of the field point variable for which a time history is desired. See Table 7 for the number used to represent each variable.</td>
<td>1615 one card for variable/point combination</td>
</tr>
<tr>
<td></td>
<td>IFLPT</td>
<td>The I,J mesh point numbers of the point in the field for which a time history is desired. See Fig. 6 for the numbering sequence used for field points.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>If MXRPF = 0 the following card set is to be omitted. (That is, no space profiles are requested.)</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>NVAR</td>
<td>The number of the field variable for which a space profile is desired.</td>
<td>1615 one card for each set of four variable</td>
</tr>
<tr>
<td></td>
<td>MTPT</td>
<td>The time step number at which the space profile is to be observed.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NRORZ</td>
<td>= 1 If the space profile is to be taken in the R-direction = 2 If the space profile is to be taken in the Z-direction</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LINE</td>
<td>The index number of the line along which the space profile is to be taken. If NRORZ = 1 (profile in the R-direction) then LINE will be a value to the I index in the field. If NRORZ = 2 (profile in the Z-direction) then LINE will be a value of the J index in the field.</td>
<td></td>
</tr>
<tr>
<td>CARD SET</td>
<td>VARIABLE</td>
<td>DESCRIPTION</td>
<td>FORMAT</td>
</tr>
<tr>
<td>----------</td>
<td>----------</td>
<td>-----------------------------------------------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>14</td>
<td>LAMUN</td>
<td>Non-dimensional unloading wave speed $\lambda_{UN}$; See Table I</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LAMS</td>
<td>Non-dimensional shear wave speed $\lambda_s$; See Table I</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NULD</td>
<td>Poisson ratio for loading conditions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NUUN</td>
<td>Poisson ratio for unloading conditions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DENS</td>
<td>Material density, relative to DENS, M.</td>
<td></td>
</tr>
<tr>
<td>CARD SET</td>
<td>VARIABLE</td>
<td>DESCRIPTION</td>
<td>FORMAT</td>
</tr>
<tr>
<td>----------</td>
<td>----------</td>
<td>-------------</td>
<td>--------</td>
</tr>
<tr>
<td>12</td>
<td>DT</td>
<td>Time increment in consistent time units with SPEED for a dimensional run.</td>
<td>8F10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>If NOSH = 0 the following card set is omitted. (That is, if no structure lines the cavity.)</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>HSH</td>
<td>Thickness of the shell</td>
<td>8F10</td>
</tr>
<tr>
<td></td>
<td>HPL</td>
<td>Thickness of the plate</td>
<td>Last two quantities are put on a second card</td>
</tr>
<tr>
<td></td>
<td>SHNU</td>
<td>Poisson ratio of material in the shell</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PLNU</td>
<td>Poisson ratio of material in the plate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ESH</td>
<td>Young's modulus of material in the shell</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EPL</td>
<td>Young's modulus of material in the plate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>OMSH</td>
<td>Fundamental frequency of the shell</td>
<td></td>
</tr>
<tr>
<td></td>
<td>OMPL</td>
<td>Fundamental frequency of the plate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SHDEN</td>
<td>Density of shell material</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PLDEN</td>
<td>Density of plate material</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>For a non-dimensional run: thicknesses, frequencies and densities should be given as non-dimensional quantities as listed in Item 6, Table 1. Young's modulus and the implied unit $P_0$ value should have the same units.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>For a dimensional run: material and structural densities should have the same units. Thicknesses, densities, moduli, frequencies and applied pressure should have consistent units of length, time and force.</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>ILYR</td>
<td>The index of the line forming the lower boundary of the layer. See Fig. 6</td>
<td>one card containing 8 quantities, for Maximum number of cards = NLYR</td>
</tr>
<tr>
<td></td>
<td>DZ</td>
<td>Space increment in the Z-direction relative to $R_0$.</td>
<td>15,5X,7F10.0</td>
</tr>
<tr>
<td></td>
<td>LAMLD</td>
<td>Non-dimensional loading wave speed $\lambda_L$. See Table 1</td>
<td></td>
</tr>
<tr>
<td>VARIABLE NUMBER</td>
<td>VARIABLE DESCRIPTION</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------------</td>
<td>----------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Structure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$\sigma_r$ Radial stress at the structure/material interface</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\sigma_z$ Longitudinal Stress</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$\sigma_{rz}$ Shear stress</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$V_r$ Radial velocity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$V_z$ Longitudinal velocity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$W_r$ Radial displacement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$W_z$ Longitudinal displacement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$J_1$ First invariant of stress</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>$\sigma_{00}$ Hoop stress</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$\sigma_r$ Radial stress</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\sigma_z$ Longitudinal stress</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$\sigma_{rz}$ Shear stress</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$V_r$ Radial velocity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$V_z$ Longitudinal velocity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$J_1$ First invariant of stress</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix B

LIST OF COMMON VARIABLE NAMES

AJ1  The first stress invariant, $J_1$ evaluated at points in the field.
AJ1MX  Last maximum compressive value of $J_1$
ALPHP, ALPHS  Values of $\alpha$ for plate and shell as defined in Table 1, Item 5.
APDSC  Load applied at each point on the interior of the shell.
ARML1, ARLP, AZLP
AZLP1, AZLS  Intermediate quantities used in computation for right angle at shell-plate interface. See Eqs. (21)-(24)
CLD  Coefficient array computed with loading P-wave parameters.
CS  Coefficient array computed with S-wave parameters
CUN  Coefficient array computed with unloading P-wave parameters.
CNR1---CNR12  Coefficients defined for computations of motion of shell-plate structure as follows:

\[
\begin{align*}
\text{CNR1} &= \frac{\omega^2}{s} \Delta T \\
\text{CNR2} &= \frac{\lambda}{s} \Delta T \\
\text{CNR3} &= \frac{\omega^2}{p} \Delta T \\
\text{CNR4} &= \frac{\omega^2}{p} \Delta T \\
\text{CNR5} &= \frac{1}{1+y} \\
\text{CNR6} &= \frac{r_{\text{LC}}}{y_{\text{LC}-1}} \\
\text{CNR7} &= \frac{\gamma}{1+y} \\
\text{CNR8} &= \frac{\lambda}{p} \Delta T \\
\text{CNR11} &= \frac{\Delta r}{\Delta z_{\text{LC}}} \\
\text{CNR12} &= \frac{1}{2[1+(\frac{\Delta r}{\Delta z_{\text{LC}}})^2]} \\
\text{DENS} &= \text{Density of each layer, relative to a base value } \rho.
\end{align*}
\]

DR  Increment of space in the R-direction; $\Delta R$.

DR2, DR3, DR4  $(\Delta R)^2$, $(\Delta R)^3$, $(\Delta R)^4$.

DT  Increment of time.

DZ  Increment of space in the Z-direction. Possibly different for each layer.

DZ2, DZ3, DZ4  $(\Delta Z)^2$, $(\Delta Z)^3$, $(\Delta Z)^4$. 

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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1RWR, D1RWZ, D2RWR</td>
<td>Intermediate quantities used in computing shell velocities in the R and Z directions. See Tables 3a, 3b, 4a, 4b.</td>
</tr>
<tr>
<td>EPL, ESH</td>
<td>Young’s modulus for the plate and shell, respectively.</td>
</tr>
<tr>
<td>HPL, HSH</td>
<td>Thickness of the plate and shell respectively.</td>
</tr>
<tr>
<td>I</td>
<td>Index in the Z-direction measured from the top.</td>
</tr>
<tr>
<td>IFLPT</td>
<td>I-index of field point at which graph output is desired.</td>
</tr>
<tr>
<td>ILOAD</td>
<td>Indicator for type of loading. See Table 6, card set 2.</td>
</tr>
<tr>
<td>ILYR</td>
<td>I-index at the bottom of each layer.</td>
</tr>
<tr>
<td>IPTS</td>
<td>Total number of mesh points in the field, = IRIZ-IRZSH</td>
</tr>
<tr>
<td>IPR</td>
<td>I-index of point in the field at which printed output is desired.</td>
</tr>
<tr>
<td>IR</td>
<td>Maximum number of mesh points in the R-direction.</td>
</tr>
<tr>
<td>IRIZ</td>
<td>The product of IR and IZ.</td>
</tr>
<tr>
<td>IRSHL</td>
<td>Total number of points on the plate.</td>
</tr>
<tr>
<td>IRSP1</td>
<td>= IRSHL+1</td>
</tr>
<tr>
<td>IRZSH</td>
<td>The product of IRSHL and IZSHL</td>
</tr>
<tr>
<td>ISHPR</td>
<td>Index of point on shell at which printed output is desired.</td>
</tr>
<tr>
<td>ISHPT</td>
<td>Total number of points around the cavity = IRSHL+IZSHL-1.</td>
</tr>
<tr>
<td>ITGRF</td>
<td>Time index increment for graphs of time histories.</td>
</tr>
<tr>
<td>IZ</td>
<td>Maximum number of mesh points in the Z-direction.</td>
</tr>
<tr>
<td>IZSHL</td>
<td>Maximum number of points on the shell.</td>
</tr>
<tr>
<td>IZSP1</td>
<td>= IZSHL+1</td>
</tr>
<tr>
<td>JFLPT</td>
<td>J-index of field point at which graphic output is desired.</td>
</tr>
<tr>
<td>JPR</td>
<td>J-index of field point at which printed output is desired.</td>
</tr>
</tbody>
</table>
JPTS
Number of mesh points in the R-direction in the field between the shell and the outer boundary. JPTS = IR - IRSL.

JO
Index in the R-direction measured from the origin.

KODES
Indicator for state of load, unload, reload. See Table 5.

LAML, LAM, LAMUN
Non-dimensional wave speeds for loading P-waves, unloading P-waves and S-waves, respectively.

LINE
Index of line along which a space profile is to be plotted.

LR
Array LR(I) defines the number of mesh points to be included in the integration in the r-direction along each I-th horizontal line.

LRMX
Maximum values of LR.

LYRSH
Number of the layer in which the plate sits.

LZ
Array LZ(J) defines the number of mesh points to be included in the integration in the z-direction along each J-th vertical line.

LZMX
Maximum values of LZ.

MTPT
Time step number at which space profile is to be plotted.

MXGRF
Total number of time histories desired for field points.

MXGRS
Total number of time histories desired for structure points.

MXPRF
Total number of space profiles desired.

NFLV
Field variable number for which graphic output is desired. See Table 7.

NGRPH
Indicator for graphic output or not. See Table 6, card set 4.

NLAYR
Total number of layers.

NODIM
Indicator for non-dimensional or dimensional results. See Table 6, card set 4.

NORIT
Indicator for right angle preservation at shell plate interface. See Table 6, card set 4.
NOSHL  Indicator for structure or no-structure lining the cavity.

NPAIR  Total number of field points for which printed output is desired.

NPRSH  Total number of points around cavity at which printed output is desired.

NPRT   Time step index increment at which printed output is desired.

NRORZ  Indicator for direction in which space profile is to be plotted. See Table 6, card set 11.

NSHPT  Index of the point on the structure at which a time history is to be plotted.

NSHV   Number of the structure variable for which a time history is to be plotted. See Table 7.

NTIME  Maximum number of time steps to be computed.

NULD   Poisson ratio for loading conditions. See Table 1, Item 4.

NUUN   Poisson ratio for unload-reload conditions. See Table 1, Item 4.

NVAR   Number of the field variable for which a space profile is to be plotted. See Table 7.

OMPL, OMSH  Fundamental frequencies $\omega_p$, $\omega_s$ for plate and shell respectively. See Table 1, Item 6.

OM2PL, OM2SH $\frac{\omega^2_p}{\omega^2_s}$

PLDEN, PLNU  Density and Poisson ratio for plate material.

R       Distance in the r-direction measured from the origin.

SGR   Radial stress, $\sigma_r$ at points in the field.

SGZ   Longitudinal stress, $\sigma_z$ at points in the field.

SHAJ1  First invariant at the structure-field interface.

SHDEN  Density of shell material.

SHL1, SHL2 Coefficients for shell computations, $SHL1 = 1 + \alpha_s$, $SHL2 = 1 - \alpha_s$.

SHNU   Poisson ratio of shell material.
SHSGR, SHSZ, SHSRZ

Stresses $\sigma_r$, $\sigma_z$, $\sigma_{rz}$ measured in the field at the structure-material interface.

SHVR, SHVZ

Velocities of points on the structure in the R- and Z-directions, respectively.

SHWR, SHWZ

Displacements of points on the structure in the R- and Z-directions, respectively.

TIME

Cumulative time value from start of computation.

VR, VZ

Velocities of points in the field in the R- and Z-directions, respectively.

Z

Distance measured in the Z-direction from the top boundary.
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